Information, Misallocation and Aggregate Productivity*

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Abstract

We propose a theory linking imperfect information to resource misallocation and hence to aggregate productivity and output. In our setup, firms look to a variety of noisy information sources when making input decisions. We devise a novel empirical strategy that uses a combination of firm-level production and stock market data to pin down the information structure in the economy. Even when only capital is chosen under imperfect information, applying this methodology to data from the US, China, and India reveals substantial losses in productivity and output due to the informational friction. Our estimates for these losses range from 7-10% for productivity and 10-14% for output in China and India, and are smaller, though still significant, in the US. Losses are substantially higher when labor decisions are also made under imperfect information. We find that firms turn primarily to internal sources for information; learning from financial markets contributes little, even in the US.

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1 Introduction

In a frictionless environment, the optimal allocation of factor inputs across productive units requires the equalization of marginal products. Deviations from this outcome represent a misallocation of resources and translate into sub-optimal aggregate outcomes, specifically, depressed levels of productivity and output. A recent body of work empirically documents the presence of substantial misallocation and points out its potentially important role in accounting for large observed cross-country differences in productivity and income per-capita. With some notable exceptions, however, the literature has remained largely silent about the underlying factors driving this misallocation.

In this paper, we propose just such a theory, linking imperfect information to resource misallocation and hence to aggregate productivity and output. Our point of departure is a standard general equilibrium model of firm dynamics along the lines of Hopenhayn (1992). The key modification here is that firms choose inputs under limited information about their idiosyncratic fundamentals, specifically, demand conditions in their own markets. This informational friction leads to a misallocation of factors across firms in an *ex-post* sense, reducing aggregate productivity and output. The size of this effect depends on the residual uncertainty at the time of the input choice, which, in turn, is a function of the volatility of the fundamental shocks and the quality of information at the firm level. Our analytical framework enables a sharp characterization of these relationships and yields simple closed-form expressions linking informational parameters at the micro-level to aggregate outcomes.

The second piece of our theoretical framework focuses on the firm's learning problem. Our flexible information structure assumes that firms learn from a variety of sources. In addition to those within the firm (which we will refer to as 'private information'), firms also observe movements in their own stock prices, which aggregate the information of financial market participants about the firm's future prospects. We capture this aggregation using an explicit model of financial market trading in the noisy rational expectations paradigm of Grossman and Stiglitz (1980).[1] Informed investors and noise traders trade shares of the firm's stock, generating imperfectly revealing equilibrium prices.

The presence of informative asset prices serves two purposes in our analysis: first, as we describe next, the informational content of observed market prices is at the core of our empirical approach and allows us to identify the severity of otherwise unobservable informational frictions. Second, we are able to quantitatively evaluate the contribution of financial markets to allocative efficiency through an informational channel, i.e., by providing useful information to decision-makers within firms. Our analysis is, to the best of our knowledge, the first to measure and shed

[1] We rely particularly on recent work by Albagli et al. (2011b) for our specific modeling structure.

light on the aggregate consequences of this channel in a standard macroeconomic framework.

Any attempt at quantifying uncertainty runs into an obvious difficulty - the econometrician seldom observes agents' information directly. In our setting, one approach would be to use the observed degree of misallocation, i.e., dispersion in marginal products. This would lead to an accurate measure of uncertainty if misallocation was entirely driven by informational frictions. In reality, however, a host of other factors are likely to contribute to dispersion in marginal products.[2] Without explicitly modeling these factors (and quantitatively disciplining their magnitudes), disentangling the severity of informational frictions is not a straightforward exercise. This concern applies more broadly to attempts at inferring uncertainty purely from production-side moments (i.e., data on inputs and outputs). For example, the volatility of investment and its covariance with measures of fundamentals are affected by (and therefore, informative about) firm-level uncertainty, but are also likely to be influenced by other factors.

One of the contributions of this paper is a novel empirical strategy that is robust to these concerns. The main insight behind this strategy is that we can draw sharp inferences about the degree of uncertainty faced by a firm by observing a subset of its information set. Stock market prices allow us to do just that. The first step is to measure the informational content of prices and the extent to which input decisions covary with them. The correlation of stock returns with future fundamentals and investment, respectively, are natural and intuitive candidates. The second and key step is to note that for a given level of noise, the extent to which firm decisions comove with the price signal is a function of the overall quality of their information (from *all* sources, including those we do not observe). The poorer this quality, the higher is the correlation between investment and stock market returns. Intuitively, the information contained in stock returns plays a bigger role in the firm's investment decision when the firm is more uncertain. It is worth emphasizing the need to analyze these moments together - the correlation of returns with investment alone does not tell us much about the extent of uncertainty.[3]

For special cases of our model, this intuition can be formalized quite sharply. When firm-level fundamentals are i.i.d. or follow a random walk, we prove that the informational parameters of our model are identified by these two correlations (of stock returns and fundamentals and investment, respectively, as well as the volatility of returns in the latter case). Our use of correlations, as opposed to variances or covariances, makes this strategy robust to some important perturbations of our baseline model; for example, to the inclusion of additional factors that have

[2]To cite a few examples, these could be technological factors (e.g., adjustment costs), financial constraints, taxes or regulations.

[3]To give a simple example, the correlation between returns and investment can be high either because firms and investors are both perfectly informed, in which case all firm-level variables are functions of a single fundamental shock, or alternatively, because firms are poorly informed and therefore learn much from market prices.

a 'scaling' effect on the firm's capital choice, e.g., distortions that dampen the responsiveness of investment to fundamentals. For the analytically tractable special cases, we show that the scaling effects introduced by such distortions do not affect our measure of the total uncertainty faced by firms, despite generating dispersion in marginal products. Our identification strategy is also robust to the presence of correlation between firm and market information. Again, we prove this result to be exact in the special cases: here, our measure of firm uncertainty remains valid for any degree of correlation. In fact, even if the information in stock prices is fully redundant from the perspective of the firm, implying that firms do not learn anything new from them, our strategy uncovers the true extent of uncertainty. This is a reassuring finding because it shows that our estimates of the severity of informational frictions are robust to assumptions about the extent of commonality between firm and market information sets, an object which is difficult to measure given the richness of information flows between firms and market participants (financial statements, announcements, regulatory filings, etc.).

In our quantitative work, we depart from these polar cases and consider an intermediate level of persistence in fundamentals that is in line with the data. However, we show numerically that the informational parameters are still well identified by the same set of moments and that our estimates of uncertainty display a similar robustness to perturbations. In particular, we demonstrate this robustness by considering the effect of capital adjustment costs as well as correlation in firm and market signal errors.

We apply our methodology to data on publicly traded firms from 3 countries - the US, China and India. Our results show substantial uncertainty at the micro level, particularly in China and India, leading to significant levels of misallocation. Even in the US, which has the highest degree of learning, our most conservative estimate for the posterior variance of the firm is about 40% of the ex-ante, or prior, uncertainty.[4] The corresponding estimates for the other two countries range from about 60-80%. To put these results in context, we compare them to direct measures of misallocation in our sample and find that informational frictions account for anywhere from 20-50% of observed dispersion in the marginal (revenue) product of capital, a fraction that goes up once we control for firm-fixed effects. The associated implications for aggregate productivity and output are then quite significant. In China and India, TFP losses (relative to the first best) are in the range of 7-10%, while losses in steady state output (again relative to the first best) range from 10% to almost 15%. The corresponding values in the US are noticeably smaller but still significant - 4% for productivity and 5% for output. Importantly, these baseline calculations assume that only investment decisions are made under imperfect information while labor can adjust perfectly to contemporaneous conditions. In this sense, they are conservative estimates

[4]Given our AR(1) structure on fundamentals, the prior uncertainty is simply the variance of contemporaneous innovations.

of the total impact of informational frictions. Assuming that the friction affects labor inputs to the same degree as capital leads to losses that are substantially higher. For example, in this case, the gap between status quo and first best increases to about 55-80% in TFP and 80-100% in output for China and India. We interpret this as an upper bound on the total effect of the friction, with reality likely falling somewhere in between this and the baseline version.[5]

Our framework also enables us to quantify the sources of learning, particularly the contribution of financial markets. To the extent that prices reflect information not otherwise available from firms' internal sources, stock markets provide firms with valuable information and guide real activity. Note that this does not require investors to know more than firms; only that they are privy to different information that may also be relevant for firm decisions.[6] Here, we arrive at a striking conclusion - learning from stock prices is at best only a small part of total learning at the firm level, even in a relatively well-functioning financial market like the US.[7] Thus, the contribution of financial markets to overall allocative efficiency and aggregate performance through this channel is quite limited. We show that this is primarily due to the high levels of noise in market prices, making them relatively poor signals of fundamentals.[8] A counterfactual experiment delivering access to US-quality financial markets (in a purely informational sense) to firms in China and India generates only small improvements in allocative efficiency.[9] In contrast, a significant amount of learning occurs from private sources, i.e., those internal to the firm. Moreover, disparities along this dimension, that is, in the quality of such information, are the primary drivers of cross-country differences in the severity of informational frictions, much more so than access to well-functioning financial markets. This finding, in spirit, parallels those in Bloom et al. (2013), who highlight the role of manager skill and/or better management practices in explaining cross-country differences in performance. Finally, we show that differences in the volatility of firm-level fundamentals also play a meaningful role in explaining cross-country differences in the severity of informational frictions. Firms in China and India are subject to larger shocks to fundamentals than firms in the US, making the inference problem

[5]We provide suggestive evidence that this is the case using observed dispersion in the marginal product of labor among US firms.

[6]The informational content of stock returns and their role in guiding real activity is the focus of an active body of work in corporate finance, both theoretical and empirical. We will briefly survey this literature later in this section.

[7]This is true even under our baseline assumption that the information in stock prices is conditionally independent of the firm's own signals, which, in a sense, is the most optimistic case. Allowing for correlation would further reduce the extent of new information in prices.

[8]This is related to the concepts of Forecasting Price Efficiency (FPE), namely, to what extent do prices reflect and predict fundamentals, and Revelatory Price Efficiency (RPE), namely, to what extent do prices promote real efficiency by providing new information to firms, as defined in Bond et al. (2012). Our results imply that the poor RPE of stock prices stems from their poor FPE.

[9]Of course, this does not consider other channels through which informative prices, and more generally, well-functioning stock markets may improve efficiency. See the discussion in Section 4.4.

more difficult in those countries even without the differences in signal qualities.

Our paper relates to several existing branches of literature. We bear a direct connection to recent work on the aggregate implications of misallocated resources, for example, Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Guner et al. (2008), and Bartelsman et al. (2013). Indeed, we can map our measure of informational frictions directly into the measures of misallocation studied in these papers, i.e., into the dispersion in marginal products and the covariance between firm-level fundamentals (productivity, for example) and activity (i.e., factor use or output). We differ from these papers in our explicit modeling of a specific friction as the source of misallocation, a feature we share with Midrigan and Xu (2013), Moll (2014), Buera et al. (2011), and Asker et al. (2012), who study the role of financial frictions and capital adjustment costs, respectively. We view our analysis as complementary to those investigating the role of adjustment costs; specifically, in predicting a sluggish response of investment to fundamentals, our theory provides a foundation for adjustment costs, one that may help guide future modeling strategies along these lines and takes a step toward disentangling technological frictions from informational ones, factors that standard estimates of adjustment costs would seem to confound.[10] As an example, Asker et al. (2012) assume in their baseline specification that adjustment cost parameters are the same for all countries. Our analysis provides a primitive that would justify considering cross-country differences in these costs. Our focus on the role of imperfect information is related to that of Jovanovic (2013), who studies an overlapping generations model where informational frictions impede the efficient matching of entrepreneurs and workers.

Our structure of firm learning holds some similarity with Jovanovic (1982) and our linking of financial markets, information transmission, and real outcomes is reminiscent of Greenwood and Jovanovic (1990). The informational role of financial markets has been the subject of much study, dating back at least to Tobin (1982). We discuss a few particularly relevant examples below and refer the reader to Bond et al. (2012) for an excellent survey. One strand of this literature focuses on measuring the informational content of stock prices. Durnev et al. (2003) show that firm-specific variation in stock returns, i.e., 'price-nonsynchronicity,' is useful in forecasting future earnings and Morck et al. (2000) find that this measure of price informativeness is higher in richer countries. A related body of work closer to our own and recently surveyed by Bond et al. (2012) looks directly at the feedback from stock prices to investment and other decisions. Chen et al. (2007), Luo (2005), and Bakke and Whited (2010) are examples of studies that find evidence of managers learning from markets while making investment decisions. Bai et al. (2013) combines a simple investment model with a noisy rational expectations framework

[10]Fuchs et al. (2013) investigate another type of informational friction as a foundation for adjustment costs, namely, adverse selection in the market for existing capital.

to assess whether US stock markets have become more informative over time. Our analysis complements these papers by placing information aggregation through financial markets into a standard macroeconomic setting, which allows us to make precise statements about the quantitative importance of this channel for information transmission, real activity, and aggregate outcomes. Our results on the limited role for stock market information bear some resemblance to the well-known results in Morck et al. (1990), who find a limited incremental role for stock prices in predicting investment, once fundamentals are controlled for.[11] Our focus here is different - we are interested in measuring the contribution of stock market information to aggregate allocative efficiency. In a sense, our analysis provides a structural interpretation of their results; but more importantly, it allows us to quantify the implications for resource allocations and aggregate outcomes.

The paper is organized as follows. Section 2 describes our model of production and financial market activity under imperfect information. Section 3 spells out our approach to identifying informational frictions using the two analytically tractable special cases, while Section 4 details our numerical analysis and presents our quantitative results. We summarize our findings and discuss directions for future research in Section 5. Details of derivations and data work are provided in the Appendix.

2 The Model

In this section, we develop our model of production and financial market activity under imperfect information. We turn first to the production side of the economy, where we derive sharp relationships linking the extent of micro level uncertainty to aggregate outcomes. Next, we flesh out the information structure, including a fully specified financial market in which dispersed private information of investors and noise trading interact to generate imperfectly informative price signals.

2.1 Production

We consider a discrete time, infinite-horizon economy, populated by a representative large family endowed with a fixed quantity of labor supplied inelastically. The aggregate labor endowment is denoted by N. The household has preferences over consumption of a final good and rents capital to firms. The household side of the economy is deliberately kept simple as it plays a limited role in our analysis.

[11] More precisely, they find very small improvements in R^2 when adding stock returns to an investment regression that already includes fundamentals.

Technology. A continuum of firms of fixed measure one, indexed by i, produce intermediate goods using capital and labor according to

$$Y_{it} = K_{it}^{\widehat{\alpha}_1} N_{it}^{\widehat{\alpha}_2}, \qquad \widehat{\alpha}_1 + \widehat{\alpha}_2 \leq 1$$

These intermediate goods are bundled to produce the single final good using a standard CES aggregator

$$Y_t = \left(\int A_{it} Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

The term A_{it} represents an idiosyncratic demand shifter and is the only source of fundamental uncertainty in the economy (i.e., we abstract from aggregate risk). We assume that A_{it} follows an AR(1) process in logs:

$$a_{it} = (1-\rho)\,\overline{a} + \rho a_{it-1} + \mu_{it}, \qquad \mu_{it} \sim \mathcal{N}\left(0, \sigma_\mu^2\right) \tag{1}$$

where we use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $a_{it} = \log A_{it}$. In this specification, $\overline{a}$ represents the unconditional mean of a_{it}, ρ the persistence, and μ_{it} an i.i.d. innovation with variance σ_μ^2.

Market structure and revenue. The final good is produced by a competitive firm under perfect information. This yields a standard demand function for intermediate good i

$$Y_{it} = P_{it}^{-\theta} A_{it}^{\theta} Y_t \quad \Rightarrow \quad P_{it} = \left(\frac{Y_{it}}{Y_t} \right)^{-\frac{1}{\theta}} A_{it}$$

where P_{it} denotes the relative price of good i in terms of the final good, which serves as numeraire.

The elasticity of substitution θ indexes the market power of intermediate good producers. Our specification nests various market structures. In the limiting case of $\theta = \infty$, we have perfect competition, i.e., all firms produce a homogeneous intermediate good. In this case, the survival of heterogenous firms requires decreasing returns to scale in production to limit firm size, that is, $\widehat{\alpha}_1 + \widehat{\alpha}_2 < 1$. When $\theta < \infty$, we have monopolistic competition, with constant or decreasing returns to scale. No matter the assumption here, however, firm revenue can be expressed as

$$P_{it} Y_{it} = Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} \tag{2}$$

where

$$\alpha_j = \left(1 - \frac{1}{\theta}\right) \widehat{\alpha}_j \qquad j = 1, 2$$

This framework accommodates two alternative interpretations of the idiosyncratic component A_{it}: as a firm-specific shifter of either demand or productive efficiency. Neither the theory nor our empirical strategy requires us to differentiate between the two, so we will simply refer to A_{it} as a firm-specific fundamental.

Input choices under imperfect information. The key element of our theory is the effect of imperfect information on the firm's choice of factor inputs, that is, capital and labor. These are modeled as static and otherwise frictionless decisions, i.e., firms rent capital and/or hire labor period-by-period, but with potentially imperfect knowledge of their fundamental A_{it}. Clearly, the impact of the informational friction will depend on whether it affects both inputs or just one. Rather than take a particular stand on this important issue regarding the fundamental nature of the production process, we present results for two cases: in case 1, both factors of production are chosen simultaneously under the same (imperfect) information set; in case 2, only capital is chosen under imperfect information whereas labor is freely adjusted after the firm perfectly learns the current state.

Case 1: Both factors chosen under imperfect information. In this case, the firm's profit-maximization problem is given by

$$\max_{K_{it},N_{it}} \quad Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left[A_{it}\right] K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} - R_t K_{it} \tag{3}$$

where $\mathbb{E}_{it}\left[A_{it}\right]$ denotes the firm's expectation of fundamentals conditional on its information set $\mathcal{I}_{it}$, which we make explicit below. Standard optimality and market clearing conditions imply

$$\frac{N_{it}}{K_{it}} = \frac{\alpha_2 R_t}{\alpha_1 W_t} = \frac{N}{K_t} \tag{4}$$

i.e., the capital-labor ratio is constant across firms.

Our empirical analysis uses moments of firm-level investment data and with this in mind, we use the optimality conditions characterized in (4) to rewrite (3) simply as a capital input choice problem:

$$\max_{K_{it}} \quad \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left[A_{it}\right] K_{it}^{\alpha} - \left(1 + \frac{\alpha_2}{\alpha_1}\right) R_t K_{it} \tag{5}$$

where

$$\alpha = \alpha_1 + \alpha_2 = (\widehat{\alpha}_1 + \widehat{\alpha}_2)\left(\frac{\theta - 1}{\theta}\right)$$

Notice that the firm's expected revenues depend only on the aggregate capital-labor ratio, its conditional expectation of A_{it}, and the chosen level of its capital input. The curvature parameter

α depends both on the returns to scale in production as well as on the elasticity of demand, and will play an important role in our quantitative analysis below. Solving this problem and imposing capital market clearing gives the following expression for the firm's capital choice (the labor choice exactly parallels that of capital):

$$K_{it} = \frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di} K_t \tag{6}$$

Case 2: Only capital chosen under imperfect information. The firm's problem now is

$$\max_{K_{it}} \quad \mathbb{E}_{it}\left[\max_{N_{it}} \quad Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it}\right] - R_t K_{it}$$

and optimizing over N_{it} gives

$$N_{it} = \left(\frac{\alpha_2}{W} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}} \tag{7}$$

Note that, in contrast to (4), capital-labor ratios are now functions of the firm's fundamental A_{it} and chosen level of capital K_{it}, the former fully observed when making the labor choice and the latter fixed. Imposing labor market clearing and substituting, we can write the firm's capital choice problem as:

$$\max_{K_{it}} \quad (1-\alpha_2)\left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}} \mathbb{E}_{it}\left[\tilde{A}_{it}\right] K_{it}^{\tilde{\alpha}} - R_t K_{it} \tag{8}$$

where

$$\tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha_2}}, \qquad \tilde{\alpha} = \frac{\alpha_1}{1-\alpha_2}$$

Thus, the firm's capital choice problem here has the same structure as in case 1 (compare equations (5) and (8)), but with a slightly modified fundamental and overall curvature. This will make the two cases qualitatively very similar, though, as we will see, the quantitative implications will be quite different. We mark with a $\sim$ the transformed objects that are relevant in case 2, a convention we will carry thoughout this section. The firm's input choices can be shown to satisfy

$$K_{it} = \frac{\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}}{\int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di} K_t, \qquad N_{it} = \frac{\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int \tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di} N \tag{9}$$

While the capital choice looks similar to case 1, the labor choice now depends on the joint distribution of $\tilde{A}_{it}$ and $\mathbb{E}_{it}\left[\tilde{A}_{it}\right]$. Despite this, the analysis remains quite tractable and we will

derive simple expressions for aggregate objects.

To complete our characterization of the firm's problem and therefore of the production-side equilibrium in the economy, we need to spell out the firm's information set $\mathcal{I}_{it}$. We defer this discussion to the following subsection and for now directly make conjectures about firm beliefs, which we will later show to be true. Specifically, we assume the conditional distribution of the fundamental to be log-normal in both case 1 and 2, i.e.,

$$\begin{aligned} a_{it}|\mathcal{I}_{it} &\sim \mathcal{N}\left(\mathbb{E}_{it}\left[a_{it}\right], \mathbb{V}\right) \\ \tilde{a}_{it}|\mathcal{I}_{it} &\sim \mathcal{N}\left(\mathbb{E}_{it}\left[\tilde{a}_{it}\right], \tilde{\mathbb{V}}\right) \end{aligned}$$

where $\mathbb{E}_{it}\left[a_{it}\right]$ and $\mathbb{V}$ denote the posterior mean and variance of a_{it} in case 1, respectively, and similarly $\mathbb{E}_{it}\left[\tilde{a}_{it}\right]$ and $\tilde{\mathbb{V}}$ in case 2. Further, as we will show, the cross-sectional distribution of the posterior mean $\mathbb{E}_{it}\left[a_{it}\right]$ is also normal, centered around the true mean $\overline{a}$ with associated variance $\sigma_a^2 - \mathbb{V}$. Focusing on case 1 for a moment, the variance $\mathbb{V}$ indexes the severity of informational frictions in the economy and will turn out to be a sufficient statistic for misallocation and the associated productivity/output losses. It is straightforward to show that $\mathbb{V}$ is closely related to commonly used measures of allocative efficiency. For example, it maps exactly into the dispersion of the marginal revenue product of capital (in logs),

$$\sigma_{mrpk}^2 = \mathbb{V} \tag{10}$$

Similarly, it has a negative effect on the covariance between fundamentals and firm activity as examined, for example, in Bartelsman et al. (2013) and Olley and Pakes (1996), i.e., the covariance between a_{it} and k_{it} satisfies $\sigma_{ak} = \frac{\sigma_a^2 - \mathbb{V}}{1-\alpha}$. Thus, our measure of informational frictions is easily related to measures of misallocation studied in the literature. An analogous correspondence holds for case 2.

Aggregation. We now turn to the aggregate economy, and in particular, measures of total factor productivity (TFP) and output. Given our focus on misallocation, we abstract from aggregate risk and restrict our attention to a stationary equilibrium, in which all aggregate variables remain constant through time. From here on, we will also assume constant returns to scale in production, i.e., $\widehat{\alpha}_1 + \widehat{\alpha}_2 = 1$. Though not essential to our results, this greatly simplifies the expressions.[12] Relegating the lengthy but straightforward derivations to Appendix A.1 and A.2, we use (6) and (9), along with the fact that $Y = \int P_{it} Y_{it} di$, as well as standard properties of

[12]It is straightforward to relax this assumption and work in the more general case; we do so in our derivations in the Appendix.

the log-normal distribution, to derive the following simple representation for aggregate output:

$$\log Y \equiv y = a + \widehat{\alpha}_1 k + \widehat{\alpha}_2 n \tag{11}$$

Aggregate TFP, denoted a, is endogenous and is given by

$$\text{Case 1:} \qquad a = \ a^* - \frac{1}{2}\theta \mathbb{V} \tag{12}$$

$$\text{Case 2:} \qquad a = \ a^* - \frac{1}{2}\left(\theta \widehat{\alpha}_1 + \widehat{\alpha}_2\right)\widehat{\alpha}_1 \tilde{\mathbb{V}} \tag{13}$$

where

$$a^* = \frac{\theta}{\theta - 1}\overline{a} + \frac{1}{2}\left(\frac{\theta}{\theta - 1}\right)\frac{\sigma_a^2}{1 - \alpha}$$

is aggregate TFP under full information, which is identical in the two cases.

These expressions are at the heart of our mechanism and reveal a sharp connection between the micro-level uncertainty summarized by $\mathbb{V}$ (or $\tilde{\mathbb{V}}$) and aggregate TFP: in both cases, aggregate productivity monotonically decreases in uncertainty, with the magnitude of the effect depending on the elasticity of substitution (i.e., the degree of curvature) θ. The higher is θ, that is, the closer we are to perfect competition, the more severe are the losses from misallocated resources. The intuition is easy to see - when goods are highly subsitutable, misallocation is particularly costly. In case 1, only θ plays a role. In case 2, the relative shares of capital and labor in the production function also matter. Intuitively, the greater is labor's share $\widehat{\alpha}_2$ (and so the lower capital's share $\widehat{\alpha}_1$), the greater the ability of firms to mitigate the effects of imperfect capital choice by adjusting labor. To take the extreme case, as $\widehat{\alpha}_2$ approaches one and so $\widehat{\alpha}_1$ zero, the multiplier on $\tilde{\mathbb{V}}$ approaches zero, that is, the informational friction has no effect on aggregate TFP. This flexibility is absent in case 1, in which both inputs are subject to the same friction. Notice that the two cases are equivalent at the opposite extreme, i.e. when $\widehat{\alpha}_1 = 1$ and $\widehat{\alpha}_2 = 0$. It is easy to see that for a fixed set of parameters, the coefficient on uncertainty in case 2 is smaller than that in case 1.

Holding the aggregate factor stocks fixed, the effect of informational frictions on aggregate productivity a is also the effect on aggregate output y. However, the aggregate capital stock in the steady state is not invariant to informational frictions: misallocation reduces incentives for capital accumulation and so the steady state stock of capital decreases with uncertainty. Incorporating this additional effect, we obtain the familiar expression showing the amplified

impact of allocative inefficiencies on aggregate output due to changes in the capital stock:

$$\frac{dy}{d\mathbb{V}} = \frac{da}{d\mathbb{V}}\left(\frac{1}{1-\widehat{\alpha}_1}\right) \tag{14}$$

$$\frac{dy}{d\tilde{\mathbb{V}}} = \frac{da}{d\tilde{\mathbb{V}}}\left(\frac{1}{1-\widehat{\alpha}_1}\right) \tag{15}$$

2.2 Information

We have shown that $\mathbb{V}$ (or in case 2, $\tilde{\mathbb{V}}$), i.e., the variance of the firm's posterior beliefs, is a sufficient statistic for the impact of informational frictions on resource misallocation and the resulting consequences for aggregate outcomes. We now make explicit the information structure in the economy, that is, the elements of the firm's information set $\mathcal{I}_{it}$, which in turn will allow us to characterize $\mathbb{V}$ in terms of the primitives of the economy - specifically, the variances of fundamentals and signal errors.

The firm's information set $\mathcal{I}_{it}$ has three elements. The first is the entire history of its fundamental shock realizations, i.e., $\{a_{it-s}\}_{s=1}^{\infty}$. Within the context of the model, this follows from the fact that *ex-post* revenues reveal the fundamental perfectly.[13] Second, the firm also observes a noisy private signal of its contemporaneous fundamental

$$s_{it} = a_{it} + e_{it}, \qquad e_{it} \sim \mathcal{N}\left(0, \sigma_e^2\right)$$

where e_{it} is an i.i.d. mean-zero and normally distributed noise term. The third and last element of the firm's information set is the price of its own stock, P_{it}. The final piece of our theory then is to outline how the stock price is determined and to characterize its informational content.

The stock market. Our formulation of the stock market and its informational properties follows the noisy rational expectations paradigm in the spirit of Grossman and Stiglitz (1980). For our specific model structure, we draw heavily from recent work by Albagli et al. (2011a) and Albagli et al. (2011b). For each firm i, there is a unit measure of outstanding stock or equity, representing a claim on the firm's profits. These claims are traded by two groups of agents - imperfectly informed investors and noise traders.[14]

[13] In our numerical analysis, we interpret a period as relatively long (3 years), making this a very natural assumption.

[14] There are several possible interpretations of the precise nature of these agents. One is that they are intermediaries investing on behalf of the representative family. Some of them are rational, optimizing investors, while others trade randomly. Because the household cannot distinguish between the two, they co-exist. An alternative is that they are members of the representative large family, again, with some members trading rationally and some randomly. The exact interpretation of these agents in the model is not crucial for our purposes - only that some trade rationally given their information and others randomly, with the net result that

There is a unit measure of risk-neutral investors for each stock. Every period, each investor decides whether or not to purchase up to a single unit of firm i's stock at the current market price P_{it}. This assumption is standard in the literature; without it, risk neutral investors would take unbounded positions in the stock. The market is also populated by noise traders who purchase a random quantity $\Phi(z_{it})$ of stock i each period, where $z_{it} \sim \mathcal{N}(0, \sigma_z^2)$ is i.i.d. and Φ denotes the standard normal CDF. This convenient transformation ensures that the total demand of these traders is positive and less than one, the total supply.

Like firms, investors also observe the entire history of fundamental realizations, and in particular, know a_{it-1} at time t. They also see the current stock price P_{it}, or equivalently, place limit orders conditional on P_{it}. Finally, each investor j is endowed with an independent noisy private signal about the firm's contemporaneous fundamental a_{it}:

$$s_{ijt} = a_{it} + v_{ijt}, \qquad v_{ijt} \sim \mathcal{N}\left(0, \sigma_v^2\right)$$

Our baseline assumption is that the random variables v_{ijt} and z_{it} are independent of the fundamental a_{it} and the noise in the firm's private signal.[15] The total demand of investors for stock i is then given by

$$D(a_{it-1}, a_{it}, P_{it}) = \int d(a_{it-1}, s_{ijt}, P_{it})\, dF(s_{ijt}|a_{it})$$

where $d(a_{it-1}, s_{ijt}, P_{it}) \in [0,1]$ is the demand of investor j and F is the conditional distribution of investors' private signals. The market clearing condition is

$$D(a_{it-1}, a_{it}, P_{it}) + \Phi(z_{it}) = 1$$

The expected payoff to investor j from purchasing the stock is given by

$$\mathbb{E}_{ijt}[\Pi_{it}] = \int \left[\pi(a_{it-1}, a_{it}, P_{it}) + \beta \bar{P}(a_{it})\right] dH(a_{it}|a_{it-1}, s_{ijt}, P_{it})$$

The term $\pi(a_{it-1}, a_{it}, P_{it})$ denotes the expected current profit of the firm as a function of history, the current realization of the fundamental a_{it} and the current stock price P_{it}.[16] The expected current profit is a function of the current price P_{it} because it enters the firm's information set

prices only imperfectly reflect firm fundamentals.

[15]This implies that the noise in the stock price is orthogonal to the firm's information and in this sense, maximizes the potential for learning from prices. We relax this assumption later in our robustness section.

[16]Given the assumption of an AR(1) structure for fundamentals and an i.i.d. process for z_{it}, the most recent fundamental realization, a_{it-1}, is a sufficient statistic for historical information.

and through that, influences firm decisions.[17] The distribution $H\left(a_{it}|a_{it-1}, s_{ijt}, P_{it}\right)$ is investor j's posterior over a_{it} and $\bar{P}\left(a_{it}\right)$ is the expected price in period $t+1$, conditional on the current fundamental a_{it}. Formally,

$$\bar{P}\left(a_{it}\right) = \int \mathcal{P}\left(a_{it}, a_{it+1}, z_{it+1}\right) dG\left(a_{it+1}, z_{it+1}|a_{it}\right)$$

is obtained by integrating the price function $\mathcal{P}\left(\cdot\right)$ over (a_{it+1}, z_{it+1}) (using the conditional distribution G). Clearly, optimality implies:

$$d\left(a_{it-1}, s_{ijt}, P_{it}\right) = \begin{cases} 1 & \text{if } \mathbb{E}_{ijt}\left[\Pi_{it}\right] > P_{it} \\ \in [0,1] & \text{if } \mathbb{E}_{ijt}\left[\Pi_{it}\right] = P_{it} \\ 0 & \text{if } \mathbb{E}_{ijt}\left[\Pi_{it}\right] < P_{it} \end{cases} \tag{16}$$

that is, an investor purchases the maximum quantity allowed (1 share) when the expected payoff (conditional on her information) strictly exceeds the price, does not purchase any shares when the expected payoff is strictly less than the price, and is indifferent when the two are equal.

A rational expectations equilibrium is then a set of functions for prices $\mathcal{P}\left(\cdot\right)$, expected profits $\pi\left(\cdot\right)$, investor decision rules $d\left(\cdot\right)$, and firm capital choice $k\left(\cdot\right)$, such that, for any history of shocks, (i) $\pi\left(\cdot\right)$ is consistent with firm optimization and the price function $\mathcal{P}\left(\cdot\right)$; (ii) $d\left(\cdot\right)$ is consistent with investor optimality as in (16) above; (iv) $k\left(\cdot\right)$ is optimal given the firm's information set; and (v) markets for capital and each firm's stock clear.

We conjecture that equilibrium outcomes have the following 2 properties: (a) trading decisions of investors are characterized by a threshold rule, i.e., there is a signal $\widehat{s}_{it}$ such that only investors observing signals higher than $\widehat{s}_{it}$ choose to buy, and (b) the market price P_{it} is an invertible function of $\widehat{s}_{it}$.[18]

Aggregating the demand decisions of all investors, market clearing then implies

$$1 - \Phi\left(\frac{\widehat{s}_{it} - a_{it}}{\sigma_v}\right) + \Phi\left(z_{it}\right) = 1$$

which leads to a simple characterization of the threshold signal

$$\widehat{s}_{it} = a_{it} + \sigma_v z_{it} \tag{17}$$

This defines a monotonic relationship between P_{it} and $\widehat{s}_{it}$, implying that observing the stock

[17] Appendix A.3 explicitly characterizes $\pi\left(\cdot\right)$ in terms of the firm's problem studied in the previous subsection.

[18] Given that we have unbounded shocks, there are always histories where this conjecture does not hold. However, these realizations are extremely unlikely - in fact, they do not show up at all in any our simulated sample paths. In this sense, we verify this conjecture in our numerical results.

price is informationally equivalent to observing $\widehat{s}_{it}$: in other words, from an informational standpoint, the stock price is simply a signal of firm fundamentals of the form (17). The precision of this signal, $\frac{1}{\sigma_v^2\sigma_z^2}$, is decreasing in both the variance of the noise in investors' private signals and the size of the noise trader shock.

The simple expression for price informativeness in (17) is the key payoff of the structure we have imposed on stock market trading: we now have a complete characterization of the firm's information set and hence the posterior variance $\mathbb{V}$, even without an explicit solution for the price function. Formally, the firm's information set is $\mathcal{I}_{it} = (a_{it-1}, s_{it}, \widehat{s}_{it})$. It is straightforward to show that the conditional and cross-sectional distributions are log-normal under this information set, exactly as conjectured. Direct application of Bayes' rule implies the following formula for the conditional expectation of the fundamental a_{it},

$$\mathbb{E}_{it}\left[a_{it}\right] = \frac{\mathbb{V}}{\sigma_\mu^2}\left[(1-\rho)\,\bar{a} + \rho a_{it-1}\right] + \frac{\mathbb{V}}{\sigma_e^2}s_{it} + \frac{\mathbb{V}}{\sigma_v^2\sigma_z^2}\widehat{s}_{it}$$

where $\mathbb{V}$ is the posterior variance given by:

$$\mathbb{V} = \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}\right)^{-1}$$

Thus, $\mathbb{V}$, our sufficient statistic, is increasing in the noisiness of the two signals, private and market (σ_e^2 and $\sigma_v^2\sigma_z^2$, respectively). In the absence of any learning, $\mathbb{V} = \sigma_\mu^2$, that is, all fundamental uncertainty remains unresolved at the time of the firm's input choice. At the other extreme, under perfect information, $\mathbb{V} = 0$.

Finally, note that the marginal investor, i.e., the investor whose signal $s_{ijt} = \widehat{s}_{it}$, must be exactly indifferent between buying and not buying. It follows then that the price P_{it} must equal her expected payoff from holding the stock:

$$\begin{aligned} P_{it} &= \int \left[\pi\left(a_{it-1}, a_{it}, P_{it}\right) + \beta\bar{P}\left(a_{it}\right)\right] dH\left(a_{it}|a_{it-1}, \widehat{s}_{it}, P_{it}\right) \\ &= \int \left[\pi\left(a_{it-1}, a_{it}, \widehat{s}_{it}\right) + \beta\bar{P}\left(a_{it}\right)\right] dH\left(a_{it}|a_{it-1}, \widehat{s}_{it}, \widehat{s}_{it}\right) \end{aligned}$$

where, with a slight abuse of notation, we replace P_{it} with its informational equivalent $\widehat{s}_{it}$ in the arguments of $H(\cdot)$. Rewriting this equation in recursive form yields a fixed-point charac-

terization of the price function:

$$\begin{aligned}\mathcal{P}(a_{-1}, a, z) &= \int \pi(a_{-1}, a, a+\sigma_v z)\, dH(a|a_{-1}, a+\sigma_v z, a+\sigma_v z) \\ &\quad +\beta \int \left[\int \mathcal{P}(a, a', z')\, dG(a', z'|a)\right] dH(a|a_{-1}, a+\sigma_v z, a+\sigma_v z) \qquad (18)\end{aligned}$$

In our numerical analysis, we solve this functional equation using an iterative procedure with a discrete grid of shock realizations. We then verify that the threshold and invertibility properties of the equilibrium hold for all points on the grid.

3 Identifying Informational Frictions

The main hurdle in quantifying the effects of uncertainty is imposing discipline on the information structure, given that we do not directly observe agents' signals. One approach would be to use the model's implications for observable moments in production-side variables. For example, the model implies a tight connection between uncertainty and the cross-sectional dispersion of investment - see equations (6) and (9). Similarly, equation (10) is a direct mapping between the dispersion in marginal products and our measure of uncertainty. However, these relationships rely heavily on the assumption that the capital choice is a static and otherwise undistorted function of the firm's expectation of fundamentals. In reality, investment decisions may be affected, or 'distorted,' by a number of other factors. These may originate, for example, from technological limitations (e.g., adjustment costs), contracting frictions (e.g., financial constraints), or distortionary government policies. All of these can lead to dispersion in marginal products. Quantifying their contribution - whether individually or jointly - to observed cross-country differences is certainly the overall objective of the growing literature on misallocation, but one that is well beyond the scope of this paper. Our more limited goal here is to isolate and quantify the degree of firm-level uncertainty and its particular role in generating misallocation.

To this end, we develop an empirical strategy that allows us to draw robust inferences about uncertainty, even without explicitly modeling these other factors. Our approach combines moments from firm-level production and stock market data to pin down the informational parameters of our model. In this section, we develop the intuition for that strategy by analyzing two special cases of our model - when firm level shocks are i.i.d. and when they follow a random walk. When we return to our general model in the following section, we will verify numerically that the properties of the special cases analyzed here extend to the general model used there.

We prove that in these two cases, the informational parameters of our model are identified by three readily observable moments - the correlations of stock returns with both fundamentals

and investment, and the volatility of stock returns.[19] In both cases, we derive intuitive expressions mapping these moments to the informational parameters. Two key insights underlie this strategy and the choice of moments: first, correlations are invariant to scaling effects, which makes our strategy robust to the presence of other factors that dampen (or exacerbate) the responsiveness of investment to fundamentals; second, the Gaussian structure of the model allows us to say a lot about the extent of uncertainty by looking at the comovement of investment with any element of the firm's information set (in our case, this element is stock returns). We exploit the tractability of the two special cases to analytically demonstrate both these insights.

We proceed in two steps. We first derive the identification equations in our baseline setup laid out in the previous sections. We then show that our approach remains valid under two important modifications: the first introduces other 'distortions' into the firm's investment choice problem; the second allows for a general correlation structure between firm and market information. These exercises also serve to highlight some merits of our approach relative to reduced-form, regression-based strategies used extensively in existing work on the relationship between stock markets and investment.

3.1 Identification

Transitory shocks. Consider the case where shocks to fundamentals are i.i.d., i.e., $\rho = 0$ in equation (1). A log-linear approximation of the stock price (around the deterministic case) leads to[20]

$$p_{it} \equiv \log P_{it} \approx \xi \tilde{\mathbb{E}}_{it} [a_{it}] + \text{Constant}$$

where $\xi = \frac{1-\beta}{1-\alpha}$ and $\tilde{\mathbb{E}} [a_{it}]$ is the expectation of a_{it} conditional on the marginal investor's information set. It is then straightforward to derive[21]

$$\tilde{\mathbb{E}}_{it} [a_{it}] = \tilde{\mathbb{E}}_{it} [\mu_{it}] = \psi (\mu_{it} + \sigma_v z_{it})$$

where

$$\psi = \frac{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_v^2 \sigma_z^2}}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_v^2} + \frac{1}{\sigma_v^2 \sigma_z^2}}$$

[19] Note that we can use the structure of the model to directly back out the fundamental a_{it} from data on revenues and capital.

[20] See Appendix A.4 for derivations. From here on, in a slight abuse of notation, we use $\mathbb{V}$ to denote the uncertainty in both case 1 and case 2, where it should be understood that $\mathbb{V}$ in case 2 corresponds to $\tilde{\mathbb{V}}$ in the theory. We similarly use a_{it} to denote the fundamental in both cases and α the relevant curvature parameter.

[21] Note that both of the signals in the marginal investor's information set are equal to $u_{it} + \sigma_v z_{it}$.

Similarly, capital is a log-linear function of the firm's expectation of the current innovation

$$k_{it} = \frac{\mathbb{E}_{it}\left[\mu_{it}\right]}{1-\alpha} + \text{Constant} \tag{19}$$

which is a precision-weighted average of its private signal and the information in prices:

$$\mathbb{E}_{it}\left[\mu_{it}\right] = \phi_1\left(\mu_{it} + e_{it}\right) + \phi_2\left(\mu_{it} + \sigma_v z_{it}\right)$$

where

$$\phi_1 = \frac{\mathbb{V}}{\sigma_e^2}, \qquad \phi_2 = \frac{\mathbb{V}}{\sigma_v^2 \sigma_z^2}$$

From here, we derive the following expressions for the two correlations of interest, that between returns and changes in fundamentals, denoted ρ_{pa}, and between returns and investment, denoted ρ_{pk}:

$$\rho_{pa} \equiv \text{Corr}\left(p_{it} - p_{it-1}, a_{it} - a_{it-1}\right) = \frac{1}{\sqrt{1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}}} \tag{20}$$

$$\rho_{pk} \equiv \text{Corr}\left(p_{it} - p_{it-1}, k_{it} - k_{it-1}\right) = \frac{1}{\sqrt{\left(1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}\right)\left(1 - \frac{\mathbb{V}}{\sigma_\mu^2}\right)}} \tag{21}$$

Equation (20) shows that the higher is $\frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}$, the noise-to-signal ratio in prices, the lower is the correlation of returns with fundamentals. Equation (21) then implies that for a given level of noise in prices, ρ_{pk} is increasing in the firm's posterior variance $\frac{\mathbb{V}}{\sigma_u^2}$ - investment choices covary more strongly with the signal when firms are more uncertain. Note that we work with $\frac{\mathbb{V}}{\sigma_u^2}$ for convenience - combined with σ_u^2 and $\sigma_v^2 \sigma_z^2$ (from the expression for ρ_{pa}), this bears a one-to-one relationship with σ_e^2, the noise in the firm's private signal.

Notice that a high ρ_{pk} is not by itself indicative of the degree of uncertainty. Firm choices can be highly correlated with returns either because they both track fundamentals very closely or because firms are uncertain.[22] Observing ρ_{pa} allows us to isolate the effect of the latter. To see this more clearly, substitute for ρ_{pa} in (21) to derive

$$\frac{\rho_{pa}}{\rho_{pk}} = \sqrt{1 - \frac{\mathbb{V}}{\sigma_\mu^2}} \qquad \Rightarrow \qquad \frac{\mathbb{V}}{\sigma_\mu^2} = 1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2 \tag{22}$$

Thus, with i.i.d. fundamentals, the severity of informational frictions is pinned down directly by the ratio of the correlations, $\frac{\rho_{pa}}{\rho_{pk}}$. Under full information, this ratio takes a value of 1. This

[22] For example, suppose we make prices more informative, i.e., decrease $\sigma_v^2 \sigma_z^2$. Then ρ_{pk} rises even though uncertainty decreases.

sharp link between the relationship between the correlations and the severity of informational frictions guides our empirical approach in our quantitative analysis below.[23] The more general version of the model precludes an analytical mapping between these correlations and $\mathbb{V}$, but numerical simulations reveal a very similar positive relationship between the relative correlation and $\mathbb{V}$.

Relationship to investment-Q regressions. This special case also leads to a reduced-form representation of investment as a log-linear function of fundamentals, signal errors and prices:

$$\Delta k_{it} = \lambda_1 \left(\Delta \mu_{it} + \Delta e_{it} \right) + \lambda_2 \Delta p_{it} \tag{23}$$

where Δ's denote changes. This is in the spirit of specifications widely used in the empirical corporate finance literature to examine the role of learning from stock markets. For example, Morck et al. (1990) regress investment growth on stock returns, sales growth and other controls.[24] Our structural model enables us to interpret the coefficients from these reduced-form regressions in terms of the informational primitives of the economy. For example, consider the coefficient on stock returns, λ_2:

$$\lambda_2 = \frac{1}{(1-\beta)\,\psi} \left(\frac{\mathbb{V}}{\sigma_v^2 \sigma_z^2} \right) \tag{24}$$

Equation (24) reveals the same intuition as in (21): λ_2 can be high either because firms are subject to a good deal of uncertainty, i.e., $\mathbb{V}$ is large, and so rely heavily on any information that can be gleaned from markets, or because markets are highly informative, i.e., $\sigma_v^2 \sigma_z^2$ is low.

The regression implied by (23) consistently identifies the coefficients only in the case of orthogonality between the error e_{it} and the regressors μ_{it} and p_{it}. If the noise terms in the signals of firms and investors are correlated, this orthogonality assumption is violated, leading to endogeneity biases in the regression estimates. A similar issue arises if the choice of capital is affected by additional factors that are correlated with fundamentals.[25] As we will show later in this section, our approach is robust to these concerns.

[23] For completeness, we show in Appendix A.4 that the volatility of returns and their correlation with fundamentals can be used in a final step to separately identify σ_v^2 and σ_z^2.

[24] Chen et al. (2007) use Q and cash flow growth as their independent variables.

[25] Consider, for example, the effects of introducing a 'correlated distortion' $\tau_{it} = \gamma u_{it}$ into the firm's capital choice in (19), so that $k_{it} = \frac{(1+\gamma)\mathbb{E}[\mu_{it}]}{1-\alpha} + \text{Constant}$. The coefficient λ_2 is given by $\frac{1+\gamma}{(1-\beta)\psi} \frac{\mathbb{V}}{\sigma_v^2 \sigma_z^2}$. In other words, inferring $\mathbb{V}$ from λ_2 requires knowledge of (or at the very least, an adjustment for) γ. Note that 'uncorrelated distortions' will not affect λ_2.

Permanent shocks. Our second special case is that of permanent shocks, i.e., $\rho = 1$. The main insights from the i.i.d. case extend to this case as well.[26] We start by deriving an expression for the stock price, which takes a similar form to the i.i.d. case:[27]

$$p_{it} = \frac{1}{1-\alpha}\tilde{\mathbb{E}}_{it}\left[a_{it}\right] + \text{Constant}$$

We can then derive the following expressions. As in the i.i.d. case, they demonstrate a sharp mapping between the three moments - ρ_{pk}, ρ_{pa} and σ_p^2 - and the informational parameters:

$$\begin{aligned} \frac{\mathbb{V}}{\sigma_\mu^2} &= \frac{\rho_{pk} - \rho_{pa}}{\eta} \\ \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2} &= \frac{(1-\eta^2)}{2\rho_{pa}^2} + \frac{\eta}{\rho_{pa}} - 1 \\ \frac{\sigma_z^2+1}{\sigma_z^2+1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}} &= \frac{1}{\eta} \end{aligned} \tag{25}$$

where $\eta = \left(\frac{1}{1-\alpha}\right)\frac{\sigma_\mu}{\sigma_p}$. In contrast to the i.i.d. case, all three moments are now necessary to infer the extent of uncertainty, but otherwise the intuition is very similar: all else equal, a higher relative correlation ($\rho_{pk} - \rho_{pa}$) implies greater uncertainty, a lower ρ_{pa} higher levels of noise in prices, and higher return volatility, larger noise trader shocks.

3.2 Robustness

We now turn to two important exercises aimed at demonstrating the robustness of the identification strategy outlined above. In the first, we generalize the information structure to allow for arbitrary correlations between signal errors. In the second, we introduce other factors into the firm's investment decision. For simplicity, we discuss only the i.i.d. case here (the Appendix repeats the analysis for permanent shocks).

An alternative information structure. In our baseline setup, the noise terms in the signals received by firms and investors are assumed to be orthogonal to each other. This may be an unrealistic assumption - for example, in practice, firms routinely release forecasts and announce their investment plans to investors/analysts. This could induce correlation in the signal errors (specifically, e_{it}, v_{ijt}, and z_{it}), raising a potential concern - this is a source of co-movement between investment and returns and so may bias the inference of $\mathbb{V}$. However, this turns out

[26] With permanent shocks and no exit, there is no stationary distribution. Since our goal here is primarily to provide intuition for our empirical strategy, we ignore this complication and interpret this as a limiting case.

[27] All derivations are in Appendix A.4.

not to be the case - the ratio of correlations still leads us to the true $\mathbb{V}$. More precisely, (22) holds for an arbitrary correlation structure between market and firm information.[28] Therefore, our assessment of firm-level uncertainty is not sensitive to assumptions about the structure of information flows between firms and investors. Note that this only applies to our estimate of $\mathbb{V}$ - our conclusions about the individual variances (σ_e^2, σ_v^2 and σ_z^2) do change with assumptions about correlations.

Other distortions. So far, we have maintained the somewhat extreme assumption that informational frictions are the only impediment to marginal product equalization. By abstracting from other factors that potentially enter the firm's investment decision, we raise a potential concern about the validity of our measurement strategy.

To address this concern, we introduce other distortions drawn from a flexible, albeit stylized, class. The goal is to incorporate some essential features of these other factors without sacrificing analytical tractability. Specifically, we directly modify the firm's optimality condition (19):

$$k_{it} = \frac{(1+\gamma)\,\mathbb{E}_{it}\left[\mu_{it}\right] + \varepsilon_{it}}{1-\alpha} + \text{Constant}$$

This is equivalent to introducing a 'distortion' τ_{it} of the form:[29]

$$\tau_{it} = \gamma\mu_{it} + \varepsilon_{it} \qquad \varepsilon_{it} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$$

The parameter γ indexes the extent to which τ_{it} covaries with the fundamental μ_{it}. For example, if $\gamma < 0$, the distortion discourages (encourages) investment by firms with stronger (weaker) fundamentals. The second term, ε_{it}, captures factors that are uncorrelated with firm-specific fundamentals. The two parameters γ and σ_ε^2 together pin down the amplitude (measured by the standard deviation) of these distortions and their correlation with fundamentals. We assume that τ_{it} is idiosyncratic (though it is straightforward to add a common component). More importantly, firms know γ and observe ε_{it}, but both objects are unknown to the econometrician.

To understand how the presence of τ_{it} affects our inference strategy, we ask whether the ratio of the two correlations in equation (22) continues to identify the true extent of uncertainty. Consider first the case with only 'correlated distortions,' i.e., $\sigma_\varepsilon^2 = 0, \gamma \neq 0$. We show that equation (22) is unaffected, that is, the ratio of the two correlations still uncovers the true $\mathbb{V}$.[30] In other words, equation (22) allows us to measure $\mathbb{V}$ directly even if we do not know γ. Note

[28] See Appendix A.5.

[29] This flexible specification can capture a rich structure of distortions. For example, it is possible to uniquely map a set of revenue and capital 'taxes' (lognormally distributed and arbitrarily correlated with fundamentals), as in Hsieh and Klenow (2009), to the two parameters $\left(\gamma, \sigma_\varepsilon^2\right)$.

[30] Derivations for this section are in Appendix A.6.

that this is despite the fact that γ does contribute to misallocation. For example, the dispersion in the marginal revenue product of capital (MRPK) is equal to

$$\sigma^2_{mrpk} = \gamma^2 \left(\sigma^2_\mu - \mathbb{V}\right) + \mathbb{V}$$

which is increasing in (the absolute value of) γ. Similarly, the covariance between fundamentals and investment is now

$$\sigma_{ak} = \left(\frac{1+\gamma}{1-\alpha}\right)\left(\sigma^2_\mu - \mathbb{V}\right)$$

Thus, a strategy which targets these measures directly (i.e., chooses $\mathbb{V}$ to match these moments without adjusting for γ) would lead to a biased estimate of the severity of informational frictions. In particular, inferring $\mathbb{V}$ from σ^2_{mrpk} overstates the extent of uncertainty, while using σ_{ak} can lead to an upward or downward bias, depending on the sign of γ. Similar concerns also apply to inferences made directly from the cross-sectional variance of investment or revenue - these moments also confound the effects of uncertainty with other factors, and so using them without taking a stand on the nature of these other distortions is problematic. Using the relative correlation as outlined above, on the other hand, continues to identify the true level of uncertainty and in that sense, is robust to the presence of distortions which have a scaling effect on the capital choice.[31]

Next, we turn to the case where distortions are uncorrelated with fundamentals, i.e., $\sigma^2_\varepsilon \neq 0, \gamma = 0$. In this case, we can show

$$\frac{\mathbb{V}}{\sigma^2_\mu} = \left[1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2\right] + \frac{\sigma^2_\varepsilon}{\sigma^2_\mu}$$

The terms inside the square brackets on the right hand side is our estimate of uncertainty using (22). As the expression shows, this underestimates the true extent of uncertainty (by $\frac{\sigma^2_\varepsilon}{\sigma^2_\mu}$). In other words, factors uncorrelated with fundamentals tend to make our estimate of $\mathbb{V}$ more conservative, in that we would infer less uncertainty than is truly the case. This is despite the fact that these distortions exacerbate misallocation, as the following expression shows

$$\sigma^2_{mrpk} = \mathbb{V} + \sigma^2_\varepsilon$$

Again, a strategy of choosing $\mathbb{V}$ to directly match observed misallocation would lead to an overstatement of the severity of informational frictions.[32]

[31] However, the effect of uncertainty on aggregate outcomes does depend on γ.

[32] An important concern, one which applies not just to this paper but to the entire literature on misallocation, is measurement error (see the discussion in section V of Hsieh and Klenow (2009)). In our setting, classical

Note that in both instances (correlated and uncorrelated), if the true $\mathbb{V}$ were 0 (i.e., firms were perfectly informed), using (22) would not lead us to a positive estimate. In other words, we would not find evidence of uncertainty in a full information economy, even one with disperion in marginal products.

4 Quantitative Analysis

Our analytic results in the previous section demonstrated a tight relationship between the moments $\left(\rho_{pk}, \rho_{pa}, \sigma_p^2\right)$ and the informational parameters $(\sigma_e^2, \sigma_v^2, \sigma_z^2)$, and through them, the degree of micro-level uncertainty. With this intuition in hand, we now return to our general model and use an identification approach quite similar in spirit to infer these parameters using data from 3 countries - the US, China, and India. We use the results to quantify the extent of informational frictions, the degree of resulting misallocation, and the impact on aggregate outcomes in all 3 countries. Our analysis also sheds light on the role of learning from various channels, including the role of financial markets in improving allocative efficiency by delivering useful information to firms. We also make use of the multi-country aspect of our analysis to perform a number of counterfactual experiments assessing the sources of cross-country variation in uncertainty and their respective effects on allocative efficiency.

4.1 Parameterization

We begin by assigning values to the more standard parameters in our model - specifically, those governing the preference and production structure of the economy. Throughout our analysis, we will hold these constant across countries; cross-country differences will come only from the parameters governing the stochastic processes on firm fundamentals and learning. Although a simplification, we feel that this is a natural starting point and allows us to focus on the role of imperfect information in leading to differences in aggregate outcomes across countries.

An important issue here is the choice of period length in the model. The focus of this paper - investment decisions - and our specific modeling choices push us towards larger period lengths. There is significant empirical evidence of long lags in planning and implementing investment projects, with estimates of the mean duration time between the planning stage and project completion of between 2 and 3 years.[33] It seems reasonable then to assume that firms

measurement error in y_{it} or k_{it} leads to exaggerated estimates of both the extent of misallocation (i.e., σ^2_{mrpk}) as well as the severity of information frictions (i.e., $\mathbb{V}$). Whether the overstatement in the former is more or less than the latter, however, is ambiguous and depends on parameters.

[33]The classic reference here is Mayer (1960), who uses survey data on new industrial plants and additions to existing plants to find a mean gestation lag between the drawing of plans and the completion of construction of about 22 months. More recently, Koeva (2000) finds the average length of time-to-build lags to be about 2

are required to forecast fundamentals over a relatively long horizon when making large investment decisions and have only limited flexibility to adjust capital choices ex-post in response to additional information. One approach would be to explicitly model these lags as well as other features (such as adjustment costs/irreversibilities) that are likely relevant for investment decisions over shorter horizons. Largely for tractability, we take a different route and model the capital choice as a static one, but interpret a period in the model as spanning a relatively long horizon, specifically 3 years. This makes the omission of explicit lags/irreversibilities, etc. somewhat less of a concern, while, importantly, allowing us to preserve the simple expressions linking uncertainty to aggregate outcomes.[34]

In line with our choice of period length, we set the discount rate β equal to 0.9. We assume constant returns to scale in production and set the production parameters $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ to standard values of 0.33 and 0.67, respectively. Given our CRS assumption, firm scale is then limited only by the curvature in demand, captured by the elasticity of substitution θ. This will be a key parameter in influencing the quantitative impact of informational frictions. The literature contains a wide range of estimates for this parameter. We set our baseline value at 6, roughly in the middle of the commonly used range.[35] Our choice of θ translates into an $\tilde{\alpha} = 0.62$ in case 2 and $\alpha = 0.83$ in case 1.

Next, we turn to the country-specific parameters. We begin with those governing the process on firm fundamentals a_{it}: the persistence ρ and variance of the innovations σ^2_μ. In both of our cases, we can directly construct the fundamental for each firm (up to an additive constant) as $rev_{it} - \alpha k_{it}$, where rev_{it} denotes the log of revenues, and the value of α depends on whether we are in case 1 or case 2.[36] We then estimate the parameters of the fundamental process by performing the autoregression implied by (1), additionally including a time fixed-effect in order to isolate the idiosyncratic component of the innovations. The resulting coefficient delivers an estimate for ρ and the variance of the residuals for σ^2_μ.[37]

Finally, it remains to pin down the three informational parameters, i.e., the variances of the error terms in firm and investor signals σ^2_e and σ^2_v, and the variance of the noise trader shock σ^2_z. We follow almost directly the strategy outlined above, i.e., target second moments in the growth rates of firm-level investment, fundamentals, and prices.[38] The precise moments

years in most industries, defined as the period between the announcement of new construction and the ensuing date of completion. Given that this excludes the planning period prior to the announcement date, the total gestation lag is likely somewhat longer.

[34]Morck et al. (1990) make a similar argument and perform their baseline empirical analysis using 3-year spans. They also point out that the explanatory power of investment growth regressions at shorter horizons (e.g., 1 year) are quite low.

[35]Additionally, we report our results, at least in case 2, for two other values of θ, namely 4 and 10.

[36]Substitute either (4) or (7) into (2).

[37]See Appendix C for details.

[38]We follow the literature examining the feedback effect of stock prices and use growth rates of investment

we use are the cross-sectional correlations of stock returns with investment growth (denoted ρ_{pi}) and changes in fundamentals, as well the variance of returns (as above, denoted ρ_{pa} and σ_p^2, respectively). Importantly, stock returns are lagged by one period, so that the correlations reflect the comovement of investment and fundamentals with stock returns over the preceding period. This avoids feedback from investment and fundamentals to returns, the reverse of the relationship in which we are interested. Table 1 summarizes our empirical approach.

Table 1: Parameterization - Summary

Parameter	Description	Target/Value
Common		
	Time period	3 years
β	Discount rate	0.90
$\widehat{\alpha}_1$	Capital share	0.33
$\widehat{\alpha}_2$	Labor share	0.67
θ	Elasticity of substitution	6
Country-specific		
ρ	Persistence of fundamentals	Estimates of (1):
σ_μ	Shocks to fundamentals	$a_{it} = (1-\rho)\,\overline{a} + \rho a_{it-1} + \mu_{it}$
σ_e^2	Firm private signal	ρ_{pi}
σ_v^2	Investor private signal	ρ_{pa}
σ_z^2	Noise trading	σ_p^2

We use a simulated method of moments (SMM) approach to assign values to the informational parameters. Formally, we search over the parameter vector $(\sigma_e^2, \sigma_v^2, \sigma_z^2)$ to find the combination that minimizes the (unweighted) sum of squared deviations of the model-implied moments from the target moments. Before reporting the parameter estimates, we provide a numerical analogue of the identification argument in the previous section. This more general version of the model does not yield analytical expressions for the moments of interest, but we show in Figure 1 that the relationships between moments and parameters highlighted above go through almost exactly. The first panel shows a positive relationship between $\mathbb{V}$ and the relative correlation, as suggested by equations (22) and (25).[39] Similarly, in the second panel, we see that higher levels of noise in prices are associated with lower ρ_{pa}, exactly as we saw

(in the analytical cases studied earlier, we used investment, i.e. the growth rate of capital). By working with growth rates, we partly cleanse the data of firm fixed-effects, which are a significant component in cross-sectional differences in these variables. See Morck et al. (1990) for a more detailed discussion of these issues.

[39]Here, we plot the difference in correlations, i.e. $\rho_{pi} - \rho_{pa}$, but using the ratio yields a very similar picture. All moments in the figure are computed using simulated firm-level data from the model as we vary the corresponding parameter, holding the remaining parameters fixed at their estimated values (for the US).

in the analytical cases.[40] Finally, the bottom panel shows that, holding fixed the total noise ($\sigma_v^2\sigma_z^2$), increasing the size of the noise trader shock makes returns more volatile.

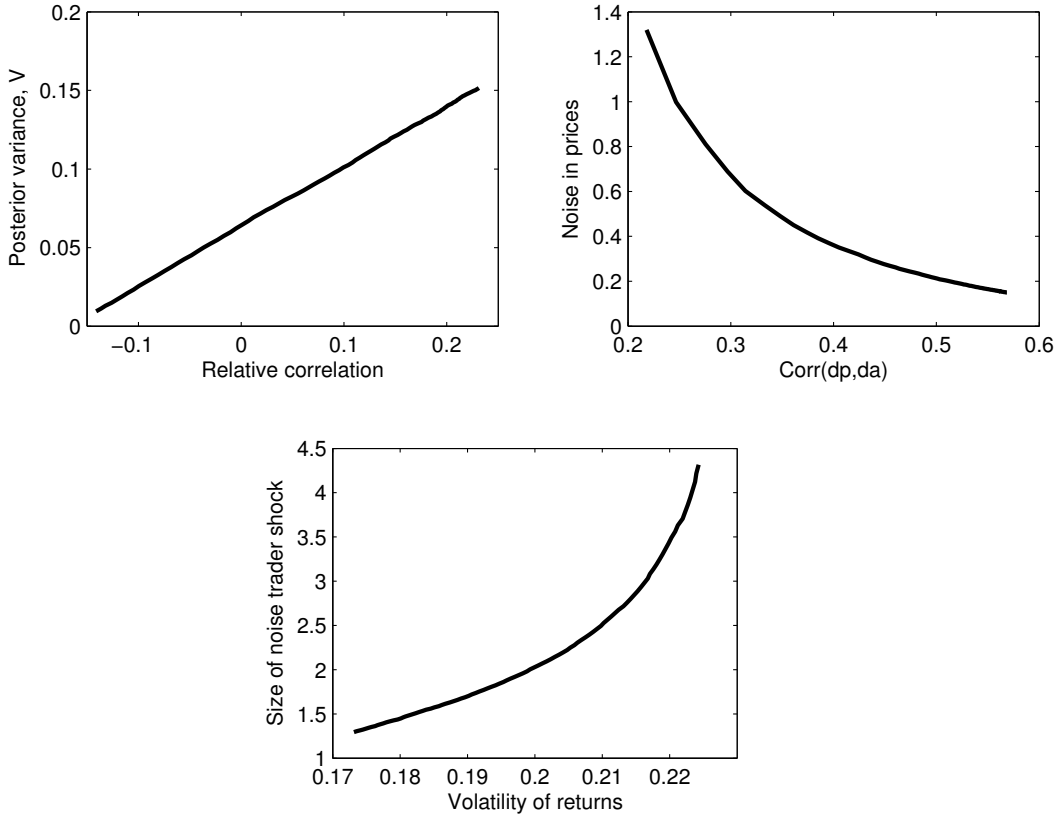

Figure 1: Identification - Quantitative Model

4.2 Data and Parameter Values

We construct the target moments using data on firm-level production variables and stock returns from Compustat North America for the US and Compustat Global for China and India. We focus on a single cross-section of firms in each country for the year 2012. This is the period with the largest number of observations, particularly so for China and India.[41] Note that our data requirements are quite stringent: due to our focus on 3 year horizons and our use of growth rates, we require at least 9 consecutive years of data for a firm in order to construct two 3 year periods and include it in our sample, with 2012 representing the final year of the second period.

[40]Here, we hold V and the ratio $\frac{\sigma_v}{\sigma_z}$ fixed, so as to focus on the overall informational content of prices.

[41]This is less of an issue for the US, and so as a robustness exercise, we recomputed the moments using a larger sample with more years. While there is some time variation, the results are quite similar to those from the single cross-section.

We compute the firm's capital stock k_{it} in each period as the average of its property, plant and equipment (PP&E) over the relevant 3 years, and investment as the change in the capital stock relative to the preceding 3 year period. To construct our measure of the fundamental a_{it}, we compute sales/revenue analogously as the average over the 3 year period and calculate $a_{it} = rev_{it} - \alpha k_{it}$. First-differencing gives investment and changes in fundamentals between the two periods. Stock returns are constructed as the change in the firm's stock price over each 3 year period, adjusted for splits and dividend distributions. In order to be comparable to the unlevered returns in our model, stock returns in the data need to be adjusted for financial leverage. To do so, we assume that claims to firm profits are sold to investors in the form of debt and equity in a constant proportion (within each country). Observed return variances must then be multiplied by a constant factor in order to make them comparable to returns in the model, where the factor depends on the ratio of debt to total assets (or alternatively, debt to equity). Computing the debt-asset ratio from our sample gives average values of about 0.30 in both the US and India, and about 0.16 in China, with corresponding adjustment factors of about 0.5 and 0.7, respectively. All values reported below reflect this adjustment.[42] To isolate the firm-specific variation in our data series, we extract a time fixed-effect from each and utilize the residual as the component that is idiosyncratic to the firm. This is equivalent to demeaning each series from the unweighted average in each time period.[43] As mentioned above, the target moments are computed using returns lagged by one period.[44] This avoids the simultaneity problems associated with comparing price movements to contemporaneous investment/fundamentals numbers. We trim the 2% tails of each series in order to mitigate measurement error. Appendix C provides further details on how we build our sample and construct the variables.

We report the three target moments in the left hand panel of Table 2 both for case 2 in the upper panel and case 1 in the lower.[45] In both cases, the moments exhibit significant cross-country variation. The US and India show similar levels of return volatility and a similar relation between returns and investment growth, but quite different comovements with fundamentals - returns in the US are more highly correlated with future changes in fundamentals. China has a return variance that is almost half that of the other two countries, along with the lowest correlations of returns with investment and fundamentals. As made clear by the analytic results in Section 3, none of these moments is a sufficient statistic to identify the informational role

[42] In brief, letting d denote the debt-asset ratio, the observed return variances must be multiplied by $(1-d)^2$ to obtain the variance of unlevered returns. We describe the details of the calculations in Appendix C.

[43] An alternative is to use CAPM $\beta's$ to remove the aggregate component from individual stock returns. This approach yields very similar results.

[44] For example, ρ_{pi} is the correlation between 2006-09 returns and investment growth during 2009-12.

[45] ρ_{pa} changes across cases since α affects our estimates of a. The remaining two moments change almost imperceptibly due to trimming.

of markets or infer the extent of micro-level uncertainty; rather, it is the joint pattern of the three moments that matters and our explicit modeling of both production and financial market activity is precisely what allows us to tease this out.

In the right hand panel of Table 2, we report the resulting parameter values. The parameters governing the process on fundamentals ρ and σ_μ are inferred from the regression implied by (1) as detailed above and the informational parameters from the target moments and SMM procedure just described. As we would expect from the cross-country variation in the target moments, the parameter estimates also vary markedly across countries. The US has less volatile fundamental shocks and lower levels of noise both in private signals at the firm level (lower σ_e) and generally in the stock market as well (lower σ_v and σ_z). In the next section, we gauge the detrimental impact of the estimated frictions on productivity and output and in leading to differences in these aggregates across countries.

Table 2: Target Moments and Parameter Estimates

	Target moments			Parameters				
	ρ_{pi}	ρ_{pa}	σ_p^2	ρ	σ_μ	σ_e	σ_v	σ_z
Case 2								
US	0.23	0.18	0.23	0.92	0.45	0.39	0.37	3.50
China	0.16	0.06	0.14	0.78	0.51	0.67	0.74	4.24
India	0.25	0.08	0.23	0.93	0.53	1.04	0.69	4.36
Case 1								
US	0.24	0.10	0.23	0.88	0.46	0.63	0.65	3.16
China	0.15	0.02	0.14	0.75	0.53	0.74	1.18	3.14
India	0.26	0.00	0.22	0.88	0.55	1.39	1.69	4.14

4.3 Results

We report our baseline results in Table 3. The first three columns present the implied value for $\mathbb{V}$ based on the parameter estimates in Table 2, both in absolute terms and as a percentage of the underlying fundamental uncertainty, σ_μ^2, and of the total dispersion in the MRPK in our sample, σ_{mrpk}^2.[46] In the last two columns, we compute the implied losses in aggregate TFP and output relative to a full information benchmark. The top panel contains results for case 2, in which only capital is chosen under imperfect information, and the bottom panel the analogous results for case 1, in which both capital and labor are. Case 2 is the more conservative scenario (in the sense that it leads to lower TFP/output losses). We return to this issue in our discussion below and provide some suggestive evidence that reality likely falls in between the two cases.

[46] σ_{mrpk}^2 is computed as the average of within-industry dispersion in each country.

Table 3: The Impact of Informational Frictions

	$\mathbb{V}$	$\frac{\mathbb{V}}{\sigma_\mu^2}$	$\frac{\mathbb{V}}{\sigma_{mrpk}^2}$	$a^* - a$	$y^* - y$
Case 2 ($\alpha = 0.62$)					
US	0.08	0.41	0.22	0.04	0.05
China	0.16	0.63	0.34	0.07	0.10
India	0.22	0.77	0.48	0.10	0.14
Case 1 ($\alpha = 0.83$)					
US	0.13	0.63	0.35	0.40	0.60
China	0.18	0.65	0.39	0.55	0.83
India	0.26	0.86	0.56	0.77	1.15

Turning to the first two columns, there is substantial uncertainty in all cases: as a percent of the fundamental uncertainty σ_μ^2, the residual uncertainty ranges from a low of 41% in the US to a high of 77% in India in case 2, and similarly in case 1, although China and the US move much closer together on this score. In other words, firm learning ($\sigma_\mu^2 - \mathbb{V}$) eliminates from a high of about 60% of total uncertainty in the US to a low of about 20% in India in case 2, with a generally similar pattern in case 1, although the estimated degree of learning falls in the US and India. As we would expect from the cross-country variation in the parameter estimates in Table 2, the level of uncertainty varies systematically across countries: US firms are the most informed and Indian firms the least, with Chinese firms falling in the middle.

Next, we ask, how much of total MRPK dispersion do informational frictions account for? The answer is a significant portion: as a percentage of the total σ_{mrpk}^2, $\mathbb{V}$ represents between about 20% and almost 60%, with the share generally lower in the US than in the two emerging markets. Later, we will decompose the observed MRPK into a firm fixed effect and a transitory component and assess the role of informational frictions in explaining the latter. In a sense, this is a more meaningful comparison because informational frictions really cannot speak to fixed effects, which seem to an important component of observed MRPK dispersion.

Finally, the last two columns of Table 3 show that the substantial degree of residual uncertainty implies significant losses in productivity and output. Compared to the full-information benchmark, in case 2, losses in steady state TFP range from 4% in the US (more precisely, 0.04 log-points) to 10% in India, with corresponding output losses of 5% to 14%.[47] Estimated losses are significantly higher in case 1, ranging from 40% to about 80% in productivity and 60% to over 1 in output. In both cases, the US exhibits the smallest losses, reflecting the fact that US firms exhibit the smallest degree of ex-post uncertainty, and India the largest, with China falling in the middle. The differential impact of informational frictions leads to signif-

[47] In what follows, we adopt the convention of referring to log points as percentages.

icant differences between the US and the two emerging markets in productivity and output, ranging from 3% to almost 40% for the former (across the two cases) and from 5% to over 50% for the latter (subtract $a^* - a$ and $y^* - y$ for the US from the corresponding values for China and India). These are somewhat modest, however, in comparison to standard measures of cross-country differences in aggregate TFP and income per-capita, but very much in line with other estimates based on particular frictions - for example, Midrigan and Xu (2013). In Section 4.5, we will use the structural parameters of our model to investigate in more detail the differences in informational frictions across countries; specifically, we perform a number of counterfactual experiments exploring the potential gains in the emerging markets from having access to a US-quality information structure.

Case 1 vs Case 2. Table 3 shows that under either scenario, informational frictions have a significant detrimental impact on aggregate performance, both when comparing the parameterized economies to their full information benchmarks and when comparing the consequences of these frictions in emerging markets to the US. The magnitude of the effects depend to a great extent on the nature of the firm's input decisions, that is, whether all inputs are chosen subject to the friction or only traditionally quasi-fixed inputs like capital.[48] The intuition for these differences comes directly from equations (12) and (13): in case 1, given the level of uncertainty $\mathbb{V}$, the aggregate consequences depend only on the elasticity of substitution θ; in case 2, the relative shares of capital and labor in production also matter. The greater is labor's share $\widehat{\alpha}_2$, the greater the ability of firms to mitigate the effects of capital misallocation due to the informational friction by reallocating labor in a compensating fashion. Any such flexibility is absent in case 1, in which both inputs are subject to the same friction. Thus, the aggregate "cost" of a given level of uncertainty is larger in case 1 than case 2.

To dig a bit deeper into this issue, note from equations (7) and (6) that in case 2, the marginal revenue product of labor (MRPL) is equalized across firms so that there should be no dispersion, while in case 1, dispersion in the MRPL exactly equals that in the MRPK. This is a direct result of our assumptions that in the former, all information is revealed before the labor choice, while in the latter, labor is chosen under the same limited information as capital. This implies a tight link between the cases and the ratio of the dispersion in the MRPL to that in the MRPK, i.e., $\frac{\sigma^2_{mrpl}}{\sigma^2_{mrpk}} = 0$ in case 2 and $= 1$ in case 1, which suggests that the empirical counterpart of this ratio may be useful in assessing how close we are to the two cases. We compute this ratio for the US firms in our sample, using total employment as an admittedly rough measure

[48] It is straightforward to extend our methodology to labor moments. The main hurdle is availability of data on labor inputs, especially in India and China. Even for the US, the coverage and quality of data on labor variables is lower than for capital.

of labor inputs.[49] The ratio is equal to 0.57, suggesting that reality is indeed in the middle of the two extremes we have analyzed.[50] In this sense, we view our results as providing bounds on the adverse consequences of imperfect information for aggregate performance. Certainly, an important direction for future work is a more in-depth analysis of the precise nature of the firm's input choices.

Transitory MRPK dispersion. Table 3 shows that uncertainty can account for a significant portion of the total MRPK dispersion observed in the data, ranging from 20-60% across countries and cases. Note, however, that the dispersion induced by uncertainty is short-lived. In the data, on the other hand, there is some evidence of permanent deviations - in other words, measured MRPK deviations seem to comprise both a firm-specific fixed effect (which we cannot speak to with our theory) and a transitory component. In a sense, a more appropriate gauge of the contribution of informational frictions would be a comparison to this latter piece. To do so, we separate the two components for the US firms in our sample.[51] We restrict the sample to firms with at least 15 years of data and extend the data as far back as 1985, so that we can construct at least 5 and as many as 10 3-year observations for each firm. We compute the MRPK for each firm in each period and regress the result on a firm fixed-effect. The residuals from this regression capture the purely transitory component of MRPK deviations. We then compute the variance of this object, i.e., the dispersion in the transitory component of MRPK deviations, and ask how much of this dispersion do informational frictions account for? This exercise reveals, first, that dispersion in the transitory component is substantial, representing about one-third of the total. Moreover, our estimated $\mathbb{V}$ for the US accounts for about 60% of the transitory dispersion in case 2, and just about the entirety in case 1, again pointing to the empirical relevance of informational frictions.[52]

The effect of curvature. The impact of informational frictions is sensitive to the degree of curvature, captured here by the elasticity of substitution θ, set equal to 6 in our baseline computations. Table 4 reports results for case 2 under two alternative values: $\theta = 4$, which is on the low end of the commonly used range, and $\theta = 10$, which is on the high end. Changes in θ lead to significant changes in the effects of the friction. Both the estimates of $\mathbb{V}$ and their impact on TFP are lower with a smaller θ, with TFP losses ranging from 2% in the US to 6% in India and higher, but similarly ordered, output losses. In contrast, the higher value of θ shows

[49] σ^2_{mrpl} is computed analogously to σ^2_{mrpk}.

[50] Data on the wage bill is available only for a small set of the US firms in our dataset; however, using wage bill to measure labor input for this small set of firms gives a very similar ratio of 0.53. Reliable employment data is not available in the emerging markets.

[51] Sufficient data to perform this analysis are not available in the other two countries.

[52] We again compute the average of within-industry dispersion.

a more severe aggregate impact, ranging from 8% in the US to 16% in India for TFP and from 11% to almost 25% for output.[53] The impact of the friction varies with θ for two reasons: first, as can be seen in (13), for a given $\mathbb{V}$ the aggregate consequences are larger for higher values of θ (i.e., higher α). Second, the estimated $\mathbb{V}$ itself increases with θ, due to the fact that the ρ_{pa} we measure from the data falls as θ increases but the other two moments (importantly, ρ_{pi}) are unchanged.

Table 4: The Impact of Informational Frictions - Alternative Values of θ

	$\theta = 4$					$\theta = 10$				
	$\mathbb{V}$	$\frac{\mathbb{V}}{\sigma_\mu^2}$	$\frac{\mathbb{V}}{\sigma_{mrpk}^2}$	$a^* - a$	$y^* - y$	$\mathbb{V}$	$\frac{\mathbb{V}}{\sigma_\mu^2}$	$\frac{\mathbb{V}}{\sigma_{mrpk}^2}$	$a^* - a$	$y^* - y$
US	0.05	0.25	0.14	0.02	0.03	0.12	0.57	0.31	0.08	0.11
China	0.13	0.52	0.28	0.04	0.06	0.17	0.63	0.36	0.11	0.17
India	0.17	0.60	0.37	0.06	0.08	0.24	0.83	0.53	0.16	0.24

4.4 The Sources of Learning

Decomposing $\mathbb{V}$. Table 3 shows that firm learning can be quite significant and potentially mitigates a substantial portion of the underlying fundamental uncertainty. Here, we explore the relative importance of the two sources of learning present in our model, i.e., private sources within the firm versus market prices. We begin by reporting in the left-hand panel of Table 5 the total extent of learning and its aggregate consequences. To do so, we compute the reduction in $\mathbb{V}$ both in absolute and percentage terms due to learning from both channels, i.e., $\Delta\mathbb{V} = \mathbb{V} - \sigma_\mu^2$, and the resulting effects on aggregate productivity and output. The table shows that total learning can be quite important and translates into significant improvements in TFP and output: in case 2 these range from 3% in India to 5% in the US for the former and from 4% to 8% for the latter, with substantially higher gains in case 1.[54] Interestingly, the contribution of learning in China in case 1 appears comparable to that in the US, but this does not imply that Chinese firms are necessarily as well-informed as US firms: due to the convexity of $\mathbb{V}$, it is possible that a noisier signal leads to a greater reduction in uncertainty. Intuitively, if there is simply a greater amount of underlying uncertainty, as is the case in China, even a signal with identical precision is in a sense more valuable.

[53] The sensitivity of our results to θ is not particular to our framework, but rather, is common when using this class of model to study the aggregate implications of misallocation. See, for example, Hsieh and Klenow (2009), who find that gains from marginal product equalization approximately double in China and India when moving from $\theta = 3$ to $\theta = 5$.

[54] Note the difference between these calculations and those in Table 3. There, we compute losses compared to a full-information benchmark. Here, we are computing gains versus a no-information (about innovations to fundamentals) benchmark.

Table 5: The Importance and Sources of Learning

	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	Share from source	
					Private	Market
Case 2						
US	−0.12	−0.59	0.05	0.08	0.92	0.08
China	−0.10	−0.37	0.04	0.06	0.96	0.04
India	−0.06	−0.23	0.03	0.04	0.89	0.11
Case 1						
US	−0.08	−0.37	0.23	0.35	0.91	0.09
China	−0.10	−0.35	0.30	0.45	0.96	0.04
India	−0.04	−0.14	0.12	0.19	0.96	0.04

Notes: The left hand panel shows the total effect of learning, i.e., from all sources: the reduction in $\mathbb{V}$ and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The right hand panel shows the relative shares of private and market sources in total learning.

To break down the sources of learning, it is easier to work with the inverse of $\mathbb{V}$, i.e., the total precision of the firm's information, which lends itself to a simple linear decomposition:

$$\frac{1}{\mathbb{V}} = \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}$$

We focus on the latter two terms, which capture the contributions of private and market learning to overall precision and compute their relative import as $\frac{1/\sigma_e^2}{1/\sigma_e^2+1/\sigma_v^2\sigma_z^2}$ and $\frac{1/\sigma_v^2\sigma_z^2}{1/\sigma_e^2+1/\sigma_v^2\sigma_z^2}$, respectively. In this way, we calculate the fraction of the total increase in precision due to each source. We report the results in the right hand panel of Table 5. Strikingly, learning is due overwhelmingly to private sources: at best, markets account for about 10% of the increase in precision in the US and India, and about half of this in China. Clearly, these results point to firm private sources as the dominant channel for learning, with markets making only a small additional contribution. As we will see, the limited informational role of markets will prove to be a robust finding throughout our analysis.

The role of market information. To explore in greater depth the importance of new information in stock prices, we recompute $\mathbb{V}$ under the assumption that firms learn nothing from these prices, i.e., that they contain no information. This simply entails sending the noise in markets, $\sigma_v^2\sigma_z^2$, to infinity. The change in $\mathbb{V}$ is a measure of the contribution of stock market

information to firm learning, given by

$$\Delta\mathbb{V} = \mathbb{V} - \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2}\right)^{-1}$$

We report this value in the first columns of Table 6, along with the associated aggregate consequences. Even for the US, which has the most informative prices, market information reduces uncertainty only modestly, eliminating about 2% of the fundamental uncertainty, with associated output gains as small as 0.3% and at a maximum, 2%. For China and India, the contribution of market-produced information is even smaller, reducing uncertainty by between 0.5% and 2%, which represents a maximum gain in output of about 1%.

Table 6: The Contribution of Market Information

	With both sources				With only market learning			
	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy
Case 2								
US	−0.004	−0.021	0.002	0.003	−0.021	−0.107	0.009	0.014
China	−0.003	−0.010	0.001	0.002	−0.007	−0.025	0.003	0.004
India	−0.005	−0.019	0.002	0.004	−0.009	−0.030	0.004	0.006
Case 1								
US	−0.004	−0.021	0.013	0.020	−0.010	−0.051	0.031	0.046
China	−0.003	−0.009	0.008	0.011	−0.006	−0.022	0.017	0.026
India	−0.001	−0.005	0.004	0.006	−0.002	−0.006	0.005	0.008

Notes: The table shows the reduction in $\mathbb{V}$ due to market information and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The left hand panel shows the effects of market learning in the presence of private learning and the right hand panel when markets are the only source of information.

Next, we ask whether the limited informational role of markets is due to the level of noise in prices or to the fact that firms already have considerable information about fundamentals, mitigating the incremental contribution of market information. This distinction is related to the concepts of forecasting price efficiency (FPE) versus revelatory price efficiency (RPE) as put forth in Bond et al. (2012). The former captures the extent to which prices reflect and predict fundamentals, i.e., the absolute level of information in prices; the latter, the extent to which prices promote real efficiency by revealing new information to the firm. In our framework, $\Delta\mathbb{V}$ in the first columns of Table 6 is the natural measure of RPE, since it is the marginal impact of the information in prices on uncertainty, given the other sources of firm information. As the table shows, at the estimated parameter values, markets in all 3 countries exhibit relatively low RPE. To measure FPE, we compute the reduction in $\mathbb{V}$ from the information in stock prices

alone, i.e.,

$$\Delta\mathbb{V} = \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_v^2\sigma_z^2}\right)^{-1} - \sigma_\mu^2$$

This measures the contribution of market information assuming it is the only source of learning for firms. We report the results in the right hand panel of Table 6. In general, stock markets are rather weak predictors of fundamentals - even as the only source of learning, the information they provide leads to only modest reductions in uncertainty. For the US, which has the most informative prices, market generated information reduces fundamental uncertainty by between 5% and 10%. Compare this to the US values in Table 5, where a total of between 35% and 60% of uncertainty is eliminated. The associated TFP and output effects of markets as a standalone source of information in the US are, at highest, 3% and 5%, respectively, in case 1. The aggregate impact of markets is even more modest for the US in case 2, and is lower in the two emerging markets than in the US in both cases. Even in a forecasting sense then, the efficiency of stock market prices is fairly low, suggesting that the limited role of market information is in large part due to its poor quality; in other words, the poor RPE of stock markets can largely be attributed to their low FPE. Comparing these values to the corresponding ones in the left hand panel shows that the already modest impact of the information in prices is even further diluted by the presence of alternative information available to firms, leading in net to the low levels of RPE shown.

While the limited informational role of markets is a robust pattern across countries and cases, these results should be interpreted somewhat cautiously. Our analysis focuses exclusively on information and decisions over the medium term. It is silent on the role of stock markets in guiding longer-term, strategic decisions (e.g., the decision to enter a new market or acquire another company). In fact, one possible source of 'noise' in stock returns is information about the firm's prospects over a much longer horizon. So long as it is orthogonal to fundamentals over the medium term, the relevant object for the firm's current decisions, our strategy will still lead us to the right measure of uncertainty and the associated misallocation - our estimates for the specific informational parameters and the informational role of financial markets, however, are likely to be sensitive to assumptions about the nature of this 'noise' term.

Our analysis also abstracts from the contribution of financial market information in mitigating uncertainty about aggregate (or industry) conditions, particularly for outsiders (e.g., potential entrants, creditors, regulators, etc.). Explicitly modeling these features in a fully-fledged general equilibrium environment is a challenging task, but may well be essential for a comprehensive evaluation of the informational role of well-functioning financial markets.[55]

[55]See section 5 in Bond et al. (2012) for a related discussion.

The role of private information. Next, we explore the contribution of learning from information generated within the firm (i.e., firm 'private information') by performing experiments analogous to those above: specifically, we calculate the marginal contribution of private information to allocative efficiency, both in the presence of market information and when it is the only source of learning for firms.

We report the results in Table 7. In contrast to the effect of market learning, firm private information plays a much larger role in reducing uncertainty and improving aggregate performance. Turning to the first set of columns, private information eliminates between about 30% and 50% of the fundamental uncertainty in the US. The associated aggregate gains are substantial, ranging from 4-20% for TFP and from 6-30% for output. The corresponding figures are smaller for China and India in case 2 (although only slightly so in China), reflecting a worse quality of firm private information in those countries, and are smaller for India in case 1. Having said that, they are still quite significant across countries and are noticeably larger than the effects of market information. Similar to what we saw in Table 5, the contribution of private learning in China in case 1 is on par with that in US, again because even a less precise signal can have more value if the extent of uncertainty is greater.

Table 7: The Contribution of Private Information

	With both sources				With only private learning			
	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy
Case 2								
US	−0.10	−0.48	0.04	0.06	−0.11	−0.57	0.05	0.07
China	−0.09	−0.35	0.04	0.06	−0.09	−0.36	0.04	0.06
India	−0.06	−0.20	0.02	0.04	−0.06	−0.21	0.03	0.04
Case 1								
US	−0.07	−0.32	0.20	0.30	−0.07	−0.37	0.22	0.33
China	−0.09	−0.33	0.28	0.42	−0.10	−0.38	0.29	0.44
India	−0.04	−0.13	0.12	0.18	−0.04	−0.14	0.12	0.18

Notes: The table shows the reduction in $\mathbb{V}$ due to private information and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The left hand panel shows the effects of private learning in the presence of market learning and the right hand panel when private sources are the only source of information.

The right hand panel of Table 7 reports the contribution of firm private information were it the only source of learning. A comparison of the two panels shows that the values are quite similar - the presence of market information does not significantly alter the importance of private information for overall learning.

4.5 Cross-country counterfactuals

Next, we use our estimates to perform a number of instructive counterfactual experiments. Specifically, we assess the potential gains to China and India from having access to US quality information, whether through more informative financial markets or from better firm-level private information. In the first experiment, we compute the change in $\mathbb{V}$ and associated aggregate gains in China and India under the assumption that the informativeness of financial markets - summarized by $\sigma_\nu^2\sigma_z^2$ - is equal to that in the US, leaving all other country-specific parameters fixed. Second, we perform the same exercise for firm private information, that is, compute the change in $\mathbb{V}$ and aggregate improvements under the assumption that firms in China and India have the same σ_e^2 as their US counterparts (again leaving the other country-specific parameters fixed). In a final experiment, we turn away from learning and study the role of fundamentals in leading to a differential impact of informational frictions across countries; to do so, we recompute $\mathbb{V}$ in China and India assuming that firms in these countries face the same fundamental shocks as do US firms.

We report the results from these experiments in Table 8. The top panel shows that delivering US-quality markets to the emerging economies results in potentially significant yet modest reductions in uncertainty. As a percent of total fundamental uncertainty, these range from 2% to 7% (across cases) with a corresponding aggregate impact ranging, for example, from 1% to 6% in output. The middle panel shows that access to US-quality private information would have a much larger impact, reducing $\mathbb{V}$ by as much as 40% of the underlying uncertainty in the most optimistic case, and, excepting China in case 1 which is an outlier here, more than 25% in the other cases. The resulting aggregate gains can be substantial, ranging, for example, from 4% to almost 40% in output. Across the board, the gains from accessing US-quality private information are much larger than from US-quality market information. To the extent that differences in learning lead to cross-country variation in economic aggregates, these disparities appear to be largely due to a lack of high quality firm private information, rather than to a lack of well-functioning (in an informational sense) financial markets in emerging markets. This is yet another instance of a result we have now seen several times: financial markets play only a modest informational role in promoting aggregate efficiency.

Finally, we recompute $\mathbb{V}$ in China and India under the assumption that firms in those countries face shocks of the same volatility as those in the US, i.e., we replace σ_μ^2 in the two countries with that of the US. We report the results in the bottom panel of Table 8, which shows that disparities in the volatility of shocks also contribute to differences in the impact of informational frictions.[56] Exposure to a fundamental process such as that in the US reduces $\mathbb{V}$

[56] Asker et al. (2012) also highlight the role of different firm-specific shock processes in generating misallocation in a model with capital adjustment costs.

Table 8: The Effects of a US Information Structure

	Case 2				Case 1			
	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy
Market Information								
China	−0.01	−0.05	0.01	0.01	−0.01	−0.02	0.02	0.02
India	−0.02	−0.07	0.01	0.01	−0.01	−0.05	0.04	0.06
Private Information								
China	−0.07	−0.26	0.03	0.04	−0.02	−0.08	0.06	0.09
India	−0.12	−0.43	0.05	0.08	−0.09	−0.31	0.26	0.39
Shocks								
China	−0.02	−0.09	0.01	0.02	−0.03	−0.13	0.10	0.15
India	−0.05	−0.19	0.02	0.03	−0.07	−0.24	0.20	0.31

Notes: The table shows the reduction in $\mathbb{V}$ and the corresponding gains in the aggregates (hence the pattern of negatives and positives) in India and China from having access to US-quality financial market information (top panel), private information (middle panel) and facing US fundamental shocks (bottom panel).

between about 10% and 25% of the underlying uncertainty, leading to TFP and output gains that are potentially substantial, ranging from 1-20% and 2-30% respectively.[57]

4.6 Robustness

We now turn to two important robustness exercises. In Section 3, we used our special cases to demonstrate the validity of our measurement approach under two variants of the baseline model: first, in the presence of other factors entering the firm's capital choice decision, and second, with a richer correlation structure between firm and market information. To explore the extent to which that intuition carries over to the general model, in this section we conduct two numerical experiment in the same spirit. Specifically, we explore the effects of (1) an alternative information structure in which investors observe a noisy signal of the firm's private signal (in contrast to the conditional independence of signals in the baseline model), and (2) a specific friction in the firm's capital choice, namely adjustment costs. In both experiments, our goal is to assess how the presence of these alternative factors affects our measure of the severity of informational frictions; the results from both continue to demonstrate the robustness of our baseline results.

Correlated information. Recall that in the special cases analyzed in Section 3, our identification strategy proved robust to an arbitrary correlation structure, i.e., no matter how rich

[57]These numbers correspond to $\Delta(a - a^*)$ and $\Delta(y - y^*)$. Recall that a^*, the full information productivity level, is also affected by the size of fundamental shocks, σ_μ^2.

or complex the flow of information between firms and markets, our measure of uncertainty remained valid. Here, the lack of an analytical characterization means that we have to resort to numerical experiments to demonstrate a similar robustness. Without direct data on firm/market information, it is not obvious how to discipline the extent of the commonality of information. We take a different route and show results from a rather extreme modification of our baseline information structure - we assume that the signal observed by investors is a noisy signal of the firm's private signal, i.e.,

$$s_{ijt} = a_{it} + e_{it} + v_{ijt}$$

Investors then have 2 sources of information - imperfect readings of the firm's own information and the stock price. Notice that by construction, firms have nothing to learn from market prices. Despite that, investment and returns will comove in part due to the common component in the two information sets. We estimate $\mathbb{V}$ in this modified version of the model using the same procedure and data as above. We report the results in Table 9. The estimates for firm-level uncertainty are very close to those in our base case (which, of course, implies that the associated aggregate implications are also virtually unchanged). Thus, we conclude that our estimates of uncertainty are not particularly sensitive to assumptions about correlation in firm and market information.[58]

Table 9: Uncertainty with Correlated Information

	σ_e	σ_v	σ_z	$\mathbb{V}$	Baseline $\mathbb{V}$
US	0.37	0.17	8.01	0.08	0.08
China	0.60	0.63	4.65	0.15	0.16
India	0.79	0.45	6.37	0.19	0.22

Notes: The table shows parameter estimates and the resulting extent of uncertainty when investors are assumed to observe a noisy signal of the firm's private signal. All parameters not displayed retain the same values as the baseline case.

Capital adjustment costs. In the special case analyzed in Section 3, we showed that our measure of uncertainty was robust to the presence of other distortions, both correlated and uncorrelated with fundamentals, that may affect the firm's capital choice decision. That analysis, while reassuring, was done with a stylized representation of these factors, primarily for analytical tractability. To explore this result in the general model, we ask how the presence of a particular friction, capital adjustment costs, affects our assessment of the severity of in-

[58]Our estimates of the individual error variances do change, however.

formational frictions.[59] We focus on a polar case - an economy where fully informed firms are subject to convex costs of adjustment. We then take our model with informational frictions to moments generated from this hypothetical economy - our goal is to see whether our empirical strategy will lead us to an incorrect inference about uncertainty.

This experiment requires two modifications to our baseline model. First, following Bloom (2009), Cooper and Haltiwanger (2006), and Asker et al. (2012), we subject firms to quadratic costs of adjusting capital $\zeta K_{t-1}\left(\frac{K_t-K_{t-1}}{K_{t-1}}\right)^2$.[60] Second, we assume that firms have full information ($\sigma_e^2 = 0$) but that stock markets are noisy ($\sigma_v^2 \neq 0, \sigma_z^2 \neq 0$). We then parameterize and solve the model in general equilibrium.[61] We choose values of $(\sigma_v^2, \sigma_z^2, \zeta)$ to match 3 moments in the data - the volatility of returns and their correlation with fundamentals, σ_p^2 and ρ_{pa}, and the cross-sectional variation in firm-level investment. The first two were also used in our baseline analysis to measure the information in prices; the last is added to discipline the adjustment cost parameter ζ.[62] We simulate data using this parameterized model and compute the correlation of prices with investment (growth), ρ_{pi}. Since ρ_{pa} is directly targeted, the model-implied ρ_{pi} reveals the extent to which adjustment costs generate a large *relative* correlation, the statistic at the heart of our inference about $\mathbb{V}$. Finally, we reestimate the parameters of our model with informational frictions using this simulated value for ρ_{pi}, along with the other two moments (namely, σ_p^2 and ρ_{pa}).

We report the results in Table 10.[63] The correlation of returns with investment choices implied by the full-information adjustment cost model is significantly lower than that observed in the data, leading to the substantially lower values for the relative correlation reported in the table. In other words, in the absence of firm-level uncertainty, investment decisions covary much less with stock returns even in the presence of adjustment costs. The relative correlation numbers suggest that applying our empirical strategy to these simulated moments would result in substantially lower estimates of $\mathbb{V}$. The third column of the table confirms that this is

[59] We put particular focus on adjustment costs since Asker et al. (2012) find that these types of costs can account for a significant portion of MRPK dispersion across industries and countries, whereas Midrigan and Xu (2013) find much more modest effects stemming from financial frictions. Moreover, our analysis focuses on large publicly traded firms for whom financial constraints are less likely to be binding.

[60] We focus on convex costs and abstract from the fixed-type costs also considered in the literature, as the former are most capable of delivering a configuration of moments that resemble those under informational frictions. Intuitively, convex costs imply that investment decisions respond to innovations in fundamentals only in part contemporaneously and then with a lag. This latter feature resembles the pattern implied by informational frictions.

[61] Since the firm's capital choice problem is now a dynamic one, we can no longer analytically characterize the joint distribution of fundamentals and capital across firms, but rather, solve numerically for this distribution (and the associated general equilibrium implications) in steady state. We refer the reader to Appendix B for details.

[62] This is along the lines of the estimation strategy employed in Bloom (2009), Cooper and Haltiwanger (2006), and Asker et al. (2012).

[63] We show in Appendix B the full set of target moments and resulting parameter values.

indeed the case: the estimated $\mathbb{V}$'s from the simulated data are only about one-third of those in our baseline analysis (compare to the fifth column). The resulting aggregate effects are also substantially lower: for example, TFP losses are only between 1 and 4%, compared to 4-10% in our baseline. These results provide some reassurance that our estimates are not simply picking up the effects of adjustment costs, but are indeed robust measures of firm-level uncertainty.

Table 10: A Full-Information Adjustment Cost Model

	Relative Correlation		Implied Uncertainty		Baseline Uncertainty	
	Data	Model	$\mathbb{V}$	$a^* - a$	$\mathbb{V}$	$a^* - a$
US	0.05	−0.03	0.03	0.01	0.08	0.04
China	0.10	−0.08	0.06	0.03	0.16	0.07
India	0.17	−0.02	0.08	0.04	0.22	0.10

Notes: The table shows the relative correlation of returns with investment growth and returns with fundamentals ($\rho_{pi} - \rho_{pa}$), both as computed from the data and as predicted by the full-information adjustment cost model, as well as the implied extent of uncertainty from the latter and from the baseline model. ρ_{pa} is a targeted moment and so is the same in the data and model.

While this analysis had the admittedly limited goal of confirming the robustness of our estimates of uncertainty, it also suggests that both types of frictions (adjustment costs and imperfect information) might be necessary to match a broader set of empirical moments. Such a unified framework would also allow us to disentangle their effects on aggregate outcomes as well as explore how they interact with each other. At some level, it is not even clear that they should be thought of as two independent frictions: imperfect information leads to delayed responses to fundamentals and in this sense, provide a deeper theory of adjustment costs.[64] Exploring these issues, both theoretically and quantitatively, is an important, if challenging, direction for future research.

5 Conclusion

In this paper, we have laid out a theory of informational frictions that distort the allocation of factors across heterogeneous firms, leading to reduced aggregate productivity and output. Taking this theory to the data, we find evidence of substantial micro-level uncertainty and associated aggregate losses, particularly so in China and India.[65]

[64] Fuchs et al. (2013) provide a microfoundation for adjustment costs based on asymmetric information and adverse selection.

[65] Strictly speaking, our sample covers only large, publicly traded firms. However, these are also likely to be the most well-informed. In this sense, one can view our estimates as a lower bound for the total effect of information frictions in the economy.

There are several promising directions for future research. In our modeling approach, we have aimed to strike a balance between realism and transparency of the economic forces at play as well as of our empirical methodology. In doing so, we have made a couple of admittedly extreme assumptions. For example, the investment choice is modeled as a static and otherwise undistorted decision problem. Similarly, the learning process is rather stark, with perfect revelation at the end of each period, implying that firms are able to quickly 'correct' their past errors. As a result, investment in any given period includes a component related to the forecast error in the previous period. When shocks are highly persistent, this adjustment makes a substantial contribution to investment volatility. These assumptions limit our ability to match a larger set of micro moments on investment dynamics. Relaxing them is straightforward from a conceptual standpoint - for example, by introducing financial constraints, adjustment costs, or a richer stochastic process for fundamentals[66] - however, there are substantial computational and empirical challenges. First, this introduces additional state variables into the firm's problem and precludes the simple analytical mapping between uncertainty and aggregate outcomes. Second, this requires additional data to infer the additional parameters - particularly on the time series behavior of firm-level investment. This is less of an issue for the US, but in China and India, both availability and quality of the data decline dramatically as we go back in time.

Another important direction for future work is towards a deeper theory of differences in information quality. This is valuable both from a conceptual point of view as well as from the perspective of designing policies. For example, much of the cross-country variation we find seems to stem from differences in the quality of firms' internal information sources and only marginally from the quality of information from financial markets. These results suggest that policies aimed at improving the quality of firm-level information may be more fruitful in improving aggregate performance than those intended to bring financial markets to a level closer to that in the US. Given that we follow the standard approach of modeling information as exogenous noisy signals, we are left to speculate on the exact form of such policies. In our view, the most likely sources of variation in the quality of firm private information are differences in manager skill and/or information collection/processing within the firm. Under this interpretation, improving manager training and/or accounting and information systems within firms holds the most promise of generating substantial gains in aggregate TFP. Our results also show that a reduction in fundamental volatility would also help mitigate the informational disadvantage of firms in less developed countries.

[66]Suppose, for example, that the fundamental a_{it} consists of a persistent component and a purely transitory one. Then, even if a_{it-1} is observed perfectly ex-post, firms will need to form beliefs about the persistent component (e.g., by using a Kalman filter) while forecasting a_{it}. This implies that innovations to the persistent component are revealed slowly.

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Appendix

A Detailed Derivations

A.1 Case 1: Both factors chosen under imperfect information

As we show in equation (5) in the text, the firm's capital choice problem can be written as

$$\max_{K_{it}} \quad \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left[A_{it}\right] K_{it}^{\alpha} - \left(1+\frac{\alpha_2}{\alpha_1}\right) R_t K_{it}$$

and optimality requires

$$\begin{aligned} \alpha\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)\left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} K_{it}^{\alpha-1}\mathbb{E}_{it}\left[A_{it}\right] &= R_t \\ \Rightarrow \qquad \left[\alpha\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)\left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \frac{\mathbb{E}_{it}\left[A_{it}\right]}{R_t}\right]^{\frac{1}{1-\alpha}} &= K_{it} \end{aligned}$$

Capital market clearing then implies

$$\begin{aligned} \int K_{it} di &= \left[\left(\frac{N}{K_t}\right)^{\alpha_2} \alpha\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)\frac{Y_t^{\frac{1}{\theta}}}{R_t}\right]^{\frac{1}{1-\alpha}} \int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di = K_t \\ \Rightarrow \qquad \left[\left(\frac{N}{K_t}\right)^{\alpha_2} \alpha\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)\frac{Y_t^{\frac{1}{\theta}}}{R_t}\right]^{\frac{1}{1-\alpha}} &= \frac{K_t}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di} \end{aligned}$$

from which we can solve for

$$K_{it} = \frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di} K_t$$

From here, it is straightforward to express firm revenue as

$$P_{it}Y_{it} = K_t^{\alpha_1} N^{\alpha_2} Y_t^{\frac{1}{\theta}} A_{it} \left(\frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di} \right)^{\alpha}$$

and noting that aggregate revenue must equal aggregate output, we have

$$Y_t = \int P_{it}Y_{it}di = K_t^{\alpha_1} N^{\alpha_2} Y_t^{\frac{1}{\theta}} \frac{\int A_{it} \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}} di}{\left(\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di\right)^{\alpha}}$$

or in logs,

$$y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \log \int A_{it} \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}} di - \alpha \log \int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di$$

Now, note that under conditional log-normality,

$$a_{it}|\mathcal{I}_{it} \sim \mathcal{N}\left(\mathbb{E}_{it}\left[a_{it}\right], \mathbb{V}\right) \quad \Rightarrow \quad \mathbb{E}_{it}\left[A_{it}\right] = \exp\left(\mathbb{E}_{it}a_{it} + \frac{1}{2}\mathbb{V}\right)$$

The true fundamental a_{it} and its conditional expectation $\mathbb{E}_{it}a_{it}$ are also jointly normal, i.e.

$$\begin{bmatrix} a_{it} \\ \mathbb{E}_{it}a_{it} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \bar{a} \\ \bar{a} \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_a^2 - \mathbb{V} \\ \sigma_a^2 - \mathbb{V} & \sigma_a^2 - \mathbb{V} \end{bmatrix} \right)$$

We then have that

$$\begin{aligned} \log \int A_{it} \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}} di &= \log \int \exp\left(a_{it} + \frac{\alpha}{1-\alpha}\mathbb{E}_{it}a_{it} + \frac{1}{2}\frac{\alpha}{1-\alpha}\mathbb{V} \right) di \\ &= \frac{1}{1-\alpha}\bar{a} + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)^2 \left(\sigma_a^2 - \mathbb{V}\right) \\ &\quad + \frac{\alpha}{1-\alpha}\left(\sigma_a^2 - \mathbb{V}\right) + \frac{1}{2}\frac{\alpha}{1-\alpha}\mathbb{V} \\ &= \frac{1}{1-\alpha}\bar{a} + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\alpha\frac{2-\alpha}{(1-\alpha)^2}\left(\sigma_a^2 - \mathbb{V}\right) + \frac{1}{2}\frac{\alpha}{1-\alpha}\mathbb{V} \end{aligned}$$

and

$$\log \int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di = \frac{1}{1-\alpha}\bar{a} + \frac{1}{2}\left(\frac{1}{1-\alpha}\right)^2 \left(\sigma_a^2 - \mathbb{V}\right) + \frac{1}{2}\frac{1}{1-\alpha}\mathbb{V}$$

Combining these,

$$\log \int A_{it} \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}} di - \alpha \log \int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di = \bar{a} + \frac{1}{2}\frac{\sigma_a^2}{1-\alpha} - \frac{1}{2}\frac{\alpha}{1-\alpha}\mathbb{V}$$

Substituting and rearranging, we obtain the expressions in (11) and (12) in the text,

$$\begin{aligned} y_t &= \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \bar{a} + \frac{1}{2}\frac{\sigma_a^2}{1-\alpha} - \frac{1}{2}\frac{\alpha}{1-\alpha}\mathbb{V} \\ &= \hat{\alpha}_1 k_t + \hat{\alpha}_2 n + \frac{\theta}{\theta-1}\bar{a} + \frac{1}{2}\left(\frac{\theta}{\theta-1}\right)\frac{\sigma_a^2}{1-\alpha} - \frac{1}{2}\theta\mathbb{V} \end{aligned}$$

with aggregate productivity given by

$$a = \underbrace{\frac{\theta}{\theta-1}\bar{a} + \frac{1}{2}\left(\frac{\theta}{\theta-1}\right)\frac{\sigma_a^2}{1-\alpha}}_{a^*} - \frac{1}{2}\theta\mathbb{V}$$

It now remains to endogenize K_t. The rental rate in steady state satisfies

$$R = \frac{1}{\beta} - 1 + \delta$$

Then, from the optimality and market clearing conditions, we have from (4) that

$$\frac{\alpha_1 N}{\alpha_2 K} = \frac{R}{W} \quad \Rightarrow \quad K \propto W$$

i.e., the aggregate capital stock is proportional to the wage. To characterize wages, we return to the firm's profit maximization problem

$$\max_{K_{it}, N_{it}} \quad Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left[A_{it}\right] K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W N_{it} - R K_{it}$$

which, after maximizing over capital, can be written as

$$\max_{N_{it}} \quad (1-\alpha_1)\left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1}} \left(Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left(A_{it}\right) N_{it}^{\alpha_2}\right)^{\frac{1}{1-\alpha_1}} - W N_{it}$$

Optimality and labor market clearing imply

$$\begin{aligned} \left(\frac{\alpha_2}{W}\right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}} \left(Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha_1-\alpha_2}} &= N_{it} \\ \left(\frac{\alpha_2}{W}\right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}} \int \left(Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha_1-\alpha_2}} di &= N \end{aligned}$$

As before, letting $\alpha = \alpha_1 + \alpha_2$, we see that

$$\begin{aligned} W &\propto \left(\int \mathbb{E}_{it} \left[A_{it} \right]^{\frac{1}{1-\alpha}} di \right)^{\frac{1-\alpha}{1-\alpha_1}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_1}} \\ &= \left[\left(\int \left(\exp\left(\mathbb{E}_{it} a_{it} + \frac{1}{2}\mathbb{V} \right) \right)^{\frac{1}{1-\alpha}} di \right)^{1-\alpha} Y_t^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}} = \left[\exp\left(\bar{a} + \frac{1}{2}\left(\frac{\sigma_a^2 - \mathbb{V}}{1-\alpha} \right) + \frac{1}{2}\mathbb{V} \right) Y_t^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}} \\ &= \left[\exp\left(\bar{a} + \frac{1}{2}\left(\frac{\sigma_a^2 - \alpha\mathbb{V}}{1-\alpha} \right) \right) Y_t^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}} \end{aligned}$$

or in logs,

$$w \propto \left(\frac{1}{1-\alpha_1} \right) \bar{a} + \frac{1}{1-\alpha_1}\frac{1}{2}\left(\frac{\sigma_a^2 - \alpha\mathbb{V}}{1-\alpha} \right) + \frac{1}{\theta}\frac{1}{1-\alpha_1} y_t$$

Recalling that

$$K \propto W \quad \Rightarrow \quad \frac{dk}{d\mathbb{V}} = \frac{dw}{d\mathbb{V}}$$

which, in conjunction with (11) and (12), implies

$$\begin{aligned} \frac{dy}{d\mathbb{V}} &= \hat{\alpha}_1 \left(\frac{dk}{d\mathbb{V}} \right) - \frac{1}{2}\theta \\ &= \frac{\hat{\alpha}_1}{1-\alpha_1} \left[-\frac{1}{2}\frac{\alpha}{1-\alpha} + \frac{1}{\theta}\frac{dy}{d\mathbb{V}} \right] - \frac{1}{2}\theta \end{aligned}$$

and finally, collecting terms and rearranging, and using the fact that $\widehat{\alpha}_1 = \frac{\theta}{\theta-1}\alpha_1$, we obtain

$$\frac{dy}{d\mathbb{V}} = -\frac{1}{2}\theta \frac{1}{1-\hat{\alpha}_1} = \frac{da}{d\mathbb{V}} \frac{1}{1-\hat{\alpha}_1}$$

A.2 Case 2: Only capital chosen under imperfect information

The firm's labor choice problem can be written as

$$\max_{N_{it}} \quad Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it}$$

and optimality requires

$$N_{it} = \left(\frac{\alpha_2}{W} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1-\alpha_2}}$$

Labor market clearing then implies

$$\begin{aligned}\int N_{it}\, di &= \int \left(\frac{\alpha_2}{W} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}} di = N \\ \Rightarrow \quad N_{it} &= \frac{(A_{it} K_{it}^{\alpha_1})^{\frac{1}{1-\alpha_2}}}{\int (A_{it} K_{it}^{\alpha_1})^{\frac{1}{1-\alpha_2}} di} N\end{aligned}$$

Letting $\tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha_2}}$ and $\tilde{\alpha} = \frac{\alpha_1}{1-\alpha_2}$, we have

$$N_{it} = \frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di} N$$

which implies

$$\begin{aligned}W_t &= Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} \alpha_2 \left(\frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di} N\right)^{\alpha_2 - 1} \\ &= \frac{\alpha_2}{N^{1-\alpha_2}} Y_t^{\frac{1}{\theta}} \left(\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}}\, di\right)^{1-\alpha_2} A_{it} K_{it}^{\alpha_1} \left(A_{it}^{\frac{1}{1-\alpha_2}} K_{it}^{\frac{\alpha_1}{1-\alpha_2}}\right)^{\alpha_2 - 1} \\ &= \frac{\alpha_2}{N^{1-\alpha_2}} Y_t^{\frac{1}{\theta}} \left(\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}}\, di\right)^{1-\alpha_2}\end{aligned}$$

From here, it is straightforward to express the firm's capital choice problem as in (8):

$$\max_{K_{it}} \quad (1-\alpha_2) \left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}} \mathbb{E}_{it}\left[\tilde{A}_{it}\right] K_{it}^{\tilde{\alpha}} - R_t K_{it}$$

Optimality requires

$$\begin{aligned}R_t &= (1-\alpha_2) \left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}} \mathbb{E}_{it}\left[\tilde{A}_{it}\right] \tilde{\alpha} K_{it}^{\tilde{\alpha}-1} \\ \Rightarrow \quad K_{it} &= \left[\frac{(1-\alpha_2)\tilde{\alpha}}{R}\right]^{\frac{1}{1-\tilde{\alpha}}} \left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}\frac{1}{1-\tilde{\alpha}}} Y_t^{\frac{1}{\theta(1-\alpha_2)(1-\tilde{\alpha})}} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}\end{aligned}$$

Capital market clearing then implies

$$\begin{aligned}\int K_{it}di &= \left[\frac{(1-\alpha_2)\tilde{\alpha}}{R}\right]^{\frac{1}{1-\tilde{\alpha}}}\left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}\frac{1}{1-\tilde{\alpha}}}Y_t^{\frac{1}{\theta(1-\alpha_2)(1-\tilde{\alpha})}}\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di = K_t \\ \Rightarrow \quad K_{it} &= \frac{\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}}{\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di}K_t\end{aligned}$$

and we can rewrite the labor choice as

$$N_{it} = \frac{\tilde{A}_{it}K_{it}^{\tilde{\alpha}}}{\int\tilde{A}_{it}K_{it}^{\tilde{\alpha}}di}N = \frac{\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di}N$$

Combining the solutions for capital and labor, we can express firm revenue as

$$\begin{aligned}P_{it}Y_{it} &= Y_t^{\frac{1}{\theta}}A_{it}\left\{\frac{\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}}{\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di}K_t\right\}^{\alpha_1}\left\{\frac{\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di}N\right\}^{\alpha_2} \\ &= Y_t^{\frac{1}{\theta}}K_t^{\alpha_1}N^{\alpha_2}\frac{A_{it}\tilde{A}_{it}^{\alpha_2}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\alpha_1+\alpha_2\tilde{\alpha}}{1-\tilde{\alpha}}}}{\left\{\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di\right\}^{\alpha_1}\left\{\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right\}^{\alpha_2}} \\ &= Y_t^{\frac{1}{\theta}}K_t^{\alpha_1}N^{\alpha_2}\frac{\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\left\{\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di\right\}^{\alpha_1}\left\{\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right\}^{\alpha_2}}\end{aligned}$$

and again using the fact that $y_t = \log\int P_{it}Y_{it}di$, we can write

$$y_t = \frac{1}{\theta}y_t + \alpha_1 k_t + \alpha_2 n - \alpha_1\log\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di + (1-\alpha_2)\log\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di$$

Again, we exploit log-normality to obtain

$$\begin{aligned}\log\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di &= \log\int\exp\left(\tilde{a}_{it} + \frac{\tilde{\alpha}}{1-\tilde{\alpha}}\mathbb{E}_{it}\tilde{a}_{it} + \frac{1}{2}\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\tilde{\mathbb{V}}\right)di \\ &= \frac{1}{1-\tilde{\alpha}}\tilde{a} + \frac{1}{2}\sigma_{\tilde{a}}^2 + \frac{1}{2}\tilde{\alpha}\frac{2-\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}}\right) + \frac{1}{2}\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\tilde{\mathbb{V}}\end{aligned}$$

and similarly,

$$\begin{aligned}\log\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di &= \log\int\left(\exp\left(\mathbb{E}_{it}\tilde{a}_{it}+\frac{1}{2}\tilde{\mathbb{V}}\right)\right)^{\frac{1}{1-\tilde{\alpha}}}di\\ &= \frac{1}{1-\tilde{\alpha}}\tilde{a}+\frac{1}{2}\frac{\sigma_{\tilde{a}}^2-\tilde{\mathbb{V}}}{(1-\tilde{\alpha})^2}+\frac{1}{2}\frac{1}{1-\tilde{\alpha}}\tilde{\mathbb{V}}\end{aligned}$$

Combining, and using the fact that $\tilde{a}_{it}=\frac{a_{it}}{1-\alpha_2}$:

$$\begin{aligned}&-\alpha_1\log\int\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}di+(1-\alpha_2)\log\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\\ =\;&\frac{1-\alpha_2-\alpha_1}{1-\tilde{\alpha}}\tilde{a}+\frac{(1-\alpha_2)}{2}\sigma_{\tilde{a}}^2+\frac{1}{2}\left[\frac{(1-\alpha_2)(2\tilde{\alpha}-\tilde{\alpha}^2)}{(1-\tilde{\alpha})^2}\right]\left(\sigma_{\tilde{a}}^2-\tilde{\mathbb{V}}\right)\\ =\;&\frac{1-\alpha}{1-\tilde{\alpha}}\frac{\bar{a}}{1-\alpha_2}+\frac{(1-\alpha_2)}{2}\sigma_{\tilde{a}}^2+\frac{1}{2}\frac{\alpha_1}{1-\tilde{\alpha}}\left(\sigma_{\tilde{a}}^2-\tilde{\mathbb{V}}\right)\\ =\;&\bar{a}+\frac{1}{2}\frac{1}{1-\alpha}\sigma_a^2-\frac{1}{2}\frac{\alpha_1}{1-\alpha}(1-\alpha_2)\tilde{\mathbb{V}}\end{aligned}$$

Substituting and collecting terms, we obtain expressions (11) and (13) in the text,

$$y_t=\widehat{\alpha}_1k_t+\widehat{\alpha}_2n+\frac{\theta}{\theta-1}\bar{a}+\frac{1}{2}\left(\frac{\theta}{\theta-1}\right)\frac{\sigma_a^2}{1-\alpha}-\frac{1}{2}\left(\theta\widehat{\alpha}_1+\widehat{\alpha}_2\right)\widehat{\alpha}_1\tilde{\mathbb{V}}$$

with aggregate productivity given by

$$a=\underbrace{\frac{\theta}{\theta-1}\bar{a}+\frac{1}{2}\left(\frac{\theta}{\theta-1}\right)\frac{\sigma_a^2}{1-\alpha}}_{a^*}-\frac{1}{2}\left(\theta\widehat{\alpha}_1+\widehat{\alpha}_2\right)\widehat{\alpha}_1\tilde{\mathbb{V}}$$

To endogenize K_t, we begin by characterizing the steady state wage W in terms of $\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di$ and $Y^{\frac{1}{\theta}}$:

$$\begin{aligned}W &= \frac{\alpha_2}{N^{1-\alpha_2}}Y^{\frac{1}{\theta}}\left(\int\tilde{A}_{it}K_{it}^{\tilde{\alpha}}di\right)^{1-\alpha_2}\\ &= \frac{\alpha_2}{N^{1-\alpha_2}}Y^{\frac{1}{\theta}}\left\{\int\tilde{A}_{it}\left(\frac{(1-\alpha_2)\tilde{\alpha}}{R}\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}Y^{\frac{1}{\theta(1-\alpha_2)}}\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right\}^{1-\alpha_2}\\ &= \frac{\alpha_2}{N^{1-\alpha_2}}Y^{\frac{1}{\theta}}\left[\frac{(1-\alpha_2)\tilde{\alpha}}{R}\right]^{\frac{\alpha_1}{1-\tilde{\alpha}}}\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2\tilde{\alpha}}{1-\tilde{\alpha}}}Y^{\frac{1}{\theta}\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}\left\{\int\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right\}^{1-\alpha_2}\end{aligned}$$

and rearranging,

$$\begin{aligned}
W &= \left(\frac{\alpha_2}{N^{1-\alpha_2}}\right)^{\frac{1-\tilde{\alpha}}{1-\alpha_1}} \left[\frac{(1-\alpha_2)\tilde{\alpha}}{R}\right]^{\frac{\alpha_1}{1-\alpha_1}} \alpha_2^{\frac{\alpha_2\tilde{\alpha}}{1-\alpha_1}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_1}} \left\{\int \tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di\right\}^{\frac{(1-\alpha_2)(1-\tilde{\alpha})}{1-\alpha_1}} \\
&\propto \left\{\int \tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di\right\}^{\frac{1-\alpha}{1-\alpha_1}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_1}} \\
&= \left\{\left[\exp\left(\frac{1}{1-\tilde{\alpha}}\tilde{a} + \frac{1}{2}\sigma_{\tilde{a}}^2 + \frac{1}{2}\frac{2\tilde{\alpha}-\tilde{\alpha}^2}{(1-\tilde{\alpha})^2}\left(\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}}\right) + \frac{1}{2}\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\tilde{\mathbb{V}}\right)\right]^{1-\alpha} Y^{\frac{1}{\theta}}\right\}^{\frac{1}{1-\alpha_1}} \\
&= \left\{\exp\left[\bar{a} + \frac{1}{2}\frac{1}{(1-\alpha_2)(1-\tilde{\alpha})}\sigma_a^2 - \frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{1-\tilde{\alpha}}\tilde{\mathbb{V}}\right] Y^{\frac{1}{\theta}}\right\}^{\frac{1}{1-\alpha_1}}
\end{aligned}$$

or in logs,

$$w \propto \left(\frac{1}{1-\alpha_1}\right)\bar{a} + \frac{1}{2}\frac{1}{(1-\alpha_1)}\frac{1}{(1-\alpha)}\sigma_a^2 - \frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})}\tilde{\mathbb{V}} + \frac{1}{\theta}\frac{1}{1-\alpha_1}y_t$$

As before,

$$K \propto W \quad \Rightarrow \quad \frac{dk}{d\tilde{\mathbb{V}}} = \frac{dw}{d\tilde{\mathbb{V}}} = -\frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})} + \frac{1}{\theta}\frac{1}{1-\alpha_1}\frac{dy}{d\tilde{\mathbb{V}}}$$

and substituting into the derivative of aggregate output,

$$\begin{aligned}
\frac{dy}{d\tilde{\mathbb{V}}} &= \hat{\alpha}_1\left(\frac{dk}{d\tilde{\mathbb{V}}}\right) - \frac{1}{2}\left(\theta\widehat{\alpha}_1 + \widehat{\alpha}_2\right)\widehat{\alpha}_1 \\
&= \hat{\alpha}_1\left[-\frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})} + \frac{1}{\theta}\frac{1}{1-\alpha_1}\frac{dy}{d\tilde{\mathbb{V}}}\right] - \frac{1}{2}\left(\theta\widehat{\alpha}_1 + \widehat{\alpha}_2\right)\widehat{\alpha}_1
\end{aligned}$$

Finally, collecting terms and rearranging, and using the facts that $1-\alpha = (1-\tilde{\alpha})(1-\alpha_2)$ and $\left(\frac{\theta}{\theta-1}\right)\alpha_1 = \widehat{\alpha}_1$, we obtain

$$\frac{dy}{d\tilde{\mathbb{V}}} = -\frac{1}{2}\left(\theta\widehat{\alpha}_1 + \widehat{\alpha}_2\right)\widehat{\alpha}_1\frac{1}{1-\widehat{\alpha}_1} = \frac{da}{d\tilde{\mathbb{V}}}\left(\frac{1}{1-\hat{\alpha}_1}\right)$$

A.3 The stock market

Here, we connect expected profits $\pi(\cdot)$ in the price function to the firm's problem. For brevity, we only show the derivation for case 1 (case 2 is very similar). The firm's profit is

$$
\begin{aligned}
&= \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha} - \left(1+\frac{\alpha_2}{\alpha_1}\right) RK_{it} \\
&= \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} A_{it} \left(\frac{(\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}}}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di} K_t\right)^{\alpha} - \left(1+\frac{\alpha_2}{\alpha_1}\right) \frac{(\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}}}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di} RK_t \\
&= \underbrace{\left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \left(\frac{K_t}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di}\right)^{\alpha}}_{\Gamma_1} A_{it} (\mathbb{E}_{it}[A_{it}])^{\frac{\alpha}{1-\alpha}} - \underbrace{\frac{\left(1+\frac{\alpha_2}{\alpha_1}\right) RK_t}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di}}_{\Gamma_2} (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} \\
&= \Gamma_1 A_{it} (\mathbb{E}_{it}[A_{it}])^{\frac{\alpha}{1-\alpha}} - \Gamma_2 (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}}
\end{aligned}
$$

From the definition of the firm's information set,

$$
\begin{aligned}
\mathbb{E}_{it}[A_{it}] &= \mathbb{E}[A_{it}|a_{it-1}, a_{it}+e_{it}, a_{it}+\sigma_v z_{it}] \\
&= \frac{\mathbb{V}}{\sigma_\mu^2}\rho(a_{it-1}-\bar{a})
\end{aligned}
$$

The profit function $\pi(\cdot)$ is obtained after integrating out the noise term in the firm's signal

$$
\begin{aligned}
\pi(a_{it-1}, a_{it}, a_{it}+\sigma_v z_{it}) &= \Gamma_1 \int A_{it} (\mathbb{E}[A_{it}|a_{it-1}, a_{it}+e_{it}, a_{it}+\sigma_v z_{it}])^{\frac{\alpha}{1-\alpha}} d\Phi\left(\frac{e_{it}}{\sigma_e}\right) \\
&\quad -\Gamma_2 \int ((\mathbb{E}[A_{it}|a_{it-1}, a_{it}+e_{it}, a_{it}+\sigma_v z_{it}]))^{\frac{1}{1-\alpha}} d\Phi\left(\frac{e_{it}}{\sigma_e}\right)
\end{aligned}
$$

A.4 Special cases: Identification

Transitory shocks. A log-linear approximation of prices (around the deterministic case):[67]

$$
\begin{aligned}
P\exp(p_{it}) &= \tilde{\mathbb{E}}_{it} AK^{\alpha} \exp(a_{it}+\alpha k_{it}) - \tilde{\mathbb{E}}_{it} RK \exp(k_{it}) + \beta \tilde{\mathbb{E}}_{it} P_{it+1} + \text{Const.} \\
\Rightarrow P + Pp_{it} &\approx AK^{\alpha} - RK + \beta P + AK^{\alpha} \tilde{\mathbb{E}}_{it}(a_{it}+\alpha k_{it}) - RK\tilde{\mathbb{E}}_{it} k_{it} + \beta \tilde{\mathbb{E}}_{it} Pp_{it+1} + \text{Const.} \\
Pp_{it} &\approx AK^{\alpha} \tilde{\mathbb{E}}_{it} a_{it} + (\alpha AK^{\alpha} - RK) \frac{\tilde{\mathbb{E}}_{it}[\mathbb{E}_{it} a_{it}]}{1-\alpha} + \beta \tilde{\mathbb{E}}_{it} Pp_{it+1} + \text{Const.} \\
&= AK^{\alpha} \tilde{\mathbb{E}}_{it} a_{it} + \beta \tilde{\mathbb{E}}_{it} Pp_{it+1} + \text{Const.} \\
p_{it} &= \frac{AK^{\alpha}}{P} \tilde{\mathbb{E}}_{it} a_{it} + \beta \tilde{\mathbb{E}}_{it} p_{it+1} + \text{Const.}
\end{aligned}
$$

where $\tilde{\mathbb{E}}$ denotes the marginal investor's expectations.

Guess-and-verify

[67] The aggregate constant multiplying revenues is normalized to 1.

$$p_{it} = \xi \tilde{\mathbb{E}}_{it} a_{it} + \text{Const.}$$

Substituting,

$$\xi \tilde{\mathbb{E}}_{it} a_{it} + \text{Const.} = \frac{AK^{\alpha}}{P} \tilde{\mathbb{E}}_{it} a_{it} + \beta \xi \tilde{\mathbb{E}}_{it} a_{it+1} + \text{Const.} \qquad \Rightarrow \qquad \xi = \frac{Y}{P} = \frac{1-\beta}{1-\alpha}$$

Variances (of growth rates):

$$\begin{aligned}
\sigma_p^2 &\equiv \text{Var}\left(p_{it} - p_{it-1}\right) = 2\psi^2\xi^2\left(\sigma_\mu^2 + \sigma_v^2\sigma_z^2\right) = 2\psi^2\xi^2\left(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\sigma_u^2 \\
\sigma_k^2 &\equiv \text{Var}\left(k_{it} - k_{it-1}\right) = \left[2\left(\phi_1+\phi_2\right)^2\sigma_\mu^2 + \phi_1^2\sigma_e^2 + \phi_2^2\sigma_v^2\sigma_z^2\right]\left(\frac{1}{1-\alpha}\right)^2 \\
&= 2\frac{\left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}\right)^2\sigma_u^2 + \left(\frac{1}{\sigma_e^2}\right)^2\sigma_e^2 + \left(\frac{1}{\sigma_v^2\sigma_z^2}\right)^2\sigma_v^2\sigma_z^2}{\left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}\right)^2}\left(\frac{1}{1-\alpha}\right)^2 \\
&= 2\left(\frac{\frac{1}{\sigma_v^2\sigma_z^2} + \frac{1}{\sigma_e^2}}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}}\right)\sigma_\mu^2 = 2\left(\sigma_\mu^2 - \mathbb{V}\right)\left(\frac{1}{1-\alpha}\right)^2 \\
\sigma_a^2 &\equiv \text{Var}\left(a_{it} - a_{it-1}\right) = 2\sigma_\mu^2
\end{aligned}$$

Covariances (of growth rates):

$$\begin{aligned}
\text{Cov}\left(p,k\right) &\equiv \text{Cov}\left(p_{it} - p_{it-1}, k_{it} - k_{it-1}\right) \\
&= 2\xi\psi\left(\phi_1+\phi_2\right)\sigma_\mu^2 + \phi_2\sigma_v^2\sigma_z^2\left(\frac{1}{1-\alpha}\right) \\
&= 2\xi\psi\left(\frac{\left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}\right)\sigma_\mu^2 + \left(\frac{1}{\sigma_v^2\sigma_z^2}\right)\sigma_v^2\sigma_z^2}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}}\right)\left(\frac{1}{1-\alpha}\right) = 2\xi\psi\sigma_\mu^2\left(\frac{1}{1-\alpha}\right) \\
\text{Cov}\left(p,a\right) &= 2\xi\psi\sigma_\mu^2
\end{aligned}$$

The correlations:

$$\begin{aligned}
\rho_{pa} &= \frac{\mathrm{Cov}\left(p,a\right)}{\sigma_p\sigma_a} = \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2+\sigma_v^2\sigma_z^2\right)\sigma_\mu^2}} = \frac{1}{\sqrt{1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}} \\
\rho_{pk} &= \frac{\mathrm{Cov}\left(p,k\right)}{\sigma_p\sigma_k} = \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2+\sigma_v^2\sigma_z^2\right)\left(\sigma_u^2-\mathbb{V}\right)}} \\
&= \frac{1}{\sqrt{\left(1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right)}}
\end{aligned}$$

The volatility of returns:

$$\sigma_p^2 = \xi^2\left(\frac{\sigma_z^2+1}{\sigma_z^2+1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}\right)^2\left(1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\sigma_u^2 = \xi^2\left(\frac{\sigma_z^2+1}{\sigma_z^2+\frac{1}{\rho_{pa}^2}}\right)^2\frac{1}{\rho_{pa}^2}\sigma_\mu^2$$

Permanent shocks. We start with the price function, (suppressing the time subscript and normalizing the GE term multiplying revenues to 1),

$$\mathcal{P}\left(a_{-1},a,z\right) = \tilde{\mathbb{E}}\left(e^aK^\alpha - RK\right) + \beta\tilde{\mathbb{E}}\mathcal{P}\left(a,a',z'\right)$$

For small information frictions, profits are approximately

$$e^aK^\alpha - RK \approx e^{\frac{a}{1-\alpha}}$$

Substituting,

$$\mathcal{P}\left(a_{-1},a,z\right) \approx \tilde{\mathbb{E}}e^{\frac{a}{1-\alpha}} + \beta\tilde{\mathbb{E}}\mathcal{P}\left(a,a',z'\right)$$

Guess

$$\mathcal{P}\left(a_{-1},a,z\right) = \hat{\mathcal{P}}\cdot\tilde{\mathbb{E}}e^{\frac{a}{1-\alpha}}$$

To verify the guess

$$\begin{aligned}
\hat{\mathcal{P}}\cdot\tilde{\mathbb{E}}e^{\frac{a}{1-\alpha}} &\approx \tilde{\mathbb{E}}e^{\frac{a}{1-\alpha}} + \beta\hat{\mathcal{P}}\cdot\tilde{\mathbb{E}}\left(\tilde{\mathbb{E}}'e^{\frac{a'}{1-\alpha}}\right) \\
&= \tilde{\mathbb{E}}e^{\frac{a}{1-\alpha}} + \beta\hat{\mathcal{P}}\cdot\tilde{\mathbb{E}}\Phi\left(u',z'\right)\cdot\tilde{\mathbb{E}}e^{\frac{a}{1-\alpha}} \\
&= \left(1+\beta\hat{\mathcal{P}}\cdot\tilde{\mathbb{E}}\Phi\left(u',z'\right)\right)\tilde{\mathbb{E}}e^{\frac{a}{1-\alpha}}
\end{aligned}$$

where $\tilde{\mathbb{E}}'$ denotes the expectation of marginal investor in the following period, which in turn is a function Φ of the future shock realizations (u',z'). Since (u',z') are iid, our guess is verified.

$$\log \mathcal{P}\left(a_{-1}, a, z\right) = \text{Const.} \ + \frac{1}{1-\alpha}\tilde{\mathbb{E}}a$$

or equivalently,

$$p_{it-1} = \frac{1}{1-\alpha}\tilde{\mathbb{E}}_{it-1}a_{it-1} + \text{const.}$$

Next, when $\rho = 1$, firm and investor beliefs are given by

$$\mathbb{E}_{it}a_{it} = a_{it-1} + \mathbb{E}_{it}\mu_{it} \qquad\qquad \tilde{\mathbb{E}}_{it}a_{it} = a_{it-1} + \tilde{\mathbb{E}}_{it}\mu_{it},$$

which leads to

$$\begin{aligned} p_{it} &= \frac{1}{1-\alpha}a_{it-1} + \frac{1}{1-\alpha}\psi\left(u_{it} + \sigma_v z_{it}\right) + \text{Const} \\ \left(1-\alpha\right)k_{it} &= a_{it-1} + \phi_1\left(\mu_{it} + e_{it}\right) + \phi_2\left(\mu_{it} + \sigma_v z_{it}\right) + \text{Const.} \end{aligned}$$

where the coefficients ψ, ϕ_1 and ϕ_2 are the same as in the i.i.d. case. In growth rates,

$$\begin{aligned} \Delta a_{it} &= \mu_{it} \\ \Delta p_{it} &= \frac{1}{1-\alpha}\left(a_{it-1} - a_{it-2}\right) + \frac{1}{1-\alpha}\psi\left(\mu_{it} + \sigma_v z_{it} - \mu_{it-1} - \sigma_v z_{it-1}\right) \\ &= \frac{1}{1-\alpha}\left(1-\psi\right)\mu_{it-1} + \frac{1}{1-\alpha}\psi\mu_{it} + \xi\psi\sigma_v\left(z_{it} - z_{it-1}\right) \\ \left(1-\alpha\right)\Delta k_{it} &= \left(a_{it-1} - a_{it-2}\right) + \left(\phi_1 + \phi_2\right)\left(\mu_{it} - \mu_{it-1}\right) + \phi_1\left(e_{it} - e_{it-1}\right) + \phi_2\sigma_v\left(z_{it} - z_{it-1}\right) \\ &= \left(1 - \phi_1 - \phi_2\right)\mu_{it-1} + \left(\phi_1 + \phi_2\right)\mu_{it} + \phi_1\left(e_{it} - e_{it-1}\right) + \phi_2\sigma_v\left(z_{it} - z_{it-1}\right) \end{aligned}$$

Second moments (of growth rates):

$$\begin{aligned} \sigma_p^2 &= \left(\frac{1}{1-\alpha}\right)^2\left(1-\psi\right)^2\sigma_\mu^2 + \left(\frac{1}{1-\alpha}\right)^2\psi^2\sigma_\mu^2 + 2\left(\frac{1}{1-\alpha}\right)^2\psi^2\sigma_v^2\sigma_z^2 \\ &= \left(\frac{1}{1-\alpha}\right)^2\sigma_\mu^2\left[1 - 2\psi + 2\psi^2(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2})\right] \qquad (26) \\ \sigma_k^2 &= \left(\frac{1}{1-\alpha}\right)^2\left[\left(1 - \phi_1 - \phi_2\right)^2\sigma_\mu^2 + \left(\phi_1 + \phi_2\right)\sigma_\mu^2 + 2\phi_1^2\sigma_e^2 + 2\phi_2^2\sigma_v^2\sigma_z^2\right] \qquad (27) \\ &= \left(\frac{1}{1-\alpha}\right)^2\left[\sigma_\mu^2 - 2\left(\sigma_\mu^2 - \mathbb{V}\right) + 2\sigma_\mu^2 - 4\mathbb{V} + 2\frac{\mathbb{V}^2}{\sigma_\mu^2} + 2\frac{\mathbb{V}^2}{\sigma_e^2} + 2\frac{\mathbb{V}^2}{\sigma_v^2\sigma_z^2}\right] \qquad (28) \\ &= \left(\frac{1}{1-\alpha}\right)^2\sigma_\mu^2 \qquad (29) \end{aligned}$$

Thus, the variance of investment is invariant to uncertainty ! This is because, with persis-

tence, undoing the effects of past forecast errors also contributes to investment volatility. When shocks are permanent, this exactly offsets the contemporaneous dampening that occurs due to imperfect information. Thus, with permanent shocks, uncertainty shifts the timing of the response of capital to fundamentals, but not the overall magnitude.

Covariances:

$$\begin{aligned}
\text{Cov}(p,a) &= \frac{1}{1-\alpha}\psi\sigma_\mu^2 \\
\text{Cov}(p,k) &= \left(\frac{1}{1-\alpha}\right)^2\left[(1-\psi)(1-\phi_1-\phi_2)\sigma_\mu^2+\psi(\phi_1+\phi_2)\sigma_\mu^2+2\psi\phi_2\sigma_v^2\sigma_z^2\right] \\
&= \left(\frac{1}{1-\alpha}\right)^2\left[(1-\psi-\phi_1-\phi_2)\sigma_\mu^2+2\psi(\phi_1+\phi_2)\sigma_\mu^2+2\psi\phi_2\sigma_v^2\sigma_z^2\right] \\
&= \left(\frac{1}{1-\alpha}\right)^2\left[(1-\psi-\phi_1-\phi_2)\sigma_\mu^2+2\psi\left(\sigma_\mu^2-\mathbb{V}\right)+2\psi\mathbb{V}\right] \\
&= \left(\frac{1}{1-\alpha}\right)^2\left(\mathbb{V}+\psi\sigma_\mu^2\right)
\end{aligned}$$

Directly,

$$\begin{aligned}
\text{Cov}(p,k)-\left(\frac{1}{1-\alpha}\right)\text{Cov}(p,a) &= \left(\frac{1}{1-\alpha}\right)^2\mathbb{V} \\
\rho_{pk}\sigma_p\sigma_k-\left(\frac{1}{1-\alpha}\right)\rho_{pa}\sigma_p\sigma_a &= \\
\rho_{pk}\sigma_p\sigma_u\left(\frac{1}{1-\alpha}\right)-\left(\frac{1}{1-\alpha}\right)\rho_{pa}\sigma_p\sigma_u &= \\
(\rho_{pk}-\rho_{pa})\left(\frac{1}{1-\alpha}\right)\sigma_p\sigma_\mu &=
\end{aligned}$$

Re-arranging yields the expression in the text.

$$\frac{\mathbb{V}}{\sigma_\mu^2}=\frac{(\rho_{pk}-\rho_{pa})\,\sigma_p}{\sigma_\mu}(1-\alpha)$$

Correlations:

$$\begin{aligned}\rho_{pk} &= \frac{\psi + \frac{\mathbb{V}}{\sigma_\mu^2}}{\sqrt{1 - 2\psi + 2\psi^2\left(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)}} \\ \rho_{pa} &= \frac{\psi}{\sqrt{1 - 2\psi + 2\psi^2\left(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)}} = \frac{1}{\sqrt{\left(\frac{1}{\psi} - 1\right)^2 + 1 + 2\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}}\end{aligned}$$

Then,

$$\frac{\rho_{pk}}{\rho_{pa}} = 1 + \frac{1}{\psi}\frac{\mathbb{V}}{\sigma_\mu^2} \qquad \Rightarrow \qquad \psi = \frac{\frac{\mathbb{V}}{\sigma_\mu^2}}{\frac{\rho_{pk}}{\rho_{pa}} - 1} = \frac{\rho_{pa}}{\eta}$$

Given ψ, we then use the expression for ρ_{pa} to solve for $\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}$:

$$\begin{aligned}\rho_{pa}^2 &= \frac{1}{\left(\frac{\eta}{\rho_{pa}} - 1\right)^2 + 1 + 2\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}} \\ \Rightarrow \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2} &= \frac{1}{2\rho_{pa}^2} - \frac{1}{2}\left(\frac{\eta}{\rho_{pa}} - 1\right)^2 - \frac{1}{2} \\ &= \frac{(1-\eta^2)}{2\rho_{pa}^2} + \frac{\eta}{\rho_{pa}} - 1\end{aligned}$$

Finally, combining this with the definition of $\psi \equiv \frac{\sigma_z^2+1}{\sigma_z^2+1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}$ allows us to disentangle σ_z^2 and σ_v^2.

A.5 Correlated signals

Here, we present alternative, more direct derivations of equations (22) and (25), which rely only on conditional normality and not on the correlation structure.

Transitory shocks. We start with the i.i.d. case. Then, p_{it} is an i.i.d. random variable, uncorrelated with past and future realizations of a_{it}.

$$\begin{aligned}\text{Cov}\left(p_{it} - p_{it-1}, a_{it} - a_{it-1}\right) &= \text{Cov}\left(p_{it}, a_{it}\right) + \text{Cov}\left(p_{it-1}, a_{it-1}\right) \\ &= 2\text{Cov}\left(p_{it}, a_{it}\right) = 2\text{Cov}\left(p_{it}, \mathbb{E}_{it}a_{it} + \varpi_{it}\right) \\ &= 2\text{Cov}\left(p_{it}, \mathbb{E}_{it}a_{it}\right) + 2\text{Cov}\left(p_{it}, \varpi_{it}\right) \\ &= 2\text{Cov}\left(p_{it}, \mathbb{E}_{it}a_{it}\right)\end{aligned}$$

where ϖ_{it} denotes the firm's forecast error, which is uncorrelated with firm information - in particular, with any element in the firm information set. Therefore, $\text{Cov}(p_{it}, \varpi_{it}) = 0$. Note that this is true independent of the correlation structure between p_{it} and the other elements of that information set. Now, we use $\text{Cov}(p_{it}, k_{it}) = \frac{1}{1-\alpha}\text{Cov}(p_{it}, \mathbb{E}_{it}a_{it})$ and divide both sides of the equation by $\sigma_p\sigma_a$ to get

$$\begin{aligned}
\frac{\text{Cov}\left(p_{it} - p_{it-1}, a_{it} - a_{it-1}\right)}{\sigma_p\sigma_a} &= \frac{2\text{Cov}\left(p_{it}, k_{it}\right)(1-\alpha)}{\sigma_p\sigma_a}\frac{\sigma_k}{\sigma_k} \\
&= \frac{\text{Cov}\left(p_{it} - p_{it-1}, k_{it} - k_{it-1}\right)}{\sigma_p\sigma_k}\frac{(1-\alpha)\sigma_k}{\sigma_a} \\
\Rightarrow \frac{\rho_{pa}}{\rho_{pk}} &= \frac{(1-\alpha)\sigma_k}{\sigma_a}
\end{aligned}$$

Since

$$\begin{aligned}
\sigma_k^2 &= \left(\frac{1}{1-\alpha}\right)^2 2\sigma^2\left(\mathbb{E}_{it}a_{it}\right) = \left(\frac{1}{1-\alpha}\right)^2 2\left(\sigma_\mu^2 - \mathbb{V}\right) \\
&= \left(\frac{1}{1-\alpha}\right)^2 2\sigma_\mu^2\left(1 - \frac{\mathbb{V}}{\sigma_\mu^2}\right) = \left(\frac{1}{1-\alpha}\right)^2 \sigma^2\left(a_{it} - a_{it-1}\right)\left(1 - \frac{\mathbb{V}}{\sigma_\mu^2}\right) \\
\Rightarrow \frac{\sigma_k}{\sigma_a} &= \frac{1}{1-\alpha}\sqrt{1 - \frac{\mathbb{V}}{\sigma_\mu^2}}
\end{aligned}$$

Combining yields (22).

Permanent shocks. Next, consider the case with $\rho = 1$. Investment now becomes

$$\begin{aligned}
(1-\alpha)\left(k_{it} - k_{it-1}\right) &= a_{it-1} + \mathbb{E}_{it}u_{it} - a_{it-2} + \mathbb{E}_{it-1}u_{it-1} \\
&= \mathbb{E}_{it}u_{it} + u_{it-1} - \mathbb{E}_{it-1}u_{it-1}
\end{aligned}$$

$$\begin{aligned}
p_{it} &= \frac{1}{1-\alpha}\left(a_{it-1} + \tilde{\mathbb{E}}_{it}u_{it}\right) \\
&= \frac{1}{1-\alpha}\left(a_{it-1} + \tilde{\psi}\mathbb{E}_{it}^p\left(u_{it}\right)\right)
\end{aligned}$$

where $\mathbb{E}_{it}^p\left(u_{it}\right)$ is the expectation conditional on p_{it} alone. Here, we make use of the fact that the marginal investor's expectation is proportional to $\mathbb{E}_{it}^p$. The constant of proportionality $\tilde{\psi}$ depends on the variance parameters, but, as we will see, will play no role in the determination

of $\mathbb{V}$. Stock returns are given by:

$$\begin{aligned}(1-\alpha)(p_{it}-p_{it-1}) &= a_{it-1}+\tilde{\psi}\mathbb{E}_{it}^{p}u_{it}-a_{it-2}-\tilde{\psi}\mathbb{E}_{it-1}^{p}u_{it-1}\\ &= u_{it-1}+\tilde{\psi}\mathbb{E}_{it}^{p}u_{it}-\tilde{\psi}\mathbb{E}_{it-1}^{p}u_{it-1}\\ &= \tilde{\psi}\mathbb{E}_{it}^{p}u_{it}+\left(1-\tilde{\psi}\right)u_{it-1}+\tilde{\psi}\left(u_{it-1}-\mathbb{E}_{it-1}^{p}u_{it-1}\right)\end{aligned}$$

Define

$$\omega_{it-1}\equiv\mathbb{E}_{it-1}u_{it-1}-\mathbb{E}_{it-1}^{p}u_{it-1}$$

Since ω_{it-1} is in the firm's information set, it must be orthogonal to the firm's forecast error $\varpi_{it-1}\equiv u_{it-1}-\mathbb{E}_{it-1}u_{it-1}$. We can use this to write the forecast error component in stock returns as the sum of two orthogonal components

$$\begin{aligned}u_{it-1}-\mathbb{E}_{it-1}^{p}u_{it-1} &= (u_{it-1}-\mathbb{E}_{it}u_{it-1})+\omega_{it-1}\\ &= \varpi_{it-1}+\omega_{it-1}\end{aligned}$$

Substituting into the expression for the growth rates, we get

$$\begin{aligned}(1-\alpha)(k_{it}-k_{it-1}) &= \mathbb{E}_{it}^{p}u_{it}+\omega_{it}+\varpi_{it-1}\\ (1-\alpha)(p_{it}-p_{it-1}) &= \tilde{\psi}\mathbb{E}_{it}^{p}u_{it}+\left(1-\tilde{\psi}\right)u_{it-1}+\tilde{\psi}\left(\varpi_{it-1}+\omega_{it-1}\right)\end{aligned}$$

Covariances

$$\begin{aligned}Cov(\Delta p,\Delta a) &= Cov(p_{it}-p_{it-1},a_{it}-a_{it-1})=\left(\frac{1}{1-\alpha}\right)\tilde{\psi}Var\left(\mathbb{E}_{it}^{p}u_{it}\right)\\ Cov(\Delta k,\Delta p) &= \left(\frac{1}{1-\alpha}\right)^{2}\tilde{\psi}Var\left(\mathbb{E}_{it}^{p}u_{it}\right)+\left(\frac{1}{1-\alpha}\right)^{2}\left(1-\tilde{\psi}\right)Cov\left(u_{it-1},\varpi_{it-1}\right)\\ &\quad+\left(\frac{1}{1-\alpha}\right)^{2}\tilde{\psi}Var\left(\varpi_{it-1}\right)\\ &= \left(\frac{1}{1-\alpha}\right)Cov(\Delta p,\Delta a)+\left(\frac{1}{1-\alpha}\right)^{2}\left(1-\tilde{\psi}\right)\mathbb{V}+\left(\frac{1}{1-\alpha}\right)^{2}\tilde{\psi}\mathbb{V}\\ &= \left(\frac{1}{1-\alpha}\right)Cov(\Delta p,\Delta a)+\left(\frac{1}{1-\alpha}\right)^{2}\mathbb{V}\end{aligned}\tag{30}$$

Which gives us

$$\begin{aligned}
\rho_{kp}\sigma_p\sigma_k &= \left(\frac{1}{1-\alpha}\right)\rho_{pa}\sigma_p\sigma_\mu + \left(\frac{1}{1-\alpha}\right)^2\mathbb{V} \\
\frac{1}{1-\alpha}\rho_{kp}\sigma_p\sigma_\mu &= \left(\frac{1}{1-\alpha}\right)\rho_{pa}\sigma_p\sigma_\mu + \left(\frac{1}{1-\alpha}\right)^2\mathbb{V} \\
\rho_{kp}\sigma_p\sigma_\mu &= \rho_{pa}\sigma_p\sigma_\mu + \frac{1}{1-\alpha}\mathbb{V}
\end{aligned}$$

Re-arranging yields equation (25).

A.6 Other distortions

Distortions affect the capital choice

$$k_{it} = \frac{(1+\gamma)\,\mathbb{E}_{it}a_{it} + \varepsilon_{it}}{1-\alpha}$$

but have only a second order effect on profits. Therefore, our approximations for stock prices are unaffected under both cases, i.e. under iid and permanent shocks. In other words, the only moments affected by distortions are $\mathrm{Cov}(p,k)$ and σ_k. With only correlated distortions, both these moments are scaled by $1+\gamma$. Uncorrelated distortions have no effect on $\mathrm{Cov}(p,k)$ but increase σ_k.

Transitory shocks. With correlated distortions, since both $\mathrm{Cov}(p,k)$ and σ_k are scaled up by the same factor, ρ_{pk} remains unchanged:

$$\begin{aligned}
\rho_{pk} &= \frac{\mathrm{Cov}\,(p,k)}{\sigma_p\sigma_k} = \frac{2\xi\psi\sigma_\mu^2\,(1+\gamma)}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2+\sigma_v^2\sigma_z^2\right)\left(\sigma_u^2-\mathbb{V}\right)(1+\gamma)^2}} \\
&= \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2+\sigma_v^2\sigma_z^2\right)\left(\sigma_u^2-\mathbb{V}\right)}} = \frac{1}{\sqrt{\left(1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right)}}
\end{aligned}$$

the same as before.

With uncorrelated distortions,

$$\begin{aligned}\rho_{pk} &= \frac{\mathrm{Cov}\,(p,k)}{\sigma_p\sigma_k} = \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2+\sigma_v^2\sigma_z^2\right)\left(\sigma_u^2+\sigma_\varepsilon^2-\mathbb{V}\right)}} \\ &= \frac{1}{\sqrt{\left(1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}+\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}\right)}} = \frac{\rho_{pa}}{\sqrt{1-\frac{\mathbb{V}}{\sigma_\mu^2}+\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}}}\end{aligned}$$

so that using (22) will yield an underestimate of $\frac{\mathbb{V}}{\sigma_\mu^2}$ by an amount $\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}$.

Permanent shocks. With correlated distortions, equation (30) becomes

$$\begin{aligned}\frac{Cov\,(\Delta k,\Delta p)}{1+\gamma} &= \left(\frac{1}{1-\alpha}\right)Cov\,(\Delta p,\Delta a)+\left(\frac{1}{1-\alpha}\right)^2\mathbb{V} \\ \frac{\rho_{pk}\sigma_p\sigma_k}{1+\gamma} &= \frac{1}{1-\alpha}\rho_{pa}\sigma_p\sigma_\mu+\left(\frac{1}{1-\alpha}\right)^2\mathbb{V} \\ \left(\frac{1}{1-\alpha}\right)\rho_{pk}\sigma_p\sigma_\mu &=\end{aligned}$$

yielding the same relationship between $\mathbb{V}$ and the other moments as before. In other words, using equation (25) leads us to the correct $\mathbb{V}$.

With uncorrelated distortions, we still have

$$\rho_{pk}\sigma_p\sigma_k = \left(\frac{1}{1-\alpha}\right)\rho_{pa}\sigma_p\sigma_\mu+\left(\frac{1}{1-\alpha}\right)^2\mathbb{V}$$

but now $\sigma_k = \frac{1}{1-\alpha}\sqrt{\sigma_\mu^2+2\sigma_\varepsilon^2}$. Substituting,

$$\rho_{pk}\sigma_p\sigma_\mu\sqrt{1+2\left(\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}\right)} = \rho_{pa}\sigma_p\sigma_\mu+\frac{1}{1-\alpha}\mathbb{V}$$

so that now, the formula $(\rho_{kp}\sigma_p-\rho_{pa}\sigma_p)\,\sigma_\mu$ underestimates $\frac{\mathbb{V}}{1-\alpha}$.

B Adjustment Costs

When firms are fully informed but subject to capital adjustment costs, the dynamic optimization problem of the firm is characterized by the value function:

$$V\left(\tilde{A}_{it}, K_{it-1}\right) = \max_{K_{it}, N_{it}} G\tilde{A}_{it}K_{it}^{\tilde{\alpha}} - K_{it} - \zeta K_{it-1}\left(\frac{K_{it}}{K_{it-1}} - 1\right)^2 + \beta \mathbb{E}V\left(\tilde{A}_{it+1}, (1-\delta)K_{it}\right)$$

To characterize G and $\tilde{A}_{it}$, we start with the firm's labor choice problem

$$\max_{N_{it}} \quad P_{it}Y_{it} - WN_{it} = \max_{N_{it}} \quad Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - WN_{it}$$

where α's are as defined in (2). Optimizing over N_{it} and substituting back into the objective gives

$$P_{it}Y_{it} - WN_{it} = (1-\alpha_2)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}} \tilde{A}_{it}K_{it}^{\tilde{\alpha}}$$

where $\tilde{\alpha}$ and $\tilde{A}_{it}$ are defined as in case 2 in the text. We can then solve for

$$G = (1-\alpha_2)\left[\left(\frac{\alpha_2}{W}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}}\right]^{\frac{1}{1-\alpha_2}}$$

Next, using the firm's labor optimality condition, labor market clearing implies

$$\left(\frac{\alpha_2}{W}\right)^{\frac{1}{1-\alpha_2}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}} \int \tilde{A}_{it}K_{it}^{\tilde{\alpha}} di = N$$

from which we can solve for

$$\left(\frac{\alpha_2}{W}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} = \left[\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}} \ di\right]^{\frac{1-\alpha_2}{\theta-1}(1-\theta\alpha_2)} N^{\frac{\theta}{\theta-1}\alpha_2(1-\alpha_2)}$$

and so

$$G = (1-\alpha_2)\left[\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}} \ di\right]^{\frac{1-\theta\alpha_2}{\theta-1}} N^{\frac{\theta}{\theta-1}\alpha_2}$$

Note that under full information, the production side of the economy is completely decoupled from stock markets, so we can now solve the model laid out above. Starting with a guess for the general equilibrium term G, we solve for the value functions (using a standard iterative procedure and a discretized grid for capital), simulate to obtain the steady state distributions and verify that our initial guess for G is consistent with that distribution. If not, we update the guess and iterate until convergence.

We then solve for stock prices, again using the conjecture that the informed investors follow a threshold rule. Proceeding exactly as in the baseline model, we can then derive the following functional equation for the price:

$$\begin{aligned} P\left(a_{it-1}, k_{it-1}, a_{it}, z_{it}\right) &= \int \pi\left(a_{it}, k_{it-1}\right) H\left(a_{it} | a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it}\right) \\ &\quad + \beta \int \bar{P}\left(a_{it}, k^*\left(a_{it}, k_{it-1}\right)\right) H\left(a_{it} | a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it}\right) \end{aligned}$$

The model is parameterized as described in the text. Table 11 reports the full set of target moments and parameter estimates. Notice that the first two columns are identical to those in Table 2 (case 2). σ_k^2 denotes the variance of investment rates, which is used to pin down the size of the adjustment cost ζ.

Table 11: Targets and Parameters - Adjustment Cost Model

	Target moments			Parameters		
	ρ_{pa}	σ_p^2	σ_k^2	ζ	σ_v	σ_z
US	0.18	0.23	0.10	0.45	0.37	4.25
China	0.06	0.14	0.15	0.18	1.00	4.00
India	0.08	0.23	0.10	0.52	0.69	6.36

C Data

We use annual data on firm-level production variables and stock market returns from Compustat North America (for the US) and Compustat Global (for China and India). For each country, we exclude duplicate observations (firms with multiple observations within a single year), firms not incorporated within that country, and firms not reporting in local currency. We build three year production periods as the average of firm sales and capital stock over non-overlapping 3-year horizons (i.e., $K_{2012} = \frac{K_{2010}+K_{2011}+K_{2012}}{3}$, and analogously for sales). We measure the capital stock using gross property, plant and equipment (PPEGT in Compustat terminology), defined as the "valuation of tangible fixed assets used in the production of revenue." We then calculate investment as the change in the firm's capital stock relative to the preceding period. We construct the firm fundamental a_{it} as the log of revenue less the relevant α (which depends on the case) multiplied by the log of the capital stock. Finally, we first-difference the investment and fundamental series to compute investment growth and changes in fundamentals.

Stock returns are constructed as the change in the firm's stock price over the three year period, adjusted for splits and dividend distributions. We follow the procedure outlined in the

Compustat manual. In Compustat terminology, and as in the text, using p_{it} as shorthand for log returns, returns for the US are computed as

$$p_{it} = \log\left(\frac{\text{PRCCM}_{it} * \text{TRFM}_{it}}{\text{AJEXM}_{it}}\right) - \log\left(\frac{\text{PRCCM}_{it-1} * \text{TRFM}_{it-1}}{\text{AJEXM}_{it-1}}\right)$$

where periods denote 3 year spans (i.e., returns for 2012 are calculated as the adjusted change in price between 2009 and 2012), PRCCM is the firm's stock price, and TRFM and AJEXM adjustment factors needed to translate prices to returns from the Compustat monthly securities file. Data are for the last trading day of the firm's fiscal year so that the timing lines up with the production variables just described. The calculation is analogous for China and India, with the small caveat that global securities data come daily, so that the Compustat variables are PRCCD, TRFD, and AJEXDI, where "D" denotes days. Again, the data are for the last trading day of the firm's fiscal year.

To extract the firm-specific variation in our variables, we regress each on a time fixed-effect and work with the residual. This eliminates the component of each series common to all firms in a time period and leaves only the idiosyncratic variation. As described in the text, we limit our sample to a single cross-section, namely 2012, and finally, we trim the 2% tails of each series. It is then straightforward to compute the target moments, i.e., σ_p^2, ρ_{pi}, and ρ_{pa}. As described in the text, we lag returns by one period, so that, e.g., ρ_{pi} is the correlation of 2006-09 returns with investment growth from 2009-12.

To estimate the parameters governing the evolution of firm fundamentals, i.e., the persistence ρ and variance of the innovations σ_μ^2, we perform the autoregression implied by (1). Here we use annual observations on a_{it} at a 3-year frequency in order to simplify issues of time aggregation. We estimate the process using our data from 2012 and 2009. We include a time fixed-effect in order to isolate the idiosyncratic component of the innovations in a_{it}. Differences in firm fiscal years means that different firms within the same calendar year are reporting data over different time periods, and so the time fixed-effect incorporates both the reporting year and month. The results from this regression deliver an estimate for ρ and σ_μ^2, from which we trim the 1% tails.

Our leverage adjustment is as follows: we assume that claims to firm profits are sold to investors in the form of both debt and equity in a constant proportion (within each country). This implies that the payoff from an equity claim is $S_{it} = V_{it} - D_{it-1}$, where V_{it} is the value of the unlevered firm and $D_{it-1} = d\mathbb{E}_t[V_{it}]$, where $d \in (0,1)$ represents the share of expected firm value in the hands of debt-holders. In other words, firm value is allocated to investors as a debt claim that pays off a constant fraction of its ex-ante expected value and as a residual claim to equity holders. The change in value of an equity claim is then equal to $\Delta S_{it} = \Delta V_{it}$ and dividing both

sides by the ex-ante expected value of the claim (i.e., the price) $\overline{S} = \overline{V} - d\overline{V}$, where $\overline{V} = \mathbb{E}_t[V_{it}]$, gives returns as $\frac{\Delta S_{it}}{\overline{S}} = \frac{\Delta V_{it}}{(1-d)\overline{V}}$. Taking logs and computing variances shows $\sigma^2_{v_{it}} = (1-d)^2 \sigma^2_{s_{it}}$, i.e., the volatility in (unlevered) firm value is a fraction $(1-d)^2$ of the volatility in (levered) equity returns. To assign values to d in each country, we examine the debt-asset and debt-equity ratios of the set of firms in Compustat over the period 2006-2009. Because these vary to some degree from year to year and depend to some extent on the precise approach taken (i.e., whether we use debt-assets or debt-equity and whether we compute average ratios or totals), we simply take the approximate midpoints of the ranges for each country, which are about 0.30 for the US and India and 0.16 for China, leading to adjustment factors of about 0.5 and 0.7, respectively.