Estimating Fiscal Limits: The Case of Greece *

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Abstract

This paper uses Bayesian methods to estimate the 'fiscal limit' distribution for Greece implied by a rational expectations framework. We build a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic 'fiscal limit,' and sovereign default. A fiscal limit measures the debt level beyond which the government is no longer willing to finance, causing a partial default to occur. The fiscal policy specification takes into account government spending, lump-sum transfers, and distortionary taxation. Using the particle filter to perform likelihood-based inference, we estimate the full nonlinear model with post-EMU data until 2010Q4. We find that the probability of default on Greek debt remained close to zero from 2001 until 2009, and then rose sharply to the range of 5% to 10% by 2010Q4. The model also predicts a probability of default between 60-80% by 2011Q4, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the real interest rate in Greece in 2011 is within forecast bands of our rational expectations model. Finally, model comparisons based on Bayes factors strongly favor the nonlinear model specification with an endogenous probability of default over a linearized specification without default.

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1. Introduction

During the past five years, there has been a concern over the fiscal positions of several Eurozone nations, as evidenced by the rapid increase in long-term interest rate spreads of several countries' bonds against German bonds. For instance, the spread between Greek bonds and German bonds rose from 2.35 percentage points in 2009 to nearly 30 points by the end of 2011.

Such marked increases in spreads have stimulated a debate over whether financial markets have mispriced default risks in the Eurozone periphery countries before and/or after the start of the crisis. Aizenman, Hutchison, and Jinjarak (2012) find a dynamic panel model can hardly predict the magnitude of sovereign risk premia in the Eurozone periphery countries during the crisis, either in or out of sample. They conjecture that the market has either mispriced the risk, or has priced based on future, not current, fundamentals. Using a similar approach, De Grauwe and Ji (2012) take a stronger stand and argue that the financial markets underpriced the sovereign risk from 2001 to 2008 and have been overpricing the risk since then.

One issue with these approaches is that the reduced-form panel regressions lack the structure to extensively explore how macroeconomic fundamentals explain interest rate dynamics. This paper introduces a different approach to contribute to the pricing debate by estimating a rational expectations model that allows for sovereign default. Using Bayesian method, the structural model is estimated for Greece during the post-EMU period until the end of 2010.

Two key findings emerge. First, the rapid deterioration of confidence in Greek debt over 2011 is consistent with behavior from a simple rational expectations model. Using the estimated structural model, we find that the model-implied default probability remained close to zero from 2001 until 2009, and then rose sharply to the range of 5% to 10% by the fourth quarter of 2010. The model also predicts a probability of default between 60% to 80% by the end of 2011, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the real interest rate observed in Greece over 2011 is well within the forecast bands of our rational expectations model. These results suggest that Greek debt was not mispriced in 2011, as the interest rate path can be accounted for by macroeconomic fundamentals.

Second, model comparisons based on Bayes factors strongly favor the nonlinear model specification with an endogenous probability of default over a log-linearized model specification that does not allow for default. In addition, some parameters are more precisely estimated with the nonlinear specification.

We consider a closed economy in which the government finances transfers and expenditures by collecting distortionary income taxes and issuing bonds.[1] The bond contract is not enforceable and depends on the maximum level of debt that the government is politically able to service, a so-called 'fiscal limit.' Given the stochastic environment, we do not assume that the fiscal limit is a deterministic point, but rather follows a distribution. Previous studies have shown that economic models can give rise endogenously to such a distribution through dynamic Laffer curves and the political inability to cut public expenditures (see Bi (2012), and Juessen, Linnemann, and Schabert (2011)). In the case of Greece, the protests against austerity measures in 2010 and 2011 suggest the relevance of political considerations. Instead of estimating a structural political economy model that captures such underlying political factors, we follow the approach of Davig, Leeper, and Walker (2010) and take the distribution of the fiscal limit as exogenously given. The parameters associated with the distribution are estimated, so that the fiscal limit is data-driven in our framework.

We do not model default as a strategic decision made by a benevolent government. Instead, we appeal to political frictions that make the default decision intrinsically uncertain. Default is possible at any point on the distribution of the fiscal limit. Randomness inherent in the politically-determined default decision is modeled as a random draw of the 'effective fiscal limit' from the distribution. Each period, the government reneges on a fraction of its debt if the level of government debt surpasses the effective fiscal limit; otherwise, it fully honors the debt obligations.

Since the economy switches between the default and the no-default regimes endogenously depending upon the level of government debt, the model cannot be solved using a first-order approximation. Instead, it is solved using the monotone mapping method (see Davig (2004) and Coleman (1991)) and estimated using Bayesian inference methods and a sequential Monte Carlo approximation of the likelihood (see Fernandez-Villaverde and Rubio-Ramirez (2007), Doh (2011), and Amisano and Tristani (2010)).

2. Contacts with the Literature

An important branch of the sovereign default literature builds on Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008). In those models, sovereign default is the optimal decision made by a benevolent social planner in response to unlucky external shocks. Our approach abstracts from the government's strategic incentives to default; instead, it lets

[1] Ceteris paribus, the assumption that the economy is open and all debt is held by foreigners raises the observed risk premium relative to the closed economy environment, as foreigners do not experience the negative wealth effects of debt and, in turn, have less incentive to hold debt. Therefore, the estimates from our closed economy framework can be thought as the *lower* bound to estimates of the fiscal limit. See section 4.2.3 for more details.

the data determine the distribution of the fiscal limit, which can be driven by *both* bad policy and bad luck.

A related branch of the empirical literature focuses on estimating first order approximations of small, open economy DSGE models to study real business cycle properties. Examples include Garcia-Cicco, Pancrazi, and Uribe (2010) and Guerron-Quintana (forthcoming). In contrast, we estimate a full, nonlinear model that incorporates a richer fiscal policy specification than often is encountered in this literature to study properties related to default.[2]

This paper is also related to a large empirical literature that studies the determinants of sovereign default risk premia through reduced-form panel regressions. Examples include Alesina, De Broeck, Prati, and Tabellini (1992), Bernoth, von Hagen, and Schuknecht (2006), Aizenman, Hutchison, and Jinjarak (2012), and De Grauwe and Ji (2012). This literature has not fully agreed upon the relative importance of different factors in determining risk premia, suggesting there is an importance of country-specific macroeconomic fundamentals in explaining sovereign risk premia. In addition, linear regressions that are often used in this line of work cannot capture the strong nonlinearity in sovereign risk premia that rational expectations models predict, see Arellano (2008) and Bi (2012) among others.[3] Nonlinearity may be particularly relevant in a crisis, such as the one the Eurozone is facing now.

Using a VAR model based on rolling-window estimation, Polito and Wickens (2012) compute the time-varying forecasts of the debt-to-GDP ratio and their volatility for the United States, and then calculate the time-varying default probabilities for different ad-hoc debt limits. Related work by Ostry, Ghosh, Kim, and Qureshi (2010) estimate historical fiscal responses to construct debt limits which are backward-looking by construction.

This paper builds upon our earlier work of Bi and Traum (2012), which provides a coherent framework for estimating forward-looking fiscal limits, but does not examine the extent to which a rational expectations model can explain the interest rates surges on sovereign debt.

3. Model

Following Bi (2012), our model is a closed economy with linear production technology, whereby output depends on the level of productivity (A_t) and the labor supply (n_t). Private consumption (c_t) and government purchases (g_t) satisfy the aggregate resource constraint,

$$c_t + g_t = A_t n_t. \tag{1}$$

[2]Our modeling of private behavior is simpler than the frameworks often found in this literature, which is a trade-off made to ensure the full nonlinear estimation is computationally feasible.

[3]Some papers in this literature allows weak nonlinearity by including quadratic terms.

The deviation of technological productivity (A_t) from its steady state value (A) follows an $AR(1)$ process,

$$A_t - A = \rho^A(A_{t-1} - A) + \varepsilon_t^A \qquad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2). \tag{2}$$

3.1 Government

The government finances lump-sum transfers to households (z_t) and exogenous, unproductive purchases by levying a tax (τ_t) on labor income and issuing one-period bonds (b_t). We denote q_t as the price of the bond in units of consumption at time t. For each unit of the bond, the government promises to pay the household one unit of consumption in the next period. However, the bond contract is not enforceable. At time t, the government may partially default on its liability (b_{t-1}) by a fraction of Δ_t, with the post-default liability being denoted as b_t^d,

$$\tau_t A_t n_t + b_t q_t = \underbrace{(1 - \Delta_t)b_{t-1}}_{b_t^d} + g_t + z_t. \tag{3}$$

The default scheme at each period depends on an effective fiscal limit (s_t^*), specified in terms of the debt-to-GDP ratio. If current outstanding debt obligations, relative to GDP, are below the effective fiscal limit, then the government fully repays its liabilities; otherwise, the government partially defaults on its obligations by a fixed share of δ. The amount of unpaid bonds in any period (Δ_t), which we call the default rate, is summarized by

$$\Delta_t = \begin{cases} 0 & \text{if } s_{t-1} < s_t^* \\ \delta & \text{if } s_{t-1} \geq s_t^* \end{cases}$$

where $s_{t-1} = b_{t-1}/y_{t-1}$. The effective fiscal limit (s_t^*) is stochastic and drawn from an exogenous distribution, $s_t^* \sim \mathcal{S}^*$. Bi (2012) shows that a similar rational expectations model can give rise to an endogenous distribution of the fiscal limit that is strongly nonlinear — once the default probability begins to rise, it does so rapidly. To capture this strong nonlinearity, we follow Davig, Leeper, and Walker (2010) and model the cumulative density function of the fiscal limit distribution as a logistical function with parameters η_1 and η_2 dictating its shape,[4]

$$p_{t-1} \equiv P(s_{t-1} \geq s_t^*) = \frac{\exp(\eta_1 + \eta_2 s_{t-1})}{1 + \exp(\eta_1 + \eta_2 s_{t-1})}, \tag{4}$$

[4]Davig, Leeper, and Walker (2010) specify the fiscal limit in terms of a tax rate, while we specify the fiscal limit in terms of the debt-to-GDP ratio.

where p_{t-1} is the default probability associated with the debt-to-GDP ratio of s_{t-1}. The higher the government liability, the higher the default probability. The distribution is uniquely determined by the η_1 and η_2 parameters. As elaborated in section 4.2.1, these parameters are inferred from the estimation, so that the associated distribution reflects the economic and political fiscal limit implied by the data. With the estimated distribution, we can compute the default probability for Greek government debt.

We assume that the government follows simple fiscal rules: it may raise the income tax rate or reduce government spending when the level of debt increases:

$$\tau_t = \underbrace{(1-\rho^\tau)\tau + \rho^\tau \tau_{t-1} + \varepsilon_t^\tau}_{u_t^\tau} + \gamma^\tau \left(b_t^d - b\right) \qquad \gamma^\tau > 0, \quad \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2) \tag{5}$$

$$g_t = \underbrace{(1-\rho^g)g + \rho^g g_{t-1} + \varepsilon_t^g}_{u_t^g} - \gamma^g \left(b_t^d - b\right) \qquad \gamma^g > 0, \quad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2) \tag{6}$$

$AR(1)$ components are denoted as u_t^τ and u_t^g, and x denoting the steady state level of any variables x_t. γ^τ and γ^g are the fiscal adjustment parameters. A larger γ^τ or γ^g implies that the government is more willing to retire debt by raising the tax rate or cutting government spending.

The non-distortionary transfers are modeled as a residual in the government budget constraint, exogenously determined by an AR(1) process,

$$z_t - z = \rho^z (z_{t-1} - z) + \varepsilon_t^z \qquad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2). \tag{7}$$

Since transfers are not included as an observable in our estimation, they can be thought of as a residual capturing all movements in government debt that are not explained by the model.

3.2 Household

With access to the sovereign bond market, a representative household chooses consumption (c_t), hours worked (n_t), and bond purchases (b_t) by solving,

$$\max \qquad E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t - h\bar{c}_{t-1}\right) + \phi \log(1-n_t)\right) \tag{8}$$

$$s.t. \qquad A_t n_t (1-\tau_t) + z_t - c_t = b_t q_t - (1-\Delta_t) b_{t-1} \tag{9}$$

where β is the discount factor. The household's utility for consumption is relative to a habit stock that is given by a fraction of aggregate consumption from the previous period, $h\bar{c}_{t-1}$

with $h \in [0, 1]$.

The household's first-order condition requires the marginal rate of substitution between consumption and leisure equates to the after-tax wage. The bond price reflects the household's expectation about the probability and magnitude of sovereign default in the next period.

$$\phi \frac{c_t - h\bar{c}_{t-1}}{1 - n_t} = A_t(1 - \tau_t) \tag{10}$$

$$q_t = \beta E_t\Big((1 - \Delta_{t+1})\frac{c_t - h\bar{c}_{t-1}}{c_{t+1} - h\bar{c}_t}\Big). \tag{11}$$

The optimal solution to the household's maximization problem must also satisfy the transversality condition,

$$\lim_{j \to \infty} E_t \beta^{j+1} \frac{u_c(t + j + 1)}{u_c(t)}(1 - \Delta_{t+j+1})b_{t+j} = 0. \tag{12}$$

3.3 Model Solution

The complete model consists of a system of nonlinear equations, including the household's optimization conditions, household and government budget constraints, specifications of fiscal policy, the default scheme, shock processes, and the transversality condition. The solution, based on Coleman (1991) and Davig (2004), conjectures candidate decision rules that reduce the equilibrium conditions to a set of expectation first-order difference equations.

At time t, the state vector (ψ_t) includes $(b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$, and the decision rule of the bond price can be written as $q_t = f^q(\psi_t)$. The core equilibrium conditions are

$$q_t = \frac{b_t^d + z_t + g_t - \tau_t A_t n_t}{b_t} \tag{13}$$

$$q_t = \beta(c_t - hc_{t-1})E_t \frac{1 - \Delta_{t+1}}{c_{t+1} - hc_t}. \tag{14}$$

The government bond demand equation, (13), is derived from the government budget constraint, while the bond supply equation, (13), is from the household's first-order condition. Appendix A discusses the solution procedure in detail.

4. Estimation

The model is estimated for Greece over the post-EMU period, 2001Q1-2010Q4, as interest rates during the pre-Euro period are susceptible to exchange rate risk from which our model abstracts. Five observables are used for the estimation: real output, the government spending-to-GDP ratio, the tax revenue-to-GDP ratio, the government debt-to-GDP ratio, and the 10-year sovereign real interest rate. For the estimation, we use percentage deviations of each observable from its mean value over the sample. Figure 3 depicts the data used for estimation. Appendix B.1 provides a detailed description of the data.

4.1 Methodology

We estimate the model using Bayesian methods. The equilibrium system is written in the nonlinear state-space form, linking observables v_t to model variables x_t:

$$x_t = f(x_{t-1}, \epsilon_t, \theta) \tag{15}$$

$$v_t = Ax_t + \xi_t, \tag{16}$$

where θ denotes model parameters and ξ_t is a vector of measurement errors distributed $N(0, \Sigma)$. We assume that Σ is a diagonal matrix and calibrate the standard deviation of each measurement error to be 20% of the standard deviation of the corresponding observable variable.[5]

We use a particle filter to approximate the likelihood function. For a given sequence of observations up to time t, $v^t = [v_1, ..., v_t]$, the particle filter approximates the density $p(x_t|v^t, \theta)$ by applying a law of large numbers to a series of simulations using a swarm of particles x_t^i $(i = 1, ..., N)$, see appendix B.2 for more details. The particle filter is applicable for nonlinear and non-Gaussian distributions, and it is increasingly used to estimate nonlinear DSGE models, to which class our model belongs. Recent examples include An and Schorfheide (2007), Fernandez-Villaverde and Rubio-Ramirez (2007), Amisano and Tristani (2010), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), and Doh (2011).

We combine the likelihood $L(\theta|v^T)$ with a prior density $p(\theta)$ to obtain the posterior kernel, which is proportional to the posterior density. We assume that parameters are independent a priori. However, we discard any prior draws that do not deliver a unique rational expectations equilibrium, as we restrict the analysis to the determinacy parameter subspace.[6] We construct the posterior distribution of the parameters using the random walk Metropolis-Hastings algorithm, see appendix B.3 for more details. In each estimation, we sample 500,000 draws from the posterior distribution and discard the first 300,000 draws.[7] The likelihood is

[5] Estimating measurement errors provides complications with nonlinear estimation techniques. See Doh (2011) for more discussion of the role of measurement error in nonlinear DSGE model estimation.

[6] A technical appendix of the authors provides more discussion on this point. Assuming a 30% annualized default rate, only 0.38% of the prior distribution falls outside the determinacy region.

[7] We use Fortran MPI code compiled in Intel Visual Fortran for the estimation. We use the computer

computed using 60,000 particles, and posterior analysis is conducted using every 25th draw from the chains.

4.2 Prior Distributions

As shown in table C, we impose dogmatic priors over some parameters. The discount rate is 0.99, so that the deterministic net interest rate is 1%.[8] We calibrate the household's leisure preference parameter ϕ such that a household spends 25% of its time working at the steady state. We calibrate the deterministic debt-to-GDP ratio, government spending-to-GDP ratio, and tax rate to the mean values of the data sample.

The prior for habit persistence h is similar to those in the linear DSGE estimation literature, for instance Smets and Wouters (2007). For the remaining parameters, we first use ordinary least squares and estimate an AR(1) process for GDP and processes for government spending, the tax rate, and transfers given by equations (5)-(7).[9] The results are used as general guidance for the parameter space of the shocks' persistence and standard deviations.

For the government spending and the tax adjustment parameters, we form priors for the long-run responses in terms of percentage deviations from the steady state, that is

$$\gamma^{g,L} \equiv \frac{\bar{b}}{\bar{g}} \frac{\gamma^g}{(1-\rho^g)}, \quad \gamma^{t,L} \equiv \frac{\bar{b}}{\bar{\tau}} \frac{\gamma^\tau}{(1-\rho^\tau)}$$

These values are more comparable to estimates in the literature. Since determinacy is sensitive to the combination of the $\gamma^{\tau,L}$ and $\gamma^{g,L}$ parameters, we restrict the lower bound of the $\gamma^{\tau,L}$ ($\gamma^{g,L}$) prior to a value that ensures determinacy when only $\gamma^{\tau,L}$ ($\gamma^{g,L}$) finances debt.

For the standard deviations of shocks, we form priors relative to relevant steady-state variables: $\sigma_{k,p} \equiv \sigma_k / \bar{J}$ for $J = \{A, g, \tau, z\}$ and $k = \{a, g, \tau, z\}$. This gives standard deviations as percentage deviations, which provides more intuitive comparisons across values.

4.2.1 Fiscal Limit

As discussed in section 3.1, the fiscal limit distribution, given by equation (4), is defined by the η_1 and η_2 parameters. A property of the logistic function is that for any given two points on the distribution, $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$, the parameters η_1 and η_2 can be uniquely determined

server system at the Bank of Canada, each CPU of which uses Xeon CPU X5680 at 3.33GHz and has 23 processors with 64G RAM. One evaluation using the particle filter takes 5 seconds. These computational constraints limit the number of draws from the Metropolis-Hastings algorithm.

[8]The mean of our data is 0.8% for Greece.

[9]We back out the model-consistent tax rate and transfers series implied by our observables for this exercise.

by

$$\eta_2 = \frac{1}{\tilde{s} - \hat{s}} \log\left(\frac{\tilde{p}}{\hat{p}}\frac{1-\hat{p}}{1-\tilde{p}}\right), \quad \eta_1 \;=\; \log\frac{\tilde{p}}{1-\tilde{p}} - \eta_2\tilde{s}. \tag{17}$$

For a given default rate δ, $\tilde{p}$ and $\hat{p}$ represent the default probabilities at the debt-to-GDP ratios of $\tilde{s}$ and $\hat{s}$ respectively. Since they provide a more intuitive description about the fiscal limit distribution than η_1 and η_2, we work directly with $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$.

We choose $\tilde{p} = 0.3$ and $\hat{p} = 0.999$. Unfortunately, since defaults are not observed in our data sample, the data is unlikely to be informative about the upper bound of the fiscal limit distribution. In order to estimate $\tilde{s}$, we fix the difference between $\tilde{s}$ and $\hat{s}$ to be 60% of steady-state output, which captures the observation that once risk premia begin to rise, they do so rapidly.[10] This assumption fixes the shape of the fiscal limit distribution, while the estimate of $\tilde{s}$ determines the intercept of the distribution. Figure 1 illustrates this concept by plotting the fiscal limit distributions that are associated with various $\tilde{s}$ values. Given the lack of guidance for the parameter $\tilde{s}$, we adopt a diffuse uniform prior over the interval 1.4 to 1.8, implying that the debt level associated with a 30% probability of default ranges from 140 to 180% of GDP.[11] The red dashed line of figure 1 depicts the fiscal limit distribution associated with the lower bound of $\tilde{s}$, while the black dotted line shows the distribution associated with the upper bound of $\tilde{s}$.

4.2.2 δ Identification

To our knowledge, aside from Bi and Traum (2012), this paper is the only attempt to estimate a DSGE model of sovereign default. Thus, prior to estimating the model with real data, we performed several estimations with simulated data.[12] Unfortunately, the results revealed that we cannot jointly identify the default rate (δ) and the fiscal limit parameter ($\tilde{s}$) when the data exclude observed defaults, since various combinations of δ and $\tilde{s}$ are consistent with the same risk premium. Given this limitation, we estimate our model for two different calibrations: δ fixed at 0.05 and 0.075. These calibrations imply annualized rates of default (δ^A) of 20% and 30% respectively, which falls within the range of actual default rates in emerging market economies over the period 1983 to 2005, as documented by Bi (2012).

[10]The difference of 60% of output, albeit ad-hoc, should not change the key estimation results as the data are unlikely to be informative about the upper bound of distribution.

[11]Posterior estimates are robust to raising the upper bound of $\tilde{s}$ (results are available from the authors). Decreasing the lower bound of $\tilde{s}$ dramatically increases the parameter region that falls outside of determinacy, as it increases the likelihood of the government encountering a situation where it repeatedly defaults on debt.

[12]The results are available in a technical appendix from the authors.

4.2.3 Modeling Alternatives

We assume a closed economy model for the estimation. In contrast, the literature on sovereign default in emerging markets often utilizes an open economy framework where all sovereign debt is held by foreigners, see Eaton and Gersovitz (1981), and Arellano (2008).

In the closed economy, domestic households have incentive to hold debt because of the negative wealth effects of deficit-financed fiscal policies. Such policies cause households to increase their current bond holdings (i.e. savings), as they know that future taxes will increase to finance the accumulated debt. In contrast, foreigners do not face the same negative wealth effects and, therefore, the interest rate reflects only the probability of default. Ceteris paribus, in our model the assumption that the economy is open and all debt is held by foreigners raises the observed risk premium relative to the closed economy environment. Thus, estimates from our closed economy framework can be thought as the *lower* bound to estimates of the fiscal limit associated with certain probabilities of default.

4.3 Posterior Estimates

Figure 2 plots the prior and posterior distributions for parameters when the annualized default rate is set to 30%. Table 2 compares the medians and 90% credible intervals of the posterior distributions estimated under both default rate specifications. For comparison, the means and 90% intervals from the priors are also listed. The data appear informative for all of the parameters, as the 90% credible intervals are smaller than those from the prior distributions.

The estimates of $\tilde{s}$ vary slightly across the two default rate specifications. If the partial default rate, δ^A, is 30%, the debt-to-GDP ratio that is associated with a 30% probability of default is between 1.53-1.58, with the median being 1.56. If the partial default rate is 20%, the debt-to-GDP ratio that is associated with a 30% probability of default is between 1.5-1.55, with the median being 1.52. The lower $\tilde{s}$ estimates for a lower δ^A calibration are consistent with theory. The model tries to match the interest rate data through the values of δ^A and $\tilde{s}$. For higher values of δ^A, agents expect to lose more of the face-value of debt following a default, and therefore households demand a higher interest rate to compensate for this risk. For the given risk premium implied by the data, a slightly higher $\tilde{s}$ is needed to offset a higher δ^A value.

For comparison, we also list the estimates implied by a log-linearized version of our model in which the government is not allowed to default. The system of equations for the log-linearized model is listed in appendix C. We use the Kalman filter to calculate the likelihood function and initialize the Metropolis-Hastings algorithm using the posterior mode and inverse Hessian at the posterior mode. The estimates from the linear model suggest that the data is not informative about $\gamma^{g,L}$ and σ^z, as the 90% credible interval from the posterior distribution mirrors that from the prior distribution, shown in table (2). $\gamma^{\tau,L}$ is also estimated less precisely than in the nonlinear specifications. It appears that allowing default in the standard RBC model may help to identify the fiscal policy responses in Greece. More formal comparisons of the linear and nonlinear model specifications are provided in the next section.

5. Analysis

5.1 Model Fit

To examine how well the model fits the data, we compute smoothed estimates of model variables using the sequential monte carlo approximation of the forward-backward smoothing recursion, see Doh (2011) for a detailed explanation of the procedure. Figure 3 compares the smoothed values from the nonlinear model with $\delta^A = 0.3$ and from the linearized model without default to the observable variables. For each specification, the fitted values are computed using the corresponding posterior median. The fit for most variables is quite accurate, with the real interest rate being the least precise.

We also compute smoothed estimates of the measurement errors $E(\xi_t|v^T, \theta)$ and report their mean absolute values and relative standard deviations in table 3. For the estimation, the standard deviation of each measurement error was fixed to be 20% of the standard deviation of the respective observable variable. For most observables, the actual estimated relative standard deviation is less than 20%, suggesting that the measurement error did not introduce many constraints for the model fit. The exception is the measurement error for the real interest rate in the nonlinear model and for output in the linear model. Table 3 also shows that mean absolute values of measurement error are close to zero.

Given that the different model specifications imply similar smoothed estimates, we perform posterior odds comparisons to determine which model is favored by the data. Bayes factors are used to evaluate the relative model fit of the linear and two nonlinear specifications. Table 4 presents the results. Bayes factors are based on log-marginal data densities calculated using Geweke's (1999) modified harmonic mean estimator with a truncation parameter of 0.5. The results demonstrate that the data strongly prefer the nonlinear model specifications over the linearized framework. In addition, it appears the data cannot distinguish between the two nonlinear models with alternative calibrated default rates, as the log Bayes factor is close to zero.

5.2 Default Probability and Interest Rate Dynamics

In this section, we use the estimated structural estimates to evaluate the historical probability of default in Greece and explore the model's ability to forecast the fiscal deterioration in Greece since 2010Q4, which is the end of our estimation period.

5.2.1 Default Risk

Figure 4 depicts model-implied historical sovereign default probabilities for Greece, based upon the estimated fiscal limit distribution with an annualized default rate of 30%, $\delta^A = 0.3$. Solid lines show the median and 90% posterior interval for the probability of default, calculated using the actual debt-to-GDP ratios from our debt and output observables for the estimated sample period.[13] Figure 4 shows that Greek debt had virtually zero probability of default from 2001 until 2009. Starting in 2009, the probability of default rose steadily, and ranged from 5-10% by the end of the estimated period 2010Q4, reflecting the deterioration in confidence in Greek debt in 2010.

In addition, the dashed lines provide the median and 90% posterior interval for the model-implied default probabilities for the out-of-sample debt-to-GDP ratios observed in 2011. The probabilities increase dramatically, ranging from 60-80% by 2011Q4. The striking build-up reflects the unsustainability of the Greek fiscal position and suggests imminent default. The predictions are consistent with the debt restructuring arrangements that took place at the beginning of 2012, which can be viewed as an effective default on Greek debt.

5.2.2 Out-of-Sample Interest Rate Forecasts

Over the course of 2011, the long-term nominal interest rate in the secondary market for Greek government bonds rose from 9.1 percentage points in December 2010 to 21.2 in December 2011 based on BIS data, and to 31.2 based on Bloomberg data.[14] In this section, we examine the ability of the estimated model to predict this sharp increase in the Greek interest rate.

To examine this issue, we use the posterior median estimates with an annualized default rate of 30% to simulate 10,000 time series for four quarters, starting from the fitted values for model variables in 2010Q4. This gives a distribution for the forecasted real interest rate in 2011. Figure 5 displays the median (blue, circled line) and 90% interval (blue, dashed lines)

[13] For the estimation, we use data in terms of percentage deviations from the sample average. In contrast, we need level variables to back out model-implied probabilities of default. For model consistency, we convert the percentage deviations of the data to level variables using the steady states of model variables.

[14] Quarterly averages of Bloomberg and BIS data differ only in 2011Q3 and 2011Q4, dues to differences in pricing such as the PSI introduction.

of these interest rate forecasts. The figure also plots the observed real interest rate during the same period (black solid line for BIS data and dotted-dashed red line for Bloomberg data).[15]

Figure 5 shows that the surge in the real interest rate in Greece is well within forecast bands of our rational expectations model. The fact that model forecasts can be consistent with the 2011 interest rate path suggests that the actual interest rate surge in 2011 can be explained by macroeconomic fundamentals in a rational expectation framework.

The model-implied forecast band for 2011 ranges dramatically from 6 to 35 percentage points. One might be tempted to conclude that the model forecast band is too large to be of much use. To explore this issue, we repeat our forecasting exercise starting from the fitted values for model variables in 2006Q4. That is, we calculate the model-implied distribution for the interest rate path in 2007, assuming that only information through 2006 was known. Figure 6 plots the median (blue, circled line) and 90% interval (blue, dashed lines) of the interest rate forecasts, along with the observed interest rate (black solid line). Again, the actual path is well within the forecast band, and the band, ranging a bit over 2 percentage points, is much tighter than the 2011 band. This is because macroeconomic fundamentals were more stable in 2006Q4 than in 2010Q4, as reflected by the virtually zero probability of default in 2006Q4. Thus, while forecast bands appear tight in good times, large forecast bands may emerge at a turning point from a deteriorating fiscal position, which is inherent in the nonlinear model.

The results suggest that the surge in the Greek interest rate premium and the rapid deterioration of confidence in Greek debt in 2011 are consistent with behaviors from a simple rational expectations model, and that Greek debt was not mispriced in 2011.

6. Conclusion

This paper uses Bayesian methods to estimate the fiscal limit distribution and the associated sovereign default probability for Greece. We build a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic fiscal limit, and sovereign default risk. The fiscal policy specification takes into account government spending, lump-sum transfers, and distortionary taxation. We model the fiscal limit distribution with a logistical function, which illustrates the market's belief about the government's ability and willingness to service its debt at various debt levels.

Using the particle filter to perform likelihood-based inference, we estimate the full nonlinear model with post-EMU data. We find that the probability of default on Greek debt

[15] We constructed the real interest rate in 2011 using the same procedure documented in appendix B.1.

remained close to zero from 2001 until 2009, and then rose sharply to the range of 5% to 10% by the fourth quarter of 2010. The model also predicts a probability of default between 60% to 80% by the end of 2011, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the Greek real interest rate in 2011 is within the forecast bands of our rational expectations model. The results suggest that Greek debt was not mispriced in 2011, as the interest rate path can be accounted for by macroeconomic fundamentals. Finally, model comparisons based on Bayes factors strongly favor our nonlinear model specification with an endogenous probability of default over a linearized specification without default.

In current ongoing research, we are estimating the model for other European countries, so as to allow cross-country comparisons of default probabilities. Although our nonlinear model allows for interactions among fiscal policy instruments and the fiscal limit, it is only a first step in our understanding of default probabilities for developed countries. Exploration of other model features are worthy of attention, including the interaction of monetary and fiscal policies, the interaction of financial sector and sovereign debt, and open economy considerations.

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A Solving the Nonlinear Model

Other than the end-of-period government debt, all other variables are either exogenous or can be computed in terms of the current state, $\psi_t = (b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$.

$$\tau_t = u_t^\tau + \gamma^\tau\left(b_t^d - b\right) \tag{A.1}$$

$$g_t = u_t^g + \gamma^g\left(b_t^d - b\right) \tag{A.2}$$

$$z_t = (1-\rho^z)z + \rho^z z_{t-1} + \varepsilon_t^z \tag{A.3}$$

$$A_t = (1-\rho^A)A + \rho^A A_{t-1} + \varepsilon_t^A \tag{A.4}$$

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta & \text{if } b_{t-1} \geq b_t^* \end{cases}$$

Given the utility function, consumption is determined by,

$$c_t = \frac{\phi h c_{t-1} + (A_t - g_t)(1-\tau_t)}{1+\phi-\tau_t}. \tag{A.5}$$

In terms of computation, the most time-consuming part is the loop iterations of the numerical integration in equation (14) in the text.

$$\begin{aligned} E_t\frac{1-\Delta_{t+1}}{c_{t+1}-hc_t} &= \int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\int_{s_{t+1}^*}\frac{1-\Delta_{t+1}}{c_{t+1}-hc_t} \\ &= \left(1-\Phi(s_t \geq s_{t+1}^*)\right)\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1}{c_{t+1}-hc_t}\Big|_{\text{no default}} \\ &+ \Phi(s_t \geq s_{t+1}^*)\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1-\delta}{c_{t+1}-hc_t}\Big|_{\text{default}} \end{aligned} \tag{A.6}$$

Thus, the integration in Equation (A.6) can be re-written as

$$\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1}{c_{t+1}-hc_t} = \int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1+\phi-\tau_{t+1}}{(1-\tau_{t+1})(A_{t+1}-g_{t+1}-hc_t)} \tag{A.7}$$

$$= \int_{\varepsilon_{t+1}^\tau}\frac{1+\phi-\tau_{t+1}}{1-\tau_{t+1}}\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\frac{1}{A_{t+1}-g_{t+1}-hc_t} \tag{A.8}$$

The logarithmal utility function helps to reduce the 4-dimension integration into 1- and 2-dimension integrations. The decision rule for government debt, $b_t = f^b(\psi_t)$, is solved in the following steps:

- Step 1: Discretize the state space ψ_t with grid points of $n_b = 25, n_c = 5, n_A = 5, n_g =$

$7, n_z = 12, n_\tau = 11$.[16] Make an initial guess of the decision rule f_0^b over the state space.

- Step 2: At each grid point, solve the following core equation and obtain the updated rule f_i^b using the given rule f_{i-1}^b. The integral in the right-hand side is evaluated as described above using numerical quadrature.

$$\frac{b_t^d + z_t + g_t - \tau_t A_t n(\psi_t)}{f_i^b(\psi_t)} = \beta E_t \frac{c(\psi_t) - hc_{t-1}}{c(\psi_{t+1}) - hc(\psi_t)}(1 - \Delta_{t+1}) \tag{A.9}$$

where $\psi_{t+1} = \left(\underbrace{(f_{i-1}^b(\psi_t), \Delta_{t+1})}_{b_t^d}, c_t, A_{t+1}, u_{t+1}^g, z_{t+1}, u_{t+1}^\tau \right)$.

- Step 3: Check the convergence of the decision rule. If $|f_i^b - f_{i-1}^b|$ is above the desired tolerance (set to $1e-5$), go back to step 2; otherwise, f_i^b is the decision rule and used to evaluate the particle filter as described below.

B Estimation

B.1 Data Description

Five observables for Greece over the period 2001Q1-2010Q4 are used for the estimation: real output, government spending to GDP ratio, tax revenue to GDP ratio, government debt to GDP ratio, and 10-year real interest rate. This appendix provides documentation for constructing these series.

Real GDP. Constructed by dividing the nominal quarterly gross domestic product from the OECD quarterly National Accounts (using the expenditure approach, series B1_GE) by the gross domestic product deflator (constructed using the expenditure approach, series B13).

Real Gov. Spending. Constructed using general government final consumption expenditure from the OECD quarterly National Accounts (series P3S13) divided by the gross domestic product deflator (constructed using the expenditure approach, series B13).

Real Tax Revenue. A quarterly tax revenue series is not provided by the OECD. Thus, we construct a quarterly measure in the following way. First, we construct a measure of total tax revenue by combining Eurostat quarterly series for tax receipts on income/wealth, production and imports, capital taxes, and social contributions. We seasonally adjust this series

[16] The grid boundaries are fixed throughout the estimation to ensure the same solution precision for different parameter draws.

using Demetra+ and the tramo-seat RSA4 specification. Next, using an annual nominal tax revenue series from the OECD volume 90 (consisting of indirect and direct taxes and social security contributions, TIND + TY + SSRG), we interpolate a quarterly frequency series using the method of Chow and Lin (1971)[17] and the seasonally adjusted quarterly Eurostat tax revenue series for the interpolation. Finally, we construct a quarterly real tax revenue series by dividing the interpolated series by the OECD's gross domestic product deflator (constructed using the expenditure approach, series B13).

Real Gov. Debt. A quarterly government debt series is not provided by the OECD. Thus, we construct a quarterly measure in the following way. First, we seasonally adjust using Demetra+ and the tramo-seat RSA4 specification the Eurostat quarterly series for nominal gross government consolidated debt. Next, using the annual nominal gross public debt series (under the Maastricht criterion) from the OECD volume 90, we interpolate a quarterly frequency series using the method of Chow and Lin (1971) and the seasonally adjusted quarterly Eurostat tax revenue series for the interpolation. Finally, we construct a quarterly real debt series by dividing the interpolated series by the OECD's gross domestic product deflator (constructed using the expenditure approach, series B13).

Real Interest Rate. To construct a 10-year real interest rate measure, we use data for the nominal interest rate, i_t (taken from the BIS) and the expected inflation rate, π_t^e. Our measure of expected inflation for Greece is the expected inflation series from the Survey of Professional Forecasts EU-area five year ahead expected inflation series. The gross real interest rate is constructed using the relation

$$R_t = \frac{1 + i_t}{1 + \pi_t^e}$$

Data for Estimation. We calculate the government spending to GDP ratio, tax revenue to GDP ratio, and government debt to GDP ratio by taking each real fiscal series described above and dividing by our real GDP series. The estimation uses these three series, along with the real GDP and real interest rate series described above. For each series, we transform the series into percentage deviations from its mean value over the period 2001Q1-2010Q4. In addition, the real GDP series is linearly detrended. The black sold lines of figure 3 graphs the observables.

[17] Forni, Monteforte, and Sessa (2009) use a similar approach.

B.2 Particle Filter Algorithm

Let v^T denote $\{\hat{v}_t\}_{t=1}^T$, which evolves according to equations (15) and (16) in the text. To evaluate the likelihood function $L(\theta|v^T)$, we use a sequential Monte Carlo filter, specifically, the sequential importance resampling filter of Kitagawa (1996). The algorithm is as follows:

- Step 1. Initialize the state variable x_0. Instead of drawing particles of x_0 from the unconditional distribution $p(x_0|\theta)$, we utilize pre-sample data to construct values x_0. We use data for the observables in 2000Q3 and 2000Q4 to back out the model state vector in 2000Q4, assuming no measurement error.[18]

  Denote these particles by x_0^i for $i = 1, ..., 60,000$. Draw $60,000$ values from standard normal distributions for each of the structural shocks (ϵ^A, ϵ^g, ϵ^t, ϵ^z) and 60,000 values from a standard uniform distribution for fiscal limit probabilities. Denote the vector of these particles by u^i. By induction, in period t these are particles $u^{t|t-1,i}$.

- Step 2. Construct $x^{t|t-1,i}$ using equation (15) in the text. Assign to each draw ($u^{t|t-1,i}$, $x^{t|t-1,i}$) a weight defined as:

$$w_t^i = \frac{1}{(2\pi)^{5/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\left(y_t - Ax^{t|t-1,i}\right)' \Sigma \left(y_t - Ax^{t|t-1,i}\right)\right] \tag{B.1}$$

- Step 3. Normalize the weights:

$$\tilde{w}_t^i = \frac{w_t^i}{\sum_{i=1}^N w_t^i}$$

  Update the values of $x^{t|t-1,i}$ by sampling with replacement 60,000 values of $x^{t|t-1,i}$ using the relative weights $\tilde{w}_t^i$ and the residual resampling algorithm.

- Repeat steps 2-3 for $t \leq T$.

The log-likelihood function is approximated by

$$L(\theta|v^T) \simeq \sum_{t=1}^{T} \ln\left(\frac{1}{60,000}\sum_{i=1}^{60,000} w_t^i\right) \tag{B.2}$$

B.3 MCMC Algorithm

The random walk Metropolis-Hastings algorithm used for estimation works as follows:

[18]Fixing the initial state vector x_0 with the data improves the accuracy of parameter estimates, which is especially useful gvien our short sample period.

- Step 1. Compute the posterior log-likelihood for 500 draws from the priors. Call the draw with the highest posterior log-likelihood value θ^*.

- Step 2. Starting from θ^*, generate a MCMC chain using the following random-walk proposal density

$$\theta_{j+1}^{prop} = \theta_j^{prop} + c\mathcal{N}(0, \Lambda), \quad j = 1, ..., 100,000$$

  where Λ is the covariance matrix of 500 draws from the priors and $c > 0$ is a tuning parameter set to determine the acceptance ratio.

- Step 3. Compute the acceptance ratio $\varphi = \min\left\{\frac{p(\theta_{j+1}^{prop}|v^T)}{p(\theta_j|v^T)}, 1\right\}$. Given a draw u from the standard uniform distribution, $\theta_{j+1} = \theta_{j+1}^{prop}$ if $u < \varphi$; and $\theta_{j+1} = \theta_j$ otherwise. Repeat for $j = 1, ..., 100,000$.

- Step 4. Update the random walk proposal density in the following way. Update Λ to be the covariance matrix from the previous draws $\{\theta_j\}_1^{100,000}$. Update θ^* to be the mean of previous draws $\{\theta_j\}_1^{100,000}$. Starting from the new θ^*, proceed through steps 2 and 3 for 500,000 draws from the new MCMC chain.

We burn the first 300,000 draws from the final MCMC chain and thin every 25 draws.

C Log-Linearized Model Equations

The log-linearized system of equations for the basic variant of the model without default are:

$$\hat{c}_t - \frac{1}{1+h}E_t\hat{c}_{t+1} + \frac{1-h}{1+h}\hat{R}_t = \frac{h}{1+h}\hat{c}_{t-1} \tag{C.1}$$

$$\frac{1}{1-h}\hat{c}_t + \frac{n}{1-n}\hat{n}_t - \hat{A}_t + \frac{\tau}{1-\tau}\hat{\tau}_t = \frac{h}{1-h}\hat{c}_{t-1} \tag{C.2}$$

$$\frac{c}{y}\hat{c}_t + \frac{g}{y}\hat{g}_t = \hat{A}_t + \hat{n}_t \tag{C.3}$$

$$\frac{b}{y}\hat{b}_t - \frac{g}{y}\hat{g}_t - \frac{z}{y}\hat{z}_t + \tau(\hat{\tau}_t + \hat{A}_t + \hat{n}_t) = R * \frac{b}{y}(\hat{R}_{t-1} + \hat{b}_{t-1}) \tag{C.4}$$

$$\hat{g}_t = (1-\rho^g)\hat{g}_{t-1} - \gamma^{g,L}(1-\rho^g)b_{t-1} + \sigma_{g,p}\epsilon_t^g, \;\; \epsilon_t^g \sim N(0,1) \tag{C.5}$$

$$\hat{\tau}_t = (1-\rho^\tau)\hat{\tau}_{t-1} + \gamma^{\tau,L}(1-\rho^\tau)b_{t-1} + \sigma_{\tau,p}\epsilon_t^\tau, \;\; \epsilon_t^\tau \sim N(0,1) \tag{C.6}$$

$$\hat{z}_t = (1-\rho^z)\hat{z}_{t-1} + \sigma_{z,p}\epsilon_t^z, \;\; \epsilon_t^z \sim N(0,1) \tag{C.7}$$

$$\hat{A}_t = (1-\rho^a)\hat{A}_{t-1} + \sigma_{a,p}\epsilon_t^a, \;\; \epsilon_t^z \sim N(0,1) \tag{C.8}$$

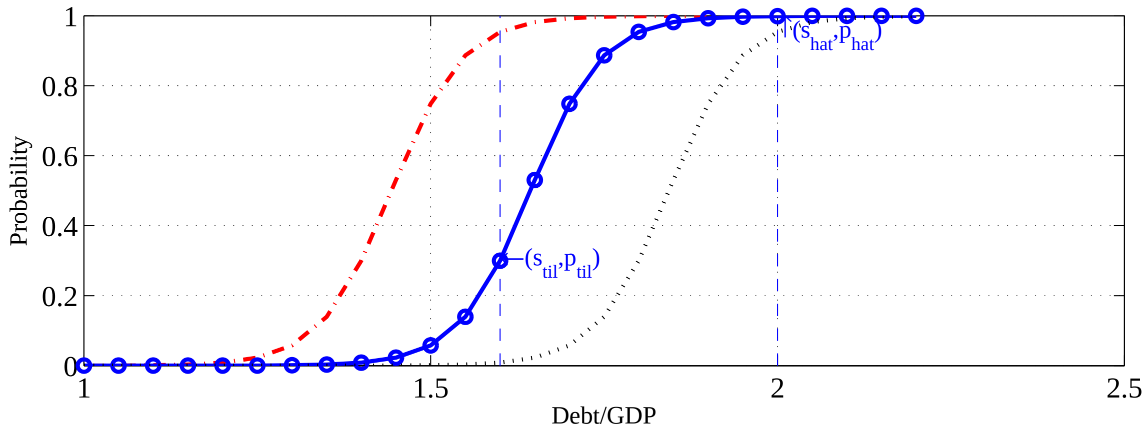

Figure 1: The fiscal limit distribution associated with various $\tilde{s}$ values. Red dashed line: $\tilde{s}$ =1.4; blue solid line: $\tilde{s}$ = 1.6; black dotted line: $\tilde{s}$ = 1.8.

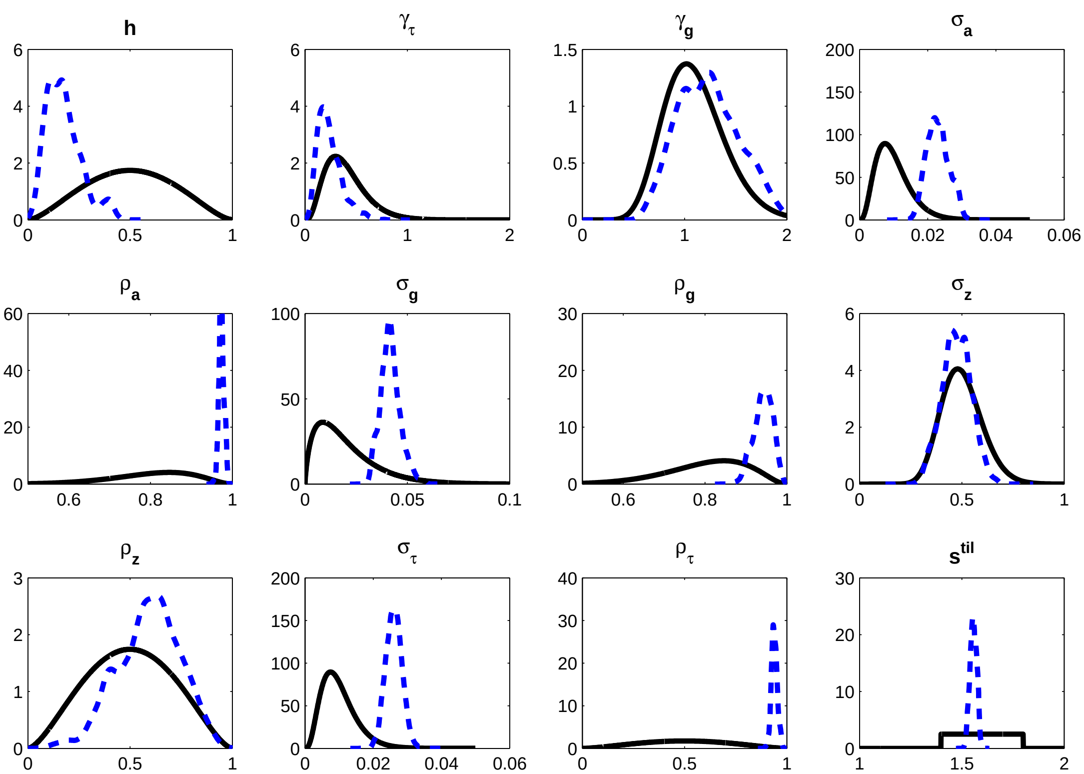

Figure 2: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with annualized default rate of 30%, $\delta^A = 0.3$.

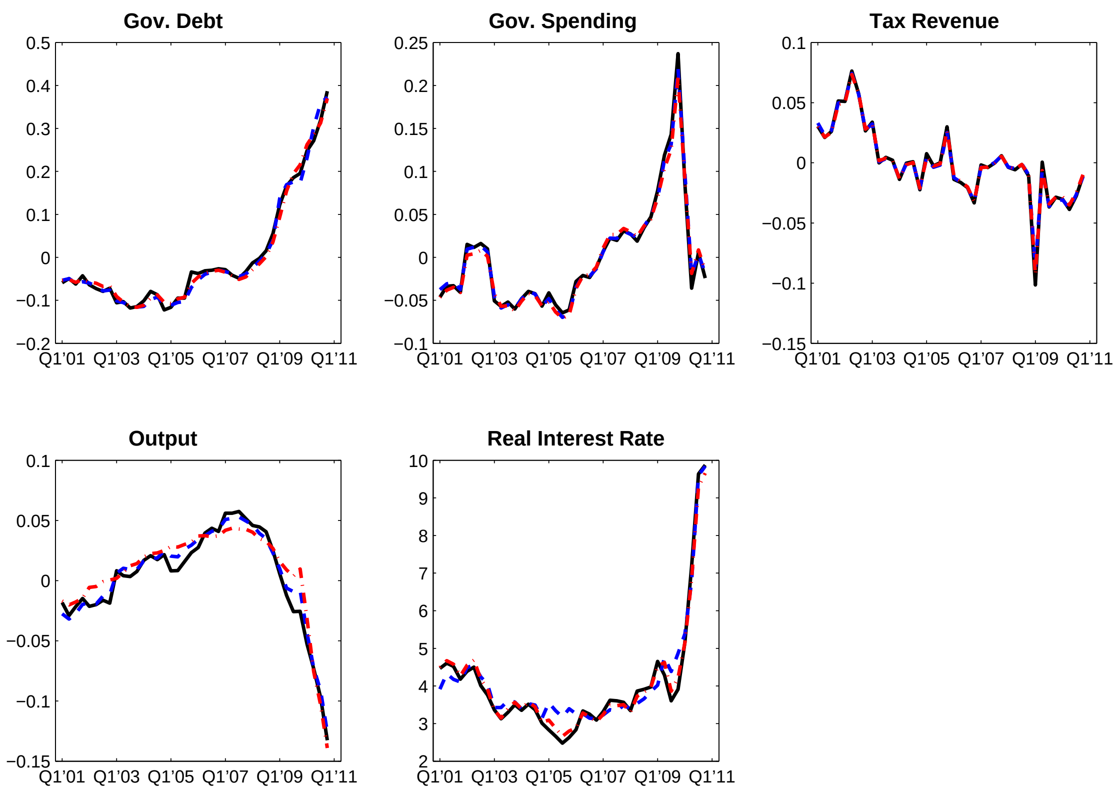

Figure 3: Fitted values for various estimations for Greece. Black, solid lines: data. Blue, dashed lines: nonlinear model with annualized default rate of 30%, $\delta^A = 0.3$. Red, dotted-dashed lines: linear model.

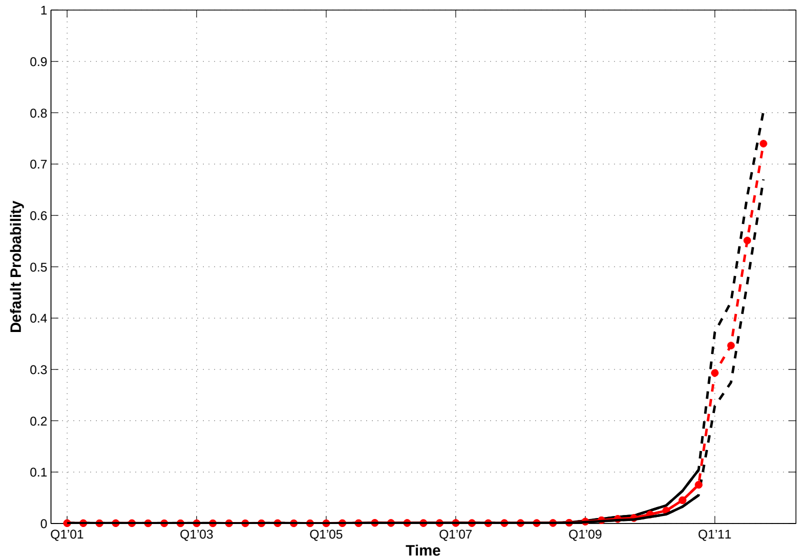

Figure 4: Model-implied sovereign default probabilities for Greece. Solid lines denote the median and 90% confidence interval probabilities for in-sample debt-to-GDP ratios. Dashed lines denote the median and 90% confidence interval probabilities for out-of-sample debt-to-GDP ratios.

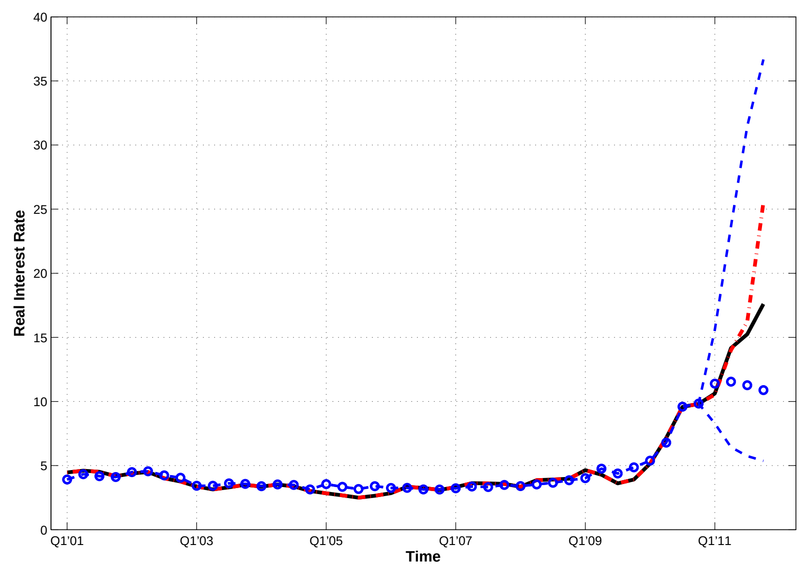

Figure 5: Data versus fitted and forecast values for the Greek interest rate. The median (blue, circled line) and 90% interval (blue, dashed lines) of the interest rate forecasts for 2011 are calculated based on the posterior median parameter estimates. The black solid line shows the BIS data, and red dotted-dashed line shows the Bloomberg data.

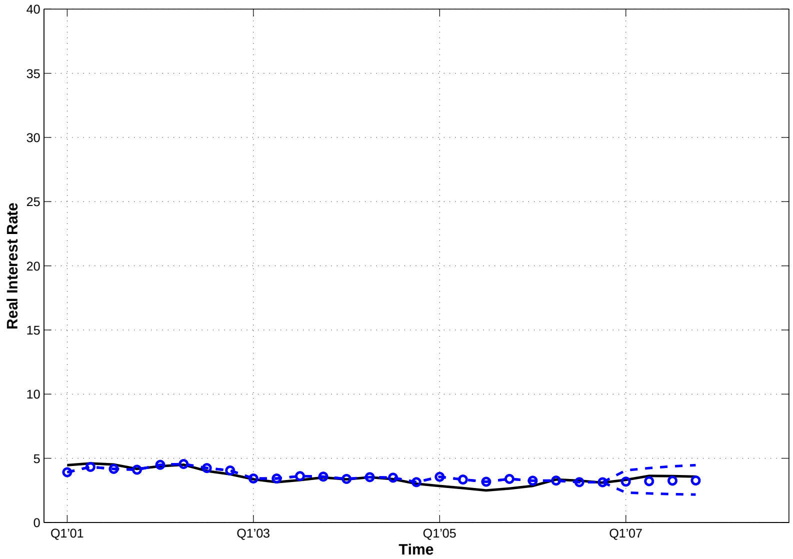

Figure 6: Data versus fitted and forecast values for the Greek interest rate. The median (blue, circled line) and 90% interval (blue, dashed lines) of the interest rate forecasts for 2007 are calculated based on the posterior median parameter estimates. The black solid line shows the BIS data.

Table 1: Calibration and Priors for Greece

Calibration

β	0.99
$\bar{n}$	0.75
$\bar{g}/\bar{y}$	0.181
$\bar{b}/\bar{y}$	1.095*4
τ	0.333

Priors

	Function	Mean	St. Dev.
h	Beta	0.5	0.2
$\tilde{s}$	Uniform	1.6	0.013
$\gamma^{\tau,L}$	Gamma	0.4	0.2
$\gamma^{g,L}$	Gamma	1.1	0.3
ρ^a	Beta	0.8	0.1
ρ^g	Beta	0.8	0.1
ρ^τ	Beta	0.5	0.2
ρ^z	Beta	0.5	0.2
$\sigma_{a,p}$	Gamma	0.005	0.01
$\sigma_{g,p}$	Gamma	0.02	0.015
$\sigma_{\tau,p}$	Gamma	0.005	0.01
$\sigma_{z,p}$	Gamma	0.5	0.1

Table 2: Greece Estimates.

	Prior		Posterior: $\delta^A = 0.3$		Posterior: $\delta^A = 0.2$		Posterior: Linear	
	mean	[5, 95]	median	[5, 95]	median	[5, 95]	median	[5, 95]
h	0.5	[0.17, 0.83]	0.16	[0.06, 0.36]	0.14	[0.06, 0.37]	0.73	[0.59, 0.83]
$\tilde{s}$	1.6	[1.42, 1.78]	1.56	[1.53, 1.58]	1.52	[1.50, 1.55]	-	-
$\gamma^{\tau,L}$	0.4	[0.14, 0.78]	0.21	[0.09, 0.47]	0.19	[0.07, 0.39]	0.61	[0.30, 1.05]
$\gamma^{g,L}$	1.1	[0.66, 1.64]	1.22	[0.77, 1.75]	1.07	[0.80, 1.35]	1.07	[0.66, 1.62]
ρ^a	0.8	[0.61, 0.94]	0.97	[0.96, 0.98]	0.97	[0.96, 0.98]	0.95	[0.94, 0.96]
ρ^g	0.8	[0.61, 0.94]	0.94	[0.90, 0.98]	0.94	[0.90, 0.98]	0.96	[0.91, 0.98]
ρ^z	0.5	[0.17, 0.83]	0.60	[0.33, 0.83]	0.69	[0.30, 0.87]	0.63	[0.39, 0.82]
ρ^τ	0.5	[0.17, 0.83]	0.94	[0.92, 0.97]	0.94	[0.91, 0.96]	0.93	[0.90, 0.95]
$\sigma_{a,p}$	0.01	[0.003, 0.02]	0.023	[0.018, 0.029]	0.024	[0.019, 0.03]	0.019	[0.015, 0.024]
$\sigma_{g,p}$	0.02	[0.003, 0.05]	0.041	[0.034, 0.051]	0.040	[0.033, 0.049]	0.041	[0.034, 0.051]
$\sigma_{z,p}$	0.5	[0.35, 0.68]	0.47	[0.35, 0.59]	0.41	[0.29, 0.58]	0.50	[0.38, 0.66]
$\sigma_{\tau,p}$	0.01	[0.003, 0.02]	0.026	[0.022, 0.030]	0.026	[0.022, 0.031]	0.022	[0.018, 0.027]

Table 3: Smoothed estimates of measurement error.

Greece		$\frac{b_t}{y_t}$	$\frac{g_t}{y_t}$	$\frac{T_t}{y_t}$	y_t	R_t
Nonlinear $\delta^A = 0.3$	mean absolute value	0.01	0.004	0.002	0.005	0.001
	relative standard deviation	0.12	0.10	0.11	0.16	0.22
Linear	mean absolute value	0.01	0.006	0.002	0.01	0.0003
	relative standard deviation	0.09	0.13	0.08	0.29	0.10

Table 4: Model Fit Comparisons

Model Specification	Bayes Factor Rel. to M1
M1: Nonlinear Model w/ $\delta^A = 0.3$	1
M2: Nonlinear Model w/ $\delta^A = 0.2$	exp[0.02]
M3: Linear	exp[55.5]