Macro-prudential Policy in a Neo-Fisherian Model of Financial Innovation*

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Abstract

The interaction between credit frictions, financial innovation, and a switch from optimistic to pessimistic beliefs is thought to have played a central role in the 2008 financial crisis. This paper uses a quantitative general equilibrium framework in which this interaction drives the financial amplification mechanism to study macro-prudential policy. Financial innovation enhances the ability of agents to collateralize assets into debt, but the riskiness of this new regime can only be learned over time. Beliefs about the transition probabilities across states with high and low ability to borrow change as agents learn from the observed realizations of financial conditions. At the same time, the collateral constraint introduces a pecuniary externality, because agents fail to internalize the effect of their borrowing decisions on asset prices. The interaction of learning and credit frictions strengthens the financial amplification mechanism, making macro-prudential policy more desirable. Quantitative analysis shows that the effectiveness of this policy depends on the government's information set, the tightness of credit constraints and the pace at which optimism surges in the early stages of financial innovation. Macro-prudential policy is effective except when the government is as uninformed as private agents, credit constraints are tight, and optimism builds quickly.

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1 Introduction

Policymakers have responded to the lapses in financial regulation in the years before the 2008 global financial crisis and the unprecedented systemic nature of the crisis itself with a strong push to revamp financial regulation following a "macro-prudential" approach. This approach aims to focus on the macro (i.e. systemic) implications that follow from the actions of credit market participants and to implement policies that influence behavior in "good times" in order to make financial crises less severe and less frequent. The design of macro-prudential policy is hampered, however, by the need to develop models that are reasonably good at explaining the macro dynamics of financial crises and at capturing the connection between potential macro-prudential policy instruments and the choices of agents in credit markets.

The task of developing these models is particularly challenging because of the fast pace of financial development. Indeed, the decade before the 2008 crash was a period of significant financial innovation, which included both the introduction of a large set of complex financial instruments, such as collateralized debt obligations, mortgage backed securities and credit default swaps, and the enactment of major financial reforms of a magnitude and scope unseen since the end of the Great Depression. Thus, models of macro-prudential regulation have to take into account the changing nature of the financial environment, and hence deal with the fact that credit market participants, as well as policymakers, may be making decisions lacking perfect information about the true riskiness of a changing financial regime.

This paper proposes a dynamic stochastic general equilibrium model in which the interaction between financial innovation, credit frictions and imperfect information is at the core of the financial transmission mechanism, and uses it to study its quantitative implications for the design and effectiveness of macro-prudential policy. In the model, a collateral constraint limits the agents ability to borrow to a fraction of the market value of the assets they can offer as collateral. Financial innovation enhances the ability of agents to securitize assets into debt, but also introduces risk because of the possibility of fluctuations in collateral coefficients or loan-to-value ratios.

We take literally the definition of financial innovation as a the introduction of a truly new financial regime. This forces us to deviate from the standard assumption that agents formulate rational expectations with full information about the stochastic process driving fluctuations in credit conditions. In particular, we assume that agents learn (in Bayesian fashion) about the transition probabilities of financial regimes only as they observe regimes with high and low ability to borrow over time. In the long run, and in the absence of new waves of financial innovation, they learn the true transition probabilities and form standard rational expectations, but in the short run agents' beliefs display waves of optimism and pessimism depending on their initial priors and on the market conditions they observe. These changing beliefs influence agents borrowing decisions and equilibrium asset prices, and together with the collateral constraint they form an amplification feedback loop: optimistic (pessimistic) expectations lead to over-borrowing (under-borrowing) and increased (reduced) asset prices, and as asset prices change the ability to borrow changes as well.

Our analysis focuses in particular on a learning scenario in which the arrival of financial innovation starts an "optimistic phase," in which a few observations of enhanced borrowing ability lead agents to believe that the financial environment is stable and risky assets are not "very risky." Hence, they borrow more and bid up the price of risky assets more than in a full-information rational expectations equilibrium. The higher value of assets in turn relaxes the credit constraint. Thus, the initial increase in debt due to optimism is amplified by the interaction with the collateral constraint via optimistic asset prices. Conversely, when the first realization of the low-borrowing-ability regime is observed, a "pessimistic phase" starts in which agents overstate the probability of continuing in poor financial regimes and overstate the riskiness of assets. This results in lower debt levels and lower asset prices, and the collateral constraint amplifies this downturn.

Macro-prudential policy action is desirable in this environment because the collateral constraint introduces a pecuniary externality in credit markets that leads to more debt and financial crises that are more severe and frequent than in the absence of this externality. The externality exists because individual agents fail to internalize the effect of their borrowing decisions on asset prices, particularly future asset prices in states of financial distress (in which the feedback loop via the collateral constraint triggers a financial crash).

There are several studies in the growing literature on macro-prudential regulation that have examined the implications of this externality, but typically under the assumption that agents form rational expectations with full information (e.g. Lorenzoni (2008), Stein (2011), Bianchi (2010), Bianchi and Mendoza (2010), Korinek (2010), Jeanne and Korinek (2010), Benigno, Chen, Otrok, Rebucci, and Young (2010)). In contrast, the novel contribution of this paper is in that we study the effects of macro-prudential policy in an environment in which the pecuniary externality is influenced by the interaction of the credit constraint with learning about the riskiness of a new financial regime. The findings of Boz and Mendoza (2010) suggest that taking this interaction into account can be very important, because the credit constraint in the learning setup produces significantly larger effects on debt and asset prices than in a full-information environment with the same credit constraint. Their study, however, focused only on quantifying the properties of the decentralized competitive equilibrium and abstracted from normative issues and policy analysis.

The policy analysis of this paper considers social planners under different informational assumptions. First, an *uninformed planner* (SP1) who has to learn about the true riskiness of the new financial environment, and faces the set of feasible credit positions supported by the collateral values (i.e. asset pricing function) of the competitive equilibrium with learning. In the baseline scenario, private agents and SP1 have the same initial priors and thus form the same sequence of beliefs, but we also study a scenario in which private agents and the uninformed planner form different beliefs. Second, an *informed planner* with full information, who therefore knows the true transition probabilities across financial regimes. We consider two types of informed planner, one (SP2) facing the same set of feasible credit positions as SP1, and one (SP3) facing a set of feasible credit positions consistent with the collateral values of the full-information, rational expectations competitive equilibrium.

We compute the decentralized competitive equilibrium of the model with learning (DEL) and contrast this case with the above social planner equilibria. We then compare the main features of these equilibria, in terms of the behavior of macroeconomic aggregates and asset pricing indicators, and examine the characteristics of macro-prudential policies that support the allocations of the planning problems as competitive equilibria. This analysis emphasizes the potential limitations of macro-prudential policy in the presence of significant financial innovation (which makes the informational friction relevant), and highlights the relevance of taking into account informational frictions in evaluating the effectiveness of macro-prudential policy.

The quantitative analysis indicates that the interaction of the collateral constraint with optimistic beliefs in the DEL equilibrium can strengthen the case for introducing macro-prudential regulation compared with the same equilibrium under full information (DEF). This is because, as Boz and Mendoza (2010) showed, the interaction of these elements produces larger amplification both of the credit boom in the optimistic phase and of the financial crash when the economy switches to the bad financial regime. The results also show, however, that the effectiveness of macro-prudential policy varies sharply with the assumptions about the information set and collateraly pricing function used by the social planner. Moreover, for the uninformed planner, the effectiveness of macro-prudential policy also depends on the tightness of the borrowing constraint and the pace at which optimism builds in the early stages of financial innovation.

Consider first SP1. For this planner, the undervaluation of risk weakens the incentives to build precautionary savings against states of nature with low-borrowing-ability regimes over the long run, because this planner underestimates the probability of landing on and remaining in those states. In contrast, SP2 assesses the correct probabilities of landing and remaining in states with good and bad credit regimes, so its incentives to build precautionary savings are stronger than SP1, but SP2 cannot correct the agent's mispricing of collateral under imperfect information, and hence allows larger debt positions than SP3. Finally, SP3's optimal macro-prudential policy features a precautionary component that lowers borrowing levels at given asset prices, and a component that influences portfolio choice of debt v. assets to address the effect of the agents' mispricing of risk on collateral prices.

It is important to note that even the uninformed planner SP1 has the incentive to use macro-prudential policy to tackle the pecuniary externality and alter debt and asset pricing dynamics. In our baseline calibration, however, the borrowing constraint becomes tightly binding (first for DEL and then for SP1) and optimism builds quickly in the early stages of financial innovation, and as a result macro-prudential policy is not very effective (i.e. debt positions and asset prices differ little between DEL and SP1). Since when the constraint binds debt equals the high-credit-regime fraction of the value of collateral, debt levels in SP1 and DEL are similar once the constraint becomes binding for SP1. But this is *not* a general result.[1] Variations of SP1 in which optimism builds more gradually produce outcomes in which macro-prudential policy is effective even when the planner has access to the same information set. On the other hand, it is generally true that the uninformed planner allows larger debt positions than the informed planners because of the lower precautionary savings incentives.

Our model follows a long and old tradition of models of financial crises in which credit frictions and imperfect information interact. This notion dates back to the classic work of Fisher (1933), in which he described his well-known debt-deflation financial amplification mechanism as the result of a feedback loop between agents' beliefs and credit frictions (particularly those that force fires sales of assets by distressed borrowers). Minsky (1992) is along a similar vein. More recently, macroeconomic models of financial accelerators (e.g. Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Aiyagari and Gertler (1999)) have focused on modeling financial amplification

[1] It is also important to note that this result is not due to the fact that SP1 faces the same collateral pricing function as DEL. Working under the same pricing assumption in a model with full information, but using a different calibration of collateral coefficients, Bianchi and Mendoza (2010) found that the planner supports very different debt allocations and asset prices than the decentralized equilibrium.

but typically under rational expectations with full information about the stochastic processes of exogenous shocks.

The particular specification of imperfect information and learning that we use follows closely that of Boz and Mendoza (2010) and Cogley and Sargent (2008a), in which agents observe regime realizations of a Markov-switching process without noise but need to learn its transition probability matrix. The imperfect information assumption is based on the premise that the U.S. financial system went through significant changes beginning in the mid-90s as a result of financial innovation and deregulation that took place at a rapid pace. As in Boz and Mendoza (2010), agents go through a learning process in order to "discover" the true riskiness of the new financial environment as they observe realizations of regimes with high or low borrowing ability.

Our quantitative analysis is related to Bianchi and Mendoza (2010)'s quantitative study of macro-prudential policy. They examined an asset pricing model with a similar collateral constraint and used comparisons of the competitive equilibria vis-a-vis a social planner to show that optimal macro-prudential policy curbs credit growth in good times and reduces the frequency and severity of financial crises. The government can accomplish this by using Pigouvian taxes on debt and dividends to induce agents to internalize the model's pecuniary externality. Bianchi and Mendoza's framework does not capture, however, the role of informational frictions interacting with frictions in financial markets, and thus is silent about the implications of differences in the information sets of policy-makers and private agents.

Our paper is also in a similar vein as Gennaioli, Shleifer, and Vishny (2010), who study financial innovation in an environment in which "local thinking" leads agents to neglect low probability adverse events (see also Gennaioli and Shleifer (2010)). As in our model, this distorts decision rules and equilibrium asset prices, but the modeling of the informational friction is different. In the model of Gennaioli et al. agents ignore part of the state space relevant for pricing risk by assumption, assigning zero probability to rare negative events. In contrast, in our learning framework agents always assign non-zero probability to all the regimes that are part of the realization vector of the Markov switching process of the loan-to-value limit in the collateral constraint.[2] However, agents do assign lower (higher) probability to tight credit regimes than they would under full information rational expectations when they are optimistic (pessimistic), and this lower probability is an outcome of a Bayesian learning process. Moreover, learning yields realizations of equilibrium pricing functions in future dates, after learning progresses, that agents did not consider possible

[2]Beliefs that assign zero probability to a state can only be achieved as a limiting case.

with the beliefs of previous dates. In addition, the welfare analysis of Gennaioli, Shleifer, and Vishny (2010) focuses on the effect of financial innovation under local thinking, while we emphasize the interaction between a fire-sale externality and informational frictions.

Finally, our work is also related to Stein (2011) argument in favor of a cap and trade system to address a pecuniary externality that leads banks to issue excessive short-term debt in the presence of private information. Our analysis differs in that we study the implications of a form of model uncertainty (i.e. uncertainty about the transition probabilities across financial regimes) for macro-prudential regulation, instead of private information, and we focus on Pigouvian taxes as a policy instrument to address the pecuniary externality.

The rest of the paper is organized as follows: Section 2 describes the model. Section 3 conducts the quantitative analysis comparing the decentralized competitive equilibrium with the various planning problems. Section 4 provides the main conclusions.

2 A Fisherian Model of Financial Innovation

The setup of the model's competitive equilibrium and learning environment is similar to Boz and Mendoza (2010). The main difference is that we extend the analysis to characterize social planning problems under alternative information sets and collateral pricing functions.

2.1 Decentralized Competitive Equilibrium

The economy inhabited by a continuum of identical agents who maximize a standard constant-relative-risk-aversion utility function. Agents choose consumption, c_t, holdings of a risky asset k_{t+1} (i.e. land), and holdings of a one-period discount bond, b_{t+1}, denominated in units of the consumption good. Land is a risky asset traded in a competitive market, where its price q_t is determined, and is in fixed unit supply. Individually, agents see themselves as able to buy or sell land at the market price, but since all agents are identical, at equilibrium the price clears the land market with all agents choosing the same land holdings.

Bonds carry an exogenous price equal to $1/R$, where R is an exogenous gross real interest rate. Thus, the model can be interpreted as a model of a small open economy, in which case b represents the economy's net foreign asset position and R is the world's interest rate, or as a partial equilibrium model of households or a subset of borrowers in a closed economy, in which case b represents these borrowers' net credit market assets and R is the economy's risk free real interest rate. Under either interpretation, the behavior of creditors is not modeled from first principles. They are simply assumed to supply of funds at the real interest rate R subject to the collateral constraint described below.

The bond market is imperfect because creditors require borrowers to post collateral that is "marked to market" (i.e. valued at market prices). In particular, the collateral constraint limits the agents'debt (a negative position in b) to a fraction κ of the market value of their individual land holdings.[3] The collateral coefficient κ is stochastic and follows a Markov regime-switching process. Information is imperfect with respect to the true transition probability matrix governing the evolution of κ, and the agents learn about it by observing realizations of κ over time. We will model learning so that in the long-run the agents' beliefs converge to the true transition probability matrix, at which point the model yields the same competitive equilibrium as a standard rational-expectations asset pricing model with a credit constraint.

Agents operate a production technology $\varepsilon_t Y(k_t)$ that uses land as the only input, and facing a productivity shock ε_t. This shock has compact support and follows a finite-state, stationary Markov process about which agents are perfectly informed.

The agents' preferences are given by:

$$E_0^s\left[\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\sigma}}{1-\sigma}\right]. \tag{1}$$

E^s is the subjective conditional-expectations operator that is elaborated on further below, β is the subjective discount factor, and σ is the coefficient of relative risk aversion.

The budget constraint faced by the agents is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + \varepsilon_t Y(k_t) \tag{2}$$

The agents' collateral constraint is:

$$-\frac{b_{t+1}}{R_t} \leq \kappa_t q_t k_{t+1} \tag{3}$$

[3]This constraint could follow, for example, from limited enforcement of credit contracts, by which creditors can only confiscate a fraction κ of the value of a borrower's land holdings. In actual credit contracts, this constraint resembles loans subject to margin calls or loan-to-value limits, value-at-risk collateralization and mark-to-market capital requirements.

Using μ_t for the Lagrange multiplier of (3), the first-order conditions of the agents optimization problem are given by:

$$u'(t) = \beta R E_t^s \left[u'(t+1) \right] + \mu_t \tag{4}$$

$$q_t(u'(t) - \mu_t \kappa_t) = \beta E_t^s \left[u'(t+1) \left(\varepsilon_{t+1} Y_k(k_{t+1}) + q_{t+1} \right) \right] \tag{5}$$

A *decentralized competitive equilibrium with learning* (DEL) is a sequence of allocations $[c_t, k_{t+1}, b_{t+1}]_{t=0}^{\infty}$ and prices $[q_t]_{t=0}^{\infty}$ that satisfy the above conditions, using the agents' beliefs about the evolution of κ to formulate expectations, together with the collateral constraint (3) and the market-clearing conditions for the markets of goods and assets:

$$\begin{aligned} c_t + \frac{b_{t+1}}{R_t} &= b_t + \varepsilon_t Y(k_t) \\ k_t &= 1 \end{aligned}$$

The *decentralized competitive equilibrium with full information* (DEF) is defined in the same way, except that expectations are formulated using the true distribution of κ.

2.2 Learning and its Interaction with the Credit Constraint

Expectations in the payoff function (1) are based on Bayesian beliefs agents form based on initial priors and information they observe over time. As mentioned in the Introduction, we model learning following closely Boz and Mendoza (2010) and Cogley and Sargent (2008a). Hence, we provide here only a short description and refer the interested reader to those other articles for further details.

The stochastic process of κ follows a classic two-point, Hamilton-style regime-switching Markov process. There are two realizations of κ, a regime with high ability to borrow κ^h and a regime with low ability to borrow κ^l. The "true" regime-switching Markov process has continuation transition probabilities defined by F^a_{hh} and F^a_{ll}, with switching probabilities thus given by $F^a_{hl} = 1 - F^a_{hh}$ and $F^a_{lh} = 1 - F^a_{ll}$. Hence, learning in this setup is about forming beliefs regarding the distributions of the transition probabilities F^s_{hh} and F^s_{ll} by combining initial priors with the observations of κ that arrive each period. After observing a sufficiently long and varied set of realizations of κ^h and κ^l, agents learn the true regime-switching probabilities of κ. Modeling of learning in this fashion is particularly useful for representing financial innovation as the introduction of a brand-new financial regime for which there is no data history agents could use to infer the true transition distribution of

κ, while maintaining a long-run equilibrium that converges to a conventional rational expectations equilibrium.

Agents learn using a beta-binomial probability model starting with exogenous initial priors. Take as given a history of realizations of κ that agents will observe over T periods, $\kappa^T \equiv \{\kappa_0, \kappa_1, ..., \kappa_{T-1}, \kappa_T\}$, and initial priors, F^s, of the distributions of F^s_{hh} and F^s_{ll} for date $t = 0$, $p(F^s)$. Bayesian learning with beta-binomial distributions will yield a sequence of posteriors $\{f(F^s \mid \kappa^t)\}_{t=1}^T$.

To understand how the sequence of posteriors is formed, consider first that at every date t, from 0 to T, the information set of the agent includes κ^t as well as the possible values that κ can take (κ^h and κ^l). This means that agents also know the number of times a particular regime has persisted or switched to the other regime (i.e. agents know the set of counters $\left[n_t^{hh}, n_t^{hl}, n_t^{ll}, n_t^{lh}\right]_{t=0}^T$ where each n_t^{ij} denotes the number of transitions from state κ^i to κ^j that have been observed prior to date t).[4] These counters, together with the priors, form the arguments of the Beta-binomial distributions that characterize the learning process. For instance, the initial priors are given by $p(F^s_{ii}) \propto (F^s_{ii})^{n_0^{ii}-1}(1 - F^s_{ii})^{n_0^{ij}-1}$. As in Cogley and Sargent (2008a), we assume that the initial priors are independent and determined by n_0^{ij} (i.e. the number of transitions assumed to have been observed prior to date $t = 1$).

The agents' posteriors about F^s_{hh} and F^s_{ll} have Beta distributions as well. The details of how they follow from the priors and the counters are provided in Cogley and Sargent (2008a) and Boz and Mendoza (2010). The posteriors are of the form $F^s_{hh} \propto Beta(n_t^{hh}, n_t^{hl})$ and $F^s_{ll} \propto Beta(n_t^{lh}, n_t^{ll})$, and the posterior means satisfy:

$$E_t[F^s_{hh}] = n_t^{hh}/(n_t^{hh} + n_t^{hl}), \quad E_t[F^s_{ll}] = n_t^{ll}/(n_t^{ll} + n_t^{lh}) \tag{6}$$

This is a key result for the solution method we follow, because, as will be explained later in this Section, the method relies on knowing the evolution of the posterioe means as learning progresses.

An important implication of (6) is that the posterior means change only when that same regime is observed at date t. Since in a two-point, regime-switching setup continuation probabilities also determine mean durations, it follows that the beliefs about both the persistence and the mean durations of the two financial regimes can be updated only as the economy actually experiences κ^l or κ^h.

[4]The number of transitions across regimes is updated as follows: $n_{t+1}^{ij} = n_t^{ij} + 1$ if both $\kappa_{t+1} = \kappa^j$ and $\kappa_t = \kappa^i$, and $n_{t+1}^{ij} = n_t^{ij}$ otherwise.

The potential for financial innovation to lead to significant underestimation of risk can be inferred from the evolution of the posterior means implied by (6). Specifically, if we start from very low values of n_0^{ij}, which implies that the new financial regime is truly new, the initial sequence of realizations of κ^h observed until just before the first realization of κ^l generates substantial optimism (i.e. a sharp increase in $E_t[F_{hh}^s]$ relative to F_{ll}^a). Moreover, it also follows that the magnitude of the optimism that any subsequent sequence of realizations of κ^h generates will be smaller than in the initial optimistic phase. This is because it is only after observing the first switch to κ^l that agents rule out the possibility of κ^h being an absorbent state. Similarly, the first realizations of κ^l generate a pessimistic phase, in which $E_t[F_{ll}^s]$ is significantly higher than F_{ll}^a, so the period of optimistic expectations is followed by a period of pessimistic expectations.

Following Boz and Mendoza (2010), the effects of the interaction between the collateral constraint and learning can be explained intuitively by combining the Euler equations on land and bonds (Equations (4) and (5)) to obtain an expression for the model's land premium, $E_t^s[R_{t+1+i}^q]$, and then solving forward for the price of land in Equation (5).

The expected land premium one-period ahead is given by:

$$E_t^s\left[R_{t+1}^q - R\right] = \frac{(1-\kappa_t)\mu_t - cov_t^s(\beta u'(c_{t+1}), R_{t+1}^q)}{E_t^s\left[\beta u'(c_{t+1})\right]} \tag{7}$$

This land premium rises in every state in which the collateral constraint binds because of a combination of three effects: the increased shadow value of land as collateral (which is limited to the fraction $(1-\kappa_t)$ of μ_t because only the fraction κ_t of land can be collateralized into debt), the lower covariance between marginal utility and land returns, and the increased expected marginal utility of future consumption. The latter two effects occur because the binding credit constraint hampers the agents' ability to smooth consumption and tilts consumption towards the future.

Consider now a state at date t in which the collateral constraint binds even at κ^h, and compare what the land premium would look like in the learning economy ($E_t^s[R_{t+1}^q|\kappa_t^h = \kappa^h, \mu_t > 0]$) v. a perfect information economy ($E_t^a[R_{t+1}^q|\kappa_t^h = \kappa^h, \mu_t > 0]$). If beliefs are optimistic (i.e. $E_t[F_{hh}^s] > F_{hh}^a$), agents assign lower probability to the risk of switching to κ^l at $t+1$ (which has higher land returns because the constraint is more binding than with κ^h) than they would under perfect information. This lowers the expected land premium in the learning model because agents' beliefs put more weight on states with lower land returns.

To see how this affects asset prices, consider the forward solution of q_t:

$$q_t = E_t^s \left[\sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \left(\frac{1}{E_t^s[R_{t+1+i}^q]} \right) \right) \varepsilon_{t+1+j} Y_k(k_{t+1+j}) \right]. \tag{8}$$

This expression shows that the lower land returns of the model with learning under optimistic beliefs, either at date t or expected along the equilibrium path for any future date, translate into higher land prices at t (and higher than under full information). But if the constraint was already binding at t with κ^h, and κ^h is the current state, the value of collateral rises and agents borrow more. Hence, optimistic beliefs and the credit constraint interact to amplify the total upward effects on credit and prices. Notice, however, this feedback process is nonlinear and features a dampening mechanism, because as collateral values rise the constraint becomes relatively less binding (i.e. μ_t falls).

When the financial regime does switch to κ^l after a spell of $\kappa^{h\prime}s$, the opposite process is set in motion, and instead of dampening mechansim, there is an additional amplification mechanism via a Fisherian deflation. Now agents become pessimistic (i.e. $E_t[F_{ll}^s] > F_{ll}^a$), so they assign excessive probability to staying in κ^l. This increases the expected land premium because now agents' beliefs put more weight on states with higher land returns, and the higher expected premia lower asset prices relative to full information. As asset prices fall, and if κ^l is the current state, the collateral constraint becomes even more binding, which triggers a Fisherian deflation and fire sales of assets, which in turn put further upward pressure on land premia and downward on land prices, and agents continue to put higher probability in these states with even higher land returns and lower land prices. Unlike the case of the optimistic beliefs in the κ^h state, in which there is a dampening effect because the price increase lowers the shadow value of the borrowing constraint, with pessimistic beliefs in the κ^l state the Fisherian deflation of asset prices increases this shadow value and feeds back into larger land premia and lower land prices, and thus larger contractions in credit.

2.3 Recursive Anticipated Utility Competitive Equilibrium

The fact that this learning setup involves learning from and about an exogenous variable (κ) allows us to separate the solution of the evolution of beliefs from the agents' dynamic optimization problem. Thus, we solve for the equilibrium dynamics following a two-stage solution method. In the first stage, we use the above Bayesian learning framework to generate the agents' sequence of posterior means determined by (6). In the second stage, we characterize the agents' optimal plans as a recursive equilibrium by adopting Kreps's Anticipated Utility (AU) approach to approximate dynamic optimization with Bayesian learning. The AU approach focuses on combining the sequences of posterior means obtained in the first stage with chained solutions from a set of "conditional" AU optimization problems (AUOP).[5] Each of these problems solves what looks like a standard optimization problem with full information and rational expectations, but using the posterior means of each date t instead of the true transition probabilities (see Boz and Mendoza (2010) for further details).

The AU competitive equilibrium in recursive form is constructed as follows. Consider the date-t AUOP. At this point agents have observed κ_t , and use it to update their beliefs so that (6) yields $E_t[F^s_{hh}]$ and $E_t[F^s_{ll}]$. Using this posterior means, they construct the date-t beliefs about the transition probability matrix across financial regimes $E^s_t[\kappa'|\kappa] \equiv \begin{bmatrix} E_t[F^s_{hh}] & 1-E_t[F^s_{hh}] \\ 1-E_t[F^s_{ll}] & E_t[F^s_{ll}] \end{bmatrix}$. The solution to the date-t AUOP is then given by policy functions $(b'_t(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa))$ and a pricing function $q_t(b,\varepsilon,\kappa)$ that satisfy the following recursive equilibrium conditions:

$$u'\left(c_t(b,\varepsilon,\kappa)\right) = \beta R\left[\sum_{\varepsilon'\in E}\sum_{\kappa'\in\{\kappa^h,\kappa^l\}} E^s_t[\kappa'|\kappa]\pi(\varepsilon'|\varepsilon)u'(c_t(b',\varepsilon',\kappa'))\right] + \mu_t(b,\varepsilon,\kappa) \tag{9}$$

$$q_t(b,\varepsilon,\kappa)\left[u'(c_t(b,\varepsilon,\kappa)) - \mu_t(b,\varepsilon,\kappa)\kappa\right] = \tag{10}$$

$$\beta\left[\sum_{z'\in Z}\sum_{\kappa'\in\{\kappa^h,\kappa^l\}} E^s_t[\kappa'|\kappa]\pi(\varepsilon'|\varepsilon)u'(c_t(b',\varepsilon',\kappa'))\left[\varepsilon' Y(1) + q_t(b',\varepsilon',\kappa')\right]\right]$$

$$c_t(b,\varepsilon,\kappa) + \frac{b'_t(b,\varepsilon,\kappa)}{R} = \varepsilon Y(1) + b \tag{11}$$

$$\frac{b'_t(b,\varepsilon,\kappa)}{R} \geq -\kappa q_t(b,\varepsilon,\kappa)1 \tag{12}$$

The time subscripts that index the policy and pricing functions indicate the date of the beliefs used to form the expectations (which is also the date of the most recent observation of κ, date t). Notice that these equilibrium conditions already incorporate the market clearing condition of the land market.

[5] Cogley and Sargent (2008b) show that the AU approach is significantly more tractable than full Bayesian dynamic optimization and yet produces very similar quantitative results unless risk aversion coefficients are large. The full Bayesian optimization problem uses not just the posterior means but the entire likely evolution of posterior density functions to project the effects of future κ realizations on beliefs. This problem runs quickly into the curse of dimensionality because it requires carrying the counters $\left[n^{hh}_t, n^{hl}_t, n^{ll}_t, n^{lh}_t\right]^T_{t=0}$ as additional state variables.

It is critical to note that solving for date-t policy and pricing functions means solving for a full set of optimal plans over the entire (b, ε, κ) domain of the state space and conditional on date-t beliefs. Thus, we are solving for the optimal plans agents "conjecture" they would make over the infinite future acting under those beliefs. For characterizing the "actual" equilibrium dynamics to match against the data, however, the solution of the date-t AUOP determines optimal plans for date t only. This is crucial because beliefs change as time passes, and each subsequent κ_t is observed, which implies that the policy and pricing functions that solve each AUOP also change.

The model's recursive AU equilibrium is defined as follows:

Definition Given a T-period history of realizations $\kappa^T = (\kappa_T, \kappa_{T-1}, ..., \kappa_1)$, a recursive AU competitive equilibrium for the economy is given by a sequence of decision rules $[b'_t(b, \varepsilon, \kappa), c_t(b, \varepsilon, \kappa), \mu_t(b, \varepsilon, \kappa)]_{t=1}^T$ and pricing functions $[q_t(b, \varepsilon, \kappa)]_{t=1}^T$ such that: (a) the decision rules and pricing function for date t solve the date-t AUOP conditional on $E_t^s[\kappa'|\kappa]$; (b) $E_t^s[\kappa'|\kappa]$ is the conjectured transition probability matrix of κ produced by the date-t posterior density of F^s determined by the Bayesian passive learning as defined in (6).

Intuitively, the complete solution of the recursive equilibrium is formed by chaining together the solutions for each date-t AUOP. For instance, the sequence of equilibrium bond holdings that the model predicts for dates $t = 1, ..., T$ is obtained by chaining the relevant decision rules as follows: $b_2 = b'_1(b, \varepsilon, \kappa),\ b_3 = b'_2(b, \varepsilon, \kappa), ...,\ b_{T+1} = b'_T(b, \varepsilon, \kappa)$.

2.4 Constrained Planners' Problems

We examine macro-prudential policy by studying three versions of the optimal policy problem faced by a benevolent social planner who maximizes the agents' utility subject to the resource constraint and the collateral constraint. The key difference between the equilibria attained by these planner problems and the DEL is that the former internalize the effects of borrowing decisions on the market prices of collateral assets. Following Bianchi and Mendoza (2010), we assume that the government values collateral taking as given an asset pricing function determined in a decentralized equilibrium (either DEL or DEF). This is sensible because, as explained earlier, the government represents either the government of a small open economy that is an atomistic participant in the global credit market, or a partial-equilibrium planner maximizing the payoff of households in a large national credit market. As Bianchi and Mendoza (2010) argued, this assumption also has the advantage that it makes the planners' optimization problem time-consistent, which guarantees that macro-prudential policy, if effective, improves welfare across all states and dates in a time-consistent fashion.[6] Moreover, the allocations of these planners' are decentralized with Pigouvian taxes that support the planners' pricing functions as an equilibrium outcome of the "regulated" DEL.

The three planner problems follow from different assumptions about the information set of the government and the collateral pricing function it faces:

Social planner SP1 is subject to a similar learning problem as private agents. This planner observes the same history κ^T but starts learning off date-0 priors that may or may not be the same as those of the private sector. This planner is assumed to price collateral using the DEL's collateral pricing functions ($q_t^{DEL}(b, \varepsilon, \kappa)$), which means that SP1 faces the same set of feasible credit positions as private agents in the DEL.

Social planner SP2 is a fully informed planner (a planner who knows F_{hh}^a and F_{ll}^a) but who, like SP1, values collateral using the DEL's collateral pricing functions $q_t^{DEL}(b, \varepsilon, \kappa)$ and thus faces the corresponding set of feasible credit positions.

Social planner SP3 is also a fully informed planner, but in addition this planner prices collateral using the time-invariant pricing function of the DEF $q^{DEF}(b, \varepsilon, \kappa)$, which means that this planner can implement the set of feasible credit positions of the full-information competitive equilibrium.[7]

The three planners' optimization problems in standard intertemporal form can be summarized as follows:

$$E_0^i \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right] \quad \text{for} \quad i = SP1, SP2, SP3 \tag{13}$$

$$s.t. \quad c_t + \frac{b_{t+1}}{R_t} = b_t + \varepsilon_t Y(1) \tag{14}$$

$$-\frac{b_{t+1}}{R_t} \leq \kappa_t q_t^i \tag{15}$$

with $q_t^i = q_t^{DEL}$ for $i = SP1,\ SP2$ and $q_t^{SP3} = q_t^{DEF}$. Note that in SP1, the planner solves a similar Bayesian learning problem as private agents observing the same history of credit regimes κ^T. This planner's initial priors are denoted p_0^{ij} for $i, j = h, l$. If $p_0^{ij} = n_0^{ij}$, which will be our baseline scenario, both SP1 and private agents have identical beliefs at all times. In SP2 and SP3 the planners use the true transition probabilities F_{hh}^a and F_{ll}^a.

[6] In contrast, if a planner can manipulate the collateral pricing function, the planner would internalize not only how the choice of debt at t affects the land price at $t+1$, but also how it affects land prices and the tightness of the collateral constraint in previous periods.

[7] These pricing functions are time invariant because they correspond to the solutions of a standard recursive rational expectations equilibrium. The resulting planning problem is analogous to the one solved in Bianchi and Mendoza (2010).

We solve the problem of each planner in recursive form, and to simplify the exposition we represent all three as AU recursive equilibria.[8] For each planner $i = SP1, SP2, SP3$ the solution to the date-t AUOP is given by policy functions $(b_t'(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa))$ that satisfy the following recursive equilibrium conditions:

$$u'(c_t(b,\varepsilon,\kappa)) - \mu_t(b,\varepsilon,\kappa) = \tag{16}$$

$$\beta R \left[\sum_{\varepsilon' \in E} \sum_{\kappa' \in \{\kappa^h,\kappa^l\}} E_t^i[\kappa'|\kappa]\pi(\varepsilon'|\varepsilon) \left[u'(c_t(b',\varepsilon',\kappa')) + \kappa'\mu_t(b',\varepsilon',\kappa') \frac{\partial q_t^i(b',\varepsilon',\kappa')}{\partial b'} \right] \right]$$

$$c_t(b,\varepsilon,\kappa) + \frac{b_t'(b,\varepsilon,\kappa)}{R} = \varepsilon Y(1) + b \tag{17}$$

$$\frac{b_t'(b,\varepsilon,\kappa)}{R} \geq -\kappa q_t^i(b,\varepsilon,\kappa)1 \tag{18}$$

where the pricing functions for each planner are $q_t^i(b,\varepsilon,\kappa) = q_t^{DEL}(b,\varepsilon,\kappa)$ for $i = SP1,\ SP2$ and $q_t^{SP3}(b,\varepsilon,\kappa) = q^{DEF}(b,\varepsilon,\kappa)$ Moreover, expectations in each planner's date-t AUOP are taken using $E_t^{SP1}[\kappa'|\kappa] \equiv \begin{bmatrix} E_t[F_{hh}^g] & 1 - E_t[F_{hh}^g] \\ 1 - E_t[F_{ll}^g] & E_t[F_{ll}^g] \end{bmatrix}$ and $E_t^i[\kappa'|\kappa] \equiv \begin{bmatrix} F_{hh}^a & 1 - F_{hh}^a \\ 1 - F_{ll}^a & F_{ll}^a \end{bmatrix}$ for $i = SP2,\ SP3$.[9] Note also that in these problems the time subindexes of expectations operators, decision rules and pricing functions represent the date of the AUOP to which they pertain, and not the indexing of time within each AUOP. That is, in the date-t AUOP the planner creates expectations of the prices and allocations of all future periods using the date-t recursive decision rules and pricing functions (e.g. in the date-t AUOP, consumption projected for $t+1$ is given by the expectation of $c_t(b',\varepsilon',\kappa')$). Moreover, for SP3, since the planner has full information and can implement the credit feasibility set of the DEF, the decision rules are actually time-invariant at equilibrium (all date-t AUOP's for SP3 are identical because they use the true Markov process of κ and the DEF time-invariant pricing functions).

We can now define the three recursive social planner equilibria for a given history of realizations κ^T:

SP1 Equilibrium Given the DEL time-varying asset pricing functions $[q_t^{DEL}(b,\varepsilon,\kappa)]_{t=1}^T$, a recursive AU equilibrium for the SP1 planner is given by a sequence of decision rules $[b_t'(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa)]_{t=1}^T$

[8]This is redundant for SP3 because this planner solves a standard full-information rational expectations recursive equilibrium with time-invariant decision rules and pricing functions.

[9]By analogy with the results in (6), the posterior means of the government's learning dynamics satisfy: $E_t[F_{hh}^g] = p_t^{hh}/(p_t^{hh} + p_t^{hl})$, $E_t[F_{ll}^g] = p_t^{ll}/(p_t^{ll} + p_t^{lh})$. Note that, since both the private sector and the government observe the same κ sequence, these counters can differ from those of private agents only because of differences in date-0 priors.

such that: (a) the decision rules for date t solve SP1's date-t AUOP conditional on $E_t^g[\kappa'|\kappa]$; and (b) the elements of $E_t^g[\kappa'|\kappa]$ are the posterior means produced by the date-t posterior densities of F_{hh}^g and F_{ll}^g determined by the Bayesian passive learning process.

SP2 Equilibrium Given the DEL time-varying asset pricing functions $[q_t^{DEL}(b,\varepsilon,\kappa)]_{t=1}^T$, a recursive AU equilibrium for the SP2 planner is given by a sequence of decision rules $[b_t'(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa)]_{t=1}^T$ such that the decision rules for date t solve SP2's date-t AUOP conditional on $E^a[\kappa'|\kappa]$.

SP3 Equilibrium Given the DEF time-invariant asset pricing function $q^{DEF}(b,\varepsilon,\kappa)$, a recursive AU equilibrium for the SP3 planner is given by time-invariant decision rules $[b'(b,\varepsilon,\kappa), c(b,\varepsilon,\kappa), \mu(b,\varepsilon,\kappa)]$ such that the decision rules solve SP3's date-t AUOP conditional on $E^a[\kappa'|\kappa]$ for all t.

2.5 Pecuniary Externality and Decentralization of Planners' Allocations

The key difference between the first-order conditions of the social planners and those obtained in the private agents' DEL is the pecuniary externality reflected in the right-hand-side of the Euler equation for bonds: The planners' internalize how, in states in which the collateral constraint is expected to bind next period (i.e. $\mu_t(b',\varepsilon',\kappa') > 0$ for at least some states), the choice of debt made in the current period, b', will alter prices in the next period ($\frac{\partial q_t^i(b',\varepsilon',\kappa')}{\partial b'}$). This derivative represents the response of the land price tomorrow to changes in the debt chosen today, which can be a very steep function when the collateral constraint binds because of the Fisherian deflation mechanism.

While the three planning problems internalize the above price derivative, they differ sharply in how they do it. Consider for example a period of optimism produced by the effect of a spell of κ^h realizations on the private agents' beliefs. SP1 (assuming $p_0^{ij} = n_0^{ij}$ so its beliefs are identical to those of private agents) shares in the agent's optimism both in terms of beliefs about transition probabilities of κ and in terms of facing the feasible set of credit positions implied by optimistic collateral prices in the DEL pricing function. This planner still wants to use macro-prudential policy to dampen credit growth because it internalizes the slope of the asset pricing function when the collateral constraint on debt is expected to bind, but this planner's expectations are as optimistic as the private agents' and hence it assigns very low probability to a financial crash (i.e. a transition from κ^h to κ^l), and it internalizes a pricing function inflated by optimism. Our quantitative findings show that, if optimism builds quickly (i.e. $E_t[F_{hh}^s]$ approaches 1) and the collateral constraint binds tightly in the early stages of financial innovation, these limitations can result in SP1 attaining equilibrium debt and land prices close to those of the DEL, thus reducing the effectiveness of macro-prudential policy. But if optimism builds gradually and/or the collateral constraint is not tightly binding, SP1 attains lower debt positions than private agents in the DEL.

SP2 and SP3 differ sharply because they do not share in the private agents' optimistic beliefs and thus assign higher probability to the likelihood of observing a κ^h-to-κ^l transition, which therefore strengthens their incentive to build precautionary savings and borrow less. But SP2 and SP3 do not adopt identical policies. SP2 faces the collateral pricing functions (or credit feasibility sets) of the DEL, which again are influenced by the agents' optimism. This planner is more cautious than SP1, because it assigns higher probability to transitions from states with optimistic prices to those with pessimistic crash prices, but the collateral pricing functions it faces are the same as for SP1. In contrast, SP3 faces the time-invariant pricing function (or credit feasibility set) of the DEF, which "endogenously" constraints the set of feasible debt positions relative to those SP2 deals with. Hence, SP3 assigns higher probability to switches from κ^h to κ^l than private agents just like SP2, but facing pricing functions across those two states that display sharply smaller collapses than those captured in the pricing functions of the DEL (which are the ones SP2 uses). Again depending on whether the constraint binds and how optimistic are beliefs, at equilibrium both SP2 and SP3 acquire less debt and experience lower land price booms than both SP1 and the DEL, but for the same reason their use of macro prudential policy is more intensive.

Given the model's pecuniary externality, the most natural choice to model the implementation of macro-prudential policies are Pigouvian taxes. In particular, using taxes on debt ($\tau^i_{b,t}$) and land dividends ($\tau^i_{l,t}$) we can fully implement each of the constrained planner problems' allocations (for $i = SP1, SP2, SP3$). With these taxes, the budget constraint of private agents becomes:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t(1+\tau^i_{b,t})} = q_t k_t + b_t + \varepsilon_t Y(k_t)(1-\tau^i_{l,t}) + T^i_t. \tag{19}$$

T^i_t represents lump-sum transfers by which the government rebates to private agents all its tax revenue (or a lump-sum tax in case the tax rates are negative, which is not ruled out).

The Euler equations of the competitive equilibrium with the macro-prudential policy in place are:

$$u'(t) = \beta R(1+\tau^i_{b,t}) E^s_t \left[u'(t+1)\right] + \mu_t \tag{20}$$

$$q_t(u'(t) - \mu_t \kappa) = \beta E^s_t \left[u'(t+1)\left(\varepsilon_{t+1} Y_k(k_{t+1})(1-\tau^i_{l,t}) + q_{t+1}\right)\right]. \tag{21}$$

We compute the state-contingent, time-varying schedules of these taxes by replacing each planner's allocations in these optimality conditions and then solving for the corresponding tax rates, so that the DEL *with* macro-prudential policy supports both the same allocations of each planner's problem and the corresponding asset pricing functions that each planner uses to value collateral. The debt tax is needed to replicate the planner's debt choices, and the dividends tax is needed to support the pricing functions that the planner used to value collateral as the competitive equilibrium asset pricing functions. The tax schedules in recursive form are denoted $\tau^i_{b,t}(b, \varepsilon, \kappa)$ and $\tau^i_{l,t}(b, \varepsilon, \kappa)$. There will be one of these schedules for each date-t AUOP solved by private agents in the DEL with Pigouvian taxes.

It is important to note that when the collateral constraint is binding at t, one can construct multiple representations of the tax schedules that implement the contrained planning problems. This is because when $\mu_t > 0$ the value of b_{t+1} is determined by the collateral constraint and not by the Euler equation for bonds. In particular, if we plug a given planner's consumption and debt plans and collateral pricing function in conditions (20) and (21), there are different schedules of debt and dividend taxes depending on a chosen schedule for the shadow value μ_t in the DEL with taxes. For simplicity, we chose the tax schedules such that $\tau^i_{b,t} = 0$ when the collateral constraint binds. Hence, when $\mu_t > 0$, the shadow value of the constraint is set at $\mu_t = u'(t) - \beta R E^s_t \left[u'(t+1)\right]$, and given this the corresponding tax on dividends when the constraint binds follows from condition (21).

The debt tax can be decomposed into three terms that are useful for interpreting how macro-prudential policy responds to the effects of imperfect information, the pecuniary externality and the interaction of these two. In particular, combining (20) and (16) and rearranging terms, the debt tax for each planner can be expressed as follows:

$$\tau^i_{b,t} = \underbrace{\frac{E^i_t[u'(t+1)]}{E^s_t[u'(t+1)]} - 1}_{\text{information}} + \underbrace{\frac{E^i_t\left[\kappa'\mu(b', \varepsilon', \kappa')\frac{\partial q^i_t(.)}{\partial b'}\right] - E^s_t\left[\kappa'\mu(b', \varepsilon', \kappa')\frac{\partial q^i_t(.)}{\partial b'}\right]}{E^s_t[u'(t+1)]}}_{\text{interaction}} + \underbrace{\frac{E^s_t\left[\kappa'\mu(b', \varepsilon', \kappa')\frac{\partial q^i_t(.)}{\partial b'}\right]}{E^s_t[u'(t+1)]}}_{\text{externality}} \tag{22}$$

where $q^i_t(.) = q^i_t(b', \varepsilon', \kappa')$ and E^i are calculated using the information set and collateral pricing functions of planner i. The first term in the right-hand-side of this expression is labeled "information" because it reflects the contribution to the debt tax that arises from deviations in the one-period-ahead expected marginal utilities of private agents and planner i, which arise because of the beliefs formed with their different information sets. If the two information sets, and hence beliefs, are identical, as they are in our baseline SP1, this term vanishes, but for SP2 and SP3 it does not vanish. The second term, labeled "interaction", reflects differences in the expected value of the externality when evaluated using the beliefs of each planner v. the private agents' beliefs. This term is zero when either the information sets are the same or the DEL is far from the region where the constraint binds, and hence the externality term is zero for all possible states in $t+1$. Thus, the label "interaction" is reflects the fact that both the informational difference and the externality need to be present for this term to be nonzero. Finally, the third term labeled "externality" is simply the value of the externality evaluated using the beliefs of private agents.

3 Quantitative Analysis

This Section explores the quantitative implications of the model. We discuss first the baseline calibration, and then compare the DEL with the three planning problems. We also quantify the macro-prudential tax schedules that decentralize the planners' allocations and decompose them into their three components.

3.1 Baseline Calibration

We borrow the baseline calibration from Boz and Mendoza (2010), so we keep the description here short. Note, however, that in contrast with their work, our aim here is to study how the interaction between the learning friction and the pecuniary externality affect the design of macro-prudential policy in the aftermath of financial innovation. This implies that the uncertainty surrounding the values of the parameters driving the learning process and the collateral constraint takes particular relevance, so we view this initial calibration more as a baseline to begin the quantitative analysis than as a calibration intended to judge the model's ability to match the data.

The model is calibrated to U.S. quarterly data at annualized rates and assuming a learning period of length T in which $\kappa = \kappa^h$ from $t = 1, ..., J$ (the optimistic phase) and $\kappa = \kappa^l$ from $J+1$ to T (the pessimistic phase). The parameter values are listed in Table 1.

As in Boz and Mendoza (2010), we set the start of the learning dynamics and the dates T and J by following observations from a timeline of the financial innovation process and events leading to the U.S. financial crisis. Financial innovation is defined as a structural change from a regime with only a time-invariant collateral coefficient κ^l to one where the can be switches between κ^h and κ^l. We set the date of this structural change in 1997Q1 to be consistent with two important facts.

Table 1: Baseline Parameter Values

β	Discount factor (annualized)	0.91
σ	Risk aversion coefficient	2.0
c	Consumption-GDP ratio	0.670
A	Lump-sum absorption	0.321
r	Interest rate (annualized)	2.660
ρ	Persistence of endowment shocks	0.869
σ_e	Standard deviation of TFP shocks	0.008
α	Factor share of land in production	0.025
κ^h	Value of κ in the high securitization regime	0.926
κ^l	Value of κ in the low securitization regime	0.642
F^a_{hh}	True persistence of κ^h	0.964
F^a_{ll}	True persistence of κ^l	0.964
n_0^{hh},n_0^{hl}	Priors	0.0205

First, 1997 was the year of the first publicly-available securitization of mortgages under the New Community Reinvestment Act and the first issuance of corporate CDS's by JPMorgan. Second, this was also the year in which the U.S. households' net credit assets-GDP ratio started on a declining trend that lasted until the end of 2008, while prior to 1997 this ratio was quite stable at about -30 percent. We date the start of the financial crisis at 2007Q1, consistent with the initial nation-wide decline in home prices and the early signs of difficulties in the subprime mortgage market. The experiment ends two years later. These assumptions imply setting $T = 48$ and $J = 40$ (i.e. 40 consecutive quarters of κ^h realizations followed by 8 consecutive quarters of κ^l).

The model's parameters are calibrated as follows: First, the values of $(\sigma, R, \rho, \sigma_e, \kappa^l, \kappa^h, F^a_{hh}, F^a_{ll})$ are calculated directly from the data or set to standard values from the quantitative DSGE literature. Second, the values of (α, β) are calibrated such that the model's pre-financial innovation stochastic stationary state is consistent with various averages from U.S. data from the pre-financial-innovation period (i.e. pre-1997), assuming that in that period the financial constraint with κ^l was binding on average. Finally, the initial priors are calibrated assuming that they are symmetric, with the common value of n_0 for all transitions targeted to match an estimate of observed excess land returns, as described later in this Section.

We set the real interest rate to the average ex-post real interest rate on U.S. three-month T-bills during the period 1980Q1-1996Q4, which is 2.66 percent annually. The utility function is of the constant-relative-risk aversion (CRRA) type $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. The value of σ is set to $\sigma = 2.0$, the standard value in DSGE models of the U.S. economy.

To pin down κ^l and κ^h, we use the data on net credit market assets of U.S. households and non-profit organizations from the *Flow of Funds* as a proxy for b in the model. The proxy for ql is obtained from the estimates of the value of residential land provided by Davis and Heathcote (2007). On average over the 1980Q1-1996Q4 period, the ratios of the value of residential land and net credit market assets relative to GDP were stable around 0.477 and -0.313, respectively. Next, we construct a macro estimate of the household leverage ratio, or the loan-to-value ratio, by dividing net credit market assets by the value of residential land. We set the value of κ^l by combining the 1980Q1-1996Q4 average of this ratio with the calibrated value of R which yields $\kappa^l = 0.659/1.0266 = 0.642$. Following a similar idea, we set κ^h to the 2006Q4 value of the estimated leverage ratio, hence $\kappa^h = 0.926$.

We calibrate F^a_{hh} based on Mendoza and Terrones (2008)'s finding that the mean duration of credit booms in industrial economies is 7 years. To match this mean duration, we set $F^a_{hh} = 0.964$. We assume a symmetric process by setting $F^a_{ll} = 0.964$. Notice that the true transition probability matrix is not needed to solve the model with learning, but F^a_{hh} and F^a_{ll} necessary for solving the planner problems with full information and the asset pricing function of the DEF.

We assume a standard Cobb-Douglas production function: $Y(k_t) = k_t^\alpha$. Using the 1980Q1-1996Q4 average of the value of residential land to GDP, the value of R, and the condition that arbitrages the returns on land and bonds, which follows from the optimality conditions (4)-(5), the implied value for α is $\alpha = 0.0251$ [10].

The stochastic process for ε is set to approximate an AR(1) ($\ln(\varepsilon_t) = \rho \ln(\varepsilon_{t-1}) + e_t$) fitted to HP-filtered real U.S. GDP per capita using data for the period 1965Q1-1996Q4. We estimate $\rho = 0.869$ and $\sigma_e = 0.00833$, which imply a standard deviation of TFP of $\sigma_\varepsilon = 1.68$ percent.

The value of β is set so that in the pre-financial-innovation stochastic steady state the model matches the observed standard deviation of consumption relative to output over the 1980Q1-1996Q4 period, which is 0.8. This yields $\beta = 0.91$.

We introduce an exogenous, time-invariant amount of autonomous spending in order to make the model's average consumption-output ratio and average resource constraint consistent with the data. As noted earlier, the *Flow of Funds* data show that the observed average ratio of net credit

[10] Since the model with a single financial regime set at κ^l (i.e., the pre-financial-innovation regime) yields a collateral constraint that is almost always binding and a negligible excess return on land, we use the approximation $E[R^q] \approx R$, and then conditions (4) and (5) imply: $\alpha = (ql/l^\alpha)[R - 1 + \beta^{-1}(1 - \beta R)(1 - \kappa^l)]$

assets to GDP in the 1980Q1-1996Q4 period was very stable at $\bar{b} = -0.313$. In the case of the consumption-GDP ratio, the data show a slight trend, so we use the last observation of the pre-financial-innovation regime (1996Q4), which implies $\bar{c} = 0.670$.[11] To make these ratios consistent with the model's resource constraint in the average of the stochastic stationary state for that same financial regime, we introduce autonomous spending by the share A of GDP, so that the long-run average of the resource constraint is given by $1 = \bar{c} + A - \bar{b}(R-1)/R$. Given the values for $\bar{b}$, $\bar{c}$ and R, A is calculated as a residual $A = 1 - \bar{c} + \bar{b}(R-1)/R = 0.321$.[12] This adjustment represents the averages of investment and government expenditures, which are not explicitly modeled.

The remaining parameters are the counters of the beta-binomial distribution that determine the initial priors. As noted earlier, the assumption of symmetric priors implies $n_0 = n_0^{hl} = n_0^{hh} = n_0^{ll} = n_0^{lh}$, so that there is only one parameter to calibrate. We set n_0 so that the implied expected excess return one period ahead at $t = 0$ from the DEL matches the annualized 1997Q2 spread on the Fannie Mae residential MBS with 30-year maturity over the T-bill rate. This excess return was equal to 47.6 basis points and the model matches it with $n_0 = 0.0205$.

3.2 Quantitative Findings from the Baseline Calibration

The main quantitative experiment compares the time-series dynamics triggered by financial innovation over the learning period ($t = 1, ..., 48$) in the DEL with those of the three planning problems. These dynamics are computed by solving the sequence of AUOPs for each date t that define each equilibrium, and constructing forecast functions that chain together the decision rules of each date-t AUOP as described in Boz and Mendoza (2010).[13] The forecast functions keep TFP unchanged at its mean value ($\varepsilon = 1$) and start from the initial condition $b_0 = -0.345$, which corresponds to the net credit market assets-GDP ratio of U.S. households observed in the data in 1996Q4. Figure 1 plots the forecast functions for bonds and land prices (Panels (a) and (b)), the shadow value of collateral μ_t (Panel (c)), the evolution of beliefs (Panel (d)), and the externality, defined as $E_t[\kappa' \mu_t(b', \varepsilon', \kappa') \frac{\partial q_t^i(b', \varepsilon', \kappa')}{\partial b'}]$ for $i = SP1, SP2, SP3$ (Panel (e)).

Consider first the evolution of beliefs. As shown in Panel (d), $E_t[F_{hh}^s]$ rises from 0.980 to 0.999 from $t = 1$ to $t = 40$, as agents observe the long spell of κ^hs. Since there are no observations of

[11] Consumption and GDP data were obtained from the *International Financial Statistics* of the IMF.

[12] Note that, since land is in fixed unit supply and the unconditional mean of ε equal to 1, the mean of output in the model is also 1.

[13] Recall that, as explained in Section 2, the decision rules of DEL and SP1 change every period as their beliefs evolve, and hence the dynamics shown for these scenarios result from chaining together the corresponding period's bond decision rules and equilibrium prices. The decision rules of SP2 also change every period because this planner internalizes the time-varying DEL asset pricing function.

κ^l, the beliefs about κ^l do not change during this time (recall Equation (6). At date 41, when the economy switches to κ^l for the first time, $E_{41}[F^s_{hh}]$ falls to 0.975, and more importantly $E_{41}[F^s_{ll}]$ rises sharply from 0.5 to 0.98. Hence, beliefs turn pessimistic very quickly after the first realization of κ^l. Panel (d) also shows the time-invariant true transition probabilities F^a_{hh} and F^a_{ll}, which are the same because we assumed a symmetric process for κ. The excesses of $E_t[F^s_{hh}]$ over F^a_{hh} and F^a_{ll} over $E_t[F^s_{ll}]$, for $t = 1, ..., 40$, measure the degree of optimism built during the optimistic phase.

The increase in $E_t[F^s_{hh}]$ from date 1 to 40 may appear small (from 0.98 to 0.999) and the difference relative to F^a_{hh} (which is set at 0.964) may also seem small. However, even these small differences have important implications for the perception of riskiness of the financial environment, particularly for the expected mean duration of the κ^h regime and the "perceived" variability of the κ process. The expected mean duration of κ^h rises from 50 quarters with $E_1[F^s_{hh}] = 0.98$ to 1,000 quarters with $E_{40}[F^s_{hh}] = 0.999$ at the peak of the optimistic phase, and the coefficient of variation of κ based on date-40 beliefs is about 1/4 of that based on date-1 beliefs. Thus, agents' expectations of the riskiness of the new financial environment drop dramatically as the optimistic phase progresses. This is also true relative to $F^a_{hh} = 0.964$, which implies that in the true regime-switching Markov process the κ^h regime has a significantly shorter mean duration of 28 periods. This is about half of what the agents that are learning perceive already at $t = 1$ of the optimistic phase, and a negligible fraction of the mean duration they expect by $t = 40$.

The difference between $E_t[F^s_{ll}]$ as learning progresses and F^a_{ll} has a similar implication. During the optimistic phase, in which $E_t[F^s_{ll}]$ remains constant at 1/2, and since $F^a_{ll} = 0.964$, agents' beliefs imply a projected mean duration for the κ^l regime of only 2 periods, whereas the true mean duration is 28 periods.

These sharp differences in projected mean durations of both κ regimes play a key role in driving the much stronger incentives for precautionary savings of SP2 and SP3. These two planners anticipate that κ^h (κ^l) will arrive less (more) often and that sequences of κ^h (κ^l) are likely to be of much shorter (longer) duration than what agents in the DEL and SP1 believe. Thus, DEL and SP1 perceive much less riskiness in the new financial environment than SP2 and SP3.

In line with the above description of the evolution of beliefs, Panel (a) of Figure 1 shows that in DEL there is a large and sustained increase in debt for the first 40 periods and a very sharp correction at date 41. This increase in debt accounts for about 2/3rds of the observed rise in net credit liabilities of U.S. households. Panel (b) shows that the surge in debt in the DEL is accompanied by a sharp increase in the price of the risky asset, which is about 44 percent the observed rise in U.S. residential land prices. These two results are reassuring, because they show that the model's baseline DEL is a reasonable laboratory in which to conduct macro-prudential policy experiments inasmuch as the Fisherian interaction of the financial friction and financial innovation produce sizable, sustained booms in debt and land prices. Moreover, as Boz and Mendoza (2010) showed, these booms are twice as large as what the model would predict by either removing the debt-deflation amplification mechanism or the informational friction.

Panel (a) of Figure 1 also shows that all three social planners choose lower debt positions than the DEL during the optimistic phase, but the size of the adjustment differs across the three planners. SP1 chooses only slightly smaller debt (higher bonds) than DEL, while both SP2 and SP3 choose similar debt levels that are much smaller than those of SP1 and DEL. To understand these differences, recall that there are two key factors driving the three planners' actions: differences in beliefs, which are used to project the future evolution of κ, and differences in the pricing functions that support feasible credit positions, which are used to evaluate the derivatives that reflect the planners' efforts to internalize how projected prices respond to debt choices when the collateral constraint binds.

In Panel (a), SP1 borrows slightly less than DEL in the early periods after financial innovation is introduced, but then bond holdings and asset prices become nearly identical starting at about $t = 6$. This may seem puzzling, because in principle SP1 still has the incentive to use macro-prudential policy, and this is reflected in the positive externality terms that SP1 displays in Panel (e). In fact, as we show later in the sensitivity analysis, the nearly identical bond and price dynamics in the baseline DEL and SP1 is *not* a general result. It is the outcome for the baseline calibration because the borrowing constraint binds tightly as a result of fast growing optimism (i.e. initial priors are such that $E_1[F^s_{hh}]$ is close to 1 and $E_1[F^s_{ll}]$ is far from 1) soon after financial innovation starts. Thus, households' willingness to borrow induces them to face a high shadow value from relaxing the collateral constraint, and hence, since SP1 also considers the high value of current consumption attributed by households, it also decides to borrow up to the limit. Notice that although date-t borrowing decisions for DEL and SP1 coincide, the fact that the collateral constraint is expected to bind one period ahead still generates an externality for SP1 (see Panel (e)), but this is not strong enough to offset the high value assigned to date-t borrowing which pushes both private agents and SP1 to borrow up to the limit.

Panel (e) also shows that SP1's externality becomes weaker over time as it approaches the end of the optimistic phase at $t = 40$. The weak externality at this date is easier to interpret by examining Figure 2, which plots the bond decision rules and pricing functions for $t = 40$ in the two κ regimes (for $\varepsilon = 1$). The externality term is given by $E_{40}[\kappa' \mu_{40}(b', \varepsilon', \kappa') \frac{\partial q_{40}^{DEL}(b',\varepsilon',\kappa')}{\partial b'}]$. As Panel (b) of Figure 2 shows, the pricing function $q_{40}^{DEL}(b, 1, \kappa^h)$ is relatively flat, which means that land prices do not differ much for different choices of b'. Thus, in the κ^h state that SP1 believes most likely to continue, the price derivative driving the externality, $\frac{\partial q_{40}^{DEL}(b',\varepsilon',\kappa')}{\partial b'}$, is small. In the other financial regime, κ^l, the pricing function $q_t^{DEL}(b, 1, \kappa^l)$ is very steep (see Panel (d)), but this carries a very small weight because SP1 assigns a negligible probability to switching from κ^h to κ^l. A similar dynamic is at play as the externality weakens from $t = 6$ to $t = 40$. The externality weakens because optimistic beliefs imply that, conditional on having observed $\kappa_t = \kappa^h$ at each date of the optimistic pase, SP1's perceived probability of a switch to κ^l is very low (i.e. $E_t[F^s_{hl}]$ is close to zero). Hence, SP1 evaluates the externality assigning a large and increasing weight to the one-period-ahead state with the small derivative $\frac{\partial q_t^{DEL}(b',1,\kappa^h)}{\partial b'}$ and nearly zero weight to the state with the large derivative $\frac{\partial q_t^{DEL}(b',1,\kappa^l)}{\partial b'}$. Notice that there is another effect that goes in the opposite direction. At higher levels of debt along the transition path, the slope of the pricing functions becomes relatively steeper, which would make the externality term larger. But the previous effect on the increasing weight on κ^h regime dominates.

Using the true regime-switching transition probabilities across the κ regimes, SP2 and SP3 perceive higher risk in the new financial environment (both in terms of the likelihood of switching to κ^l one period ahead of each date $t = 1, ..., 40$ and in terms of the long-run perceived mean duration of the κ^h regime and the volatility of the κ process). Thus, they have significantly stronger precautionary savings motives, and choose much lower debt levels than SP1 and DEL during the optimistic phase (see Panel (a) of Figure 1). In fact, on average they stay away from the borrowing constraint during the entire optimistic phase (see Panel (e)). The asset prices SP2 and SP3 support are, however, quite different. SP2 supports prices very similar to DEL and SP1 while SP3 yields significantly lower prices.

The prices of SP1 and SP2 are similar despite SP2 choosing significantly less debt because both of SP1 and SP2 use the same collateral pricing functions $q_t^{DEL}(b, 1, \kappa)$, *and* because these pricing functions have a flat slope in the κ^h state. In particular, since $\frac{\partial q_t^{DEL}(b',1,\kappa^h)}{\partial b'}$ is small for SP1 and SP2 for $t = 1, ..., 40$, their different choice of bonds translates into small differences in land prices. If the pricing functions were steeper at the optimal debt choices, the equilibrium dynamics of land prices would be very different even though both SP1 and SP2 use the same pricing functions, because the lower debt levels chosen by SP2 would imply different date-t prices picked from the same pricing function (i.e. the same $q_t^{DEL}(b', 1, \kappa^h)$ would return different prices for each planner because of the different choices of b').

The equilibrium prices for SP3 are lower than the rest, and particularly SP2 (who uses the same beliefs based on the true transition probabilities) because SP3 carries lower debt levels *and* faces the full-information pricing functions, $q_t^{DEF}(b', 1, \kappa^h)$. These pricing functions are also relatively flat, but they return uniformly lower prices because asset pricing is not influenced by the optimistic beliefs of the DEL (see Panel (b) of Figure 2). Hence, SP3 chooses lower debt levels because of precautionary reasons, and these credit positions support lower land prices because this planner can attain credit positions that undo the effect of optimistic expectations on prices.

The dynamics of consumption are easy to infer from the debt and price dynamics. During the early periods of the optimistic phase, consumption in DEL and SP1 exceeds that of SP2 and SP3, in line with the faster debt buildup in those equilibria. Consumption for SP3 lies slightly above the other equilibria after about period 25 due to the lower level of debt this economy is converging to before the switch of financial regime, which thus entails low debt service payments.

Consider now the outcomes predicted by the DEL and the three planners when the first switch to κ^l arrives at $t = 41$, which we define as a "crisis episode." To illustrate more clearly the dynamics around this crisis episode, Figure 4 shows event windows for seven quarters before and after the crisis. As shown in panel (a) of this figure, SP3, who chose the lowest levels of debt in the optimistic phase, experiences the smallest correction in debt. This is consistent with the macro-prudential behavior that led SP3 to take precautionary action and choose lower debt levels, because SP3 can correct the optimism of private beliefs and their effect on the set of feasible credit positions (i.e. it can support collateral values consistent with those of the DEF). With both sources of overborrowing shut down, the smaller correction in debt at $t = 41$ is in response to the exogenous tightening of the constraint due to a lower realization of κ. This exogenous debt correction cannot be avoided because even with full information about the transition probability matrix across financial regimes, κ remains stochastic.

The realization of κ^l in period 41 leads to a change in the beliefs of SP1 about the persistence of the κ^h regime making the debt correction for SP1 more pronounced. Since this social planner could not undo the informational source of overborrowing, this planner cannot avoid arriving at date 40 with debt levels that leave the economy vulnerable to large adjustments in case of a transition to κ^l. As a result, the change in debt that takes place in date 41 is more than twice as large as that of SP2 or SP3.

The ranking of the price decline in SP2, SP1 and DEL (with SP2 smaller and DEL and SP1 the largest) follows the ranking of the debt correction. Consistent with the sharp change in beliefs at date 41, having built a larger debt than the other SPs and facing the same set of feasible credit positions as DEL, SP1 cannot avoid falling on the relatively steep portion of the pricing function, as plotted in panel (d) of Figure 3. In the region where SP1 chose debt levels, prices vary significantly across debt positions in the κ^l regime, leading to a large in decline in the asset price for SP1,as shown in panel (b) of Figure 4. Notice also how the differences across the different equilibria shrink for all the macroeconomic series plotted in this figure towards the end of the time series experiment, as the beliefs get closer to rational expectations.

Figure 5 shows the taxes on debt, τ_b, and dividends, τ_l, necessary to support each planners' allocations as DELs with taxes. In line with the above results showing that SP1's debt and land prices deviate only slightly from the outcomes of the DEL without taxes, SP1 makes very limited use of these taxes. In the first seven periods after financial innovation starts, it uses a debt tax of about 2-3 percent and a small subsidy on dividends. After that, as the collateral constraint becomes binding for SP1, the debt tax drops to zero and the dividends subsidy becomes negligible. In contrast, SP2 and SP3 use macro-prudential taxes very actively. They use similar debt taxes, which increase gradually from about 3 percent to close to 9 percent in the optimistic phase, and then drop to zero as the financial crisis erupts, making the collateral constraint bind for these planners. Their dividends tax policies are similar qualitatively, with subsidies that increase gradually during the optimistic phase, but quantitatively SP3 uses smaller subsidies, because SP3 aims to support the DEF asset pricing functions, which are uniformly lower than the DEL pricing functions supported by SP2. Thus, SP3 taxes debt just as much as SP2 to weaken the incentives of private agents to borrow, but subsidizes land dividends less to deflate the effect of optimistic beliefs on land prices.

Figure 6 decomposes the above tax policy dynamics in terms of the information, interaction, and externality terms of Equation 22. Since SP1 goes through the same learning process as private agents in the DEL without taxes, the information and interaction terms of the debt taxes for this planner are always zero. The externality term (which captures the expected value of the pecuniary externality using the DEL's beliefs in units of expected marginal utility) accounts for the full amount of SP1's debt taxes shown in Figure 5. This terms rises up to a maximum of about 3 percent, before vanishing after the seventh period. Again, the results that the externality tax term (and the tax itself) are small are consistent with the finding that SP1's debt and land prices deviate slightly from those obtained in the DEL without taxes.

The dynamics of the tax components for SP2 and SP3 show that the externality component remains small even for these two planners.[14] In the early stages after financial innovation starts, this component is even smaller for these two planners than for SP1, but in contrast with SP1, the externality component remains slightly positive throughout the optimistic phase. The debt taxes of SP2 and SP3 are significantly higher than those of SP1 because of larger information and interaction components. Both the information and interaction tax components of SP2 and SP3 rise gradually during the optimistic phase. Interestingly, SP3 displays a higher information component than SP2, while SP2 displays a higher interaction component, and these differences net out into the very similar debt taxes shown for both planners in Figure 5. The higher information term for SP3 is due to the fact that consumption booms less under this planner than under SP2, which results in higher expected marginal utility. Given that the externality components for SP2 and SP3 are similar, it follows from Equation 22 that the higher interaction term for SP2 is due to the fact that this planner has higher expected externality terms ($E_t^{SP2}\left[\kappa'\mu(b',\varepsilon',\kappa')\frac{\partial q_t^{DEL}(.)}{\partial b'}\right] > E_t^{SP3}\left[\kappa'\mu(b',\varepsilon',\kappa')\frac{\partial q_t^{DEF}(.)}{\partial b'}\right]$), which in turn result from the steeper DEL collateral pricing functions in the κ^l regime than in the DEF (see Figure 2) and the fact that both SP2 and SP3 assign more weight to κ^l using the true Markov-switching probabilities across financial regimes.

3.3 Sensitivity Analysis

This sub-section presents the results of two alternative parameterizations aimed to illustrate how the findings of the baseline scenario vary as we change the key parameter values related to the structure of the learning dynamics (particularly the initial priors). This is important because there is a great deal of uncertainty about the values of these parameters, and as we indicated earlier, in this regard the baseline parameterization is more a benchmark to start the quantitative analysis than a calibration tied closely to robust empirical estimates. We will show in particular that the baseline result showing that debt and land price dynamics of SP1 and DEL are similar, and hence that SP1's macro-prudential policy makes little difference during the optimistic phase, is not a general result.

(a) Gradual Optimism Scenario

Consider a parameterization under which initial priors are still the same for the government and the private sector, but they are constructed so that optimism builds more gradually in the

[14] Interestingly, the size of the externality tax component is comparable in magnitude to the debt taxes estimated by Bianchi and Mendoza (2010) in a model with a similar collateral constraint but with a constant κ, production with labor and working capital financing of wages, and rational expectations formed with full information.

early stages of financial innovation than in the baseline. In particular, the priors are set so that the mean beliefs are equal to the true transition probabilities and the initial counters are not symmetric across the four transitions. We assume that $n_0^{hh} = 7.6$, higher than in the baseline, but then set $n_0^{hl} = 0.4$ so that $E_0[F_{hh}^s] = F_{hh}^a = 0.95$, and we keep $n_0^{hl} = 0.02$ as in the baseline, but then set $n_0^{ll} = 0.38$ so that $E_0[F_{ll}^s] = F_{ll}^a = 0.95$. Note that we reduced F_{hh}^a and F_{ll}^a slightly relative to the 0.964 baseline value. This lower persistence of the financial regimes corresponds to a 5-year mean duration for each regime, in line with the more recent evidence in Mendoza and Terrones (2011), where they updated the duration estimates of credit booms with data through 2010. With these priors, learning starts from $E_0[F_{hh}^s] = 0.95$ and rises gradually towards 1, while in the baseline it starts at $E_0[F_{hh}^s] = 0.5$ and jumps to 0.98 with just the first observation of κ^h (by contrast, the gradual optimism scenario reaches 0.98 after 12 observations of κ^h). Keep in mind also that while beliefs start at the true transition probabilities, both private agents and SP1 do not know that, and hence their beliefs shift away from the true probabilities as realizations of κ arrive, until they converge back to the true values in the long run.

This scenario with gradual optimism yields significantly larger differences between the outcomes attained by DEL and SP1 (see Figure 7). In the run-up to the crisis, debt levels in SP1 are about 4 percentage points of GDP smaller than DEL. Moreover, during the crash, asset prices are 16 percent higher for SP1 due to the lower leverage at the time of the crisis. Clearly, SP1 accumulates less debt during the transition phase which leads to a smaller crash at $t = 41$, and hence in this scenario macro-prudential policy is more effective even when both private agents and the government face the same learning problem and the same collateral pricing functions.

The key reason for the difference between the baseline and this scenario is that in the baseline the combination of the rapid surge in optimism and the households' impatience leads them to borrow up to the limit, and attain a high shadow value from relaxing the collateral constraint. Since the benevolent planner also considers the high value assigned to current consumption by the households, it also decides to borrow up to the limit. In line with this reasoning, Figure 7 shows that the differences in bond positions between DEL and SP1 narrow as the optimistic phase progresses and the shadow value of relaxing the collateral constraint increases.

In general, we find that the more gradual is the build-up of optimism, the more the collateral constraint is likely to remain slack or be marginally binding during the optimistic phase, and the more effective is macro-prudential policy, even if the planner is as uninformed as private agents. The planners that have full information continue to be significantly more cautious, however, and hence implement more active macro-prudential policies. Interestingly, with gradual optimism SP2 actually reduces the level of debt over time during the entire optimistic phase.

The prices under the DEL, SP1 and SP2 continue to be very similar because the DEL pricing function in the κ^h regime continues to be relatively flat (see Figure 8). The externality itself, however, is actually larger, because the pricing function is again very steep for the κ^l regime and the gradual buildup of optimism means that SP1 assigns higher probability to switching to this regime than it did in the baseline. Hence, SP1 levies larger debt taxes in this scenario than in the baseline (see Figure 9). In contrast, SP2 and SP3 charge slightly lower debt taxes. In terms of the components of the debt tax (Figure 10), gradual optimism lowers the magnitude of the information component for SP2 and SP3 (recall it is always zero for SP1). The interaction term is now positive and large for SP1, compared with zero in the baseline, and the interaction terms for SP2 and SP3 now display higher values in the initial stages of the optimistic phase, declining to values in the 0.035-0.04 range, whereas in the baseline they started around 0.01 and rose gradually during the optimistic phase to the 0.05-0.06 range. By contrast, the externality component of the taxes rises sharply for all three planners under the gradual optimism scenario, relative to the baseline. This is in line with the previous findings indicating that the gradual buildup of optimism enlarges the externality and creates more room for macro-prudential policy.

(b) Asymmetric priors between government and private agents

Consider now a scenario in which we return to the baseline parameterization, but the government and the private sector do not start with the same priors. In particular, consider a planner with $p_0^{hh} = p_0^{ll} = 0.2$ which we label SP4 to facilitate comparison with SP1, who had the same initial beliefs as the private sector. We maintain the assumption of a symmetric distribution of priors for the planner, so that at date $t = 0$ we have $E_0^p[F_{hh}] = E_0^p[F_{ll}] = 0.5$.

With SP4 having higher initial counters for the persistence of each regime, we alter significantly the perception of riskiness of the financial environment that SP4 has, compared with DEL and SP1. For instance, at date $t = 1$ after the first realization of κ^h is observed, SP4 expects the mean duration of the κ^h regime to be about 6 quarters while the private agents and SP1 expect a mean duration of 50 quarters under the baseline calibration of $n_0^{hh} = 0.0205$. Hence, SP4 perceives more riskiness in the financial environment inasmuch as he believes the mean duration of the good credit regime will be significantly shorter. Moreover, this scenario has a feature similar to the previous one in that optimism builds more gradually for SP4 than for the DEL and SP1. The evolution of the mean beliefs under SP4 and the DEL are plotted in Panels (f) and (d) of Figure 11, respectively.

Notice also that SP4 continues to face the DEL pricing function distorted by the agents' optimism, which remains the same as in the baseline.

Panel (a) of Figure 11 reveals that introducing asymmetric beliefs to make SP4 perceive more risk once again results in an outcome in which a planner subject to learning and facing the same collateral prices of the DEL chooses lower debt positions than in the DEL. In fact, SP4's debt levels are also lower than those of SP1 plotted in Figure 1.[15] This is because of the combination of the higher perception of risk, the more gradual buildup of optimism, and the fact that under the influence of these forces the collateral constraint is not binding for SP4 under the entire optimistic phase. Given a uniformly higher externality term, as plotted in Panel (e), and uniformly less optimistic beliefs than SP1, SP4 chooses lower debt levels, and not up to the point where the constraint binds. In fact, SP4 only hits the borrowing limit when the economy switches to the κ^l state. This is evident in the shadow price being almost always zero in Panel (c) of Figure 11 except in period 41 and the last few periods of the experiment.

It is also interesting to note in Panel (e) of Figures 11 and 1 that the dynamics of the externality term have similar shapes for SP4 and SP1. The main difference is in that the levels are uniformly higher for SP4. Similar forces are at play for these planners, initially the fast buildup of debt relative to the buildup of optimism increases the probability assigned to the constraint becoming binding at date $t+1$. After about period 10, the buildup of debt slows down and this effect is dominated by the beliefs becoming more optimistic over time leading to a weakening of the externality as these planners assign smaller probabilities to a switch to κ^l, where the derivative of the pricing function, $\frac{\partial q_t^{DEL}(b',1,\kappa^l)}{\partial b'}$, is large. The externality term is uniformly larger for SP4 than SP1 because its beliefs are uniformly less optimistic. SP4 always assigns a higher weight to the κ^l regime where the derivative of the pricing function, $\frac{\partial q_t^{DEL}(b',1,\kappa^l)}{\partial b'}$ is large.

The price dynamics of SP4 are very similar to those of DEL. The lower debt choices of this planner do not translate into large differences in the price given the flatness of the pricing function.[16] In fact, SP4 prices are slightly above those of DEL since lower debt positions are associated with higher land prices.

Consistent with the externality being large, SP4 levies taxes on debt that are higher than in the baseline and also higher than in the gradual optimism scenario. Moreover, for SP4 the interaction component of the debt tax is the largest almost throughout the entire experiment, as was the

[15] The comparison of SP4 with SP1 is very relevant because both of them go through a learning process and face the same collateral pricing function.

[16] Since DEL prices do not change when we change the priors of the planner, we do not re-plot them here.

case for SP2 and SP3 in the baseline. Hence, our finding that the interaction of financial and information frictions play a key role in the design of macro-prudential policy remains robust to considering a social planner with beliefs different from those of private agents or from the true rational expectations, and this was also the case in the gradual optimism scenario.

4 Conclusion

This paper provides a quantitative dynamic stochastic general equilibrium framework for studying macro-prudential policy that incorporates two key elements of the financial amplification mechanism: Imperfect information about the true riskiness of new financial regimes and a credit constraint that limits the debt of agents to a fraction of the market value of their assets. The fraction of the value of assets that can be pledged as collateral increases with financial innovation, but risk also increases because this collateral coefficient also becomes stochastic, and the persistence of regimes with high and low ability to borrow needs to be learned over time. As learning progresses, agents go through waves of optimism and pessimism which distort their debt decisions and hence equilibrium asset prices. In addition, the credit constraint introduces a pecuniary externality whereby individual agents do not internalize the effect of their borrowing decisions on equilibrium prices. The interaction of waves of optimistic and pessimistic beliefs with this pecuniary externality produces a powerful amplification mechanism that can yield large increases in debt and asset prices in a decentralized competitive equilibrium.

We study the effects of macroprudential policies in the form of Pigouvian taxes on debt and dividends in this environment, considering three different social planners that face different information sets and feasible credit positions. The first planner faces a similar learning problem as private agents and faces the collateral pricing function of the DEL (i.e., the same set of feasible credit positions). The second planner has full information about transition probabilities across financial regimes but faces the same collateral pricing function as the first planner. The third planner has full information and in addition it faces the collateral pricing function of a rational expectations equilibrium with full information (i.e. the set of feasible credit positions of this equilibrium).

In a baseline calibration to U.S. data, the third social planner supports debt positions and land prices that are much lower than those in the DEL, and hence faces smaller corrections in debt, consumption and land prices when financial crises hit. The second planner supports debt allocations only slightly larger than the third, but land prices are as inflated as in DEL. Finally, the first planner supports allocations and prices that deviate only slightly from those of DEL. Thus, in the baseline parameterization, macro-prudential policy is significantly more effective when the planner has full information and can support collateral values free from the effect of optimistic beliefs, and has negligible effects when the planner is subject to the same subjective beliefs and collateral pricing conditions of the DEL. Sensitivity analysis shows, however, that by varying the structure of the learning setup, particularly the pace at which optimism builds in the early stages of financial innovation, and the tigthness of the borrowing constraint during the optimistic phase of a credit boom, it is possible even for the first planner to use macro-prudential policy to attain different equilibrium debt than DEL and thus improve the performance of the economy during financial crises.

These results highlight the importance of considering the information set of policymakers in the design of macro-prudential policies. If regulators operate with the same incomplete information set as the private agents, the effects of these policies are more limited and can even be negligible. This is particularly important in a boom-bust cycle in credit largely driven by financial innovation, about which the regulators are likely to be just as uninformed as the private agents. If on the other hand, in a credit boom episode where the private agents operate under incomplete or misleading information while the regulators can acquire better information, say by looking at similar previous episodes in the history of the country or other countries in similar situations, then the macro-prudential policy has good potential to contain the amplitude of the boom-bust cycle.

An interesting extension of our framework would be to model agents with heterogenous beliefs as in Cao (2011) who shows that heterogeneous beliefs under incomplete markets can have important implications for asset prices and investment variability. Financial innovation in such a framework may generate asset price and debt dynamics that are closer to data and potentially shed further light on the effectiveness of macro-prudential policy.

5 Appendix: Recursive Optimization Problems

We assume that agents make decisions according to the anticipated utility approach. Accordingly, the recursive optimization problem can be written as

$$
\begin{aligned}
V_t(b,k,B,\varepsilon) &= \max_{b',k',c} \frac{c^{1-\sigma}}{1-\sigma} + \beta E_t^s \left[V_{t+1}(b',k',B',\varepsilon')\right] \qquad (23)\\
s.t. \quad q(B,\varepsilon)k' + c + \frac{b'}{R} &= q(B,\varepsilon)k + b + \varepsilon F(k)\\
B' &= \Gamma(B,\varepsilon)\\
\frac{b'}{R} &\leq \kappa q(B,\varepsilon)k'
\end{aligned}
$$

Notice that the value function is indexed by t because beliefs are changing over time. In rational expectations, instead, the value function would be a time-invariant function of the individual and aggregate state variables.

(**Recursive Competitive Equilibrium**)
The (AU) recursive competitive equilibrium under is defined by a subjective conditional-expectation operator E_t^s, an asset pricing function $q_t(B,\varepsilon)$, a perceived law of motion for aggregate bond holdings $\Gamma_t(B,\varepsilon)$, and a set of decision rules $\left\{\hat{b}_t'(b,k,B,\varepsilon), \hat{k}_t'(b,k,B,\varepsilon), \hat{c}_t(b,k,B,\varepsilon)\right\}$ with associated value function $V_t(b,k,B,\varepsilon)$ such that:

1. $\left\{\hat{b}_t'(b,k,B,\varepsilon), \hat{k}_t'(b,k,B,\varepsilon), \hat{c}_t(b,k,B,\varepsilon))\right\}$ *and* $V_t(b,k,B,\varepsilon)$ *solve (23), taking as given* $q_t(B,\varepsilon)$, $\Gamma_t(B,\varepsilon)$.

2. *The perceived law of motion for aggregate bonds is consistent with the actual law of motion:* $\Gamma_t(B,\varepsilon) = \hat{b}_t'(B,\bar{K},B,\varepsilon)$.

3. Land prices satisfy $q(B,\varepsilon) = E_{\varepsilon'|\varepsilon}\left\{\frac{\beta u'\left(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right)\ \left[\varepsilon' F_k(\bar{K},\varepsilon') + q(\Gamma_t(B,\varepsilon),\varepsilon')\right]}{u'\left(\hat{c}(B,\bar{K},B,\varepsilon)\right) - \kappa \max\left[0, u'\left(\hat{c}(B,\bar{K},B,\varepsilon)\right) - \beta R E_{\varepsilon'|\varepsilon} u'(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right]}\right\}$

4. Goods and asset *markets clear:* $\frac{\hat{b}'(B,\bar{K},B,\varepsilon)}{R} + c(B,\bar{K},B,\varepsilon) = \varepsilon f(\bar{K}) + B_t$ and $\hat{k}(B,\bar{K},B,\varepsilon) = \bar{K}$

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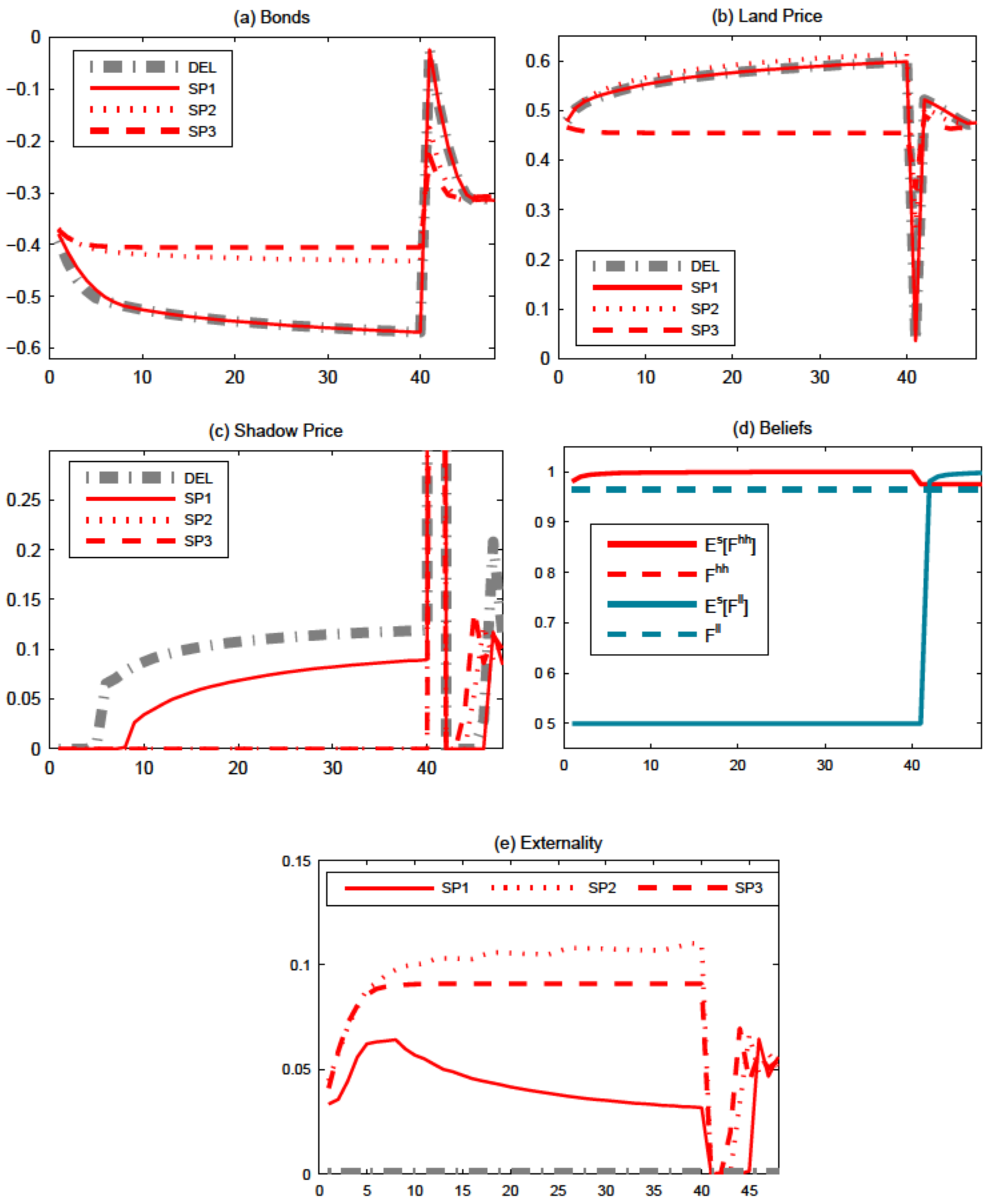
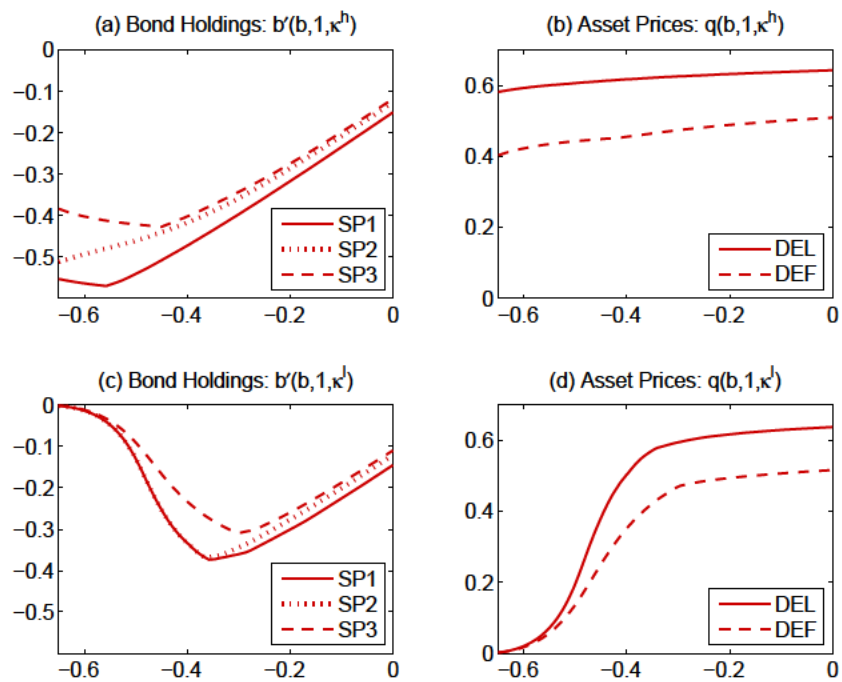
Figure 1: Forecast Functions: Baseline Calibration



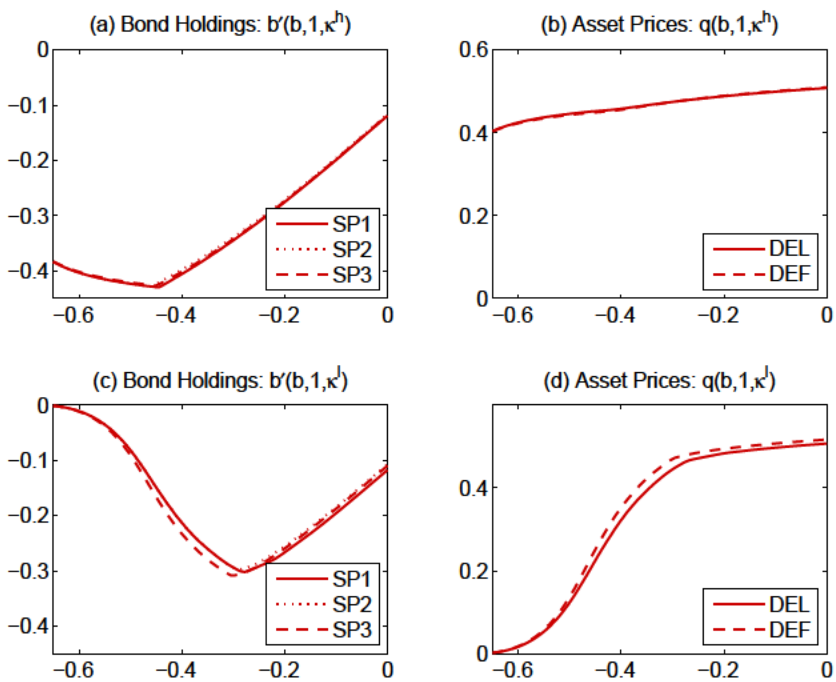
Notes: DEL: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEL, SP3: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 2: Period 40 Bond Holdings and Asset Prices



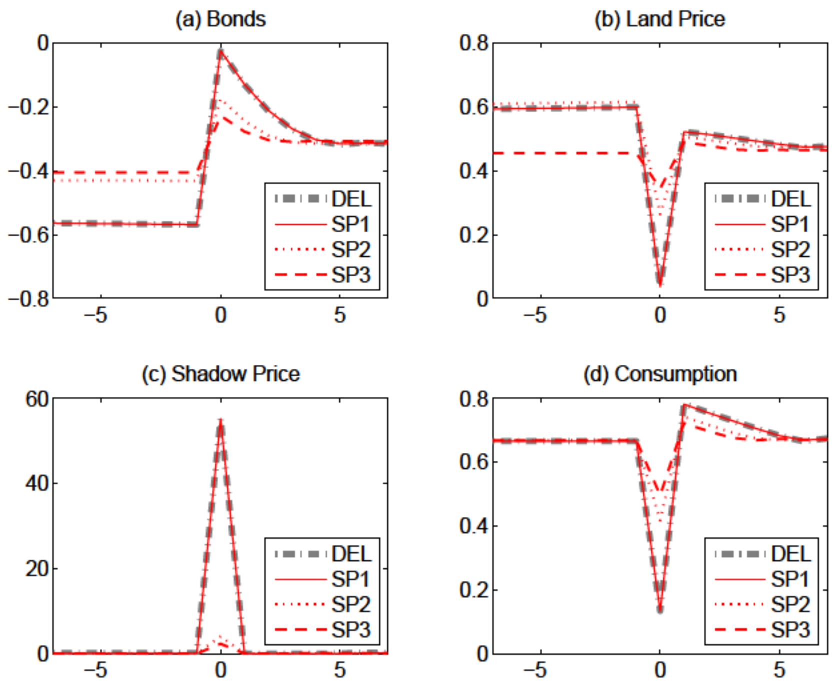
Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP3: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.

Figure 3: Period 41 Bond Holdings and Asset Prices



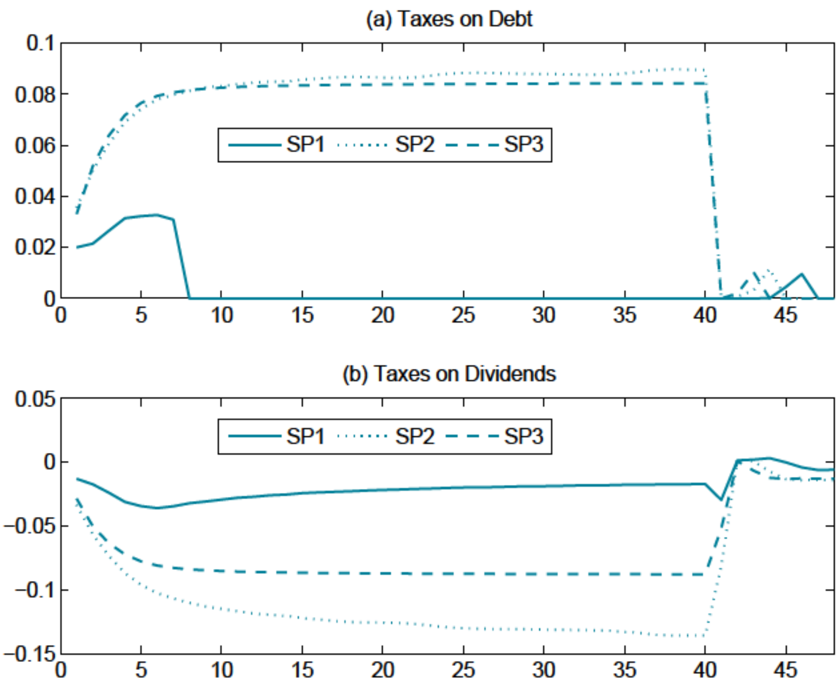
Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP3: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.

Figure 4: Crisis Episode



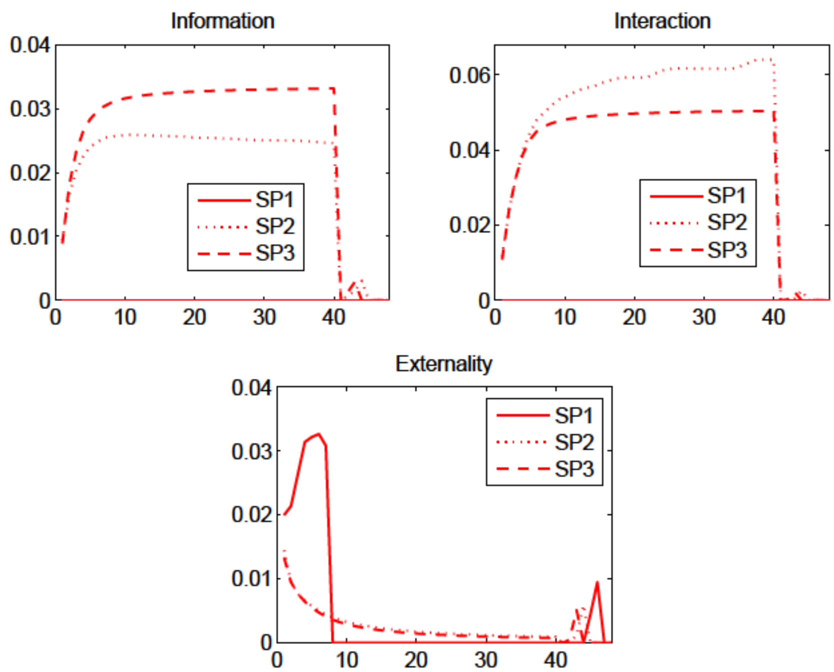
Notes: This figure plots the time series dynamics in periods 41 $\pm$ 7. DEL: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEL, SP3: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 5: Taxes on Debt and Land Dividends



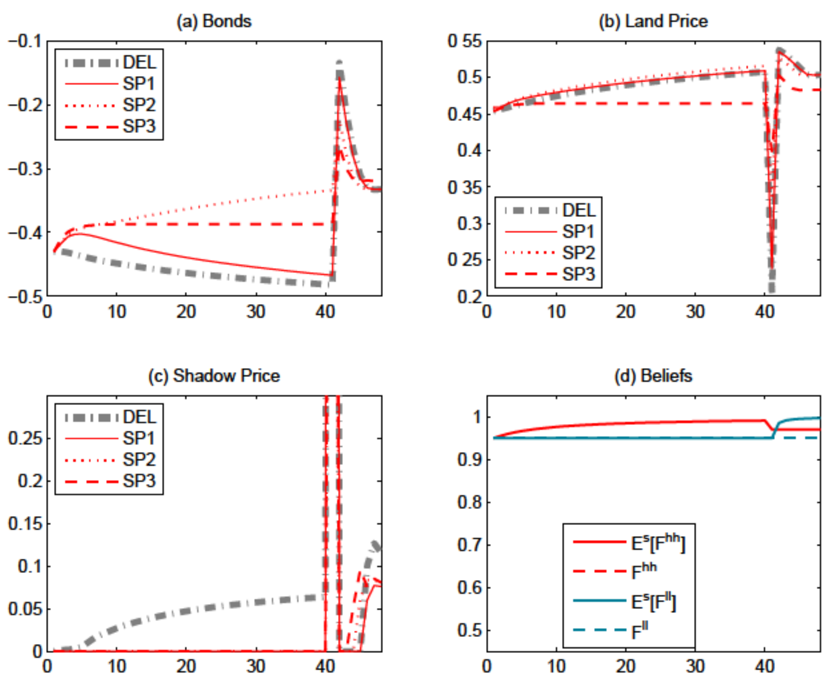
Notes: This figure plots the taxes on debt and on land dividends that support the corresponding planners allocations as competitive equilibrium. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEL, SP3: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 6: Decomposition of Taxes on Debt



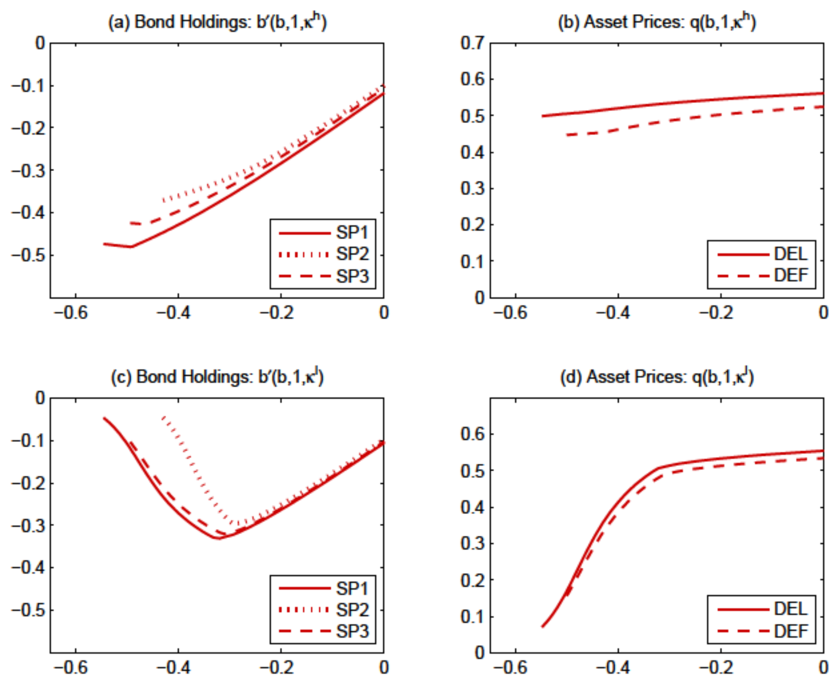
Notes: This figure plots the decomposition of taxes on debt to three distinct parts: 'information' arises due to the differences in the expectation of one period ahead consumption between private agents and the social planner, 'externality' captures the pecuniary externality, 'interaction' is due to the differences in the expectation of the one period ahead externality between private agents and the social planner. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEL, SP3: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 7: Forecast Functions: Gradual Optimism



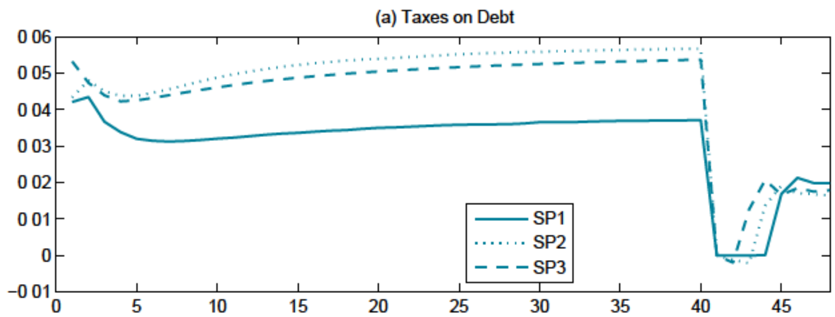
Notes: DEL: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEL, SP3: Social planner with full information implementing the set of feasible credit positions of DEF.

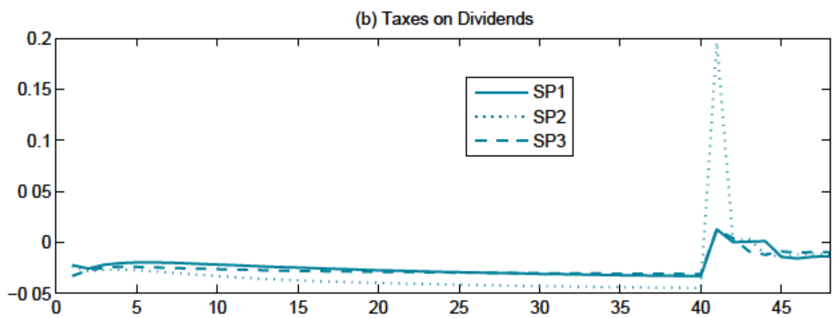
Figure 8: Period 40 Bond Holdings and Prices: Gradual Optimism



Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP3: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.

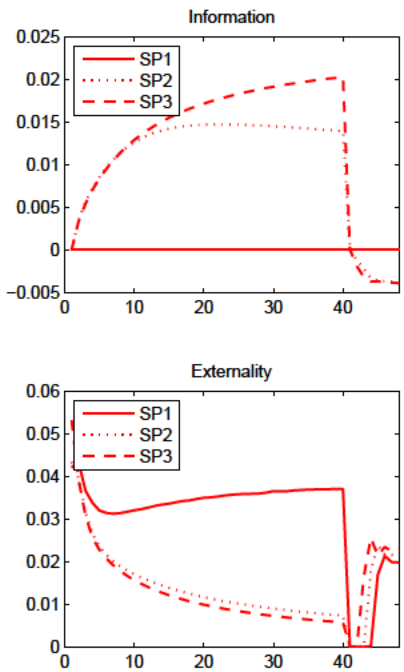
Figure 9: Taxes on Debt and Land Dividends: Gradual Optimism





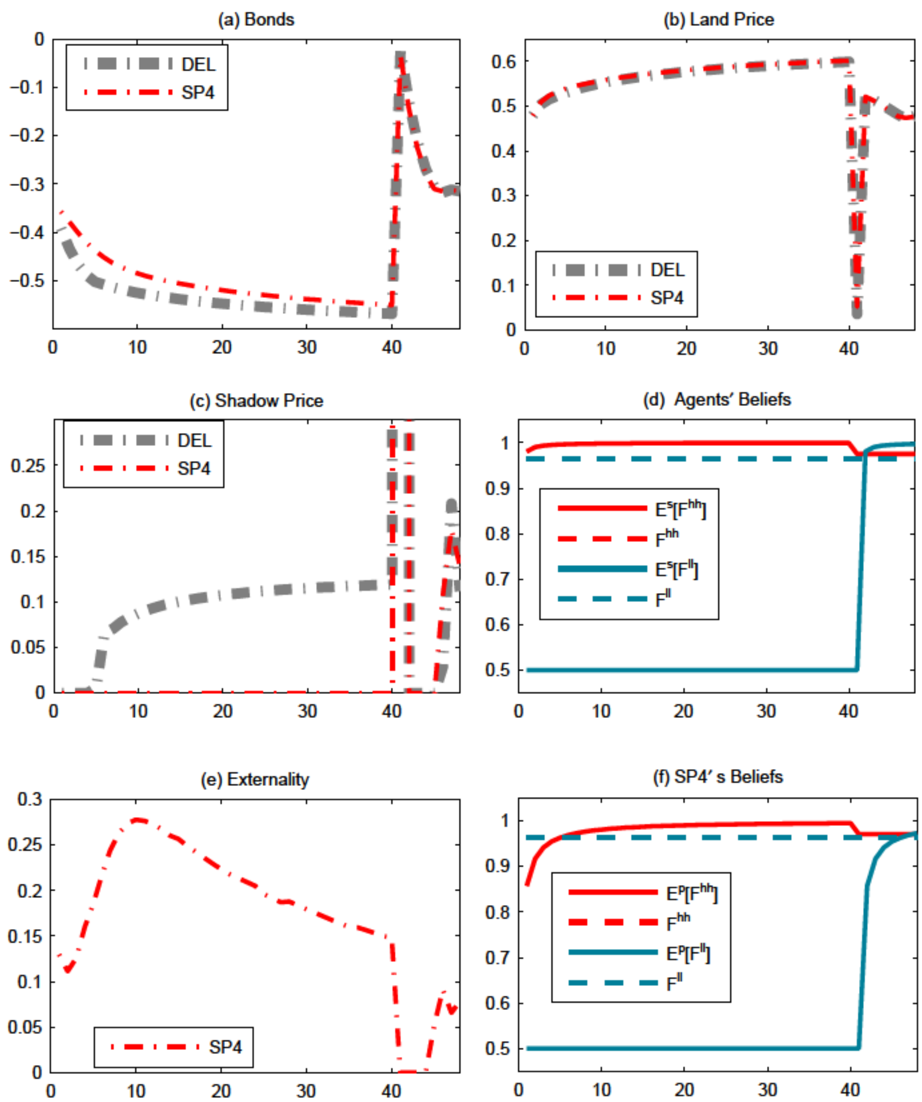
Notes: This figure plots the taxes on debt and on land dividends that support the corresponding planners allocations as competitive equilibrium. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEL, SP3: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 10: Decomposition of Taxes on Debt: Gradual Optimism



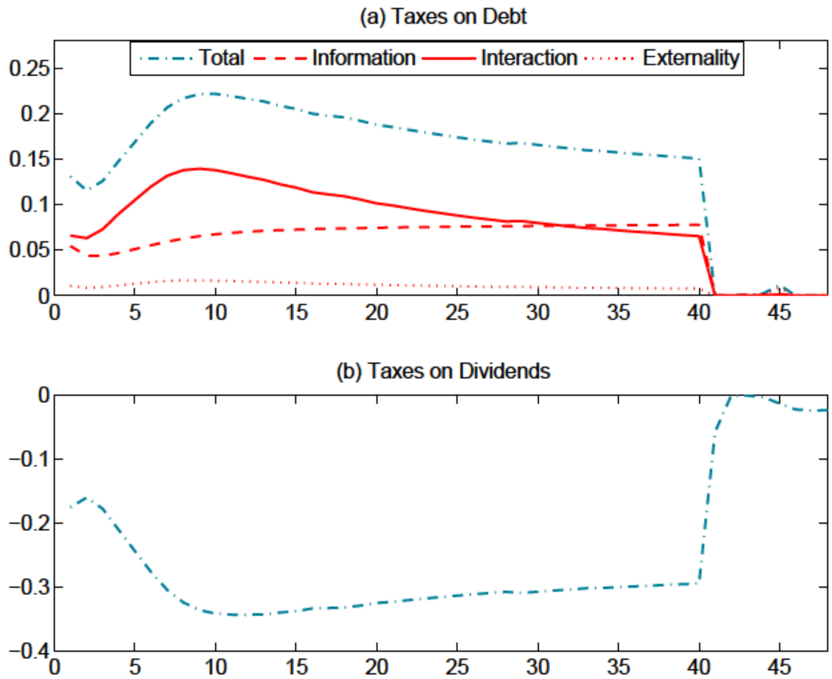
Notes: This figure plots the decomposition of taxes on debt to three distinct parts: 'information' arises due to the differences in the expectation of one period ahead consumption between private agents and the social planner, 'externality' captures the pecuniary externality, 'interaction' is due to the differences in the expectation of the one period ahead externality between private agents and the social planner. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEL, SP3: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 11: Forecast Functions: Asymmetric Priors



Notes: DEL: Imperfect information decentralized equilibrium, SP4: Social planner with imperfect information and different priors than private agents implementing the set of feasible credit positions of DEL.

Figure 12: Taxes on Debt and Land Dividends: Asymmetric Priors



Notes: This figure plots the taxes on debt and on land dividends that support the corresponding planners allocations as competitive equilibrium for SP4. (SP4: Social planner with imperfect information and different priors than private agents implementing the set of feasible credit positions of DEL.) Top panel decomposes taxes on debt to three distinct parts: 'information' arises due to the differences in the expectation of one period ahead consumption between private agents and the social planner, 'externality' captures the pecuniary externality, 'interaction' is due to the differences in the expectation of the one period ahead externality between private agents and the social planner.