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OPTIMAL MONETARY POLICY WITH ENDOGENOUS CAPITAL AND A CREDIT FRICTION

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Abstract

This paper investigates optimal monetary policy in a New Keynesian economy with endogenous capital and a credit friction. Introducing a credit friction affects the normative objectives of the central bank. It provides a central bank with the incentive to stabilise volatility in the net worth of borrowers. This is in addition to the traditional objectives of monetary policy. Reducing volatility in net worth implies a reduction in the volatility of default rates, asset prices and investment. Quantitatively, inflation targeting remains a good approximation of monetary policy when steady state credit frictions are small. For larger frictions, in the order of the magnitude estimated for the US and Euro areas, some tolerance of inflation may be optimal in response to contractionary financial shocks.

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1. Introduction

> A relevant question is why should we be interested in whether an asset price, or indeed any other variable for that matter, appears in some ad hoc class of feedback rule, even though the coefficients of that rule may have been optimised? It seems more instructive to ask first what an optimal rule looks like, and then consider how asset prices ought to figure in it. One might go on to consider whether particular simple rules represent sufficiently close approximations to the optimal rule to be useful guideposts for policy...
>
> Charles Bean (2003)

This paper is concerned with the objectives of a central bank when implementing monetary policy optimally in an economy with capital, and a credit friction. The aim is to provide insight into whether volatility in financial variables, such as asset prices, affect the path of interest rates optimally chosen by a central bank. In particular, I consider whether asset prices or other financial variables form part of a central bank's stabilisation goals in their own right; or, alternatively, whether such variables are only important to the extent that they are useful for forecasting the output gap and inflation.

There has been much debate on the objectives of a central bank when faced with asset price and financial volatility. Prior to the recent financial crisis, this debate has typically focused on the question of whether asset price volatility should be incorporated into a policymaker's objective. For example, much of the discussion has centered on whether a central bank should look to smooth or "lean against" large movements in asset prices, in either direction, or whether the central bank should respond asymmetrically, using policy only to 'clean up' or stabilise the economy after an asset price bust. Following the recent financial crisis, this debate has been re-invigorated. There is now a wider discussion of the appropriate objectives for policy when faced with volatility in asset and financial markets, and whether monetary policy ought to be used, in conjunction with other macroeconomic tools, to help promote financial stability.

Notwithstanding extensive discussion in policymaking circles, theoretical literature on a central bank's objectives in an economy with capital and financial frictions is limited. There is analytical work addressing the central bank's objectives in a New Keynesian economy with capital but no credit friction – see for example, Edge (2003), and Takamura, Watanabe and Kudo (2006). There is also work addressing optimal monetary policy in an economy with a credit friction but no capital – see for example, Cúrdia and Woodford (2010) and De Fiore and Tristani (2009). However, the central bank's objectives in an economy where both capital and financial frictions play an important role remains an open question. Providing analytical insight that addresses this question is important for determining the objectives of monetary policy, especially if policymakers believe endogenous capital and financial frictions are important factors that influence economic volatility and inflation.

The methodology used in this paper is the Linear Quadratic or LQ approach empahsised by Woodford (2003). Specifically, I derive a first-order approximation of the solution to an optimal monetary policy problem for a central bank concerned with maximising social welfare.[1] The advantage of the LQ approach is that it provides insight into the objectives of a central bank, and whether there exists a meaningful trade-off in achieving these objectives. This analysis complements previous analytical work undertaken, such as that of Cúrdia and Woodford, and De Fiore and Tristani, and provides additional insight into the findings of previous numerical work on optimal policy, such as that undertaken by Faia and Monacelli (2007).

To preview the main result, I find that the main financial variable of interest to the central bank in an economy with a credit friction and capital is the net worth of borrowers. A central bank implementing policy optimally will look to mitigate

1 Optimal monetary policy here refers to "optimal monetary policy from a timeless perspective" as discussed by Woodford (2003).

volatility in net worth precisely because it is the net worth of borrowers that affects the extent to which a credit friction distorts the economy over time. The incentive to smooth net worth exists in addition to the traditional objectives for monetary policy, that include stabilising volatility in inflation and volatility in the composition of output. The existence of the credit friction, in addition to the price friction, introduces a trade-off for policy, and thus a strict inflation target is no longer equivalent to the optimal policy plan.

Although a policymaker is concerned with stabilising net worth qualitatively, there remains the question of whether this incentive is important quantitatively. The analytical results established here, derived under the assumption that the steady state credit friction is small, highlight that although there is a trade-off between stabilising inflation and mitigating the credit distortion, quantitatively this trade-off is not found to be large. As a consequence, inflation targeting remains a good approximation of optimal policy when the credit friction is not highly distortionary.

I also examine the case of a more distortionary steady state credit friction numerically. Using a calibration of the model similar to that estimated by Queijo von Heideken (2009), for the US and Euro area economies, I find that, in contrast to the small friction case, monetary policy can find it optimal to tolerate some deviation of inflation from target in response to financial shocks. This result confirms that the extent to which policymakers wish to stabilise net worth in the economy is contingent on the extent to which the credit friction is distortionary.

The next section provides a brief sketch of the New Keynesian model with a flexible rental market for capital and an endogenous credit friction. Section 3 discusses an analytical representation of the approximate optimal policy problem, including the objectives of the policymaker and the optimal policy plan. Section 4 considers how sensitive the findings are to the magnitude of the credit friction. Conclusions are drawn in the final section.

2. The Credit Friction Model

The economy I consider is a New Keynesian economy with capital and a credit friction. The economic environment is very similar to those studied by Carlstrom and Fuerst (1997; 2001) and Faia and Monacelli (2007). However, some small departures from the standard setup are considered to facilitate simplification of the optimal policy analysis. I include a brief review of the full economic environment, and draw attention to the departures from the standard credit friction model.[2]

Households

There is a continuum of identical individual households uniformly distributed on the interval $[0,1]$. Each household, indexed by h, supplies specialised labour $(H_t(h))$ and consumes $(C_t(h))$. Households are able to save by purchasing investment goods $\left(I_t^h(i)\right)$ from entrepreneurs that deliver physical capital $\left(K_{t+1}^h(i)\right)$ for use in an industry i in the period $t+1$, where $i \in [0,1]$. Alternatively, households may save by purchasing a portfolio of state-contingent securities $E_t\left(M_{t,t+1}A_{t+1}^h\right)$ where M_{t+1} is the price of a security delivering one unit of consumption if state s_{t+1} is realised.[3] Households receive income from their portfolio of previously purchased state-contingent securities, A_t^h, rents from capital held across industries $\left(P_t \int_0^1 R_t(i) K_t^h(i)\, di\right)$, wages for their specialised labour $(P_t W_t(h))$, and are the residual claimants to any profits from production $P_t \int_0^1 D_t^h(i)\, di$. Formally, a household h chooses a sequence of specialised labour supply, consumption, Arrow-Debreu securities and investment

2 For the reader already familiar with the Carlstrom and Fuerst model, the main departures are:

(a) I focus on an equilibrium where entrepreneurs save all of their available resources until retirment (see Carlstrom and Fuerst 2001). This is in contrast to the more standard assumption in the literature that entrepreneurs are indifferent between saving and consuming;

(b) I assume that in steady state the credit friction distortion is small (of second-order); and

(c) I assume a social welfare function that ensures that monetary policy abstracts from any incentive to redistribute consumption between households and entrepreneurs.

3 I make the usual assumptions regarding the stochastic structure of the economy; specifically, I assume it exists on a non-degenerate probability space $\{\Omega, \mathscr{F}, P\}$ where a history at time t is denoted by $s^t \in \Omega$,and a state, s_t, is an element of a given history (for example, $s^t \equiv \{s_1, \ldots, s_t\}$). E_t is the time t conditional expectation. I use the generic notation for a random variable $X_t \equiv X_t(s_t)$.

goods $\left\{H_t(h), C_t(h), A^h_{t+1}(s_{t+1}), I^h_t(i)\right\}_{t=t_0}^{\infty}$, for all possible states $s_{t+1} \in \Omega$ and $i \in [0,1]$, that solve

$$\max E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U(C_t(h), \psi_t) - V(H_t(h))\right]$$

subject to the flow budget constraint

$$\begin{aligned} P_t C_t(h) + \int_0^1 Q_t(i) I^h_t(i)\, di + E_t\left(M_{t,t+1} A^h_{t+1}\right) \leq\ & A^h_t + P_t W_t(h) H_t(h) - P_t T_t \\ & + P_t \int_0^1 R_t(i) K^h_t(i)\, di \\ & + P_t \int_0^1 D^h_t(i)\, di \end{aligned}$$

and the capital accumulation constraint

$$K^h_{t+1}(i) = (1-\delta) K^h_t(i) + I^h_t(i)$$

where T_t is a real lump-sum tax, $W_t(h)$ is the real wage paid for household h's labour, $R_t(i)$ is the real return to capital held in industry i, $Q_t(i)$ is the nominal price of an investment good in industry i, δ is the depreciation rate of capital that is common across industries, and ψ_t is a taste shock common to all households.

The Euler equations in a symmetric equilibrium where all households are the identical, including in their initial endowments ($A^h_{t_0} = A_{t_0}$ for all h), and are indifferent to their allocation of investment across industries, are given by

$$U_c(C_t, \psi_t) = \beta E_t\left(U_c(C_{t+1}, \psi_{t+1})\left(1 + i^b_t\right)\frac{P_t}{P_{t+1}}\right) \tag{1}$$

$$q_t(i) = \beta E_t\left(\frac{U_c(C_{t+1}, \psi_{t+1})}{U_c(C_t, \psi_t)}\left(R_{t+1}(i) + (1-\delta) q_{t+1}(i)\right)\right) \tag{2}$$

$$\frac{V_H(H_t(h))}{U_c(C_t, \psi_t)} = W_t(h) \tag{3}$$

where $C_t = C_t(h)$ for all $h \in [0,1]$, and $q_t(i) \equiv \frac{Q_t(i)}{P_t}$ is the real price of an investment good. It should be noted that Tobin's $q_t(i)$ reflects the shadow price of a unit of capital installed for use in $t+1$ in industry i (or equivalently, is the share price of a firm that holds capital in industry i on the households' behalf).

Entrepreneurs (Investment Supply)

There is a continuum of entrepreneurs in the economy, who lie on the interval $[1, 1+\zeta]$. Entrepreneurs are risk neutral and supply investment goods to households through their access to a risky investment technology. An entrepreneur, indexed by j, is able to purchase $I_t(j)$ units of final output and invest it in a risky project that yields $\omega_t(j)I_t(j)$ units of final capital, which is then be sold to households at price Q_t.[4] The stochastic technology underpinning an entrepreneurs' risky project is idiosyncratic and identically distributed with an exogenous cumulative distribution function $\Phi_\omega(x) \equiv \Pr(\omega_t \leq x)$, that is common for all entrepreneurs, and satisfies the first and second moment conditions $\int_{-\infty}^{\infty} x d\Phi_\omega(x) = 1$ and $\int_{-\infty}^{\infty} (x-1)^2 d\Phi_\omega(x) = \sigma_\omega^2$.

Entrepreneurs have limited net worth, and use financial intermediaries (banks) to leverage their project. The amount they borrow, $L_t(j)$, is defined by the total value of their investment, $P_t I_t(j)$, less their net worth, $NW_t(j)$ so that

$$L_t(j) \equiv P_t I_t(j) - NW_t(j)$$

Entrepreneurs' net worth is determined by their capital income and a small government fiscal subsidy (F^e) financed by a lump-sum tax on households,

$$NW_t(j) = P_t F_t^e + (P_t R_t + Q_t(1-\delta)) K_t^e(j) \tag{4}$$

where $K_t^e(j)$ is the stock of physical capital held by entrepreneur j. The government subsidy is a simplification that provides some starting capital for entrepreneurs, and allows me to abstract from their labour supply decision (see for example, De Fiore and Tristani 2009). This is a simplification that assists the optimal policy derivations that follow.

4 I assume that all investment goods are perfectly substitutable. And so, entrepreneurs will all receive the same price for investment goods they sell.

I assume that entrepreneurs borrow from perfectly competitive banks.[5] The default threshold, $\overline{\omega}_t(j)$, for an entrepreneur is defined by

$$\overline{\omega}_t(j) \equiv \frac{\left(1 + R_t^L(j)\right)(P_t I_t(j) - NW_t(j))}{Q_t I_t(j)}$$

where $R_t^L(j)$ is the interest rate charged by banks on a loan to entrepreneur j. Entrepreneurs are only able to repay their loans when their investment return satisfies $\omega_t(j) \geq \overline{\omega}_t(j)$. In the event that $\omega_t(j) < \overline{\omega}_t(j)$, entrepreneurs default and receive a zero payoff.

Financial Intermediaries

To model the credit friction I assume asymmetric information in the form of costly state verification for banks (Carlstrom and Fuerst 1997). Although entrepreneurs can observe $\omega_t(j)$ after their investment return has been realised, for banks to observe $\omega_t(j)$ they must pay a verification or bankruptcy cost, $\nu_t \in [0,1]$, that is proportional to the nominal resale value of an entrepreneurs investment income. I allow for exogenous time variation in the proportion of a project that is lost in the event of bankruptcy. Or, more conveniently in the optimal policy derivations that follow, I focus on the renormalised exogenous shock $\xi_t \equiv \ln(1-\nu_t)$, which can be considered, approximately, as a shock to the proportion of funds recovered by a bank in the event of default. This recovery rate shock is always negative.

Under the optimal contract, banks only pay the verification cost when an entrepreneur defaults and does not repay their loan (Gale and Hellwig 1985). Following Faia and Monacelli (2007), the banks participation constraint can be written as

$$Q_t I_t(j)\, g(\overline{\omega}_t(j), \nu_t) \geq P_t I_t(j) - NW_t(j)$$

5 Provided that the expected return from the investment project is large enough, entrepreneurs will be willing to invest their entire net worth in their project and to borrow from banks.

where

$$g\left(\overline{\omega}_t(j), \nu_t\right) \equiv 1 - \nu_t \Phi\left(\overline{\omega}_t(j)\right) - f\left(\overline{\omega}_t(j)\right) \tag{5}$$

$$f\left(\overline{\omega}_t(j)\right) \equiv \int_{\overline{\omega}_t(j)}^{\infty} \left(\omega_t(j) - \overline{\omega}_t(j)\right) d\Phi\left(\omega_t(j)\right) \tag{6}$$

$g\left(\overline{\omega}_t(j), \nu_t\right)$ can be interpreted as the expected share of investment income accruing to banks that lend to entrepreneur j, and $f\left(\overline{\omega}_t(j)\right)$ can be interpreted as the expected share of investment income accruing to entrepreneur j.

The Optimal Contract

The optimal contract consists of a default threshold and investment level $\{\overline{\omega}_t(j), I_t(j)\}$, chosen by the entrepreneur, that maximises their return subject to the banks' participation constraint. Entrepreneurs solve

$$\max_{\{\overline{\omega}_t(j), I_t(j)\}} Q_t I_t(j) f\left(\overline{\omega}_t(j)\right)$$
$$\text{subject to: } Q_t I_t(j) g\left(\overline{\omega}_t(j), \nu_t\right) \geq P_t I_t(j) - NW_t(j)$$

The optimal contracting conditions, after aggregation, are

$$q_t f\left(\overline{\omega}_t\right) = \frac{f_\omega\left(\overline{\omega}_t\right)}{g_\omega\left(\overline{\omega}_t, \nu_t\right)} \left(q_t . g\left(\overline{\omega}_t, \nu_t\right) - 1\right) \tag{7}$$

$$I_t = \frac{nw_t}{1 - q_t . g\left(\overline{\omega}_t, \nu_t\right)} \tag{8}$$

where $nw_t \equiv \frac{NW_t}{P_t}$, $f_\omega\left(\overline{\omega}_t\right) \equiv \frac{\partial f\left(\overline{\omega}_t\right)}{\partial \overline{\omega}_t}$ and $g_\omega\left(\overline{\omega}_t, \nu_t\right) \equiv \frac{\partial g\left(\overline{\omega}_t, \nu_t\right)}{\partial \overline{\omega}_t}$. It should be noted that in equilibrium all entrepreneurs choose the same default threshold, $\overline{\omega}_t(j) = \overline{\omega}_t$ for all $j \in [1, 1+\zeta]$ from (7). Also note that investment in effect becomes proportional to the aggregate net worth of entrepreneurs, see (8).

Entrepreneurs (Consumption)

Non-defaulting entrepreneurs consume after the returns from their investment decision have been realised.[6] To ensure a well defined and stable equilibrium, I assume entrepreneurs face an exogenous retirement shock, consistent with Carlstrom and Fuerst (2001). Stochastic retirements ensures that when considering equilibria where entrepreneurs save resources through capital, this saving is not sufficient for entrepreneurs to become self-financing. The consumption decision problem for an individual entrepreneur is

$$\max_{\{C_t^e\}} E_{t_0} \sum_{t=t_0}^{\infty} \left(\kappa\beta^e\right)^{t-t_0} C_t^e\left(j\right)$$

$$\begin{aligned} \text{subject to: } & Q_t K_{t+1}^e\left(j\right) + P_t C_t^e\left(j\right) \leq \left(\omega_t - \overline{\omega}_t\right) Q_t I_t\left(j\right) \\ & K_{t+1}^e\left(j\right) \geq 0 \\ & C_t^e\left(j\right) \geq 0 \end{aligned}$$

where $\beta^e \in (0,1)$ is the discount factor for entrepreneurs, $\kappa \in [0,1]$ is the probability than an entrepreneur is not affected by the retirement shock, and $C_t^e\left(j\right)$ denotes consumption by entrepreneur j. It should be emphasised that entrepreneurs are liquidity constrained in the sense that they cannot borrow against expected future project returns to fund additional consumption or saving. It is straightforward to verify that the entrepreneurs' budget constraint will bind, and thus

$$q_t K_{t+1}^e\left(j\right) + C_t^e\left(j\right) = \left(\omega_t - \overline{\omega}_t\right) q_t I_t\left(j\right)$$

To complete the description of the entrepreneurs' first-order conditions, there are several possible equilibria associated with this decision problem. The appropriate choice can vary depending on the steady state that one chooses to approximate around. To provide insight on optimal monetary policy, I focus on two possible equilibria. The first is an equilibrium where entrepreneurs consume all of their available resources, and so

$$K_{t+1}^e\left(j\right) = 0$$

This optimality condition can be supported in a steady state where $\beta^e < \beta$ and $\kappa = 1$. Thus, stochastic retirement is not required since entrepreneurs are assumed to be sufficiently impatient that they are always willing to consume all of their available resources.

As an alternative, I also consider an equilibrium where entrepreneurs save all of their available resources until they are stochastically forced to retire.[7] That is, prior

6 Defaulting entrepreneurs forfeit all their investment proceeds and have no resources to consume. Provided these entrepreneurs are not also hit by the retirement shock, they enter the investment supply market in the next period with only the government fiscal subsidy.

to retirement,

$$C_t^e(j) = 0$$

which can be supported in a steady state where $\kappa < \beta$ and $1 > \kappa\beta^e > \beta$, given that I will latter assume that steady state monitoring costs are small. In this case, the restriction that $\kappa\beta^e > \beta$ ensures that entrepreneurs are sufficiently patient to be willing to save until their retirement. The restriction that $\kappa < \beta$ ensures that entrepreneurs retire frequently enough so that they are unable to become self-financing.[8] The measure of entrepreneurs is assumed to be constant over time so that any entrepreneur forced to retire is replaced by the birth of a new entrepreneur that begins life with capital F_t^e.

Aggregating across the measure of entrepreneurs, the equilibrium conditions are

$$q_t K_{t+1}^e + C_t^e = f(\overline{\omega}_t) q_t I_t \tag{9}$$

and, for the no-saving and saving equilibria respectively,

$$K_{t+1}^e = 0 \text{ if } \beta^e < \beta \text{ and } \kappa = 1 \tag{10}$$

or

$$C_t^e = (1-\kappa) f(\overline{\omega}_t) q_t I_t \text{ if } 1 > \kappa\beta^e > \beta \text{ and } \kappa < \beta \tag{11}$$

Whether entrepreneurs save or not will have a substantial bearing on optimal monetary policy. The reason for this is that if (10) is used, the endogenous

7 One can also consider a third equilibrium where entrepreneurs are indifferent between saving and consuming

$$1 = \beta^e E_t \left(\frac{R_{t+1} + q_{t+1}(1-\delta)}{q_t} \frac{q_{t+1} f(\overline{\omega}_{t+1})}{1 - q_{t+1} g(\overline{\omega}_{t+1}, \nu_{t+1})} \right)$$

However, when approximating equilibria with small monitoring costs, this equilibrium is not stable when solved numerically (i.e. perturbations to the economy result in a switch to either a saving or no saving equilibrium). For the case where entrepreneurs are indifferent, and monitoring costs are large, the equilibrium is stable and results are considered in subsequent numerical comparisons in Section 4.

8 The implied restriction that entrepreneurs are very patient, $\beta^e > 1$, may at first seem counterintuitive. However, it arises because I assume that steady state monitoring costs, and returns to saving, are small This is in contrast to the usual assumption made for numerical work, where steady state monitoring costs, and returns to saving are high, and so entrepreneurs are assumed to be impatient.

evolution of net worth is shut down in the model since entrepreneurs do not save. In contrast, if (11) is used, the endogenous evolution of net worth is retained and will have important consequences for optimal policy. I now briefly review the supply side of the economy, which is standard.

Final Producers

Final good producers combine intermediate goods to produce a final good that can be either consumed either by households or entrepreneurs, or used by entrepreneurs in their risky investment project. Assuming perfect competition in the market for final goods, each period final good producers solve

$$\max_{X_t(i)} P_t Y_t - \int_0^1 p_t(i) X_t(i) di$$

subject to a Dixit-Stiglitz transformation technology between intermediate inputs and final goods

$$Y_t = \left[\int_0^1 X_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{12}$$

where Y_t is output (the final good), $p_t(i)$ is the price of intermediate good i, $X_t(i)$ denotes the quantity of intermediate good i used in production, and θ is the constant elasticity of substitution between intermediate goods used. The demand for intermediate goods produced by firm i is given by

$$X_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t$$

Intermediate Firms

There is a continuum of monopolistically competitive intermediate good producers (firms) on the unit interval. Each firm produces a single differentiated intermediate good using firm-specific labour, and capital. For tractability, I assume that a fully flexible rental market for capital exists that allows capital to be instantaneously reallocated between firms. Consistent with the standard New Keynesian model, only a fraction of firms γ (randomly selected) are able to choose their price optimally each period (Calvo 1983). Firms not able to choose their prices optimally, retain the price they set in the previous period.

Each period all firms, whether they are able to reset their price or not, choose a cost-minimising bundle of labour and capital subject to a Cobb-Douglas production technology. Firm i solves

$$\min_{K_t(i),H_t(i)} \quad R_t(i)K_t(i) + W_t(i)H_t(i)$$

subject to

$$X_t(i) = K_t(i)^{\alpha} \left(Z_t H_t(i)\right)^{1-\alpha} \tag{13}$$

where Z_t can be interpreted as a technology or productivity innovation. Firm i will choose a capital to labour ratio that satisfies

$$\frac{K_t(i)}{H_t(i)} = \frac{W_t(i)}{R_t(i)} \frac{\alpha}{1-\alpha} \tag{14}$$

and the real marginal cost for firm i, $S_t(i)$, is given by

$$S_t(i) = \frac{1}{Z_t^{1-\alpha}} \left(\frac{W_t(i)}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t(i)}{\alpha}\right)^{\alpha} \tag{15}$$

At this point it should be noted that although I assume households supply specialised labour, the capital rental market is perfectly flexible. This assumptions ensures that capital rents across industries are the same in each period $R_t(i) = R_t$, and by implication that there is a unique measure of asset prices across all industries, $Q_t(i) = Q_t$ (see (2)).

There is a second decision for those firms able to reset their price optimally. These firms choose the price of their intermediate good to maximise the expected value of profits distributed to (and thus discounted on behalf of) households. The program for those firms able to choose their price is

$$\max_{\{p_t(i)\}_{t=t_0}^{\infty}} E_t \sum_{t=t_0}^{\infty} \gamma^{t-t_0} M_{t_0,t} \frac{P_t}{P_{t_0}} \left(\left(\frac{p_{t_0}(i)}{P_t} - (1-\chi) S_t(i) \right) X_t(i) \right)$$

subject to the demand for their intermediate good,

$$X_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$$

where the nominal stochastic discount factor is defined as,

$$M_{t_0,t} \equiv \beta^{t-t_0} \frac{U_c(C_t, \psi_t)}{U_c(C_{t_0}, \psi_{t_0})} \frac{P_{t_0}}{P_t}$$

real marginal costs are given by (15), and χ is a fiscal subsidy on real marginal costs that is financed by a lump-sum tax on households. The reason for including this subsidy will be discussed in the next section.

The optimal price chosen by the subset of intermediate firms who can choose their price freely is given by

$$\frac{p_t(i)}{P_t} = (1-\chi)\frac{\theta}{\theta-1}\frac{E_t\sum_{\tau=t}^{\infty}(\beta\gamma)^{\tau-t}\frac{U_c(C_\tau,\psi_\tau)}{U_c(C_t,\psi_t)}P_\tau^{\theta}Y_\tau S_\tau(i)}{E_t\sum_{\tau=t}^{\infty}(\beta\gamma)^{\tau-t}\frac{U_c(C_\tau,\psi_\tau)}{U_c(C_t,\psi_t)}P_\tau^{\theta}Y_\tau} \tag{16}$$

where the aggregate price index is a Dixit-Stiglitz aggregator of the form

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta}\,di\right]^{\frac{1}{1-\theta}} \tag{17}$$

Market Clearing Conditions

The aggregate resource constraint, and market clearing conditions are[9]

$$Y_t = C_t + \zeta C_t^e + \zeta I_t \tag{18}$$
$$K_{t+1} = (1-\delta) K_t + \zeta I_t - \zeta \nu_t \Phi(\overline{\omega}_t) I_t \tag{19}$$
$$K_t = \int_0^1 K_t^h(i)\, di + \zeta K_t^e \tag{20}$$
$$K_t(i) = K_t^h(i) + K_t^e(i) \tag{21}$$
$$\zeta C_t^e = \int_1^{1+\zeta} C_t^e(j)\, dj \tag{22}$$
$$\zeta K_t^e = \int_1^{1+\zeta} K_t^e(j)\, dj \tag{23}$$
$$\zeta I_t = \int_1^{1+\zeta} I_t(j)\, dj \tag{24}$$
$$\zeta NW_t = \int_1^{1+\zeta} NW_t(j)\, dj \tag{25}$$

Definition 1. *A rational expectations (RE) equilibrium is defined as a set of sequences* $\left\{K_t(i), K_t^h(i), H_t(i), W_t(i), X_t(i), S_t(i), p_t(i), R_t(i), Q_t(i)\right\}_{t=t_0}^{\infty}$ *for all* $i \in [0,1]$, $\{\overline{\omega}_t(j), NW_t(j), K_t^e(j), C_t^e(j), g(\overline{\omega}_t(j), \nu_t), f(\overline{\omega}_t(j)), I_t(j)\}_{t=t_0}^{\infty}$ *for* $j \in [1, 1+\zeta]$, *and* $\left\{C_t, Y_t, I_t, K_t, P_t, C_t^e, K_t^e, NW_t, \overline{\omega}_t, Q_t, R_t, i_t^b\right\}_{t=t_0}^{\infty}$ *given initial endowments* $K_{t_0}^h(i) = K_{t_0}^h, K_{t_0}^e(j) = K_{t_0}^e, A_{t_0}(i) = A_{t_0}$ *and bounded shock processes* $\{\psi_t, Z_t, \nu_t, F_t^e\}_{t=t_0}^{\infty}$ *such that (1) to (9), (12) to (25), and either of (10) or (11) are satisfied.*

Note that in equilibrium all entrepreneurs choose the same default threshold, $\overline{\omega}_t(j) = \overline{\omega}_t$, and that $R_t(i) = R_t$ and $Q_t(i) = Q_t$ given the assumption of perfectly flexible capital markets. The properties of the RE equilibrium, such as uniqueness

9 Alternatively, one could re-specify the credit friction to affect the marginal rate of transformation of output into investment by assuming that monitoring costs are incurred in the form of reduced output rather than reduced capital accumulation. The results that follow are robust to re-writing the model in this way.

and determinacy, will in general depend on how monetary policy is implemented by the central bank through its choice of nominal interest rates over time, $\left\{i_t^b\right\}_{t=t_0}^{\infty}$.

3. Optimal Monetary Policy in the Credit Friction Model

I now consider the implementation of optimal monetary policy in the economy with a credit friction. In particular, I focus on the analytical LQ solution approach, emphasised by Woodford (2003), to attain insight into the objectives of monetary policy, and what policies will achieve these objectives. Numerically, the LQ approach is equivalent to a log-linear approximation of the solution to the Ramsey policy problem for the central bank.

A Second-Order Approximation of Welfare

To begin, one must define the appropriate normative objective for the central bank. I assume that the central bank is concerned with maximising a measure of aggregate social welfare, that incorporates both the welfare of households and entrepreneurs. In particular, I assume

$$SW_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\int_0^1 U\left(C_t\left(i\right)\right) di - \int_0^1 V\left(H_t\left(i\right)\right) di \right) + E_{t_0} \sum_{t=t_0}^{\infty} \Lambda_t \left(\kappa \beta^e\right)^{t-t_0} \int_1^{1+\zeta} C_t^e\left(j\right) dj$$

The above social welfare measure implies that all households receive an equal weighting in social welfare, as do all entrepreneurs. In terms of the relative weighting applied across household and entrepreneurs, Λ_t represents a time-varying weight on the welfare of entrepreneurs (the household weight is normalised to one). I choose a deterministic process for Λ_t, which ensures that monetary policy does not have an incentive to redistribute mean consumption between household and entrepreneurs, as will become clearer below.

Taking a second-order approximation[10] of households' contribution to social welfare (Appendix A) I have[11]

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(U\left(C_t\right)-\int_0^1 V\left(H_t\left(i\right)\right)di\right)=-\left(U_cY\right)E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t^H$$
$$+t.i.p+O\left(\|\vartheta\|^3\right) \qquad (26)$$

where

$$\begin{aligned}L_t^H \equiv\ & \omega_y\frac{y_t^2}{2}+\omega_i\frac{i_t^2}{2}+\omega_\pi\frac{\pi_t^2}{2}\\ &+\omega_k\frac{k_t^2}{2}+\widetilde{\omega}_e\frac{\left(c_t^e\right)^2}{2}\\ &+s^e c_t^e-\omega_{yi}y_t i_t-\omega_{yk}y_t k_t-\omega_{ey}c_t^e y_t\\ &-\omega_{ei}c_t^e i_t-\omega_\xi\left(\xi_t\widehat{\Phi}\left(\overline{\omega}\right)+\xi_t i_t\right)\end{aligned}$$

and all lower case variables denote their log deviation from steady state. It should be noted that $y,i,\pi,k,c^e,\widehat{\Phi}\left(\overline{\omega}\right),\xi_t$ map to output, investment, inflation, capital, entrepreneurial consumption, the default rate and the recovery rate shock respectively. s^e denotes the steady state share of entrepreneurial consumption in output, and all other welfare coefficients, the ω's are functions of structural parameters defined in Appendix A. For brevity, and without loss of generality, I abstract from the effects of productivity and taste shocks.[12]

10 The approximations here are taken with respect to the natural logarithms of variables.

11 Note that $C_t\left(i\right)=C_t$ in the symmetric equilibrium I focus on, $t.i.p$ stands for terms that are independent of monetary policy, and that ϑ is a vector consisting of the sequences of all exogenous shocks in the economy. I use the notation $O\left(\|\vartheta\|^3\right)$ to denote the approximation residual which is of third-order or higher in the bound $\|\vartheta\|$ on the amplitude of the exogenous shocks (see Benigno and Woodford (2006) for further discussion).

12 Results that include these shocks are available on request.

A second-order approximation of entrepreneurial welfare yields

$$E_{t_0}\sum_{t=t_0}^{\infty}\left(\kappa\beta^e\right)^{t-t_0}\int_1^{1+\zeta}C_t^e(j)\,dj=-(Y)\,E_{t_0}\sum_{t=t_0}^{\infty}\left(\kappa\beta^e\right)^{t-t_0}L_t^e + t.i.p + O\left(\|\vartheta\|^3\right) \tag{27}$$

where

$$L_t^e \equiv -s^e c_t^e - s^e\frac{\left(c_t^e\right)^2}{2}$$

Comparing the first-order terms in the household and entrepreneurial loss functions, the presence of first-order terms in entrepreneur consumption, c_t^e, implies that there can be an incentive for the central bank to use monetary policy to redistribute consumption between entrepreneurs and households. The amount of redistribution considered optimal is a function of the weight on entrepreneurial welfare in social welfare, Λ_t, the effective discount factor for entrepreneurs' $\left(\kappa\beta^e\right)$ relative to that for households (β), and the slope of the household utility function evaluated at the steady state (U_c). To ensure that monetary policy abstracts from any incentive to redistribute consumption between households and entrepreneurs, I assume that the weight on entrepreneurial welfare evolves over time according to

$$\Lambda_t \equiv U_c\left(\frac{\beta}{\kappa\beta^e}\right)^{t-t_0}$$

This assumption ensures that the first-order terms relating to entrepreneurial consumption in both household and entrepreneurial welfare cancel in every time period t, and is similar to the approach used by De Fiore and Tristani (2009). The approximation of social welfare in this case simplifies to

$$SW_t = -E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}U_cY\left(L_t\right) + t.i.p + O\left(\|\vartheta\|^3\right) \tag{28}$$

where

$$
\begin{aligned}
L_t \equiv\ & \omega_y \frac{y_t^2}{2} + \omega_i \frac{i_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} \\
& + \omega_k \frac{k_t^2}{2} + \omega_e \frac{\left(c_t^e\right)^2}{2} \\
& - \omega_{yi} y_t i_t - \omega_{yk} y_t k_t - \omega_{ey} c_t^e y_t \\
& - \omega_{ei} c_t^e i_t - \omega_\xi \left(\xi_t \widehat{\Phi}\left(\overline{\omega}\right) + \xi_t i_t \right)
\end{aligned} \tag{29}
$$

and $\omega_e \equiv \widetilde{\omega}_e - s^e$.

A useful property of the above social welfare measure is that it does not contain first-order terms. This stems from the specific choice of the weight on entrepreneurial welfare, Λ_t, and that I choose to approximate around a steady state where the distortions associated with monopolistic competition and bankruptcy are assumed to be small. These assumptions imply that monetary policy will focus on time variation in the costs of these distortions associated with economic shocks, rather than using monetary policy to mitigate steady state distortions. These assumptions also contribute significantly to the simplification of the optimal policy problem, as the absence of first-order terms in (29) implies that the central bank's objective can be maximised subject to a first-order approximation of the constraints that describe the economy's decentralised equilibrium. For those readers who prefer analysis with large steady state distortions, this question is addressed numerically in Section 4.

A First-Order Approximation of the Constraints

Using a first-order approximation of the constraints in a symmetric decentralised equilibrium, I have[13]

$$k_{t+1} = (1-\delta)k_t + \delta i_t + \delta\Phi(\overline{\omega})\xi_t \tag{30}$$

$$\sigma_c c_t^h = \sigma_c E_t c_{t+1}^h - \left(i_t^b - E_t\left(\pi_{t+1}\right)\right) + \upsilon\psi_t - \upsilon E_t\psi_{t+1} \tag{31}$$

$$\begin{aligned}\sigma_c c_t^h &= \sigma_c E_t c_{t+1}^h - (1-\beta(1-\delta))E_t r_{t+1} + \widehat{q_t} \\ &\quad -\beta(1-\delta)E_t\widehat{q_{t+1}} + \upsilon\psi_t - \upsilon E_t\psi_{t+1}\end{aligned} \tag{32}$$

$$\widehat{q_t} = \widehat{\widetilde{f_\omega}(\overline{\omega}_t)} - g_\omega\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right) + \left(1 - q_t.g\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right)\right) - \widehat{f(\overline{\omega}_t)} \tag{33}$$

$$i_t = \widehat{nw_t} - \left(1 - q_t.g\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right)\right) \tag{34}$$

$$\begin{aligned}r_t &= \left(\sigma + \frac{1+\eta}{1-\alpha}\right)y_t - s\sigma i_t - \sigma s^e c_t^e - \frac{1+\alpha\eta}{1-\alpha}k_t \\ &\quad -(1+\eta)z_t - \upsilon\psi_t\end{aligned} \tag{35}$$

$$\pi_t = \Theta r_t - \Theta y_t + \Theta k_t + \beta E_t\pi_{t+1} \tag{36}$$

$$\sigma_c c_t^h = \sigma y_t - \sigma s\delta i_t - \sigma s^e c_t^e \tag{37}$$

$$c_t^e = \widehat{f(\overline{\omega}_t)} + \widehat{q_t} + i_t \tag{38}$$

where $\widetilde{f_\omega}(\overline{\omega}_t) \equiv -f_\omega(\overline{\omega}_t), s \equiv \frac{K}{Y}, \upsilon \equiv \frac{U_{c\psi}}{U_c}, \eta \equiv \frac{V_{HH}H}{V_H}\sigma_c \equiv -\frac{U_{cc}C}{U_c}$, $\sigma \equiv \sigma_c\frac{Y}{C}$ and variables with hats denote log deviations of functionals from their steady state value, or where the lower case of a variable has already been used to denote its real value.[14] The first three equations describe capital accumulation, the household consumption Euler equation and the household investment Euler equation respectively. The next two equations are the optimal contracting conditions, that determine the relationships between asset prices, the default threshold, bankruptcy costs, investment and net worth. This is followed by the equation for rental returns to capital, the New Keynesian Phillips curve, the aggregate resource constraint and the equation for entrepreneurial consumption.

13 For brevity, I omit the approximation errors here.

14 For example, $\widehat{f(\overline{\omega}_t)} \equiv \ln\frac{f(\overline{\omega}_t)}{f(\overline{\omega})}$ where $f(\overline{\omega}_t)$ is a function of the default threshold, or real net worth is defined by $nw_t \equiv \frac{NW_t}{P_t}$ and thus $\widehat{nw_t} \equiv \ln\frac{nw_t}{nw}$.

Entrepreneurial capital holdings and net worth follow either

$$k_{t+1}^{e} = 0 \tag{39}$$

$$\widehat{nw_t} = \widehat{F_t^e} \tag{40}$$

if $\beta^e < \beta$ and $\kappa = 1$ in the equilibrium where entrepreneurs do not save, or

$$k_{t+1}^{e} = \widehat{f(\overline{\omega}_t)} + i_t \tag{41}$$

$$\widehat{nw_t} = \frac{F^e}{nw}\widehat{F_t^e} + \beta^{-1}\frac{K^e}{nw}k_t^e + \left(\beta^{-1} - 1 + \delta\right)\frac{K^e}{nw}r_t + (1-\delta)\frac{K^e}{nw}\widehat{q}_t \tag{42}$$

if $1 > \kappa\beta^e > \beta$ and $\kappa < \beta$ in the equilibrium where entrepreneurs save and then consume at retirement.[15]

Optimal Monetary Policy

The LQ approximation of the solution to the optimal policy problem is to maximise (28) and (29) subject to the above constraint system (30 to 38 and either of 39 and 40, or 41 and 42). To understand optimal policy, I re-write the LQ problem in terms of gap variables, where the latter are defined in terms of log deviations from their zero-inflation equilibrium values.[16] That is, gap variables measure the deviation from the policy that would be considered optimal in the absence of the price and credit frictions.[17] The loss function (29) can be re-written as (see Appendix C)

$$\begin{aligned} L_t = {} & \omega_y \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} \\ & + \omega_i \frac{g_t^2}{2} + \omega_k \frac{j_t^2}{2} - \omega_{yi} x_t g_t - \omega_{yk} x_t j_t \\ & + \omega_e \frac{\left(n_t - n_t^*\right)^2}{2} - \omega_{ey} n_t x_t + \omega_{ei} n_t g_t \end{aligned} \tag{43}$$

15 nw, F^e and K^e denote the steady state values of net worth, the lump-sum fiscal transfer, and capital all with respect to entrepreneurs.

16 See Appendix B for the system of first-order constraints in the zero-inflation equilibrium.

17 This definition is consistent with the fact that I abstract from the effects of cost-push shocks.

where x_t, g_t, j_t and n_t are the output, investment, capital and net worth gaps respectively,

$$\begin{aligned} x_t &= y_t - y_t^n \\ g_t &\equiv i_t - i_t^n \\ j_t &= k_t - k_t^n \\ n_t &\equiv \widehat{nw}_t - \widehat{nw}_t^n \end{aligned}$$

n_t^* is a target for the net worth gap that is zero when entrepreneurs do not save, and is non-zero when entrepreneurs save,

$$n_t^* = \left\{ \begin{array}{cc} 0 & \text{if } \beta^e < \beta \text{ and } \kappa = 1 \\ \frac{-\left(\frac{s\delta}{s^e}\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}f(\overline{\omega})v_t + \sigma_c c_t^n\right)}{\sigma s^e} & \text{if } 1 > \kappa\beta^e > \beta \text{ and } \kappa < \beta \end{array} \right\}$$

and where c_t^n is the equilibrium value for household consumption that would be chosen in the absence of price and credit frictions.

Comparing (43) with the loss function in the economy with capital and exogenous capital adjustment costs, but no credit friction,[18]

$$\begin{aligned} L_t^{No\ CF} &= \omega_y \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} \\ &+ \omega_i \frac{g_t^2}{2} + \omega_k \frac{j_t^2}{2} - \omega_{yi} x_t g_t - \omega_{yk} x_t j_t \end{aligned} \quad (44)$$

it is clear that only difference is in the third line of (43), and thus the new variable of interest to the policymaker in the economy with a credit friction is net worth.

More specifically, a central bank will have an incentive to smooth movements in net worth in the equilibrium where entrepreneurs save. This is because the central bank has a non-trivial policy trade-off in this equilibrium. At the margin, the central bank can choose between stabilising inflation, reducing price dispersion that is socially costly, or following an alternative policy that mitigates time

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18 See Hansen (2010) for the derivation of the loss function, $L_t^{No\ CF}$, in the New Keynesian economy with capital and capital adjustment costs, but no credit friction.

variation in the credit friction, that is also socially costly. Importantly, both of these goals cannot be addressed simultaneously, and so there is a trade-off for monetary policy.[19]

The reason that net worth volatility is the key additional variable of interest to the policy maker, is that it is net worth that determines how socially costly the credit friction is. In particular, when net worth is high entrepreneurs are able to choose investment allocations that are similar to those that would be chosen in the absence of a credit friction, and so the distortionary effects of this friction are small. In contrast, when net worth is low, entrepreneurs are constrained in their ability to obtain external finance. This implies that entrepreneurs' investment allocations depart more significantly from those that would be chosen if the credit friction were not present.

Another way to see this point is to analyse the equilibrium where entrepreneurs do not save, and thus the amplification mechanism associated with net worth is shut down. In this case it can be verified that net worth becomes a term that is independent of policy when entrepreneurs do not save,[20] $n_t = t.i.p$ and $n_t^* = 0$, and so the social loss function for the central bank becomes equivalent to that derived in an economy with capital but no credit friction ($L_t = L_t^{No\ CF}$). In this case, as emphasised by Hansen (2010), a zero-inflation target is optimal and the policymaker is only concerned with stabilising dispersion in prices. This makes sense, as although the credit friction still exists when entrepreneurs do not save, the central bank is in fact unable to use monetary policy to mitigate time variation in this distortion.

To understand the determinants of net worth volatility when it is responsive to changes in monetary policy, the net worth gap can be written as

$$n_t = \frac{f_{\overline{\omega}_t}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\left(\widehat{\overline{\omega}}_t - \widehat{\overline{\omega}^n}_t\right) + \left(\widehat{q}_t - \widehat{q}_t^{\,n}\right) + g_t + O\left(\|\vartheta\|^2\right)$$

19 This can be seen clearly from analysis of the first-order conditions of the optimal policy problem, and noting that, in general, the target around which net worth volatility is stabilised is non-zero $\left(n_t^* \neq 0\right)$. See Appendix D for further detail.

20 See Appendix C.

in the equilibrium where entrepreneurs save. Thus, when net worth responds to policy, the incentive to smooth movements in net worth is equivalent to an incentive to smooth movements in default rates, the asset price gap, and the investment gap. Interestingly, only investment gap volatility, g_t, and default rate volatility, $\widehat{\overline{\omega}}_t - \widehat{\overline{\omega}^n}_t$, are in fact variables that are endogenous to policy and respond to changes in interest rates. Asset prices volatility, or strictly speaking volatility in the asset prices gap, can be shown to be a term that is in fact independent of policy.[21] Although the policymaker has an incentive to smooth volatility in asset prices, a topic that has received much attention in previous literature, the policymaker in this economy is in fact unable to address this incentive, when the steady state credit friction is small.

Comparing these results with recent literature, it should be noted that the incentive to smooth volatility in the default rates of entrepreneurs is equivalent to an incentive to smooth credit spreads, when spreads are appropriately defined in this economy.[22] The importance of spreads (default) is a finding that is similar to those made by Cúrdia and Woodford (2010) and De Fiore and Tristani (2009). These authors also show that spreads feature in the objective of the policymaker in economies with a credit friction but no capital. However, a key difference, between their results and those emphasised here, is that spreads are not the primitive variable in the policymakers objective in this economy. Spreads only matter to the extent that they influence net worth, or more fundamentally the social cost of the credit friction. Although this distinction may appear subtle, a central bank that is concerned with smoothing spreads, as opposed to net worth, would not in fact be implementing monetary policy optimally in this economy.

21 Appendix C, Lemma 2 verifies that $\widehat{q}_t = t.i.p$ and so $\widehat{q}_t - \widehat{q}_t^n = 0$.

22 The credit spread (external finance premium) is defined as

$$\begin{aligned} \widetilde{sp}_t &\equiv \frac{1+R_t^L}{P_t} - 1 \\ &= \frac{\overline{\omega}_t}{g\left(\overline{\omega}_t, 1-e^{\xi_t}\right)} - 1 \end{aligned}$$

Focusing on the normalised spread, $sp_t = 1 + \widetilde{sp}_t$, it is straightforward to verify that deviations in the spread gap, $sp_t - sp_t^n$, are proportional to deviations in the default rate gap, $\widehat{\overline{\omega}}_t - \widehat{\overline{\omega}^n}_t$ up to a first-order approximation.

Qualitatively, these results are broadly consistent with the set of monetary policies used across countries in the most recent financial crisis. Arguably, one interpretation is that a number of countries implemented monetary policies, including conventional monetary policy, with the objective of stabilising borrowers' net worth playing some part in policymakers' overall approach. Possible examples include the sharp reduction in interest rates used in the US, UK and Europe at the onset of the crisis, as well as the additional macroeconomic tools used, such as the purchase of financial securities and lending programs, that assisted in stabilising the net worth of borrowers and lenders, and in reducing the incidence of default.[23]

4. Quantitative Analysis

I now examine the extent to which these incentives matter quantitatively. For a baseline calibration, I calibrate a similar default rate (3 percent quarterly) and external finance premium (150 basis points) as that used by Faia and Monacelli (2007). All other parameters are calibrated at values that are similar to those used by Faia and Monacelli, or in line with previous empirical literature that estimates the New Keynesian model analysed here (see Table 1).

Table 1: Calibration for the Benchmark Model

Parameter	Steady State Value[(a)]	Interpretation
β	0.99	Discount factor
δ	0.025	Depreciation rate
α	0.33	Share of factor payments to capital
σ_c	1.00	Coefficient of relative risk aversion
η	2	Inverse of Frisch elasticity of labour supply
γ	0.65	Proportion of firms that cannot change their price each quarter
θ	8	Elasticity of substitution between intermediate goods
κ	0.90	Entrepreneurs' probability of not retiring
$\frac{\zeta F^e}{Y}$	0.02	Aggregagate entrepreneur subsidy relative to income[(b)]
$\phi(\overline{\varpi})$	0.5	Uniform PDF of investment projects[(c)]

Notes: (a) All parameters are chosen to be consistent with a Balanced Growth Path in non-detrended variables, and to match empirical estimates using non-detrended data.
(b) Consistent with a steady state government subsidy to entrepreneurs of 9 per cent (as a share of steady state net worth).
(c) On the $[0,2]$ interval.

23 See for example Bernanke (2009), Kohn (2009) and Stark (2009).

Given the recent financial crisis, it is topical to consider two shocks that are financial in nature.[24] The first shock is a negative one percentage point shock to the proportion of funds recovered when an entrepreneur declares default.[25] This results in a decline in the willingness of banks to lend to entrepreneurs (i.e. an increase in the cost of asymmetric information). The second shock is a negative shock to the lump-sum government subsidy to entrepreneurs, which reduces their net worth directly by one percent. This acts as an exogenous reduction in entrepreneurs' pledgable collateral, increasing their external financing requirement and reducing their ability to borrow funds for an investment project.

As a useful benchmark for comparison, Figure 1 reports the impulse response functions if monetary policy follows a zero-inflation target. With a small credit friction, it can be observed that a decrease in recovery rates results in an expansionary monetary policy that stabilises inflation, increases asset prices and the net worth of entrepreneurs, and results in a fall in default rates.[26] Interestingly, notwithstanding the increase in net worth, investment still falls because although entrepreneurs are now more able to finance projects from their internal funds, the contraction in external finance provided by banks is larger. The net effect is a decline in credit and investment. Banks are less willing to lend precisely because they recover less when an entrepreneur defaults.

I now consider the extent to which optimal policy deviates from a zero-inflation target. Figure 2 reports the deviation of variables from their zero-inflation values. The results highlight that although it is optimal to run a more expansionary monetary policy than that of a zero-inflation target, the order of magnitude of this deviation is small. For example, with a one percent fall in the recovery rate, optimal monetary policy would call for stimulating quarterly inflation in the order of 0.005 per cent.

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24 The implications of the following results are unchanged if productivity shocks or taste shocks are considered.

25 Recall that, by assumption, the steady state recovery rate is close to one, when monitoring costs are small in steady state

26 In the economy I describe, the external finance premium always moves in the same direction as the default rate.

Figure 1: Decline in the Recovery Rate – Zero Inflation Target

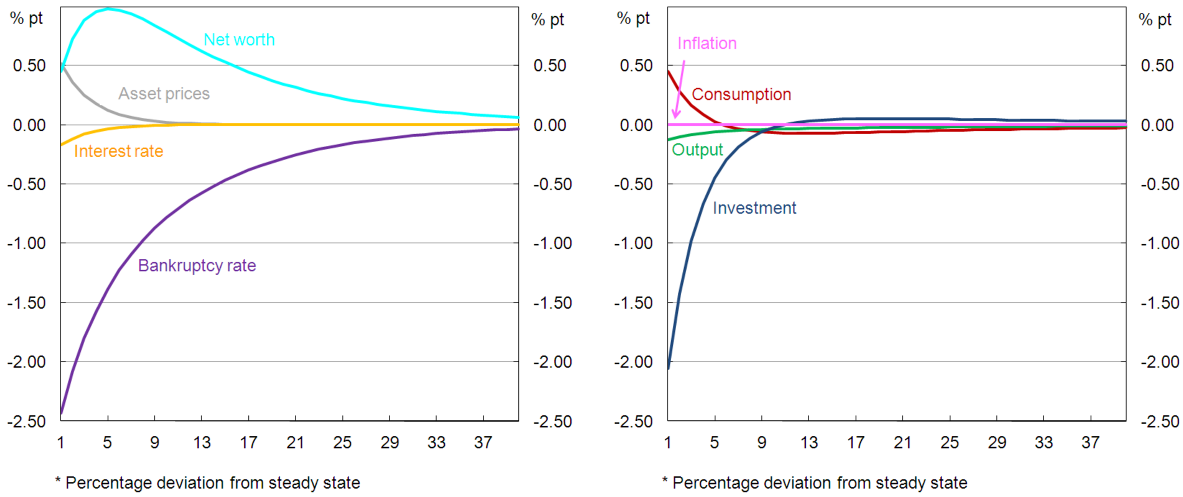
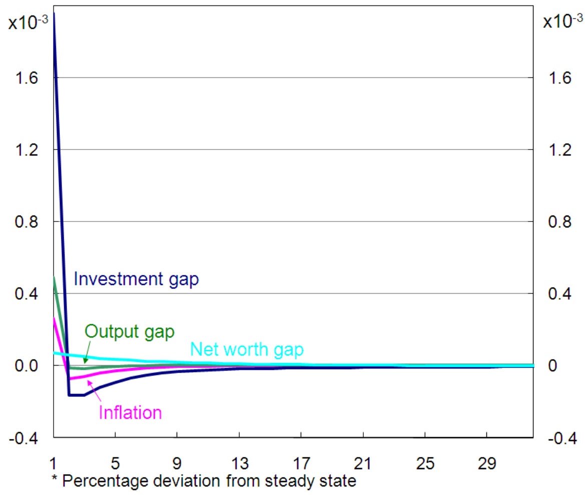


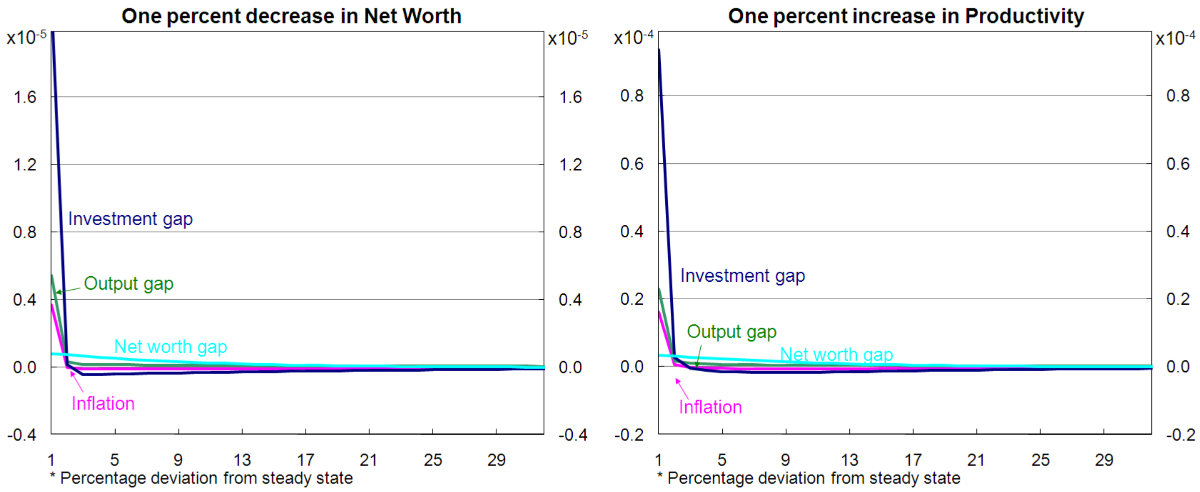
Figure 2: Decline in the Recovery Rate – Optimal Monetary Policy



To be clear, the very small deviations from a strict inflation target are not specific to the type of shock to fundamentals considered. Figure 3 reports the deviations from the zero-inflation target in response to a negative net worth shock, and a positive productivity shock. Again, though a more expansionary policy is called for in response to both shocks, the optimal deviation of inflation from zero is very small.

The LQ framework provides further insight into these results, as it allows the relative weights of variables in the loss function to be examined. Table 2 highlights that inflation is by far the most important variable for social welfare. Of secondary

Figure 3: Net Worth and Productivity Shocks – Optimal Monetary Policy



importance is the output gap, the investment and capital gaps, and then the net worth gap. Clearly, the credit friction objective for policy is noticeably subordinate to the objective of stabilising inflation.

Table 2: Loss Function Weights

Variable	Weight on Variance Term in L_t [(a)]
Output gap (ω_y)	1.55×10^{-2}
Investment gap (ω_i)	2.39×10^{-4}
Inflation (ω_π)	1
Capital gap (ω_k)	2.63×10^{-3}
Net worth gap (ω_e)	2.12×10^{-6}

Notes: (a) Normalised by weight on inflation.

Robustness: Does the size of the friction matter?

A natural question is whether the previous quantitative results rely on the assumption of a small steady state credit friction. To address this, I also consider the optimal inflation response to shocks in models with a larger steady state credit friction. Specifically, I compare the benchmark economy previously described, with economies that have steady state credit frictions that are similar in size to those estimated for the US and Euro area economies by Queijo von Heideken (2009).

To be clear there are three main distinctions between the large friction economies I now consider, and the benchmark economy previously discussed. These are:

(A) The large friction economies now have monitoring costs that distort the economy to the first-order;

(B) Since steady state returns are higher in the credit friction economies, I focus on an equilibrium where entrepreneurs are impatient, and indifferent to saving and consuming; and

(C) I assume that all firms are able to optimise their price each period, but must pay a quadratic adjustment cost when doing so.

(A) is the distinction of interest I wish to investigate. That is, does the magnitude of the credit friction affect the extent to which a central bank wishes to deviate from a zero-inflation target in response to either productivity, net-worth or bankruptcy rate shocks. (B) and (C) are technical assumptions required when solving for optimal (Ramsey) policy numerically. (B) ensures that entrepreneurs do not become self-financing in an equilibrium where monitoring costs are significant, and steady state investment returns are high. (C) is included as an alternative assumption to Calvo pricing that facilitates a numerical solution.[27]Concerning the measure of social welfare in the larger frictions economies, I continue to assume that the social welfare measure maximised by the central bank incorporates both the welfare of households and entrepreneurs, and that monetary policy is not concerned with first-order or mean consumption redistribution between these groups.[28]

Table 3 summarises the alternative models I compare. Specifically, I compare the benchmark economy with an economy that assumes steady state bankruptcy costs in the order of 0.25, which is comparable to that level of bankruptcy costs

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27 It overcomes the technical difficulty that, in principle, the whole distribution of prices matters for welfare when solving for numerical Ramsey policy with Calvo pricing (see Faia and Monacelli (2007) for further discussion).

28 In particular, I normalise the weight on households as a group to one, and set $\Lambda_t = U_c \left(\frac{\beta}{\kappa\beta^e}\right)^t$, which is identical in form to the weighting used in the benchmark model.

estimated for the Euro area (see Queijo von Heideken 2009). The second model I consider assumes steady state bankrupcty costs at 0.16, which is similar to estimates for the US, but is otherwise identical to the Euro area model. The third model, US Alt, is identical to the US model with the exception that entrepreneurs are assumed to be more patient. This latter assumption ensures a more realistic steady state rate of default for the US.

Table 3: Alternative Credit Friction Models

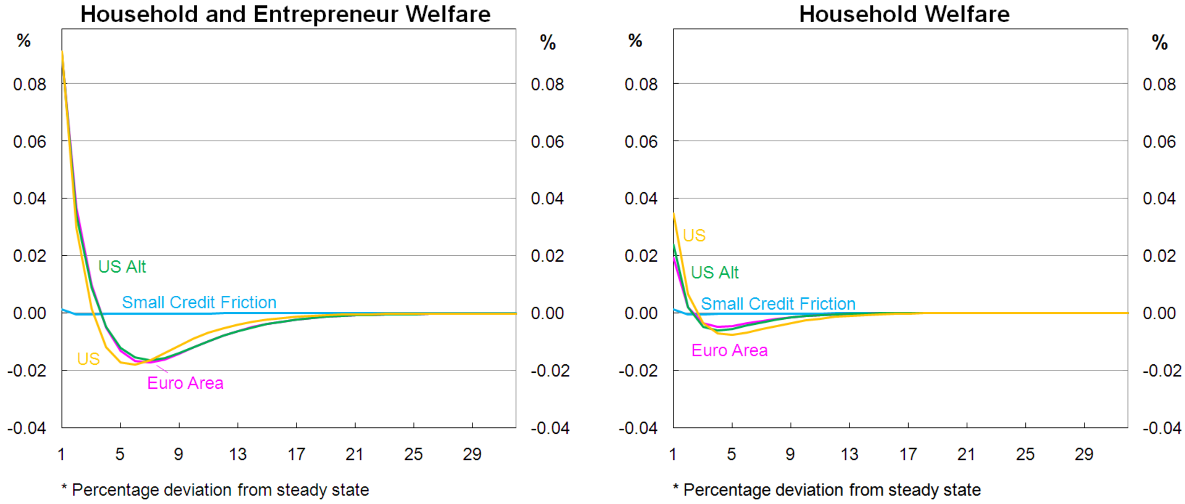
	Benchmark	Euro Area	US	US Realistic
Entrepreneurs	Save[(a)]	Indifferent[(b)]	Indifferent[(b)]	Indifferent[(b)]
SS Recovery Rate	1.00	0.75	0.84	0.84
SS Default Rate	0.03	0.03	0.38	0.03
Entrepreneur Discount Factor	1.1	0.87	0.87	0.87

Notes: (a) Entrepreneurs save until stochastic retirement.
(b) Entrepreneurs are indifferent between consuming and saving.
(c) Firms pay a quadratic adjustment cost when changing their price.
(d) Social welfare includes household and entrepreneur welfare.

The left hand panel of Figure 4 reports results from comparing the response of the inflation rate to a one percentage point decline in the recovery rate, assuming that the central bank implements Ramsey optimal policy. The results highlight that the size of the steady state credit friction does affect the extent to which the central bank deviates from a zero-inflation target. In particular, in the larger friction models a concern with stabilising movements in net worth implies that the policymaker now chooses an inflation rate that is approximately 0.1 percentage points higher than steady state inflation. This is a non-trivial deviation from a zero-inflation target, given that that this response is measured at a quarterly frequency, and is in response to a one percentage point decline in the recovery rate.

The right hand panel of Figure 4 recomputes the same responses assuming that the central bank only maximises household welfare. A common assumption made in previous numerical literature on optimal policy, the results highlight that the extent of deviation from the strict inflation target is roughly halved. These results are more in line with the findings of Faia and Monacelli (2007), for example, and emphasise that the results are sensitive to the weights attached to households and borrowers (entrepreneurs) in the policymakers social weflare function. If policymakers, are concerned only with the welfare of households, and not entrepreneurs, then a strict inflation target becomes a better approximation

Figure 4: Optimal (Ramsey) Inflation Paths Across Models



of optimal policy monetary in an economy with a first-order distortionary credit friction.

Overall, these results confirm that credit frictions are potentially important for optimal policy both qualitatively, and quantitatively when considering economies that have non-trivial credit distortions. Although inflation remains the primary objective of importance, the quantitative results in this paper suggest that some deviation of inflation from target may be appropriate if credit frictions are thought to be having a highly distortionary effect on economic activity.

It is informative to compare these results with those obtained in previous work by Faia and Monacelli (2007). Faia and Monacelli consider numerical welfare comparisions of different monetary policy rules with optimal (Ramsey) policy for a similar New Keynesian economy with a credit friction and productivity and government spending shocks. These authors find that a high weight on inflation in an interest-rate rule is socially optimal. One assumption that appears important in their findings is that the central bank is only concerned with the welfare of households. The findings here suggest that if the policymaker also takes into consideration the welfare of entrepreneurs (borrowers), motivated by the idea that policymakers abstract from redistribution, then a greater tolerance for non-zero inflation can be induced in response to financial shocks such as increases in the costs of bankruptcy.

It is also informative to compare these results with the findings of Vleighe (2010). In Vlieghe's analysis, the credit friction is assumed to affect the ability of low productivity agents to transfer resources to high productivity agents, a setup that builds on the work of Kiyotaki and Moore (1997). Interestingly, although Vlieghe models quite a different financial friction, he also finds that optimal monetary policy implies small, though not trivial, departures from inflation targeting when solving for optimal policy numerically.

5. Conclusion

There has been much debate in the literature regarding optimal monetary policy in response to fluctuations in asset prices and financial volatility. However, much of this debate has been based on a numerical approach, which has left the objectives of monetary policy somewhat unclear. This paper aims to provide additional insight into this question by taking an analytical approach to the question of optimal monetary policy in a New Keynesian economy with endogenous capital and a credit friction.

The results highlight that policymakers can have an incentive to stabilise volatility in the net worth of borrowers to help mitigate time variation in the distortionary costs of a credit friction in the economy. This incentive exists in addition to the more traditional objectives of stabilising the composition of output and inflation. Interestingly, for the type of credit friction modelled here, credit spreads and asset prices are only variables of interest to policymakers to the extent that they influence the net worth of borrowers in the economy.

Quantitatively, I find that inflation targeting remains a good approximation of optimal monetary policy when the credit friction is not highly distortionary. However, for a more distortionary friction, such as that which can be observed during episodes of financial stress, these results imply that a more expansionary monetary policy with some tolerance of inflation is optimal.

Appendix A: Approximation of Household Welfare

Approximation of Household Utility from Consumption

Taking a quadratic approximation of household utility from consumption, which is identical across all households, and using quadratic and linear approximations of the aggregate resource constraint (18) to eliminate the linear and quadratic terms in c_t respectively, I have[29]

$$U\left(C_t\right)=U_cY\left(\begin{array}{c} y_t-s\delta i_t-s^e c_t^e \\ \frac{(1-\sigma)y_t^2}{2}-\left(s\delta+\sigma s^2\delta^2\right)\frac{i_t^2}{2}-s^e\left(1+\sigma s^e\right)\frac{(c_t^e)^2}{2} \\ +s\sigma\delta y_t i_t+\sigma s^e c_t^e y_t-s\sigma\delta s^e c_t^e i_t \end{array}\right)$$
$$+t.i.p+O\left(\|\vartheta\|^3\right)$$

where $s\equiv\frac{K}{Y}$, $s^e\equiv\frac{\zeta C^e}{Y}$, $\sigma\equiv\sigma_c\frac{Y}{C}$, and $\sigma_c\equiv-\frac{U_{cc}(C)C}{U_c(C)}$.

Using a second-order approximation of capital accumulation (19), where I use $\xi_t\equiv\ln\left(1-\nu_t\right)$ as a convenient renormalisation of the bankruptcy cost shock, I eliminate the first-order term in investment in the above approximation to obtain

$$U\left(C_t\right)=U_cY\left(\begin{array}{c} y_t+\frac{(1-\sigma)y_t^2}{2}-s\left(k_{t+1}+\frac{k_{t+1}^2}{2}\right) \\ +s\left(1-\delta\right)\left(k_t+\frac{k_t^2}{2}\right)-s^e c_t^e \\ -\sigma s^2\delta^2\frac{i_t^2}{2}-s^e\left(1+\sigma s^e\right)\frac{(c_t^e)^2}{2} \\ +s\sigma\delta y_t i_t+\sigma s^e c_t^e y_t-s\sigma\delta s^e c_t^e i_t \\ +s\delta\Phi\left(\overline{\omega}\right)\left(\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)+\xi_t i_t\right) \end{array}\right)$$
$$+t.i.p+O\left(\|\vartheta\|^3\right) \qquad \text{(A1)}$$

29 For brevity, and without loss of generality, I omit taste and productivity shocks. All approximations are taken with respect to the natural logarithms of variables unless otherwise specified.

where I approximate around a stead state such that $K_t = K, \overline{\omega}_t = \overline{\omega}, \xi = 0, \frac{I}{K} = \frac{\delta}{\zeta}$, treat the recovery share parameter ξ_t as an expansion parameter that can be perturbed, and collect terms that are independent of policy in $t.i.p$.

Approximation of the Disutility of labour

Approximating $\int_0^1 V(H_t(i))di$, I have

$$\int_0^1 V(H_t(i))di = V_H H\left(E_i h_t(i) + \frac{1}{2}(1+\eta)E_i h_t(i)^2\right) + t.i.p + O\left(\|\vartheta\|^3\right) \quad \text{(A2)}$$

where $\eta \equiv \frac{V_{HH}H}{V_H}$, $E_i h_t(i) \equiv \int_0^1 h_t(i)di$ and $E_i h_t(i)^2 \equiv \int_0^1 h_t(i)^2 di$.

Using the steady state values for the intratemporal household condition (3), the production technology (13), the optimal choice of the labour to capital ratio (14), real marginal cost (15), and the optimal pricing decision (16), it follows that

$$V_H H = \mu^{-1}(1-\chi)^{-1}(1-\alpha)YU_c$$

where $\mu^{-1} \equiv (1-\theta^{-1})$ is the inverse of the mark-up. Thus, (A2) can be written as

$$\int_0^1 V(H_t(i))di = \frac{U_c Y}{\mu(1-\chi)}\left((1-\alpha)E_i h_t(i) + \frac{1}{2}(1-\alpha)(1+\eta)E_i h_t(i)^2\right) \quad \text{(A3)}$$

Using the production function (13), and a second-order approximation of the final producer's demand function for intermediate goods (12), I have

$$(1-\alpha)E_i h_t(i) = y_t - \alpha E_i k_t(i) - (1-\alpha)z_t - \frac{1}{2}\mu^{-1} var_i x_t(i) + O\left(\|\vartheta\|^3\right) \quad \text{(A4)}$$

Substituting (A4) into (A3) I obtain

$$\int_0^1 V(h_t(i))di = \frac{U_c Y}{\mu(1-\chi)}\begin{pmatrix} y_t - \alpha E_i k_t(i) - \frac{1}{2}\mu^{-1} var_i x_t(i) \\ +\frac{1}{2}(1-\alpha)(1+\eta)E_i h_t(i)^2 \end{pmatrix} \quad \text{(A5)}$$
$$+ t.i.p + O\left(\|\vartheta\|^3\right)$$

Using a second-order approximation of the capital aggregation condition across industries (21), (A5) can be re-written as

$$\int_0^1 V(H_t(i))di = \frac{U_c Y}{\mu(1-\chi)} \begin{pmatrix} y_t - \alpha k_t - \frac{1}{2}\mu^{-1} var_i x_t(i) \\ +\frac{\alpha}{2} var_i k_t(i) + \frac{(1-\alpha)(1+\eta)}{2} var_i (h_t(i)) \\ +\frac{(1-\alpha)(1+\eta)}{2} (E_i h_t(i))^2 \end{pmatrix} + t.i.p + O\left(\|\vartheta\|^3\right) \tag{A6}$$

It is straightforward to show that $var_i(k_t(i))$ and $var_i(h_t(i))$ are proportional to $var_i(x_t(i))$ using first-order approximations of (3), (13) and (14),

$$var_i(h_t(i)) = \frac{1}{(1+\alpha\eta)^2} var_i(x_t(i)) + O\left(\|\vartheta\|^3\right)$$
$$var_i(k_t(i)) = \left(\frac{1+\eta}{1+\alpha\eta}\right)^2 var_i(x_t(i)) + O\left(\|\vartheta\|^3\right)$$

Substituting these relationships into (A6) yields

$$\int_0^1 V(H_t(i))di = \frac{U_c Y}{\mu(1-\chi)} \begin{pmatrix} y_t - \alpha k_t + \frac{1}{2}\omega_x var_i x_t(i) \\ +\frac{(1-\alpha)(1+\eta)}{2} (E_i h_t(i))^2 \end{pmatrix} + t.i.p + O\left(\|\vartheta\|^3\right) \tag{A7}$$

where $\omega_x \equiv \frac{\eta+1}{\alpha\eta+1} - \mu^{-1}$. Finally using first-order approximations of (12) and (13) to substitute out $E_i h_t(i)$, (A7) can be written as

$$\int_0^1 V(H_t(i))di = \frac{U_c Y}{\mu(1-\chi)} \begin{pmatrix} y_t - \alpha k_t + \frac{\omega_x}{2} var_i x_t(i) \\ +\frac{(1-\alpha)(1+\eta)}{2} \left(\frac{y_t - \alpha k_t}{1-\alpha} - z_t\right)^2 \end{pmatrix} + t.i.p + O\left(\|\vartheta\|^3\right) \tag{A8}$$

Combining (A1) and (A8) it follows that the quadratic approximation of average household felicity is given by

$$U(C_t) - \int_0^1 V(H_t(i))\,di = U_c Y \begin{pmatrix} \frac{(1-\sigma)y_t^2}{2} - s^2\sigma\delta^2\frac{i_t^2}{2} - \alpha\frac{k_t^2}{2} - s^e\left(1+\sigma s^e\right)\frac{(c_t^e)^2}{2} \\ -s^e c_t^e + s\sigma\delta y_t i_t + \sigma s^e c_t^e y_t - s\sigma\delta s^e c_t^e i_t \\ +s\delta\Phi(\overline{\omega})\left(\xi_t\widehat{\Phi}(\overline{\omega}) + \xi_t i_t\right) \\ -\frac{(1-\alpha)(1+\eta)}{2}\left(\frac{y_t-\alpha k_t}{1-\alpha} - z_t\right)^2 - \frac{\omega_x}{2}var_i x_t(i) \end{pmatrix}$$
$$+t.i.p + O\left(\|\vartheta\|^3\right)$$

To obtain this result I have assumed that the subsidy on intermediate production ensures that the distortion associated with monopolistic competition is eliminated $(1-\chi)\mu = 1$, and that the steady state recovery rate is one ($e^{\xi} = 1$). I have also iterated out linear and quadratic terms involving capital using the result

$$\begin{aligned} E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(-sk_{t+1} + s(1-\delta)k_t + \alpha k_t\right) &= -sE_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(k_{t+1} - \beta^{-1}k_t\right) \\ &= s\beta^{-1}k_{t_0} \\ &= t.i.p \end{aligned} \tag{A9}$$

which also applies to second-order terms in capital, and is consistent with Takamura *et al* (2006).

For brevity, and without loss of generality, I abstract from productivity shocks (as well as taste shocks) assuming $z_t = 0$. I can then write the approximation of household welfare as

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(U(C_t) - \int_0^1 V(H_t(i))\,di\right) = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}U_c Y\left(L_t^h\right)$$
$$+t.i.p + O\left(\|\vartheta\|^3\right)$$

where

$$L_t^h \equiv \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right)\frac{y_t^2}{2} + s^2\sigma\delta^2\frac{i_t^2}{2} + \omega_\pi\frac{\pi_t^2}{2}$$
$$+\left(\alpha + \frac{\alpha^2(1+\eta)}{1-\alpha}\right)\frac{k_t^2}{2} + s^e\left(1+\sigma s^e\right)\frac{\left(c_t^e\right)^2}{2}$$
$$+ s^e c_t^e - s\sigma\delta y_t i_t - \frac{\alpha(1+\eta)}{1-\alpha}y_t k_t - \sigma s^e c_t^e y_t$$
$$+ s\sigma\delta s^e c_t^e i_t - s\delta\Phi(\overline{\omega})\left(\xi_t\widehat{\Phi}(\overline{\omega}_t) + \xi_t i_t\right)$$

which corresponds to (26) in the main text and where $\omega_\pi \equiv \frac{\theta^2\gamma\omega_x}{(1-\gamma)(1-\gamma\beta)}$. In obtaining this last result I follow the approach discussed in Woodford (2003), Chapter 6, where it can be verified in the context of this paper that $var_i(x_t(i)) = \theta^2 var_i(p_t(i))$ and that

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}var_i(p_t(i)) = \frac{\gamma}{(1-\gamma)(1-\gamma\beta)}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\pi_t^2 + t.i.p + O\left(\|\vartheta\|^3\right)$$

Appendix B: The Constraints in the Zero-Inflation Equilibrium

For the model with the credit friction and endogenous variation in net worth, the zero-inflation equilibrium values must satisfy the system

$$
\begin{aligned}
k_{t+1}^{n} &= (1-\delta)\,k_t^n + \delta i_t^n + \delta\Phi(\overline{\omega})\,\xi_t \\
\sigma_c c_t^n &= \sigma_c E_t c_{t+1}^n + \upsilon\psi_t - \upsilon E_t\psi_{t+1} - r_t^{bn} \\
\sigma_c c_t^n &= \sigma_c E_t c_{t+1}^n - (1-\beta(1-\delta))\,E_t r_{t+1}^n + \widehat{q}_t^{\,n} - \beta(1-\delta)\,E_t\widehat{q_{t+1}}^{\,n} \\
&\quad + \upsilon\psi_t - \upsilon E_t\psi_{t+1} \\
\widehat{q}_t^{\,n} &= -\frac{\phi(\overline{\omega})\,(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t - \Phi(\overline{\omega})\,\xi_t \\
0 &= \widehat{nw}_t^n - i_t^n - \frac{\phi(\overline{\omega})\,g(\overline{\omega},0)}{1-\Phi(\overline{\omega})}\xi_t + \Phi(\overline{\omega})\,\xi_t - \frac{f_{\overline{\omega}}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t^{\,n} \\
\widehat{nw_t} &= \varphi_F\widehat{F_t^e} + \varphi_k k_t^{en} + \varphi_r r_t^n + \varphi_q\widehat{q}_t^{\,n} \\
r_t^n &= y_t^n - k_t^n \\
0 &= \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_t^n - s\sigma\delta i_t^n - \sigma s^e c_t^{en} - \frac{\alpha(1+\eta)}{1-\alpha}k_t^n - (1+\eta)\,z_t - \upsilon\psi_t \\
\sigma_c c_t^n &= \sigma y_t^n - \sigma s\delta i_t^n - \sigma s^e c_t^{en} \\
c_t^{en} &= \frac{f_{\overline{\omega}}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t^{\,n} + \widehat{q}_t^{\,n} + i_t^n \\
k_{t+1}^{en} &= \frac{f_{\overline{\omega}}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t^{\,n} + i_t^n
\end{aligned}
$$

This is a first-order difference system with 11 equations in 11 unknowns that can be used to solve for the real rate of interest, r_t^{bn}, consistent with the absence of price and credit frictions.

Appendix C: Re-writing the Loss Function

I begin with the loss function (29)

$$
\begin{aligned}
L_t \equiv & \left(\sigma + \frac{\alpha+\eta}{1-\alpha}\right)\frac{y_t^2}{2} + s^2\sigma\delta^2\frac{i_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} \\
& + \left(\alpha + \frac{\alpha^2(1+\eta)}{1-\alpha}\right)\frac{k_t^2}{2} + \sigma\left(s^e\right)^2\frac{\left(c_t^e\right)^2}{2} \\
& - s\sigma\delta y_t i_t - \frac{\alpha(1+\eta)}{1-\alpha} y_t k_t - \sigma s^e c_t^e y_t \\
& + s\sigma\delta s^e c_t^e i_t - s\delta\Phi(\overline{\omega})\left(\xi_t\widehat{\Phi}(\overline{\omega}_t) + \xi_t i_t\right)
\end{aligned}
$$

As noted in the main text, it is useful to re-write variables in terms of the log deviation from their respective zero-inflation equilibrium values. Abstracting from the effects of productivity shocks (z_t) and taste shocks (ψ_t) without loss of generality, conditional expectations of zero-inflation (hereafter natural) variables must satisfy the following real marginal cost condition in a zero-inflation equilibrium[30]

$$
0 = \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_{t|t_0}^n - s\sigma\delta i_{t|t_0}^n - \sigma s^e c_{t|t_0}^{en} - \frac{\alpha(1+\eta)}{1-\alpha} k_{t|t_0}^n \qquad \text{(C1)}
$$

where I use the notation $E_{t_0}\left(y_t^n\right) = y_{t|t_0}^n$. Using this condition, it follows that the loss function (29), for any given period t, can be written as

$$
\begin{aligned}
L_t \equiv & \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right)\frac{\left(y_t - y_t^n\right)^2}{2} + s^2\sigma\delta^2\frac{\left(i_t - i_t^n\right)^2}{2} + \omega_\pi\frac{\pi_t^2}{2} \\
& + \alpha\frac{\left(k_t - k_t^n\right)^2}{2} + \left(\frac{\alpha^2(1+\eta)}{1-\alpha}\right)\frac{\left(k_t - k_t^n\right)^2}{2} + \sigma\left(s^e\right)^2\frac{\left(c_t^e - c_t^{en}\right)^2}{2} \\
& - s\sigma\delta\left(y_t - y_t^n\right)\left(i_t - i_t^n\right) - \frac{\alpha(1+\eta)}{1-\alpha}\left(y_t - y_t^n\right)\left(k_t - k_t^n\right) \\
& - \sigma s^e\left(c_t^e - c_t^{en}\right)\left(y_t - y_t^n\right) + s\sigma\delta s^e\left(c_t^e - c_t^{en}\right)\left(i_t - i_t^n\right) + \widetilde{R}_t \qquad \text{(C2)}
\end{aligned}
$$

30 This result can be derived from analysis of the first-order approximation of the constraints in the zero-inflation equilibrium (see Appendix B).

and where the remainder, $\widetilde{R}_t$ is given by

$$\begin{aligned}\widetilde{R}_t \equiv & \frac{\alpha(1+\eta)}{1-\alpha}\alpha k_t^n k_t - \frac{\alpha(1+\eta)}{1-\alpha} y_t^n k_t - s\sigma\delta y_t^n i_t \\ & + s^2\sigma\delta^2 i_t^n i_t + \sigma\left(s^e\right)^2 c_t^{en} c_t^e - \sigma s^e y_t^n c_t^e \\ & + s\sigma\delta s^e c_t^{en} i_t + s\sigma\delta s^e i_t^n c_t^e + \alpha k_t^n k_t \\ & - s\delta\Phi(\overline{\omega})\left(\xi_t\widehat{\Phi}(\overline{\omega}_t) + \xi_t i_t\right)\end{aligned}$$

The next two propositions establish that (C2) can be rewritten as

$$\begin{aligned}L_t \equiv & \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right)\frac{\left(y_t - y_t^n\right)^2}{2} + s^2\sigma\delta^2\frac{\left(i_t - i_t^n\right)^2}{2} + \omega_\pi\frac{\pi_t^2}{2} \\ & + \alpha\frac{\left(k_t - k_t^n\right)^2}{2} + \left(\frac{\alpha^2(1+\eta)}{1-\alpha}\right)\frac{\left(k_t - k_t^n\right)^2}{2} + \sigma\left(s^e\right)^2\frac{\left(nw_t - nw_t^n - n_t^*\right)^2}{2} \\ & - s\sigma\delta\left(y_t - y_t^n\right)\left(i_t - i_t^n\right) - \frac{\alpha(1+\eta)}{1-\alpha}\left(y_t - y_t^n\right)\left(k_t - k_t^n\right) \\ & - \sigma s^e\left(nw_t - nw_t^n\right)\left(y_t - y_t^n\right) + s\sigma\delta s^e\left(nw_t - nw_t^n\right)\left(i_t - i_t^n\right) + t.i.p\end{aligned}$$

where

$$n_t^* = \left\{\begin{array}{cc} 0 & \text{if } \beta^e < \beta \text{ and } \kappa = 1 \\ \frac{-\left(\frac{s\delta}{s^e}\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}f(\overline{\omega})v_t + \sigma_c c_t^n\right)}{\sigma s^e} & \text{if } 1 > \kappa\beta^e > \beta \text{ and } \kappa < \beta \end{array}\right\}$$

and that $n_t = 0$ if $\beta^e < \beta$ and $\kappa = 1$, and $n_t \neq 0$ if $1 > \kappa\beta^e > \beta$ and $\kappa < \beta$, which is consistent with (43) and the surrounding discussion in the main text.

I first consider the case where entrepreneurs do not save and so $\beta^e < \beta$ and $\kappa = 1$.

Proposition 1. *In the case that entrepreneurs do not save ($\beta^e < \beta$ and $\kappa = 1$) and entrepreneur consumption behaviour is described by (9) and (10) in the main text, to the first-order*

$$\begin{aligned}\widetilde{R}_t &= t.i.p \\ c_t^e &= \widehat{nw_t} = t.i.p \\ nw_t - nw_t^n &= 0\end{aligned}$$

Proof. It is sufficient to show that $\widetilde{R}_t$ is made up of terms that are independent of policy and $c_t^e = \widehat{nw_t} = t.i.p.$ To begin, I substitute out $-\frac{\alpha(1+\eta)}{1-\alpha}\alpha k_t^n k_t$ from the definition of $\widetilde{R}_t$ using the property that natural value of real marginal costs is zero (see C1)

$$\begin{aligned}\widetilde{R}_t = & \left(s\sigma\delta i_t^n - \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_t^n + \sigma s c_t^{en}\right)\alpha k_t \\ & - \frac{\alpha(1+\eta)}{1-\alpha} y_t^n k_t - s\sigma\delta y_t^n i_t \\ & + s^2\sigma\delta^2 i_t^n i_t + \sigma\left(s^e\right)^2 c_t^{en} c_t^e - \sigma s^e y_t^n c_t^e \\ & + s\sigma\delta s^e c_t^{en} i_t + s\sigma\delta s^e i_t^n c_t^e + \alpha k_t^n k_t \\ & - s\delta\Phi(\overline{\omega})\left(\xi_t\widehat{\Phi}(\overline{\omega}_t) + \xi_t i_t\right)\end{aligned}$$

Next I substitute for δi_t using the first-order approximation of capital accumulation and drop terms that are independent of policy to obtain

$$\begin{aligned}\widetilde{R}_t = & \left(\left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_t^n - s\sigma\delta i_t^n - \sigma s^e c_t^{en}\right)\alpha k_t \\ & - \frac{\alpha(1+\eta)}{1-\alpha} y_t^n k_t - \sigma y_t^n s\left(k_{t+1} - (1-\delta)k_t\right) \\ & + s\sigma\delta i_t^n s\left(k_{t+1} - (1-\delta)k_t\right) + \sigma\left(s^e\right)^2 c_t^{en} c_t^e - \sigma s^e y_t^n c_t^e \\ & + \sigma s^e c_t^{en} s\left(k_{t+1} - (1-\delta)k_t\right) + s\sigma\delta s^e i_t^n c_t^e + \alpha k_t^n k_t \\ & - s\delta\Phi(\overline{\omega})\xi_t\widehat{\Phi}(\overline{\omega}_t) - \Phi(\overline{\omega})\xi_t s\left(k_{t+1} - (1-\delta)k_t\right)\end{aligned}$$

Now I iterate out terms that are multiplicative in capital using

$$\begin{aligned}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\begin{array}{c} s\left(m_{t+1}k_{t+1}\right) - s(1-\delta) \\ m_t k_t - \alpha m_t k_t\end{array}\right) & = sE_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(m_{t+1}k_{t+1} - \beta^{-1}m_t k_t\right) \\ & = -s\beta^{-1}m_{t_0}k_{t_0} + s\lim_{j\to\infty}\beta^j E_{t_0}\left(m_{t_0+j}k_{t_0+j}\right) \\ & = t.i.p\end{aligned}$$

for *any* given endogenous variable m_t in the system under consideration. This follows from the property that I approximate around a non-distorted steady state where $\beta^{-1} = 1 - \delta + \frac{\alpha}{s}$ (noting $s \equiv \frac{K}{Y}$), that I focus on stationary policies such that $\lim_{j\to\infty}\beta^j E_{t_0}\left(m_{t_0+j}k_{t_0+j}\right)=0$, and that I restrict attention to policies that are optimal from a timeless perspective ($m_{t_0}k_{t_0}=t.i.p$). Thus, for the purposes of dealing with the remainder in period t, I can use the expression

$$m_t\left(sk_{t+1}-s\left(1-\delta\right)k_t\right)=\alpha m_t k_t+s\left(m_t-m_{t+1}\right)k_{t+1}$$

for *any* given endogenous variable m_t. Using this result, the remainder can be written as

$$\begin{aligned}\widetilde{R}_t=&-\alpha k_t y_t^n+\alpha k_t^n k_t\\&+\sigma\left(y_{t+1}^n-y_t^n\right)sk_{t+1}-s\sigma\delta\left(i_{t+1}^n-i_t^n\right)sk_{t+1}\\&-\sigma s^e\left(c_{t+1}^{en}-c_t^{en}\right)sk_{t+1}\\&+s^e c_t^e\left(\sigma s^e c_t^{en}-\sigma y_t^n+s\sigma\delta i_t^n\right)\\&-s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)-\Phi\left(\overline{\omega}\right)\xi_t\alpha k_t\\&-\Phi\left(\overline{\omega}\right)\left(\xi_t-\xi_{t+1}\right)sk_{t+1}\end{aligned}\tag{C3}$$

I now use the first-order approximations of the household Euler equation and the aggregate resource constraint (in the zero-inflation equilibrium) to obtain

$$\begin{aligned}\widehat{q_{t|t_0}^n}=&\ \sigma\left(y_{t|t_0}^n-y_{t|t_0+1}^n\right)-s\sigma\delta\left(i_{t|t_0}^n-i_{t|t_0+1}^n\right)-\sigma s^e\left(c_{t|t_0}^{en}-c_{t|t_0+1}^{en}\right)\\&+\left(1-\beta\left(1-\delta\right)\right)\left(y_{t|t_0+1}^n-k_{t|t_0+1}^n\right)+\beta\left(1-\delta\right)\widehat{q_{t|t_0+1}^n}\end{aligned}$$

which I can then use to eliminate the following term in $\widetilde{R}_t$

$$\left(\sigma\left(y_t^n-y_{t+1}^n\right)-s\sigma\delta\left(i_t^n-i_{t+1}^n\right)-\sigma s^e\left(c_t^{en}-c_{t+1}^{en}\right)\right)sk_{t+1}$$

The remainder (C3) becomes

$$\begin{aligned}\widetilde{R}_t=&-\alpha k_t y_t^n+\alpha k_t^n k_t\\&+\left(\left(1-\beta\left(1-\delta\right)\right)\left(y_{t+1}^n-k_{t+1}^n\right)-\widehat{q}_t^{\,n}+\beta\left(1-\delta\right)\widehat{q_{t+1}}^{\,n}\right)sk_{t+1}\\&+s^e c_t^e\left(\sigma s^e c_t^{en}-\sigma y_t^n+s\sigma\delta i_t^n\right)\\&-s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)-\Phi\left(\overline{\omega}\right)\xi_t\alpha k_t-\Phi\left(\overline{\omega}\right)\left(\xi_t-\xi_{t+1}\right)sk_{t+1}\end{aligned}$$

Iterating out $(1-\beta(1-\delta))\left(y_{t+1}^{n}-k_{t+1}^{n}\right)sk_{t+1}-\alpha\left(y_t^n-k_t^n\right)$
(noting $1-\beta(1-\delta)=\alpha\beta$ in steady state) I have

$$\begin{aligned}\widetilde{R}_t &= \left(\beta(1-\delta)\widehat{q_{t+1}}^{n}-\widehat{q}_t^{\,n}\right)sk_{t+1}\\ &\quad+s^e c_t^e\left(\sigma s^e c_t^{en}-\sigma y_t^n+s\sigma\delta i_t^n\right)\\ &\quad-s\delta\Phi(\overline{\omega})\xi_t\widehat{\Phi}(\overline{\omega}_t)-\Phi(\overline{\omega})\xi_t\alpha k_t-\Phi(\overline{\omega})\left(\xi_t-\xi_{t+1}\right)sk_{t+1}\end{aligned}\tag{C4}$$

Using Lemma (1) it follows that

$$\begin{aligned}c_t^e &= \widehat{nw}_t\\ &= t.i.p\end{aligned}$$

with respect to the loss function. This follows directly from the fact that I focus on an equilibrium where non-defaulting entrepreneurs are assumed to be sufficiently impatient that they consume all of their available resources once their investment return has been realised. Using this property of the equilibrium where entrepreneurs do not save, the remainder can be written as

$$\begin{aligned}\widetilde{R}_t &= \left(\beta(1-\delta)\widehat{q_{t+1}}^{n}-\widehat{q}_t^{\,n}-\Phi(\overline{\omega})\left(\xi_t-\xi_{t+1}\right)\right)sk_{t+1}\\ &\quad-s\delta\Phi(\overline{\omega})\xi_t\widehat{\Phi}(\overline{\omega}_t)-\Phi(\overline{\omega})\xi_t\alpha k_t\end{aligned}\tag{C5}$$

Using Lemma (2) it follows that in the flexible price equilibrium, asset prices are determined by shocks in the recovery rate

$$\widehat{q}_t^{\,n}=-\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t-\Phi(\overline{\omega})\xi_t$$

Substituting this expression into (C5) and iterating out the term
$(1-\beta(1-\delta))\Phi(\overline{\omega})\left(\xi_{t+1}\right)sk_{t+1}-\alpha\Phi(\overline{\omega})\xi_t k_t$ I obtain

$$\begin{aligned}\widetilde{R}_t &= \left(\beta(1-\delta)\left(-\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_{t+1}\right)+\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t\right)sk_{t+1}\\ &\quad-s\delta\Phi(\overline{\omega})\xi_t\widehat{\Phi}(\overline{\omega}_t)\end{aligned}$$

Using $\Phi(\overline{\omega})\widehat{\Phi(\overline{\omega}_t)} = \phi(\overline{\omega})\overline{\omega}\widehat{\overline{\omega}}_t$ and Lemma (3), it follows that the above expression can be re-written as

$$\begin{aligned}\widetilde{R}_t &= \left(\beta(1-\delta)\left(-\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_{t+1}\right)+\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t\right)sk_{t+1}\\ &\quad -s\delta\xi_t\phi(\overline{\omega})\frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\left(\widehat{nw_t}-i_t-\frac{\phi(\overline{\omega})g(\overline{\omega},0)}{1-\Phi(\overline{\omega})}\xi_t+\Phi(\overline{\omega})\xi_t\right)\end{aligned}$$

Again noting that $\widehat{nw_t} = c_t^e = t.i.p$ it follows that

$$\begin{aligned}\widetilde{R}_t &= \left(\beta(1-\delta)\frac{\phi(\overline{\omega})f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\xi_{t+1}-\frac{\phi(\overline{\omega})f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\xi_t\right)sk_{t+1}\\ &\quad +s\delta\phi(\overline{\omega})\frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}i_t\xi_t+t.i.p\end{aligned}$$

where I have used that in the steady state without monitoring costs

$$\begin{aligned}f(\overline{\omega}) &= 1-g(\overline{\omega},0)\\ f_{\overline{\omega}}(\overline{\omega}) &= \Phi(\overline{\omega})-1\end{aligned}$$

Again substituting for investment using a linear approximation of capital accumulation and then iterating out the remaining terms that are multiplicative in capital I have

$$\begin{aligned}\widetilde{R}_t &= \beta s(1-\delta)\frac{\phi(\overline{\omega})f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\left(\xi_{t+1}k_{t+1}-\beta^{-1}\xi_t k_t\right)\\ &= t.i.p\end{aligned}$$

which verifies the desired result. □

I now consider the case where entrepreneurs save ($1 > \kappa\beta^e > \beta$ and $\kappa < \beta$).

Proposition 2. *In the case that entrepreneurs do save* ($1 > \kappa\beta^e > \beta$ *and* $\kappa < \beta$) *and their consumption behaviour is described by (9) and (11),*

$$\widetilde{R}_t = -\sigma \left(s^e\right)^2 \left(n^*\right) n_t + t.i.p$$
$$\widehat{nw}_t - \widehat{nw}_t^n = c_t^e - c_t^{en} + O\left(\|\vartheta\|^2\right)$$
$$n_t^* = \frac{s\delta \frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})} f\left(\overline{\omega}\right) \nu_t + s^e \sigma_c c_t^n}{\sigma \left(s^e\right)^2}$$

Proof. The steps in this proof are almost identical to those used in the proof of Proposition (1). The only exception is that the policymaker must now keep track of entrepreneurial consumption and net worth, given that these variables are no longer independent of policy.

Using Lemma (1), it is straightforward to establish (irrespective of whether entrepreneurs save or not)

$$c_t^e = \frac{\phi\left(\overline{\omega}\right)}{1-\Phi\left(\overline{\omega}\right)} \xi_t + \widehat{nw}_t + O\left(\|\vartheta\|^2\right) \tag{C6}$$

and so

$$\widehat{nw}_t - \widehat{nw}_t^n = c_t^e - c_t^{en} + O\left(\|\vartheta\|^2\right)$$

I now re-evaluate the remainder for the equilibrium in which entrepreneurs save all of their available resources before consuming at retirement. Applying the same reasoning that is used in Proposition (1), but retaining terms in c_t^e and $\widehat{nw}_t$, that are no longer independent of policy, it can be observed that

$$\widetilde{R}_t = s^e c_t^e \left(\sigma s^e c_t^{en} - \sigma y_t^n + s\sigma\delta i_t^n\right) - s\delta\phi\left(\overline{\omega}\right) \frac{f\left(\overline{\omega}\right)}{f_{\overline{\omega}}\left(\overline{\omega}\right)} \widehat{nw}_t \xi_t + t.i.p$$

Using (C6) and a first-order approximation of the resource constraint (in the zero-inflation equilibrium),

$$-\sigma_c c_t^n = \sigma s^e c_t^{en} - \sigma y_t^n + s\sigma\delta i_t^n$$

I have

$$\widetilde{R}_t = \left(-s^e \sigma_c c_t^n - s\delta\phi\left(\overline{\omega}\right) \frac{f\left(\overline{\omega}\right)}{f_{\overline{\omega}}\left(\overline{\omega}\right)} \xi_t\right) \widehat{nw}_t + t.i.p$$

where

$$\widehat{nw_t} = \varphi_F \widehat{F_t^e} + \varphi_k k_t^e + \varphi_r r_t + \varphi_q q_t$$

From a first-order approximation of the definition of the recovery rate shock $e^{\xi_t} \equiv 1 - \nu_t$

$$\xi_t = -\nu_t + O\left(\|\vartheta\|^2\right)$$

where ν_t is the proportion of funds lost in the case of bankruptcy (with $\nu = 0$).[31] Using this result, and that in steady state, $f_\omega(\overline{\omega}) = \Phi(\overline{\omega}) - 1$, the desired result

$$\widetilde{R}_t = \left(-s\delta \frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})} f(\overline{\omega})\nu_t - s^e \sigma_c c_t^n\right) n_t + t.i.p$$

follows. □

The rest of this appendix discusses the lemmas used in Propositions (1) and (2).

Lemma 1. *In either of the saving or no saving equilibriums,* $c_t^e = \frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}\xi_t + \widehat{nw}_t$. *Moreover, specific to the equilibrium where entrepreneurs do not save (*$\beta^e < \beta$ *and* $\kappa = 1$*),* $\widehat{nw}_t = t.i.p$ *and so* $c_t^e = t.i.p$ *.*

Proof. To establish the first result, I begin by taking a first-order approximation of the derivative of the expected share of investment returns accruing to financial intermediaries (5), with respect to the default threshold,

$$g_{\overline{\omega}_t}\left(\widehat{\overline{\omega}_t, 1 - e^{\xi_t}}\right) = \frac{\phi(\overline{\omega})}{\widetilde{f_{\overline{\omega}_t}}(\overline{\omega})}\xi_t + \widehat{\widetilde{f_{\overline{\omega}_t}}(\overline{\omega}_t)} \quad \text{(C7)}$$

Combining (C7) with the optimal choice of default threshold, (33), I have

$$\widehat{q}_t + \widehat{f(\overline{\omega}_t)} = \frac{-\phi(\overline{\omega})}{\widetilde{f_{\overline{\omega}_t}}(\overline{\omega})}\xi_t + \left(1 - q_t . g\left(\widehat{\overline{\omega}_t, 1 - e^{\xi_t}}\right)\right) \quad \text{(C8)}$$

31 ν_t is in levels and not in logarithmic form.

Using the investment supply equation (34) and (C8)

$$\widehat{q}_t + \widehat{f(\overline{\omega}_t)} = \frac{-\phi(\overline{\omega})}{\widetilde{f}_{\overline{\omega}_t}(\overline{\omega})}\xi_t + \widehat{nw}_t - i_t \tag{C9}$$

Combining (C9) with (38), I obtain

$$c_t^e = \widehat{nw}_t - \frac{\phi(\overline{\omega})}{\widetilde{f}_{\overline{\omega}_t}(\overline{\omega})}\xi_t \tag{C10}$$

which establishes the first-result. Note that this result applies in both the equilibrium where entrepreneurs do not save, and in the equilibrium where entrepreneurs save.

To establish the second result, that is specific to the equilibrium where entrepreneurs do not save, recall that entrepreneurial net worth is given by

$$nw_t = F_t^e + (R_t + q_t(1-\delta))K_t^e$$

Combining the initial condition that entrepreneurs have no starting capital,[32] with the optimality condition that $K_{t+1}^e = 0$ for all $t \geq t_0$ *in the equilibrium where entrepreneurs do not save*, it follows that

$$nw_t = F_t^e \quad \text{for all } t \geq t_0$$

and thus

$$\begin{aligned} \widehat{nw}_t &= \widehat{F_t^e} \\ &= t.i.p \end{aligned} \tag{C11}$$

since the lump-sum transfer to entrepreneurs is not a control variable for the central bank. Using (C11) and (C10), it also follows that

$$c_t^e = t.i.p$$

which establishes the second result. □

32 Note that this is consistent with the economy beginning at its deterministic steady state where $K_{t_0}^e = 0$, when entrepreneurs do not save.

Lemma 2. $\widehat{q_t} = -\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t - \Phi(\overline{\omega})\xi_t$

Proof. I begin with the definition

$$\left(1-q_t.g\widehat{\left(\overline{\omega}_t,1-e^{\xi_t}\right)}\right) \equiv \ln\left(\frac{1-q_t.g\left(\overline{\omega}_t,1-e^{\xi_t}\right)}{1-q.g\left(\overline{\omega},0\right)}\right)$$

Taking a first approximation of $\ln\left(1-q_t.g\left(\overline{\omega}_t,1-e^{\xi_t}\right)\right)$ around the deterministic steady state without monitoring costs where $q=1$ and $\xi=0$ it follows that

$$\left(1-q_t.g\widehat{\left(\overline{\omega}_t,1-e^{\xi_t}\right)}\right) = -\frac{g\left(\overline{\omega},0\right)}{1-g\left(\overline{\omega},0\right)}\widehat{q_t} - \frac{g\left(\overline{\omega},0\right)}{1-g\left(\overline{\omega},0\right)}g\widehat{\left(\overline{\omega}_t,1-e^{\xi_t}\right)} \quad \text{(C12)}$$

Using a first-order approximation of the expected share of investment returns that accrues to banks (5),

$$g\widehat{\left(\overline{\omega}_t,1-e^{\xi_t}\right)} = \frac{-f\left(\overline{\omega}\right)}{1-f\left(\overline{\omega}\right)}\widehat{f\left(\overline{\omega}_t\right)} + \frac{\Phi\left(\overline{\omega}\right)}{1-f\left(\overline{\omega}\right)}\xi_t \quad \text{(C13)}$$

Substituting (C13) into (C12) implies

$$\left(1-q_t.g\widehat{\left(\overline{\omega}_t,1-e^{\xi_t}\right)}\right) = -\frac{g\left(\overline{\omega},0\right)}{1-g\left(\overline{\omega},0\right)}\widehat{q_t} + \widehat{f\left(\overline{\omega}_t\right)} - \frac{\Phi\left(\overline{\omega}\right)}{1-g\left(\overline{\omega},0\right)}\xi_t \quad \text{(C14)}$$

where $f\left(\overline{\omega}\right) = 1-g\left(\overline{\omega},0\right)$. Combining (C8) with (C14) it follows that

$$\widehat{q_t} = -\frac{\phi\left(\overline{\omega}\right)\left(1-g\left(\overline{\omega},0\right)\right)}{1-\Phi\left(\overline{\omega}\right)}\xi_t - \Phi\left(\overline{\omega}\right)\xi_t \quad \text{(C15)}$$

where I have used the property that $\widetilde{f}_{\overline{\omega}_t}\left(\overline{\omega}\right) = 1-\Phi\left(\overline{\omega}\right)$ in steady state. □

Lemma 3. $\left(\overline{\omega}\right)\widehat{\overline{\omega}}_t = \frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\left(\widehat{nw_t} - i_t - \frac{\phi(\overline{\omega})g(\overline{\omega},0)}{1-\Phi(\overline{\omega})}\xi_t + \Phi\left(\overline{\omega}\right)\xi_t\right)$

Proof. I begin by rearranging (C14)

$$\widehat{f(\overline{\omega}_t)} = \left(1 - q_t . g\left(\widehat{\overline{\omega}_t, 1} - e^{\xi_t}\right)\right) + \frac{g(\overline{\omega},0)}{1-g(\overline{\omega},0)}\widehat{q_t} + \frac{\Phi(\overline{\omega})}{1-g(\overline{\omega},0)}\xi_t$$

Using the investment supply condition (34) and (C15), the above expression becomes

$$\widehat{f(\overline{\omega}_t)} = \widehat{nw_t} - i_t + \frac{g(\overline{\omega},0)}{1-g(\overline{\omega},0)}\left(-\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t - \Phi(\overline{\omega})\xi_t\right) + \frac{\Phi(\overline{\omega})}{1-g(\overline{\omega},0)}\xi_t \tag{C16}$$

Using $\widehat{f(\overline{\omega}_t)} = \frac{f_\omega(\overline{\omega})\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t$ in (C16) I have

$$(\overline{\omega})\widehat{\overline{\omega}}_t = \frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\left(\widehat{nw_t} - i_t - \frac{\phi(\overline{\omega})g(\overline{\omega},0)}{1-\Phi(\overline{\omega})}\xi_t + \Phi(\overline{\omega})\xi_t\right)$$

as required. □

Appendix D: The Optimal Policy Problem

Using a first-order approximation (for brevity the steps are omitted here, but are available from the author on request), the constraints describing the decentralised equilibrium of the economy (measured in terms of deviations from zero-inflation target values) are given by

$$\begin{aligned}
0 = {} & \sigma x_t - \left(\beta(1-\delta)\sigma - (1-\beta(1-\delta))\frac{1+\eta}{1-\alpha}\right)E_t x_{t+1} \\
& - \left(s\sigma\left(1+\beta(1-\delta)^2\right) + (1-\beta(1-\delta))\frac{1+\alpha\eta}{1-\alpha}\right)j_{t+1} \\
& + s\sigma(1-\delta)j_t + \beta(1-\delta)s\sigma E_t j_{t+2} \\
& - \sigma s^e n_t + \beta(1-\delta)\sigma s^e E_t n_{t+1} \\
0 = {} & \varphi_k n_{t-1} + \varphi_r\left(\sigma + \frac{1+\eta}{1-\alpha}\right)x_t - \varphi_r s\sigma j_{t+1} \\
& + \varphi_r\left(s\sigma(1-\delta) - \frac{1+\alpha\eta}{1-\alpha}\right)j_t - \left(1+\varphi_r\sigma s^e\right)n_t
\end{aligned}$$

and

$$0=\Theta\left(\sigma+\frac{\alpha+\eta}{1-\alpha}\right)x_t-\Theta s\sigma j_{t+1}+\Theta\left(s\sigma\left(1-\delta\right)-\frac{\alpha\left(1+\eta\right)}{1-\alpha}\right)j_t$$
$$-\Theta\sigma s^e n_t+\beta E_t\pi_{t+1}-\pi_t$$
$$0=\sigma\left(x_t-E_tx_{t+1}\right)+s\sigma\left(1-\delta\right)j_t-s\sigma\left(1-\delta\right)E_tj_{t+1}$$
$$-\sigma s^e\left(n_t-E_tn_{t+1}\right)+i_t^b-r_t^{bn}-E_t\left(\pi_{t+1}\right)$$

where

$$\varphi_k\equiv\beta^{-1}\frac{K^e}{nw}$$
$$\varphi_r\equiv\left(\beta^{-1}-1+\delta\right)\frac{K^e}{nw}$$

The optimal policy problem is to minimise $E_0\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t$ where L_t is given by (43), subject to the above four constraints, and choosing sequences $\{i_t^b,g_t,j_t,n_t,\pi_t\}_{t=t_0}^{\infty}$ and where r_t^{bn} is given by the system defined in Appendix B. It is straightforward to show from the first-order conditions of this problem (again results omitted for brevity but are available on request), that a zero gap solution for all variables does not satisfy these conditions in the general case that $n_t^*\neq 0$, and given an economy that is initially at its efficient steady state. Therefore, in general, optimal policy will deviate from a zero-inflation policy.

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