Exchange Rates and Fundamentals: A Generalization

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Abstract

Exchange rates have raised the ire of economists for more than 20 years. The problem is that few, if any, exchange rate models are known to systematically beat a naive random walk in out of sample forecasts. Engel and West (2005) show that these failures can be explained by the present value model (PVM) because it predicts random walk exchange rate dynamics if the discount factor approaches one and fundamentals have a unit root. This paper broadens and generalizes the Engel and West (EW) hypothesis. We use standard time series tools to broaden analysis of the PVM. For example, our analysis exploits a common feature implication of the PVM and a discount near unity to show that the exchange rate follows a random walk. A PVM of the exchange rate is also constructed from an open economy dynamic stochastic general equilibrium (DSGE) model. The DSGE-PVM predicts that the exchange rate exhibits random walk behavior. Bayesian estimates reveal that the Canadian dollar-U.S. dollar exchange rate is dominated by permanent monetary and productivity shocks as the discount factor becomes close to one. Thus, our results generalize the EW hypothesis to the larger class of open economy DSGE models, while presenting new challenges for future research.

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1. Introduction

The search for satisfactory exchange rate models continues to be elusive. Since the seminal work of Meese and Rogoff (1983a, 1983b), a train of models have been tried in vain to improve on naive random walk forecasts of exchange rates. These range from linear rational expectations models examined by Meese (1986) to nonlinear models proposed by Diebold and Nason (1990), Meese and Rose (1991), Gençay (1999), and Kilian and Taylor (2003).

The Journal of International Economics volume edited by Engel, Rogers, and Rose (2003) indicates that there has been a split between theoretical exchange rate models and what is considered a useful forecasting model. For example, Kilian and Taylor (2003) argue that there are specific nonlinear forecasting models that can vie with a naive random walk of exchange rates. This approach maybe useful to obtain candidates for a forecast competition. Nonetheless, there are limits because, as Diebold and Nason (1990) note, the class of nonlinear exchange rate models might be infinite.

This paper steps back from the exchange rate forecasting problem. Rather, we study a workhorse theory of currency market equilibrium determination, the present-value model (PVM) of exchange rates. Actual data most often rejects the exchange rate PVM. Typical are tests Meese (1986) reported that are based on the first ten years of the floating rate regime. He finds that exchange rates are infected with persistent deviations from fundamentals, which reject the PVM and its cross-equation restrictions. However, Meese is unable to uncover the source of the rejections. Instead of a condemnation of the PVM, we view results such as Meese's as a challenge to update and deepen analysis of the PVM.

Engel and West (2005) take a similar position. Starting with the PVM and using uncontroversial assumptions about fundamentals and the discount factor, Engel and West (EW) hypothesize that the PVM generates an approximate random walk in exchange rates if currency traders are highly interest rate sensitive and fundamentals are $I(1)$. An important implication of the EW hypothesis is that fundamentals through date t have no power to forecast the future path of the exchange rate although the PVM dictates equilibrium in the currency market. EW support their hypothesis with a key theorem and empirical and simulation evidence.

This paper complements Engel and West (2005). We broaden and generalize the EW hypothesis. Standard time series tools are used to broaden the EW hypothesis to show that the standard-PVM predicts a random walk in the exchange rate when it and fundamentals share a common feature. We extend and generalize analysis of the PVM by linearizing and solving the uncovered interest rate parity (UIRP) condition and money demand function of a canonical monetary dynamic stochastic general equilibrium (DSGE) model. The linearized UIRP and money demand equations yield the DSGE-PVM, which we use to generalize the EW hypothesis. Table 1 reviews the key elements of the standard- and DSGE-PVMs.

Table 2 summarizes eight propositions that generalize the EW hypothesis. The first five propositions are constructed from the standard-PVM of the exchange rate, given fundamentals are $I(1)$ and fundamental growth has a Wold representation. The propositions are: (1) the exchange rate and fundamental cointegrate [Campbell and Shiller (1987)], (2) the PVM yields an error correction model (ECM) for currency returns in which the lagged cointegrating relation is the only regressor, (3) if fundamental growth depends only on the lagged cointegrating relation, the exchange rate and fundamental have a common trend-common cycle decomposition [Vahid and Engle (1993)], (4) the PVM predicts a limiting economy (*i.e.*, the PVM discount factor approaches one from below) in which the exchange rate is a martingale, and (5) the EW hypothesis is also satisfied in the limiting economy of (4) when the exchange rate and fundamental fail to cointegrate, but share a common feature. We report evidence using Canadian–, Japanese–, and U.K.–U.S. samples that reject (1), (3), but support (5). This represents more support for the EW hypothesis because we find a common feature exists between the Canadian dollar–, Yen–, and Pound–U.S. dollar exchange rates and the relevant fundamentals.

We generalize the EW hypothesis to the DSGE-PVM. Two propositions extend propositions (1) and (4) to the DSGE-PVM. We also show DSGE-PVM supports the EW hypothesis of proposition (5). A corollary is that the exchange rate is unpredictable when the DSGE-PVM discount factor goes to one. Thus, the standard- and DSGE-PVMs share many predictions. However, the DSGE-PVM suggests a richer set of testable restrictions because the class of open economy monetary DSGE models is large.

The DSGE-PVM imposes cross-equation restrictions on the exchange rate and fundamentals.

The observable fundamentals are cross-country money and consumption for the DSGE-PVM that we restrict with permanent-transitory decompositions. Under this assumption, the DSGE-PVM is cast as an unobserved components (UC) model. The UC model incorporates DSGE-PVM cross-equation restrictions conditional on whether the DSGE-PVM discount factor is calibrated or estimated. Three UC models calibrate the DSGE-PVM discount factor to one, which disconnects the exchange rate from transitory fundamentals. Transitory fundamentals restrict the exchange rate in three other UC models that estimate the DSGE-PVM discount factor. Within this dichotomy of the DSGE-PVM discount factor, six UC models are identified by restrictions on transitory cross-country money and consumption shocks.

We estimate six UC models on a Canadian-U.S. sample running from $1976Q1$ to $2004Q4$. The UC model yields a state space system for the DSGE-PVM, which allows us to exploit the Kalman filter to evaluate the likelihood. We compute likelihoods of the UC models using the Metropolis-Hastings (MH) simulator of Rabanal and Rubio-Ramírez (2005) to draw Markov chain Monte Carlo (MCMC) replications from the posteriors. The Canadian-U.S. data favor the UC model that calibrates the DSGE-PVM discount factor to one with the only transitory shock to cross-country consumption. Next is the UC model with the same transitory shock, but the estimated posterior mean of the DSGE-PVM discount factor is 0.9962. The posterior of this UC model revels that permanent shocks to fundamentals dominate Canadian dollar–U.S. dollar exchange rate fluctuations. Thus, the Canadian-U.S. data prefer UC models that are consistent with the EW hypothesis. Further, we find that the Canadian–U.S. data fail to support UC models that tie the exchange rate to the transitory monetary shock. Rogoff (2007) also notes that exchange rates appear disconnected from 'mean reverting monetary fundamentals', but our results are still puzzling because of the key roles assigned to nominal rigidities, UIRP shock persistence, and monetary disturbances in open economy monetary DSGE models.

The outline of the paper follows. The next section solves the standard-PVM of the exchange rate and presents its five propositions. Section 3 constructs the DSGE-PVM and dicusses its three propositions. Our Bayesian econometric strategy is discussed in section 4. Section 5 reports the posterior estimates of the six UC models. We conclude in section 6.

2. The Standard Present-Value Model of Exchange Rates

The standard-PVM determines the equilibrium exchange rate by combining a liquidity-money demand function, uncovered interest rate parity (UIRP) condition, purchasing power parity (PPP), and flexible prices. This is a workhorse exchange rate model used by, among others, Dornbusch (1976), Bilson (1978), Frankel (1979), Meese (1986), Mark (1995), and Engel and West (2005).

The standard-PVM of the exchange rate starts with the liquidity-money demand function

$$m_{h,t} - p_{h,t} = \psi y_{h,t} - \phi r_{h,t}, \quad 0 < \psi, \phi, \tag{1}$$

where $m_{h,t}$ $p_{h,t}$, $y_{h,t}$, and $r_{h,t}$ denote the home country's natural logarithm of money stock, price level, output, and the level of the nominal interest rate. The parameter ψ measures the income elasticity of money demand. Since the nominal interest rate is in its level, ϕ is the interest rate semi-elasticity of money demand. Define cross-country differentials $m_t = m_{h,t} - m_{f,t}$, $p_t = p_{h,t} - p_{f,t}$, $y_t = y_{h,t} - y_{f,t}$, and $r_t = r_{h,t} - r_{f,t}$, where f denotes the foreign country. Assuming PPP holds, $e_t = p_t$, where e_t is the log of the (nominal) exchange rate in which the U.S dollar is the home country's currency.

Under UIRP, the law of motion of the exchange rate is approximately

$$\mathbf{E}_t e_{t+1} - e_t = r_t. \tag{2}$$

Substitute for r_t in the law of motion of the exchange rate (2) with the money demand function (1) and impose PPP to produce the Euler equation $e_t - \omega \mathbf{E}_t e_{t+1} = (1-\omega) z_t$, where the standard-PVM discount factor is $\omega \equiv \dfrac{\phi}{1+\phi}$ and $z_t \equiv m_t - \psi y_t$ is the standard-PVM fundamental, which nets cross-country money with its income demand. Iterate on the Euler equation through date T and recognize that the transversality condition $\lim_{T\to\infty} \omega^{T+1} \mathbf{E}_t e_{t+T} = 0$ to obtain the present-value relation

$$e_t = (1-\omega) \sum_{j=0}^{\infty} \omega^j \mathbf{E}_t z_{t+j}. \tag{3}$$

The present-value relation (3) sets the log exchange rate equal to the annuity value of the fundamental z_t at the standard-PVM discount factor $\omega \equiv \dfrac{\phi}{1+\phi}$.[1]

[1] The present-value relation (3) yields the weak prediction that e Granger-causes z. Engel and West (2005) and Rossi (2007) report that this prediction is often not rejected in G –7 data.

2a. Cointegration Restrictions

The present-value relation (3) provides several predictions given

Assumption 1: $z_t \sim I(1)$.

Assumption 2: $(1-\mathbf{L})z_t$ has a Wold representation, $(1-\mathbf{L})z_t = \Delta z^* + \zeta(\mathbf{L})\upsilon_t$, where $\mathbf{L}z_t = z_{t-1}$.[2]

Given Assumptions 1 and 2, the first prediction is that e_t and z_t share a common trend. This follows from subtracting the latter from both sides of the equality of the present-value relation (3) and combining terms to produce the exchange rate-fundamental cointegrating relation

$$e_t - z_t = \sum_{j=1}^{\infty} \omega^j \mathbf{E}_t \Delta z_{t+j}, \quad \Delta \equiv 1 - \mathbf{L}. \tag{4}$$

Equation (4) reflects the forces – expected discounted value of fundamental growth – that push the exchange rate toward long-run PPP.

Proposition 1: *If z_t satisfies Assumptions 1 and 2, $\mathcal{X}_t = \beta' q_t$ forms a cointegrating relation with cointegrating vector $\beta' = [1 \quad -1]$, where $q_t \equiv [e_t \quad z_t]'$.*

The proposition is a variation of results found in Campbell and Shiller (1987). Note that the cointegrating relation becomes $\mathcal{X}_t = \zeta(\omega)\, \upsilon_t$, under Assumptions 1 and 2.

The cointegrating relation $\mathcal{X}_t$ equals the expected present discounted value of Δm_t minus $\psi \Delta y_t$. Thus, $\mathcal{X}_t$ is stationary, given Assumption 1 (*i.e.*, m_t and y_t are $I(1)$ and fail to share a common trend). We interpret $\mathcal{X}_t$ as the 'adjusted' exchange rate because it eliminates cross-country money stock movements netted for its income demand. The 'adjusted' exchange rate is a forward-looking function of the expected path of fundamental growth. This suggests the cointegrating relation is a "*cycle generator*", as described by Engle and Issler (1995), with the serial correlation of fundamental growth its source.

2b. Equilibrium Currency Return Dynamics

The second PVM prediction begins by writing the present-value relation (3) as

[2]The restrictions on the moving average are Δz^* is linearly deterministic, $\zeta_0 = 1$, $\zeta(\mathbf{L})$ is an infinite order lag polynominal with roots outside the unit circle, the ζ_is are square summable, and υ_t is mean zero, homoskedastic, linearly independent given history and is serially uncorrelated with itself and the past of Δz_t. Assumption 2 restricts fundamentals more than Engel and West (2005) require, but is standard for linear rational expectation models; see Hansen, Roberds, and Sargent (1991).

$$e_t \; - \; (1-\omega)z_t \;\; = \;\; (1-\omega)\sum_{j=1}^{\infty}\omega^j\mathbf{E}_t z_{t+j}.$$

Next, difference this equation, $\Delta e_t \; - \; (1-\omega)\Delta z_t \;\; = \;\; (1-\omega)\sum_{j=1}^{\infty}\omega^j\left[\mathbf{E}_t z_{t+j} \; - \; \mathbf{E}_{t-1}z_{t+j-1}\right]$, add and subtract $\mathbf{E}_t z_{t+j-1}$ inside the brackets, and use the present-value relation (4) to obtain

$$\Delta e_t \; - \; \frac{1-\omega}{\omega}\mathcal{X}_{t-1} \;\; = \;\; (1-\omega)\sum_{j=0}^{\infty}\omega^j[\mathbf{E}_t \; - \; \mathbf{E}_{t-1}]z_{t+j}. \tag{5}$$

Currency returns are driven by the lagged cointegrating relation and innovations to fundamentals.

Proposition 2: *Assume Proposition 1 holds. The PVM predicts that in equilibrium $\Delta e_t \sim$ ECM(0), an error correction model in which only the lagged cointegrating relation and forecast innovation appears.* The ECM(0) of currency returns is $\Delta e_t = \vartheta\mathcal{X}_{t-1} + u_{\Delta e,t}$, where $\vartheta = \dfrac{1-\omega}{\omega}$ and the present-value term of equation (5) is $u_{\Delta e,t} = (1-\omega)\;\zeta\,(\omega)\;\upsilon_t$, under Assumption 2.[3]

2c. The Common Trend and Common Cycle of Exchange Rates and Fundamentals

Proposition 2 provides an easy method to compute a BNSW common trend-common cycle decomposition for q_t. Given Δz_t is also an ECM(0), a BNSW decomposition of q_t relies on the cointegrating vector β' and the relationship between currency returns and fundamental growth.

Proposition 3: *Assume fundamental growth has an ECM(0) process $\Delta z_t = \eta\mathcal{X}_{t-1} + u_{\Delta z,t}$, where the forecast innovation $u_{\Delta z,t}$ is Gaussian. Given Proposition 2, q_t has a common feature, $\mathcal{F}_t = \overline{\beta}'\Delta q_t$, in the sense of Engle and Kozicki (1993), where $\overline{\beta}' = [1 \;\; -\frac{\vartheta}{\eta}]$. The cointegrating and common feature vectors β and $\overline{\beta}$ restrict the trend-cycle decomposition of q_t, as described by Vahid and Engle (1993).*

The currency return-fundamental growth common feature is apparent in the VECM(0)

$$\begin{bmatrix} \Delta e_t \\ \Delta z_t \end{bmatrix} = \begin{bmatrix} \vartheta \\ \eta \end{bmatrix}\mathcal{X}_{t-1} + \begin{bmatrix} u_{\Delta e,t} \\ u_{\Delta z,t} \end{bmatrix}.$$

Pre-multiply the bivariate ECM(0) by $\overline{\beta}'$ to obtain the common feature vector $\mathcal{F}_t$. According to Engle and Kozicki (1993), $\overline{\beta}$ creates a common feature in Δq_t because a linear combination of Δe_t and Δz_t are

[3]The error $u_{\Delta e,t}$ is also justified if the econometrician's information set is strictly within that of currency traders.

unpredictable based on the relevant history (*i.e.* , $u_{\Delta e,t}$ and $u_{\Delta z,t}$ are uncorrelated at all non-zero leads and lags). Hecq, Palm, and Urbain (2006) note that $\mathcal{F}_t$ restricts the spectra of Δq_t to be flat.

Proposition 3 predicts $q_t = [e_t \;\; z_t]'$ has a BNSW decomposition with one common trend and one common cycle. This mimics a result in Vahid and Engle (1993), which sets the trend of q_t to $\mathbf{I}_2 - \overline{\beta}(\beta'\overline{\beta})^{-1}\beta'$.[4] These restrictions decompose e_t into trend and cycle components $\frac{-\omega\eta}{1-\omega(1+\eta)}\overline{\beta}'q_t$ and $\frac{1-\omega}{1-\omega(1+\eta)}\beta' q_t$, respectively. Since Δe_t and Δz_t share a strong form common feature, the cycles common to e_t and z_t arise in the short-, medium-, and long-run. Thus, no long-run predictability exists for the exchange rate. A prediction at odds with the empirical evidence of Mark (1995).

2d. A Limiting Model of Exchange Rate Determination

Proposition 2 relies on $\omega < 1$ to define short- to medium-run currency return dynamics. This raises the question of the impact of relaxing this bound.

PROPOSITION 4: *The exchange rate approaches a martingale (in the strict sense) as* $\omega \longrightarrow 1$*, according to the present-value relation* (5) *assuming Proposition 1.*

Proposition 4 suggests an equilibrium path for e_{t+1} in which its best forecast is e_t, given relevant information.[5] The hypothesis of Proposition 4 drives the error u_t and slope coefficient ϑ of the ECM(0) regression to $u_t \xrightarrow{p} 0$ and $\vartheta \xrightarrow{p} 0$, which implies $\mathbf{E}_t e_{t+1} = e_t$. The martingale result implies random walk behavior for the exchange rate.[6]

2e. PVM Exchange Rate Dynamics Redux

Engel and West (2005) show that the PVM of the exchange rate yields an approximate random walk as ω approaches one. This section affirms the EW hypothesis, but unlike Proposition 3 does not rely on Proposition 2. Rather than follow the EW proof exactly, we invoke Assumptions 1 and 2, the present-value relation (3), the Weiner-Kolmogorov prediction formula, and the *conjecture* $e_t = \mathtt{a}z_t$ to find that currency returns are unpredictable.

[4]Vahid and Engle show a n–dimension VAR(1) with d cointegrating relations has $n - d$ common feature relations.

[5]Hansen, Roberds, and Sargent (1991) study linear rational expectations models that anticipate Proposition 4.

[6]Maheswaran and Sims (1993) show that the martingale restriction has little empirical content for tests of asset pricing models when data is sampled at discrete moments in time.

The EW hypothesis is $plim_{\omega \longrightarrow 1}[\Delta e_t - \mathsf{a}\zeta(\mathbf{1})\upsilon_t] = 0$. Its hypothesis test begins by noting $e_t = z_{t-1} + \sum_{j=0}^{\infty} \omega^j \mathbf{E}_t \Delta z_{t+j}$, which is obtained from the present-value relation (3). Use this equation to construct $\Delta e_t - \mathbf{E}_{t-1}\Delta e_t = \zeta(\omega)\upsilon_t$, given Assumptions 1 and 2 and the Weiner-Kolmogorov prediction formula. The present-value relation (3) also implies currency returns equal the annuity value of fundamental growth, $\Delta e_t = (1-\omega)\sum_{j=0}^{\infty} \omega^j \, \mathbf{E}_t \Delta z_{t+j}$. The last two equations yield

$$\Delta e_t \;=\; \zeta(\omega)\,\upsilon_t \;+\; (1-\omega)\sum_{j=0}^{\infty}\omega^j \mathbf{E}_{t-1}\Delta z_{t+j}. \tag{6}$$

By letting $\omega \xrightarrow{p} 1$, the random walk hypothesis of EW is verified independent of the ECM(0) of Proposition 2 (and cointegrating relation of Proposition 1).[7]

We show that the EW hypothesis is satisfied by exploiting the common feature implication of the PVM for currency returns and fundamental growth. This result relies on the assumption that Δq_t is $I(0)$ and has a Wold representation, $\Delta q_t = \lambda(\mathbf{L})\xi_t$. When q_t is $I(1)$ consisting of independent trends, the exchange rate and fundamental possess a bivariate BN decomposition, $q_t = \lambda(\mathbf{1})\Xi_t + \Lambda(\mathbf{L})\xi_t$, where $\lambda(\mathbf{1})$ has full rank, $\Lambda(\mathbf{L}) = \sum_{i=0}^{\infty}\Lambda_i$, $\Lambda_i = -\sum_{j=i+1}^{\infty}\lambda_j$, and $\Xi_t = \sum_{j=0}^{\infty}\xi_{t-j}$. The bivariate BN decomposition is

$$\Delta q_t \;=\; \lambda(\mathbf{1})\xi_t \;+\; \Delta\Lambda(\mathbf{L})\xi_t. \tag{7}$$

for the currency returns-fundamental growth rate system, which gives us

Proposition 5: *The exchange rate-random walk hypothesis of Engel and West (2005) requires currency returns and fundamental growth share a common feature, as well as* $\omega \longrightarrow 1$.

The EW hypothesis eliminates the BN cycle, $\Lambda(\mathbf{L})\xi_t$, from equation (7). Serial correlated common cycles are annihilated by the common feature vector $\overline{\beta}'$, which for the multivariate BN growth rates representation (7) sets $\overline{\beta}'\Delta q_t = \overline{\beta}'\lambda(\mathbf{1})\xi_t$. All that remains to drive Δq_t is $\lambda(\mathbf{1})\xi_t$. When $\overline{\beta}' \xrightarrow{p} [1 \;\; 0]$, Proposition 5 predicts that the limiting behavior of the exchange rate is a random walk driven by its own forecast innovations. Thus, the EW hypothesis is affirmed by Proposition 5. Proposition 5 also reveals that the EW hypothesis is consistent with a common feature restriction on short-, medium-, and long-run movements in the exchange rate and fundamentals.

[7] This analysis matches equations $A.3 - A.11$ and the surrounding discussion of Engel and West (2005).

2f. Tests of the PVM of the Exchange Rate

Propositions 1, 3, and 5 yield testable restrictions on exchange rates and fundamentals. If the lag length of the levels VAR of the exchange rate and fundamental exceeds one, the VECM(0) required by Proposition 3 is rejected. Cointegration tests suffice to examine Proposition 1. Vahid and Engel (1993) and Engel and Issler (1995) provide common feature tests that yield information about the EW hypothesis and Proposition 5. Table 3 summarizes the results and details the tests involved.

We estimate VARs of foreign currency-U.S. dollar exchange rates and fundamentals using Canadian, Japanese, U.K., and U.S. data on a $1976Q1$ – $2004Q4$ sample.[8] VAR lag lengths are chosen using likelihood ratio (LR) statistics, given a VAR(8), ..., VAR(1).[9] The Canadian–, Japanese–, and U.K.–U.S. samples yield a VAR(8), VAR(5), and VAR(4), respectively.[10] Thus, the Canadian, Japanese, U.K., and U.S. data reject a weak implication of Proposition 3.

Engel and West (2005) argue there is little evidence that exchange rates and fundamentals cointegrate. Table 3 presents Johansen (1991, 1994) trace and $\lambda-$max statistics that support this conclusion. Since these tests reject a cointegrating relation for the exchange rate and fundamental, we find no evidence to confirm Proposition 3.

Table 3 includes squared canonical correlations of currency returns and fundamental growth. The common feature null is that the smallest correlation equals zero. We use a χ^2 statistic found in Vahid and Engle (1993) and a $F-$statistic suggested by Rao (1973) to test this null. The tests reject the null for the largest canonical correlation, but not for the smaller one on the three samples. This supports Proposition 5 – the EW hypothesis that the exchange rate is a random walk – because currency returns and fundamental growth have a common feature.

[8] Fundamentals equal cross-country money minus cross-country output, which implies an income elasticity of money demand, ψ, calibrated to one. This calibration is consistent with estimates reported by Mark and Sul (2003). The money stocks (outputs) are measured in current (constant) local currency units and per capita terms.

[9] The VARs include a constant and linear time trend. The LR statistics employ the Sims (1980) correction and have standard asymptotic distribution according to results in Sims, Stock, and Watson (1990).

[10] The Canadian-U.S. and Japanese-U.S. VARs are selected when the $p-$value of the LR test is five percent or less. Since the U.K.-U.S. VAR offers ambiguous results, we settle on a VAR(4).

3. The DSGE Present-Value Model of the Exchange Rate

Propositions 1 – 5 broaden our understanding of the EW hypothesis. Standard time series tools are used to obtain restrictions on the joint behavior of exchange rates and fundamentals from the standard-PVM. For example, Proposition 1 predicts that the exchange rate and fundamentals cointegrate. However, we estimate VARs that reject the common trend prediction for the Canadian dollar-, Yen-, or Pound-U.S. dollar exchange rate and fundamentals. The estimated VARs also expose greater serial correlation in these series than predicted by the currency returns-fundamental growth VECM(0) of Proposition 3. Nonetheless, we find currency returns and fundamental growth share a common feature in the Canadian–, Japanese–, U.K.–U.S. data which supports the EW random walk hypothesis.

Rejection of the PVM is often given as a reason to discard linear rational expectations models of exchange rates. This paper does not. In this section, we develop a PVM model of the exchange rate derived from a canonical optimizing two-country monetary DSGE model. Our aim is to generalize the PVM and EW hypothesis to this wider class of DSGE models. We meet this goal with the DSGE-PVM which yields an equilibrium exchange rate model whose short-, medium-, and long-run predictions are testable on actual data. We address the empirical implications of the DSGE-PVM in the next two sections.

3a. The DSGE Model

The optimizing monetary DSGE model consists of the preferences of domestic and foreign economies and their resource constraints. For the home (h) and foreign (f) countries, the former objects take the form

$$\text{(8)} \qquad \mathcal{U}\left(C_{i,t},\ \frac{M_{i,t}}{P_{i,t}}\right) \ = \ \frac{\left[C_{i,t}^{\nu}\left(\frac{M_{i,t}}{P_{i,t}}\right)^{(1-\nu)}\right]^{(1-\varphi)}}{1-\varphi}, \quad 0 < \nu < 1, \quad 0 < \varphi,$$

where $C_{i,t}$ and $M_{i,t}$ represent the ith country's consumption and the ith country's holdings of its money stock. The resource constraint of the home country is

$$\text{(9)} \quad B_{h,t}^{h} + s_t B_{h,t}^{f} + P_{h,t}C_{h,t} + M_{h,t} = (1+r_{h,t-1})B_{h,t-1}^{h} + s_t(1+r_{f,t-1})B_{h,t-1}^{f} + M_{h,t-1} + P_{h,t}Y_{h,t},$$

where $B_{i,t}^{i}$, $B_{i,t}^{\ell}$, $r_{i,t-1}$, $r_{\ell,t-1}$, $Y_{i,t}$, and s_t denote the ith country's nominal holding of its own bonds at the end of date t, the ith country's nominal holding of the ℓth country's bonds at the end of date t, the return on the ith country's bond, the return on the ℓth country's bond, the output level of the ith country, and the level of the exchange rate. The two-country DSGE model is closed with $B^h_{h,t} + B^f_{h,t} + B^h_{f,t} + B^f_{f,t} = 0$. This condition forces the world stock of nominal debt to be in zero net supply, period-by-period, along the equilibrium path.

In section 2, analysis of the standard-PVM relies on $I(1)$ fundamentals. Likewise, we assume that the processes for labor-augmenting total factor productivity (TFP), $A_{i,t}$, and $M_{i,t}$ satisfy

Assumption 3: $ln[A_{i,t}]$ and $ln[M_{i,t}] \sim I(1)$, $i = h, f$.

Assumption 4: Cross-country TFP and money stock differentials are $I(1)$ and do not cointegrate.

Assumptions 3 and 4 impose stochastic trends on the two-country DSGE model.

3b. Optimizing UIRP and Money Demand

The home country maximizes its expected discounted lifetime utility over uncertainty streams of consumption and real balances,

$$\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} (1+\rho)^{-j} \mathcal{U}\left(C_{h,t+j},\ \frac{M_{h,t+j}}{P_{h,t+j}} \right) \right\}, \quad 0 < \rho,$$

subject to (9). The first-order necessary conditions of economy i yield optimality conditions that describe UIRP and money demand. The utility-based UIRP condition of the home country is

$$\mathbf{E}_t \left\{ \frac{\mathcal{U}_{C,h,t+1}}{P_{h,t+1}} \right\} (1 + r_{h,t}) = \mathbf{E}_t \left\{ \frac{\mathcal{U}_{C,h,t+1}}{P_{f,t+1}} \right\} \frac{(1 + r_{f,t})}{s_t}, \tag{10}$$

where $\mathcal{U}_{C,h,t}$ is the marginal utility of consumption of the home country at date t. Given the utility specification (8), the exact money demand function of country i is

$$\frac{M_{i,t}}{P_{i,t}} = C_{i,t} \left(\frac{1-\nu}{\nu} \right) \frac{1 + r_{i,t}}{r_{i,t}}, \quad i = h, f. \tag{11}$$

The consumption elasticity of money demand is unity, while the interest elasticity of money demand is a nonlinear function of the steady state bond return.

The UIRP condition (10) and money demand equation (11) can be stochastically detrended and then linearized to produce an equilibrium DSGE-law of motion for the exchange rate. Begin by combining the utility function (8) and the UIRP condition (10) to obtain

$$\mathbf{E}_t\left\{\frac{\mathcal{U}_{h,t+1}}{P_{h,t+1}C_{h,t+1}}\right\}(1 + r_{h,t}) \;=\; \mathbf{E}_t\left\{\frac{\mathcal{U}_{h,t+1}}{P_{f,t+1}C_{h,t+1}}\right\}\frac{(1 + r_{f,t})}{s_t},$$

where $\mathcal{U}_{i,t}$ is the utility level of country i at date t. Prior to stochastically detrending the previous expression, define $\widehat{\mathcal{U}}_{i,t} = \mathcal{U}_{i,t}/A_{i,t}$, $\hat{P}_{i,t} = P_{i,t}A_{i,t}/M_{i,t}$, $\hat{C}_{i,t} = C_{i,t}/A_{i,t}$, $\gamma_{A,i,t} = A_{i,t}/A_{i,t-1}$, $\gamma_{M,i,t} = M_{i,t}/M_{i,t-1}$, $\hat{s}_t = s_t A_t/M_t$, $A_t = A_{h,t}/A_{f,t}$, and $M_t = M_{h,t}/M_{f,t}$. Note that $\hat{C}_{i,t}$ is the transitory component of consumption of the ith economy, $\gamma_{A,i,t}(\gamma_{M,i,t})$ is the TFP (money) growth rate of country i, and the cross-country TFP (money stock) differential A_t (M_t) are $I(1)$. Applying the definitions, the stochastically detrended UIRP condition becomes

$$\mathbf{E}_t\left\{\frac{\widehat{\mathcal{U}}_{h,t+1}\gamma_{A,h,t+1}^{1-\varphi}}{\gamma_{M,h,t+1}\hat{P}_{h,t+1}\hat{C}_{h,t+1}}\right\}(1 + r_{h,t}) \;=\; \mathbf{E}_t\left\{\frac{\widehat{\mathcal{U}}_{h,t+1}\gamma_{A,f,t+1}}{\gamma_{A,h,t+1}^{\varphi}\gamma_{M,f,t+1}\hat{P}_{f,t+1}\hat{C}_{h,t+1}}\right\}\frac{(1 + r_{f,t})}{\hat{s}_t}.$$

A log linear approximation of the stochastically detrended UIRP condition yields

$$\mathbf{E}_t\tilde{e}_{t+1} - \tilde{e}_t \;=\; \frac{r^*}{1+r^*}\tilde{r}_t + \mathbf{E}_t\left\{\tilde{\gamma}_{A,t+1} - \tilde{\gamma}_{M,t+1}\right\}, \tag{12}$$

where, for example, $\tilde{e}_t = \ln[\hat{s}_t] - \ln[s^*]$ and $r^*(= r_h^* = r_f^*)$ denotes the steady state (or population) world real rate, for example.

3c. A DSGE-PVM of the Exchange Rate

We use the linear approximate law of motion of the exchange rate (12), and a stochastically detrended version of the money demand equation (11) to produce the DSGE-PVM. The unit consumption elasticity-money demand equation (11) implies the money demand equation $-\tilde{p}_t = \tilde{c}_t - \frac{1}{1+r^*}\tilde{r}_t$. Impose PPP on the stochastically detrended version of the money demand equation and combine it with the law of motion (12) of the transitory component of the exchange rate to find

$$\left[1 - \frac{1}{1+r^*}\mathbf{E}_t\mathbf{L}^{-1}\right]\tilde{e}_t \;=\; \frac{1}{1+r^*}\mathbf{E}_t\left\{\tilde{\gamma}_{M,t+1} - \tilde{\gamma}_{A,t+1}\right\} - \frac{r^*}{1+r^*}\tilde{c}_t.$$

Solving this stochastic difference equation forward yields the DSGE-PVM

$$(13) \qquad \tilde{e}_t = \sum_{j=1}^{\infty} \kappa^j \mathbf{E}_t \left\{ \tilde{\gamma}_{M,t+j} - \tilde{\gamma}_{A,t+j} \right\} - (1-\kappa) \sum_{j=0}^{\infty} \kappa^j \mathbf{E}_t \tilde{c}_{t+j},$$

where the relevant tranversality conditions are invoked and the DSGE-PVM discount factor $\kappa \equiv \frac{1}{1+r^*}$. Note that the DSGE-PVM and permanent income hypothesis discount factors are equivalent.

The DSGE-PVM relation (13) is the equilibrium law of motion of the cyclical component of the exchange rate. Transitory movements in the exchange rate are equated with the future discounted expected path of cross-country money and TFP growth and the (negative of the) annuity-value of the transitory component of cross-country consumption. The DSGE model identifies the exchange rate's unobserved time-varying risk premium with expected path of cross-country TFP growth and transitory consumption, which suggest additional sources of exchange rate fluctuations.

3d. DSGE-PVM Cointegration Restrictions

The DSGE-PVM produces an ECM of the exchange rate. The cointegrating relation follows from a balanced growth restrictions of the DSGE model, $e_t \equiv \ln[s_t] = \ln[\hat{s}_t] + m_t - a_t$, where $m_t = \ln[M_t]$ and $a_t = \ln[A_t]$. Thus, the DSGE-PVM produces the cointegrating relation

$$(14) \qquad x_{DSGE,t} = \tilde{e}_t + \tilde{c}_t, \quad x_{DSGE,t} \equiv e_t - (m_t - c_t),$$

where constants are ignored, $c_t = \ln[C_t]$, and stochastic detrending implies $a_t = c_t - \tilde{c}_t$.

The ECM reflects the forces that push the exchange rate toward long-run PPP plus sources of short- and medium-run PPP deviations. Persistent PPP deviations depend on the forward-looking component $\tilde{e}_t$ and transitory date t cross-country consumption, $\tilde{c}_t$. Nonetheless, the DSGE-PVM restricts PPP deviations to be stationary, which leads to

Proposition 6: *If m_t and A_t satisfy Assumptions 3 and 4, $x_{DSGE,t} = \beta'_{DSGE} q_{DSGE,t}$ forms a cointegrating relation with cointegrating vector $\beta'_c = [1 \;\; -1 \;\; 1]$, where $q_{DSGE,t} \equiv [e_t \;\; m_t \;\; c_t]'$.*

The DSGE-PVM predicts a forward-looking cointegrating relation, but with new sources of transitory dynamics. Unobserved $\tilde{e}_t$ and $\tilde{c}_t$ movements create persistence and volatility in the "*cycle genera-*

tor" $\mathcal{X}_{DSGE,t}$ of (14). Thus, the DSGE-PVM engages unobserved sources of serial correlated short- and medium-run PPP deviations not found in the standard-PVM to drive exchange rate fluctuations.

3e. DSGE-PVM Equilibrium Currency Return Dynamics

The DSGE model produces an equilibrium currency return generating equation that resembles the standard-PVM (5). Thus, the DSGE model predicts martingale-like behavior for the exchange rate given Assumptions 3 and 4, Proposition 6, and that κ approaches one. This result follows from unwinding the stochastic detrending of the DSGE-PVM (13) to find

$$(15) \qquad e_t \;=\; (1-\kappa)\sum_{j=0}^{\infty}\kappa^j \mathbf{E}_t\left\{m_{t+j} - c_{t+j}\right\}.$$

Subsequent to some algebra, the levels DSGE-PVM (15) yields the ECM(0)

$$(16) \qquad \Delta e_t - \frac{(1-\kappa)}{\kappa}\mathcal{X}_{DSGE,t-1} \;=\; (1-\kappa)\sum_{j=0}^{\infty}\kappa^j\left[\mathbf{E}_t - \mathbf{E}_{t-1}\right]\left\{m_{t+j} - c_{t+j}\right\}.$$

This suggests

Proposition 7: *Assume e_t, m_t, and c_t share a common trend. As $\kappa \xrightarrow{p} 1$, the exchange rate approximates a martingale.*

The DSGE model restricts currency returns to follow the ECM(0) of (16). When the DSGE-PVM discount factor near unity, current returns becomes independent of fundamentals. Thus, the DSGE-PVM predicts, as does the standard-PVM, an equilibrium in which the current exchange rate is the best forecast of the future exchange rate, $\mathbf{E}_t e_{t+1} = e_t$. Proposition 7 suggests why open economy DSGE models have trouble producing exchange rate forecasts that beat a naive random walk. The exchange rate decouples from fundamentals at a small world real interest rate r^*, or $\kappa \longrightarrow 1$.

The levels DSGE-PVM (15) also generates exchange rate dynamics that satisfy the EW hypothesis. From this version of the DSGE-PVM, it is straightforward to obtain

$$(17) \qquad \Delta e_t \;=\; \sum_{j=0}^{\infty}\kappa^j\left[\mathbf{E}_t - \mathbf{E}_{t-1}\right]\left\{\Delta m_{t+j} - \Delta c_{t+j}\right\} + (1-\kappa)\sum_{j=0}^{\infty}\kappa^j\mathbf{E}_t\left\{\Delta m_{t+j} - \Delta c_{t+j}\right\}.$$

It is obvious from (17) that as κ goes to one currency returns become unpredictable in the forecast innovations of cross-country money and consumption growth, which verifies the EW hypothesis. We make this explicit by endowing m_t with the permanent component μ_t, where $\mu_{t+1} = \mu_t + \varepsilon_{\mu,t+1}$, $\varepsilon_{\mu,t+1} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_\mu})$, and likewise let $a_{t+1} = a_t + \varepsilon_{a,t+1}$, $\varepsilon_{a,t+1} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_a})$, which satisfies Assumption 4. Also, take the transitory components of m_t and c_t to be Wold, where $\varsigma_i(\mathbf{L})\varepsilon_{i,t}$, $i = \widetilde{m}, \widetilde{c}$ (with $\varsigma_{i,0} = 1$). Applying these permanent-transitory decompositions to (17) produces

$$\begin{aligned}(18) \qquad \Delta e_t \; = \; & \varepsilon_{\mu,t} + \varepsilon_{\widetilde{m},t} - (\varepsilon_{a,t} + \varepsilon_{\widetilde{c},t}) + (1-\kappa)\Big[[1 - \varsigma_{\widetilde{m}}(\kappa)]\varepsilon_{\widetilde{m},t} \\ & - [1 - \varsigma_{\widetilde{c}}(\kappa)]\varepsilon_{\widetilde{c},t} + \sum_{j=0}^{\infty} \kappa^j \mathbf{E}_t \left\{\Delta m_{t+j} - \Delta c_{t+j}\right\}\Big].\end{aligned}$$

The equilibrium currency return generating equation (18) gives us

Proposition 8: *Let m_t and c_t have random walk permanent and transitory Wold components. As the DSGE-PVM discount factor $\kappa \xrightarrow{p} 1$, the exchange rate becomes a random walk driven by contemporaneous innovations to the permanent and transitory components of fundamentals.*

Let κ go to one (from below). The equilibrium currency return generating equation (18) collapses to $\Delta e_t = \varepsilon_{\mu,t} + \varepsilon_{\widetilde{m},t} - \varepsilon_{a,t} - \varepsilon_{\widetilde{c},t}$. The currency return is white noise tied to innovations in the random walk and transitory Wold components of fundamentals, as the DSGE-PVM discount factor nears one. Thus, Proposition 8 generalizes the EW hypothesis that the exchange rate mimics a random walk to the larger class of DSGE models. A corollary is that changes in fundamentals do not Granger cause currency returns as $\kappa \longrightarrow 1$. Only if $\kappa \in (0, 1)$ do movements in fundamentals have predictive power for currency returns according to the DSGE-PVM (18). However, currency returns Granger cause growth in fundamentals as long as it is predicted by lags of permanent or transitory innovations to cross-country money and TFP. The equilibrium currency return generating equation (18) shows that this holds even if $\kappa \longrightarrow 1$.

In summary, this section constructs and studies a DSGE-PVM of the exchange rate that generalizes the EW hypothesis. We show that the DSGE-PVM predicts the exchange rate and fundamentals cointegrate, an ECM(0) generates equilibrium currency returns, and the exchange rate mimics a random walk as the DSGE-PVM discount factor goes to one. Since the DSGE-PVM predictions match those of the standard-PVM, we extend the EW hypothesis to the large class of open economy monetary DSGE models. The next two sections describe the empirical content of the DSGE-PVM and draw its implications for a Canadian–U.S. sample.

4. Econometric Models and Methods

This section describes the Bayesian methods we employ to estimate six UC multivariate models. The UC models represent different combinations of restrictions imposed by the DSGE-PVM on the exchange rate, cross-country money, and cross-country consumption. For example, κ is estimated for three UC models, which ties the exchange rate to the transitory component(s) of fundamentals. The exchange rate is disconnected from transitory shocks in remaining three UC models because κ is calibrated to one. We cast the UC models in state space form to evaluate numerically the likelihoods on a $1976Q1$–$2004Q4$ sample of the Canadian dollar–U.S. dollar ($CDN\$/US\$$) exchange rate and the Canadian–U.S. money and consumption differentials. The random walk MH simulator is used to generate MCMC draws from the UC model posterior distributions conditional on this sample. We compute model moments, such as parameter means, unconditional variance ratios, permanent-transitory decompositions, and forecast error variance decompositions (FEVDs), from the posterior distributions. Model comparisons are based on marginal likelihoods, which we construct by integrating the likelihood function of each model across its parameter space where the weighting function is the model prior.

4a. State Space Systems of the UC Models

The state space systems of the six UC models begin with the balanced growth restriction the DSGE model imposes on the exchange rate. This restriction is equivalent to the permanent-transitory decomposition $e_t = m_t - a_t + \widetilde{e}_t$. The DSGE-PVM (13) puts restrictions on the stationary component of the exchange rate, $\widetilde{e}_t$. These elements form the balanced growth version of the DSGE-PVM

$$e_t = \mu_t - a_t + (1-\kappa)\sum_{j=0}^{\infty}\kappa^j \mathbf{E}_t\left\{\widetilde{m}_{t+j} - \widetilde{c}_{t+j}\right\}, \tag{19}$$

which also relies on the DSGE balanced growth path restrictions on cross-country money and consumption. The balanced growth DSGE-PVM (19) implies the cointegrating relation Proposition 6 places on the exchange rate and cross-country money and consumption. Thus, the exchange rate responds only to trends in cross-country money, μ_t, and TFP, a_t, in the long-run. Serial correlation in the exchange rate is produced by the transitory components of cross-country money and consumption, $\widetilde{m}_t$ and $\widetilde{c}_t$. Also, note that if a common cycle generates these transitory components, the exchange does shares the restriction. Thus, the permanent and transitory components of cross-country money and consumption drive exchange rate fluctuations, which give rise to cross-equations in the UC models.

Cross-equation restrictions are conditioned on the permanent and transitory components of cross-country money and cross-country consumption. We assume $\widetilde{m}_t$ is a MA($k_{\widetilde{m}}$), $\widetilde{m}_t = \sum_{j=0}^{k_{\widetilde{m}}} \alpha_j \varepsilon_{\widetilde{m},t-j}$, where $\alpha_0 \equiv 1$ and $\varepsilon_{\widetilde{m},t} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_{\widetilde{m}}})$. For $\widetilde{c}_t$, we employ a AR($k_{\widetilde{c}}$), $\widetilde{c}_t = \sum_{j=1}^{k_{\widetilde{c}}} \theta_j \widetilde{c}_{t-j} + \varepsilon_{\widetilde{c},t}$, where $\varepsilon_{\widetilde{c},t} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_{\widetilde{c}}})$. As in the previous section, the permanent components of money and consumption are $\mu_{t+1} = \mu^* + \mu_t + \varepsilon_{\mu,t+1}$, $\varepsilon_{\mu,t+1} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_\mu})$, and $a_{t+1} = a^* + a_t + \varepsilon_{a,t+1}$, $\varepsilon_{a,t+1} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_a})$. Note that μ^* and a^* are the deterministic trend growth rates of cross-country money and TFP.

We classify the UC models according to whether there are two cycles or a common cycle and whether κ is calibrated to one or estimated. Thus, the DSGE-PVM (19) is solved for the exchange rate given $\widetilde{m}_t \sim$ MA($k_{\widetilde{m}}$) and $\widetilde{c}_t \sim$ AR($k_{\widetilde{c}}$) or a common cycle is imposed using either the MA($k_{\widetilde{m}}$) or AR($k_{\widetilde{c}}$). We double these three UC models when κ is calibrated to one or not. However, the six UC models have in common the cross-country money and TFP trends, μ_t and a_t.

The richest set of cross-equation restrictions arises in the 2-trend, 2-cycle UC model with $\kappa \in$ (0, 1). In part, its state space system consists of the observation equations

$$(20) \qquad \begin{bmatrix} e_t \\ m_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 & -1 & \delta_{\widetilde{m},0} & \delta_{\widetilde{m},1} & \dots & \delta_{\widetilde{m},k_{\widetilde{m}}} & \delta_{\widetilde{c},0} & \delta_{\widetilde{c},1} & \dots & \delta_{\widetilde{c},k_{\widetilde{c}}-1} \\ 1 & 0 & 1 & \alpha_1 & \dots & \alpha_{k_{\widetilde{m}}} & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 \end{bmatrix} S_{\widetilde{m},c,t},$$

where $S_{\widetilde{m},c,t} = \begin{bmatrix} \mu_t & a_t & \varepsilon_{\widetilde{m},t} & \varepsilon_{\widetilde{m},t-1} & \dots & \varepsilon_{\widetilde{m},t-k_{\widetilde{m}}} & \widetilde{c}_t & \widetilde{c}_{t-1} & \dots & \widetilde{c}_{t-k_{\widetilde{c}}+1} \end{bmatrix}'$, the factor loadings on $\varepsilon_{\widetilde{m},t}$ and its lags are

$$\delta_{\widetilde{m},i} \;=\; (1-\kappa)\sum_{j=i}^{k_{\widetilde{m}}}\kappa^{j-i}\alpha_j, \quad i \;=\; 0,\ldots,k_{\widetilde{m}}, \tag{21}$$

the factor loadings on $\widetilde{c}_t, \ldots, \widetilde{c}_{t-k_{\widetilde{c}}}$ are elements of the row vector

$$\delta_{\widetilde{c}} \;=\; -s_{\widetilde{c}}(1-\kappa)\Big[\mathbf{I}_{k_{\widetilde{c}}} \;-\; \kappa\Theta\Big]^{-1}, \quad s_{\widetilde{c}} \;=\; [1 \;\; \mathbf{0}_{1\times k_{\widetilde{c}}-1}], \tag{22}$$

and Θ is the companion matrix of the AR($k_{\widetilde{c}}$) of $\widetilde{c}_t$. The system of first-order state equations is

$$S_{\widetilde{m},c,t+1} \;=\; \begin{bmatrix} \mu^* \\ a^* \\ 0 \\ \vdots \\ 0 \\ \vdots \end{bmatrix} + \begin{bmatrix} 1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \mathbf{I}_{k_{\widetilde{m}}} & \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & 0 & \theta_1 & \ldots & \theta_{k_{\widetilde{c}}} \\ \vdots & \vdots & & \vdots & & \mathbf{I}_{k_{\widetilde{c}}-1} & \mathbf{0}_{(k_{\widetilde{c}}-1)\times 1} \\ & & & & & & \end{bmatrix} S_{\widetilde{m},c,t} \;+\; \begin{bmatrix} \varepsilon_{\mu,t+1} \\ \varepsilon_{a,t+1} \\ \varepsilon_{\widetilde{m},t+1} \\ \mathbf{0}_{k_{\widetilde{m}}\times 1} \\ \varepsilon_{\widetilde{c},t+1} \\ \mathbf{0}_{(k_{\widetilde{c}}-1)\times 1} \end{bmatrix}, \tag{23}$$

with the covariance matrix $\Omega_{m,\widetilde{c}} = \varepsilon_{\widetilde{m},c,t}\varepsilon'_{\widetilde{m},c,t}$ where $\varepsilon_{\widetilde{m},c,t} = [\varepsilon_{\mu,t+1} \;\; \varepsilon_{a,t+1} \;\; \varepsilon_{\widetilde{m},t+1} \;\; \mathbf{0}_{k_{\widetilde{m}}\times 1} \;\; \varepsilon_{\widetilde{c},t+1} \;\; \mathbf{0}_{(k_{\widetilde{c}}-1)\times 1}]'$.

We also study the implications of imposing one common transitory factor on m_t and c_t. When this common component is $\widetilde{m}_t$, the response of c_t to $\widetilde{m}_t$ is denoted $\pi_{m,\widetilde{c}}$. This implies $\widetilde{c}_t = \pi_{m,\widetilde{c}}\widetilde{m}_t = \pi_{m,\widetilde{c}}\sum_{j=0}^{k_{\widetilde{m}}}\alpha_j\varepsilon_{\widetilde{m},t-j}$. For the 2-trend, money cycle UC model, the state vector and observer system are $S_{\widetilde{m},t} = \Big[\mu_t \;\; a_t \;\; \varepsilon_{\widetilde{m},t} \;\; \varepsilon_{\widetilde{m},t-1} \;\ldots\; \varepsilon_{\widetilde{m},t-k_{\widetilde{m}}}\Big]'$ and

$$\begin{bmatrix} e_t \\ m_t \\ c_t \end{bmatrix} \;=\; \begin{bmatrix} 1 & -1 & (1-\pi_{c,\widetilde{m}})\delta_{\widetilde{m},0} & (1-\pi_{c,\widetilde{m}})\delta_{\widetilde{m},1} & \ldots & (1-\pi_{c,\widetilde{m}})\delta_{\widetilde{m},k_{\widetilde{m}}} \\ 1 & 0 & 1 & \alpha_1 & \ldots & \alpha_{k_{\widetilde{m}}} \\ 0 & 1 & \pi_{c,\widetilde{m}} & \pi_{c,\widetilde{m}}\alpha_1 & \ldots & \pi_{c,\widetilde{m}}\alpha_{k_{\widetilde{m}}} \end{bmatrix} S_{\widetilde{m},t}, \tag{24}$$

respectively. The state equation of this system is

$$
(25) \qquad S_{\widetilde{m},t+1} = \begin{bmatrix} \mu^* \\ a^* \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix} S_{\widetilde{m},t} + \begin{bmatrix} \varepsilon_{\mu,t+1} \\ \varepsilon_{a,t+1} \\ \varepsilon_{\widetilde{m},t+1} \\ 0 \\ \vdots \\ 0 \end{bmatrix},
$$

with covariance matrix $\Omega_{\widetilde{m}} = \varepsilon_{\widetilde{m},t}\varepsilon'_{\widetilde{m},t}$, where $\varepsilon_{\widetilde{m},t} = [\varepsilon_{\mu,t+1} \;\; \varepsilon_{a,t+1} \;\; \varepsilon_{\widetilde{m},t+1} \;\; 0 \dots 0]'$.

Identifying the common transitory component with $\widetilde{c}_t$ restricts $\widetilde{m}_t = \pi_{m,\widetilde{c}}\widetilde{c}_t$. This yields the system of observer equations of the 2-trend, consumption cycle UC model

$$
(26) \qquad \begin{bmatrix} e_t \\ m_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 & -1 & (1-\pi_{m,\widetilde{c}})\delta_{\widetilde{c},0} & (1-\pi_{m,\widetilde{c}})\delta_{\widetilde{c},1} & \dots & (1-\pi_{m,\widetilde{c}})\delta_{\widetilde{c},k_{\widetilde{c}}-1} \\ 1 & 0 & \pi_{m,\widetilde{c}} & 0 & \dots & 0 \\ 0 & 1 & 1 & 0 & \dots & 0 \end{bmatrix} S_{\widetilde{c},t},
$$

and the system of state equations

$$
(27) \qquad S_{\widetilde{c},t+1} = \begin{bmatrix} \mu^* \\ a^* \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \theta_1 & \theta_2 & \dots & \theta_{k_{\widetilde{c}}-1} & \theta_{k_{\widetilde{c}}} \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} S_{\widetilde{c},t} + \begin{bmatrix} \varepsilon_{\mu,t+1} \\ \varepsilon_{a,t+1} \\ \varepsilon_{\widetilde{c},t+1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},
$$

where $S_{\widetilde{c},t} = \left[\mu_t \;\; a_t \;\; \widetilde{c}_t \;\; \widetilde{c}_{t-1} \;\dots\; \widetilde{c}_{t-k_{\widetilde{c}}+1}\right]'$, $\Omega_{\widetilde{c}} = \varepsilon_{\widetilde{c},t}\varepsilon'_{\widetilde{c},t}$, and $\varepsilon_{\widetilde{c},t} = [\varepsilon_{\mu,t+1} \;\; \varepsilon_{a,t+1} \;\; \varepsilon_{\widetilde{c},t+1} \;\; 0 \dots 0]'$.

The three remaining UC models set $\kappa = 1$ in the state space systems of the 2-trend, 2-cycle UC model, the 2-trend, money cycle UC model, and the 2-trend, consumption cycle UC model. These are versions of the limiting DSGE models in which $\kappa \longrightarrow 1$. The restriction on the state space of these UC models is that beyond the second column only zeros occupy the first row of the observation equations (20), (24), and (26). Thus, we are able to compare DSGE-PVMs with κ estimated on (0, 1) to limiting DSGE models in which $\kappa \longrightarrow 1$ on our Canadian-U.S. sample. This provides an empirical appraisal of the EW hypothesis.

4b. The UC Model and Its Likelihood Function

We label the 2-trend, 2-cycle UC model with $\kappa \in (0, 1)$ $UC_{2,2,\kappa}$. Likewise, $UC_{2,\widetilde{m},\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ denote the 2-trend, money cycle and 2-trend, consumption cycle, $\kappa \in (0, 1)$ UC models. The state space systems of $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$, and $UC_{2,\widetilde{c},\kappa}$ are (20) and (23), (24)–(25), and (26)–(27), respectively. These state space systems represent the dynamics of $\mathcal{Y}_t = [e_t \;\; m_t \;\; c_t]'$ restricted by the DSGE-PVM and permanent-transitory specifications of m_t and c_t. These state space systems are mapped into the Kalman filter to evaluate the likelihood function as proposed by Harvey (1989) and Hamilton (1994).[11] Denote the likelihood $\mathcal{L}(\mathcal{Y}_t \mid \Gamma_{2,i,\kappa},\ UC_{2,i,\kappa})$, where $i = 2,\ \widetilde{m},\ \widetilde{c}$ and $\Gamma_{2,i,\kappa}$ is the parameter vector of $UC_{2,i,\kappa}$. The parameter vector of $UC_{2,2,\kappa}$ contains $11 + k_{\widetilde{m}} + k_{\widetilde{c}}$ elements, $\Gamma_{2,2,\kappa} = [\kappa \;\; \alpha_1 \;\; \ldots \;\; \alpha_{k_{\widetilde{m}}} \;\; \theta_1 \;\; \ldots \;\; \theta_{k_{\widetilde{c}}}$ $\mu^* \;\; a^* \;\; \sigma_\mu \;\; \sigma_a \;\; \sigma_{\widetilde{m}} \;\; \sigma_{\widetilde{c}} \;\; \varrho_{a,\widetilde{c}} \;\; \pi_{e,0} \;\; \pi_{e,t} \;\; \pi_{e,a}]'$.

We add the parameters $\varrho_{a,\widetilde{c}}$, $\pi_{e,0}$, $\pi_{e,t}$, and $\pi_{e,a}$ to $\Gamma_{2,2,\kappa}$ to better fit the UC models to the data. For example, the Canadian-U.S. TFP differential exhibits more variation than c_t if the correlation coefficient of innovations to a_t and $\widetilde{c}_t$, $\mathbf{E}\{\varepsilon_{a,t}\, \varepsilon_{\widetilde{c},t}\} = \varrho_{a,\widetilde{c}}$, is negative.[12] The remaining three parameters allow for an unrestricted exchange rate intercept, $\pi_{e,0}$, a linear exchange rate time trend, $\pi_{e,t}$, and a factor loading on the Canadian-U.S. TFP differential, $\pi_{e,a}$, that differs from negative one for the (1, 2) element in the matrix of the observation equation systems (20), (24), and (26). Note that the factor loading on the permanent component of m_t remains (normalized to) one.

[11] A related example is Harvey, Trimbur, and van Dijk (2007) who use Bayesian methods to estimate permanent-transitory decompositions of aggregate time series, but without rational expectations cross-equation restrictions.

[12] Morley, Nelson, and Zivot (2003) show that this restriction applied to an univariate UC model resolves its differences with the Beverage and Nelson (1981) decomposition.

The parameter vectors of the other five UC models are smaller. The $UC_{2,\widetilde{m},\kappa}$ model drops two plus $k_{\widetilde{c}}$ parameters from $\Gamma_{2,\widetilde{m},\kappa} = [\kappa \;\; \alpha_1 \;\; \ldots \; \alpha_{k_{\widetilde{m}}} \;\; \mu^* \;\; a^* \;\; \sigma_\mu \;\; \sigma_a \;\; \sigma_{\widetilde{m}} \;\; \pi_{e,0} \;\; \pi_{e,t} \;\; \pi_{e,a} \;\; \pi_{c,\widetilde{m}}]'$, while adding the factor loading on $\widetilde{m}_t$ for c_t, $\pi_{c,\widetilde{m}}$. The factor loading $\pi_{m,\widetilde{c}}$ enters the parameter vector of $UC_{2,\widetilde{m},\kappa}$, while $\alpha_1 \; \ldots \; \alpha_{k_{\widetilde{m}}}$ and $\sigma_{\widetilde{m}}$ are dropped from $\Gamma_{2,\widetilde{c},\kappa} = [\kappa \;\; \theta_1 \; \ldots \; \theta_{k_{\widetilde{c}}} \;\; \mu^* \;\; a^* \;\; \sigma_\mu \;\; \sigma_a$ $\sigma_{\widetilde{c}} \;\; \varrho_{a,\widetilde{c}} \;\; \pi_{e,0} \;\; \pi_{e,t} \;\; \pi_{e,a} \;\; \pi_{m,\widetilde{c}}]'$. The parameter vectors of the UC models $UC_{2,2,\kappa=1}$, $UC_{2,\widetilde{m},\kappa=1}$, and $UC_{2,\widetilde{c},\kappa=1}$ are identical to $\Gamma_{2,2,\kappa}$, $\Gamma_{2,\widetilde{m},\kappa}$, and $\Gamma_{2,\widetilde{c},\kappa}$ except that $\kappa = 1$.

4c. *The Data*

The sample runs from 1976Q1 to 2004Q4, $T = 116$. We have observations on the Canadian dollar – U.S. dollar exchange rate (average of period). The Canadian monetary aggregate is M1 in current Canadian dollars, while for the U.S. it is the Board of Governors Monetary Base (adjusted for changes in reserve requirements) in current U.S. dollars. Consumption is the sum of non-durable and services expenditures in constant local currency units.[13] The aggregate quantity data is seasonally adjusted and converted to per capita units. The data is logged and multiplied by 100, but is neither demeaned nor detrended.

4d. *Estimation Methods*

The likelihood function of the UC models do not have analytic solutions. We approximate the likelihoods $\mathcal{L}(\mathcal{Y}_t \,|\, \Gamma_{2,i,\kappa=1},\; UC_{2,i,\kappa=1})$ and $\mathcal{L}(\mathcal{Y}_t \,|\, \Gamma_{2,i,\kappa},\; UC_{2,i,\kappa})$ with posterior distributions of $\Gamma_{2,i,\kappa=1}$ and $\Gamma_{2,i,\kappa}$, generated by the MCMC replications of the random walk MH simulator. Our estimates of $\Gamma_{2,i,\kappa=1}$ and $\Gamma_{2,i,\kappa}$ and marginal likelihoods build on the Bayesian estimation tools of Fernández-Villaverde and Rubio-Ramírez (2004), Rabanal and Rubio-Ramírez (2005), Geweke (1999, 2005), and Gelman, Carlin, Stern, and Rubin (2004). The MH simulator is asked to create 1.5 million MCMC draws from the posterior. The initial 750,000 draws are treated as a burn-in sample and therefore discarded. We base our estimates on the remaining 750,000 draws from the posteriors of the $UC_{2,2,\kappa=1}$, $UC_{2,\widetilde{m},\kappa=1}$, $UC_{2,\widetilde{c},\kappa=1}$, $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$, and $UC_{2,\widetilde{c},\kappa}$ models.[14]

[13]Canadian consumption includes semi-durable expenditures.

[14]The posterior distributions are based on acceptance rates of between 25 and 36 percent. Besides the 750,000 MCMC draws used to compute the moments reported below, four more sequences of 750,000 MCMCs are generated from disparate starting

4e. Priors

The second column of table 4 (5) list the priors of $\Gamma_{2,i,\kappa=1}$ $(\Gamma_{2,i,\kappa})$, $i = 2,\ \widetilde{m},\ \tilde{c}$. Under a normal prior, the first element is the degenerate mean and second its standard deviation. The inverse-gamma priors are parameterized by its degrees of freedom, the first element, and its mean, the second element. The left and right end points of a uniform prior is denoted by its first and second elements.

We choose degenerate priors for the lag lengths of the MA($k_{\widetilde{m}}$) of $\widetilde{m}_t$ and AR($k_{\tilde{c}}$) of $\tilde{c}_t$ that set $k_{\widetilde{m}} = k_{\tilde{c}} = 2$. Normal priors for the MA (α_1 and α_2) and AR (θ_1 and θ_2) coefficients allow for disparate transitory behavior in $\widetilde{m}_t$ and $\tilde{c}_t$. The prior means of α_1, α_2, θ_1, and θ_2 guarantee that the relevant eigenvalues are strictly less than one. The eigenvalues of the MA(2) (AR(2)) of $\widetilde{m}_t$ ($\tilde{c}_t$) are $0.60 \pm 0.20i$ (0.95 and -0.10). The standard deviation of the normal priors of the MA and AR coefficients provide for a wide set of realizations for α_1, α_2, θ_1, and θ_2. However, when a draw generates an eigenvalue greater than one (in absolute value) for either the MA or AR coefficients, the draw is discarded. Nonetheless, the MA and AR priors admit transitory cycles in cross-country money and consumption that allow for power at the business cycle frequencies, if the data wants.

We opt for priors of μ^* and a^* that rely on the Canadian–U.S. money stock and consumption differentials samples. Since μ^* and a^* represent deterministic trend growth, we ground the priors on normal distributions. The prior standard deviations of μ^* and a^* match sample moments.

Priors on the standard deviations of the shock innovations reflect standard practice for estimating DSGE models with Bayesian methods. For example, Adolfson, Laséen, Lindé, and Villani (2007) employ inverse-gamma priors for the standard deviations of the shock innovations of their sticky price open economy DSGE model. However, there is a lack of good information about σ_μ, σ_a, $\sigma_{\widetilde{m}}$, and $\sigma_{\tilde{c}}$. This explains why we impose a prior with two degrees of freedom, which forces these standard deviations to

values to assess across chain and with chain convergence. We compute the $\hat{R}$ statistic of Gelman, Carlin, Stern, and Rubin (2004) to evaluate across chain across and the separated partial means test of Geweke (2005) convergence, which is distributed asymptotically χ^2. Across the 77 parameters of the six UC models, the two largest $\hat{R}$s are 1.20 and 1.04, while Gelman, et al suggest a $\hat{R}$ of about 1.10. On five subsamples, The Geweke separated partial means test has no $p-$value smaller than 0.21 across the six UC models and five MCMC simulation sequences.

be positive. On the other hand, we attach a normally distributed prior to the correlation of innovations to a_t and $\widetilde{c}_t$, $\varrho_{a,\widetilde{c}}$. Its mean is negative to capture our prior that a_t is smoother than c_t. Since we have no information about the extent of the smoothness, the mean is -0.5 with a standard deviation of 0.2 that places draws close to negative one or zero in the 95 percent coverage interval of the prior. Draws greater than one or less than negative one are ignored. The correlation of innovations to μ_t and $\widetilde{m}_t$ is fixed at zero because our belief is that the sources and causes of permanent and transitory monetary shocks are orthogonal.

The exchange rate intercept and linear time trend priors are set according to a linear regression of the sample Canadian dollar–U.S. dollar exchange rate on these objects. This motivates our choice of normally distributed priors for $\pi_{e,0}$ and $\pi_{e,t}$ and is the source of their degenerate means and standard deviations.

The remaining factor loadings have priors that reflect a dearth of information on our part. The uniform priors of $\pi_{e,a}$, $\pi_{c,\widetilde{m}}$, and $\pi_{c,\widetilde{m}}$ are wide and include zero. If, for example, $\pi_{e,a}$ is small it indicates the inadequacy of the balanced growth restriction and the impact of permanent fluctuations in Canadian–U.S. TFP differentials on the exchange rate. The same holds for the response of c_t (m_t) to transitory movements in the Canadian–U.S. money stock (consumption) differential.

The $UC_{2,i,\kappa}$ models have only one 'economic' parameter, the DSGE-PVM discount factor $\kappa \equiv \dfrac{1}{1+r^*}$, in common. We adopt the Engel and West (2005) prior for κ. They argue that it is necessary for $\kappa \in [0.9,\ 0.999]$ to generate an approximate random walk exchange rate from the standard-PVM. Hence, our prior on κ is constructed to provide information about the EW hypothesis from the posteriors of the $UC_{2,i,\kappa}$ models. We impose an inverse-gamma prior on the DSGE-PVM discount factor κ, which follows Del Negro and Schorfheide (2006). The degenerate prior means of $\kappa = 0.988$ and $\exp([a^* = 0.158]/400)$ imply an annual average real world interest rate of about five percent. Although a five percent real world interest rate is large for the floating rate period, the standard deviation of 0.038 guarantees draws for κ that cover a wide interval. However, MCMC draws from the random walk MH simulator of the $UC_{2,i,\kappa}$ models obey the EW prior because we ignore draws for which $\kappa \notin [0.9,\ 0.999]$.

5. Results

This section presents the results of implementing our empirical strategy. Tables 4 and 5 provide the posterior means and standard deviations of the $\Gamma_{2,i,\kappa=1}$ and $\Gamma_{2,i,\kappa}$ vectors, $i = 2, \widetilde{m}, \widetilde{c}$, for the $UC_{2,2,\kappa=1}$, $UC_{2,\widetilde{m},\kappa=1}$, $UC_{2,\widetilde{c},\kappa=1}$, $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$, and $UC_{2,\widetilde{c},\kappa}$ models. We present densities of the prior and posteriors of κ for the latter three UC models in figure 1. The posterior distributions of the six UC models are used to construct the marginal likelihoods of the six UC model, as described by Fernández-Villaverde and Rubio-Ramírez (2004), Rabanal and Rubio-Ramírez (2005), and Geweke (1999), to conduct inference across these models. We also report the factor loadings on the $CDN\$/US\$$ exchange rate of the transitory components of the Canadian-U.S. money stock and consumption differentials, unconditional variance ratios of the present discounted value of the shock innovations to the $CDN\$/US\$$ exchange rate, FEVDs of the trend-cycle decomposition of the exchange rate with respect to these shocks, and summary statistics of trend-cycle decompositions in tables 6, 7, 8, and 9, respectively. Figures 2, 3, and 4 plot the trend-cycle decomposition of the $CDN\$/US\$$ exchange rate.

5a. Parameter Estimates

Tables 4 and 5 list the posterior means and standard deviations of the parameters of the six UC models. Estimates of the limiting DSGE models appear in table 4. These are $UC_{2,2,\kappa=1}$, $UC_{2,\widetilde{m},\kappa=1}$, $UC_{2,\widetilde{c},\kappa=1}$. These three models exhibit persistence in the transitory components of the Canadian-U.S. money and consumption differentials, $\widetilde{m}_t$ and $\widetilde{c}_t$. For example, the $UC_{2,\widetilde{m},\kappa=1}$ ($UC_{2,\widetilde{c},\kappa=1}$) model yield AR (MA) estimates that imply the half life of shock to $\widetilde{c}_t$ ($\widetilde{m}_t$) is 17 (7) years.[15] However, only $\widetilde{c}_t$ is persistent in the $UC_{2,2,\kappa=1}$ model. The half life of a shock to $\widetilde{m}_t$ is less than two quarters, while for $\widetilde{c}_t$ it is between nine and ten years. Note also that the priors and posterior means of the MA coefficients, α_1 and α_2, only differ for the $UC_{2,\widetilde{m},\kappa=1}$ model. Although the posterior means of the AR coefficients have moved away from the prior means, a one standard deviation of the posterior of θ_2 covers zero for the $UC_{2,2,\kappa=1}$ and $UC_{2,\widetilde{c},\kappa=1}$ models.

The posterior means of μ^* and a^* show that Canada experiences slower (faster) trend money

[15]The half life equals $\log[0.5]/\log[q]$, where q is the largest modulus of the companion matrix of the AR or MA coefficients.

(TFP) growth than for the U.S. from $1976Q1$ to $2004Q4$. Trend U.S. money growth is on average about 0.05 percent higher annually according to the posteriors of the $UC_{2,2,\kappa=1}$, $UC_{2,\widetilde{m},\kappa=1}$, and $UC_{2,\widetilde{c},\kappa=1}$ models. Across these models, $a^* \approx 0.16$ indicates Canadian deterministic trend TFP growth dominates its U.S. counterpart by about 0.06 percent at an annual rate.

The $UC_{2,2,\kappa=1}$, $UC_{2,\widetilde{m},\kappa=1}$, and $UC_{2,\widetilde{c},\kappa=1}$ models show differences across estimates of the posterior means of the shock innovation standard deviations the Canadian-U.S. sample. Only the estimated impulse structure of the $UC_{2,\widetilde{m},\kappa=1}$ model is dominated by movements in the permanent innovations of the Canadian–U.S. money differential shock, σ_{μ}. The converse is that this model yields the smallest posterior means of the standard deviation of the Canadian–U.S. TFP differential shock and $\widetilde{m}_t$ shock innovations, σ_a and $\sigma_{\widetilde{m}}$. The $UC_{2,\widetilde{c},\kappa=1}$ model yields the largest estimates of σ_a and $\sigma_{\widetilde{c}}$, but these posterior means are about the same magnitude. Note also that the correlation of the innovations to the Canadian–U.S. TFP differential shock and $\widetilde{c}_t$ shock is estimated to be $\varrho_{a,\widetilde{c}} = -0.88$ and -0.95 by the $UC_{2,2,\kappa=1}$ and $UC_{2,\widetilde{c},\kappa=1}$ models, respectively. Thus, these models are consistent with the Canadian–U.S. TFP trend differential being more volatile than observed Canadian–U.S. consumption.

Estimates of the exchange rate intercept and linear time trend indicate that the $UC_{2,\widetilde{m},\kappa=1}$ model provides the largest value for the $US\$$ in steady state and the largest deterministic growth rate for the $CDN\$/US\$$ exchange rate. The posterior means of $\pi_{e,0}$ and $\pi_{e,t}$ imply that the steady state $CDN\$/US\$$ exchange rate is 1.23 with a deterministic annual growth rate of about 0.8 percent. For the $UC_{2,2,\kappa=1}$ ($UC_{2,\widetilde{c},\kappa=1}$) model, the analogous values are 1.10 (1.03) and 0.3 (0.2) percent per annum. Thus, the $UC_{2,\widetilde{m},\kappa=1}$ model places more emphasis on deterministic elements to fit the data compared to the other two UC models of the limiting DSGE-PVM.

The remaining coefficients are the factor loadings $\pi_{e,a}$, $\pi_{e,\widetilde{m}}$, and $\pi_{e,\widetilde{c}}$. The posterior mean estimates of $\pi_{e,a}$ -2.68 and -9.02 reveal that there are statistically and economically large deviations from the balanced growth path by the $UC_{2,2,\kappa=1}$ and $UC_{2,\widetilde{m},\kappa=1}$ models. The $UC_{2,\widetilde{c},\kappa=1}$ model is closer to satisfying the balanced growth hypothesis that $\pi_{e,a} = -1$. This UC model has a posterior mean of -0.72 for $\pi_{e,\widetilde{c}}$ whose two standard deviation interval contains the balanced growth restriction. The response of the Canadian–U.S. money stock differential to $\widetilde{c}_t$ is also close to negative one for the $UC_{2,\widetilde{c},\kappa=1}$ because the posterior mean of $\pi_{m,\widetilde{c}} = -0.90$ with a standard deviation of 0.21. The $UC_{2,\widetilde{m},\kappa=1}$ model reveals that a one percent rise in $\widetilde{c}_t$ results in a 4.4 percent rise in the Canadian–U.S. consumption differential.

The key economic parameter of the DSGE-PVM is its discount factor κ. Table 5 lists the posterior means and standard deviations of the $\Gamma_{2,2,\kappa}$ and $\Gamma_{2,\widetilde{m},\kappa}$, and $\Gamma_{2,\widetilde{c},\kappa}$ vectors that include estimates of κ. Aside from the inclusion of the prior, posterior mean, and standard deviation κ at the top of table 5, the posterior means and standard deviations of the remaining coefficients resemble those reported on table 4. The only notable exceptions are that the posterior means of σ_a, $\sigma_{\widetilde{c}}$, $\pi_{e,a}$, and $\pi_{e,\widetilde{c}}$ are smaller for the $UC_{2,\widetilde{c},\kappa}$ model compared to its cousin with the calibration $\kappa = 1$. The result is that σ_μ is the largest innovation shock standard deviation of the $UC_{2,\widetilde{c},\kappa}$ model. Also, this UC model and the data produce an estimate of $\pi_{e,a}$ whose one standard deviation coverage interval contains negative one. Thus, the $UC_{2,\widetilde{c},\kappa}$ model is closer to the balanced growth hypothesis and relies to a greater extent on permanent shocks to the Canadian–U.S. money stock differential.

The posterior means of κ range from 0.966 for the $UC_{2,2,\kappa}$ model, 0.974 for the $UC_{2,\widetilde{m},\kappa}$ model, to the largest estimate of 0.9962 for the $UC_{2,\widetilde{c},\kappa}$ model. These estimates are consistent with annual world real interest rates of 15.1, 11.4, and 1.7 percent given the posteriors of the $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models, respectively. Although the $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$ models have posteriors that suggest unreasonably large world real interest rates, these UC models yield 95 percent coverage intervals whose upper end is 0.999. The $UC_{2,\widetilde{c},\kappa}$ model produces a posterior of κ with a 95 percent coverage interval whose lower end equals 0.987. This value of κ is greater than the posterior means of κ of the $UC_{2,2,\kappa}$ for the $UC_{2,\widetilde{m},\kappa}$ models. Thus, the $UC_{2,\widetilde{c},\kappa}$ model generates a posterior distribution of κ that is to the right of those produced by the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{m},\kappa}$ models.

Figure 1 reinforces the idea that the posteriors of the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{m},\kappa}$ models yield estimates of κ that are shifted to the right of those of the $UC_{2,\widetilde{c},\kappa}$ model. Posterior densities of κ are plotted in figure 1 for these UC models, along with the density of the inverse-gamma prior of κ on the EW prior of $\kappa \in [0.9, 0.999]$. The solid (black) line is the density of the κ prior and is close to the posterior density of κ derived from the $UC_{2,2,\kappa}$ model, which is the dashed (blue) line. The $UC_{2,\widetilde{m},\kappa}$ model generates a posterior density of kappa, the dot-dash (green) plot, which moves off the prior by placing less weight on κs less than 0.97 and more weight above this value. The dot-dot (red) plot is the density of κ from the posterior of the $UC_{2,\widetilde{c},\kappa}$ model. This density is deflated by ten percent for ease of comparison to the other densities. A striking feature of figure 1 is that the posterior of the $UC_{2,\widetilde{c},\kappa}$ model pushes κ off of its prior because its mass lays between 0.98 and 0.999.

Table 6 contains the posterior means of the exchange rate factor loadings with respect to $\widetilde{m}_t$ and $\widetilde{c}_t$, the $\delta_{\widetilde{m},i}$s and $\delta_{\widetilde{c},i}$s.[16] A striking aspect of the estimates of $\delta_{\widetilde{m},0}$, $\delta_{\widetilde{m},1}$, and $\delta_{\widetilde{m},2}$ is that the response of the $CDN\$/US\$$ exchange rate to innovations in $\widetilde{m}_t$ is economically small for either the $UC_{2,2,\kappa}$ or $UC_{2,\widetilde{m},\kappa}$ models. The large posterior standard errors on these factor loading also indicate the imprecision the Canadian–U.S. data give to these estimates. The data have less problems yielding a precise estimate of $\delta_{\widetilde{c},0}$ for the $UC_{2,2,\kappa}$ model. The posterior mean of this factor loading shows that the exchange rate falls by 0.6 percent given an one percent increase in $\widetilde{c}_t$. These estimates drops to -0.33 for the $UC_{2,\widetilde{c},\kappa}$ model. Also, the associated 95 percent coverage interval contains zero. In summary, the $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$, and $UC_{2,\widetilde{c},\kappa}$ models have posteriors in which there is either a negligible exchange rate response to $\widetilde{m}_t$ shocks or an economically large negative reaction by the $CDN\$/US\$$ exchange rate to $\widetilde{c}_t$ fluctuations. However, the latter exchange rate response can be estimated imprecisely.

5b. Unconditional Variance Ratios and FEVDs of the Exchange Rate

Tables 7 and 8 present unconditional variance ratios and FEVDs computed using the posteriors of the $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$, and $UC_{2,\widetilde{c},\kappa}$ models. We calculate the variances of the present discounted values (PDVs) of the Canadian–U.S. money, TFP, and consumption differential shock innovations using the equilibrium currency return generating equation (17) and the restrictions of the UC models which yield estimates of κ. The variance ratios are these values divided by the sample variance of the $CDN\$/US\$$ exchange rate (= 2.04). According to the unconditional variance ratios, only permanent shocks to the Canadian–US money differential, $\varepsilon_{\mu,t}$, and the TFP differential, $\varepsilon_{a,t}$, explain variation in the $CDN\$/US\$$

[16] For the $UC_{2,\widetilde{m},\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models, the relevant factor loadings are multiplied by $1 - \pi_{e,\widetilde{m}}$ or $1 - \pi_{e,\widetilde{c}}$.

exchange rate. The variances of the PDVs of shock innovations to $\widetilde{m}_t$ and $\widetilde{c}_t$ are small and lack precision. Note that except for the $UC_{2,\widetilde{m},\kappa}$ model, the variance of the PDV of $\varepsilon_{a,t}$ is larger than that of $\varepsilon_{\mu,t}$.

We report FEVDs in table 8 with implications similar to the unconditional variance ratios.[17] The top panel of figure 8 shows that the posterior of the $UC_{2,2,\kappa}$ yields a FEVD in which the Canadian–US TFP differential shock $\varepsilon_{a,t}$ makes a large and increasing contribution to $CDN\$/US\$$ exchange rate fluctuations at longer forecast horizons. The Canadian–US money differential shock $\varepsilon_{\mu,t}$ remains economically important for $CDN\$/US\$$ exchange rate movements out to a three to five year forecast horizon, but shocks to $\widetilde{m}_t$ and $\widetilde{c}_t$ are unimportant at any forecast horizon. Much the same is true for the FEVDs found using the posterior of $UC_{2,\widetilde{m},\kappa}$. However, the relative shares of the $\varepsilon_{\mu,t}$ and $\varepsilon_{a,t}$ shocks are unchanged at a two-thirds/one-third split from the one quarter to ten year forecast horizons.

The posterior of the $UC_{2,\widetilde{c},\kappa}$ model imbues a slowly changing dynamic to the $CDN\$/US\$$ exchange rate FEVDs as found in the bottom panel of table 8. The $\varepsilon_{\mu,t}$ and $\varepsilon_{a,t}$ shocks are responsible for about 60 and 40 percent, respectively, of fluctuations in the $CDN\$/US\$$ exchange rate at the short horizons. At a 10 year horizon, the contribution of $\varepsilon_{\mu,t}$ ($\varepsilon_{a,t}$) only falls (rises) to 55 (45) percent. Thus, only the posterior of the $UC_{2,\widetilde{c},\kappa}$ model predicts that permanent shocks to the Canadian–US money differential dominate $CDN\$/US\$$ exchange rate movements at the longer forecast horizons.

5c. Trend-Cycle Decompositions

Trend-cycle decompositions of the $CDN\$/US\$$ exchange rate and Canadian–U.S. money and consumption differentials are plotted in figures 2 and 3 with summary statistics given in table 9. The posteriors of the $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$, and $UC_{2,\widetilde{c},\kappa}$ models are run through the Kalman smoother to create the trend-cycle decompositions and summary statistics. Figures 2 and 3 and table 9 contain moments that are averages over the 750,000 draws from the UC model posteriors. We label trend exchange rate growth Δe^{τ} in table 9.

The top window of figure 2 contains plots of the $CDN\$/US\$$ exchange rate and smoothed

[17] The FEVDs are computed using the vector ECM implied by equation (16) and the permanent-transitory specifications of Canadian-U.S. money and consumption differentials. The vector ECM is placed in state space form as outlined by Heqc, Palm, and Urbain (2000) and iterated upon to create the FEVDs of table 8.

trends taken from the posteriors of the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models.[18] The solid (black) line is e_t, the log of the actual $CDN\$/US\$$ exchange rate. The smoothed trends of the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models are the dashed (blue) and dotted (red) plots, respectively. Note that these UC models generate smoothed $CDN\$/US\$$ exchange rate trends that are more volatile than the actual exchange rate. The top row of table 9 indicate that the posteriors of the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models generate standard deviations of Δe^{τ} are 2.66 and 2.44, respectively. The standard deviation of Δe^{τ} equals 2.04.

The smoothed $CDN\$/US\$$ exchange rate cycles appear in the bottom window of figure 2. The dotted (blue) line is the smoothed exchange rate cycle, $\widetilde{e}_t$, based on the posterior of the $UC_{2,2,\kappa}$ model, while the dotted (red) line is associated with the $UC_{2,\widetilde{c},\kappa}$ model. Although the former $\widetilde{e}_t$ exhibits more variability than the latter (the standard deviations are 3.68 and 2.44), these $\widetilde{e}_t$s are persistent with AR1 correlation statistics of 0.97 and 0.98. Note that only the posterior of the $UC_{2,\widetilde{m},\kappa}$ model yields (close to) a non-zero correlation for Δe^{τ} and $\widetilde{e}$, according to table 9.

Figure 3 shows the smoothed permanent-transitory decompositions of the Canadian-U.S. money and consumption differentials. The actual differentials and smoothed trends appear in the top row of windows, while the smoothed cycles are found in the bottom row of windows. The Canadian-U.S. money (consumption) differentials are the right (left) side windows. The posterior of the $UC_{2,2,\kappa}$ model produces a money trend, μ_t that almost perfectly mimics the actual Canadian-U.S. money differentials, as found in the top left window of figure 3. The result is that the smoothed $\widetilde{m}_t$ is much less volatile, a standard deviation of 0.68, compared to a standard deviation of 1.62 for μ_t. The bottom left window of figure 3 shows a saw-toothed pattern in $\widetilde{m}_t$, conditional on the posterior of the $UC_{2,2,\kappa}$ model. This explains the AR1 correlation statistic of -0.68 for $\widetilde{m}_t$ that appears in the middle of second column of table 9.

Table 9 reveals that the posterior of the $UC_{2,\widetilde{c},\kappa}$ model produces a smoothed money trend, μ_t, that is about as volatile as does the $UC_{2,2,\kappa}$ model. The relevant standard deviations are 1.62 and 1.71

[18]We do not present the trend-cycle decompositions based on the posterior of the $UC_{2,\widetilde{m},\kappa}$ model because its log marginal likelihood is far below those of the other UC models. Table 9 includes standard deviations of Δe^{τ} and $\widetilde{e}$ from the posterior of the $UC_{2,\widetilde{m},\kappa}$ model that are larger by a factor of 30 or compared to these statistics from the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models, which is a signal of its lack of acceptance by our Canadian–U.S. sample.

in the second and fourth columns of table 9 for the smoothed money growth trend of these UC models. However, smoothed $\Delta\mu$ and Δe^{τ} share a positive correlation of 0.62 only in the posterior of the $UC_{2,\tilde{c},\kappa}$ model as shown in the bottom half of the fourth column of table 9.

The posteriors of the $UC_{2,2,\kappa}$ and $UC_{2,\tilde{c},\kappa}$ models yield qualitatively similar plots for the smoothed TFP trend, a_t, and consumption cycle $\tilde{c}_t$ in the top right window of figure 3. These plots appear in the top right window of figure 3 as dashed (blue) for the and dotted (red) for the $UC_{2,2,\kappa}$ and $UC_{2,\tilde{c},\kappa}$ models, respectively, where observed cross country consumption is the solid (black) line. The smoothed Δa is 50 percent more volatile for the $UC_{2,\tilde{c},\kappa}$ model than it is for the $UC_{2,2,\kappa}$ model. The posteriors of these UC models also produce correlations of -0.71 and -0.85 for Δe^{τ} and Δa, which suggest that rising U.S. TFP is associated with a depreciation in the Canadian dollar.

The bottom right window of figure 3 presents the smoothed $\tilde{c}_t$ of the $UC_{2,2,\kappa}$ and $UC_{2,\tilde{c},\kappa}$ models. The former cycle is the dashed (blue) line and the latter is the dotted (red) plot. These cycles are persistent because their AR1 correlation statistics 0.97 and 0.98, but the $UC_{2,2,\kappa}$ model generates a third less volatility in smoothed $\tilde{c}_t$ than found for the $UC_{2,\tilde{c},\kappa}$ model.

The posteriors of the $UC_{2,2,\kappa}$ and $UC_{2,\tilde{c},\kappa}$ models generate economically meaningful trends and cycles in the Canadian–U.S. a_t and $\tilde{c}$. The smoothed TFP differential is falling in the latter 1970s, which reflects a greater productivity slowdown in the Canada. By the 1980s, Canadian TFP is growing more rapidly than in the U.S., which continues into the early 1990s. Subsequently, U.S. TFP recovers relative to Canadian TFP. At the end of the sample, the Canadian–U.S. TFP differential is expanding once more.

The smoothed $\tilde{c}_t$ has peaks and troughs that coincide with several U.S.-Canadian business cycle dates. For example, troughs in the posterior mean of $\tilde{c}_t$ appear in 1981 and 1990 which also represent recessions dates in the U.S. and Canada. Since the end of the 1990 – 1991 recession, the rise in $\tilde{c}_t$ points to persistent, but transitory, rise in U.S. consumption relative to Canada. Nonetheless, $\tilde{c}_t$ has been falling rapidly since a peak in late 2001, which corresponds to the end of the last U.S. recession.

The bottom row of table 9 shows that $\tilde{c}_t$ and $\tilde{e}_t$ are perfectly negatively correlated in the posteriors of the $UC_{2,2,\kappa}$ and $UC_{2,\tilde{c},\kappa}$ models. The negative correlation of the transitory component of the exchange rate with $\tilde{c}_t$ help to interpret the $CDN\$/US\$$ exchange rate fluctuations. Peaks in the transitory component of the $CDN\$/US\$$ exchange rate occur either at or shortly after the end of recessions. For example, the smoothed $CDN\$/US\$$ exchange rate cycles have a tendency to peak and trough around dates usually associated with U.S. and Canadian business cycle dates (*i.e.*, the late 1970s, early 1990s, and 2001). A specific case is the peak in $\tilde{e}_t$ during the 1990 – 1991 recession in the U.S., which is a moment at which the Canadian dollar approached par against the U.S. dollar. An exception is the end of the 2001 recession when the Canadian dollar reached a low of nearly 0.62 to the U.S. dollar.

5d. Comparing the UC models

The bottom row of table 4 reports the log marginal likelihoods, $\ln \hat{\mathcal{L}}$ of the $UC_{2,2,\kappa=1}$, $UC_{2,\widetilde{m},\kappa=1}$, and $UC_{2,\tilde{c},\kappa=1}$ models. These marginal likelihoods show that our Canadian-U.S. sample gives most support $UC_{2,\tilde{c},\kappa=1}$ model. The difference between this model and the $UC_{2,2,\kappa=1}$ model is about 29, or the Bayes factor prefers the UC model with only transitory consumption. For the data to give more credence to the latter model, its prior probability must be raised by the prior probability of the $UC_{2,\tilde{c},\kappa=1}$ model multiplied by 4.7×10^{12} $[= \exp(29.18)]$. Since the magnitude of this factor is large, it seems unreasonable to include the transitory money shock in the UC model when κ is calibrated to one.

The last row of table 5 contains the log marginal likelihoods of the $UC_{2,2,\kappa}$, $UC_{2,\widetilde{m},\kappa}$, and $UC_{2,\tilde{c},\kappa}$ models. The ranking of these models matches that of the UC models with the $\kappa = 1$ calibration. The $UC_{2,\widetilde{m},\kappa}$ model dominates the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{m},\kappa}$ models. A key reason is that the posteriors of these models yield economically implausible estimates of the DGSE-PVM discount factor κ.

This raises the question of whether our Canadian–U.S. sample will find it difficult to choose between the $UC_{2,\tilde{c},\kappa=1}$ and $UC_{2,\tilde{c},\kappa}$ models. Our Canadian–U.S. sample favors the $UC_{2,\tilde{c},\kappa=1}$ and $UC_{2,\tilde{c},\kappa}$ models compared to the other four. The former UC model has a larger marginal likelihood, which suggest the data support it over the latter UC model with $\kappa \in [0.9, 0.999]$. This choice relies on the belief that scaling up the prior probability of the $UC_{2,\tilde{c},\kappa}$ model by about $167.3 = \exp(5.12)$ is too large to be justified. If, on the other hand, this factor is regarded as inconclusive in rejecting the $UC_{2,\tilde{c},\kappa}$ model, it can be argued that our Canadian–U.S. sample cannot pick between the $UC_{2,\tilde{c},\kappa=1}$ and $UC_{2,\tilde{c},\kappa}$

models.[19] We suggest that this is support for the EW hypothesis.

5e. *Exchange Rate Dynamics as $\kappa \longrightarrow 1$*

Engel and West (2005) argue that the exchange rate will approximate a random walk when the discount factor is close to one and fundamentals have a unit root. Propositions 7 and 8 also predict that $\widetilde{e}_t$ will collapse to random walk, as $\kappa \longrightarrow 1$.

We extract evidence about the EW hypothesis from the posteriors of the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models. The focus is on these UC models because the $UC_{2,\widetilde{c},\kappa=1}$ model attributes all $CDN\$/US\$$ exchange rate movements to permanent shocks. The $UC_{2,2,\kappa}$ model is included for comparison. We conduct this comparison with κs at the 16th and 84th percentiles, along with the largest κs, from the $\Gamma_{2,2,\kappa}$ and $\Gamma_{2,\widetilde{c},\kappa}$ vectors. For the $UC_{2,2,\kappa}$ ($UC_{2,\widetilde{c},\kappa}$) model, the 16th percentile, 84th percentile, and largest κs are 0.9425, 0.9883, and 0.9990 (0.9943, 0.9987, and 0.9990), respectively. Fixing κ at these values, we simulate the $UC_{2,2,\kappa}$ and $UC_{2,\widetilde{c},\kappa}$ models drawing 2000 sequences from the posteriors, discard the first 1000, run the Kalman smoother on the remaining 1000 sequences, and average the ensemble to generate $CDN\$/US\$$ exchange rate cycles to respect the rational expectations hypothesis.

Figure 4 plots the smoothed $CDN\$/US\$$ exchange rate cycles. The top (bottom) window contains the $\widetilde{e}_t$ created from the posterior of the $UC_{2,2,\kappa}$ ($UC_{2,\widetilde{c},\kappa}$) model. The dot-dash (blue), dotted (green), and dotted (red) lines are conditional on the 16th percentile, 84th percentile, and largest κs, respectively. Across the top and bottom windows, the volatility of $\widetilde{e}_t$ is compressed as κ approaches 0.999. This is reflected in the standard deviations of $\widetilde{e}_t$ that equal 4.44, 2.84, and 0.57 moving from the smallest to largest κ for the $UC_{2,2,\kappa}$ model. The equivalent standard deviations are 3.74, 1.64, and 1.30 for the $UC_{2,\widetilde{c},\kappa}$ model. Although the $UC_{2,2,\kappa}$ generates $CDN\$/US\$$ exchange rate cycles that are smoother than at its posterior mean only for the largest κs, the $UC_{2,2,\kappa}$ model is able to produce smoother exchange rate cycles at the 84th percentile and largest κs. Thus, pushing κ increases the smoothness of the $CDN\$/US\$$ exchange rate cycle which we argue is evidence in support of the EW hypothesis.

[19] Jefferys (1998) contends that Bayes factors differing by five is evidence about the two models just between 'barely worth mentioning' and considerably in favor of the model with the larger marginal likelihood.

6. Conclusion

Economists have little to say about the impact of policy on currency markets without an equilibrium theory of exchange rate determination that is empirically relevant. According to Engel and West (2005), the near random walk behavior of exchange rates explain the failure of equilibrium models to fit the data or to find any model that systematically beats it at out-of-sample forecasting. They conjecture that the standard-present value model (PVM) of exchange rates yields the random walk prediction when fundamentals are persistent and the discount factor is close to one.

This paper generalizes the Engel and West hypothesis. We find that the standard-PVM model places common trend and common cycle restrictions on the exchange rate and its fundamental. Under the former restriction and a discount factor close to one, the exchange rate collapses to a martingale. We also show that the exchange rate approximates a random walk when only the common cycle restriction holds and the discount factor approaches one.

We also construct a PVM of exchange rates from a two-country monetary dynamic stochastic general equilibrium (DSGE) model. The DSGE-PVM places restrictions on the exchange rate and fundamentals similar to those of the standard-PVM. For example, the exchange rate becomes a random walk when the DSGE-PVM discount factor approaches one. Thus, we generalize the Engel and West (2005) hypothesis to the wider class of open economy DSGE models.

Our empirical results support the view that it is difficult for the data to choose between a random walk exchange rate model and a DSGE-PVM with a discount factor estimated to be near one. Bayesian estimates of the DSGE-PVM also suggest that the Canadian dollar–U.S. dollar exchange rate is dominated by permanent shocks whether the discount factor is estimated or calibrated to one. Thus, the DSGE-PVM and the Canadian-U.S. sample yield estimates that support the Engel-West hypothesis. These results matter because the Engel-West hypothesis filtered through DSGE-PVM is part of the large class of open economy DSGE models. We expect this is a challenge to current open economy DSGE theories that hope to explain exchange rates fluctuations.

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani. 2007. Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through. *Journal of International Economics* 72, 481 – 511.

Beveridge, S., C.R. Nelson. 1981. A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle. *Journal of Monetary Economics* 7, 151 – 174.

Bilson, J.F.O. 1978. Rational Expectations and the Exchange Rate. in Frenkel, J.A., H.G. Johnson, THE ECONOMICS OF EXCHANGE RATES: SELECTED STUDIES, Addison-Wesley, Reading, MA.

Campbell, J.Y., R. Shiller. 1987. Cointegration and Tests of Present Value Models. *Journal of Political Economy* 93, 1062 – 1088.

Del Negro, M., F. Schorfheide. 2006. Forming Priors for DSGE Models (and How it Affect the Assessment of Nominal Rigidites). Working Paper 2006–16, Federal Reserve Bank of Atlanta.

Diebold, F.X., J.M. Nason. 1990. Nonparametric Exchange Rate Prediction. *Journal of International Economics* 28, 315 – 332.

Dornbusch, R. 1976. Expectations and Exchange Rate Dynamics. *Journal of Political Economy* 84, 1161 – 1176.

Engel, C., J.H. Rogers, A.K. Rose. 2003. Empirical Exchange Rate Models. *Journal of International Economics* 60, 1 – 2.

Engel, C., K.D. West, 2004. Accounting for Exchange Rate Variability in Present Value Models when the Discount Factor is Near One,. *American Economic Review (Papers and Proceedings)* 94(May), 119 – 125.

Engel, C., K.D. West, 2005. Exchange Rates and Fundamentals. *Journal of Political Economy* 113, 485 – 517.

Engle, R.F., J.V. Issler, 1995. Estimating Common Sectoral Cycles. *Journal of Monetary Economics* 35, 83 – 113.

Engle, R.F., S. Kozicki, 1993. Testing for Common Features. *Journal of Business and Economics Statistics* 11, 369 – 395.

Fernández-Villaverde, J., J.F. Rubio-Ramírez. 2004. Comparing Dynamic Equilibrium Models to Data: A Bayesian Approach. *Journal of Econometrics* 123, 153 – 187.

Frankel, J.A., 1979. On the Mark: A Theory of Floating Exchange Rates Basedon Real Interest Rate Differentials. *American Economics Review* 69, 610 – 622.

Gelman, A., J.B. Carlin, H.S. Stern, D.B. Rubin, 2004, BAYESIAN DATA ANALYSIS, SECOND EDITION, Chapman and Hall, New York, NY.

Gençay, R., 1999. Linear, Non-linear and Essential Foreign Exchange Rate Prediction with Simple Technical Trading Rules. *Journal of International Economics* 47, 91 — 107.

Geweke, J., 1999. Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication. *Econometric Reviews* 18, 1 – 126.

Geweke, J., 2005. CONTEMPORARY BAYESIAN ECONOMETRICS AND STATISTICS, J. Wiley and Sons, Inc., Hoboken, NJ.

Hamilton, J.D., 1994, TIME SERIES ANALYSIS, Princeton University Press, Princeton, NJ.

Hansen, L.P., W. Roberds, T.J. Sargent. 1991. Time Series Implications of Present Value Budget Balance and of Martingale Models and Consumption and Taxes. in Hansen, L.P., T.J. Sargent (eds.), RATIONAL EXPECTATIONS ECONOMETRICS, Westview Press, Boulder, CO.

Harvey, A.C., FORECASTING, STRUCTURAL TIME SERIES MODELS AND THE KALMAN FILTER, Cambridge University Press, Cambridge, England.

Harvey, A.C., T.M. Trimbur, H.K. van Dijk. 2007. Trends and Cycles in Economic Time Series: A Bayesian Approach. *Journal of Econometrics* 140, 618 – 649.

Hecq, A., F.C. Palm, J-P. Urbain. 2000. Permanent-Transitory Decomposition in VAR Models with Cointegration and Common Cycles. *Oxford Bulletin of Economics and Statistics* 62, 511 – 532.

Hecq, A., F.C. Palm, J-P. Urbain. 2006. Common Cyclical Features Analysis in VAR Models with Cointegration. *Journal of Econometrics* 127, 117 – 141.

Jeffreys, H., 1998. THEORY OF PROBABILITY, Oxford University Press, Oxford, UK.

Johansen, S. 1991. Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica* 59, 1551 – 1580.

Johansen, S. 1994. The Role of the Constant and Linear Terms in Cointegration Analysis and Nonstationary Variables. *Econometric Reviews* 13, 205 – 229.

Kilian, L., M.P. Taylor. 2003. Why Is It So Difficult to Beat the Random Walk Forecast of Exchange Rates? *Journal of International Economics* 60, 85 – 107.

MacKinnon, J.G., A.A. Haug, L. Michelis. 1999. Numerical Distributions of Likelihood Ratio Tests of Cointegration. *Journal of Applied Econometrics* 14, 563 – 577.

Maheswaran, S., C.A. Sims. 1993. Empirical Implications of Arbitrage-Free Asset Markets. in Phillips, P.C.B. (ed.), MODELS, METHODS AND APPLICATIONS OF ECONOMETRICS, Blackwell Publishing, Oxford, UK.

Mark, N. 1995. Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability. *American Economics Review* 85, 201 – 218.

Mark, N., D. Sul, 2003. Cointegration Vector Estimation by Panel DOLS and Long-Run Money Demand. *Oxford Bulletin of Economics and Statistics* 65, 665 – 680.

Meese, R.A., 1986. Testing for Bubbles in Exchange Markets: A Case of Sparkling Rates. *Journal of Political Economy* 94, 345 – 373.

Meese, R.A., K. Rogoff. 1983a. "Empirical Exchange Rate Models of the Seventies. *Journal of International Economics* 14, 3 – 24.

Meese, R.A., K. Rogoff. 1983b. The Out-of-Sample Failure of Exchange Rate Models: Sampling Error or Misspecification. in Frenkel, J.A. (ed.), EXCHANGE RATE AND INTERNATIONAL ECONOMICS, University of Chicago Press, Chicago, IL.

Meese, R.A., A.K. Rose. 1991. Nonlinear, Nonparametric, Nonessential Exchange Rate Determination. *Review of Economic Studies* 58, 192 – 196.

Morley, J.C., C.R. Nelson, E. Zivot. 2003. Why Are Beveridge-Nelson and Unobserved Component Decompositions of GDP So Different? *Review of Economics and Statistics* 85, 235 – 234.

Osterwald-Lenum, M. 1992. Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics, *Oxford Bulletin of Economics and Statistics* 54, 461 – 472.

Rabanal, P., J.F. Rubio-Ramírez. 2005. Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach. *Journal of Monetary Economics* 52, 1151 – 1166.

Rao, C.S. 1973. Linear Statistical Inference, Wiley, Ltd., New York, NY.

Rogoff, K. 2007. Comments on: Engel, Mark, and West's Exchange Rate Models Are Not as Bad as You Think. in Acemoglu, D., K. Rogoff, M. Woodford (eds.), NBER Macroeconomics Annual 2007, vol. 22, MIT Press, Cambridge, MA.

Rossi, B. 2007. Comments on: Engel, Mark, and West's Exchange Rate Models Are Not as Bad as You Think. in Acemoglu, D., K. Rogoff, M. Woodford (eds.), NBER Macroeconomics Annual 2007, vol. 22, MIT Press, Cambridge, MA.

Sims, C.A. 1980. Macroeconomics and Reality. *Econometrica* 48, 1 – 48.

Sims, C.A., Stock, J.H., M.W. Watson. 1990. Inference in Linear Time Series Models with Some Unit Roots. *Econometrica* 58, 113 – 144.

Vahid, F., R.F. Engle. 1993. Common Trends and Common Cycles. *Journal of Applied Econometrics* 8, 341 – 360.

Table 1: Summary of Standard PVM and DSGE-PVM

Standard-PVM

ECM(0): (5) $\Delta e_t - \frac{1-\omega}{\omega}\mathcal{X}_{t-1} = (1-\omega)\sum_{j=0}^{\infty}\omega^j[\mathbf{E}_t - \mathbf{E}_{t-1}]z_{t+j}$.

EW Equation: (6) $\Delta e_t \;=\; \zeta(\omega)\,\upsilon_t \;+\; (1-\omega)\sum_{j=0}^{\infty}\omega^j\mathbf{E}_{t-1}\Delta z_{t+j}$.

Parameters: $\omega \equiv \frac{\phi}{1+\phi}$ = Discount Factor,

ϕ = Money Demand Interest Rate Semi-Elasticity,

ψ = Money Demand Income Elasticity.

Fundamentals: $\mathcal{X}_t \;=\; e_t - z_t, \quad z_t \;=\; m_t - \psi y_t$,
m_t = Cross Country Money,
y_t = Cross-Country Output.

DSGE-PVM

ECM(0): (16) $\Delta e_t \;-\; \frac{(1-\kappa)}{\kappa}\mathcal{X}_{DSGE,t-1}$

$$= \;(1-\kappa)\sum_{j=0}^{\infty}\kappa^j\left[\mathbf{E}_t - \mathbf{E}_{t-1}\right]\Big\{m_{t+j} \;-\; c_{t+j}\Big\}.$$

EW Equation: (17) $\Delta e_t \;=\; \sum_{j=0}^{\infty}\kappa^j\left[\mathbf{E}_t - \mathbf{E}_{t-1}\right]\Big\{\Delta m_{t+j} \;-\; \Delta c_{t+j}\Big\}$

$$+\;(1-\kappa)\sum_{j=0}^{\infty}\kappa^j\mathbf{E}_t\Big\{\Delta m_{t+j} \;-\; \Delta c_{t+j}\Big\}.$$

Parameters: $\kappa \equiv \frac{1}{1+r^*}$ = Discount Factor,

r^* = Steady State Real World Interest Rate.

Fundamentals: $\mathcal{X}_{DSGE,t} \;=\; e_t - m_t + c_t$,
m_t = Cross-Country Money,
c_t = Cross-Country Consumption.

Table 2: Summary of Propositions

Standard-PVM	
Proposition 1:	PVM Predicts Exchange Rate and Fundamentals Cointegrate; Campbell and Shiller (1987).
Proposition 2:	Currency Returns Are an ECM(0).
Proposition 3:	VECM(0) Imply Common Trend and Common Cycle for Exchange Rate and Fundamental.
Proposition 4:	Exchange Rate Approximates a Martingale as $\omega \longrightarrow 1$.
Proposition 5:	If Currency Returns and Fundamental Growth Share a Co-Feature and $\omega \longrightarrow 1$, Verify EW's (2005) Hypothesis.
DSGE-PVM	
Proposition 6:	DSGE-PVM Replicates **P1**.
Proposition 7:	DSGE-PVM Generalizes **P4**.
Proposition 8:	DSGE-PVM Generalizes EW Hypothesis to DSGE Models.

Table 3: Tests of Propositions 1, 3, and 5

Sample: $1976Q1 - 2004Q4$

	Canada & U.S.	Japan & U.S.	U.K. & U.S.
Proposition 3: VECM(0)			
Levels VAR Lag Length	8	5	4
LR statistic p−value	(0.02)	(0.01)	(0.09)
Proposition 1: Common Trend			
Cointegration Tests			
Model	Case 2∗	Case 1	Case 1
λ−Max statistic	4.86	0.20	2.27
	17.28	4.64	12.32
Trace statistic	4.86	0.20	2.27
	12.42	4.43	10.04
Proposition 5: Common Cycle			
Sq. Canonical Correlations	0.30	0.44	0.19
	0.09	0.08	0.07
χ^2−statistic p−value	(0.01)	(0.00)	(0.00)
	(0.69)	(0.21)	(0.12)
F−statistic p−value	(0.00)	(0.00)	(0.00)
	(0.61)	(0.19)	(0.11)

The level of fundamentals equals cross-country money netted with cross-country output calibrated to a unitary income elasticity of money demand. The money stocks (outputs) are measured in current (constant) local currency units and per capita terms. A constant and linear time trend are included in the level VARs. The LR statistics employ the Sims (1980) correction and have standard asymptotic distribution according to results in Sims, Stock, and Watson (1990). The case 2∗ and case 1 model definitions are based on Osterwald-Lenum (1992). MacKinnon, Haug, and Michelis (1999) provide five percent critical values of 8.19 (8.19) and 18.11 (15.02) for the case 2∗ model λ−max (trace) tests and 3.84 (3.84) and 15.49 (14.26) for the case 1 model. The common feature tests compute the canonical correlations of Δe_t and $\Delta m_t - \Delta y_t$. The common feature null is all or a subset of the canonical correlations are zero. See Engle and Issler (1995) and Vahid and Engle (1993) for details.

Table 4: UC Model Posterior Means, $\omega = 1$

Parameter	Priors	$UC_{2,2,\kappa=1}$	$UC_{2,\widetilde{m},\kappa=1}$	$UC_{2,\widetilde{c},\kappa=1}$
α_1	Normal [−1.2, 0.10]	−1.1906 (0.0613)	−0.8691 (0.0470)	–
α_2	Normal [0.40, 0.17]	0.4133 (0.1092)	0.9501 (0.0528)	–
θ_1	Normal [0.85, 0.10]	0.9407 (0.0491)	–	0.9830 (0.0296)
θ_2	Normal [0.10, 0.15]	0.0403 (0.0488)	–	0.0069 (0.0291)
μ^*	Normal [−0.126, 0.015]	−0.1260 (0.0150)	−0.1258 (0.0150)	−0.1258 (0.0120)
a^*	Normal [0.158, 0.025]	0.1615 (0.0229)	0.1645 (0.0213)	0.1571 (0.0199)
σ_μ	Inv-Gamma [2.0, 1.5]	1.8838 (0.1436)	2.4629 (0.1507)	1.6784 (0.1264)
σ_a	Inv-Gamma [2.0, 0.4]	1.0471 (0.2206)	0.3971 (0.0345)	1.9461 (0.3831)
$\sigma_{\widetilde{m}}$	Inv-Gamma [2.0, 0.6]	0.6002 (0.0899)	0.4728 (0.1168)	–
$\sigma_{\widetilde{c}}$	Inv-Gamma [2.0, 0.7]	1.2874 (0.2354)	–	2.0135 (0.3823)
$\varrho_{a,\widetilde{c}}$	Normal [−0.5, 0.2]	−0.8758 (0.0462)	–	−0.9475 (0.0234)
$\pi_{e,0}$	Normal [100.0, 15.0]	80.1528 (6.8283)	138.8984 (5.7281)	62.9442 (2.9991)
$\pi_{e,t}$	Normal [1.0, 0.5]	0.7038 (0.1710)	1.9366 (0.1106)	0.3831 (0.1306)
$\pi_{e,a}$	Uniform [−10.0, 0.0]	−2.6822 (0.6316)	−9.0223 (0.5377)	−0.7208 (0.1886)
$\pi_{c,\widetilde{m}}$	Uniform [−2.0, 7.5]	–	4.3973 (1.1057)	–
$\pi_{m,\widetilde{c}}$	Uniform [−7.5, 2.0]	–	–	−0.8985 (0.2099)
$\ln \widehat{\mathcal{L}}$		−53.95	−226.76	−24.76

Table 5: UC Model Posterior Means, $\omega \in [0.9, 0.999]$

Parameter	Priors	$UC_{2,2,\kappa}$	$UC_{2,\widetilde{m},\kappa}$	$UC_{2,\widetilde{c},\kappa}$
κ	Inv-Gamma	0.9658	0.9738	0.9962
	[0.988, 0.038]	(0.0219)	(0.0196)	(0.0046)
α_1	Normal	−1.1892	−0.8828	–
	[−1.20, 0.10]	(0.0673)	(0.0369)	
α_2	Normal	0.4131	0.8465	–
	[0.40, 0.17]	(0.1179)	(0.0223)	
θ_1	Normal	0.9396	–	0.9799
	[0.85, 0.10]	(0.0431)		(0.0315)
θ_2	Normal	0.0421	–	0.0018
	[0.10, 0.15]	(0.0422)		(0.0314)
μ^*	Normal	−0.1256	−0.1260	−0.1260
	[−0.126, 0.015]	(0.0148)	(0.0149)	(0.0147)
a^*	Normal	0.1621	0.1630	0.1578
	[0.158, 0.025]	(0.0227)	(0.0205)	(0.0243)
σ_μ	Inv-Gamma	1.8914	2.4241	1.7188
	[2.0, 1.5]	(0.1484)	(0.1663)	(0.1112)
σ_a	Inv-Gamma	1.0842	0.4043	1.5738
	[2.0, 0.4]	(0.2477)	(0.0378)	(0.2727)
$\sigma_{\widetilde{m}}$	Inv-Gamma	0.6068	0.5752	–
	[2.0, 0.6]	(0.0865)	(0.0975)	
$\sigma_{\widetilde{c}}$	Inv-Gamma	1.3828	–	1.6742
	[2.0, 0.7]	(0.2346)		(0.2772)
$\varrho_{a,\widetilde{c}}$	Normal	−0.8990	–	−0.9256
	[−0.5, 0.2]	(0.0409)		(0.0291)
$\pi_{e,0}$	Normal	85.2271	135.7241	67.8714
	[100.0, 15.0]	(6.7620)	(6.3363)	(3.7506)
$\pi_{e,t}$	Normal	0.7955	1.9076	0.4526
	[1.0, 0.5]	(0.1774)	(0.1211)	(0.1112)
$\pi_{e,a}$	Uniform	−3.2825	−8.8023	−1.2605
	[−10.0, 0.0]	(0.6426)	(0.5900)	(0.3087)
$\pi_{c,\widetilde{m}}$	Uniform	–	4.4186	–
	[−2.0, 7.5]		(0.7952)	
$\pi_{m,\widetilde{c}}$	Uniform	–	–	−1.0380
	[−7.5, 2.0]			(0.2179)
$\ln \widehat{\mathcal{L}}$		−53.94	−253.03	−29.88

Table 6: UC Model Posterior Means, $\omega \in [0.9, 0.999]$, Factor Loadings on Money and Consumption Cycles

Parameter	$UC_{2,2,\kappa}$	$UC_{2,\widetilde{m},\kappa}$	$UC_{2,\widetilde{c},\kappa}$
$(1-\pi_{c,\widetilde{m}})\delta_{\widetilde{m},0}$	0.0086 (0.0096)	−0.0841 (0.0676)	–
$(1-\pi_{c,\widetilde{m}})\delta_{\widetilde{m},1}$	−0.0269 (0.0188)	0.0064 (0.0082)	–
$(1-\pi_{c,\widetilde{m}})\delta_{\widetilde{m},2}$	0.0143 (0.0108)	−0.0762 (0.0624)	–
$(1-\pi_{c,\widetilde{m}})\Sigma_i\delta_{\widetilde{m},i}$	−0.0040 (0.0202)	−0.1542 (0.1237)	–
$(1-\pi_{m,\widetilde{c}})\delta_{\widetilde{c},0}$	−0.6044 (0.2042)	–	−0.3252 (0.2052)
$(1-\pi_{m,\widetilde{c}})\delta_{\widetilde{c},1}$	−0.0238 (0.0261)	–	−0.0004 (0.0111)
$(1-\pi_{m,\widetilde{c}})\Sigma_i\delta_{\widetilde{c},i}$	−0.6283 (0.2117)	–	−0.3256 (0.2053)

†The factor loadings $\pi_{c,\widetilde{m}}$ and $\pi_{m,\widetilde{c}}$ are zero for the 2-trend, 2-cycle UC model.

Table 7: UC Model Posterior Means, $\omega \in [0.9, 0.999]$, Variance(PDV–ε) / Variance(Δe)

Parameter	$UC_{2,2,\kappa}$	$UC_{2,\widetilde{m},\kappa}$	$UC_{2,\widetilde{c},\kappa}$
$Var(PDV-\varepsilon_{\mu})/Var(\Delta e)$	0.92 (0.15)	1.48 (0.22)	0.71 (0.09)
$Var(PDV-\varepsilon_{a})/Var(\Delta e)$	3.04 (0.73)	0.36 (0.06)	0.96 (0.53)
$Var(PDV-\varepsilon_{\widetilde{m}})/Var(\Delta e)$	0.00 (0.00)	0.00 (0.00)	–
$Var(PDV-\varepsilon_{\widetilde{c}})/Var(\Delta e)$	0.22 (0.16)	–	0.12 (0.22)

Table 8: UC-Models, $\kappa \in [0.9, 0.999]$, Exchange Rate FEVDs†

$UC_{2,2,\kappa}$ Model

Forecast Horizon	ε_μ	ε_A	$\varepsilon_{\tilde{c}}$
1	0.23	0.77	0.00
4	0.23	0.77	0.00
12	0.21	0.79	0.00
20	0.19	0.80	0.01
40	0.15	0.82	0.03

$UC_{2,\widetilde{m},\kappa}$ Model

Forecast Horizon	ε_μ	ε_A	$\varepsilon_{\tilde{c}}$
1	0.32	0.68	–
4	0.32	0.68	–
12	0.32	0.68	–
20	0.32	0.68	–
40	0.32	0.68	–

$UC_{2,\tilde{c},\kappa}$ Model

Forecast Horizon	ε_μ	ε_A	$\varepsilon_{\tilde{c}}$
1	0.59	0.40	0.01
4	0.58	0.41	0.01
12	0.57	0.42	0.01
20	0.57	0.43	0.01
40	0.55	0.45	0.01

†The summary statistics are the mean of the posterior distributions of the exchange rate FEVDs with respect to permanent and transitory cross-country money and cross-country consumption shocks.

Table 9: UC-Models, $\kappa \in [0.9, 0.999]$, Summary of the Trend-Cycle Decomposition†

Parameter	$UC_{2,2,\kappa}$	$UC_{2,\widetilde{m},\kappa}$	$UC_{2,\widetilde{c},\kappa}$
$STD(\Delta e^{\tau})$	2.66	104.28	2.44
$STD(\widetilde{e})$	3.68	106.84	2.55
$AR1(\widetilde{e})$	0.97	0.55	0.98
$Corr(\Delta e^{\tau}, \widetilde{e})$	−0.06	−0.37	−0.02
$STD(\Delta\mu)$	1.62	4.54	1.71
$STD(\widetilde{m})$	0.68	3.03	–
$AR1(\widetilde{m})$	−0.68	0.13	–
$Corr(\Delta\mu, \widetilde{m})$	0.26	−0.24	–
$STD(\Delta a)$	0.99	12.18	1.56
$STD(\widetilde{c})$	5.73	–	7.73
$AR1(\widetilde{c})$	0.97	–	0.98
$Corr(\Delta a, \widetilde{c})$	−0.16	–	−0.14
$Corr(\Delta e^{\tau}, \Delta\mu)$	0.01	−0.40	0.62
$Corr(\Delta e^{\tau}, \Delta a)$	−0.85	−0.93	−0.71
$Corr(\Delta\mu, \Delta a)$	0.52	0.55	0.10
$Corr(\widetilde{e}, \widetilde{m})$	0.04	−0.73	–
$Corr(\widetilde{e}, \widetilde{c})$	−1.00	–	−1.00

†The summary statistics are taken the mean of the summary statistics of the posterior distributions of the trends and cycle of exchange rate, cross-country money, and cross-country consumption. The trend growth rate of the exchange rate is denoted Δe^{τ}.

Figure 1: Prior and Posterior PDFs of DSGE-PVM Discount Factor

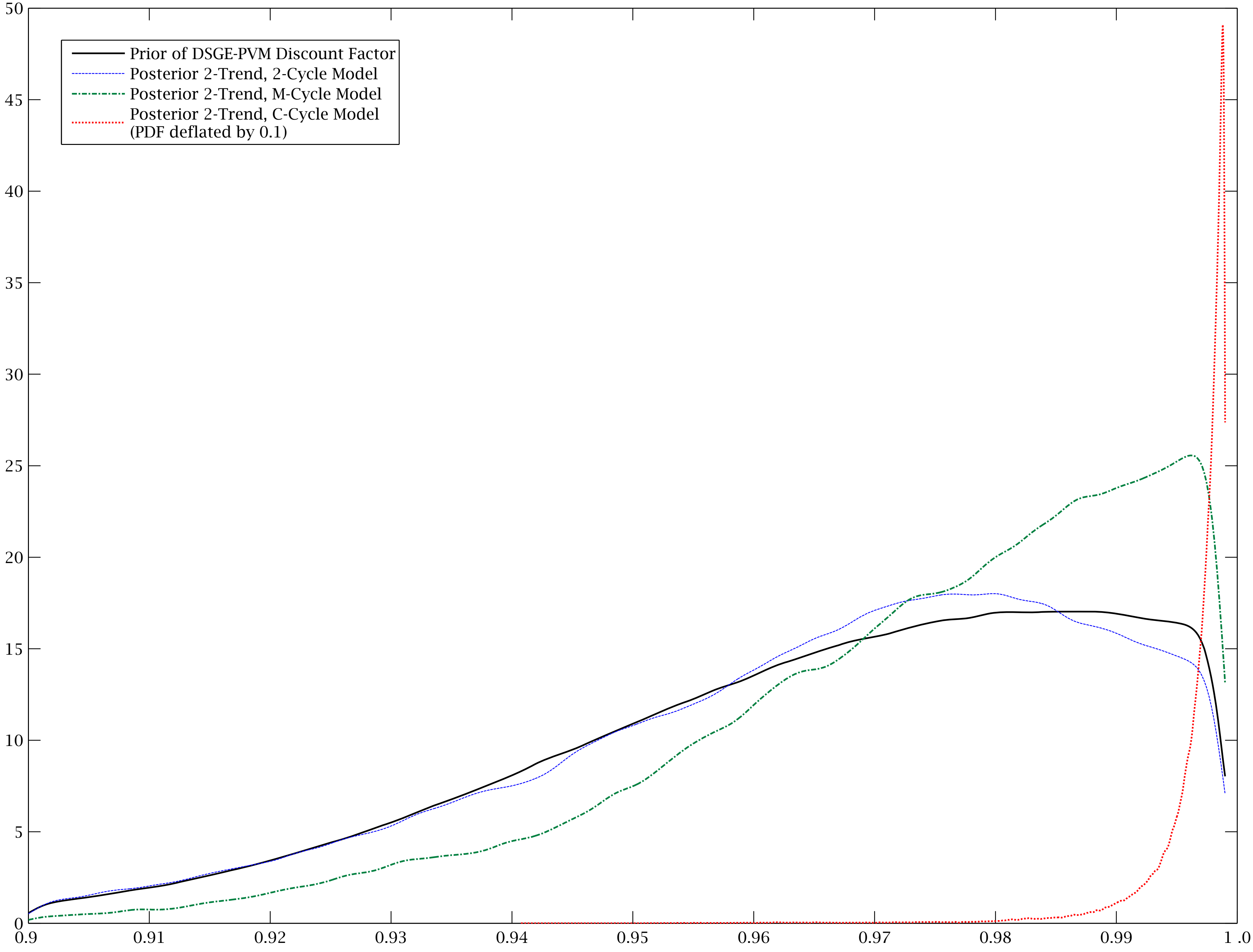

Figure 2: CDN$/US$ Exchange Rate Trend and Cycle, 1976Q1 - 2004Q4

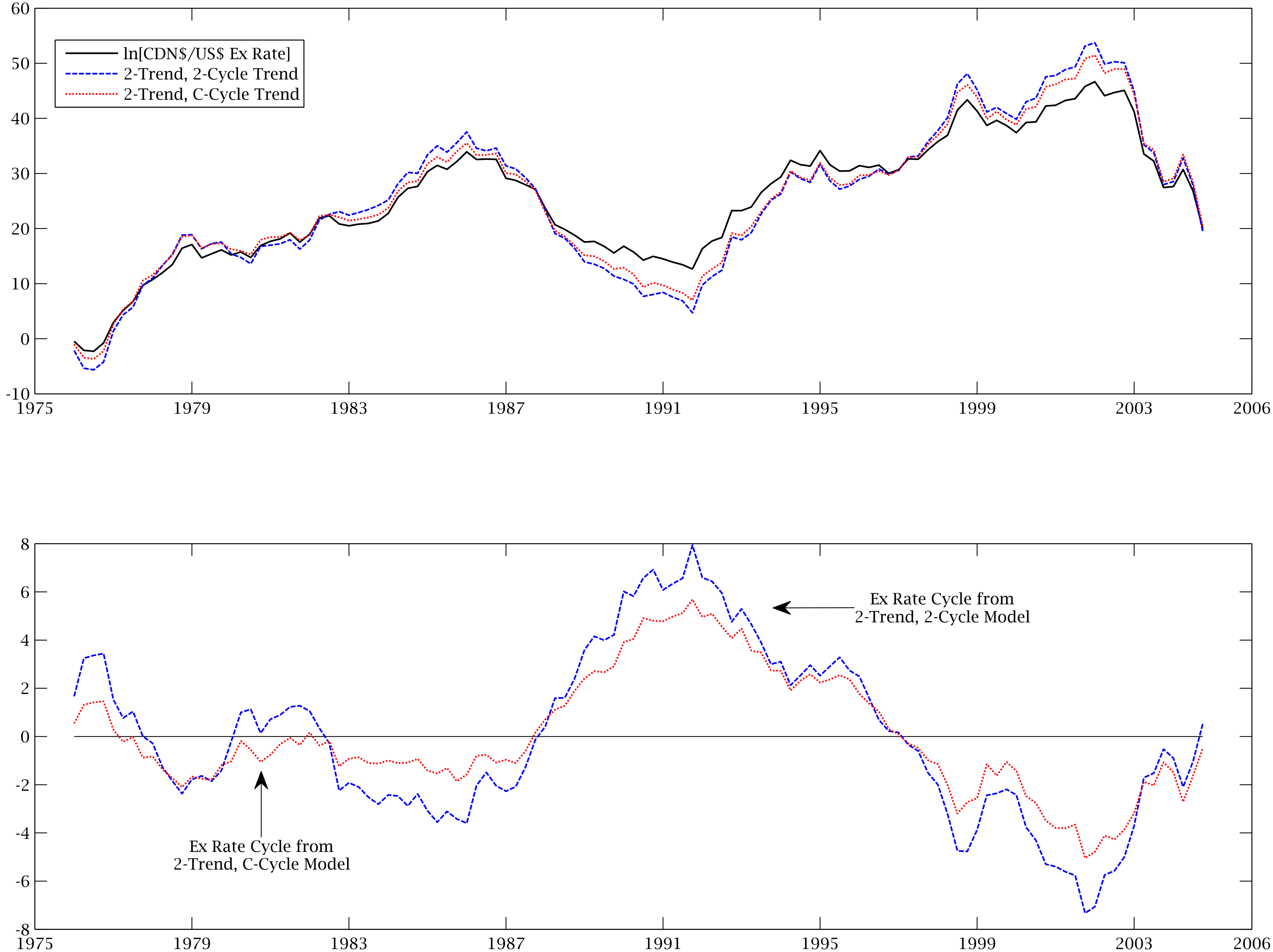

Figure 3: CDN-US Money, Consumption Trends and Cycles, 1976Q1 - 2004Q4

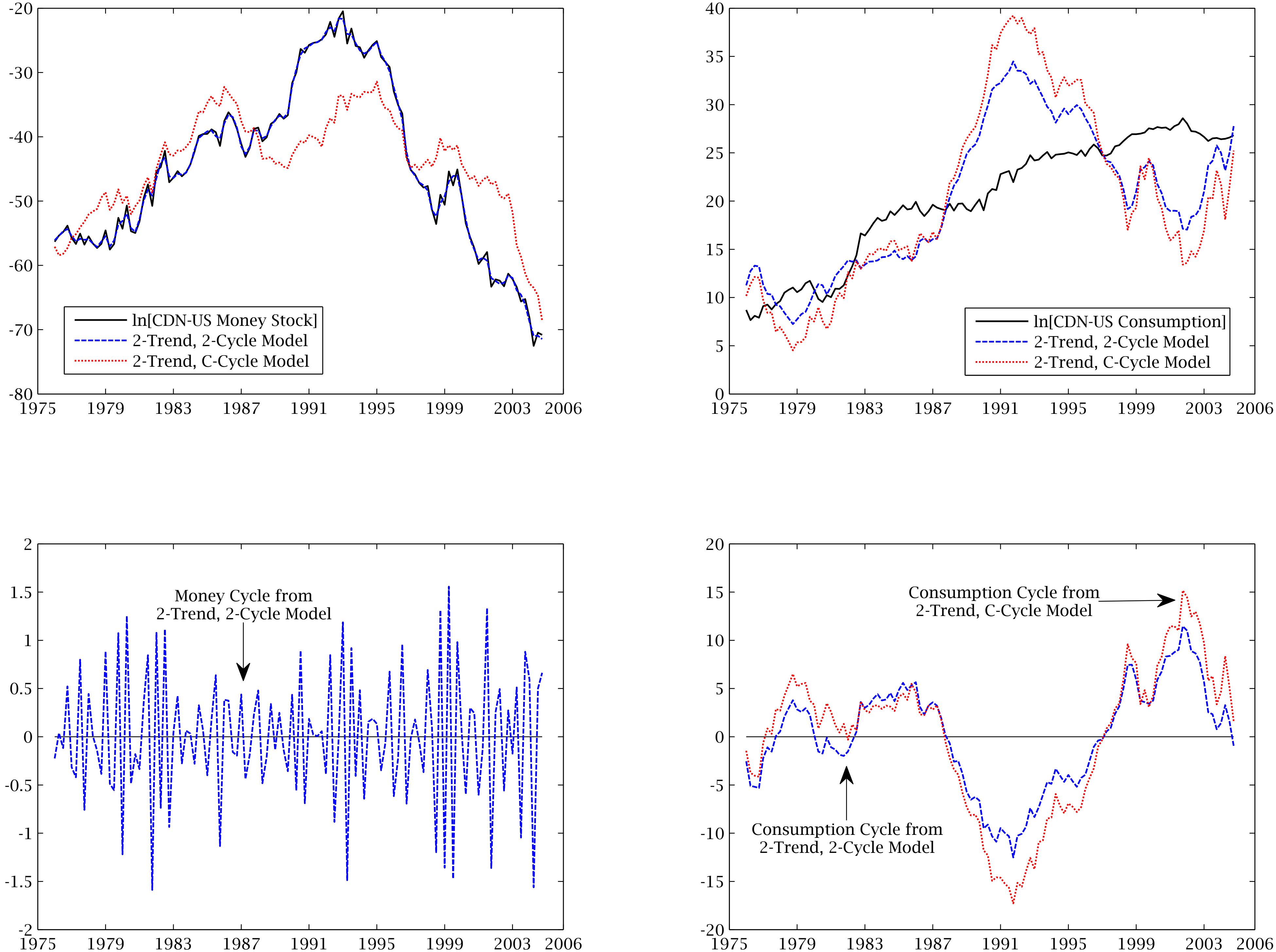

Figure 4: CDN$/US$ Ex Rate Cycles at Different DSGE-PVM Discount Factors