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Research Discussion Paper

Reserves of Natural Resources in a Small Open Economy

Isaac Gross and James Hansen

RDP 2013-14

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Author: hansenj at domain rba.gov.au

Media Office: rbainfo@rba.gov.au

Abstract

This paper studies the effect of a shock to resource prices in a small open economy where the stock of natural resources is responsive to exploration activity, and where extraction reduces the future availability of reserves. We show that the effects of a resource price shock on resource investment, labour utilisation and extraction are all amplified in the presence of endogenous reserves. We also find that spillovers to broader economic activity, including changes in domestic production, non-resource exports and consumption, are all greater in the presence of exploration activity. However, we find that incorporating endogenous reserves does not fundamentally change the effects of a resource price shock on key price measures including consumer prices, the real exchange rate and domestic interest rates.

JEL Classification Numbers: F41, Q33

Keywords: natural resources, small open economy

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1. Introduction

Large movements in commodity prices over the past decade have spurred renewed interest in the effects of commodity price shocks on a small open economy. One group of commodities that has received increasing attention, especially in the Australian case, is non-renewable resource commodities such as iron ore, coal and natural gas. We refer to this class of commodities as natural resources.[1]

Recent literature has studied the effects of shocks to the average price of natural resources by integrating a resource sector within a small open economy dynamic stochastic general equilibrium (DSGE) model. This approach provides a structural framework for studying the general equilibrium effects of these shocks, including their feedback effects and policy implications in a small open economy.

This paper adds to that work by assuming that the stock of domestic natural resource reserves is endogenous, rather than held constant as assumed in previous literature. Specifically, we allow firms to have access to an exploration technology, that can be used to increase reserves, and we assume that firms account for the effects of current extraction on the future availability of reserves (depletion). These two effects have been ignored in previous DSGE models with a natural resource sector.

Our findings suggest that allowing for endogenous reserves has substantial effects on the magnitude and persistence of the resource sector's response to a price shock in both partial and general equilibrium. The mechanism at the core of our model, the ability to accumulate newly discovered reserves through exploration, implies that resource firms respond to a price shock by increasing both extraction and exploration. Exploration that results in newly discovered reserves, in turn, leads to a permanent increase in firms' future extraction possibilities. These additional reserves provide firms with the incentive to use more labour, and

1 Throughout the paper we use the term 'natural resources' synonymously with non-renewable natural resources and abstract from renewable natural resources.

increase investment and extraction by more than in the case in which reserves are held fixed.

The larger expansion of the resource sector also has implications for the domestic allocation of goods. We find that when reserves are endogenous, there is a greater reallocation of goods between sectors in response to a resource price shock. In particular, more inputs, that would otherwise be used in the production of goods for consumption and non-resource exports, are redirected towards the resource sector where demand is stronger. However, total domestic production – measured as a weighted sum of domestic intermediate value added – is little changed relative to baseline. This is because slower growth of consumption and non-resource demand is largely offset by stronger growth in demand from the resource sector.

When comparing the behaviour of consumer prices, the real exchange rate and domestic interest rates, we find that the effects of a resource price shock are similar irrespective of whether we assume an endogenous or exogenous stock of natural resources. This suggests that the standard approach of assuming exogenous reserves can still provide a useful approximation for quantifying the price effects associated with a resource price shock.

The rest of the paper is organised as follows. Section 2 outlines our motivation and provides some simple stylised facts on the effects of a resource price shock. These stylised facts are used to help calibrate the partial and general equilibrium models that we discuss in Sections 3 and 4. Section 5 discusses the robustness of our findings in terms of identification – the mapping between our theoretical and empirical models – and whether the results are sensitive to the structure of the empirical VAR we estimate. Some conclusions from our work are drawn in Section 6.

2. Motivation

2.1 Theoretical

One approach used to study the effects of a resource price shock is to integrate a natural resource sector within a small open economy DSGE model. A common assumption used in existing literature is that the domestic economy's stock of resource reserves is held constant (or is exogenous with respect to resource prices and their effects on the domestic economy). When choosing to extract resources, firms do not account for the fact that extracting resources today reduces the amount of resources available for future extraction (depletion). In addition, firms are unable to invest in a technology that changes the level of available reserves, for example through exploration and the discovery of new reserves.

Examples of the 'exogenous reserves' approach to modelling the natural resource sector include Dib (2008), Garcia and González (2010), Bems and de Carvalho Filho (2011), Bodenstein, Erceg and Guerrieri (2011), Lama and Medina (2012) and Natal (2012). Similar abstractions are also common in DSGE models developed by central banks, including Australia (Jääskelä and Nimark 2008), Canada (Murchison and Rennison 2006), New Zealand (Lees 2009) and Spain (Andrés, Burriel and Estrada 2006).

Although a useful simplifying assumption for some purposes, a limitation of the 'exogenous reserves' approach is that there is nothing inherently natural resource-like in the behaviour of resource producers. This raises some important questions: to what extent does the assumption of exogenous reserves matter for understanding the propagation of resource price shocks? Would the responses look especially different if one allows for endogenous reserves due to exploration and depletion? We attempt to address these questions with specific reference to the effects of a resource price shock in a small open economy model.

In terms of related literature, the only paper that we are aware off that nests an endogenous reserves structure in a DSGE model is Veroude (2012) who studies business cycle correlations for Australia using a closed economy real business cycle model. Our work complements this research by studying the open economy implications of endogenous reserves with specific reference to the effects of shocks to international resource prices. We believe that openness is an important consideration because much of the resource sector's output and capital formation is exported and imported respectively, and this has implications for relative prices and the real exchange rate.

There is a separate and quite extensive literature on the optimal extraction of natural resources and investment and exploration decisions, but not within the context of a small open economy. A non-exhaustive list of useful references includes Pindyck (1978), Reiss (1990), Heal (1993), Sweeney (1993) and Bohn and Deacon (2000). There is also very informative literature studying the comparative statics of general equilibrium models with multiple sectors including resources (see, for example, Gregory (1976) and Corden (2012)), although these models do not incorporate expectations or dynamics.

2.2 Empirical

Figure 1 highlights some of the key developments in the Australian resource sector since 1976. Figure 1 shows measures of average prices, average production (extraction), real exploration expenditure, and the average stock of reserves in the sector.[2] Reserves for each resource commodity are measured to include both economically demonstrated reserves – reserves considered to be economically profitable for extraction purposes – and sub-economic reserves, which are not considered to be currently viable but that may become viable in the future with higher resource prices or an advance in technology that reduces costs.[3] The resources included in these measures are iron ore, coal, oil and petroleum (including crude oil, condensate and liquefied petroleum gas (LPG)), natural gas, five base metal ores (bauxite, copper, lead, nickel and zinc), and gold. Together these resources accounted for approximately 88 per cent of Australia's total resource exports, and 66 per cent of total goods exports in 2011/12.

Summarising the main stylised facts:

1. Real resource prices trended down for much of the sample but then increased around the turn of the millennium reflecting strong commodity demand, particularly from China.

2. Production growth was most rapid in the mid to late 1980s but has since stabilised.[4]

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2 All averages – prices, production and reserves – are export-weighted geometric averages, where the export weights used are fixed at the sample averages for 1976 to 2011.

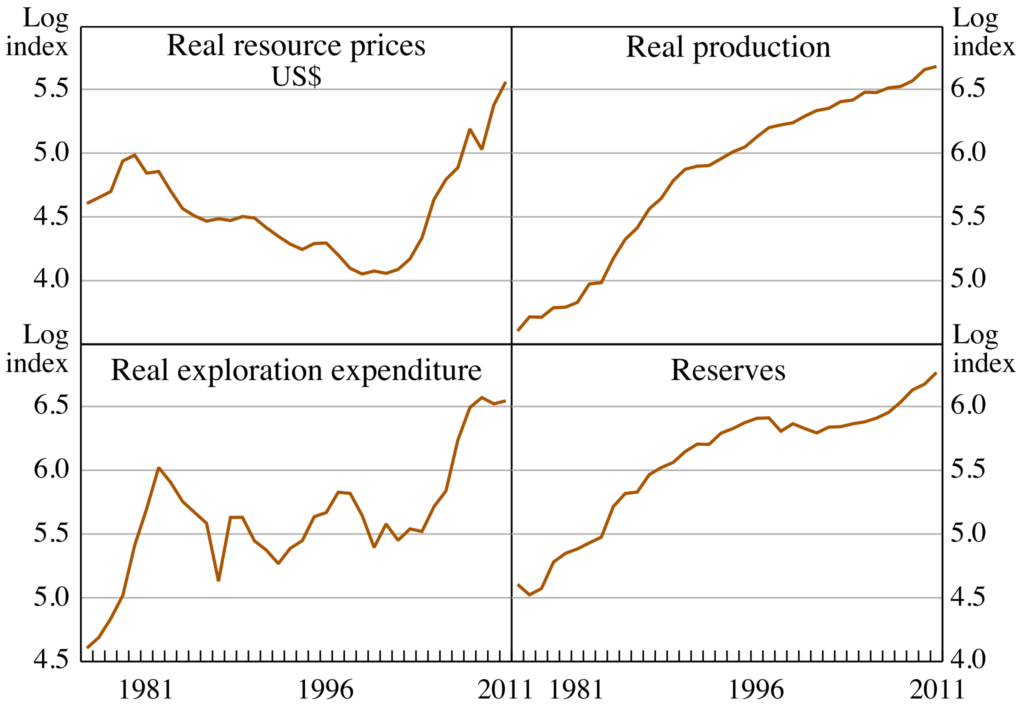
3 See Geoscience Australia (2012) for further discussion on the classification of reserves. Prior to 1992, all reserve measures are based on economically demonstrated reserves only and are spliced to the post-1992 series. Estimates for production and reserves in 2011 are inferred using the growth rates implied in Australian Bureau of Statistics (ABS) data (ABS Catalogue No 5204.0).

4 However, the recent high levels of investment in the mining sector are forecast to increase production over the medium term, see Bureau of Resources and Energy Economics (2013).

3. Real exploration initially peaked in the early 1980's and then declined for much of the period in which real resource prices fell. From around the mid 2000s, real exploration activity began to grow rapidly as the increase in resource prices became sustained.

4. The pace of growth in reserves generally slowed over the 20 years between 1980 and 2000 and accelerated from the mid 2000s. This suggests that at least some of the pick up in exploration has resulted in the discovery of new reserves.

Figure 1: Developments in the Australian Resource Sector
1976 = 100



Notes: Resource prices, production and reserves are calculated using export-weighted geometric means; exploration expenditure comprises all categories of mineral and petroleum expenditure

Sources: ABS; Australian Bureau of Agricultural and Resource Economics and Sciences (ABARES); Bloomberg; Geoscience Australia; Global Financial Data; IMF; U.S. Geological Survey (USGS); authors' calculations

To provide insight into the broader effects of a resource price shock on the Australian economy, we use a simple structural vector autoregression (VAR) with annual data. We identify the effect of a shock to resource prices using the assumption that resource prices are contemporaneously uncorrelated with domestic variables and the real exchange rate. That is, we estimate:[5]

$$\mathbf{A}_0\mathbf{z}_t = \Gamma\mathbf{z}_{t-1} + \mathbf{e}_t$$

where $\mathbf{z}_t$ is vector of observable variables including real resource prices, the real TWI, the ratio of non-mining GDP to the stock of natural reserves, the ratio of resource sector capital expenditure to the stock of natural reserves, and inflation (in that order),[6] and $\mathbf{A}_0$ is a matrix with ones along its main diagonal and zeros in the off-diagonal elements in its first row. The latter reflects the identifying assumption that resource prices are contemporaneously uncorrelated with the remaining variables in the VAR.

Figure 2 reports the impulse response functions (IRFs) due to a 1 per cent exogenous increase in resource prices. Each IRF is measured in terms of the percentage deviation from its sample mean (or percentage point deviation where appropriate), and we report the 95 per cent (asymptotic) confidence intervals. In addition to the VAR IRFs, we also report the IRFs from our theoretical model in general equilibrium with endogenous reserves. The latter are produced from a theoretical model in which we estimate a subset of the model's parameters using a Generalised Method of Moments (GMM) estimator, and the remaining parameters are calibrated (Section 4.2 provides further details).

The results in Figure 2 suggest that resource price shocks are very persistent and have significant effects on both the resource sector and the broader economy. In particular, the VAR IRFs imply that an exogenous 1 per cent increase in resource prices leads to a persistent real appreciation of the exchange rate, a temporary increase in inflation and an increase in resource capital expenditure relative to reserves. A small, though not statistically significant, decline in the ratio of non-mining GDP to reserves is also observed.

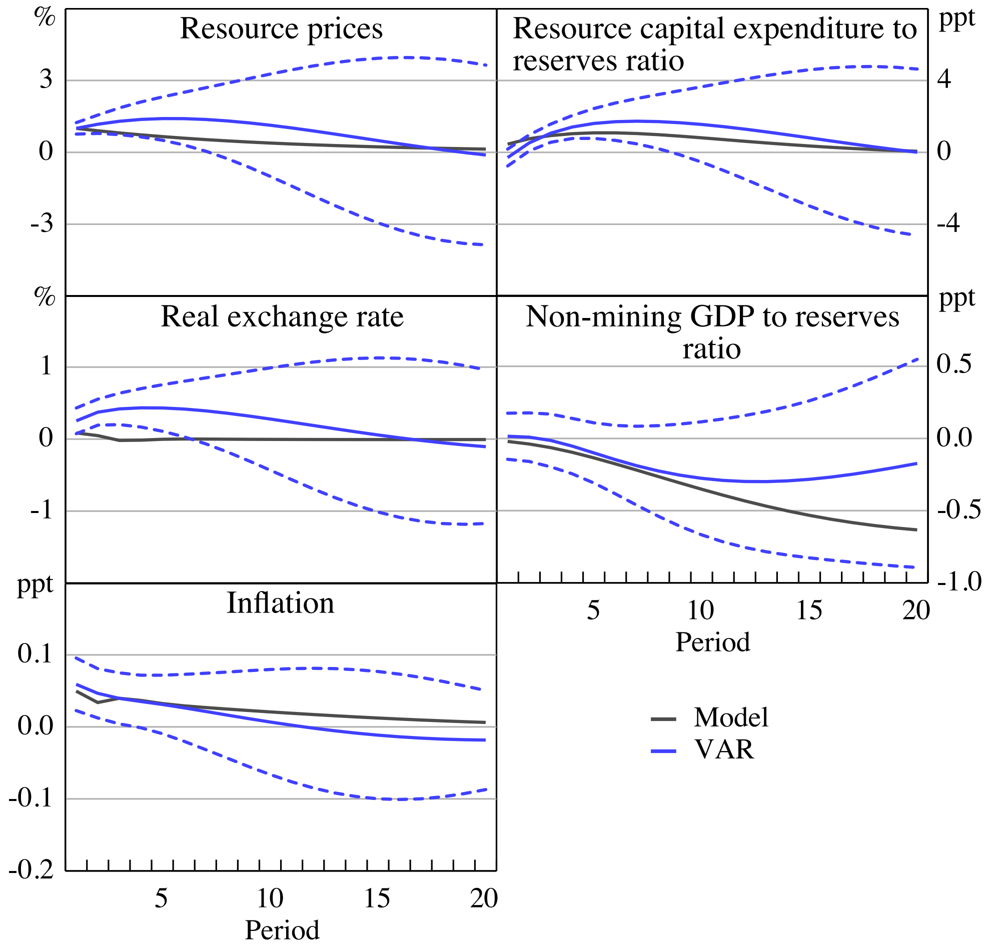
Interestingly, our theoretical model is able to reproduce these results quite well in terms of the sign, amplitude and persistence of the IRFs. The main exception is the real exchange rate. Although our model is able to reproduce an appreciation,

5 A deterministic time trend and constant are also included in each regression. It should be noted that similar estimates are obtained using HP-filtered data or differenced data.

6 For a full description of the data used, see Appendix A.

it is neither sufficiently large nor persistent when compared with the response identified in the VAR. Nevertheless, in view of the theoretical model's overall ability to match the VAR IRFs, we use these GMM estimates to help parameterise both the partial and general equilibrium models discussed below.

Figure 2: Empirical and Model Impulse Response Functions to a 1 Per Cent Increase in Resource Prices



3. Natural Resources in Partial Equilibrium

3.1 The Resource Sector

Our model of the resource sector draws on the work of Bohn and Deacon (2000). These authors allow for both endogenous exploration and depletion, and use an approach that lends itself to incorporating a resource sector into a small open economy model. For the structure of the resource sector, we assume that:

1. All resources are exported at prices that are taken as given by resource firms (that is, the resource market is globally competitive).[7]

2. Resource firms can choose to extract a commodity from existing reserves and can engage in costly exploration activity to discover new reserves.

3. Resource firms use domestic labour, imported capital, and reserves to extract their natural resource.

4. All resource firms are identical in terms of their access to exploration and extraction technologies.

These assumptions are designed to provide an approximation of the resource sector in aggregate. In the Australian context, they are consistent with the fact that the majority of extracted natural resources are exported, that firms engage in both exploration and extraction activity, and that firms import capital and use domestic labour.[8]

Formally, we assume a continuum of identical resource firms of unit measure. Each period a firm uses capital (K_t), labour (H_t^r) and its existing stock of natural reserves (R_t), to extract a natural commodity (X_t) according to a Cobb-Douglas technology:

$$X_t = \left(H_t^r A_t^r\right)^{\eta} K_t^{\gamma} R_t^{1-\eta-\gamma}$$

where A_t^r allows for labour-augmenting technical change. There are two additional constraints for a resource firm. One is the law of motion for resource-specific capital owned by the firm:

$$K_{t+1} = (1-\delta)K_t + \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

where K_t is resource-specific capital, δ is the rate of depreciation, I_t is a resource-specific investment goods (purchased from abroad), and Ξ is a real

7 For simplicity, we abstract from the use of commodities in domestic production. It should also be noted that perfect competition is the norm in literature modelling a resource sector within a small open economy.

8 See Connolly and Orsmond (2011) for further discussion.

convex investment adjustment cost function. The law of motion for resource-specific capital is standard, although we allow for adjustment costs on changes in investment (rather than the level of investment relative to the capital stock). This is a convenient reduced-form assumption for capturing time-to-build constraints and lumpiness at the level of individual investment projects. Modelling adjustment costs in this way captures the typical 'hump-shaped' response of resource-specific investment to resource price shocks (as shown in Figure 2).

The second constraint is the law of motion for reserves:

$$R_{t+1} = R_t + \omega_{t+1} D_t - \lambda X_t$$

where reserves are depleted through production (extraction) X_t and accumulated through D_t, a measure of exploration (or discovery) activity. The parameter λ is an indicator variable that is one in a model with depletion and zero in a model without depletion. This is useful for defining the equilibria with and without endogenous reserves discussed further below.

We assume that exploration activity is an uncertain process, captured by the random variable ω_{t+1}, which only becomes known at the beginning of period $t+1$. We assume ω_{t+1} is independently and identically distributed on a compact support with distribution function Γ and first moment $E\left(\omega_{t+1}\right) = 1$. This implies that the probability that a unit of exploration results in the successful discovery of a unit of new reserves is independent of the state of the economy.

Given the assumption of Cobb-Douglas production technology, the total wage bill for a resource firm, TC_t^r, is given by:

$$TC_t^r = \frac{W_t^r}{A_t^r} \left(X_t^{1+\zeta} K_t^{-\mu} R_t^{\mu-\zeta} \right)$$

where W_t^r is the wage paid to labour and, following Bohn and Deacon (2000), we define the parameters $\zeta = \frac{1}{\eta} - 1$ and $\mu = \frac{\gamma}{\eta}$. The firm chooses its investment in resource-specific capital, its extraction, and exploration expenditure by solving the following dynamic program:

$$\begin{aligned} V(K_t,R_t) &= \max_{I_t,X_t,D_t}\{S_tP_t^{r^*}X_t - \tfrac{W_t}{A_t^r}{}^r\left(X_t^{1+\zeta}K_t^{-\mu}R_t^{\mu-\zeta}\right) \\ &- S_tP_t^*I_t - C\left(D_t,\widetilde{R}_t\right) + \beta\int V\left(K_{t+1},R_{t+1}\right)d\Phi\left(\xi_{t+1}\mid\xi_t\right)\} \qquad (1) \\ \widetilde{R}_t &\equiv \int_0^1 R_t(i)\,di \end{aligned}$$

where K_{t+1} and R_{t+1} are given by the constraints previously described; $V:\mathbf{R}^2\to\mathbf{R}$ is the value function; S_t is the nominal exchange rate (measured in units of domestic currency required to purchase a single unit of foreign currency); $\widetilde{R}_t$ is the aggregate stock of domestic reserves; $P_t^{r^*}$ is the price of the extracted commodity in foreign currency terms; P_t^* is the price of investment goods (imported from abroad) that deliver resource sector-specific capital in the next period (also measured in foreign currency prices); β is a discount factor,[9] ξ_t is a state vector containing exogenous prices and aggregate reserves which are known at time t $\left(\xi_t\equiv\left[P_t^{r^*},W_t^r,P_t^*,S_t,\widetilde{R}_t,A_t^r\right]\right)$; and $C:\mathbf{R}^2\to\mathbf{R}$ is a convex cost function associated with exploration activity. The precise functional form and parametrisation of the cost function are discussed below.

Uncertainty over future prices, the aggregate stock of reserves, and the success of future exploration are captured in the expectation of the value function in the next period.[10] In the partial equilibrium analysis that follows, we assume that the factor prices and the real exchange rate $\left[W_t^r,P_t^*,S_t\right]$ remain constant in the face of a resource price shock.

Our approach is quite similar to that adopted in Bohn and Deacon (2000). However, one important difference is that we abstract from the presence of a known finite bound on the cumulative level of resources to be discovered. Consistent with Pindyck (1978), we assume that additional reserves can be discovered in perpetuity but that it is costly to discover new reserves as the stock of known reserves increases. This assumption is important for our analysis

9 In partial equilibrium we abstract from a stochastic discount factor since firm ownership is not modelled explicitly. We allow for a stochastic discount factor in the general equilibrium model in Section 4.1.

10 Note since ω_{t+1} is iid with a unitary first moment, we can integrate this variable out of the term $E_t\left(V\left(K_{t+1},R_{t+1}\right)\right)$.

because it implies that the policy functions that solve the resource firms' problem in Equation (1) are time invariant, and so can be integrated with a DSGE model. As well as increasing tractability, we think that this assumption is realistic for many countries, including Australia, given that reserves, production and exploration have continued to grow over time rather than decrease as one would expect in a model of fixed potential reserves.[11]

The first-order conditions associated with the resource firms' problem are given by:

$$S_t P_t^{r^*} = (1+\zeta)\frac{W_t^r}{A_t^r} X_t^{\zeta} K_t^{-\mu} R_t^{\mu-\zeta} + Q_t^r \tag{2}$$

$$\begin{aligned} S_t P_t^* = {} & Q_t^k \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right) - \Xi''\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\ & + \beta E_t \left(Q_{t+1}^k \Xi'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}^2}{I_t^2}\right) \end{aligned} \tag{3}$$

$$\frac{\partial C(D_t, R_t)}{\partial D_t} = Q_t^r \tag{4}$$

$$K_{t+1} = (1-\delta) K_t + \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{5}$$

$$R_{t+1} = R_t + \omega_{t+1} D_t - \lambda X_t \tag{6}$$

The marginal valuations of an extra unit of reserves and capital to the firm are respectively given by:

$$Q_t^r = \beta E_t \left((\zeta - \mu)\frac{W_{t+1}^r}{A_{t+1}^r} X_{t+1}^{1+\zeta} K_{t+1}^{-\mu} R_{t+1}^{\mu-\zeta-1} + Q_{t+1}^r\right) \tag{7}$$

$$Q_t^k = \beta E_t \left(\left(\mu \frac{W_{t+1}^r}{A_{t+1}^r} X_{t+1}^{1+\zeta} K_{t+1}^{-\mu-1} R_{t+1}^{\mu-\zeta} + Q_{t+1}^k (1-\delta)\right)\right) \tag{8}$$

Equation (2) implies that firms equate the marginal revenue of extraction with the marginal cost of extraction, where the marginal cost of extraction includes both the additional cost of extraction in period t, and the opportunity cost tied to the fact that resources extracted today cannot be extracted in future periods. Equation (3) implies that the marginal cost of purchasing resource-specific capital from abroad

11 See Appendix B for further discussion and Pindyck (1978).

is equal to the marginal return of this capital after accounting for the fact that additional investment reduces future investment-adjustment costs.

Equation (4) implies that firms equate the marginal cost of exploration with the expected marginal return, the latter being given by the shadow price of an extra unit of reserves. Equations (5) and (6) describe the law of motion for capital and the stock of natural reserves, respectively. The shadow prices in Equations (7) and (8) reflect the marginal valuations of an additional unit of reserves and an additional unit of capital respectively, and are given by the present discounted value of the additional revenue streams generated by either an extra unit of reserves or capital.

We compare two equilibria associated with these first-order conditions. The first assumes that resources are depletable ($\lambda = 1$) and exploration expenditure responds to changes in prices.

Definition 1. *A partial equilibrium for the endogenous reserves model is given by sequences for* $\left\{X_t, D_t, I_t, K_{t+1}, R_{t+1}, Q_t^R, Q_t^K\right\}$ *that solve Equations (2) to (8) taking the expected sequences* $\left\{W_t^r, S_t, P_t^*, P_t^{r^*}, A_t^r\right\}$ *as given and assuming* $\lambda = 1$.

The second equilibrium we consider assumes that the stock of resources is exogenous (fixed), and thus abstracts from both depletion and the scope for exploration activity.[12]

Definition 2. *A partial equilibrium with exogenous reserves, is given by sequences* $\left\{X_t, I_t, K_{t+1}, R_{t+1}, Q_t^R, Q_t^K\right\}$ *that solve Equations (2) to (3) and (5) to (8) taking the expected sequences* $\left\{W_t^r, S_t, P_t^*, P_t^{r^*}, A_t^r\right\}$ *as given and assuming* $\lambda = 0$ *and* $D_t = 0$ *for all* t.

3.2 Calibration

Table 1 reports the calibration of the structural parameters with the model solved at an annual frequency – the highest frequency for which production and reserves data are available. The discount factor and depreciation rate are chosen to be in line

12 It is straightforward to verify that the steady states for these equilibria exist and are identical.

with existing literature that model a resource sector.[13] The exponents on capital and labour in the resource extraction technology (γ and η) are chosen to match a steady state rate of annual extraction of two per cent, and a wage bill relative to total revenue of approximately 11 per cent.[14] The two per cent average annual extraction rate is consistent with an equally-weighted average of extraction rates in iron ore, coal, gold, lead, nickel, zinc, copper and bauxite for the sample 1976 to 2011.[15]

Table 1: Resource Sector Parameterisation

Description	Coefficient	Value
Calibrated parameters		
Discount factor	β	0.96
Labour factor exponent	η	0.13
Capital factor exponent	γ	0.49
Depreciation rate	δ	0.10
Parameters obtained from GMM estimation of general equilibrium model		
Exploration costs dynamics	ϕ_{mc}	0.5
Investment cost parameter	κ	3
AR(1) parameter (prices)	ρ_r	0.9

For the parameterisation of exploration costs, we use a function that implies that resource sector profits are homogenous of degree one: [16]

$$C\left(D_t, \widetilde{R}_t\right) = P_t^n \frac{Q^r}{\phi_{mc}} e^{\phi_{mc}\left(\frac{D_t}{\widetilde{R}_t} - \frac{D}{\widetilde{R}}\right)} \widetilde{R}_t$$

where ϕ_{mc} is a parameter that governs the sensitivity of exploration costs to shocks and, thus, the incentive to engage in exploration activity; Q^r is a normalisation used to ensure a well-defined steady state (in general equilibrium); and P_t^n is

13 See, for example, Charnavoki (2010) and Garcia and González (2010).

14 This estimate is consistent with estimates from Topp *et al* (2008) and ABS Catalogue No 8414.0.

15 More specifically, we use an arithmetic average across these industries using both the extraction weights implied when using economically demonstrated reserves (2.8 per cent per annum) and total reserves (1.65 per cent per annum), where the latter also include sub- and para-marginal reserves.

16 That is, a doubling of production and of all factor inputs, including reserves, would double revenue and double cost.

the price of a bundle of non-traded goods (held fixed for the partial equilibrium analysis).

Importantly, and as discussed further in Appendix B, this cost function satisfies the restrictions that: exploration costs are increasing in both exploration and aggregate reserves, $\frac{\partial C}{\partial D_t} > 0, \frac{\partial C}{\partial \widetilde{R}_t} > 0$; the derivative of the marginal cost of exploration is increasing in the level of exploration, $\frac{\partial^2 C}{\partial D_t^2} > 0$; and that this latter derivative is sufficiently large to outweigh any reduction in the marginal costs of exploration that are tied to larger existing reserves permitting extensions of, or new finds linked to, existing deposits, $\frac{\partial^2 C}{\partial D_t^2} + \frac{\partial^2 C}{\partial D_t \partial \widetilde{R}_t} > 0$.

For the parameteristation of investment adjustment costs, we assume a quadratic adjustment cost function satisfying $\Xi'(1) = 0$, $\Xi''(1) = \kappa$. For resource prices, we assume that the natural log of prices follows an AR(1) process with autoregressive parameter ρ_r:

$$\ln P_t^{r^*} = \rho_r \ln P_{t-1}^{r^*} + \varepsilon_t^{r^*}$$

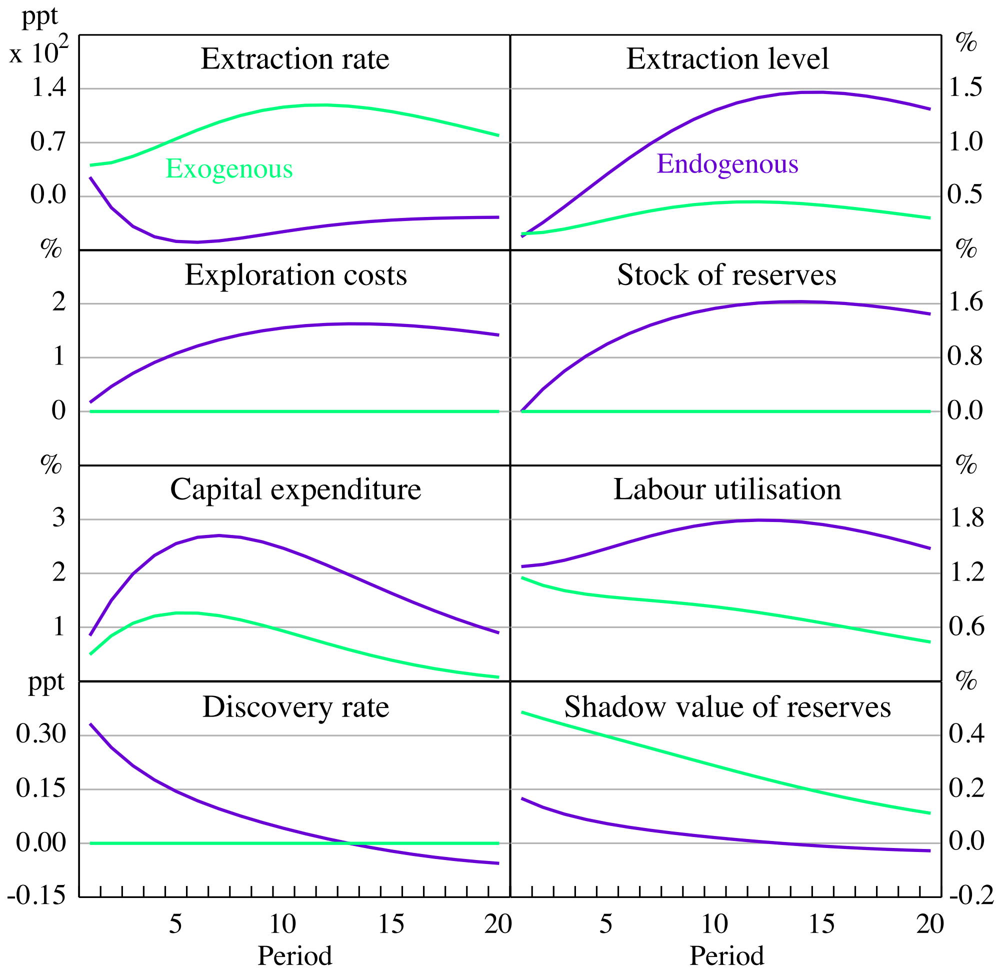
where $\varepsilon_t^{r^*}$ is iid. All other prices $\left\{W_t^r, S_t, P_t^*\right\}$ are held fixed in partial equilibrium.

The parameters ϕ_{mc}, κ and ρ_r only affect the shape of IRFs and have no bearing on the steady state of the model. For this reason, the values for these parameters are chosen to be consistent with the values implied when matching the IRFs of the general equilibrium version of our model (discussed further below) with the VAR discussed in Section 2.

3.3 Results

Figure 3 highlights the IRFs associated with a 1 per cent positive shock to resource prices in partial equilibrium – that is, holding wages, the exchange rate and the price of imported capital fixed. The IRFs are computed under both the endogenous and exogenous reserves equilibria as described in Definitions (1) and (2). Comparing these two equilibria, it is clear that the endogenous model generates additional amplification and persistence in response to a resource price shock. Factor utilisation for both labour and capital increase by more in the endogenous reserve model, as does the level of extraction. The stock of reserves also increases in the endogenous reserve model (but is held constant by assumption with exogenous reserves) as exploration activity and the discovery of new deposits results in reserves accumulating faster than they are depleted through higher extraction.

Figure 3: Response to a 1 Per Cent Increase in Resource Prices in Partial Equilibrium



The mechanism driving the amplification of the resource price shock is the feedback effects that occur through exploration. Because a persistently higher resource price provides firms with an incentive to engage in the exploration of new reserves, or to find extensions to existing deposits, the expected value of newly discovered reserves increases. This leads firms to engage in exploration activity. Importantly, any newly discovered reserves are a *permanent* addition to the resource firms' extraction opportunity set. That is, once discovered, they can be extracted either in the next period or in any future period without depreciation.

Under the assumption of Cobb-Douglas technology, reserves are complementary to both labour and capital; and so as more reserves are discovered, the marginal product of labour and capital both increase. Resource firms' respond by investing in additional capital and hiring more labour, leading to a greater expansion in all areas of mining operations.

An interesting implication of this partial equilibrium model is that reserves will have non-stationary dynamics in equilibrium. That is, transitory changes in resource prices can generate permanent changes in investment, labour utilisation, resource sector production and the stock of reserves. To see why, note that if we abstract from investment adjustment costs, the problem for a resource producer can be reformulated as:

$$V(k_t) = \max_{i^r_t, d_t, x_t} \left\{ \begin{array}{c} S_t P^r_t x_t - \frac{W^r_t}{A^r_t} x_t^{1+\zeta} k_t^{-\mu} \\ -S_t P^*_t i^r_t - C(d_t) + \beta E_t \left(r_{t+1} V\left(k_{t+1}\right)\right) \end{array} \right\}$$

subject to:

$$\begin{aligned} r_{t+1} &= 1 + \omega_{t+1} d_t - \lambda x_t \\ k_{t+1} r_{t+1} &= (1-\delta) k_t + i^r_t \end{aligned}$$

In this representation, the decision variables are reformulated in terms of the extraction rate, $x_t \equiv \frac{X_t}{R_t}$, the exploration (discovery) rate, $d_t \equiv \frac{D_t}{R_t}$, and the investment rate, $i^r_t \equiv \frac{I_t}{R_t}$; and $\left(r_{t+1} - 1\right)$ is now the growth rate in reserves.[17]

This reformulation makes it clear that one can think of a resource producer as choosing its optimal extraction rate, exploration rate, and investment rate. As discussed in further detail in Appendix C, the solution to these decision variables are a function of underlying prices and technology, $\{P^{r^*}_t, S_t, P^*_t, W^r_t, A^r_t\}$ but not the stock of reserves. This in turn implies that a resource firms' scale of operation is directly proportional to the stock of reserves and that any newly discovered reserves will imply permanent changes in the levels of extraction, exploration and investment, but only transitory changes in the optimal extraction, exploration and investment rates. This result helps to explain the degree of amplification and persistence in the IRFs, and implies that the scale of the resource sector can appear to trend over time, even if real resource prices exhibit long-run mean reversion (stationarity) as we have assumed.

17 Note that we have made use of the fact that when the constraints and return functions are both homogenous of degree one, the value function is also homogenous of degree one. For further discussion, see Appendix C.

In view of the additional amplification generated in response to a resource price shock, we now investigate whether the same results hold in general equilibrium, and whether the incorporation of endogenous reserves alters the effects of a resource price shock on the rest of the economy.

4. Natural Resources in a Small Open Economy

In general equilibrium, we assume that the resource sector is largely identical to that previously discussed:

$$S_t P_t^{r^*} = (1+\zeta)\frac{W_t^r}{A_t^r} X_t^{\zeta} K_t^{-\mu} R_t^{\mu-\zeta} + Q_t^r \tag{9}$$

$$\frac{\partial C(D_t, R_t)}{\partial D_t} = Q_t^r \tag{10}$$

$$K_{t+1} = (1-\delta) K_t + \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{11}$$

$$R_{t+1} = R_t + \omega_{t+1} D_t - \lambda X_t \tag{12}$$

$$\begin{aligned} S_t P_t^* = {} & Q_t^k \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right) - \Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\ & + \beta E_t \left(M_{t,t+1} Q_{t+1}^k \Xi'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}^2}{I_t^2}\right) \end{aligned} \tag{13}$$

$$Q_t^r = \beta E_t \left(M_{t,t+1}\left((\zeta-\mu)\frac{W_{t+1}^r}{A_{t+1}^r} X_{t+1}^{1+\zeta} K_{t+1}^{-\mu} R_{t+1}^{\mu-\zeta-1} + Q_{t+1}^r\right)\right) \tag{14}$$

$$Q_t^k = E_t \left(M_{t,t+1}\left(\mu\frac{W_{t+1}^r}{A_{t+1}^r} X_{t+1}^{1+\zeta} K_{t+1}^{-\mu-1} R_{t+1}^{\mu-\zeta} + Q_{t+1}^k (1-\delta)\right)\right) \tag{15}$$

$$\Psi_t^R = S_t P_t^{r^*} X_t - \frac{W_t^r}{A_t^r}\left(X_t^{1+\zeta} K_t^{-\mu} R_t^{\mu-\zeta}\right) - S_t P_t^* I_t - C(D_t, R_t) \tag{16}$$

$$X_t = \left(H_t^r\right)^{\eta} K_t^{\gamma} R_t^{1-\eta-\gamma} \tag{17}$$

However, to properly integrate a resource firms' problem within general equilibrium we require two further assumptions. The first assumption is that we now explicitly account for the preferences of resource firm owners. This is done by assuming that firms use a stochastic discount factor (SDF), $\beta M_{t,t+1}$, when valuing profits over time and states of the world, rather than the deterministic discount factor, β. For simplicity, and consistent with the presence of foreign ownership in the sector, we assume that the SDF for resource firms only partially updates to reflect the preferences of the domestic owners and is thus given by:

$$M_{t,t+1} = \left(\nu \frac{\Theta_{t+1}}{\Theta_t} + 1 - \nu \right) \frac{P_t^c}{P_{t+1}^c}$$

where ν is the parameter governing the importance of domestic ownership and Θ_t is the marginal utility of (domestic household) consumption in period t.

The second assumption is that exploration activity requires non-traded goods as an input:[18]

$$D_t = \frac{Y_t^{r,n}}{\frac{Q^r}{\phi_{mc}}} \tag{18}$$

where $Y_t^{r,n}$ is aggregated using a using the Dixit-Stiglitz aggregator:

$$Y_t^{r,n} = \left(\int_0^1 {Y_{it}^{r,n}}^{\frac{\theta_n - 1}{\theta_n}} di \right)^{\frac{\theta_n}{\theta_n - 1}}$$

We now briefly describe the rest of the small open economy (a more complete discussion is available in Appendix D). Concerning production in the rest of the economy, we assume that there are three sectors: a non-traded sector; an importing sector; and a non-resource exporting sector. All three sectors are assumed to operate in a monopolistically competitive environment and face a Calvo price-setting friction. Prices in all sectors are set in local (domestic) currency terms and these sectors are owned by domestic households.

Non-traded firms produce an intermediate input, which when bundled with the production of their competitors, is either consumed, used as an input in non-resource export production, or used as an input in the resource exploration process. Importers import a final good from abroad and then differentiate it to produce a specialised good that is consumed by domestic residents. Non-resource exporters transform a bundle of non-traded intermediate goods into a specialised good that is exported abroad.

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18 Specifically, we are assuming that exploration expenditure and a bundle of non-traded goods are perfect complements required for the discovery of new reserves.

For domestic (household) demand, we assume that domestic households have consumption habits in the spirit of 'keeping up with the Jones' (Abel 1990). We include this mechanism to allow for a non-unitary intertemporal elasticity of substitution (IES), which is important for matching the empirical data, while keeping the model tractable when finding its stationary representation.[19] We further assume that households view work in the resource and non-resource sectors as imperfect substitutes, which is captured through a constant elasticity of substitution (CES) function.

Although we assume complete insurance among identical individual households, allowing for the modelling device of a representative household, we assume that international financial markets are incomplete in the spirit of Benigno and Thoenissen (2008). Specifically, households can trade in either a domestic bond or a foreign bond, where the latter is subject to an endogenous risk premium.[20] This premium is governed by both the domestic economy's capacity to repay foreign debt (measured as the stock of foreign assets in domestic currency terms scaled by the stock of domestic reserves) and the relative valuation differential between the the real exchange rate and resource prices.

We include the relative valuation differential to capture the idea that changes in the real exchange rate and resource prices can have direct effects on risk premia. For example, higher resource prices and an appreciated real exchange could affect the ability to repay foreign liabilities, even with the value of these foreign liabilities (in domestic currency terms) remaining unchanged. The size of this effect is estimated and is important when matching the dynamics of the real exchange rate in response to a resource price shock (see Table 3 and Figure 5).

Our estimates suggest that a 1 per cent increase in real resource prices (or a 1 per cent appreciation in the real exchange rate) reduces the foreign risk premium by about 25 basis points after one year. This is consistent with higher resource prices increasing domestic wealth, and so the capacity to repay existing and new debt obligations. A real appreciation of the same magnitude could also imply greater capacity to repay as an appreciated real exchange rate implies that a unit

19 Note that alternatives, such as constant relative risk aversion without a habit, substantially complicate detrending of the model even though they too allow for a non-unitary IES.

20 We assume that the domestic bond is in zero net supply in equilibrium.

of domestic goods is now worth more in foreign currency terms and could, at least in principle, be pledged as greater collateral when borrowing from abroad.

For the rest of the world – defined as the foreign price level (in foreign currency terms), non-resource demand, foreign interest rates, and the price of imported resource-specific capital (again in foreign currency terms) – we assume a reduced-form VAR.[21] This is a simplification allowing us to focus on the effects of a resource price shock, holding all other international prices and quantities constant. Although we acknowledge that the source of foreign structural shocks can be important, we view this as an extension of our work given that our first-order interest is in studying the mechanism of interest, endogenous reserves, in a transparent way.[22]

We assume resource prices follow an AR(1) process, consistent with previous literature that assumes exogenous reserves. All markets clear in our economy and we assume that domestic monetary policy follows a simple Taylor rule, allowing for both interest-rate smoothing and a response to expected domestic inflation.

Overall, our approach is quite similar to existing small open economy (SOE) models such as that described in Adolfson *et al* (2007) and Jääskelä and Nimark (2008). We use a minimal level of structure to ensure that our economy is able to reproduce some basic empirical regularities such as the existence of non-tradeable production, non-resource export activity, incomplete pass-through, and a time-varying link between the marginal utility of domestic consumption and the real exchange rate. This minimal level of structure retains tractability and allows us to focus on the mechanism of interest, endogenous reserves.

4.1 Calibration

Most parameters in our general equilibrium model are calibrated. We use the same calibration for the resource sector parameters presented in the upper panel

21 The only restriction that we impose on this foreign VAR is that foreign demand for non-resource exports is cointegrated with the domestic stock of reserves. This is a technical device used to ensure that a stationary representation of our economy can be found. For further discussion on this point, see Appendix D.7.

22 Although foreign structure is interesting, it would substantially complicate interpretation of the mechanism given that all foreign prices and quantities would move simultaneously when resource prices change.

of Table 1. For the rest of the domestic economy, our calibrated parameters are chosen to be in line with the results in Jääskelä and Nimark (2008). These authors estimate a model with a relatively similar production structure to ours and Adolfson *et al* (2007) using Australian data, but with a simple reduced-form for the resource (commodity) sector.[23]

The parameters we choose are adjusted to match an annual time horizon and are summarised in Table 2. We assume identical elasticities of substitution within the non-traded goods, importing and non-resource export sectors, each consistent with a mark-up of approximately 17 per cent. We further assume identical price stickiness parameters, each implying a 20 per cent probability that a firm cannot re-optimise its price within a year's time.

We choose the home bias parameter to match a 20 per cent import share in steady state, and an elasticity of substitution between consumption of non-traded goods and imports that is close to one (Cobb-Douglas consumption preferences). We set the elasticity of substitution between resource and non-resource labour supply at 1, and fix the overall convexity parameter of labour disutility at 4. These assumptions imply that labour is relatively substitutable between sectors, but that households are averse to increasing their overall supply of labour to the economy.

Table 2: Calibration of Non-resource Economy

Description	Coefficient	Value
Household discount factor	β	0.96
Labour convexity	ξ_h	4
Labour substitution parameter	γ_h	0.5
Consumption substitution elasticity	η_c	1.01
Home-bias coefficient	$1-\alpha$	0.8
Substitution elasticity (within non-traded goods)	θ_n	7
Substitution elasticity (within imports)	θ_o	7
Substitution elasticity (within non-resource exports)	θ_x	7
Substitution elasticity (across non-resource exports)	θ_*	1
Calvo parameter (non-traded goods)	ϕ_n	0.2
Calvo parameter (imports)	ϕ_o	0.2
Calvo parameter (non-resource exports)	ϕ_x	0.2

23 The main exception is for the calibration of the elasticity of substitution on imported goods. The estimate implied in Jääskelä and Nimark (2008) implies a very large mark-up on imported goods. We abstract from concern over whether this parameter is well identified and simply fix the implied mark-ups on domestically produced and imported goods to be identical.

4.2 Estimation

The remaining parameters of the model are estimated using a GMM procedure that matches the IRFs of the empirical VAR, discussed in Section 2, and the IRFs of our theoretical model in general equilibrium with endogenous reserves. Specifically, we minimise the following measure of distance

$$\widehat{\theta} = \arg\min_{\theta} \sum_{j=1}^{5} \sum_{l=1}^{5} \left(g_{jl}^{Model}(\theta) - g_{jl}^{VAR} \right) W_{jl} \left(g_{jl}^{Model}(\theta) - g_{jl}^{VAR} \right)'$$

where θ is a vector of the parameters to be estimated, j relates to the observable variable being matched (either resource prices, the real exchange rate, inflation, the ratio of non-mining GDP to reserves, or the ratio of mining capital expenditure to reserves), l denotes the time horizon from the initial impulse (one being the period in which the resource price shock occurs), $g_{jl}^{Model}(\theta)$ is the IRF implied by our theoretical model evaluated at θ, g_{jl}^{VAR} is the estimated IRF from the VAR, and W_{jl} is a diagonal matrix that weights the deviations between the theoretical model and the VAR IRFs by the width of the 95 per cent confidence interval at each IRF point (as estimated using the VAR).

The results of this estimation procedure are reported in Table 3 and the fit of the best matching model is reported in Figure 2. The importance of domestic ownership for resource firms' stochastic discount factor is estimated at 0.35, which is similar to estimates of domestic ownership in the resource sector (see, for example, Connolly and Orsmond (2011)). The estimated coefficient of relative risk aversion is high at 10, although it is in line with the values required to rationalise the equity premium puzzle (see, for example, Mehra and Prescott (1985) and Constantinides (1990)).[24]

The elasticity of the foreign risk premium with respect to debt scaled by domestic reserves appears large but this represents the effect of scaling. When considered on the metric of the induced percentage point movement in the foreign risk premium, this parameter appears plausible (see Figure 5). Consistent with the IRFs obtained

24 In the current context, a high relative risk aversion coefficient is required to limit the sensitivity of consumption to a resource price shock. All else constant, a lower value for this coefficient implies that consumption becomes too volatile.

from the VAR, resource prices are estimated to follow a very persistent process with an autoregressive parameter of 0.9.

Interestingly, the data favour a model where non-traded firms' marginal costs respond directly to changes in resource prices, $\Upsilon = 0.33$, and so there appears to be some input-cost inflation reflecting the correlation between energy and resource prices (for further discussion, see Appendix D). Estimates for the parameters regarding investment adjustment costs and exploration costs appear plausible, with the latter suggesting exploration costs increase at a faster rate than discovered reserves.

Table 3: Parameters Estimated via GMM

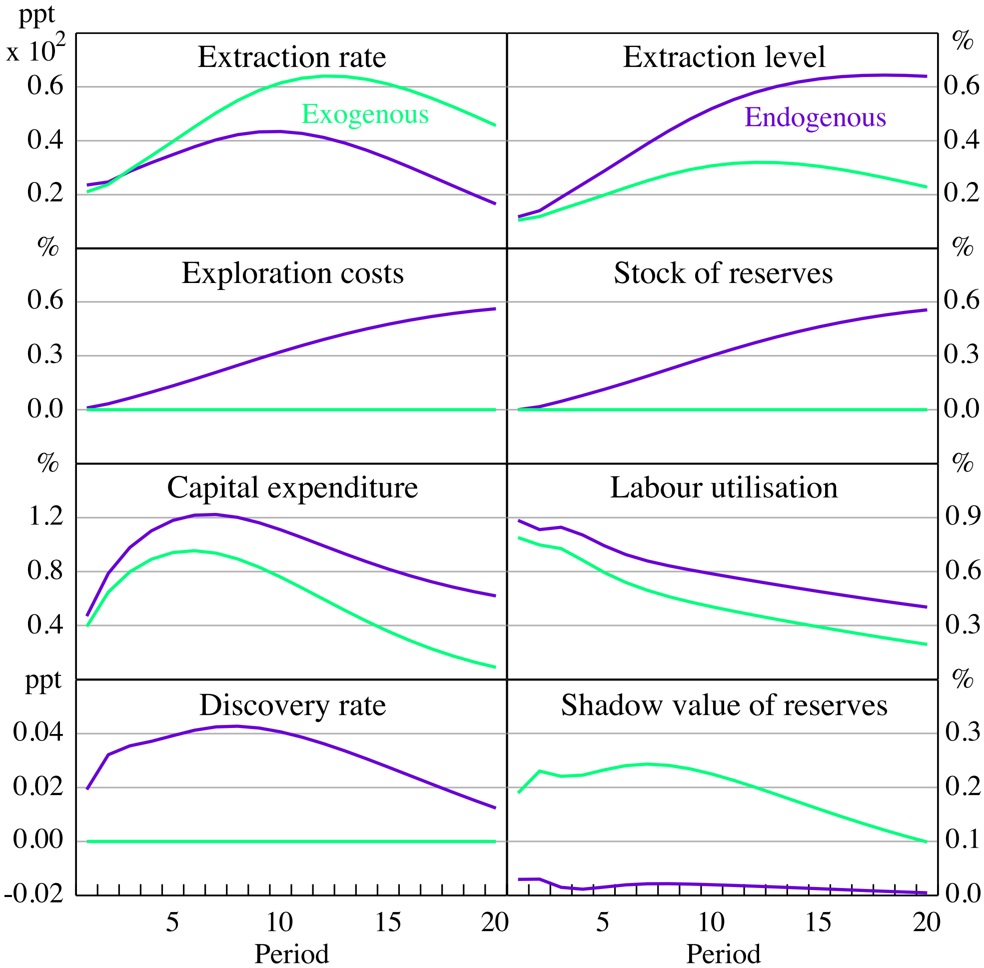
Description	Coefficient	Value
Risk aversion coefficient	ξ_c	10
Domestic ownership parameter	ν	0.35
Risk premium (repayment capacity)	φ_b	200
Risk premium (valuation dynamics)	φ_s	0.25
Exploration costs dynamics	ϕ_{mc}	0.5
Investment cost parameter	κ	3
Responsiveness parameter (marginal costs)	Υ	0.33
AR(1) parameter (prices)	ρ_x	0.9
Interest rate smoothing paramerter	ρ_i	0.2
Taylor rule parameter (inflation)	ρ_π	5

4.3 Results

Figure 4 shows the response of the resource sector to a 1 per cent increase in resource prices in general equilibrium and compares the models with exogenous reserves and endogenous reserves.[25] The first point to note is that the amplification effects associated with the inclusion of endogenous reserves remain in general equilibrium. A persistent increase in resource prices prompts firm to increase both exploration and extraction, as the marginal returns to production and the value of new reserves remain high for a period. When exploration results in the discovery of new reserves, this gives firms an additional incentive to extract more now and in future periods – leading to greater demand for labour and capital – as marginal production costs fall.

25 It should be clear that the IRFs measure changes relative to the baseline of a steady state or balanced growth path.

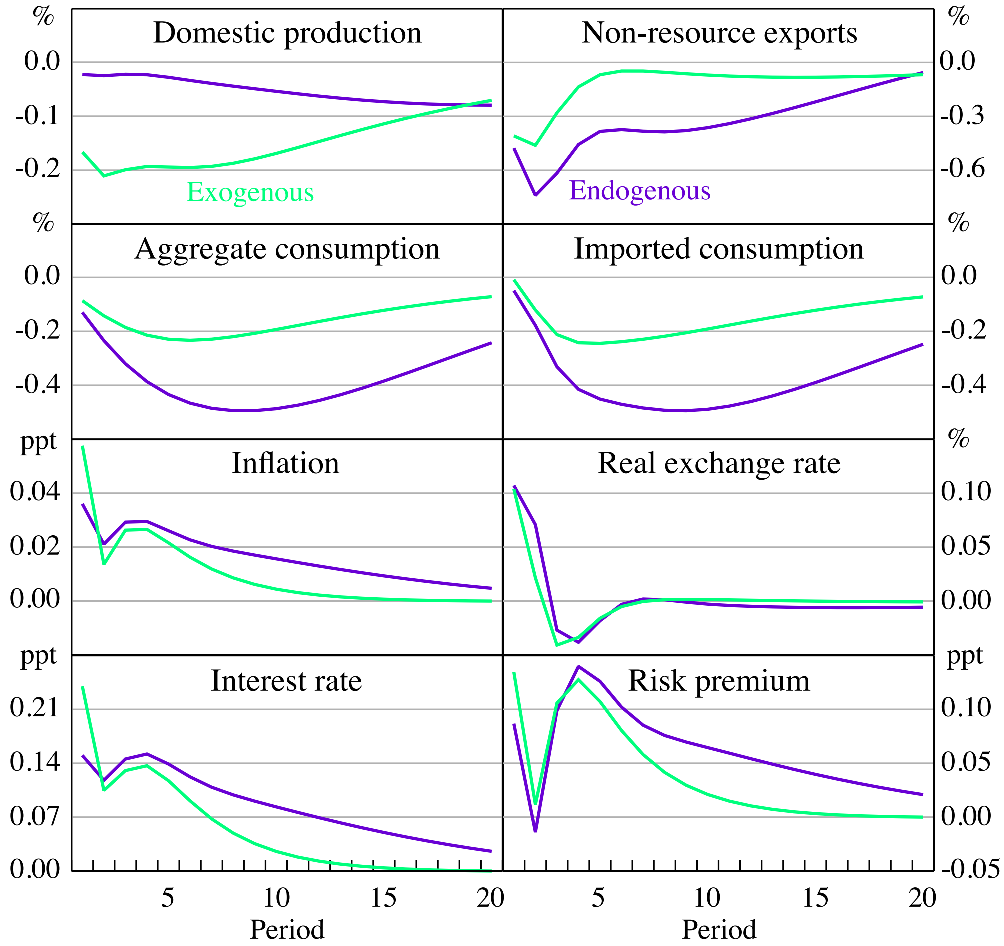
Figure 4: Resource Sector Response to a 1 Per Cent Increase in Resource Prices in General Equilibrium



Nevertheless, it is also clear that the degree of amplification attributable to endogenous reserves is smaller than in the partial equilibrium case. This occurs because the appreciation of the real exchange rate offsets part of the increase in the value of resource export receipts. Also, greater demand for labour induces an increase in wages paid in the sector and the prices of non-traded inputs rise in general equilibrium. These effects increase the costs of expansion in the resource sector, both in terms of production and exploration, and so the divergence between the IRFs in the endogenous and exogenous models, while still economically significant, are smaller than indicated by the previous partial equilibrium results.

The amplification effects of a resource price shock are also present in domestic activity (Figure 5). Comparing the responses with endogenous and exogenous reserves respectively, one can see that the declines in consumption (both aggregate and imported) and non-resource exports, relative to the steady state baseline, are larger with endogenous reserves. This is because the expansion in the resource sector absorbs a greater fraction of domestic intermediate inputs, and is consistent with the rise in expected real interest rates required to stabilise inflation. In terms of the effect on total domestic production, we find that this is close to zero because the declines in consumption and non-resource export demand are almost fully offset by the expansion of demand from the resource sector. Thus, although sectoral reallocation (the Dutch Disease) is amplified under endogenous reserves, it does not have significant implications for domestic production overall.

Figure 5: **Resource Sector Response to a 1 Per Cent Increase in Resource Prices in General Equilibrium – Rest of the Domestic Economy**



With exogenous reserves, there is no longer an additional income or aggregate demand effect associated with the discovery of new reserves. This is reflected in a smaller expansion of the resource sector, and a noticeable decline in domestic production in response to a resource price shock. In this case, although there are smaller falls in consumption and non-resource export demand (that is, less Dutch

Disease effects), the smaller expansion of the resource sector is not sufficient to offset the declines in demand from these other areas of activity. In sum, there is less sectoral reallocation of goods, but also less domestic production overall.

For domestic inflation, the domestic interest rate, and the foreign risk premium, the inclusion of endogenous reserves increases the time it takes for prices to converge back to their steady state path (i.e. the persistence of the IRFs), but does not amplify their effect when the shock first arrives (Figure 5). If anything the contemporaneous effects on these prices appear smaller when reserves are endogenous. For the real exchange rate, the propagation of the shock is largely unaltered with an initial appreciation of the real exchange rate followed by a small subsequent depreciation.[26]

5. Robustness

We now consider the robustness of our findings to some of our identifying assumptions.

5.1 Can a VAR Recover the Structural Responses?

As discussed in Section 4.2, a subset of the model parameters are estimated using a GMM procedure that matches the IRFs obtained from our theoretical model with those of the VAR discussed in Section 2. An important question is whether the estimated VAR is consistent with our theoretical model.[27] To address this question, we simulate data from the general equilibrium SOE model with endogenous reserves, and estimate a VAR on the simulated data using the same specification as that used

26 The responses for the real exchange rate are inverted so that an appreciation in the figure is a movement upwards.

27 There has been some debate on the ability of VARs (or more precisely structural VARs) to recover structural shocks. See, for example, the discussion in Christiano, Eichenbaum and Vigfusson (2007) and Chari, Kehoe and McGrattan (2008).

in Section 2.[28] We then compare the difference between the model-theoretic IRFs, and those of the VAR estimated on simulated data, to understand whether the VAR is able to identify the model-theoretic effect of a resource price shock.

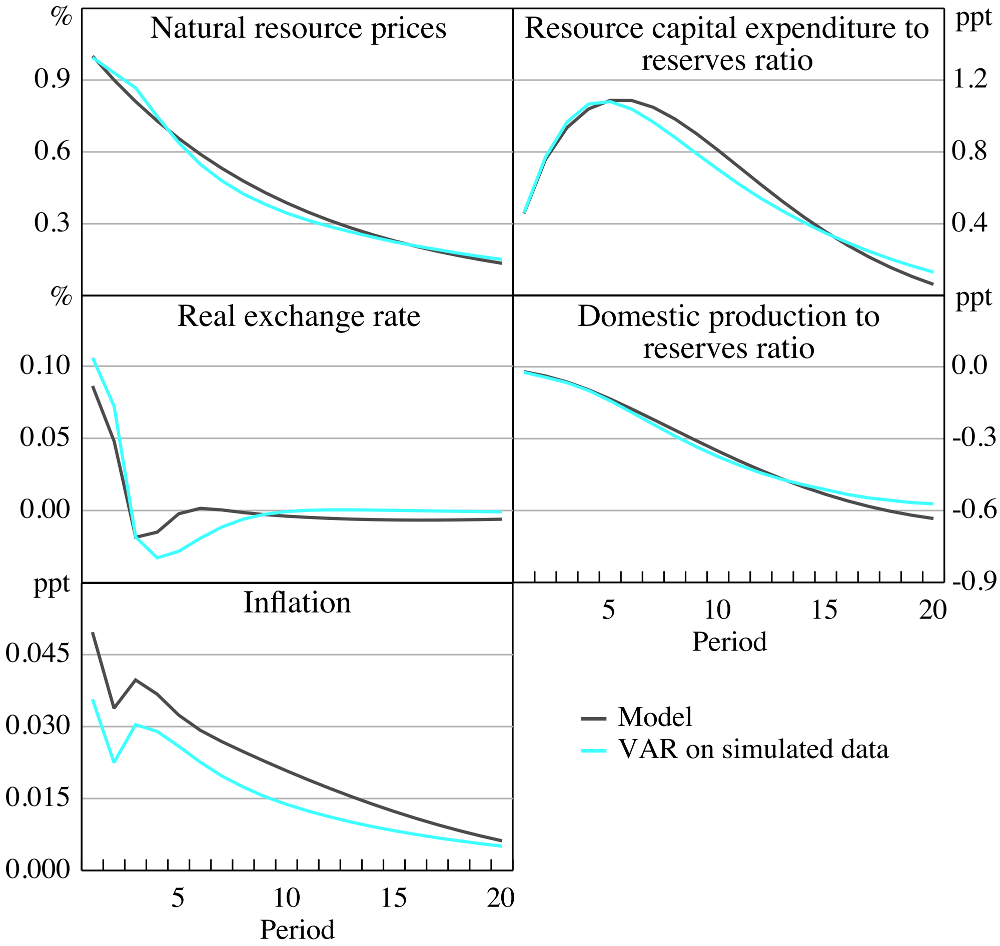
The comparison between the model-theoretic IRFs for the small open economy with endogenous reserves, and the IRFs obtained from the VAR estimated on simulated data are reported in Figure 6. The relatively small discrepancy between the IRFs confirms that the specification of the VAR in Section 2 can recover the model-theoretic effects of a shock to resource prices asymptotically. This suggests that our identification strategy is locally valid around the parameter vector used in our simulation.

The reason that the VAR specification is able to reproduce the model-theoretic IRFs is the inclusion of reserves, a key observable state variable in our model. If we did not use reserves to deflate both non-resource production and resource capital expenditure, a VAR(1) would not be able to recover the model-theoretic IRFs. Our findings can be interpreted as numerically checking the conditions for identification that are discussed more generally in Ravenna (2007).[29]

28 More precisely, we simulate the model for 1 000 periods at the parameter vector identified in Tables 1, 2 and 3 assuming equal standard deviations (at 0.01) for shocks to resource prices, non-mining technology, foreign demand for non-resource exports, the foreign price level, and the price level for investment goods that are uncorrelated. We then estimate a VAR(1) on the simulated data using resource prices, the real exchange rate, the ratio of domestic production to reserves, the ratio of resource investment to reserves and consumer price inflation in that order and including a constant but no deterministic trend. Consistent with Section 2, we assume that resource prices are contemporaneously uncorrelated with the other variables in the system and use this to identify the effects of a resource price shock.

29 To be clear, asymptotic identification should not be expected *a priori*. In general, the omission of state variables from the model, for example mining capital which is not directly observed, may imply that a finite VAR representation may not exist for the set of observables used in estimation (for further discussion, see Ravenna (2007)).

Figure 6: A 1 Per Cent Innovation in Resource Prices – Model-theoretic IRF and IRF from VAR on Simulated Data



5.2 Imposing Additional Restrictions on the VAR

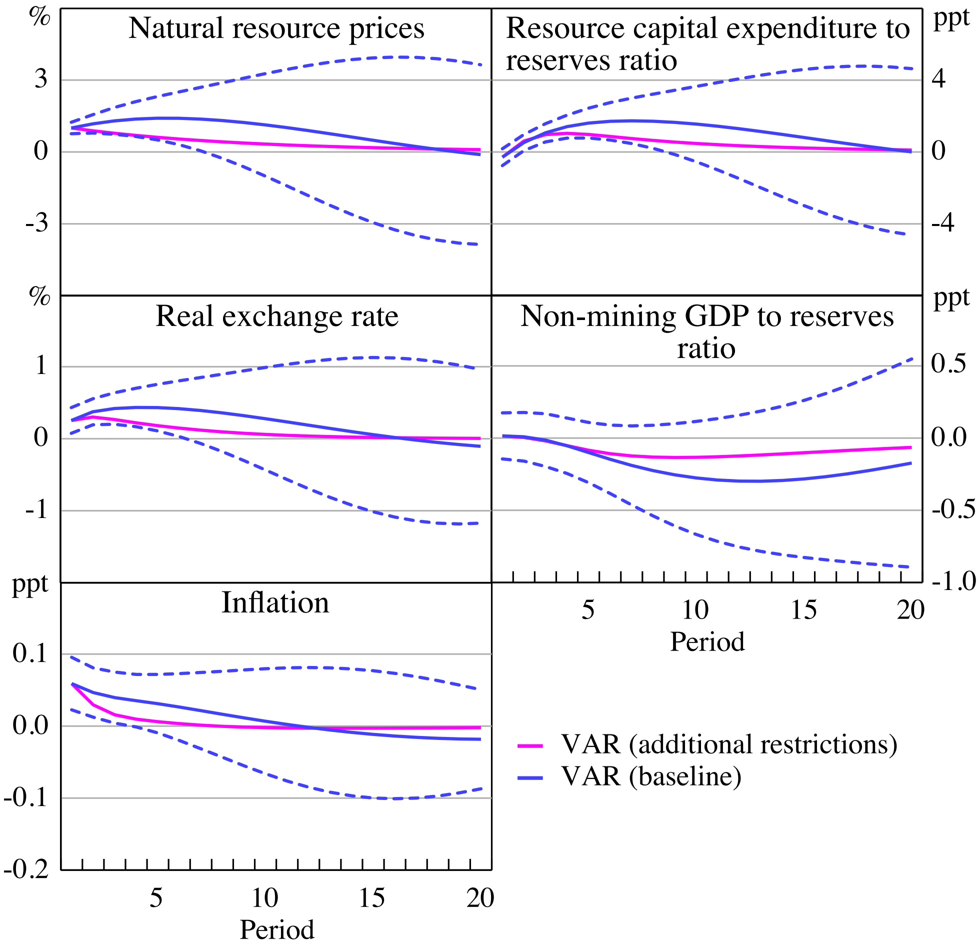
A second identification question relates to the assumption that resource prices are contemporaneously uncorrelated with domestic variables when identifying the IRFs in the VAR, but are assumed to be a statistically independent AR(1) process in the theoretical model. Revisiting the VAR discussed in Section 2,

$$\mathbf{A}_0\mathbf{z}_t = \Gamma\mathbf{z}_{t-1} + \mathbf{e}_t$$

the VAR only assumes that A_0 has ones along its main diagonal and zeros on the off diagonal elements for the first row. The theoretical model makes the same assumptions, and in addition assumes that the off-diagonal elements of the first row of Γ are also zero, implying that resource prices are an independent AR(1) process. To check whether this is important, we estimate a VAR imposing the additional restrictions on Γ. Specifically, we estimate an AR(1) for resource prices using long-run annual data, from 1900 to 2011 (making use of all available information on resource prices), and then estimate the remaining equations of the VAR using limited information methods on the sample from 1976 to 2011, for which data on annual reserves are available.

Comparing the IRFs for this alternative VAR, with the VAR used in Section 2, the IRFs in response to a resource price shock are qualitatively similar (Figure 7). Although imposing the extra restrictions reduces the amplification of the IRF functions a little, since shocks to resource prices are now estimated to be smaller and less persistent, for the purposes of estimation – that is, matching the theoretical model to either of these VARs – the results are similar and the qualitative implications of our mechanism remain the same.

Figure 7: A 1 Per Cent Innovation in Resource Prices – Baseline VAR and a VAR with Additional Restrictions on Γ



6. Conclusion

This paper examines whether the assumption of an exogenous stock of natural resources is innocuous in the context of a small open economy model. Our findings suggest that the standard exogenous reserves approximation is reasonable for quantifying the effects of a commodity or resource price shock on key prices of interest including the real exchange rate, consumer prices and the domestic interest rate.

However, our results also imply that the standard approach is likely to underestimate the effects of a resource price shock on the resource sector itself, with larger expansions in investment, labour utilisation and production occurring when reserves are responsive to exploration activity. Consistent with this, we also find that the effects of a resource price shock on the domestic allocation of productive inputs across sectors are larger under the assumption of endogenous reserves. This is because the resource sector absorbs domestic inputs into production and so consumption and non-resource export production both grow by less (relative to the baseline of a balanced growth path). The net effect on domestic production with endogenous reserves is, nevertheless, small.

Appendix A: Data Sources

Natural resource prices

For the 1976–2011 sample used in our main analysis (Sections 2 to 4), natural resource prices are an export-weighted geometric mean of iron ore, coal, gold and average base metal prices. Average base metal prices reflect an equally weighted geometric mean of aluminium, zinc, copper, lead and nickel prices.

All prices are measured in real terms (deflated by the US GDP deflator). Prices data are sourced from ABARES, Bloomberg, Global Financial Data, IMF, Pfaffenzeller, Newbold and Rayner (2007), RBA and USGS. The export weights used are the 1976–2011 sample averages, which are derived from commodity export shares data calculated by Gillitzer and Kearns (2005).

For the 1900–2011 sample used in Section 5.2, we use an equally weighted geometric mean of real aluminum, zinc, copper, lead and nickel prices.[30]

All data discussed below are only constructed over the 1976–2011 sample.

Natural resource reserves

Natural resource reserves are an equally weighted geometric mean of reserves for five base metals (aluminium, zinc, copper, lead and nickel), gold, iron ore and coal.[31] These data are sourced from Geosciences Australia (GA). ABS data are used to construct the 2011 estimate.[32]

30 We use this simpler proxy for long-run real resource prices over this time frame because there is considerable variation in the export shares of iron ore, coal and gold. Our results are similar when using an equally weighted geometrically weighted average of the real prices of the same five base metals, iron ore and coal.

31 Using an export-weighted average, based on the same export shares as used for the prices data, led to similar results.

32 Prior to 1992 data, all measures are based on economically demonstrated reserves only. From 1992 onwards, the measures include economically demonstrated, sub- and para-marginal reserves.

Non-mining GDP

Non-mining GDP is sourced from the ABS and RBA. We use non-farm GDP in chain volume terms, ABS Catalogue No 5206.0, Table 41, less an estimate of mining GDP in chain volume terms. The latter is derived from chain volume estimates of mining investment (ABS Catalogue No 5204.0, Table 64) and resource exports (derived from ABS Catalogue No 5302.0, Table 11). The measure calculated is similar to estimates produced by Rayner and Bishop (2013).

Resource-specific investment

These data are sourced from the ABS Catalogue No 5204.0, Table 64, gross fixed capital formation by industry, by asset. We compute resource-specific investment as the sum of investment in non-dwelling construction and machinery & equipment in the mining sector.

Real exchange rate

We use the real trade-weighted index as sourced from the RBA, Statistical Table F15 Real Exchange Rates Measures.

Inflation

We use a measure of underlying inflation. It is derived from quarterly data on the CPI excluding interest and health policy changes prior to September 1993, the Treasury underlying measure of inflation between September 1993 and September 1998, and the headline CPI excluding interest and tax since September 1998.

Appendix B: Discussion of the Firms' Resource Problem

One important distinction between the dynamic program we describe in Section 3 and the approach described in Bohn and Deacon (2000) is the absence of a finite upper bound on the cumulative level of resources that can be discovered. This abstraction is important for our analysis since it implies that the policy functions that solve our problem are time invariant and admit a stationary (detrended) non-stochastic steady state. Intuitively, our approach implies that firm decisions concerning investment, exploration and production are not substantively affected by the existence of a known finite level of reserves yet to be discovered. This appears to be a reasonable assumption, at least in the Australian context. It is a different problem, however, from that in which a natural resource firm simply chooses its allocations of labour, capital and production to optimally extract from a pre-defined resource stock over time (whether exploration is required or not).

Although we abstract from a known finite bound on the remaining stock of undiscovered reserves, we do not entirely abstract from the concept of resource scarcity. As an alternative, we assume that the costs associated with exploration activity are increasing in the quantity of previously accumulated aggregate reserves. Specifically, we assume that the cost function is increasing in both exploration and aggregate reserves, $\frac{\partial C}{\partial D_t} > 0, \frac{\partial C}{\partial \widetilde{R}_t} > 0$; that the derivative of the marginal cost of exploration is increasing in the level of exploration, $\frac{\partial^2 C}{\partial D_t^2} > 0$; and that this same derivative is sufficiently large that it outweighs any reduction in the marginal costs of exploration that could be associated with greater existing reserves $\frac{\partial^2 C}{\partial D_t^2} + \frac{\partial^2 C}{\partial D_t \partial \widetilde{R}_t} > 0$. Finally, we assume that exploration costs tend to infinity as the stock of accumulated reserves becomes large, $\lim_{\widetilde{R}_t \to \infty} C\left(D_t, \widetilde{R}_t\right) = \infty$.

Together, these assumptions are consistent with many of the approaches adopted in the natural resource literature including Pindyck (1978), Reiss (1990), Heal (1993) and Sweeney (1993). Although this literature covers a wider range of cases, and the quantitative implications may well be different depending on the precise structure used, our own view is that the above assumptions capture the essence of a resource firms' problem. An appealing feature of our approach, and indeed our main motivation, is that it can be directly integrated into a SOE DSGE model. This is important as it allows us to study how endogenous reserves affect the propagation of resource price shocks in general equilibrium.

Appendix C: Analysis of the Partial Equilibrium Model

Proposition 1. *In the absence of investment adjustment costs, the equilibrium law of motion for reserves*

$$R_{t+1} = R_t + \omega_{t+1} D\left(W_r^r, S_t, P_t^*, P_t^{r^*}, R_t, K_t\right) - X\left(W_r^r, S_t, P_t^*, P_t^{r^*}, R_t, K_t\right)$$

can be rewritten as

$$\ln R_{t+1} = \ln R_t + \chi_t^R$$

where

$$\begin{aligned}
\chi_t^R &\equiv \ln\left(1 + \omega_{t+1} d\left(W_t^m, S_t, P_t^*, P_t^{r^*}, k_t\right) - x\left(W_t^m, S_t, P_t^*, P_t^{r^*}, k_t\right)\right) \\
d_t &\equiv \frac{D_t}{R_t} = d\left(W_t^m, S_t, P_t^*, P_t^{r^*}, k_t\right) \\
x_t &\equiv \frac{X_t}{R_t} = x\left(W_t^m, S_t, P_t^*, P_t^{r^*}, k_t\right) \\
k_t &\equiv \frac{K_t}{R_t}
\end{aligned}$$

Proof. In the absence of investment adjustment costs the firms' decision problem is given by

$$V\left(K_t, R_t\right) = \max_{I_t(j), D_t(j), X_t(j)} \left\{ \left[\begin{array}{c} S_t P_t^{r^*} X_t - W_r^r \left(X_t^{1+\zeta} K_t^{-\mu} R_t^{\mu-\zeta}\right) \\ -S_t P_t^* I_t - C\left(D_t, \widetilde{R}_t\right) \end{array} \right] + \beta \int V\left(K_{t+1}, R_{t+1}\right) d\Phi\left(\xi_{t+1} \mid \xi_t\right) \right\}$$

subject to:

$$K_{t+1} = (1-\delta) K_t + I_t$$
$$R_{t+1} = R_t + \omega_{t+1} D_t - X_t$$

The associated first-order conditions at an interior solution are

$$
\begin{aligned}
S_t P_t^{r^*} &= (1+\zeta) W_r^r x_t^{\zeta} k_t^{-\mu} + Q_t^R \\
S_t P_t^* &= Q_t^K \\
\frac{\partial C}{\partial D_t} &= Q_t^R \\
k_{t+1} r_t &= (1-\delta) k_t + i_t \\
r_t &= 1 + \omega_{t+1} d_t - x_t \\
Q_t^R &= \beta E_t \left((\zeta - \mu) W_{t+1}^m x_{t+1}^{1+\zeta} k_{t+1}^{-\mu} + Q_{t+1}^R \right) \\
Q_t^K &= \beta E_t \left(\mu W_r^r x_t^{1+\zeta} k_t^{-\mu-1} + Q_{t+1}^K (1-\delta) \right)
\end{aligned}
$$

Note that the solutions to x_t, d_t, Q_t^R, Q_t^K, k_{t+1} can be solved from the simplified system

$$
\begin{aligned}
S_t P_t^{r^*} &= (1+\zeta) W_r^r x_t^{\zeta} k_t^{-\mu} + Q_t^R \\
S_t P_t^* &= Q_t^K \\
\frac{\partial C}{\partial D_t} &= Q_t^R \\
Q_t^R &= \beta E_t \left((\zeta - \mu) W_{t+1}^m x_{t+1}^{1+\zeta} k_{t+1}^{-\mu} + Q_{t+1}^R \right) \\
Q_t^K &= \beta E_t \left(\mu W_{t+1}^m x_{t+1}^{1+\zeta} k_{t+1}^{-\mu-1} + Q_{t+1}^K (1-\delta) \right)
\end{aligned}
$$

yielding the policy functions

$$
\begin{aligned}
x_t &= x \left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t \right) \\
d_t &= d \left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t \right) \\
Q_t^R &= Q^R \left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t \right) \\
Q_t^K &= Q^K \left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t \right) \\
k_{t+1} &= k \left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t \right)
\end{aligned}
$$

Noting that these policy functions do not include R_t in their argument, it follows that the law of motion for log reserves in equilibrium will be given by

$$\ln R_{t+1} = \ln R_t + \chi_t^R$$

where

$$\chi_t^R \equiv \ln\left(1 + \omega_{t+1} d\left(W_t^m, S_t, P_t^*, P_t^{r^*}, k_t\right) - x\left(W_t^m, S_t, P_t^*, P_t^{r^*}, k_t\right)\right)$$

is not a function of reserves, and so log reserves will have a unit root in equilibrium.

Proposition 2. *Assume the firm solves the following dynamic program*

$$V\left(K_t, R_t\right) = \max_{I_t(j), D_t(j), X_t(j)} \left\{ \begin{bmatrix} S_t P_t^{r^*} X_t - W_r^r\left(X_t^{1+\zeta} K_t^{-\mu} R_t^{\mu-\zeta}\right) \\ -S_t P_t^* I_t - C\left(D_t, R_t\right) \end{bmatrix} + \beta \int V\left(K_{t+1}, R_{t+1}\right) d\Phi\left(\xi_{t+1} \mid \xi_t\right) \right\}$$

subject to:

$$K_{t+1} = (1-\delta) K_t + \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

$$R_{t+1} = R_t + \omega_{t+1} D_t - X_t$$

and where $C\left(D_t, R_t\right)$ is homogeonous of degree one. In this case the equlibrium law of motion for log reserves can be also written as

$$\ln R_{t+1} = \ln R_t + \chi_t^R$$

where χ_t^R is defined in Proposition 1.

Proof. This proof follows noting that the profit function is homogenous of degree one when $C\left(D_t, R_t\right)$ is homogeonous of degree one. Since the constraints are also linearly homogeonous, the value function itself is linear homogenous and the associated policy functions are homogenous of degree one in the state variables

K_t and R_t (see Stokey and Lucas (1989, Section 9.3) for a formal treatment). This implies the policy functions can be written as

$$x_t = x\left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t\right)$$
$$d_t = d\left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t\right)$$
$$i_t = i\left(W_r^r, S_t, P_t^*, P_t^{r^*}, k_t\right)$$

and so the log-level of reserves in equilibrium will be given by

$$\ln R_{t+1} = \ln R_t + \ln\left(1 + \omega_{t+1} d_t - x_t\right)$$

where d_t and x_t are not direct functions of R_t in the solution to the above program.

Appendix D: The Small Open Economy in General Equilibrium

D.1 Domestic Households

We assume a continuum of identical domestic households of unit measure who are able to self-insure with each other, and so the problem we describe is isomorphic to a model with a representative agent. Each household has identical preferences given by the utility function

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{\left(\frac{C_t}{V_t}\right)^{1-\xi_c}}{1-\xi_c} + \varsigma \left(H_t^{n\frac{1}{\gamma_h}} + H_t^{r\frac{1}{\gamma_h}} \right)^{\xi_h \gamma_h} \right) \tag{D1}$$

where C_t is an aggregate consumption bundle containing domestic and imported goods, V_t is an external habit, H_t^n and H_t^r are the households' supply of labour to the non-resource and resource sectors respectively, ξ_c is the coefficient of relative risk aversion, ξ_h is a parameter governing the convexity of preferences with regard to the aggregate supply of labour, γ_h governs the elasticity of substitution between labour supplied in the resource and non-resource sectors, and ς is a scaling parameter used to obtain a well-defined steady state.

We include an external habit or 'catching up with the Jones' ($V_t = \int_0^1 C_{t-1}(j)\,dj$, see Abel (1990)) because it permits a more flexible representation of consumption preferences. In particular, it allows for a non-unitary intertemporal elasticity of substitution while still being consistent with a detrended stationary representation of the general equilibrium economy. We use a constant elasticity of substitution function for the disutlity of labour to capture the idea that working in the resource and non-resource sectors are not perfect substitutes, from the perspective of households, and so there can be relative wage dispersion between the resource and non-resource sectors.

Regarding financial markets, domestic households can trade in either of two nominal risk-less bonds denominated in the domestic, B_t, and foreign currencies, B_t^*, respectively. Following Benigno and Thoenissen (2008), we assume that when domestic residents issue claims in foreign currency they must pay a premium on this borrowing, Φ_t. The household budget constraint for an individual household is given by

$$
\begin{aligned}
P_t^c C_t + \frac{B_t}{(1+i_t)} + \frac{S_t B_t^*}{(1+i_t^*)\Phi_t} = {} & W_t^n H_t^n + W_t^r H_t^r \\
& + B_{t-1} + S_t B_{t-1}^* \\
& + \nu \Psi_t^r + \Psi_t^n + \Psi_t^o + \Psi_t^x - T
\end{aligned}
$$

where Ψ_t^r, Ψ_t^n, Ψ_t^o, Ψ_t^x are the aggregate profits (dividends) of the resource, non-resource, import and non-resource export goods sectors paid to household respectively, T is a lump-sum tax used to fund subsidies that undo the steady state distortions associated with monopolistic competition (discussed further below), and i_t and i_t^* are the domestic and foreign borrowing interest rates (the latter being measured net of the risk premium). Assuming that domestic bonds are in zero net supply, it follows that in equilibrium

$$\Theta_t = \frac{1}{C_t}\left(\frac{C_t}{V_t}\right)^{1-\xi_c} \tag{D2}$$

$$\Theta_t = \beta E_t\left((1+i_t)\frac{P_t^c}{P_{t+1}^c}\Theta_{t+1}\right) \tag{D3}$$

$$\Theta_t = \beta E_t\left(\left(1+i_t^*\right)\Phi_t \frac{P_t^c}{P_{t+1}^c}\Theta_{t+1}\frac{S_{t+1}}{S_t}\right) \tag{D4}$$

$$\frac{W_t^n}{P_t^c}\Theta_t = \varsigma H_t^{n\,\frac{1}{\gamma_h}-1}\left(H_t^{n\,\frac{1}{\gamma_h}} + H_t^{r\,\frac{1}{\gamma_h}}\right)^{\gamma_h-1} \tag{D5}$$

$$\frac{W_t^r}{P_t^c}\Theta_t = \varsigma H_t^{r\,\frac{1}{\gamma_h}-1}\left(H_t^{n\,\frac{1}{\gamma_h}} + H_t^{r\,\frac{1}{\gamma_h}}\right)^{\gamma_h-1} \tag{D6}$$

$$
\begin{aligned}
P_t^c C_t = {} & W_t^n H_t^n + W_t^r H_t^r + S_t B_{t-1}^* - \frac{S_t B_t^*}{(1+i_t^*)\Phi_t} \\
& + \nu \Psi_t^r + \Psi_t^n + \Psi_t^o + \Psi_t^x - T
\end{aligned} \tag{D7}
$$

Equation (D2) is the marginal utility of consumption. Equations (D3) and (D4) are the standard Euler equations associated with ability to trade in domestic and foreign currency-denominated bonds and imply that the return from saving (or cost of borrowing) should be equal to the forgone (additional) consumption enjoyed in the current period. Equations (D5) and (D6) are the standard intra-temporal conditions ensuring that the marginal return from working in each sector is equivalent to the households' marginal disutility from working, and Equation (D7) implies that the household budget constraint will bind.

We assume that the risk-premium on foreign borrowing is described by the following relationship

$$\Phi_t \equiv \exp\left(\left(\frac{S_t}{P_t^c}\frac{B_t^*}{R_t} - \frac{S}{P^c}\frac{B^*}{R}\right)^{-\varphi_b}\left(\ln\frac{S_{t-1}}{P_{t-1}^c} - \ln\frac{P_{t-1}^r}{P^r}\right)^{\varphi_s}\right)$$

That is, we assume that the risk premium is a function of both the domestic economy's capacity to repay its debt, and the percentage deviation between the real exchange rate and resource prices. We include the latter term to capture the idea that changes in resource prices and the real exchange rate can have direct effects on risk premia. The parameters φ_b and φ_s govern the relative importance of each of these effects and are estimated.

For their intra-temporal consumption decisions (choosing domestic and imported consumption good expenditure), each household solves

$$\min P_t^n C_t^n + P_t^o C_t^o$$

subject to:

$$C_t \leq \left[(1-\alpha)^{\frac{1}{\eta_c}}(C_t^n)^{\frac{\eta_c-1}{\eta_c}} + (\alpha)^{\frac{1}{\eta_c}}(C_t^o)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}$$

where P_t^n and P_t^o are the prices of the non-traded and imported goods purchased respectively and are taken as given. Defining the shadow price of the aggregate consumption bundle as P_t^c, the optimality conditions are given by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta_c}}(C_t^n)^{\frac{\eta_c-1}{\eta_c}} + (\alpha)^{\frac{1}{\eta_c}}(C_t^o)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} \tag{D8}$$

$$C_t^n = (1-\alpha)\left(\frac{P_t^n}{P_t^c}\right)^{-\eta_c} C_t \tag{D9}$$

$$C_t^o = \alpha\left(\frac{P_t^o}{P_t^c}\right)^{-\eta_c} C_t \tag{D10}$$

where the shadow price of consuming an additional bundle of non-traded and imported goods is given by

$$P_t^c = \left[(1-\alpha) \left(P_t^n \right)^{1-\eta_c} + \alpha \left(P_t^o \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}$$

To find the consumption allocations within the non-traded goods bundle, a household solves

$$\min \int_0^1 P_{it}^n C_{it}^n di$$

subject to:

$$C_t^n \leq \left(\int_0^1 C_{it}^{n\frac{\theta_n - 1}{\theta_n}} di \right)^{\frac{\theta_n}{\theta_n - 1}}$$

From which the shadow price and consumption allocations are given by

$$P_t^n = \left(\int_0^1 P_{it}^{n\,1-\theta_n} di \right)^{\frac{1}{1-\theta_n}}$$

$$C_{it}^n = \left(\frac{P_{it}^n}{P_t^n} \right)^{-\theta_n} C_t^n$$

$$C_t^n = \left(\int_0^1 C_{it}^{n\frac{\theta_n - 1}{\theta_n}} di \right)^{\frac{\theta_n}{\theta_n - 1}}$$

Solving the analogous problem for the imported goods consumption allocations we have

$$P_t^o = \left(\int_0^1 P_{it}^{o\,1-\theta_o} di \right)^{\frac{1}{1-\theta_o}}$$

$$C_{it}^o = \left(\frac{P_{it}^o}{P_t^o} \right)^{-\theta_o} C_t^o$$

$$C_t^o = \left(\int_0^1 C_{it}^{o\frac{\theta_o - 1}{\theta_o}} di \right)^{\frac{\theta_o}{\theta_o - 1}}$$

D.2 Domestic Non-tradeable Producers

We assume a continuum of non-tradeable consumption producers on the unit interval. Each non-tradeable producer, indexed by i, has access to a linear production technology

$$Y_{it}^{s,n} = \frac{A_t^n}{\chi_t} H_{it}^n$$

where A_t^n is a common non-traded technology, H_{it}^n is the quantity of non-traded labour used by firm i, and χ_t can be interpreted as a cost-push shock. That is, firms have to pay more for the energy they use when resource prices rise.[33] We assume that χ_t follows has the same AR(1) process as that modelled for resource prices, but that the direct response of non-traded firms' marginal costs to a change in resource prices is a free parameter to be estimated (Υ)

$$\ln \chi_t = \rho_r \ln \chi_{t-1} + \Upsilon \varepsilon_t^{r^*} \tag{D11}$$

For competitive structure, we assume non-traded firms operate under monopolistic competition and are subject to a Calvo pricing friction. For the fraction $(1-\phi_n)$ of firms able to set their price optimally, they solve

$$\max_{\overline{P}_{it_0}^n} E_{t_0} \sum_{t=t_0}^{\infty} (\phi_n \beta)^{t-t_0} \frac{P_{t_0}^c \Theta_t}{P_t^c \Theta_{t_0}} \left(\overline{P}_{it_0}^n - (1-\tau_n) MC_{it}^n \right) \left(\frac{\overline{P}_{it_0}^n}{P_t^n} \right)^{-\theta_n} Y_t^{d,n}$$

where

$$MC_{it}^n \equiv \frac{W_{it}^n}{A_t^n} \chi_t$$

is the marginal cost of production for a domestic non-traded producer, $Y_t^{d,n}$ is a measure of common non-traded demand and τ_n is a subsidy used to undo the steady state distortion associated with the assumption of monopolistic competition (this is funded by the lump-sum tax on households and simplifies the calculation

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33 Note that this assumption is consistent with the use of commodity prices as a control for expected inflation in VARs that attempt to identify the effects of monetary policy shocks and address the so called 'price puzzle' (see, for example, Sims (1992)). It is also consistent with the observed correlation between commodity prices and inflation rates across countries that cannot be explained by correlation in real activity (see, for example, Gerard (2012)).

of the steady state). A recursive formulation of the implied optimality conditions is

$$\overline{P}_t^n = (1-\tau_n)\left(\frac{\theta_n}{\theta_n - 1}\right)\frac{V_{it}^n}{U_t^n} \tag{D12}$$

$$V_t^n = Y_t^{d,n}\Theta_t \frac{\left(P_t^n\right)^{\theta_n}}{P_t^c}\frac{W_t^n}{A_t^n}\chi_t + \beta\phi_n E_t\left(V_{t+1}^n\right) \tag{D13}$$

$$U_t^n = Y_t^{d,n}\Theta_t \frac{\left(P_t^n\right)^{\theta_n}}{P_t^c} + \beta\phi_n E_t\left(U_{t+1}^n\right) \tag{D14}$$

where $\overline{P}_t^n$ is the optimal reset price for the firm. It should be noted that in equilibrium all firms will choose the same optimal reset price given that there will be a degenerate wage distribution in an equilibrium where all households are identical $\left(W_{it}^n = W_t^n\right)$ and that there are no idiosyncratic shocks. For the remaining fraction (ϕ_n) of firms not able to choose their price, they simply retain the price they offered in the previous period. Accordingly, a measure of non-traded goods prices, the shadow price of an extra bundle of non-traded consumption goods, is

$$P_t^n = \left((1-\phi_n)\left(\overline{P}_t^n\right)^{1-\theta_n} + \phi_n P_{t-1}^{n^{1-\theta_n}}\right)^{\frac{1}{1-\theta_n}} \tag{D15}$$

It is straightforward to verify that the total profit of domestically owned non-traded producers is given by

$$\int_0^1 \Psi_{it}^n di = P_t^n Y_t^{d,n} - (1-\tau_n) W_t^n H_t^n \tag{D16}$$

D.3 Domestic Importing Firms

We assume a continuum of importing firms of unit measure who are owned by domestic households. Importing firms purchase final output from the foreign sector at the foreign currency price, P_t^*, and use this output to produce a differentiated imported good. The real marginal cost, common to all importers, in domestic currency terms is

$$MC_t^o = S_t P_t^*$$

Assuming that importers operate under monopolistic competition, and that a Calvo pricing friction exists for importers resetting their domestic currency price, we have

$$\overline{P}_t^o = (1-\tau_o)\left(\frac{\theta_o}{\theta_o - 1}\right)\frac{V_t^o}{U_t^o} \tag{D17}$$

$$V_t^o = C_t^o \Theta_t \frac{\left(P_t^o\right)^{\theta_o}}{P_t^c} S_t P_t^* + \beta \phi_o E_t\left(V_{t+1}^o\right) \tag{D18}$$

$$U_t^o = C_t^o \Theta_t \frac{\left(P_t^o\right)^{\theta_o}}{P_t^c} + \beta \phi_o E_t\left(U_{t+1}^o\right) \tag{D19}$$

where $\overline{P}_t^o$ is the optimal reset price chosen by importers able to choose their price, and ϕ_o is the probability that any given firm will not be able to re-optimise its price in a given period and retains its previous period price. The shadow price relevant for imported goods is

$$P_t^o = \left((1-\phi_o)\left(\overline{P}_t^o\right)^{1-\theta_o} + \phi_o {P_{t-1}^o}^{1-\theta_o}\right)^{\frac{1}{1-\theta o}} \tag{D20}$$

For determining import firm profits we define the alternative import price index

$$\begin{aligned} \widetilde{P}_t^o &= \left[\int_0^1 P_t^o(i)^{-\theta_o} di\right]^{-\frac{1}{\theta_o}} \\ &= \left((1-\phi_o)\left(\overline{P}_t^o\right)^{-\theta_o} + \phi_o {\widetilde{P}_{t-1}^o}^{-\theta_o}\right)^{-\frac{1}{\theta o}} \end{aligned} \tag{D21}$$

where total profit in the imported sector is given by

$$\int_0^1 \Psi_{it}^o di = P_t^o C_t^o - (1-\tau_o) S_t P_t^* \left(\frac{\widetilde{P}_t^o}{P_t^o}\right)^{-\theta_o} C_t^o \tag{D22}$$

D.4 Non-resource Export Sector

Given the substantial interest in how resource sector developments can influence the non-resource export sector, we also assume a unit measure of domestically owned firms that engage in non-resource exporting (hereafter, exporters). An exporter, indexed by j, purchases a bundle of non-traded inputs from domestic producers and transforms this into a specialised export good. The demand for inputs from non-traded producer i by exporter j is given by

$$Y_{it}^{n,x}(j) = \left(\frac{P_{it}^{n}}{P_{t}^{n}}\right)^{-\theta_n} C_t^x(j)$$

where $C_t^x(j)$ is the demand for exporter j's output. The real marginal cost for an exporter is

$$MC_t^x(j) = P_t^n$$

We assume the following (reduced-form) demand function for exports of type j

$$\begin{aligned} C_t^x(j) &= \left(\frac{P_{jt}^{x^*}}{P_t^{x^*}}\right)^{-\theta_x} C_t^* \\ C_t^* &= \left(\frac{P_t^{x^*}}{P_t^*}\right)^{-\theta_*} Y_t^* \end{aligned} \tag{D23}$$

where C_t^* is a measure of the common component of demand for non-resource exports, Y_t^* is foreign output, $P_{jt}^{x^*}$ is the price of export type j in foreign currency terms, $P_t^{x^*}$ is an index of non-resource export prices in foreign currency terms, and P_t^* is the foreign price index. Note that θ_x is the within sector elasticity of non-resource export demand, and that θ_* is the cross-sector (or common) elasticity of non-resource export demand. Consistent with Adolfson, Laséen, Lindé and Villani (2007), this formulation allows for both competition effects amongst firms within the exporting sector, and competition between the export sector as a whole and the rest of the world.

Assuming that exporters are monopolistically competitive, set their prices in foreign currency terms, and are subject to a Calvo pricing friction, a recursive formulation for determining their optimal reset price is

$$\overline{P}_t^{x^*} = (1-\tau_x)\left(\frac{\theta_x}{\theta_x - 1}\right)\frac{V_t^x}{U_t^x} \tag{D24}$$

$$V_t^x = C_t^*\Theta_t\left(P_t^{x^*}\right)^{\theta_x}\frac{P_t^n}{P_t^c} + \beta\phi_x E_t\left(V_{t+1}^x\right) \tag{D25}$$

$$U_t^x = C_t^*\Theta_t\left(P_t^{x^*}\right)^{\theta_x}\frac{S_t}{P_t^c} + \beta\phi_x E_t\left(U_{t+1}^x\right) \tag{D26}$$

where the price index for non-resource exported goods is defined by

$$\begin{aligned} P_t^{x^*} &\equiv \left[\int_0^1 P_{jt}^{x^*\,1-\theta_x} dj\right]^{\frac{1}{1-\theta_x}} \\ &= \left((1-\phi_x)\left(\overline{P}_t^{x^*}\right)^{1-\theta_x} + \phi_x\left(P_{t-1}^{x^*}\right)^{1-\theta_x}\right)^{\frac{1}{1-\theta x}} \end{aligned} \tag{D27}$$

It will be useful for determining non-resource export firm profits to also define the alternative non-resource export price index

$$\begin{aligned} \widetilde{P}_t^{x^*} &\equiv \left[\int_0^1 P_{jt}^{x^*\,-\theta_x} dj\right]^{-\frac{1}{\theta_x}} \\ &= \left((1-\phi_x)\left(\overline{P}_t^{x^*}\right)^{-\theta_x} + \phi_x\left(\widetilde{P}_{t-1}^{x^*}\right)^{-\theta_x}\right)^{-\frac{1}{\theta x}} \end{aligned} \tag{D28}$$

Total profit of exporters is thus

$$\int_0^1 \Psi_{jt}^x dj = S_t P_t^{x^*} C_t^* - (1-\tau_x) P_t^n \left(\frac{\widetilde{P}_t^x}{P_t^{x^*}}\right)^{-\theta_x} C_t^* \tag{D29}$$

D.5 Monetary Policy

For domestic monetary policy we assume that the central bank follows a simple Taylor rule of the form

$$\ln(1+i_t) = (1-\rho_i)\ln\left(1+\bar{i}\right) + (1-\rho_i)\rho_\pi E_t \ln\left(\frac{P_{t+1}^c}{P_t^c}\right) + \rho_i \ln\left(1+i_{t-1}\right) \tag{D30}$$

This rule is consistent with a forward-looking central bank that targets inflation, but also allows for gradual interest rate adjustment.

D.6 Market Clearing and the Rest of the World

For market clearing, supply must meet the demand for each non-traded good i

$$Y_{it}^{s,n} = Y_{it}^{d,n}$$

Aggregating demand and supply across the continuum of goods, it follows that

$$\frac{A_t^n H_t^n}{\chi_t} = \left(\frac{\widetilde{P}_t^n}{P_t^n}\right)^{-\theta_n} Y_t^{d,n} \tag{D31}$$

and where

$$Y_t^{d,n} \equiv \int_0^1 \left(C_t^n + C_t^x + Y_t^{r,n}\right) dj - D\frac{Q^r}{\phi_{mc}} \tag{D32}$$

$$\widetilde{P}_t^n \equiv \left[\int_0^1 P_{it}^{n-\theta_n} di\right]^{-\frac{1}{\theta n}} \tag{D33}$$

Note that the common or non-idiosyncratic component of demand for non-traded goods is given by the sum of demand for non-traded consumption goods, demand for non-traded inputs which are then exported, and net demand for non-traded inputs used up in the exploration process.[34]

For the rest of the world, we assume that prices and quantities admit the following VAR(p) representation

$$\mathbf{y}_t^* = \sum_{j=0}^{p} \mathbf{A}_j \mathbf{y}_{t-j}^* + \varepsilon_t \tag{D34}$$

where $\mathbf{y}_t^* = \begin{bmatrix} Y_t^* & i_t^* & P_t^* & P_t^{r^*} \end{bmatrix}'$ is a vector collecting all foreign prices and quantities and ε_t is a 4×1 vector of reduced form shocks.

Definition 3. *A small open economy general equilibrium with endogenous reserves, and under rational expectations, is given by sequences of quantities* $\left\{C_t, C_t^n, C_t^o, Y_t^{d,n}, H_t^n, H_t^r, B_t^*, R_t, D_t, X_t, I_t, K_t, \Psi_t^n, \Psi_t^o, \Psi_t^x, \Psi_t^r, \chi_t, Y_t^{r,n}, \Theta_t, C_t^*\right\}$ *and prices* $\left\{P_t^c, P_t^n, P_t^o, P_t^{x^*}, W_t^n, W_t^r, S_t, Q_t^r, Q_t^k, \overline{P}_t^n, \overline{P}_t^o, \overline{P}_t^{x^*}, \widetilde{P}_t^n, \widetilde{P}_t^o, \widetilde{P}_t^x, v_t^n, u_t^n, v_t^o, u_t^o, v_t^x, u_t^x, i_t\right\}$ *that solve Equations (9) to (18) and (D2) to (D33) taking foreign quantities and prices* $\left\{\mathbf{y}_t^*\right\}$ *as given by Equation (D34).*

34 To simplify calculation of the steady state, we assume that the government makes a constant lump-sum allocation of exploration inputs, $D\frac{Q^r}{\phi_{mc}}$, to the resource sector. One can think of this as analogous to tax incentives or government subsidies that encourage resource exploration.

Definition 4. *A small open economy general equilibrium with exogenous reserves, and under rational expectations, is given by sequences of quantities* $\left\{C_t, C_t^n, C_t^o, Y_t^{d,n}, H_t^n, H_t^r, B_t^*, X_t, I_t, K_t, \Psi_t^n, \Psi_t^o, \Psi_t^x, \Psi_t^r, \chi_t, Y_t^{r,n}, \Theta_t, C_t^*\right\}$ *and prices* $\left\{P_t^c, P_t^n, P_t^o, P_t^{x^*}, W_t^n, W_t^r, S_t, Q_t^r, Q_t^k, \overline{P}_t^n, \overline{P}_t^o, \overline{P}_t^{x^*}, \widetilde{P}_t^n, \widetilde{P}_t^o, \widetilde{P}_t^x, v_t^n, u_t^n, v_t^o, u_t^o, v_t^x, u_t^x, i_t\right\}$ *that solve Equations (9), (11), (13) to (18), and (D2) to (D33) taking foreign quantities and prices* $\left\{\mathbf{y}_t^*\right\}$ *as given by Equation (D34) and setting* $D_t = 0$ *and* $R_t = R$ *for all* t*.*

In view of the fact that the stock of natural reserves is potentially non-stationary, as highlighted in partial equilibrium, we still need to find a stationary representation of the above economy. Appendix D.7 identifies one stationary representation that has a locally stable solution.

D.7 Stationary Representation

<u>Claim:</u> A detrended representation of the general equilibrium economy in Definition (3) with a (locally) unique stable solution exists if

1. Log foreign demand and the log of the stock of domestic natural reserves are cointegrated
$$\ln \frac{Y_t^*/Y^*}{R_t/R} = \vartheta \ln \frac{Y_{t-1}^*/Y^*}{R_{t-1}/R} - \Delta \ln R_t + \varepsilon_t^*$$
2. The logs of resource sector and non-resource sector technology are also identically cointegrated with the log of the stock of domestic natural reserves
$$\ln \frac{A_t^n}{R_t/R} = \vartheta \ln \frac{A_{t-1}^n}{R_{t-1}/R} - \Delta \ln R_t + \varepsilon_t^*$$
$$A_t^n = A_t^r$$

   and where $|\vartheta| < 1$.

Verification. We verify this claim numerically. First, we claim that the following system is a detrended representation of the economy in Definition (3) with a unique stable solution (at the paramererisation of our model referred to in the main text):

$$\frac{S_t}{P_t^c}P_t^{r^*} = (1+\zeta)\frac{W_t^r}{P_t^c R_t}\frac{R_t}{A_t^r}\left(\frac{X_t}{R_t}\right)^{\zeta}\left(\frac{K_t}{R_t}\right)^{-\mu} + \frac{Q_t^r}{P_t^c} \tag{D35}$$

$$\begin{aligned}\frac{S_t}{P_t^c}P_t^* = Q_t^k &\left(1-\Xi\left(\frac{I_t/R_t}{I_{t-1}/R_{t-1}}\frac{R_t}{R_{t-1}}\right) - \Xi'\left(\frac{I_t/R_t}{I_{t-1}/R_{t-1}}\frac{R_t}{R_{t-1}}\right)\frac{I_t/R_t}{I_{t-1}/R_{t-1}}\frac{R_t}{R_{t-1}}\right) \\ &+ \beta E_t\left(\widetilde{M}_{t,t+1}\frac{Q_{t+1}^k}{P_{t+1}^c}\Xi'\left(\frac{I_{t+1}/R_{t+1}}{I_t/R_t}\frac{R_{t+1}}{R_t}\right)\left(\frac{I_{t+1}/R_{t+1}}{I_t/R_t}\frac{R_{t+1}}{R_t}\right)^2\right)\end{aligned} \tag{D36}$$

$$\frac{\frac{\partial C(D_t,R_t)}{\partial D_t}}{P_t^c} = \frac{Q_t^r}{P_t^c} \tag{D37}$$

$$\frac{K_{t+1}}{R_{t+1}}\frac{R_{t+1}}{R_t} = (1-\delta)\frac{K_t}{R_t} + \left(1-\Xi\left(\frac{I_t/R_t}{I_{t-1}/R_{t-1}}\right)\right)\frac{I_t}{R_t} \tag{D38}$$

$$\frac{R_{t+1}}{R_t} = 1+\omega_{t+1}\frac{D_t}{R_t} - \lambda\frac{X_t}{R_t} \tag{D39}$$

$$\begin{aligned}\frac{Q_t^r}{P_t^c} &= \beta E_t\left(\widetilde{M}_{t,t+1}(\zeta-\mu)\frac{W_{t+1}^r}{R_{t+1}P_{t+1}^c}\frac{R_{t+1}}{A_{t+1}^r}\left(\frac{X_{t+1}}{R_{t+1}}\right)^{1+\zeta}\left(\frac{K_{t+1}}{R_{t+1}}\right)^{-\mu}\right) \\ &\quad + \beta E_t\left(\widetilde{M}_{t,t+1}\frac{Q_{t+1}^r}{P_{t+1}^c}\right)\end{aligned} \tag{D40}$$

$$\begin{aligned}\frac{Q_t^k}{P_t^c} &= \beta E_t\left(\widetilde{M}_{t,t+1}\mu\frac{W_{t+1}^r}{R_{t+1}P_{t+1}^c}\frac{R_{t+1}}{A_{t+1}^r}\left(\frac{X_{t+1}}{R_{t+1}}\right)^{1+\zeta}\left(\frac{K_{t+1}}{R_{t+1}}\right)^{-\mu-1}\right) \\ &\quad + \beta E_t\left(\widetilde{M}_{t,t+1}\frac{Q_{t+1}^k}{P_{t+1}^c}(1-\delta)\right)\end{aligned} \tag{D41}$$

$$\frac{\Psi_t^R}{P_t^c R_t} = \frac{S_t}{P_t^c}P_t^{r^*}\frac{X_t}{R_t} - \frac{W_t^r}{R_t P_t^c}\frac{R_t}{A_t^r}\left(\frac{X_t}{R_t}\right)^{1+\zeta}\left(\frac{K_t}{R_t}\right)^{-\mu} - \frac{S_t}{P_t^c}P_t^*\frac{I_t}{R_t} - \frac{C(D_t,R_t)}{P_t^c R_t} \tag{D42}$$

$$\frac{X_t}{R_t} = \left(\frac{A_t^r}{R_t}H_t^r\right)^{\eta}\left(\frac{K_t}{R_t}\right)^{\gamma} \tag{D43}$$

$$\frac{D_t}{R_t} = \frac{Y_t^{r,n}/R_t}{\frac{Q^r}{\phi_{mc}}} \tag{D44}$$

$$\widetilde{\Theta}_t = \frac{R_t}{C_t}\left(\frac{C_t/R_t}{V_t/R_{t-1}}\frac{R_t}{R_{t-1}}\right)^{1-\xi_c} \tag{D45}$$

$$\widetilde{\Theta}_t = \beta E_t\left((1+i_t)\frac{P_t^c}{P_{t+1}^c}\widetilde{\Theta}_{t+1}\frac{R_t}{R_{t+1}}\right) \tag{D46}$$

$$\widetilde{\Theta}_t = \beta E_t\left((1+i_t^*)\Phi_t\frac{P_t^c}{P_{t+1}^c}\widetilde{\Theta}_{t+1}\frac{S_{t+1}}{S_t}\frac{R_t}{R_{t+1}}\right) \tag{D47}$$

$$\frac{W_t^n}{R_tP_t^c}\widetilde{\Theta}_t = \varsigma\xi_h\frac{H_t^{n^{\frac{1}{\gamma_h}-1}}}{H^n}\left(\frac{H_t^{n^{\frac{1}{\gamma_h}}}}{H^n}+\frac{H_t^{r^{\frac{1}{\gamma_h}}}}{H^r}\right)^{\xi_h\gamma_h-1} \tag{D48}$$

$$\frac{W_t^r}{R_tP_t^c}\widetilde{\Theta}_t = \varsigma\xi_h\frac{H_t^{r^{\frac{1}{\gamma_h}-1}}}{H^r}\left(\frac{H_t^{n^{\frac{1}{\gamma_h}}}}{H^n}+\frac{H_t^{r^{\frac{1}{\gamma_h}}}}{H^r}\right)^{\xi_h\gamma_h-1} \tag{D49}$$

$$\begin{aligned}\frac{C_t}{R_t} &= \frac{W_t^n}{R_tP_t^c}H_t^n + \frac{W_t^r}{R_tP_t^c}H_t^r + \frac{S_t}{P_t^c}\frac{B_{t-1}^*}{R_{t-1}}\frac{R_{t-1}}{R_t} + \nu\frac{\Psi_t^r}{R_tP_t^c} \\ &\quad - \frac{S_t}{P_t^c}\frac{B_t^*}{R_t}\frac{1}{(1+i_t^*)\Phi_t} + \frac{\Psi_t^n}{R_tP_t^c} + \frac{\Psi_t^o}{R_tP_t^c} + \frac{\Psi_t^x}{R_tP_t^c} - \frac{T}{R_tP_t^c}\end{aligned} \tag{D50}$$

$$\frac{C_t}{R_t} = \left[(1-\alpha)^{\frac{1}{\eta_c}}\left(\frac{C_t^n}{R_t}\right)^{\frac{\eta_c-1}{\eta_c}} + (\alpha)^{\frac{1}{\eta_c}}\left(\frac{C_t^o}{R_t}\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} \tag{D51}$$

$$\frac{C_t^n}{R_t} = (1-\alpha)\left(\frac{P_t^n}{P_t^c}\right)^{-\eta_c}\frac{C_t}{R_t} \tag{D52}$$

$$\frac{C_t^o}{R_t} = \alpha\left(\frac{P_t^o}{P_t^c}\right)^{-\eta_c}\frac{C_t}{R_t} \tag{D53}$$

$$\ln\chi_t = \rho_r\ln\chi_{t-1} + \Upsilon\varepsilon_t^{r^*} \tag{D54}$$

$$\frac{\overline{P}_t^n}{P_t^c} = (1-\tau_n)\left(\frac{\theta_n}{\theta_n-1}\right)\frac{V_t^n/\left(P_t^c\right)^{\theta_n}}{U_t^n/\left(P_t^c\right)^{\theta_n-1}} \tag{D55}$$

$$\frac{V_t^n}{\left(P_t^c\right)^{\theta_n}} = \frac{Y_t^{d,n}}{R_t}\Theta_t\left(\frac{P_t^n}{P_t^c}\right)^{\theta_n}\frac{W_t^n}{R_tP_t^c}\frac{R_t}{A_t^n}\chi_t + \beta\phi_nE_t\left(\frac{V_{t+1}^n}{\left(P_{t+1}^c\right)^{\theta_n}}\left(\frac{P_{t+1}^c}{P_t^c}\right)^{\theta_n}\right) \tag{D56}$$

$$\frac{U_t^n}{\left(P_t^c\right)^{\theta_n-1}} = \frac{Y_t^{d,n}}{R_t}\Theta_t\left(\frac{P_t^n}{P_t^c}\right)^{\theta_n} + \beta\phi_n E_t\left(\frac{U_{t+1}^n}{\left(P_{t+1}^c\right)^{\theta_n-1}}\left(\frac{P_{t+1}^c}{P_t^c}\right)^{\theta_n-1}\right) \tag{D57}$$

$$\frac{P_t^n}{P_t^c} = \left((1-\phi_n)\left(\frac{\overline{P}_t^n}{P_t^c}\right)^{1-\theta_n} + \phi_n\left(\frac{P_{t-1}^n}{P_{t-1}^c}\frac{P_{t-1}^c}{P_t^c}\right)^{1-\theta_n}\right)^{\frac{1}{1-\theta_n}} \tag{D58}$$

$$\frac{\Psi_t^n}{R_t P_t^c} = \frac{P_t^n}{P_t^c}\frac{Y_t^{d,n}}{R_t} - (1-\tau_n)\frac{W_t^n}{R_t P_t^c}H_t^n \tag{D59}$$

$$\frac{\overline{P}_t^o}{P_t^c} = (1-\tau_o)\left(\frac{\theta_o}{\theta_o-1}\right)\frac{V_t^o/\left(P_t^c\right)^{\theta_o}}{U_t^o/\left(P_t^c\right)^{\theta_o-1}} \tag{D60}$$

$$\frac{V_t^o}{\left(P_t^c\right)^{\theta_o}} = \frac{C_t^o}{R_t}\Theta_t\left(\frac{P_t^o}{P_t^c}\right)^{\theta_o}\frac{S_t}{P_t^c}P_t^* + \beta\phi_o E_t\left(\frac{V_{t+1}^o}{\left(P_{t+1}^c\right)^{\theta_o}}\left(\frac{P_{t+1}^c}{P_t^c}\right)^{\theta_o}\right) \tag{D61}$$

$$\frac{U_t^o}{\left(P_t^c\right)^{\theta_o-1}} = \frac{C_t^o}{R_t}\Theta_t\left(\frac{P_t^o}{P_t^c}\right)^{\theta_o} + \beta\phi_o E_t\left(\frac{U_{t+1}^o}{\left(P_{t+1}^c\right)^{\theta_o-1}}\left(\frac{P_{t+1}^c}{P_t^c}\right)^{\theta_o-1}\right) \tag{D62}$$

$$\frac{P_t^o}{P_t^c} = \left((1-\phi_o)\left(\frac{\overline{P}_t^o}{P_t^c}\right)^{1-\theta_o} + \phi_o\left(\frac{P_{t-1}^o}{P_{t-1}^c}\frac{P_{t-1}^c}{P_t^c}\right)^{1-\theta_o}\right)^{\frac{1}{1-\theta o}} \tag{D63}$$

$$\frac{\widetilde{P}_t^o}{P_t^c} = \left((1-\phi_o)\left(\frac{\overline{P}_t^o}{P_t^c}\right)^{-\theta_o} + \phi_o\left(\frac{\widetilde{P}_{t-1}^o}{P_{t-1}^c}\frac{P_{t-1}^c}{P_t^c}\right)^{-\theta_o}\right)^{-\frac{1}{\theta o}} \tag{D64}$$

$$\frac{\Psi_t^o}{R_t P_t^c} = \frac{P_t^o}{P_t^c}\frac{C_t^o}{R_t} - (1-\tau_o)\frac{S_t}{P_t^c}P_t^*\left(\frac{\widetilde{P}_t^o/P_t^c}{P_t^o/P_t^c}\right)^{-\theta_o}\frac{C_t^o}{R_t} \tag{D65}$$

$$\overline{P}_t^{x^*} = (1-\tau_x)\left(\frac{\theta_x}{\theta_x-1}\right)\frac{V_t^x}{U_t^x} \tag{D66}$$

$$V_t^x = \frac{C_t^*}{R_t}\Theta_t\left(P_t^{x^*}\right)^{\theta_x}\frac{P_t^n}{P_t^c} + \beta\phi_x E_t\left(V_{t+1}^x\right) \tag{D67}$$

$$U_t^x = \frac{C_t^*}{R_t}\Theta_t\left(P_t^{x^*}\right)^{\theta_x}\frac{S_t}{P_t^c} + \beta\phi_x E_t\left(U_{t+1}^x\right) \tag{D68}$$

$$P_t^{x^*} \equiv \left((1-\phi_x)\left(\overline{P}_t^{x^*}\right)^{1-\theta_x} + \phi_x\left(P_{t-1}^{x^*}\right)^{1-\theta_x}\right)^{\frac{1}{1-\theta x}} \tag{D69}$$

$$\widetilde{P}_t^{x^*} = \left((1-\phi_x)\left(\overline{P}_t^{x^*}\right)^{-\theta_x} + \phi_x\left(\widetilde{P}_{t-1}^{x^*}\right)^{-\theta_x}\right)^{-\frac{1}{\theta x}} \tag{D70}$$

$$\frac{\Psi_t^x}{R_t P_t^c} = \frac{S_t}{P_t^c} P_t^{x^*} \frac{C_t^*}{R_t} - (1-\tau_x) \frac{P_t^n}{P_t^c} \left(\frac{\widetilde{P}_t^x}{P_t^{x^*}} \right)^{-\theta_x} \frac{C_t^*}{R_t} \tag{D71}$$

$$\frac{C_t^*}{R_t} = \left(\frac{P_t^{x^*}}{P_t^*} \right)^{-\theta_*} \frac{Y_t^*}{R_t} \tag{D72}$$

$$\ln(1+i_t) = (1-\rho_i)\ln\left(1+\bar{i}\right) + (1-\rho_i)\rho_\pi \ln\left(\frac{P_{t+1}^c}{P_t^c}\right) + \rho_i \ln\left(1+i_{t-1}\right) \tag{D73}$$

$$\frac{A_t^n}{R_t}\frac{H_t^n}{\chi_t} = \left(\frac{\widetilde{P}_t^n / P_t^c}{P_t^n / P_t^c} \right)^{-\theta_n} \frac{Y_t^{d,n}}{R_t} \tag{D74}$$

$$\frac{Y_t^{d,n}}{R_t} \equiv \frac{C_t^n}{R_t} + \left(\frac{\widetilde{P}_t^x}{P_t^{x^*}} \right)^{-\theta_x} \frac{C_t^*}{R_t} + \frac{Y_t^{r,n}}{R_t} - \frac{D}{R}\frac{Q^r}{\phi_{mc}} \tag{D75}$$

$$\frac{\widetilde{P}_t^n}{P_t^c} \equiv \left((1-\phi_n) \left(\frac{\overline{P}_t^n}{P_t^c} \right)^{-\theta_x} + \phi_n \left(\frac{\widetilde{P}_{t-1}^n}{P_{t-1}^c} \frac{P_{t-1}^c}{P_t^c} \right)^{-\theta_n} \right)^{-\frac{1}{\theta_n}} \tag{D76}$$

and where

$$\widetilde{M}_{t,t+1} = \left(\nu \frac{\Theta_{t+1}}{\Theta_t} + 1 - \nu \right)$$

Inspecting the Blanchard-Kahn (BK) conditions of the above economy (when taking a first-order appoximation), we find that the BK conditions are not satisfied when $\vartheta = 1$ (and so $\ln \frac{Y_t^*/Y^*}{R_t/R}$ and $\ln \frac{A_t^n}{R_t/R}$ become I(1) processes). Specifically, we find the presence of unit eigenvalues that are consistent with the linear approximation of this system admitting no stable solution. Imposing stationarity on either $\ln \frac{Y_t^*/Y^*}{R_t/R}$ or $\ln \frac{A_t^n}{R_t/R}$ was also not sufficient to satisfy the requirements for stability.

Some analytical intuition for this is as follows. Suppose that the above economy is a stationary representation and that in this equilibrium log reserves are an I(1) variable (as we demonstrated in partial equilibrium). In the absence of cointegration as specified above, the variables $\widehat{y}_t^* \equiv \ln \frac{Y_t^*/Y^*}{R_t/R}$ and $\widehat{a}_t^n \equiv \ln \frac{A_t^n}{R_t/R}$ become I(1) processes. Taking a first-order approximation of Equation (D72) it is clear that:

$$\widehat{c}_t^* = -\theta_* \widehat{p}_t^{x^*} + \widehat{y}_t^*$$

implying that $\widehat{c}_t^*$ will also be I(1) since $\widehat{p}_t^x$ will be I(0) under the claim that the above representation is stationary (when solved using a first-order approximation around the steady state). Taking a first-order approximation of Equation (D75):

$$\widehat{y}_t^{d,n} = \frac{\overline{C}^n}{\overline{Y}^{d,n}}\widehat{c}_t^n + \frac{\overline{C}^*}{\overline{Y}^{d,n}}\left(-\theta_x\widehat{\widetilde{p}}_t^x + \widehat{c}_t^*\right) + \frac{\overline{Y}^{r,n}}{\overline{Y}^{d,n}}\widehat{y}_t^{r,n}$$

But we see immediately that this cannot be an equilibrium solution since the right-hand side has a single variable that is I(1), while all other variables are I(0) under our claim. Consistent with this, the absence of cointegration between log foreign demand and log reserves is not consistent with the above economy admitting a unique stationary solution.

The intuition is similar when understanding why the log of domestic resource or the log of non-resource technology must also be cointegrated with the log of reserves. If either of these assumptions do not hold then inspecting the linear approximation of the above detrended representation makes it clear that some equations will represent a mixture of I(0) variables and a single I(1) variable, thus contradicting the claim that the above representation admits a unique stationary solution.

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