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Central Counterparty Links and Clearing System Exposures

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Gerard Kelly

RDP 2013-12

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Abstract

Links between central counterparties (CCPs) enable participants to clear positions in any linked CCP without needing to maintain multiple CCP memberships. While links enable exposure to be reduced by allowing netting across CCPs, CCPs become exposed to one another through the possibility of a CCP default. The change in system exposure resulting from these two effects has recently been of great interest to policymakers, and we assess this quantitatively using a straightforward extension of Duffie and Zhu (2011). Our model shows that CCP links can reduce overall system exposure in most plausible scenarios. This conclusion is robust to changes in assumptions about the nature of participants and their exposures, as well as to the existence of more than two linked CCPs.

JEL Classification Numbers: E42, G14

Keywords: clearing, netting, financial stability, central counterparty

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1. Introduction

Since the onset of the global financial crisis, a range of regulatory reforms have been introduced that focus on financial system interconnectedness and collateralisation of financial exposures. A significant element of the international reform agenda has been the effort to push financial markets towards central counterparties (CCPs).[1] CCPs can be systemically important components of the financial market infrastructure, taking on the role of buyer to every seller and seller to every buyer in the particular products they clear. In so doing, they can significantly mitigate the bilateral counterparty default risk that arises when market participants trade directly with each other. These arrangements also allow market participants to reduce a network of bilateral exposures to a single multilateral net exposure to the CCP, allowing for potentially significant reductions in the capital and collateral required to support trading activity.

But precisely how this transition occurs will determine whether the policy delivers on its objectives. For example, the multilateral netting benefits of central clearing are larger the more concentrated is central clearing. Other crucial factors include the products amenable to central clearing, and whether trades will flow to domestic or international CCPs.

With central clearing of OTC derivatives in its infancy, the eventual market outcome is uncertain. This has led to research interest in the effects of market structure on the size and location of counterparty risk. Netting and unnetting of exposures are core themes in this work, and lead to an uncertain conclusion about the overall effect of the G20's OTC derivatives reforms. Indeed, Duffie and Zhu (2011) find that mandated central clearing could increase systemic risk if activity is fragmented across multiple CCPs. Pirrong (2012) also warns that systemic risk could increase, noting that such fragmentation will increase

1 In 2009, the G20 members committed to ensuring that all standardised OTC derivatives contracts were cleared through a CCP. Australia's implementation of these measures has been overseen by the Council of Financial Regulators, with new legislation effective January 2013.

the demand for CCP-eligible collateral. Since the supply of assets suitable for collateral is largely inelastic to financial institutions' demand for them in the short run, this could result in significantly higher liquidity costs in the financial sector and disrupt market functioning in the transition period.

One proposed solution to mitigate the costs of fragmentation of clearing activity is to link competing CCPs, thereby allowing netting of obligations that a participant accrues at different CCPs. In Europe's cash equity markets, this has taken the form of interoperability, where a participant in one CCP can clear its trades in a given product at any linked CCP. Other forms of CCP links include cross-margining and mutual offset arrangements. Under cross-margining, CCPs combine their risk management arrangements to offer discounts on collateral to participants using multiple CCPs concurrently. In a mutual offset arrangement, a participant can transfer positions from one CCP to another (Garvin 2012).

While CCP links allow for the netting of participant exposures in a way that approximates the outcome of using a single CCP, they create exposures between CCPs. This is because in the event of default of one of the CCPs in the linked arrangement, a portion of its obligations would be assumed by the surviving CCP(s). This could give rise to significant losses. As noted by Singh (2013), it is therefore uncertain whether interoperability or similar link arrangements will reduce aggregate counterparty exposures. For the same reason, it is also unclear whether linking CCPs would increase or lower the collateral requirements associated with participating in centrally cleared markets.

In this paper, we consider this question quantitatively, treating exposures as expected losses given counterparty default. Using the approach in Duffie and Zhu (2011), we examine the effects of interoperability in a multiple-CCP landscape, comparing increased netting benefits to the cost of inter-CCP exposures. The results indicate that under quite general conditions, the netting benefit is likely to dominate. This in turn suggests that interoperability is unlikely to increase CCP participants' collateral requirements, relative to participating in markets with multiple CCPs that are not linked. This paper looks only at expected exposure and does not consider other possible implications of interoperability, such as complexity and contagion in periods of stress.

We find that the reduction in expected exposures brought about by a link:

- increases with the total number of participants involved in the linked CCPs
- increases with the asymmetry in the number of participants using each CCP
- increases as the correlation between participant obligations in the products cleared by the two CCPs decreases
- is greatest where participant obligations in the products cleared by two linked CCPs are similarly volatile, and decreases with the asymmetry in these obligations' volatility (either because of participant heterogeneity or differences in the products' characteristics)
- is maximised when participants of linked CCPs concentrate their clearing in one CCP only.

The analysis sheds light on the implications of interoperability for the structure of exposures in markets such as the European cash equity market, where CCP links have already emerged. It also increases the information available to regulators and CCP participants about the netting and exposure effects of introducing interoperability into new markets. European regulators have to date not entertained interoperability in derivative markets, owing to the longer duration and greater risk of these contracts. However, this issue is now being looked at more closely – under the European Market Infrastructure Regulation, the European Securities and Markets Authority must submit a report in 2014 on whether interoperability should be extended to derivative markets. While there are a number of collateral offset arrangements currently available in derivatives markets, full interoperability in either exchange-traded or OTC derivatives markets has not yet emerged.

The structure of the rest of this paper is as follows. Section 2 provides a basic overview of central counterparty interoperability and the relevant literature. Section 3 introduces a model of interlinked CCPs and Section 4 discusses the results of this model. Section 5 then develops extensions to the model to demonstrate its usefulness in a richer array of scenarios. Section 6 concludes.[2]

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2 All code used in this work has been written in Matlab and is available on request.

2. CCP Links and Exposure

Clearing is the process of transmitting, reconciling and confirming payment obligations after a trade has been negotiated. Traders in financial markets face the risk that their counterparty may default after a trade has been negotiated but before it has settled, leaving them exposed to a loss in replacing the trade if market values change adversely. To reduce exposure to this 'replacement cost' risk, trading counterparties in many securities and derivative markets clear their trades through a CCP, which intermediates between all participants in a given market through novation.[3] Novation replaces the contract between a buyer and a seller with two contracts: one between the buyer and the CCP and one between the seller and the CCP. Novation protects traders from the risk of their counterparty defaulting, as the CCP becomes responsible for meeting any defaulting trader's obligations.

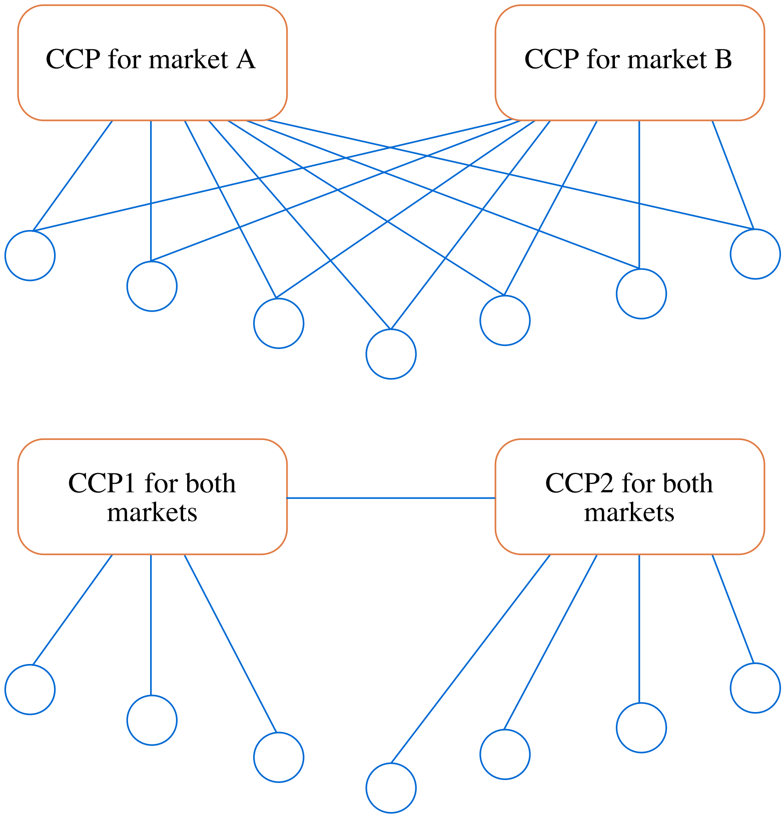
To novate trades to a CCP, traders must both be participants of that CCP, subject to the CCP's rules and requirements. These typically involve the CCP monitoring participants' financial health and trading activities, and participants posting collateral (initial margin) to cover the CCP's potential losses. By centralising all transactions through a single entity, CCPs concentrate a complex network of bilateral exposures into a single set of exposures, subject to strict risk management standards. This multilateral netting of bilateral exposures aims to reduce overall exposure in the system and economises on collateral, since less collateral is required to cover potential losses.

CCP links facilitate novated trades between participants of different CCPs. Here, novation replaces the contract between a buyer and a seller with three contracts: one between the seller and its CCP, one between the buyer and its CCP, and one between the two CCPs. Interoperability allows participants of one CCP to trade with participants of other CCPs without the costs of maintaining multiple CCP memberships. Interoperability can broaden market access, allowing trade across a wider range of products and venues, while promoting competition by lowering barriers to entry for new CCPs and allowing participants to choose their preferred CCP. By enabling participants to hold all their positions in a single

3 Three CCPs currently operate in Australia: two CCPs in the ASX Group, ASX Clear and ASX Clear (Futures), and the London-based CCP, LCH.Clearnet Ltd.

CCP, interoperability avoids the loss of netting that occurs when positions are fragmented across multiple clearing venues.

Figure 1: Unlinked and Linked CCPs



However, CCP links also generate exposures that could pose financial stability risks. In particular, links introduce credit exposure between linked CCPs. This exposure arises because all linked positions in a defaulting CCP are taken on by linked CCPs, potentially leading to very large losses. Although the probability of a CCP default is small, such an event could threaten the solvency of any surviving linked CCPs if the number of trades cleared across the link and the adverse movement of prices were significant enough. CCP links could then become the cause of significant disruption to the financial system.

It is therefore important that any inter-CCP exposures are well managed, and that linked CCPs provide sufficient collateral to each other to ensure that a default by one would not threaten the solvency of the others. As the range of financial products cleared by CCPs becomes broader and proposals for interoperability increasingly common, the effect of CCP links on exposures has become a topic of significant policy interest. In Section 3, we estimate the effect of interoperability on exposures using a stylised model in which market participants conduct centrally cleared trades through linked and unlinked CCPs.

2.1 Literature Review

There is a small but informative literature on the impact of different clearing market structures on the financial system. Both qualitative and quantitative approaches have been used to examine the effects of recent developments in market structure on collateralisation and systemic risk.

CPSS (2010) provides a broad overview of clearing market structure developments over the past decade. It identifies competition among CCPs as a key trend, noting that this could lead to the fragmentation of clearing, increasing the number of outstanding contracts and decreasing netting efficiency. While interoperable links may reduce these inefficiencies, there is a contagion risk that arises from inter-CCP exposures, particularly if linked CCPs are unable to monitor other CCPs' risk management practices effectively.

There has been some academic interest in modelling different types of market structures and examining their impact on market functioning, exposures and risk. Jackson and Manning (2007), for example, investigate the implications for risk of three alternative clearing structures: bilateral clearing, 'ring' clearing, and CCP clearing. They analyse both the magnitude and distribution of replacement cost losses across trading counterparties. By simulating trading positions and asset price changes using defined distributions, their model demonstrates that multilateral netting reduces replacement cost losses, and that CCP clearing distributes those losses more evenly across participants than the alternative arrangements considered.

Duffie and Zhu (2011) use a similar approach to Jackson and Manning (2007) to consider the implications of central clearing for netting efficiency. They find that central clearing could actually increase counterparty risk in OTC derivatives markets if netting sets of multiple products become fragmented across different CCPs. Importantly, they show that using multiple CCPs to clear different products always increases exposure relative to clearing all products through a single CCP.

An implication of higher exposures in any market is that more collateral is required to cover these exposures. This is noted by Duffie and Zhu (2011) and assessed quantitatively by Heller and Vause (2012). They estimate that a prudent CCP with a monopoly on OTC derivative clearing would need US$43 billion of initial margin for clearing interest rate swaps (IRS), and up to US$107 billion for clearing credit default swaps (CDS). The model sheds light on how these values could be affected by market structure: if, for example, three regionally focused CCPs merge, large dealers would be required to post 25 per cent less initial margin. Similar results are obtained if clearing is consolidated across products: if a single IRS CCP merged with a single CDS CCP, initial margin requirements would fall further, by a similar percentage.

While consolidation of clearing appears to be beneficial from a risk management perspective, it is unlikely, at least in the short-to-medium term, that consolidation will take place on the scale modelled in Heller and Vause (2012). Legal and regulatory frameworks differ between jurisdictions, potentially inhibiting large-scale cross-border clearing, while commercial incentives may well support a multi-CCP environment for many products in large jurisdictions.

Interoperability has been proposed as a way of avoiding fragmentation in a multi-CCP environment. Singh (2013) argues that interoperability is a desirable way of proxying a ‘first-best’ solution in which all trades are cleared by a single CCP, because trades in different CCPs can be netted against each other. However, Singh also suggests that inter-CCP exposures would need to be covered through larger default fund contributions; whether interoperability is desirable depends on the relative size of these two effects.

Collateralisation under interoperability is modelled by Mägerle and Nellen (2011). Using a model of margin requirements in a multi-CCP environment, they show that interoperable CCPs would collect the same initial margins as a single, consolidated CCP, but that these margins may not collateralise total system exposure sufficiently if they fail to cover inter-CCP exposure appropriately. This is especially likely if the linked CCPs have different coverage levels for initial margin, or if more than two CCPs are linked.

Anderson, Dion and Pérez Saiz (2013) extend the Duffie and Zhu (2011) model to examine the exposure impact on small jurisdictions of introducing a global CCP. They demonstrate that having a single global CCP clearing all trades is the most efficient from an exposure perspective, and examine an alternative scenario in which there is a domestic and a foreign CCP. In this case they assess the impact on exposure of creating a link between them. Where the number of participants in the domestic CCP is small relative to that of the foreign CCP, their model finds that total system exposure increases. This is because the inter-CCP obligations created by the link outweigh the netting benefits to domestic participants.

The underlying model used in our paper is similar to that of Anderson *et al* (2013), in that it is a straightforward application of the Duffie and Zhu (2011) model of CCP links. However, we make different assumptions about the nature of participation and exposure that allow us to generalise the results to CCPs of any size. We also consider a number of useful extensions to the model – such as more general distributional assumptions, a different model of participation, and allowing more than two CCPs to be linked – to demonstrate its applicability to a wide variety of scenarios. Similarly to Anderson *et al* (2013), our focus is on system-wide levels of exposure, and whether linking CCPs in an otherwise fragmented clearing market increases or reduces that exposure.

3. Model Framework

We extend the Duffie and Zhu (2011) framework to compare two scenarios: CCPs operating separately and CCPs operating under a link arrangement. The variable of interest is the expected value of aggregate CCP exposures in the system. Since CCPs must meet the obligations of defaulting participants and linked CCPs, the exposures of CCPs to their participants are determined by the net variation-margin obligations that participants have to each other.[4] To the extent that collateral requirements accurately reflect underlying exposures, changes in exposures would flow through to a corresponding change in collateral requirements.

To isolate the effects of a link arrangement, the number of participants and the number and types of their positions remain constant across both scenarios. The simplest way to achieve this is to assume that every participant has positions with every other participant in each product, and that all bilateral positions are cleared centrally. This section starts by describing the two scenarios, before introducing the formal model, defining its key variables and outlining the main assumptions used to derive the analytical results presented in Section 4.

4 Variation margin is paid from a participant to a CCP (or from a CCP to a participant) to cover obligations arising from marking to market open positions with other participants. Typically, variation margin is payable daily.

3.1 Participation and Positions

In the unlinked scenario, participants have full market access and join all CCPs. This could reflect participants' desire to be able to trade all products, access whichever market is offering better terms, or offer their clients the full range of clearing venues. The CCPs each clear one unique 'product'.[5] The *aggregate CCP exposure* is calculated by adding all of the expected obligations that the CCPs have to their participants.

In the linked scenario, each CCP can offer all products. Since it would be costly to maintain membership of all CCPs, each participant uses only one CCP for its full portfolio. Participant numbers in each CCP are treated as variables that always sum to the total number of participants in the unlinked case. All positions between participants using different CCPs go through the link, creating inter-CCP exposures. The aggregate CCP exposure in this case is the sum of these expected inter-CCP obligations and the obligations of the CCPs to their own participants.

In reality, not all participants are likely to be members of all CCPs in the unlinked case. However, modifying the model to allow for this would mean that the link gave participants more access to other participants than they had without the link. This would result in more positions being held, and therefore would not isolate the netting effects of a link arrangement.

In both cases, our approach to measuring aggregate exposure differs from that used by Anderson *et al* (2013), where participant exposures to CCPs are included. Our approach only accounts for CCPs' exposures, both to their participants and to each other when linked. Therefore, our conclusions about aggregate exposure will be indicative of required aggregate collateral, although we do not explicitly quantify collateral impacts. CCPs are regarded as prudent, regulated counterparties designed to withstand any individual participant default, even in extreme but plausible circumstances. Accordingly, it is uncommon for participants to receive collateral from CCPs. For this reason, participant exposure to CCPs is not included in our measure of aggregate exposure. However, collateral is required to manage inter-CCP exposures, owing to the large disruptions to the financial

5 These may be the same product traded in different venues. This is the case for the current interoperability arrangements in place in European cash equity markets. For this reason, Section 4 assumes that CCPs clear the same products as a baseline case.

system that could be caused from a CCP defaulting on its obligations to a linked CCP. Hence these obligations count towards our measure of aggregate exposure.

3.2 Notation

3.2.1 Participant obligations

Consider N market participants trading K products through a CCP. For participants i and j, let X_{ij}^k represent the net obligation of i to j in product k that could accrue in the future over the close-out period. From the CCP's perspective, X_{ij}^k is the obligation the CCP has to participant j in product k if participant i defaulted. For a close-out period similar in length to the frequency over which variation margining is calculated, X_{ij}^k can be thought of as a variation-margin obligation to the CCP. *Ex ante*, X_{ij}^k is a random variable that depends on bilateral positions in product k and product k's price movements.

If $X_{ij}^k > 0$, participant i has a positive obligation to the CCP arising from its open interest with j in product k; accordingly, it would pay variation margin to the CCP for this position. A negative value of X_{ij}^k implies that i should receive payment from the CCP on its positions with j in product k. The *net* obligation of participant i to a CCP clearing K products, however, is the sum across all j in N and k in K. Letting:

- X_{ij} be the net obligation of participant i to participant j in all products k
- X_i^k be the net obligation of participant i to all other participants j in product k
- X_i be the net obligation of participant i to all participants in all products.

We have:

$$X_i = \sum_{j \in N} X_{ij} = \sum_{j \in N} \sum_{k \in K} X_{ij}^k = \sum_{k \in K} X_i^k$$

By definition, $X_{ij}^k = -X_{ji}^k$, and $X_{ii}^k = 0$. This means that the obligations of N participants in K products is characterised by $KN(N-1)/2$ random variables X_{ij}^k, as shown in Figure 2 for the case of product k for $N = 6$.

Figure 2: Obligations in Asset Class k for $N = 6$

0	$X^k_{1,2}$	$X^k_{1,3}$	$X^k_{1,4}$	$X^k_{1,5}$	$X^k_{1,6}$
$-X^k_{1,2}$	0	$X^k_{2,3}$	$X^k_{2,4}$	$X^k_{2,5}$	$X^k_{2,6}$
$-X^k_{1,3}$	$-X^k_{2,3}$	0	$X^k_{3,4}$	$X^k_{3,5}$	$X^k_{3,6}$
$-X^k_{1,4}$	$-X^k_{2,4}$	$-X^k_{3,4}$	0	$X^k_{4,5}$	$X^k_{4,6}$
$-X^k_{1,5}$	$-X^k_{2,5}$	$-X^k_{3,5}$	$-X^k_{4,5}$	0	$X^k_{5,6}$
$-X^k_{1,6}$	$-X^k_{2,6}$	$-X^k_{3,6}$	$-X^k_{4,6}$	$-X^k_{5,6}$	0

3.2.2 Default loss and exposure

When trades in a product are novated to a CCP, the CCP is responsible for meeting the obligations of any defaulting participants. If participant i defaults, the CCP's default loss in product k, D_i^k, would be:

- the net of all participant i's obligations if $X_i^k > 0$
- zero if $X_i^k \leq 0$.

That is,

$$D_i^k = \max(X_i^k, 0) = \max\left(\sum_{j \in N} X_{ij}^k, 0\right)$$

We consider an exposure to be the expected loss given default of a participant.[6] This ignores the probability of the default occurring, which is consistent with common practice for setting collateral requirements (and which simplifies the analysis considerably). The CCP's exposure to participant i in product k is therefore:

$$Y_i^k = \mathbb{E}[D_i^k] = \mathbb{E}[\max(X_i^k, 0)]$$

Typically, the participant would be required to post collateral (initial margin) when entering the trade; this collateral is then drawn on by the CCP to cover the loss in

6 This paper focuses on loss given default, in line with CCP margining practice. Modelling the actual probability of participant or CCP default is outside the scope of this paper.

the event of a default. The level of collateral required depends on the size of the default losses the CCP expects to have to cover.[7] Accordingly, there is a clear relationship between expected losses given default and collateral requirements.

In the case of C CCPs, we denote as ${}^{c}Y_i^k$ the exposure of CCP c to participant i in product k (based on the corresponding loss given default ${}^{c}D_i^k$). We denote as ${}^{c}Y_P$ the exposure of CCP c to *all* participants in all of the products that it clears.

In the case of linked CCPs, we denote as ${}^{c}_{d}Y$ the exposure of CCP c to CCP d in all products cleared across the link.

We denote as ${}^{c}Y_U$ and ${}^{c}Y_L$ the *aggregate* exposure of CCP c (in all products), both to participants and to other CCPs, in the unlinked and linked cases respectively. Y_U and Y_L denote the corresponding total exposures for *all* CCPs.

The following scenarios consider the case of two CCPs and two products ($C = K = 2$).

3.3 Aggregate CCP Exposure: Two Unlinked CCPs

Consider N participants, all members of two unlinked CCPs, CCP1 and CCP2. These CCPs each clear only one product – products 1 and 2, respectively. CCP1 is responsible for meeting the obligations of any defaulting participant in product 1, and CCP2 is responsible for meeting the obligations of any defaulting participant in product 2.

7 In accordance with relevant international standards (CPSS-IOSCO 2012), a CCP aims to be able to cover obligations arising in the event of the default of its largest one or two participants (by exposure) in 'extreme but plausible' market conditions, which are determined with reference to stress-test scenarios. For particularly extreme circumstances where initial margin does not cover the losses from a participant default, a CCP will generally hold additional resources. This typically takes the form of a default fund, which pools the CCP's own resources and participant contributions to cover these more extreme circumstances.

The expected exposure for CCP1, ${}^1Y_U$, is the sum of the expected losses given default in product 1 over all N participants, and similarly for CCP2:

$$\begin{aligned} {}^1Y_U &= \sum_{i \in N} \mathbb{E}({}^1D_i^1) \\ &= \sum_{i \in N} \mathbb{E}\left[\max\left(\sum_{j \in N} X_{ij}^1, 0\right)\right] \end{aligned}$$

$${}^2Y_U = \sum_{i \in N} \mathbb{E}\left[\max\left(\sum_{j \in N} X_{ij}^2, 0\right)\right]$$

The relationship of the CCP exposures to the participant obligations is illustrated in Figure 3. Because the CCPs are not linked, there is no inter-CCP exposure. Therefore, the *total* exposure across all CCPs in the unlinked case is:

$$Y_U = {}^1Y_U + {}^2Y_U = \sum_{i \in N}\left(\mathbb{E}\left[\max\left(\sum_{j \in N} X_{ij}^1, 0\right)\right] + \mathbb{E}\left[\max\left(\sum_{j \in N} X_{ij}^2, 0\right)\right]\right) \quad (1)$$

Figure 3: Unlinked CCP Exposures in Products 1 and 2 for $N = 6$

$$\left.\begin{array}{|c|c|c|c|c|c|l}
\hline
0 & X_{1,2}^1 & X_{1,3}^1 & X_{1,4}^1 & X_{1,5}^1 & X_{1,6}^1 & \rightarrow {}^1Y_1^1 \\ \hline
-X_{1,2}^1 & 0 & X_{2,3}^1 & X_{2,4}^1 & X_{2,5}^1 & X_{2,6}^1 & \rightarrow {}^1Y_2^1 \\ \hline
-X_{1,3}^1 & -X_{2,3}^1 & 0 & X_{3,4}^1 & X_{3,5}^1 & X_{3,6}^1 & \rightarrow {}^1Y_3^1 \\ \hline
-X_{1,4}^1 & -X_{2,4}^1 & -X_{3,4}^1 & 0 & X_{4,5}^1 & X_{4,6}^1 & \rightarrow {}^1Y_4^1 \\ \hline
-X_{1,5}^1 & -X_{2,5}^1 & -X_{3,5}^1 & -X_{4,5}^1 & 0 & X_{5,6}^1 & \rightarrow {}^1Y_5^1 \\ \hline
-X_{1,6}^1 & -X_{2,6}^1 & -X_{3,6}^1 & -X_{4,6}^1 & -X_{5,6}^1 & 0 & \rightarrow {}^1Y_6^1 \\ \hline
\end{array}\right\} {}^1Y_U$$

$$\left.\begin{array}{|c|c|c|c|c|c|l}
\hline
0 & X_{1,2}^2 & X_{1,3}^2 & X_{1,4}^2 & X_{1,5}^2 & X_{1,6}^2 & \rightarrow {}^2Y_1^2 \\ \hline
-X_{1,2}^2 & 0 & X_{2,3}^2 & X_{2,4}^2 & X_{2,5}^2 & X_{2,6}^2 & \rightarrow {}^2Y_2^2 \\ \hline
-X_{1,3}^2 & -X_{2,3}^2 & 0 & X_{3,4}^2 & X_{3,5}^2 & X_{3,6}^2 & \rightarrow {}^2Y_3^2 \\ \hline
-X_{1,4}^2 & -X_{2,4}^2 & -X_{3,4}^2 & 0 & X_{4,5}^2 & X_{4,6}^2 & \rightarrow {}^2Y_4^2 \\ \hline
-X_{1,5}^2 & -X_{2,5}^2 & -X_{3,5}^2 & -X_{4,5}^2 & 0 & X_{5,6}^2 & \rightarrow {}^2Y_5^2 \\ \hline
-X_{1,6}^2 & -X_{2,6}^2 & -X_{3,6}^2 & -X_{4,6}^2 & -X_{5,6}^2 & 0 & \rightarrow {}^2Y_6^2 \\ \hline
\end{array}\right\} {}^2Y_U$$

3.4 Aggregate CCP Exposure: Two Linked CCPs

When CCP1 and CCP2 are linked, we assume participants consolidate their full portfolio into one CCP. Let there be N_1 and N_2 participants that consolidate in CCP1 and CCP2 respectively, where $N_1 + N_2 = N$. Since each participant's full portfolio (consisting of positions in products 1 and 2) is in a single CCP, participant obligations can be netted across products 1 and 2.

Under the link, CCP1 now faces two sources of exposure. The first arises from its N_1 participants' positions in products 1 and 2. CCP1 is responsible for meeting the obligations that any of its defaulting participants have, whether these be to other members of CCP1 or to members of CCP2. That is, where participant i is a member of CCP1, CCP1 is exposed to the net (if positive) of i's obligations X_{ij}^k across *all* j in N and *all* k in K (this is represented by the top three rows in Figure 4).

Figure 4: Linked CCP Exposures in Asset Classes 1 and 2 for $N_1 = N_2 = 3$

$$
\begin{array}{cccccccl}
 & & & {}^2_1Y \uparrow & & & & \\
0 & X^1_{1,2}+X^2_{1,2} & X^1_{1,3}+X^2_{1,3} & X^1_{1,4}+X^2_{1,4} & X^1_{1,5}+X^2_{1,5} & X^1_{1,6}+X^2_{1,6} & \rightarrow {}^1Y_1 & \multirow{3}{*}{\}\ {}^1Y_P} \\
-X^1_{1,2}-X^2_{1,2} & 0 & X^1_{2,3}+X^2_{2,3} & X^1_{2,4}+X^2_{2,4} & X^1_{2,5}+X^2_{2,5} & X^1_{2,6}+X^2_{2,6} & \rightarrow {}^1Y_2 & \\
-X^1_{1,3}-X^2_{1,3} & -X^1_{2,3}-X^2_{2,3} & 0 & X^1_{3,4}+X^2_{3,4} & X^1_{3,5}+X^2_{3,5} & X^1_{3,6}+X^2_{3,6} & \rightarrow {}^1Y_3 & \\
-X^1_{1,4}-X^2_{1,4} & -X^1_{2,4}-X^2_{2,4} & -X^1_{3,4}-X^2_{3,4} & 0 & X^1_{4,5}+X^2_{4,5} & X^1_{4,6}+X^2_{4,6} & \rightarrow {}^2Y_4 & \multirow{3}{*}{\}\ {}^2Y_P} \\
-X^1_{1,5}-X^2_{1,5} & -X^1_{2,5}-X^2_{2,5} & -X^1_{3,5}-X^2_{3,5} & -X^1_{4,5}-X^2_{4,5} & 0 & X^1_{5,6}+X^2_{5,6} & \rightarrow {}^2Y_5 & \\
-X^1_{1,6}-X^2_{1,6} & -X^1_{2,6}-X^2_{2,6} & -X^1_{3,6}-X^2_{3,6} & -X^1_{4,6}-X^2_{4,6} & -X^1_{5,6}-X^2_{5,6} & 0 & \rightarrow {}^2Y_6 & \\
 & \downarrow {}^1_2Y & & & & & &
\end{array}
$$

CCP1's exposure to its N_1 participants is therefore:

$$
{}^1Y_P = \sum_{i \in N_1} \mathbb{E}\left[\max\left(\sum_{j \in N}(X^1_{ij} + X^2_{ij}), 0\right)\right]
$$

The second source of exposure faced by CCP1 in the linked case comes from CCP2. This is the expected loss that CCP1 would experience if CCP2 defaulted. In such an event, CCP1 would be responsible for meeting all obligations passing through the link, that is, all obligations of CCP2's members to CCP1's members. The default loss experienced by CCP1 would then be the net (if positive) of the obligations X_{ij}^k across all i in N_2, j in N_1 and k in K (this is represented by the bottom left quadrant in Figure 4).

CCP1's exposure to CCP2 is therefore:

$$
{}_2^1Y = \mathbb{E}\left[\max\left(\sum_{i\in N_2}\sum_{j\in N_1}(X_{ij}^1+X_{ij}^2),0\right)\right]
$$

CCP1's overall exposure in the linked case is the sum of the exposure it has to its own participants and the exposure that it has to CCP2:

$$
\begin{aligned}
{}^1Y_L &= {}^1Y_P + {}_2^1Y \\
&= \sum_{i\in N_1}\mathbb{E}\left[\max\left(\sum_{j\in N}(X_{ij}^1+X_{ij}^2),0\right)\right] \\
&\quad +\mathbb{E}\left[\max\left(\sum_{i\in N_2}\sum_{j\in N_1}(X_{ij}^1+X_{ij}^2),0\right)\right]
\end{aligned}
$$

Similarly, CCP2 has two sources of exposure; that arising from its N_2 participants' obligations to all N participants (${}^2Y_P$, represented by the bottom three rows in Figure 4), and the inter-CCP exposure arising from CCP1's N_1 participants to its N_2 participants (${}_1^2Y$, represented by the top right quadrant in Figure 4). CCP2's overall exposure in the linked case is:

$$
\begin{aligned}
{}^2Y_L &= {}^2Y_P + {}_1^2Y \\
&= \sum_{i\in N_2}\mathbb{E}\left[\max\left(\sum_{j\in N}(X_{ij}^1+X_{ij}^2),0\right)\right] \\
&\quad +\mathbb{E}\left[\max\left(\sum_{i\in N_1}\sum_{j\in N_2}(X_{ij}^1+X_{ij}^2),0\right)\right]
\end{aligned}
$$

The total exposure across all CCPs in the linked case is therefore:

$$
\begin{aligned}
Y_L &= {}^1Y_L + {}^2Y_L \\
&= \sum_{i \in N} \mathbb{E}\left[\max\left(\sum_{j \in N}(X_{ij}^1 + X_{ij}^2), 0\right)\right] \qquad (2) \\
&+ \mathbb{E}\left[\max\left(\sum_{i \in N_2}\sum_{j \in N_1}(X_{ij}^1 + X_{ij}^2), 0\right)\right] + \mathbb{E}\left[\max\left(\sum_{i \in N_1}\sum_{j \in N_2}(X_{ij}^1 + X_{ij}^2), 0\right)\right]
\end{aligned}
$$

The first term of Equation (2) reveals the additional intra-CCP netting that a link allows. Positive and negative obligations in products 1 and 2 can now offset each other because, for a given participant, they are concentrated in the one CCP. This reduces the CCPs' exposures to their own participants relative to the unlinked case. The second two terms, however, show the additional inter-CCP exposures introduced by the link. The key question posed by this paper is which of these two effects is larger. This determines whether CCP link arrangements introduce additional exposure, which will in turn need to be covered by more collateral. To answer this question, we now impose a distribution on participant-level obligations, X_{ij}^k.

3.5 Distributional Assumptions

Assuming a distribution for X_{ij}^k with certain properties allows us to calculate aggregate exposures in the linked and unlinked cases. We will initially assume, for all $i \neq j$, that the X_{ij}^k are identically and normally distributed $\mathcal{N}(0, \sigma_k^2)$, and are independent across i and j, but possibly correlated across k (with a constant correlation for all i and j).

The assumption that the X_{ij}^k are identically distributed imposes homogeneity on the participants. For each asset class, obligations between pairs of participants are assigned randomly from the same distribution, implying that any pair of participants is as likely as any other pair to have a bilateral obligation of a given size or sign, with no pair of participants more likely than any other pair to have a large (or small) bilateral obligation.[8]

8 Section 5 extends the model to allow for heterogeneity.

Although financial time series often display excess kurtosis, a normally distributed X_{ij}^{k} can be taken as an acceptable first approximation. This is because the random variable of interest is ultimately the *net* obligations of participants to their CCP (X_{i}^{k}), which are summations of the net obligations between participants (X_{ij}^{k}). Under the central limit theorem, these summations (for a sufficiently large number of participants) lead to a normal distribution for X_{i}^{k} regardless of the distribution of X_{ij}^{k}, as long as they are identically distributed with finite variance and independent across i and j. Further, the X_{ij}^{k} are themselves *net* obligations between participants, and may represent the sums of various long and short positions in different products within contract class k. For these reasons, Section 4 will assume the normality of X_{ij}^{k} in order to derive simple analytical solutions for Equations (1) and (2), while alternative distributions (as well as non-independence) will be considered using simulation methods in Section 5.

The distribution's zero mean relates to market efficiency, implying that participants' expected profits on their obligations to individual counterparties are each zero. While participants may have an expectation that their positions will be profitable, intermediate gains and losses are, in expectation, likely to be close to zero during a short close-out period. The variance for each product may reflect the volatility of price movements in the underlying products, but may also reflect the conscious decisions of participants to target desired levels of volatility, holding the number and combination of positions that will best achieve these levels.

The independence assumption implies that one participant's exposure is not related to another participant's exposure, and that a participant's exposure to one counterparty is not related to its exposure to another counterparty. This is particularly reasonable for centrally cleared markets, as the end-counterparty to each trade has virtually no effect on the value of the trade.

Participants' obligations in different products may be correlated. This may reflect covariance in the price movements of the products, or it may reflect the decisions of participants to hedge across products, with lower correlations reflecting greater hedging.

4. Results

Consistent with the above assumptions, we can represent the obligations between i and j in products 1 and 2 using a bivariate normal distribution:

$$\begin{bmatrix} X_{ij}^1 \\ X_{ij}^2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right)$$

The correlation ρ between products does not affect aggregate CCP exposure in the unlinked case (Equation (1)). The independence of X_{ij}^k across i and j means that the netted obligations $X_i^k = \sum_{j=1}^N X_{ij}^k$ are normally distributed with mean zero and variance $(N-1)\sigma_k^2$. The default losses $D_i^k = \max(X_i^k, 0)$ then follow a mixture distribution, with probability $1/2$ of being zero, and probability $1/2$ of following a half-normal distribution, with expected value $\sigma_k\sqrt{2(N-1)}/\sqrt{\pi}$. The CCP's exposure to participant i is therefore:

$$\mathbb{E}[\max(X_i^k, 0)] = \frac{\sigma_k\sqrt{(N-1)}}{\sqrt{2\pi}} \tag{3}$$

It follows that aggregate CCP exposure in the unlinked case is:

$$Y_U = N\left(\frac{\sigma_1\sqrt{N-1}}{\sqrt{2\pi}} + \frac{\sigma_2\sqrt{N-1}}{\sqrt{2\pi}}\right) = (\sigma_1+\sigma_2)\frac{N\sqrt{N-1}}{\sqrt{2\pi}}$$

In the linked case, the correlation between product classes affects aggregate CCP exposure through multilateral netting (across products), influencing the variance of the netted obligations and therefore the expected value of default losses. The obligations are symmetrically distributed around zero, so the two terms for inter-CCP exposure in Equation (2) will be equal.[9]

9 When a random variable, Z, is symmetrically distributed around zero, $\mathbb{E}[\max(0,Z)] = -\mathbb{E}[\min(0,-Z)]$. With $X_{ij}^k = -X_{ji}^k$ (and under the assumption that these obligations are symmetrically distributed around zero), Equation (2) can be simplified to be

$$\begin{aligned} Y_L &= {}^1Y_L + {}^2Y_L \\ &= \sum_{i\in N}\mathbb{E}\left[\max\left(\sum_{j\in N}(X_{ij}^1 + X_{ij}^2), 0\right)\right] + 2\mathbb{E}\left[\max\left(\sum_{i\in N_2}\sum_{j\in N_1}(X_{ij}^1 + X_{ij}^2), 0\right)\right] \end{aligned}$$

Aggregate CCP exposure in the linked case is therefore:

$$
\begin{aligned}
Y_L &= N\left(\frac{\sqrt{(N-1)(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)}}{\sqrt{2\pi}}\right)+2\left(\frac{\sqrt{N_1N_2(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)}}{\sqrt{2\pi}}\right)\\
&= (N\sqrt{N-1}+2\sqrt{N_1N_2})\frac{\sqrt{\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2}}{\sqrt{2\pi}}
\end{aligned}
$$

The ratio of exposures in the linked scenario to exposures in the unlinked scenario is:

$$
\begin{aligned}
R \equiv \frac{Y_L}{Y_U} &= \frac{(N\sqrt{N-1}+2\sqrt{N_1N_2})\sqrt{\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2}}{(\sigma_1+\sigma_2)N\sqrt{N-1}}\\
&= \frac{\sqrt{\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2}}{(\sigma_1+\sigma_2)}\left(1+\frac{2\sqrt{N_1N_2}}{N\sqrt{N-1}}\right)
\end{aligned}
\tag{4}
$$

We use this *exposure-netting ratio*, R, to assess the effect of linking CCPs on aggregate CCP exposure. The link increases aggregate CCP exposure if $R > 1$ and reduces it if $R < 1$.

Overall, we find that the main drivers of a link's effect on aggregate CCP exposure, and ultimately on collateral requirements, are the number of total participants and the asymmetry of activity between CCPs. A higher number of participants increases the scope for multilateral netting (across products) while lowering the proportion of individual bilateral exposures that pass through the link, thereby reducing exposure. Asymmetry in product variances reduces the scope for cross-product netting, increasing exposure. Asymmetry of participant numbers means that the link arrangement plays relatively less of a role, because while any link delivers the same degree of portfolio netting benefits, the larger is one CCP relative to the other, the smaller is the number of positions that are cleared through the link, which lowers the inter-CCP exposure. The intuition behind this result is that, as shown by Duffie and Zhu (2011), the optimal market structure from a risk management perspective is a single CCP clearing all products. The greater the asymmetry in participant numbers between two linked CCPs, the more that their linked arrangement approximates this optimal case.

For the remainder of this section, we investigate how these factors affect R through the model's key variables.

4.1 Determinants of the Exposure-netting Ratio

4.1.1 Participant numbers

For a given number of total participants N, the ratio R is decreasing in the asymmetry between N_1 and N_2.[10] This is because, holding N constant, the asymmetry of membership in the linked case lowers the number of positions that are traded across the link and therefore the overall inter-CCP exposure, without reducing aggregate portfolio netting. R is also decreasing in N, the overall number of participants, although this effect diminishes as N increases.[11] We also consider a baseline case that reflects, in a stylised way, the current interoperability arrangements in European cash equity markets. Under these arrangements, the link enables a choice of CCP for a product that is already cleared by both CCPs. Assuming that the number of contracts traded by each participant is similar across the two CCPs, we can set $\sigma_1 = \sigma_2$.

Further, we treat participants' trading activities across CCP1 and CCP2 as being uncorrelated. Recall that ρ is not the correlation between returns on products 1 and 2, but rather the correlation between *obligations* (X_{ij}^k) in products 1 and 2. Accordingly, ρ reflects both the correlation between the positions taken in products 1 and 2, as well as their returns. When the two CCPs offer the same products, the returns will be perfectly correlated across the CCPs. However, participants positions are uncorrelated across the two CCPs, and so too are the obligations arising from them. Since ρ is a correlation of obligations, we set $\rho = 0$.

---

10 This can be shown by noting that, holding N constant, R is increasing in N_1N_2. Subject to the constraint that $N_1 + N_2 = N$, N_1N_2, and therefore R, falls in the asymmetry between N_1 and N_2.

11 This can be shown by taking $N_2 = aN_1$ for any constant $a > 0$ and letting $N_1 \to \infty$.

In this baseline case, where X_{ij}^1 and X_{ij}^2 are uncorrelated and drawn from distributions with equal variances ($\sigma_1 = \sigma_2$ and $\rho = 0$):

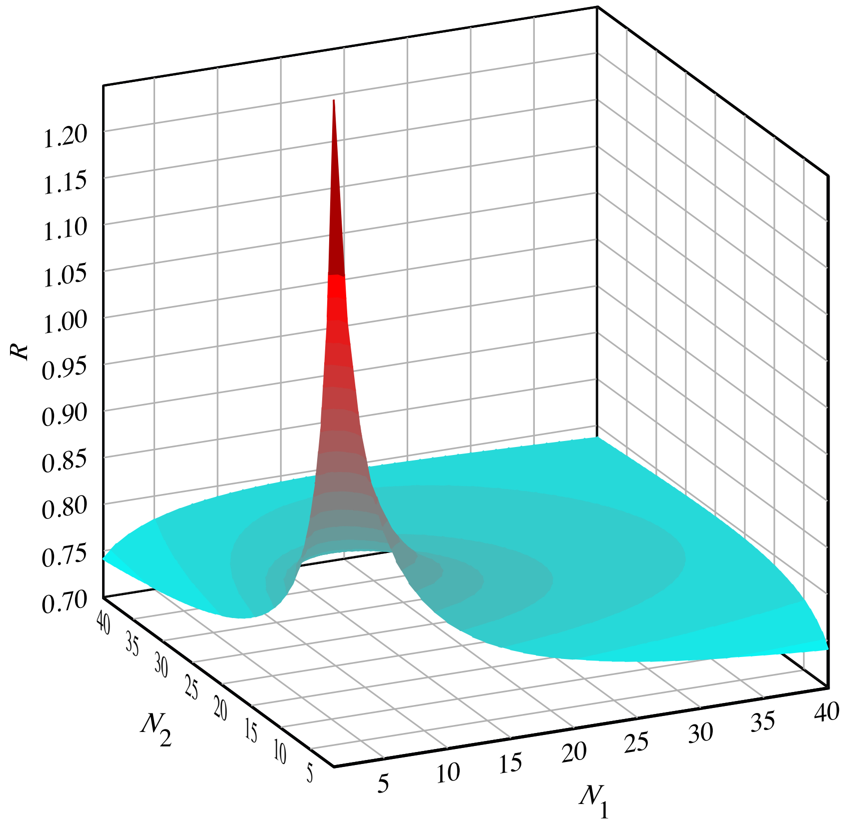
$$R = \frac{1}{\sqrt{2}} + \frac{\sqrt{2N_1N_2}}{N\sqrt{N-1}} \qquad \text{and} \qquad \lim_{N_1 \to \infty} R = \frac{1}{\sqrt{2}}$$

This gives $R < 1$ (that is, a link reduces exposures) if either N_1 or N_2 is greater than 4, which is lower than is the case for any CCP of which we are aware.[12] As an example, using $N_1 = 71$ and $N_2 = 159$ (the 2011 participation numbers for SIX x-clear and LCH.Clearnet Ltd, two linked European CCPs) gives $R = 0.75$. However, this figure assumes that all 230 participants were previously members of both CCPs, and that each resigned its membership of one of the CCPs once the link was formed. Section 5.2 explores the implications of participants retaining multiple CCP memberships after a link is formed.

The values of R over varying N_1 and N_2 in this baseline case are shown in Figure 5, which illustrates the effects of asymmetry in N_1 and N_2 and the asymptotic behavior of R as N increases. It can be seen that only for very small values of N_1 and N_2 is $R > 1$.

12 To provide some context, of the 35 CCPs internationally that provided 2011 participation data to the Committee on Payment and Settlement Systems (CPSS), the two lowest participation counts were 5 and 9. ASX Clear (Futures) was third lowest with 17 participants, and ASX Clear had 43 participants. The average number of participants was 210 after excluding an outlier in the right tail.

Figure 5: Ratio R as a Function of N_1 and N_2 – Baseline Case
$\rho = 0, \sigma_1 = \sigma_2$



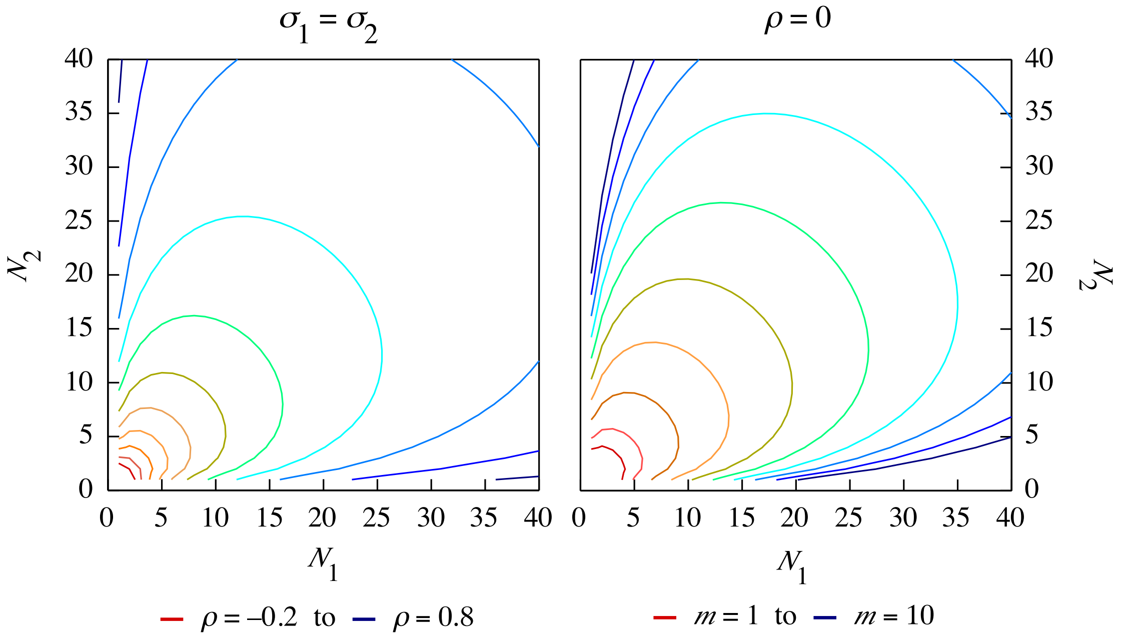
4.1.2 Variances and correlations

Although the ratio R is independent of the absolute size of the product variances σ_1^2 and σ_2^2, it is increasing in both the asymmetry between σ_1^2 and σ_2^2, and in the correlation coefficient ρ.[13] The number of participants for which aggregate exposure declines under the link arrangement depends on both ρ and m. Figure 6 plots the thresholds of R (as a function of N_1 and N_2, as in Figure 5) for which a

13 The partial derivative of R with respect to ρ is positive. Taking $\sigma_2 = m\sigma_1$, the partial derivative of R with respect to σ_1 is zero, while the partial derivative of R with respect to m is proportional to $(\rho - 1)(1 - m)$, meaning that (for $-1 \leq \rho < 1$), R decreases with m when $m < 1$ and increases with m when $m > 1$. That is, R increases in the asymmetry between σ_1 and σ_2, unless $\rho = 1$, in which case the partial derivative of R with respect to m is zero.

CCP link reduces exposures; that is, the contours where $R = 1$ as ρ (left panel) and $m = \sigma_2/\sigma_1$ (right panel) are varied.[14]

Figure 6: Thresholds of $R = 1$ for Varying ρ (left) and Varying $m = \sigma_2/\sigma_1$ (right)



Note: Lines are intervals of 0.1 for left panel and 1 for right panel

A higher (positive) correlation between X_{ij}^1 and X_{ij}^2 means that when X_{ij}^1 is positive, X_{ij}^2 will tend to be positive, and vice versa. This implies that there are fewer netting opportunities, which reduces the benefits of consolidating each participant's activity in a single CCP, and increases R. In the extreme case of perfect positive correlation, no cross-product netting is possible, and the CCPs' exposures to their own participants remain unchanged in aggregate when moving from the unlinked to the linked case. The only effect of the link, therefore, is to create an inter-CCP exposure, giving $R > 1$ for any participation numbers. As the correlation decreases, the cross-product netting benefits, and the exposure reductions introduced by the link, increase.

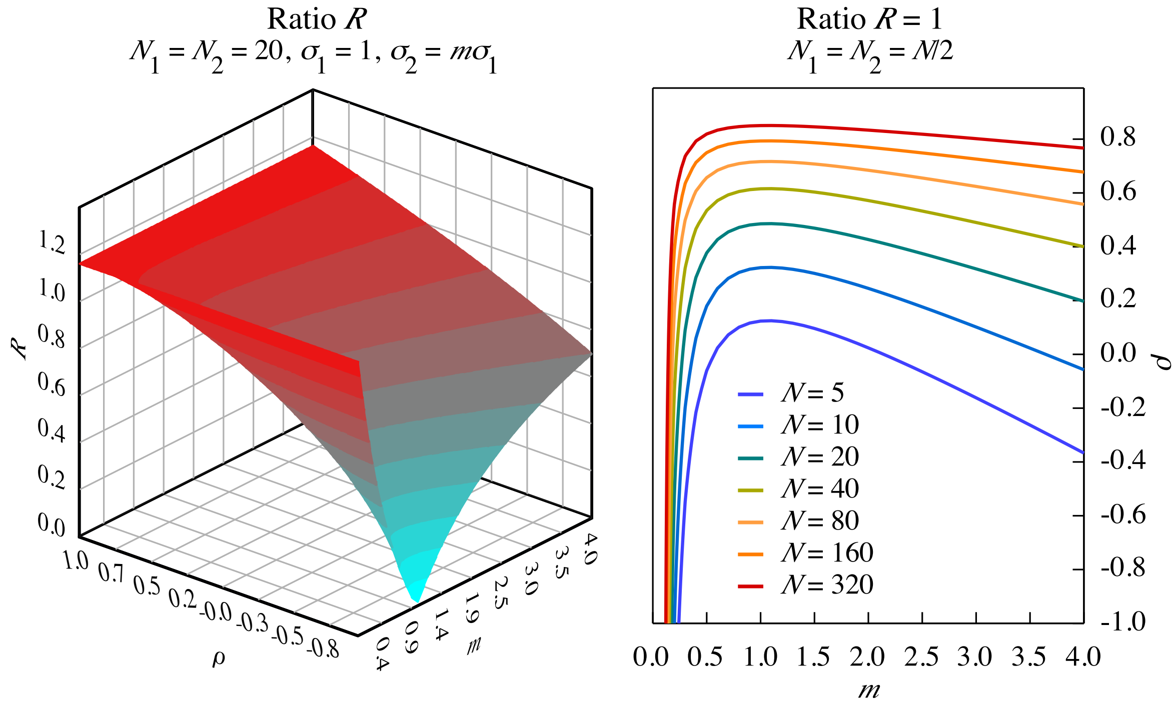
14 For example, when $\rho = 0.4$ in the left panel, the contour passes through $(N_1, N_2) = (10, 15)$ and $(N_1, N_2) = (15, 10)$. This indicates that, for $\sigma_1 = \sigma_2$ and $\rho = 0.4$, adding further participants to either N_1 or N_2 beyond this value will make the link arrangement beneficial from a CCP exposure (and, therefore, collateral) perspective. Increasing ρ or m requires more participants to make the link beneficial.

The asymmetry of the variances σ_1^2 and σ_2^2 also plays an important role in determining R.[15] If σ_1^2 and σ_2^2 differ greatly, it becomes less likely that positive and negative values of X_{ij}^1 and X_{ij}^2 will offset each other, because the probability of them being a similar absolute size is reduced. Therefore, more asymmetric variances reduce netting benefits in the same way that a higher ρ does, leading to a higher R. This is illustrated in the right panel of Figure 6, which fixes $\rho = 0$ and varies $m = \sigma_2/\sigma_1$. As m increases beyond one, the contours move outwards, meaning that more participants are required to ensure that the adoption of a link reduces exposures.

Conversely, cross-product netting benefits are large where σ_1^2 and σ_2^2 are similar (leading to no linked exposure in the extreme case of $\rho = -1$ and $m = 1$). The left panel of Figure 7 plots the ratio R as a function of ρ and $m = \sigma_2/\sigma_1$ for a fixed value of N, with $N_1 = N_2 = N/2$. It shows that the ratio R increases with the correlation between products and the asymmetry of variances. The right panel plots the contours of $R = 1$ for different N, with the area above each contour representing the range of (ρ, m) values for which a CCP link increases exposure and the area below each contour representing the range of values for which the link reduces exposure. The latter increases with the number of participants.

15 In the context of a potential link between a small domestic CCP offering product 1 and a large foreign CCP offering product 2 (as considered in Anderson *et al* (2013)), this could be thought of as the risk-adjusted relative sizes of a typical trade in the domestic and foreign product.

Figure 7: Ratio R as a Function of ρ and m (left) and Thresholds of $R = 1$ for Varying N (right)



4.1.3 Implications and summary

Our result that a main driver of R is the asymmetry of activity differs significantly from the result of Anderson *et al* (2013). Under their model, there is a domestic and a foreign CCP; if the ratio of domestic to foreign participants is sufficiently low, a CCP link increases aggregate exposure. This conclusion arises from their assumption that foreign participants do not join the domestic CCP in the unlinked case. When the CCPs link, therefore, foreign participants obtain no netting benefits. When there is a relatively high proportion of foreign participants, the result is that the relatively low netting benefits are outweighed by the inter-CCP exposure.

In our model, high asymmetry in domestic and foreign participation numbers would make a link arrangement *less* likely to increase exposures, because we assume that all participants are members of *both* CCPs in the unlinked case. The assumption in our model is the relevant one for many smaller domestic CCPs. In Australia, for instance, many large international banks and investment houses participate in the domestic CCPs, and make up the majority of participants in the futures CCP, ASX Clear (Futures). Further, while many domestic participants from smaller jurisdictions do not join foreign CCPs directly, they often use intermediaries to clear indirectly in those markets.

To summarise, our model suggests that, under the given assumptions and under most plausible parameter combinations, a move from two unlinked CCPs (where all market participants are members of both) to two linked CCPs (where no market participants are members of both) will reduce aggregate CCP exposure, because the exposure-reducing effects of cross-product netting will exceed the inter-CCP exposure. The reductions in exposure due to CCP links increase with the number of participants and the asymmetry of the number of participants using each CCP, but decrease with the correlation between products and the asymmetry of product variances.

5. Extensions

5.1 Non-normally Distributed Obligations

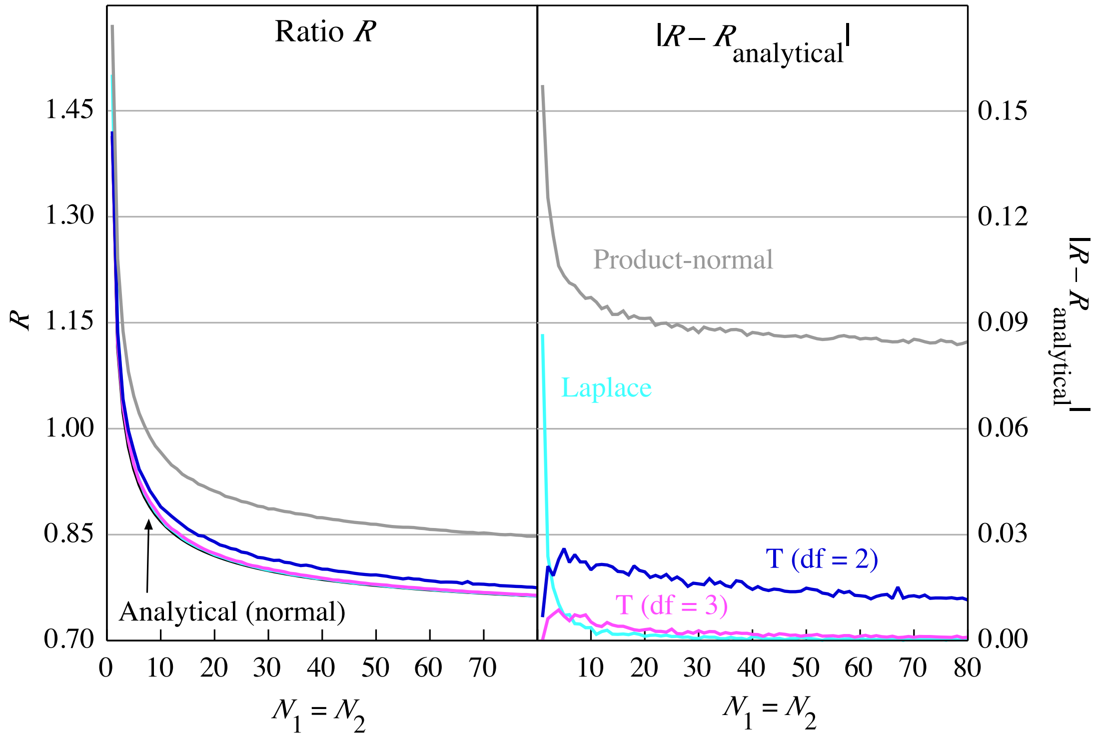
The analytical solution for R (Equation (4)) is based on the assumption that the distribution of obligations X_{ij}^k is normal. This solution may remain a fairly accurate approximation even if the assumption of normality is relaxed, provided that we retain the assumptions that the X_{ij}^k are identically distributed with finite variance and independent across i and j. These conditions allow the central limit theorem to be applied in estimating the distribution of default losses D_i^k, which are based on the sums of the random variables X_{ij}^k.

Since $X_i^k = \sum_{j \in N} X_{ij}^k$, the central limit theorem implies that for a large N, the distribution of X_i^k will be approximately normal, with mean zero and variance $(N-1)\sigma_k$ (as in Section 3), irrespective of the particular distribution of X_{ij}^k (provided they are independently and identically distributed with mean zero and a finite variance σ_k). The CCP's exposure to participant i will still be as in Equation (3), and the ratio R as in Equation (4), provided N is sufficiently large.

Symmetric, leptokurtic distributions that may be more appropriate for modelling financial returns include the Laplace (or double exponential) distribution and the t-distribution. Figure 8 compares the baseline analytical solution derived in Section 3 with the results of Monte Carlo simulations of the exposure ratio R for normal and non-normal distributions of X_{ij}^k (assuming zero correlation between product classes). Replacing the normal distribution for X_{ij}^k with a Laplace distribution or a *t*-distribution with 3 degrees of freedom (both finite-variance distributions) yields an exposure-netting ratio R that converges rapidly toward the analytical solution as N gets larger, as expected under the central limit theorem.

Relaxing the finite variance assumption, Figure 8 includes the case of the *t*-distribution with two degrees of freedom, as an example of a distribution with infinite variance. Under this distribution, the ratio R does not converge to the analytical solution, but remains slightly higher. However, the difference is sufficiently small as to be relatively insignificant in assessing whether or not CCP links increase or reduce aggregate CCP exposure.

Figure 8: Ratio R for Various Distributions of X_{ij}^k (left) and Distance from Analytical Solution of R under Normality Assumption (right)



The 'product-normal' case in Figure 8 relaxes the assumption of independence (and removes applicability of the central limit theorem), by decomposing the random variable for obligations, X_{ij}^k into the product of two random variables; V_{ij}^k, representing the size and direction of i's open position with j in product class k, and a price movement in product class k, Δp^k, which is common to all participants. In this example, it is assumed that both V_{ij}^k and Δp^k are distributed

$\mathcal{N}(0,1)$, with no correlation between k in either variable. The dependence across i and j now introduced between the X_{ij}^k means that the central limit theorem does not apply, with the ratio R remaining substantially above the analytical solution as N increases, although still falling below 1.

5.2 Multiple CCP Membership in the Linked Case

All of the previous analysis assumes that, once the two CCPs are linked, the N participants each consolidate their operations within one CCP only. While this would reduce participants' CCP membership costs, in reality participants may still choose to remain members of both CCPs even after interoperability is introduced.[16] We now extend the model to account for this possibility. Let the number of participants, N, be divided into three categories:

- N' participants who remain members of both CCPs, clearing product 1 through CCP1 and product 2 through CCP2, as in the unlinked case.
- N_1 participants who consolidate their operations in CCP1, clearing both products through that CCP.
- N_2 participants who consolidate their operations in CCP2, clearing both products through that CCP.

Accordingly, $N = N' + N_1 + N_2$. Under the same assumptions discussed in Section 3, we obtain the aggregated unlinked and linked exposures:

$$
\begin{aligned}
Y_U &= (\sigma_1+\sigma_2)\frac{N\sqrt{N-1}}{\sqrt{2\pi}} \\
Y_L &= (\sigma_1+\sigma_2)\frac{N'\sqrt{N-1}}{\sqrt{2\pi}} + (N_1+N_2)\frac{\sqrt{(N-1)(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)}}{\sqrt{2\pi}} \\
&\quad + 2\frac{\sqrt{N'(N_1\sigma_2^2+N_2\sigma_1^2)+N_1N_2(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)}}{\sqrt{2\pi}}
\end{aligned}
$$

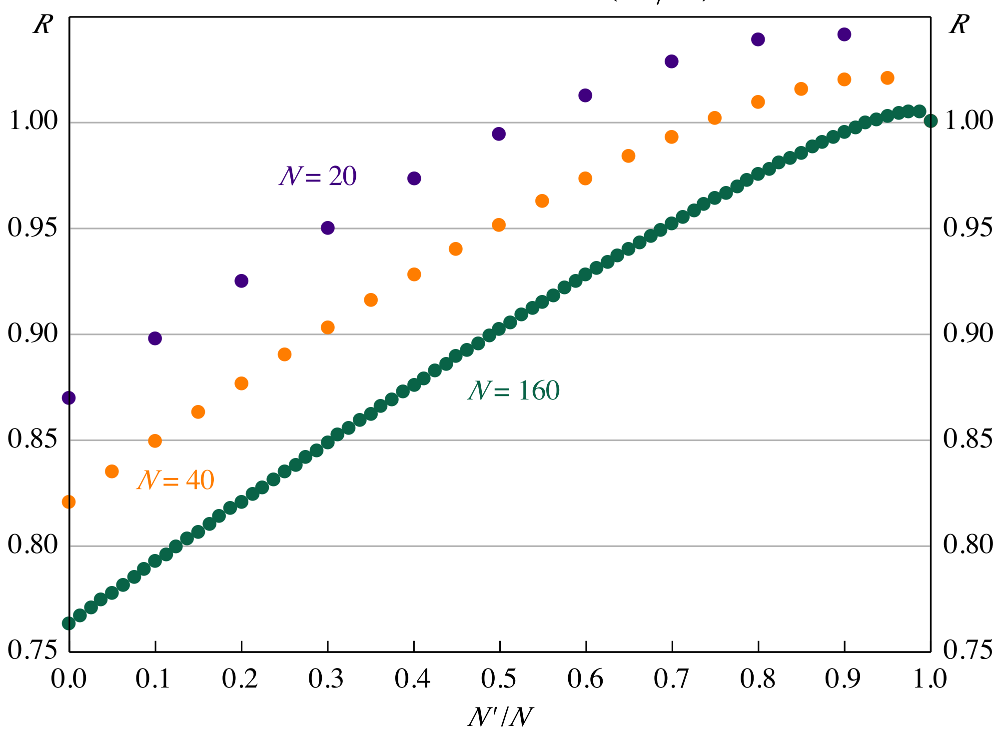
16 In Europe, this has been driven by some participants having clients that prefer to clear with particular CCPs; continuing to clear on behalf of these clients requires participants to maintain multiple CCP memberships.

Taking the base case of $\rho = 0$ and $\sigma_1 = \sigma_2$ yields

$$R = \frac{\sqrt{N-1}(2N' + \sqrt{2}(N_1 + N_2)) + 2\sqrt{N'(N_1 + N_2) + 2N_1 N_2}}{2N\sqrt{N-1}}$$

Figure 9 plots R for varying N' as a proportion of N.[17] The left edge of the graph ($N'/N = 0$) illustrates the case in which all participants consolidate their operations into either one of the two CCPs, as in Section 3. Accordingly, R is the same as previously. On the right edge of the graph, $N'/N = 1$, so that no participants change their operations. In this case, the link makes no difference because there is no change in the behaviour of the participants and no positions are put through the link. Therefore, $R = 1$.

Figure 9: **Effect on R of Changing the Proportion of Participants Joining Both Linked CCPs** (N'/N)



Generally, R increases with N'. Increasing N' means that fewer participants choose to net their obligations in a single CCP, reducing netting benefits. Once N' becomes sufficiently high, it is possible that $R > 1$. This occurs because most participants do

17 Since N is constant and $N = N' + N_1 + N_2$, the graphs show N' increasing in increments of two, while N_1 and N_2 simultaneously decrease in increments of one.

not obtain any netting gain from the link. Further, for participants that consolidate their operations (say in CCP1), the majority of their trades in product 2 must be made in CCP2, because that is the venue for the majority of trades in product 2 made by the N' participants. This means more of their trades are put through the link, which increases inter-CCP exposure.

5.3 Participant Heterogeneity

We have assumed so far that the X_{ij}^k have the same variance across all participant combinations i and j. This implies that all participants in each CCP pursue similar trading strategies with other counterparties, perhaps because they are of a similar size or have a similar level of appetite for risk. We now allow for a limited form of heterogeneity in X_{ij}^k, in which participants are either 'large' or 'small' in portfolio size or risk appetite (σ), and examine the effects in a two-CCP setting. This extension is particularly important in assessing cases such as that considered in Anderson *et al* (2013), where a smaller domestic CCP links to a larger foreign CCP. For CCP1, let there be $N_1' \leq N_1$ 'large' participants, and similarly $N_2' \leq N_2$ for CCP2. Let the ratio of standard deviations of X_{ij}^k values for 'large' and 'small' participants be g, such that:

$$\mathrm{Var}(X_{ij}^k) = \begin{cases} g^2\sigma_k^2, & \text{for } i,j \in N_1' \cup N_2' \\ \sigma_k^2, & \text{otherwise.} \end{cases}$$

Under this specification, trades between large participants involve additional variance, while the variance remains unaffected whenever a small participant is involved on either side of the trade. Assuming the X_{ij} are normally distributed, and letting $N' = N_1' + N_2'$, we have:

$$Y_U = \frac{\sigma_1 + \sigma_2}{\sqrt{2\pi}} \left[(N - N')\sqrt{N-1} + N'\sqrt{g^2(N'-1) + N - N'} \right]$$

$$\begin{aligned} Y_L = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}}{\sqrt{2\pi}} \Big[& N'\sqrt{g^2(N'-1) + N - N'} + (N - N')\sqrt{N-1} \\ & + \sqrt{N_1'[g^2 N_2' + N_2 - N_2']} + \sqrt{N_2(N_1 - N_1')} \\ & + \sqrt{N_2'[g^2 N_1' + N_1 - N_1']} + \sqrt{N_1(N_2 - N_2')} \Big] \end{aligned}$$

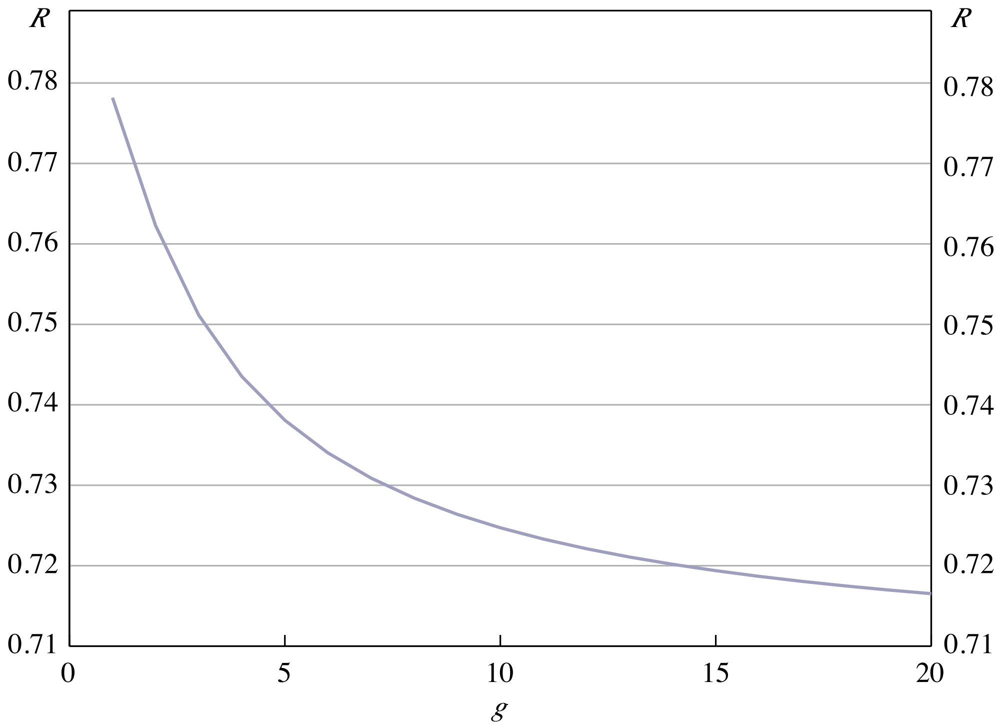
In the baseline case ($\sigma_1 = \sigma_2$, $\rho = 0$) this gives:

$$R = \frac{1}{\sqrt{2}}\left[1+\frac{\sqrt{N_1'[g^2N_2'+N_2-N_2']}+\sqrt{N_2(N_1-N_1')}+\sqrt{N_2'[g^2N_1'+N_1-N_1']}+\sqrt{N_1(N_2-N_2')}}{N'\sqrt{g^2(N'-1)+N-N'}+(N-N')\sqrt{N-1}}\right]$$

As in Section 4, we find that when participants are heterogeneous in this way, the asymmetry of activity across CCPs plays an important role in determining R, and works in a similar way to the asymmetry of participant numbers. We apply participant heterogeneity to the case examined by Anderson *et al* (2013), in which a domestic and foreign CCP establish a link. In the unlinked case, we let both N_1 domestic and N_2 foreign participants join both CCPs; they then consolidate their operations in the domestic CCP (CCP1) and the foreign CCP (CCP2) respectively.

Here, we consider a small jurisdiction in which all of the domestic participants are small on a global scale. All foreign participants are assumed to be large. We therefore have $N_1' = 0$ and $N_2' = N_2$. We now consider the effect of changing g, the ratio between the σ values for domestic and foreign participants. This is shown in in Figure 10 with $N_1 = N_2 = N_2' = 50$; increasing g lowers R. That is, increasing the asymmetry of activity across the two CCPs reduces the exposure arising from the link. This is because, given that $m = \sigma_2/\sigma_1$ is unchanged, the level of netting benefits across the two CCPs is the same. However, because a greater proportion of total activity takes place in the foreign CCP, the relative size of the inter-CCP exposures decreases. The net effect is that R decreases with the link.

Figure 10: **Effect on R of Increasing the Size Disparity between Domestic and Foreign Participants (g)**
$N_1 = N_2 = N_2' = 50$



5.4 Linking More than Two CCPs

We now extend the model to allow for link arrangements between C CCPs, where $C > 2$. In this section, we compare alternative arrangements for the netting of inter-CCP exposures: bilateral netting and multilateral netting through a meta-CCP, which acts as a CCP for inter-CCP exposures.

5.4.1 Bilateral netting

Let there be C CCPs that, in the unlinked case, clear a single product $\{1,...,K\}$ respectively, and all $K = C$ products once linked. Let $N = \sum_{c \in C} N_c$, where N_c is the number of participants that consolidate their operations in CCP c once the link is formed. If inter-CCP exposures are netted bilaterally, then the aggregate linked and unlinked CCP exposures are, respectively,

$$
\begin{aligned}
Y_U &= \sum_{k\in K}\sum_{i\in N}\mathbb{E}\left[\max\left(\sum_{j\in N}X_{ij}^k, 0\right)\right] \\
Y_L &= \sum_{c\in C}\sum_{i\in N_c}\mathbb{E}\left[\max\left(\sum_{j\in N}\sum_{k\in K}X_{ij}^k, 0\right)\right] \\
&\quad + \sum_{c\in C}\sum_{d\neq c\in C}\mathbb{E}\left[\max\left(\sum_{i\in N_c}\sum_{j\in N_d}\sum_{k\in K}X_{ij}^k, 0\right)\right]
\end{aligned}
$$

Let $\rho_{kl} = \mathrm{Corr}(X_{ij}^k, X_{ij}^l)$. For simplicity, assume the base case of $\rho_{kl} = 0$ and $\sigma_k = \sigma_l$ for all k and l. Further, assume membership is equally divided among the linked CCPs: $N_i = \overline{N} = N/C$. Assuming the X_{ij}^k are normally distributed yields:

$$
\begin{aligned}
Y_U &= \frac{\sigma NC\sqrt{N-1}}{\sqrt{2\pi}} \\
Y_L &= \frac{\sigma N\sqrt{C}(\sqrt{N-1}+C-1)}{\sqrt{2\pi}} \\
R &= \frac{1}{\sqrt{C}}\left(1+\frac{C-1}{\sqrt{N-1}}\right)
\end{aligned}
$$

As in Section 4, an increase in the total number of participants, N, increases R. As an example, the European CCPs LCH Clearnet Ltd, EuroCCP, EMCF and SIX x-clear have a four-way interoperability arrangement for many European cash equity markets. Taking the total number of participants in the linked CCPs, $N = 105$, we apply the base case of interoperability (equal σ_k values and $\rho_{kl} = 0$) to obtain $R = 0.65$. However, this result is heavily biased in favour of interoperability; in practice many clearing participants would be members of all four CCPs, to provide flexibility to their clients. As was shown in Section 5.2, maintaining multiple memberships after the link is formed results in a higher inter-CCP exposure, increasing R.

5.4.2 Multilateral netting

We could instead allow inter-CCP exposures to be multilaterally netted via a meta-CCP that takes on all inter-CCP obligations. Under this structure,

$$
\begin{aligned}
Y_U &= \sum_{k\in K}\sum_{i\in N}\mathbb{E}\left[\max\left(\sum_{j\in N}X_{ij}^{k},0\right)\right]\\
Y_L &= \sum_{c\in C}\sum_{i\in N_c}\mathbb{E}\left[\max\left(\sum_{j\in N}\sum_{k\in K}X_{ij}^{k},0\right)\right]\\
&\quad+\underbrace{\sum_{c\in C}\mathbb{E}\left[\max\left(\sum_{d\neq c\in C}\sum_{i\in N_d}\sum_{j\in N_c}\sum_{k\in K}X_{ij}^{k},0\right)\right]}_{\text{Meta-CCP exposure to } C \text{ CCPs}}
\end{aligned}
$$

Taking the same base case as in Section 5.4.1, and letting R under bilateral and multilateral netting be R_B and R_M respectively, we have:

$$
\begin{aligned}
R_B &= \frac{1}{\sqrt{C}}+\frac{C-1}{\sqrt{C(N-1)}}\\
R_M &= \frac{1}{\sqrt{C}}+\frac{\sqrt{C-1}}{\sqrt{C(N-1)}}
\end{aligned}
$$

Since $C > 2$, $R_M < R_B$, indicating that link arrangements reduce exposure when inter-CCP exposures are multilaterally netted. Further, it can be shown that $(R_B - R_M)$ is decreasing in N and increasing in C. Therefore, the additional benefits of multilateral netting fall when there are more participants, and increase when there are more linked CCPs. Taking the European example from the previous section ($C = K = 4$, $N = 105$), we have $R_B = 0.65$ and $R_M = 0.54$. As discussed previously, both these values are likely to be biased in favour of the link, but the difference between them nevertheless demonstrates the additional netting benefits of a meta-CCP arrangement.

6. Conclusion

We have explored, through a stylised model, how CCP links affect aggregate CCP exposure. CCP links cause two opposing effects on exposure: a benefit arising from inter-CCP netting and a cost arising from inter-CCP exposure. Our model finds that, under a wide variety of plausible assumptions about parameters, participation and the distribution of obligations, the netting benefit dominates, and that CCP links can decrease aggregate exposure. To the extent that collateral requirements reflect underlying exposures, this implies that CCP links can economise on collateral.

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