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Estimating and Identifying Empirical BVAR-DSGE Models for Small Open Economies

Tim Robinson

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Abstract

Different approaches to modelling the macroeconomy vary in the emphasis they place on coherence with theory relative to their ability to match the data. Dynamic stochastic general equilibrium (DSGE) models place greater emphasis on theory, while vector autoregression (VAR) models tend to provide a better fit of the data. Del Negro and Schorfheide (2004) develop a method of using a DSGE model to inform the priors of a Bayesian VAR. The resulting BVAR-DSGE model partially relaxes the relationships in the DSGE so as to fit the data better. However, their approach does not accommodate the typical restriction of small open economy models which ensures that developments in the small economy cannot affect the large economy. I develop a method that allows this restriction to be imposed and introduce a simple way, suitable for small open economies, of identifying the empirical BVAR-DSGE using information from the DSGE model. These methods are demonstrated using the Justiniano and Preston (2010a) DSGE model. Compared to the DSGE model, the empirical BVAR-DSGE model estimates that there is a larger role for foreign shocks in the small economy's business cycle.

JEL Classification Numbers: C11, C32, C51, E30

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1. Introduction

DSGE models provide an internally and theoretically consistent description of the macroeconomy. They give clear economic interpretations of the shocks affecting the economy. This interpretability, however, comes at a cost; a reduced-form VAR, for example, generally provides a better fit of the data. The aim of this paper is to develop a model in which the shocks can still be interpreted but the fit with the data may be improved.

The approach in this paper is to use an estimated DSGE model as a source of prior information about the parameters of a VAR. This effectively relaxes some of the relationships in the DSGE model, which, while theoretically well founded, may not hold in the data. One reason for this lack of fit is that all models are necessarily simplifications of reality. For example, many small open economy models include an uncovered interest parity condition, which can be derived from an arbitrage condition between foreign- and domestic-currency denominated risk-free bonds. Despite these theoretical underpinnings, uncovered interest parity has been widely found to be at odds with the data. In small open economy models other common simplifications include the assumptions that all imports are consumption goods, labour is the only input to production and financial markets operate largely without frictions. All of these assumptions can be relaxed, and whether they are of quantitative importance depends on the question to be answered.

Bayesian methods are a way of integrating prior information into parameter estimates. A Bayesian VAR (BVAR) that uses priors from an estimated DSGE model will therefore represent a compromise between the theoretically coherent DSGE and a VAR that may fit the data better. Another interpretation of this hybrid model is that the parameters of an unrestricted VAR are being pulled towards those of the DSGE model.

The DSGE model is a potentially useful way of formulating a prior for a VAR. The parameters in the DSGE model have straightforward economic interpretations.

For example, the intertemporal elasticity of substitution captures the willingness of an individual to trade off consumption today for future consumption. DSGE models are frequently estimated using Bayesian methods. One way to form a prior about the intertemporal elasticity of substitution is to look at microeconomic data, such as surveys of household consumption behaviour. However, the mapping from microeconomic to macroeconomic estimates may not be straightforward, and often priors are based on previous similar macroeconomic studies. In contrast, while priors for reduced-form VARs do exist (most notably the Minnesota prior for forecasting; see, for example, Doan, Litterman and Sims (1984)), it is more difficult to form a prior over the coefficient on, say, the second lag on a particular variable in the VAR, as it has no clear economic interpretation. The method proposed in this paper is a way of utilising information from the DSGE model, whose parameters are relatively easy to place priors over, in forming priors for a VAR.

Several studies have previously examined eliciting priors for a BVAR from general equilibrium models. The focus of Ingram and Whiteman (1994) was on forecasting using the resulting reduced-form BVAR. Of particular interest is Del Negro and Schorfheide (2004), who also estimate a reduced-form BVAR with DSGE model-based priors, and then use information from the DSGE model to identify it, a strategy I follow. This method has previously been applied to small open economies, for example by Hodge, Robinson and Stuart (2008) and Lees, Matheson and Smith (2011), for Australia and New Zealand respectively.

While the DSGE prior in these papers is based on a small open economy model, the Del Negro and Schorfheide (2004) approach does not allow for small open economy restrictions on the BVAR estimates. Hodge *et al* (2008) and Lees *et al* (2011) deal with this problem implicitly by estimating small BVARs of only five variables that do not include foreign output, interest rates or inflation, although these are included in the DSGE prior.[1] This, however, is somewhat unsatisfactory because it is necessary to include foreign variables to adequately capture the dynamics of small economies; see, for example, Dungey and Pagan (2000, 2009). However, by including foreign variables but not imposing restrictions that limit the ability of the small open economy to

1 These VARs include domestic output growth, inflation, interest rates and growth in the real exchange rate and the terms of trade.

affect the large economy may result in unrealistically large effects. The estimation approach presented here to circumvent this problem draws on DeJong, Ingram and Whiteman (1993). Filippeli, Harrison and Theodoridis (2011) independently developed a similar estimation methodology, but did not take small open economy considerations into account.

This paper is structured as follows. Section 2 describes the methodology used to estimate the empirical BVAR-DSGE model, and Section 3 discusses its identification. Section 4 provides an empirical example, by estimating an empirical BVAR-DSGE model for Australia where the Justiniano and Preston (2010a) model is used as a prior. This empirical BVAR-DSGE model estimates a larger role for the foreign shocks in the small economy's business cycle than the DSGE model. Finally, the conclusions of this paper are presented in Section 5.

2. Methodology – Estimation

The approach taken to estimate the empirical BVAR-DSGE model has several steps:

1. estimating the DSGE model using Bayesian methods
2. using the posterior of this DSGE model to construct a VAR approximation to the DSGE model by simulation methods
3. constructing a prior for the empirical reduced-form BVAR from the VAR approximation to the DSGE, and finally
4. estimating the posterior of the empirical BVAR model.

Each of these steps will be discussed in turn. Before doing so, I introduce some notation.

The reduced-form BVAR to be estimated is of the form:

$$\begin{bmatrix} y_t^L \\ y_t^S \end{bmatrix} = \Sigma_{i=1}^{p} \begin{bmatrix} \Phi_i^L & 0 \\ \Phi_i^{SL} & \Phi_i^S \end{bmatrix} \begin{bmatrix} y_{t-i}^L \\ y_{t-i}^S \end{bmatrix} + \begin{bmatrix} u_t^L \\ u_t^S \end{bmatrix}, \begin{bmatrix} u_t^L \\ u_t^S \end{bmatrix} \sim N(0, \Sigma_u), \qquad (1)$$

where y_t^j are vectors of variables and the superscript L or S denotes the large or small economy. Let $y_t \equiv \begin{bmatrix} y_t^L \\ y_t^S \end{bmatrix}$, and there be n variables in total. Φ_i^j are matrices of parameters for lags $i = 1 \dots p$; $j = SL$ denotes parameters that are the response of the small economy variables to the large economy variables. Let $\Phi_i \equiv \begin{bmatrix} \Phi_i^L & 0 \\ \Phi_i^{SL} & \Phi_i^S \end{bmatrix}$. Note that Φ_i has a block of zeros in the upper right so that the large economy does not depend on lags of the small economy variables, which is known as block exogeneity. $u_t \equiv \begin{bmatrix} u_t^L \\ u_t^S \end{bmatrix}$ are the reduced-form shocks, which are assumed to be normally distributed with a variance-covariance matrix $\Sigma_u \equiv \begin{bmatrix} \Sigma^L & \Sigma^{LS} \\ \Sigma^{LS'} & \Sigma^S \end{bmatrix}$.

2.1 Estimating the DSGE Model

The DSGE model is estimated using Bayesian methods, as is often done in the literature. The advantage of this is that it allows subjective information about the parameters to be utilised in estimation and, more pragmatically, it may lessen identification issues for some parameters. Bayesian estimation of DSGE models is summarised in An and Schorfheide (2007), and can be implemented, for example, using the pre-processor Dynare in Matlab. The observed variables used to estimate the DSGE model are the same as those included in the reduced-form BVAR.

2.2 Estimating a VAR Approximation to the DSGE Model

The solution to a DSGE model is a VAR in its variables. The structure of the DSGE model places restrictions on the parameters of the VAR. However, only a subset of the variables in the DSGE model are observed, that is, matched to actual data in estimation. The solution in these observed variables alone may be a vector-autoregressive moving average (VARMA) model rather than a VAR. This typically occurs when there is a stock variable (such as capital or net foreign assets) in the model that is not used in estimation. The VARMA model can be approximated with a low-order VAR, although the approximation is likely to become better the higher the order of the VAR. Naturally if the solution to the DSGE model has a VAR representation that can be solved for analytically, that could be used.[2]

To construct the approximation, for a particular set of DSGE parameters I solve the model, simulate long time series of the observed variables from it, and estimate the following VAR on these simulated data:

$$y_t = \Sigma_{i=1}^{p} \Phi_{DSGEi} y_{t-i} + u_{DSGEt},\ u_{DSGEt} \sim N(0, \Sigma_{DSGEu}), \tag{2}$$

where Φ_{DSGEi} are the matrices of parameters, with block exogeneity imposed, and u_{DSGEt} are the reduced-form shocks.[3]

2.3 Constructing a Prior for the Empirical BVAR-DSGE Model

There are two aspects to constructing a prior for the empirical BVAR-DSGE model from this VAR approximation. First, selecting prior distributions which will accommodate block exogeneity, and second, selecting the arguments for the prior.

2.3.1 Selecting the prior distributions

In the approach of Del Negro and Schorfheide (2004), the prior for the BVAR is formed by expressing the likelihood of data simulated from the DSGE model (for a given vector of parameters) in Normal-inverted Wishart form. This means that the prior of the variance-covariance matrix of the reduced-form shocks, given the DSGE parameters, is an inverted Wishart distribution, and the prior for the VAR parameters, conditional on the variance-covariance matrix of the shocks and the DSGE parameters, is Normal. This prior is convenient as it conjugates with normally distributed data, which means that the posterior has a known form. A disadvantage of this prior, however, is that it assumes that the same explanatory variables are in each equation; for a further discussion of this see Koop and Korobilis (2010).

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2 Fernández-Villaverde *et al* (2007) study VAR representations of DSGE models.

3 I simulate 40 000 observations, drop the first 100, and estimate the VAR using a seemingly unrelated regression (SUR).

One alternative, which does not make this assumption, is the independent Normal-Wishart prior (Koop and Korobilis 2010). This prior is similar to the Normal-inverted Wishart, except that the prior for the VAR parameters is normally distributed without being conditioned on the variance-covariance matrix of the shocks. This prior allows complete flexibility about whether variables are included or excluded in each equation of the VAR, and thus can accommodate block exogeneity.

2.3.2 Selecting the arguments of the prior

The second aspect is selecting the arguments of the prior. To do this, recall that the VAR approximation to the DSGE model in Equation (2) is conditional on a particular value of the DSGE parameters. Repeatedly sampling from the posterior of the DSGE (say 1 000 times) and constructing VAR approximations to the DSGE model, as described above, yields a set of estimates of the VAR approximation parameters $\left\{\Phi_{DSGE}^{k}\right\}_{k=1}^{1000}$ (where k indexes the estimates for each sample; the index of the lag length has been suppressed) and variance-covariance matrices of the reduced-form shocks $\left\{\Sigma_{DSGEu}^{k}\right\}_{k=1}^{1000}$.[4] These sets of parameters can be used to inform the choice of appropriate arguments for the prior distributions.

The idea of estimating a reduced-form VAR on simulated data to obtain prior parameters was first introduced by DeJong *et al* (1993), although they sample from the prior of the theoretical model, rather than its posterior, and assume different prior distributions for the VAR.[5] Also, note that in my approach the prior is more accurately described as an ‘empirical Bayes’ prior as it is constructed from the posterior of the estimated DSGE model, and therefore the same data are used to form the prior and in estimation. Sampling from the DSGE prior when constructing the VAR approximation to the DSGE model, would accord more

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4 In an attempt to ensure that the only stochastic variation in the simulation comes from the draw of the DSGE parameters, the same seed was always used for the random number generator. When drawing the parameters, restrictions to ensure the DSGE model has a deterministic solution can be added (e.g. imposing that the Taylor principle is satisfied, namely that nominal interest rates respond sufficiently aggressively to inflation).

5 DeJong *et al* (1993) mention the possibility of using independent Normal-Wishart prior, but do not do it to lessen the necessary computation. Filippeli *et al* (2011) also sample from the prior, rather than the posterior.

closely with the idea of Bayesian analysis, namely that priors should be formed before seeing the data.

2.3.3 *Independent Normal-Wishart prior*

In order to use the independent Normal-Wishart prior I rewrite the reduced-form VAR in Equation (1), in the form outlined in Koop and Korobilis (2010).

Consider the mth equation in the VAR. This is rewritten as:

$$y_{mt} = z'_{mt}\beta_m + u_{mt},$$

where y_{mt} could be either a large or small economy variable, z_{mt} is a (column) vector of its explanatory variables, β_m is a vector of their parameters, and u_{mt} is the corresponding reduced-form shock. Note that the size of z_{mt} will vary depending on whether y_{mt} is a large or small economy variable.

The n equations are stacked vertically, yielding $y_t = Z_t\beta + u_t$, where Z_t is upper triangular with z'_{mt} on the mth row. Now stacking the T observations together vertically yields y, Z and u. The VAR can then be written as:

$$y = Z\beta + u, \; u \sim N(0, I \otimes \Sigma_u).$$

Priors are placed over this formulation.

The independent Normal-Wishart prior, as presented by Koop and Korobilis (2010), is:

$$p(\beta, \, \Sigma_u^{-1}) = p(\beta)p(\Sigma_u^{-1}),$$

where $\beta \sim N(\underline{\beta}, \, \underline{V}_\beta)$, and $\Sigma_u^{-1} \sim W(\underline{S}^{-1}, \, \underline{\nu})$, with N and W denoting the Normal and Wishart distributions and the underbar the arguments for the prior. The prior is modified by constraining the parameter space of β, Θ, to include only values for which the empirical BVAR-DSGE is stable, hence $p(\beta, \, \Sigma_u^{-1}) \propto p(\beta)p(\Sigma_u^{-1})1(\beta \in \Theta)$, where 1 is an indicator function.

2.3.4 *Estimating the arguments of the prior*

Reshaping the parameter estimates Φ^i_{DSGE} into β^i, the prior for β, the empirical BVAR-DSGE model parameters, are centred at their sample mean. Similarly, $\underline{V}_\beta$

is set with reference to the variance of our set of β estimates, namely:

$$\underline{V}_{\beta} = \lambda \left(\frac{1}{1000-1} \Sigma_{k=1}^{1000} \left(\beta^{k} - \underline{\beta} \right) \left(\beta^{k} - \underline{\beta} \right)' \right) + \Xi I,$$

where I is the identity matrix and Ξ is a small positive number. λ is a parameter I have introduced for further flexibility; higher values of λ cause the prior on the mean of β to have a larger variance, effectively down-weighting it relative to the data. This is just one way the prior can be loosened; others are possible. A small amount (Ξ) has been added to the variance of each parameter to ensure that the variance-covariance matrix is not singular.[6] For a particular $\underline{\nu}$, $\underline{S}^{-1}$ is set to match the mean of Σ_{DSGEu}^{-1k}, i.e. $\underline{S}^{-1} = \frac{1}{\underline{\nu}} \frac{1}{1000} \Sigma_{k=1}^{1000} \Sigma_{DSGEu}^{-1k}$.

There are several possible approaches to selecting λ and $\underline{\nu}$. One is to simply examine plots of the prior for different values and to decide whether they appear reasonable. Another way to select them would be to examine the forecasting performance of the reduced-form empirical BVAR-DSGE model for a range of values. Finally, a natural criterion is to maximise the marginal likelihood, which can be interpreted as selecting the model (indexed by λ and $\underline{\nu}$) that maximises the likelihood of observing the data. Given that I will use a Gibbs sampler to simulate the posterior, a sensible way of estimating the marginal likelihood is the method of Chib (1995).[7]

2.4 Estimating the Reduced-form Empirical BVAR-DSGE Model Posterior

Koop and Korobilis (2010) show that with the independent Normal-Wishart prior the posterior of the VAR parameters conditional on the variance-covariance matrix

6 I use $\Xi = 1e^{-4}$.

7 In the empirical example in Section 4, $\underline{\nu}$ is set to $n+2$ and numerical difficulties were encountered when estimating the marginal likelihood. Consequently, I performed sensitivity analysis of how the results change as λ is varied. The numerical difficulties arise because for a large VAR, V_{β} will be very large and hence there will be many plausible values that have a very small determinant which Matlab treats as zero, even if it is positive definite. It is necessary to invert this determinant when using the Chib (1995) method, which is problematic. It might be possible to use a normalisation to circumvent this problem; exploring this is left for future research. Note that it is also necessary to account for the stability restriction when calculating the marginal likelihood.

of the shocks and vice-versa are normally distributed, which makes them suitable for using Gibbs sampling to produce draws from the joint posterior. In particular,

$$\beta|y,\Sigma_u^{-1} \sim N(\bar{\beta},\, \bar{V}_\beta),$$

where $\bar{V}_\beta = \left(\underline{V}_\beta^{-1} + \Sigma_{t=1}^{T} Z_t' \Sigma_u^{-1} Z_t\right)^{-1}$, and $\bar{\beta} = \bar{V}_\beta \left(\underline{V}_\beta^{-1} \underline{\beta} + \Sigma_{t=1}^{T} Z_t' \Sigma_u^{-1} y_t \right)$, and the overbars denote that these are the arguments for the posterior. The impact of the modification is only to truncate this distribution to β draws where the VAR is stable.[8]

Also,

$$\Sigma_u^{-1}|y,\beta \sim W(\bar{S}^{-1},\, \bar{v}),$$

where $\bar{v} = T + \underline{v}$, and $\bar{S} = \underline{S} + \Sigma_{t=1}^{T} (y_t - Z_t\beta)(y_t - Z_t\beta)'$.

Finally, having obtained the posterior for β, its elements can be rearranged to obtain the posterior for Φ_i, enabling us to rewrite the empirical BVAR-DSGE model as in Equation (1).

3. Identifying the Empirical BVAR-DSGE Model

In order to interpret the shocks estimated by the empirical BVAR-DSGE model it is necessary to identify the model. To do so I identify the VAR approximation to the DSGE solution, and then use this to identify the posterior of the empirical BVAR-DSGE model. In particular, I identify the latter by matching its contemporaneous impulse responses to those from the VAR approximation to the DSGE model as closely as possible. As I am matching the contemporaneous impulse responses, the focus will be on the contemporaneous matrix. In order to ensure that the small economy does not affect the large economy, the impulse responses of the large and small economies are matched separately. Ultimately,

8 This is done in the Gibbs sampler by rejecting draws where the VAR is unstable.

the desired structural form of the empirical BVAR-DSGE model is:

$$\begin{bmatrix} B^L & 0 \\ B^{SL} & B^S \end{bmatrix} \begin{bmatrix} y_t^L \\ y_t^S \end{bmatrix} = \Sigma_{i=1}^p \begin{bmatrix} F_i^L & 0 \\ F_i^{SL} & F_i^S \end{bmatrix} \begin{bmatrix} y_{t-i}^L \\ y_{t-i}^S \end{bmatrix} + \begin{bmatrix} \xi_t^L \\ \xi_t^S \end{bmatrix}, \begin{bmatrix} \xi_t^L \\ \xi_t^S \end{bmatrix} \sim N(0,I), \tag{3}$$

where B is the contemporaneous matrix, $B \equiv \begin{bmatrix} B^L & 0 \\ B^{SL} & B^S \end{bmatrix}$, F_i^j are the VAR parameters of the jth block and the ith lag, which are collected together in $F_i \equiv \begin{bmatrix} , & F_i^L & 0 \\ & F_i^{SL} & F_i^S \end{bmatrix}$, and the structural shocks are $\xi_t \equiv \begin{bmatrix} \xi_t^L \\ \xi_t^S \end{bmatrix}$.

This VAR can be written more compactly as:

$$By_t = \Sigma_{i=1}^p F_i y_{t-i} + \xi_t,\ \xi_t \sim N(0,I).$$

3.1 The Structural VAR Approximation to the DSGE Model

To identify the VAR approximation to the DSGE model's solution I normalise the structural shocks from the DSGE model and denote them as η_t, where $\eta_t \sim N(0,I)$. The structural VAR (SVAR) approximation to the DSGE solution with normalised structural shocks can then be written as

$$B_{DSGE} y_t = \Sigma_{i=1}^p F_{DSGEi} y_{t-i} + \eta_t, \tag{4}$$

where B_{DSGE} is the contemporaneous matrix and F_{DSGEi} are the coefficients on the ith lags. The reduced-form VAR approximation to the DSGE model and its shocks u_t were estimated in Equation (2). The structural shocks are related to the reduced-form shocks by $u_t = B_{DSGE}^{-1}\eta_t$. Consequently, to obtain an estimate of B_{DSGE}, I estimate this relationship as a SUR and invert B_{DSGE}^{-1}.

The contemporaneous impulse response of y_t to η_t is the inverse of the contemporaneous matrix, i.e. $\frac{\partial y_t}{\partial \eta_t} = B_{DSGE}^{-1}$. This, together with the coefficients of the reduced-form VAR approximation to the DSGE model, determine the responses at longer horizons. Finally, as B_{DSGE}^{-1} has a posterior distribution, the parameter used is based on the median of this distribution, calculated from a sample of 1 000 observations. The specific draw of B^{-1}_{DSGE} closest to the median, which is denoted by $B^{-1}_{DSGE\ median}$, is used. This can be separated into large and small economy components, as in Equation (3).

3.2 Identifying the Empirical BVAR-DSGE Model

To identify the empirical BVAR-DSGE model, the strategy I follow is similar to that used in sign-restricted VARs. In brief, the sign restriction approach is to specify the signs that the impulse responses (typically contemporaneous) should satisfy (e.g. a positive demand shock contemporaneously drives up output and inflation, whereas a positive supply shock increases output and decreases inflation). By searching over possible structural shocks, a set of shocks that satisfies these signs is constructed.

To demonstrate the identification approach used, and its relationship to that of sign-restricted VARs, initally the distinction between small and large economy variables is ignored. First, I find a set of shocks that are uncorrelated. A simple way to do this, for a particular draw from the posterior, is to apply a Cholesky decomposition to the variance-covariance matrix of the reduced-form shocks, Σ_u, obtaining $RR' = \Sigma_u$, where R is lower triangular. Pre-multiplying the reduced-form by R^{-1} yields a set of shocks, φ_t, $\varphi_t \equiv R^{-1}u_t$. While these shocks are not correlated they are not unique. If the VAR is premultiplied by the orthogonal matrix Q':

$$Q'R^{-1}y_t = Q'R^{-1}(\Phi_1 y_{t-1} + Q'\Phi_2 y_{t-2} + \ldots + Q'\Phi_p y_{t-p}) + Q'\varphi_t \qquad (5)$$

then I obtain a new set of structural shocks, $\xi_t \equiv Q'\varphi_t$, which also are uncorrelated. The variance-covariance matrix (and the likelihood) is invariant to Q, but the contemporaneous impulse responses for the empirical BVAR-DSGE model to these new structural shocks are RQ. Consequently, the identification problem has been reduced to choosing which Q matrices are appropriate.

The sign restriction approach searches over candidate Q matrices to find those that satisfy these restrictions.[9] Often there will be many such Q matrices, reflecting uncertainty about the structural model rather than uncertainty due to estimation

9 For a review (and critique) of the sign restriction literature see Fry and Pagan (2011). DSGE models have been used as a source of the sign restrictions; see, for example, Liu (2010).

of the reduced form. In the empirical BVAR-DSGE model, however, having multiple Qs for each draw of the posterior variance-covariance matrix, Σ_u, is problematic. Consequently, I consider restrictions that are stronger than sign restrictions, namely matching the contemporaneous impulse responses from the SVAR approximation to the DSGE model. Del Negro and Schorfheide (2004) also identify their model by selecting Q, based on information from the DSGE model, which in An and Schorfheide (2007) is motivated with reference to matching the impulse responses. For each vector of DSGE parameters, Del Negro and Schorfheide (2004) decompose the matrix of the contemporaneous impact of the shocks (akin to B_{DSGE}^{-1}) into Q and R matrices, and use this Q in the BVAR-DSGE model.

3.2.1 Matching contemporaneous impulse responses

The problem of selecting Q to match the impulse responses of the DSGE can be written as:

$$\begin{array}{cc} \min_Q & \left\| RQ - B_{DSGE\ median}^{-1} \right\| \\ s.t. & Q'Q = I, \end{array}$$

where $\|.\|$ denotes a matrix norm, which is a measure of the discrepancy between the contemporaneous impulse responses from the empirical BVAR-DSGE model and the SVAR approximation to the DSGE model.[10] If the discrepancy is measured as the sum of squared deviations of each element, then this problem has been extensively studied in linear algebra and is known as the 'Orthogonal Procrustes Problem' (see, for example, Golub and Van Loan (1996, p 601)).[11] The constraint simply states that Q is an orthogonal matrix. Schönemann (1966) shows that the solution to this problem can be simply found analytically using a singular-value decomposition and when Q will be unique, which it will be in this case. To emphasise, it is the matching of impulse responses which provides a criterion to select a unique Q, and this uses more information than just the sign of the impulse responses.

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10 This can be thought of as defining a prior for Q, conditional on β and Σ, which places all of its weight on the solution to this problem, Q^*, with $B_{DSGE\ median}^{-1}$ as a hyperparameter.

11 This is the Frobenius norm. Procrustes in Greek mythology would invite travellers into his house for food and a bed. However, once they entered he attached them to a bed and twisted and distorted them until they fitted it. The problem above bears his name as Q is distorting R so as to resemble, as closely as possible, $B_{DSGE\ median}^{-1}$.

The idea of identifying a SVAR (estimated using maximum likelihood) by matching the impulse responses to those of a DSGE model has previously been proposed by Liu and Theodoridis (2010). They, however, also include sign restrictions, arguing that selecting Q based solely on whether it yields impulse responses close to those from the DSGE model may yield impulse responses with counter-intuitive signs. To tackle this, they add indicator functions to the objective function that show whether the sign restrictions are met or not, and weight these. Impulse responses with the wrong sign are a potential problem, which is more likely to occur if it is present in the DSGE model itself or if the contemporaneous impact of a shock is very small. However, I do not follow this approach because including sign restrictions means that the problem can no longer be solved analytically. Alternatively, Park (2011) follows the estimation approach of Del Negro and Schorfheide (2004) for the reduced-form VAR coefficients, but specifies a prior for the contemporaneous matrix based on the impulse responses from the DSGE model rather than a prior for the variance-covariance matrix of the innovations. An advantage of this method is that Park (2011) introduces a parameter which allows the researcher to control how tightly the identifying restrictions from the DSGE model are held. However, the prior on the reduced-form BVAR-DSGE parameters is structured as in Del Negro and Schorfheide (2004), and therefore the approach of Park (2011) does not accommodate block exogeneity and the small open economy assumption.

The approach I take to matching contemporaneous impulse responses while imposing the block exogeneity restriction draws on Liu (2007). In particular, I obtain separate Q matrices for large and small economy shocks, which provides the flexibility when identifying the large economy to decide whether to match the impact of its shocks on itself alone or on the small economy as well, but does not place restrictions on the variance-covariance matrix of the reduced-form shocks.

3.2.2 Impulse responses from the BVAR

In order to pick separate Q matrices for the small and large economy shocks I first remove the large economy component of the small economy shocks, making the variance-covariance matrix of these reduced-form shocks block diagonal. This is simple to implement: for any draw of the posterior I regress $u_t^S = \kappa u_t^L + v_t^S$ using OLS, where κ is a matrix of parameters and $v_t^S \sim N(0, A)$, which effectively defines a new set of reduced-form shocks for the small economy, v_t^S, which have no large economy component.[12] The reduced-form empirical BVAR-DSGE model can be rewritten as:

$$\begin{bmatrix} y_t^L \\ y_t^S \end{bmatrix} = \Sigma_{i=1}^p \begin{bmatrix} \Phi_i^L & 0 \\ \Phi_i^{SL} & \Phi_i^S \end{bmatrix} \begin{bmatrix} y_{t-i}^L \\ y_{t-i}^S \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix} \begin{bmatrix} u_t^L \\ v_t^S \end{bmatrix}, \begin{bmatrix} u_t^L \\ v_t^S \end{bmatrix} \sim N(0, \Sigma_v),$$

where $\Sigma_v \equiv \begin{bmatrix} \Sigma^L & 0 \\ 0 & A \end{bmatrix}$.

As described in Section 3.2, Σ^L and A can be decomposed into R^L and R^S, which can be used to obtain new structural shocks ξ_t^L and ξ_t^S. The resulting VAR is:

$$\begin{aligned} \begin{bmatrix} Q^{L'} R^{L-1} & 0 \\ 0 & Q^{S'} R^{S-1} \end{bmatrix} \begin{bmatrix} y_t^L \\ y_t^S \end{bmatrix} &= \Sigma_{i=1}^p \begin{bmatrix} Q^{L'} R^{L-1} & 0 \\ 0 & Q^{S'} R^{S-1} \end{bmatrix} \begin{bmatrix} \Phi_i^L & 0 \\ \Phi_i^{SL} & \Phi_i^S \end{bmatrix} \\ &* \begin{bmatrix} y_{t-i}^L \\ y_{t-i}^S \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ Q^{S'} R^{S-1} \kappa R^L Q^L & 1 \end{bmatrix} \begin{bmatrix} \xi_t^L \\ \xi_t^S \end{bmatrix}. \end{aligned}$$

It is possible to use this form of the empirical BVAR-DSGE model to identify each economy. For the small economy, the impact of its shocks on itself is $\frac{\partial y_t^S}{\partial \xi_t^S} = R^S Q^S$. Q^{S*} can be selected to match the corresponding impulse responses from the structural VAR approximation to the DSGE model, $B_{DSGE\ median}^{S-1}$, using the method described previously. The small economy block, using the reduced form of the large economy, then is:

$$B^S y_t^S + B^{SL} y_t^L = \Sigma_{i=1}^p \left(F_i^{SL} y_{t-i}^L + F_i^S y_{t-i}^S \right) + \xi_t^s, \tag{6}$$

where: $B^S \equiv Q^{S*'} R^{S-1}$; $B^{SL} \equiv -B^S \kappa$; $F_i^{SL} \equiv B^S \left(\Phi_i^{SL} - \kappa \Phi_i^L \right)$, and $F_i^S \equiv B^S \Phi_i^S$.

To identify the large economy, that is, to select Q^{L*}, there are two possible sets of impulse responses which can be taken into account, namely the response of the large economy alone to its shocks ($\frac{\partial y_t^L}{\partial \xi_t^L} = R^L Q^L$), or the response of the small economy to these shocks ($\frac{\partial y_t^S}{\partial \xi_t^L} = \kappa R^L Q^L$). Either way, matching the impulse

12 I use 1 000 draws.

responses can be done by solving the problem as above.[13] The resulting large economy component of the empirical BVAR-DSGE model is:

$$B^L \; y_t^L \; = \Sigma_{i=1}^p F_i^L \; y_{t-i}^L \; + \; \xi_t^L \; , \; \xi_t^L \; \sim N(0,I), \tag{7}$$

where: $B^L \equiv Q^{L*'} R^{L-1}$ and $F_i^L \equiv B^L \Phi_i^L$.

Finally, writing Equations (6) and (7) together yields the structural empirical BVAR-DSGE model, Equation (3), from which the various quantities of interest, such as impulse response functions and variance decompositions, can be constructed.

4. Empirical Application

Justiniano and Preston (2010a) show that in an estimated two-country DSGE model of Canada and the United States, which includes many features typical of open economy macroeconomic models, an implausibly small amount of the variation in most Canadian series is due to shocks from the United States (less than three per cent at most forecast horizons). The model also cannot generate the correlations across the countries that exist between many of the series in the observed data. Justiniano and Preston (2010a) discuss possible reasons for these findings, including that the model has

> ... counter factual implications for the terms of trade and the real exchange rate, particularly in regards to their link with domestic inflation. When confronted with the data, this tension between fitting some cross-country correlations and the model's counter factual prediction for other moments is resolved in favor of the latter by shutting down international linkages. (p 72)

In this section I demonstrate that a similarly small impact is found when the Justiniano and Preston (2010a) model is estimated for Australia and the United States. In contrast, a structural empirical BVAR-DSGE model that uses the DSGE model as a prior and imposes block exogeneity finds a larger impact of foreign shocks on the small economy.

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13 Appendix A demonstrates that this is possible when the impact on both countries is taken into account.

Previous structural VAR studies for Australia have often found an important role for foreign shocks, for example in determining Australian GDP. This conclusion appears to be robust to the method of analysis used. Analysing a historical decomposition of Australian output, Dungey and Pagan (2009, p 14) conclude that '... the most influential of the shocks are technology, preferences and the foreign sector ...'; alternatively, in a forecast error variance decomposition Berkelmans (2005) finds that at the 1- and 4-quarter horizons, shocks to US GDP account for 14 and 27 per cent of the variance respectively. Using business cycle dating techniques Dungey and Pagan (2000) find that the ability of the structural VAR to capture the characteristics of the growth cycle is considerably impeded when foreign shocks are excluded.[14]

4.1 The Justiniano and Preston (2010a) Model

I provide only a brief sketch of the Justiniano and Preston (2010a) model. The log-linearised equations from their code are used, which was generously provided by Bruce Preston.[15] In each economy there are:

- households
- domestic producers
- retail firms
- a government
- a central bank.

Households consume a composite consumption good with external habits (the utility they derive from consumption depends on its level relative to lagged aggregate consumption – a 'keeping up with the Joneses' effect). A preference shock captures changes in preferences for consuming today relative to tomorrow. The composite good for the small economy (Australia) is composed of home-produced and foreign-produced goods, which are aggregated together using

14 The nature of these foreign shocks differ across the models. For example, Dungey and Pagan (2000) find that foreign financial shocks are particularly important.

15 The equations are listed in Appendix B.

constant elasticity of substitution (CES) technology. Each of these goods themselves is a CES aggregate of many individual goods. The US consumption bundle, alternatively, is composed only of US-produced goods. Australian households may also hold Australian dollar- and US dollar-denominated bonds, with a debt-elastic premium (which includes a stochastic component) on the latter to ensure these holdings are stationary. US households hold only US dollar-denominated bonds. Australian dollar-denominated debt in Australia (and US dollar-denominated debt in the United States) is assumed to be in zero net supply. Households receive income from the interest from their bond holdings, dividends from monopolistically competitive firms, transfer payments from the government, and working. Households supply differentiated labour, which firms aggregate using CES technology. A labour disutility shock, capturing changes in preferences with respect to the supply of labour, is also included.

Retail firms in Australia import US goods. The law of one price is assumed to hold at the docks, but retailers (importers) produce differentiated goods for consumption. Domestic producers also produce differentiated goods. A standard log-linear demand function is assumed for US consumption of Australia's domestic good. In all monopolistically competitive markets, pricing is modelled using the Calvo approach, where firms and households that do not get the opportunity to reset their prices optimally in a given quarter index them to past inflation. Cost-push shocks are included for all firms.

Monetary policy is set following a Taylor rule which allows policy to depend on contemporaneous inflation, deviations of output from its steady-state level, output growth and changes in the nominal interest rate, as well as the lagged deviation of the interest rate from its steady state level. An exogenous monetary policy shock is included. Finally, all markets clear.

To summarise, there are twelve exogenous shocks, five for the United States and seven for Australia. These include, for each country:

- technology
- cost-push
- preferences

- labour disutility
- monetary policy.

The additional shocks for Australia are a cost-push shock for retail firms and the risk-premium shock, which appears in the uncovered interest parity condition. These shocks generally follow first-order autoregressive processes, with the exception of the monetary policy and the cost-push shocks, although the cost-push shock for retailers is allowed to be persistent. The Australian economy is small in the sense that its shocks cannot affect the United States.

4.2 Data

As there are twelve shocks, and in the approach to identifying the empirical BVAR-DSGE model described previously it was assumed that the number of variables equals the number of shocks, twelve observable variables are used. This is the same number used by Justiniano and Preston (2010a). I also mostly follow their definitions of the data, but use: Australian series in place of their Canadian counterparts; non-farm GDP rather than GDP per capita as the measure of output; and total, rather than average, hours worked. The precise definitions are given in Appendix C. The observed series in both countries are:

- output
- inflation
- interest rates
- real wages
- hours worked.

For Australia, changes in the real exchange rate and the terms of trade are also included. Note that the definition of the observed real exchange rate and the terms of trade are those conventionally used in Australia and are the inverse of the model definition. Consequently, the measurement equations of these variables have the observed change as the negative of their model counterparts.[16] A further aspect to note is that when Justiniano and Preston (2010a) map the data to the model, they define the terms of trade as the ratio of retail import prices to domestically produced good prices (as these are exported), whereas in the data the import prices are measured ‘at the docks’. Justiniano and Preston state that their results are not sensitive to this. In contrast, altering the measurement equations to address this yielded implausible estimates in the Australian case.

Output and real wages are linearly detrended, as in Justiniano and Preston (2010a), and hours worked are detrended.[17] The estimation sample is 1993:Q1 to 2010:Q3, noting that inflation targeting commenced in Australia in 1993:Q1.

4.3 DSGE Posterior

The priors for the large economy are based on those used by Justiniano and Preston (2010a). I deviate slightly for the priors on the exogenous processes. They state the priors include ‘... a “tilt” towards the foreign block disturbances, which are assumed twice as volatile and more persistent than their domestic counterparts’ (p 67). I remove this tilt as, intuitively, it is difficult to conceive why a large country, namely the United States, would be more volatile. For the small economy I draw on Justiniano and Preston (2010b), but do not use a structural model for the large economy. I also deviate from the priors of Justiniano and Preston (2010a, 2010b) in a number of ways. First, I fix the inverse Frisch elasticity, which governs the labour supply response to changes in the real wage, as estimation tended to yield very small values. Second, I assume that the small economy Taylor rule does not include the exchange rate; Lubik and Schorfheide (2007) find little role for it.

Table 1 below shows the prior and posterior values for the DSGE parameters. The notation of the parameters follows Justiniano and Preston (2010a).[18] A random-

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16 That is, an increase in the observed real exchange rate is an appreciation. A further difference with Justiniano and Preston (2010a) is that I use a trade-weighted, rather than a bilateral, measure of the exchange rate. The observed terms of trade are calculated as the ratio of export to import prices.

17 Justiniano and Preston (2010a) extract a common linear trend, whereas I detrend each series individually.

18 The elasticity of substitution is between individual consumption goods (for Australia both domestic and imported) and types of labour. These are necessary to facilitate price and wage stickiness.

walk Metropolis-Hastings chain of one million observations was used to simulate the posterior. The first 85 per cent of the chain was dropped to ensure that it had converged to a stationary distribution. To check that convergence had occurred a second chain was also run.

The estimates of the preferences of the representative Australian consumer differ somewhat from those of previous studies: the intertemporal elasticity of substitution is surprisingly high (1.40), and the value of the habits parameter h suggests that there is only a limited role for external habits, which is at odds with the findings of Justiniano and Preston (2010b) (0.33) and Jääskelä and Nimark (2011) (0.76).[19] The estimated elasticity of foreign demand for Australian goods (0.38) is less than was found by Justiniano and Preston (2010b) (0.58), although given the strong demand from China for Australian resource exports in recent years, a lower elasticity may be plausible.

The Calvo parameters for domestically produced goods are lower than those of Justiniano and Preston (2010b), and are considerably smaller than generalised method of moments estimates for Australia (e.g. Kuttner and Robinson (2010)). One factor may be that there is only one good produced in the model, which can either be consumed or exported. Over the second half of the 2000s, export prices grew by considerably more than consumer prices, which is reflected in the rise in the terms of trade. The model appears to reconcile this tension by making prices relatively flexible. Wages are estimated to be much more flexible than was found by Jääskelä and Nimark (2011) (a Calvo parameter of 0.29, compared with 0.63). There is estimated to be considerable indexation of prices and wages, particularly for domestically produced goods (0.63). These figures, while broadly comparable with those found by Jääskela and Nimark, are sufficiently large to raise the question of whether it is truly indexation which is being captured.

19 All figures referring to this paper or Jääskelä and Nimark (2011) are estimates for the mean, whereas those from Justiniano and Preston (2010a, 2010b) are for the median of the posterior. The estimation samples are: Justiniano and Preston (2010b) 1984:Q1–2007:Q1; Jääskelä and Nimark (2011) 1993:Q2–2007:Q3.

Table 1: DSGE Estimation Results
(*continued next page*)

Coefficient	Description	Prior			Posterior	
Australia		Density	Mean	Std dev	Mean	90% HPD
h	Habits	B	0.5	0.25	0.18	0.03–0.32
σ	Intertemporal ES	N	1.0	0.4	1.40	0.93–1.83
λ^*	Elasticity foreign demand	G	1.5	0.5	0.38	0.24–0.54
η	Elasticity H-F goods	G	1.5	0.75	0.34	0.13–0.55
ρ_a	Technology	B	0.8	0.1	0.91	0.86–0.96
ρ_g	Preferences	B	0.8	0.1	0.81	0.73–0.89
$\rho_{cp,F}$	Cost-push imports	B	0.5	0.25	0.60	0.39–0.81
ρ_{rp}	Risk premium	B	0.8	0.1	0.95	0.92–0.99
ρ_L	Labour disutility	B	0.8	0.1	0.52	0.40–0.64
θ_i	Taylor rule, smoothing	B	0.5	0.25	0.80	0.74–0.86
α_H	Calvo domestic prices	B	0.5	0.1	0.55	0.46–0.63
α_F	Calvo import prices	B	0.5	0.1	0.56	0.45–0.68
α_W	Calvo wages	B	0.6	0.1	0.29	0.22–0.36
θ_π	Taylor rule, inflation	G	1.5	0.3	2.17	1.78–2.59
θ_{dy}	Taylor rule, growth	G	0.25	0.13	0.53	0.26–0.78
θ_y	Taylor rule, output	G	0.25	0.13	0.10	0.04–0.16
γ_H	Indexation, domestic	B	0.5	0.2	0.63	0.42–0.84
γ_F	Indexation, imports	B	0.5	0.2	0.44	0.18–0.71
γ_W	Indexation, wages	B	0.5	0.2	0.57	0.27–0.90
Standard deviations						
σ_a	Technology	IG	1	1	0.79	0.69–0.91
σ_g	Preferences	IG	1	1	1.18	0.79–1.56
$\sigma_{cp,H}$	Cost-push domestic	IG	0.25	1	0.70	0.57–0.83
$\sigma_{cp,F}$	Cost-push foreign	IG	1	1	2.69	1.68–3.74
σ_{rp}	Risk premium	IG	1	1	0.47	0.35–0.59
σ_n	Labour disutility	IG	5	1	5.96	4.23–7.66
σ_i	Monetary policy	IG	0.25	0.25	0.15	0.12–0.18
Calibrated						
φ	Inverse Frisch	1				
β	Discount factor	0.99				
χ	Interest debt elasticity	0.01				
τ	Openness	0.185				
θ	Elasticity of substitution	8				

Table 1: DSGE Estimation Results
(*continued*)

United States							
Coefficient	**Description**	**Prior**			**Posterior**	
		Density	Mean	Std dev	Mean	90% HPD
h^*	Habits	B	0.5	0.1	0.18	0.51–0.74
σ^*	Intertemporal ES	N	1.0	0.4	0.92	0.64–1.20
ρ_{a^*}	Technology	B	0.8	0.1	0.89	0.84–0.94
ρ_{g^*}	Preferences	B	0.8	0.1	0.89	0.83–0.94
ρ_{L^*}	Labour disutility	B	0.8	0.1	0.65	0.51–0.79
θ_{i^*}	Taylor rule, smoothing	B	0.8	0.1	0.88	0.85–0.91
α_{H^*}	Calvo prices	B	0.6	0.1	0.90	0.87–0.93
α_{W^*}	Calvo wages	B	0.6	0.1	0.39	0.32–0.47
θ_{π^*}	Taylor rule, inflation	N	1.8	0.3	1.90	1.48–2.32
θ_{dy^*}	Taylor rule, growth	N	0.3	0.2	0.80	0.58–1.01
θ_{y^*}	Taylor rule, output	G	0.25	0.13	0.06	0.02–0.09
γ_{H^*}	Indexation, prices	B	0.5	0.2	0.42	0.23–0.61
γ_{W^*}	Indexation, wages	B	0.5	0.2	0.56	0.25–0.88
Standard deviations						
σ_a^*	Technology	IG	1	1	0.63	0.55–0.73
σ_g^*	Preferences	IG	1	1	2.79	1.79–3.57
σ_{cp}^*	Cost-push	IG	0.25	1	0.24	0.20–0.27
σ_n^*	Labour disutility	IG	5	1	6.22	4.20–8.20
σ_i^*	Monetary policy	IG	0.25	0.25	0.12	0.10–0.13
Calibrated						
φ^*	Inverse Frisch	1				
β^*	Discount factor	0.99				
θ^*	Elasticity of substitution	8				

Notes: Prior distributions are B – Beta, N – Normal, G – Gamma, IG – inverse Gamma; HPD denotes highest probability density; ES denotes elasticity of substitution; H-F denotes home-foreign

The estimated Taylor rule for Australia exhibits a similar degree of persistence to those of Justiniano and Preston (2010b) and Jääskelä and Nimark (2011), but reacts more aggressively to inflation.[20] A one standard deviation quarterly monetary policy shock is estimated to be around 15 basis points, which is smaller than was found by Justiniano and Preston (2010b) but, as their estimation sample covers more than just the inflation-targeting period, this seems plausible.

20 Note that the inflation, output and real exchange rate terms in the Taylor rule in Jääskelä and Nimark (2011) are lagged by one period, whereas in the other models they are not (if included).

Turning to the exogenous processes, it is apparent that the standard deviation of the labour disutility shock in Australia is very large. One possible explanation is that, as the model has both a simple production and financial structure, it uses this shock to explain much of the decrease in output during the global financial crisis. This is also evident in the estimate for the United States. The standard deviations of the labour supply shocks in both countries are imprecisely estimated, despite a relatively tight prior being used. Compared to the findings of Justiniano and Preston (2010a) most of the estimates of the other US parameters are similar, although I find wages to be much more flexible, with an estimated Calvo parameter of 0.39, compared to 0.87, but indexation appears to be more prevalent. The high Calvo parameter for goods prices and the degree of indexation found imply quite a flat Phillips curve, although not as flat as that estimated by Justiniano and Preston (2010a).

The BVAR-DSGE model estimated using the methodology of Del Negro and Schorfheide (2004) yields different estimates of the DSGE parameters to those presented above and are in Appendix D. The differences arise because the DSGE and BVAR-DSGE parameters are jointly estimated in the Del Negro and Schorfheide (2004) approach. Another noticeable difference is that the standard deviations of the shocks obtained from the Del Negro and Schorflheide methodology for both economies tend to be smaller.

4.4 Empirical BVAR-DSGE Model Results

One of the main conclusions of Justiniano and Preston (2010a) is that the DSGE model does not capture the impact of the large economy shocks on the small economy, and so I do not match those impulse responses. Instead, I select Q^L to match the impact of large economy shocks on the large economy variables alone.[21]

In order to examine the dynamics of the empirical BVAR-DSGE model, and how they differ from those of the DSGE model, I now present the estimated responses to several domestic and foreign shocks. The main point of this section is to illustrate some of the dynamics of the empirical BVAR-DSGE model, particularly with respect to the impact of foreign shocks on the Australian economy, and how they differ from those of the DSGE model.

21 For the Gibbs sampler I take 30 500 draws, and discard the first 500.

In this section, BVAR-DSGE refers to the estimates obtained using the methodology of Del Negro and Schorfheide (2004), and BVAR to those from the empirical BVAR-DSGE model. Recall that λ is a parameter that was introduced to allow for some judgment about the variance of the prior in the BVAR parameters, and therefore how tightly the prior is held. Examining plots of the posterior distributions suggests that for $\lambda = 1$ the prior is very tightly held. This could partly reflect the fact that the posterior of the DSGE model was used to construct the prior for the BVAR, but it is also tightly held if I sample from the prior of the DSGE model, which suggests that the structure of the DSGE model itself places considerable restrictions on the reduced-form BVAR. Consequently, the impulse responses are also shown in the following figures for when the prior is relaxed, using $\lambda = 100$. Note that the size of a one standard deviation shock varies across the models.

In both the BVAR-DSGE and the BVAR, following Justiniano and Preston (2010a), the terms of trade and the real exchange rate are included in first differences. In the DSGE model this can be motivated by concerns about these variables possibly being non-stationary in the data, despite the predictions of the model. However, unlike the DSGE case, using first differences in the BVAR-DSGE and the BVAR models means that shocks may have a permanent impact on these variables. An alternative is to include these variables as deviation from trend, constructed outside the model (for example, using a Hodrick-Prescott filter). Another approach, which is probably preferable to off-model filtering, would be to introduce permanent shocks into the DSGE model itself.[22]

4.4.1 Impulse responses – Australian monetary policy shock

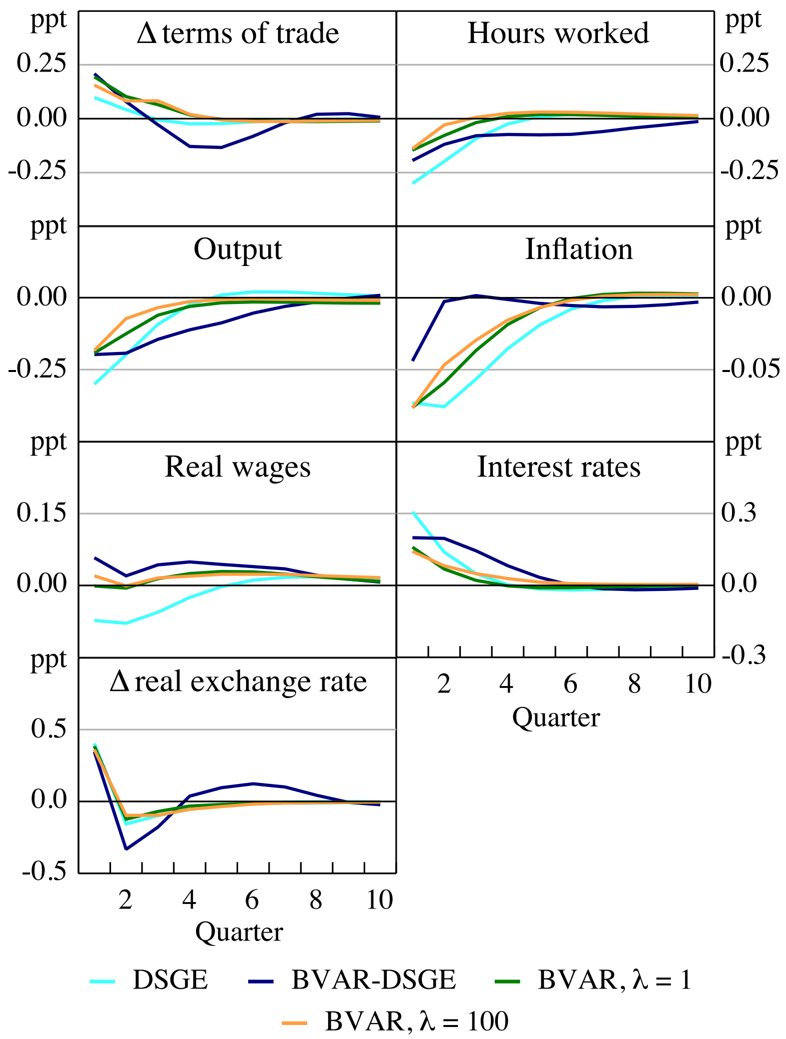
The impulse responses for most of the observed variables from the BVAR (with either value of λ) and the DSGE model, are similar for an Australian monetary policy shock (Figure 1). The size of the quarterly monetary policy shock in the

22 The estimate of the trend from, for example, a Hodrick-Prescott filter, may be inconsistent with that implied by the model and using the corresponding gap series could create econometric problems (Fukač and Pagan 2010). Alternatively, the typical way a permanent shock is introduced into a small open economy model is by introducing a common technology shock to both countries that is non-stationary. This will not produce a non-stationary real exchange rate or terms of trade, but instead predicts that many real variables, such as output in both countries, will be cointegrated, although the data do not support this. For further discussion on this see Justiniano and Preston (2010a) for Canada and Dungey and Pagan (2009) for Australia.

DSGE model is approximately 15 basis points. In all four models, the monetary policy shock leads to an immediate fall in output, which is in contrast to Dungey and Pagan (2000), where they restrict the cash rate to affect output with a lag. There is also a sizable contemporaneous appreciation of the real exchange rate and fall in inflation.

Figure 1: Response of Australian Variables to a Monetary Policy Shock

One standard deviation



The monetary policy tightening in Australia leads to a rise in the terms of trade (the ratio of export prices to import prices). This occurs in the DSGE model in the short term because the decline in inflation in domestically produced goods (exports) is less than it is for imports. While the degree of price stickiness for both domestic good producers and retailers is similar, the marginal costs of domestic goods are also sticky due to nominal wage rigidities, whereas this is not the case for imports.

One aspect of the Justiniano and Preston (2010a) DSGE model is that the terms of trade is treated as endogenous to the small economy. This is because, while the small economy cannot affect the world (US) price of its imports, importers in this model produce differentiated goods from these imports, and price them with a mark-up. When the model is mapped to the data, as previously mentioned, import prices in the terms of trade are equated to the retail price, rather than the 'at-the-dock' price. Australian domestic good producers also have some market power, and these goods are exported.

The greatest divergences between the impulse responses from the Justiniano and Preston (2010a) model and those from the BVAR models are with respect to the labour market variables. In the DSGE model there is a decline in the real wage and hours worked following a monetary policy shock, which is not surprising given the simple linear production structure for domestic goods where labour is the only input (apart from technology). The BVAR estimates, which relax this, find a smaller decline in both hours worked and the real wage.

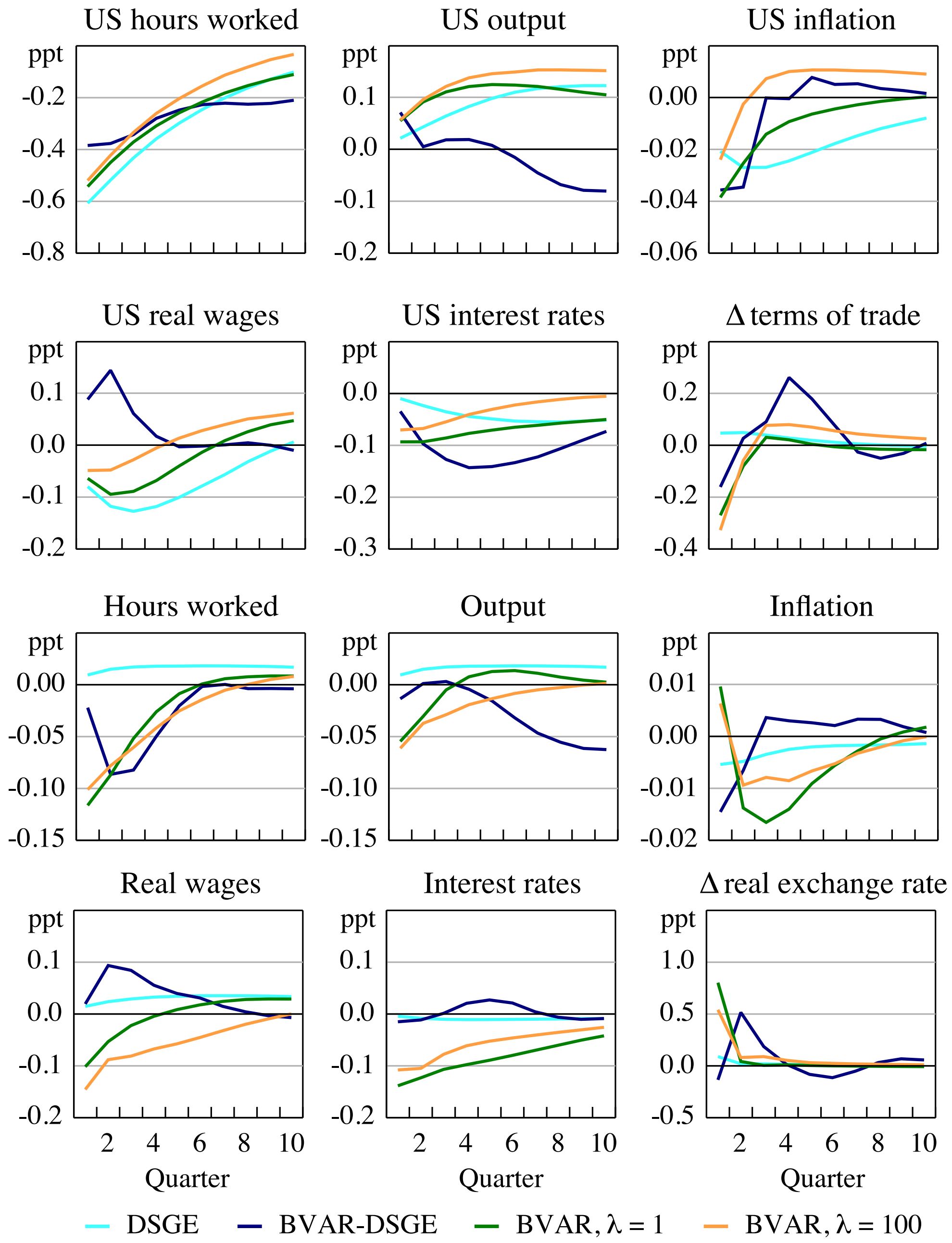
4.4.2 Impulse responses – foreign shocks

Turning now to the foreign shocks, I consider first a positive productivity shock and its impact on the US economy (Figure 2). The increase in output estimated by the BVARs is greater than that estimated by the DSGE model, whereas for most time horizons the fall in inflation is slightly less. In all models, hours worked decline in response to the productivity shock. The decrease in inflation in the DSGE model is small, reflecting the flatness of the Phillips curve, and in the BVAR models it is not very persistent, especially as the prior is relaxed. After an initial increase, the impact on output in the BVAR-DSGE model gradually becomes negative, which is difficult to rationalise as it also predicts the largest fall in the interest rate occurs after around one year.

The impact of a positive US productivity shock on the Australian variables is surprising. The estimates from the DSGE model suggest that, for all variables, the impact is slight. In contrast, the BVAR models, with either λ suggest that the real exchange rate appreciates and the terms of trade decline. Intuitively one might have expected Australia's terms of trade to improve, given that import prices should have fallen due to the nominal exchange rate appreciation, however, domestic inflation (and hence export prices), decreases after an initial rise. This could reflect the surprisingly large decreases in the interest rate, which may be an attempt to mitigate the decline in output (possibly stemming from a decrease in the volume of exports).

Figure 2: Response to US Productivity Shock

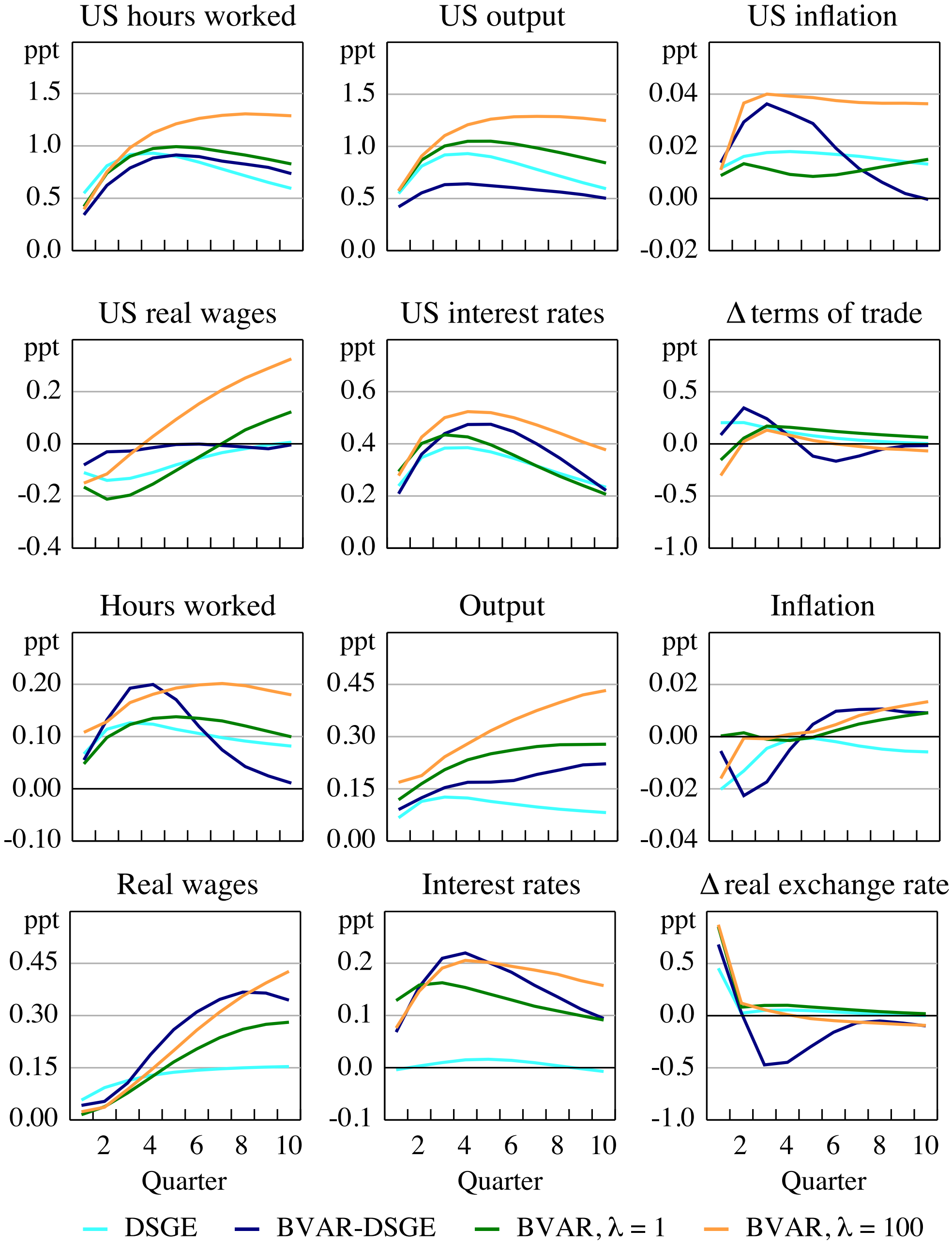
One standard deviation



A positive US preference shock, which is an example of a demand shock, leads to a rise in US hours worked and output (Figure 3). While the estimated initial impact for both variables is very similar for the DSGE and the BVAR models in the near term, loosening the prior appears to make these effects more persistent to a surprising extent. These models all suggest that there is little impact on US inflation, which is probably due to the flatness of the Phillips curve in the prior, although an accompanying factor is that the shock causes an aggressive monetary policy tightening.

Figure 3: Response to US Preference Shock

One standard deviation



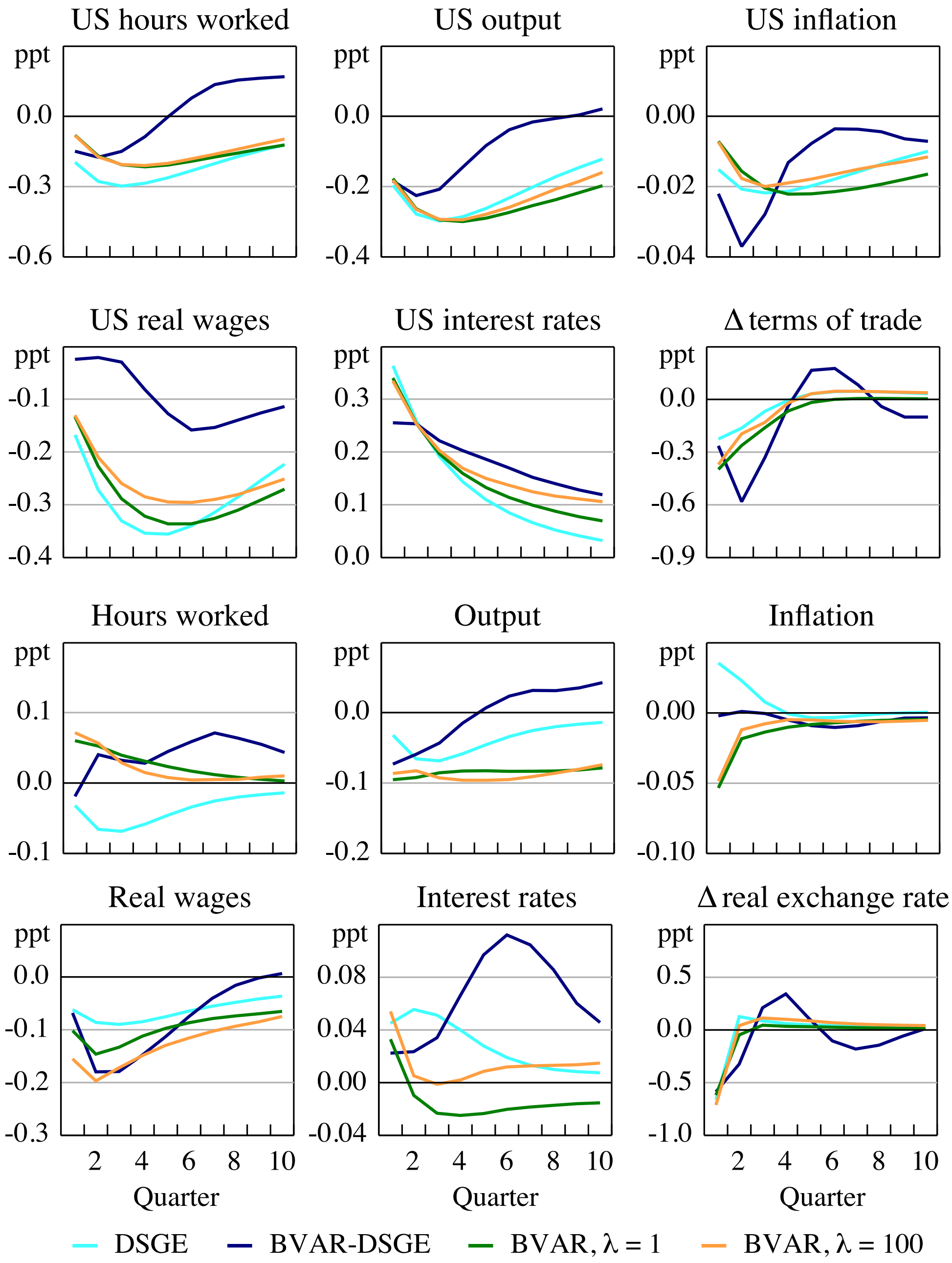
Considering now the impact on Australia of a positive US preference shock, the predictions for the terms of trade are mixed. Intuitively one would expect such a shock to raise demand for Australia's exports and thus increase the terms of trade. The DSGE and BVAR-DSGE models suggest that the terms of trade does increase contemporaneously, whereas the BVAR models predict an initial decline and then some modest growth. The models all suggest that there is a persistent increase in Australian output, meaning that the foreign demand shock increases the volume of exports by more than their value. This is surprising given Australia's recent history of strong growth in the terms of trade and subdued growth in the volume of exports.[23] The magnitude of this effect is greater in the BVAR models than in the DSGE model, and it increases with λ. Accompanying the rise in output are increases in hours worked and real wages, together with a small monetary policy tightening. The impact on inflation is slight; although output increases there is also a substantial real appreciation in the exchange rate.

The impact of a positive US monetary policy shock on the US variables are estimated to be similar by the DSGE and the BVAR models, although output declines by slightly more in the latter (and more so when $\lambda = 100$) (Figure 4). The decline in US inflation is very small. A larger, albeit still small, impact is estimated in the BVAR-DSGE model. Turning to the impact on Australia, the BVAR models suggest a depreciation in the real exchange rate, a response that is predicted by uncovered interest parity. The decline in the terms of trade predicted by the BVAR models is of a comparable magnitude to that in the DSGE model. A fall in output is predicted by all of the models, although it is more persistent in the BVARs. Intuitively, one would expect hours worked to decline as output falls, however, this only occurs in the DSGE model, with more sizable falls in real wages mitigating its decline in the BVARs. The estimated impact on Australian inflation differs across the models, with the DSGE model predicting a small increase, in part due to the real exchange rate depreciation, whereas the BVAR models suggest the opposite, perhaps as they predict a larger decline in output.

23 The latter partially reflects the depletion of existing oil and gas fields with expectations of stronger growth resulting from very high resource sector investment over recent years (Plumb, Kent and Bishop 2013).

Figure 4: Response to US Monetary Policy Shock

One standard deviation



4.4.3 Forecast error variance decomposition

One of the main findings of Justiniano and Preston (2010a) is that the estimated DSGE model suggests that US shocks account for an implausibly small share (generally less than 3 per cent) of the variation in key Canadian series at various forecast horizons. Table 2 shows that a similar result is obtained when the DSGE model is applied to Australia, although the share is larger.

The estimates in Table 2 suggest that the contribution of foreign shocks to the forecast error variance of the terms of trade is less than five per cent at most horizons.[24] Alternatively, the empirical BVAR-DSGE model, denoted in Table 2 as B-D, estimates that between 10 and 15 per cent of the variation in changes in the terms of trade is due to foreign shocks, which, while much higher than suggested by the DSGE, is still implausibly small. This may be because the foreign sector is being modelled as the United States, whereas in recent years strong demand for commodities from China has been a key factor explaining the rapid rise in Australia's terms of trade. The empirical BVAR-DSGE model suggests that US cost-push shocks play a larger role in explaining the variation in forecast errors in the terms of trade than in the DSGE model.

Table 2: Proportion of Small Economy Forecast Error Variances due to Large Economy Shocks

Per cent

	1-quarter horizon		4-quarter horizon		8-quarter horizon	
	DSGE	B-D	DSGE	B-D	DSGE	B-D
Δ Real exchange rate	4.5	18.5	4.5	18.6	4.5	18.7
Δ Terms of trade	4.1	11.4	5.4	13.3	4.9	14.1
Output	1.2	19.9	2.7	20.2	3.2	26.7
Interest rate	0.6	31.5	0.8	34.0	0.7	35.2
Inflation	1.1	15.9	1.1	14.7	1.0	17.2
Real wage	1.0	8.3	2.8	8.7	4.8	12.4
Hours worked	0.9	12.2	3.3	15.6	4.9	20.0

Notes: DSGE evaluated at mean of the posterior parameter values; B-D denotes the empirical BVAR-DSGE model and is the median share; B-D has 2 lags and $\lambda = 1$

Turning to the real exchange rate, the BVAR with $\lambda = 1$ suggests that around 19 per cent of fluctuations in the forecast error of changes in the real exchange rate are due to foreign variables, primarily reflecting US cost-push shocks and technology shocks (Table 3). Only around 20 per cent of fluctuations in Australian output are due to foreign shocks, possibly as the real exchange rate is acting as a buffer. The variable with the highest contribution from US shocks – around 30 to 35 per cent – is the interest rate, reflecting foreign cost-push and, to a lesser extent, preference and technology shocks. One interpretation of cost-push shocks is that they capture global inflation surprises. Interestingly, a recursive VAR estimated using OLS finds a similarly large role for foreign shocks for explaining the interest

24 As discussed previously, the terms of trade for Australia is not exogenous in this model.

rate. One possibility is that this reflects the rapid series of interest rate cuts that occurred in Australia during the global financial crisis. There is little variation in these shares as the forecast horizon increases, although the role of foreign shocks for output and the terms of trade does increase at longer horizons.

Table 3: **Small Economy Forecast Error Variances due to Large Economy Shocks, Varying Prior Variance**
Per cent

	DSGE	BVAR		
		$\lambda = 1$	$\lambda = 10$	$\lambda = 100$
4-quarter horizon				
Δ Real exchange rate	4.5	18.6	17.8	18.9
Δ Terms of trade	5.4	13.3	13.7	16.5
Output	2.7	20.2	25.6	28.5
Interest rate	0.8	34.0	30.3	33.2
Inflation	1.1	14.7	16.2	18.5
Real wage	2.8	8.7	12.0	16.3
Hours worked	3.3	15.6	15.7	17.2
10-quarter horizon				
Δ Real exchange rate	4.5	18.7	17.8	19.4
Δ Terms of trade	4.9	14.2	14.6	17.8
Output	3.4	30.0	41.7	47.5
Interest rate	0.7	35.5	33.0	39.0
Inflation	1.0	18.1	19.7	22.3
Real wage	5.6	15.1	22.1	28.2
Hours worked	5.4	21.4	26.2	25.1

Notes: DSGE evaluated at mean of the posterior parameter values; BVAR is the median share and has 2 lags

5. Conclusions

BVAR-DSGE models represent a compromise between theoretically coherent DSGE models and more empirically orientated VAR models. The method of Del Negro and Schorfheide (2004) uses a DSGE model to generate a prior for the parameters of a BVAR, but does not accommodate the block exogeneity restriction typically imposed in models of small open economies. Consequently the small economy ceases to be small, in the sense that its shocks can have an impact on the large economy. This paper describes an alternative method of estimating and identifying an empirical BVAR model, where a DSGE model is used as a source of prior information, that is suitable for small open economies.

Like Del Negro and Schorfheide (2004), the method developed in this paper involves estimating a reduced-form BVAR and then identifying it. However, this paper uses a prior that allows some variables to be excluded from some equations. In this way, lags of the small economy variables can be excluded from the large economy equations. Alternative approaches may well exist. One possible alternative might be to place a prior directly on the contemporaneous matrix, perhaps using some combination of the methods of Waggoner and Zha (2003) and Park (2011). I leave exploring these possibilities for future research.

One of the main findings of Justiniano and Preston (2010a) is that in an estimated small open economy model of Canada, little role is found for US shocks in explaining fluctuations in Canadian variables at various forecast horizons. I apply the method outlined in this paper to the Justiniano and Preston (2010a) model using US and Australian data. I show that a similarly small role for foreign shocks is suggested by the DSGE model for Australia, but the empirical BVAR-DSGE model that imposes block exogeneity restrictions finds a larger role. However, the contribution of foreign shocks to fluctuations in some variables, such as the terms of trade, is still much smaller than one would expect intuitively, and some of the impulse responses to foreign shocks are surprising, possibly reflecting the possibility that for Australia the United States is no longer a good proxy for the world economy for the latter part of the sample. Ultimately, while the empirical BVAR-DSGE model allows a greater role for the data, the DSGE model still has considerable influence, both in informing the prior and identifying the model, and consequently a better DSGE model should result in a better empirical BVAR-DSGE model.

Appendix A: Identifying the Large Economy

This appendix shows that the problem of selecting Q^L to match the impulse responses of the large and small economies to large economy shocks can be obtained in the same way as when only the impact on the large economy is considered, with some matrices redefined.

The problem is

$$\begin{array}{ll} \min_{Q^L} & \left\| R^L Q^L - B^{L-1}_{DSGE\ median} \right\| + \left\| \kappa R^L Q^L - B^{SL-1}_{DSGE\ median} \right\| \\ s.t. & Q^{L'} Q^L = I. \end{array}$$

The solution closely follows Schönemann (1966). In this appendix I adopt his notation, which is different to that throughout the rest of the paper. Let $T \equiv Q$, $A_1 \equiv R^L$, $B_1 \equiv B^{L-1}_{DSGE\ median}$, $A_2 \equiv \kappa R^L$, and $B_2 \equiv B^{SL-1}_{DSGE\ median}$. I can then define two residual matrices $E_1 = A_1 T - B_1$ and $E_2 = A_2 T - B_2$. The problem can then be rewritten as:

$$\begin{array}{ll} \min_T & tr(E_1) + tr(E_2) \\ s.t. & T'T = I, \end{array}$$

where $tr(.)$ denotes the trace.

The objective function is:

$$\begin{aligned} g_1 &= tr(E_1) + tr(E_2) \\ &= tr(T'A_1'A_1T - 2T'A_1'B_1 + B_1'B_1) \\ &\quad + tr(T'A_2'A_2T - 2T'A_2'B_2 + B_2'B_2), \end{aligned}$$

using that $tr(A) = tr(A')$.

The constraint can be rewritten as:

$$g_2 = tr(\mathscr{L}(T'T - I)),$$

where $\mathscr{L}$ is a matrix of Lagrange multipliers. The Lagrangian, g, then is:

$$g = g_1 + g_2.$$

Partially differentiating g with respect to the elements of T provides the first order conditions (apart from the constraint):

$$\frac{\partial g}{\partial T} = 2(A_1'A_1)T - 2A_1'B + T(\mathscr{L} + \mathscr{L}') + 2(A_2'A_2)T - 2A_2'B = 0, \tag{A1}$$

using the symmetry of $A_1'A_1$ and $A_2'A_2$, and where 0 is a matrix.

Let $P_1 \equiv A_1'A_1$, $P_2 \equiv A_2'A_2$, $S_1 \equiv A_1'B$, $S_2 \equiv A_2'B$ and $Q \equiv (\mathscr{L} + \mathscr{L}')/2$. Equation (A1) can then be rewritten as

$$\frac{\partial g}{\partial T} = 2P_1T - 2S_1 + 2TQ + 2TQ + 2P_2T - 2S_2 = 0. \tag{A2}$$

Now $P \equiv P_1 + P_2$ and $S \equiv S_1 + S_2$. Note that as the sum of two symmetric matrices is symmetric, P is symmetric. Equation (A2) can then be rewritten as

$$2PT - 2S + 2TQ = 0,$$

and hence

$$S = PT + TQ. \tag{A3}$$

Equation (A3) is the same as Equation (1.9) in Schönemann (1966). This shows that these first order conditions are the same as those to the original problem, with some of the matrices redefined. Consequently the solution is the same; the optimal T, T^*, is obtained by a singular-value decomposition of S, which yields $S = WD_s^{1/2}V'$, and setting $T^* = WV'$. In the notation used in this paper S corresponds to T^L, which equals $R^{L'}B^{L-1}_{DSGE\ median} + R^{L'}\kappa'B^{SL-1}_{DSGE\ median}$. A singular value decomposition of T^L yields $T^L = U^LW^LV^{L'}$, and the solution for T^* above corresponds to $Q^{L*} = U^LV^{L'}$.

Appendix B: Log-linearised Equations of the DSGE Model

These equations are based on code provided by Bruce Preston. I make two changes, namely eliminating the nominal exchange rate from the Taylor rule (and hence the model), which removes the need to include the real exchange rate identity, and making the steady-state mark-up of the small economy importers depend on the small economy elasticity of substitution, rather than the large economy's elasticity (although these were calibrated to be the same). The notation follows Justiniano and Preston (2010a), except that π_t denotes log-linearised inflation. I denote US variables with a $*$, log-linearised variables in lower case, the jth shock by $\varepsilon_{j,t}$, and the innovation to it by $v_{j,t}$.

B.1 Variables

The mnemonics for the variables are given in Table B1.

Table B1: Mnemonics

Mnemonic	Variable	Mnemonic	Variable
y	Output	s	Terms of trade
i	Nominal interest rate	q	Real exchange rate
π	Inflation	z	Net foreign assets
w	Real wages	π_F	Imports inflation
π^w	Nominal wage inflation	*Shocks*	
n	Hours worked	ε^*_{cp}	Foreign cost-push
c	Consumption	ε_n	Labour disutility
C_H	Consumption of home goods	ε_a	Technology
Y	Level of output	ε_i	Monetary policy
$\frac{P_H}{P}$	Rp of domestic goods	ϕ_t	Risk premium
$\frac{P_F}{P}$	Rp of imports	ε_g	Preference
ψ_F	Law on one price gap		

Note: Rp denotes relative price to consumption

B.2 Large Economy

IS curve

$$y^*_t - \frac{h^*}{1+h^*}y^*_{t-1} = \frac{1}{1+h^*}E_t(y^*_{t+1}) - \sigma^*\frac{(1-h^*)}{1+h^*}(i^*_t - E_t\pi^*_{t+1})$$
$$-\sigma^*\frac{(1-h^*)}{1+h^*}(1-\rho_g)\varepsilon^*_{g,t}$$

Price Phillips curve

$$\pi_t^* = \frac{1}{1+\beta\gamma_{H^*}}(\gamma_{H^*}\pi_{t-1}^* + \beta E_t(\pi_{t+1}^*) + \xi_{H^*}(\omega_{H^*}y_t^* + w_t^* - (1+\omega_{H^*})\varepsilon_{a,t}^*) + \varepsilon_{cp,t}^*),$$

where

$$\xi_{H^*} \equiv \frac{(1-\alpha_{H^*})(1-\alpha_{H^*}\beta)}{\alpha_{H^*}(1+\omega_{H^*}\theta^*)},$$

and $\omega_{H^*} \equiv \frac{-f^{*''}\bar{Y}_*}{\left(f^{*'}\right)^2}$, where f^* denotes the production function. Following Justiniano and Preston (2006) this is calibrated at 0.33.

Wage Phillips curve

$$\pi_t^{w*} = \beta E_t(\pi_{t+1}^{w*} - \gamma_\omega \pi_t^*) + \xi_{w^*}(\upsilon_t^* - w_t^*) + \gamma_w \pi_{t-1}^*,$$

where

$$\xi_{w^*} \equiv \frac{(1-\alpha_{W^*})(1-\alpha_{W^*}\beta)}{\alpha_{W^*}(1+\varphi^*\theta^*)},$$

and

$$\upsilon_t^* \equiv (\varphi^* + \frac{\sigma^{*-1}}{1-h^*})y_t^* - \varphi^*\varepsilon_{a,t}^* - \frac{1}{\sigma^*}\varepsilon_{g,t}^* - \frac{h^*\sigma^{*-1}}{1-h^*}y_{t-1}^* + \varepsilon_{n,t}^*.$$

Real wages

$$w_t^* = \pi_t^{w*} - \pi_t^* - w_{t-1}^*.$$

Labour

$$n_t^* = y_t^* - \varepsilon_{a,t}^*.$$

Taylor rule

$$i_t^* = \theta_{i*}i_{t-1}^* + (1-\theta_{i*})(\theta_{\pi*}\pi_t^* + (\theta_{y*} + \theta_{dy^*})y_t^* - \theta_{dy^*}y_{t-1}^*) + \varepsilon_{i,t}^*.$$

Preference shock

$$\varepsilon_{g,t}^* = \rho_{g^*}\varepsilon_{g,t-1}^* + v_{g,t}^*.$$

Technology shock

$$\varepsilon^*_{a,t} = \rho_{a^*}\varepsilon^*_{a,t-1} + v^*_{a,t}.$$

Labour disutility shock

$$\varepsilon^*_{n,t} = \rho_{n^*}\varepsilon^*_{n,t-1} + v^*_{n,t}.$$

B.3 Small Economy

Euler equation

$$c_t = \frac{1}{1+h}(hc_{t-1} + E_t(c_{t+1}) - \sigma(1-h)(i_t - E_t\pi_{t+1}) + \sigma(1-h)(1-\rho_g)\varepsilon_{g,t}).$$

Market clearing

$$y_t = \frac{\bar{C_H}}{\bar{Y}}c_t + (((1-\frac{\bar{C_H}}{\bar{Y}})\lambda^*) - \frac{\bar{C_H}}{\bar{Y}}\eta(\bar{G}-1))s_t + (1-\frac{\bar{C_H}}{\bar{Y}})\lambda^*\psi_{F,t} + (1-\frac{\bar{C_H}}{\bar{Y}})y^*_t,$$

where

$$\frac{\bar{C_H}}{\bar{Y}} = \frac{(1-\tau)\gamma_H^{-\eta}}{(1-\tau)\gamma_H^{-\eta} + \tau\gamma_H^{-1}\gamma_F^{-\eta}},$$

$$\gamma_H \equiv \frac{\bar{P_H}}{P} = \left(\frac{1-\tau(\frac{\theta}{\theta-1})^{1-\eta}}{(1-\tau)}\right)^{\frac{1}{1-\eta}},$$

$$\gamma_F \equiv \frac{\bar{P_F}}{P} = \frac{\theta}{\theta-1},$$

$$\bar{G} \equiv \frac{(1-\tau)\bar{s}^{-(1-\eta)}}{\tau + (1-\tau)\bar{s}^{-(1-\eta)}},$$

and

$$\bar{s} \equiv \frac{\bar{\gamma_F}}{\gamma_H}.$$

Terms of trade

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t}.$$

Change in the terms of trade

$$\Delta s_t \equiv s_t - s_{t-1}.$$

Law of one price

$$q_t = \psi_{F,t} + \bar{G}s_t,$$

Net foreign assets

$$z_t = \frac{1}{\beta}(z_{t-1} + q_{t-1}) - q_t - c_t - (1-\lambda^*)\psi_{F,t} - (1-\lambda^* - (1+\eta)\bar{G})s_t + y_t^*.$$

Uncovered interest parity

$$q_t = \frac{1}{1+\chi}(E_t(q_{t+1}) - (i_t - E_t\pi_{t+1}) + (i_t^* - E_t\pi_{t+1}^*) - \chi z_t + \phi_t).$$

Change in the real exchange rate

$$\Delta q_t \equiv q_t - q_{t-1}.$$

Wage inflation

$$\pi_t^w = \beta E_t(\pi_{t+1}^w - \gamma_\omega \pi_t) + \gamma_\omega \pi_{t-1} + \xi_\omega(\upsilon_t - w_t),$$

where

$$\xi_\omega \equiv \frac{(1-\alpha_W)(1-\alpha_W\beta)}{\alpha_W(1+\varphi\theta)},$$

and

$$\upsilon_t \equiv \varphi(y_t - \varepsilon_{a,t}) + \frac{\sigma^{-1}}{1-h}(c_t - hc_{t-1}) - \sigma^{-1}\varepsilon_{g,t} + \varepsilon_{n,t}.$$

Real wage

$$w_t = \pi_t^w - \pi_t - w_{t-1}.$$

Labour

$$n_t = y_t - \varepsilon_{a,t}.$$

Taylor rule

$$i_t = \theta_i i_{t-1} + (1-\theta_i)(\theta_\pi \pi_t + (\theta_y + \theta_{dy})y_t - \theta_{dy}y_{t-1}) + \varepsilon_{i,t}.$$

Domestic good inflation

$$\pi_{H,t} = \left(\frac{1}{1+\beta\gamma_H}\right)\big(\beta E_t(\pi_{H,t+1}) + \xi_H(w_t + (1-\bar{G})s_t + \omega_p y_t - (1-\omega_p)\varepsilon_{a,t})$$
$$+\gamma_H \pi_{H,t-1} + \varepsilon_{ch,t}\big),$$

where

$$\xi_H \equiv \frac{(1-\alpha_H)(1-\alpha_H\beta)}{\alpha_H(1+\varphi\omega_p)},$$

and $\omega_p \equiv \frac{-f''\bar{Y}}{(f')^2}$, where f denotes the production function. Following Justiniano and Preston (2006) this is calibrated at 0.33.

Foreign good inflation

$$\pi_{F,t} - \gamma_F \pi_{F,t-1} = \xi_F \psi_{F,t} + \beta E_t(\pi_{F,t+1} - \gamma_F \pi_{F,t}) + \varepsilon_{cf,t},$$

where

$$\xi_F = \alpha_F^{-1}(1-\alpha_F)(1-\alpha_F\beta).$$

CPI

$$\pi_t = \pi_{H,t} + \tau \triangle s_t.$$

Preference shock

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + v_{g,t}.$$

Technology shock

$$\varepsilon_{a,t} = \rho_a \varepsilon_{a,t-1} + v_{a,t}.$$

Labour disutility shock

$$\varepsilon_{n,t} = \rho_n \varepsilon_{n,t-1} + v_{n,t}.$$

Risk premium shock

$$\phi_t = \rho_{rp}\phi_{t-1} + v_{rp,t}.$$

Cost-push imports

$$\varepsilon_{F,t} = \rho_F \varepsilon_{F,t-1} + v_{F,t}.$$

Appendix C: Data Definitions

C.1 United States

Output – real GDP, 1 decimal, billions of chained 2005 dollars, seasonally adjusted at an annual rate; Federal Reserve Bank of St. Louis, FRED database: GDPC1

Inflation – chain price index of GDP; FRED database: GDPCTPI

Interest rates – Federal Funds rate, average quarter; RBA Statistical Table F13 International Official Interest Rates

Real wages – real compensation per hour, non-farm business sector, seasonally adjusted; Bureau of Labor Statistics: PRS85006153

Hours worked – non-farm business sector; Bureau of Labor Statistics: PRS85006033

C.2 Australia

Output – real non-farm GDP, chain volume, seasonally adjusted; Australian Bureau of Statistics (ABS), Australian National Acccounts: National Income, Expenditure and Product (ABS Cat No 5206.0)

Inflation – trimmed mean inflation, excluding interest payments and tax changes, seasonally adjusted; RBA

Interest rates – cash rate, average quarter; RBA Statistical Table F1 Interest Rates and Yields – Money Market

Real wages – non-farm compensation of employees, seasonally adjusted, divided by non-farm hours worked and trimmed mean inflation; compensation of employees and hours worked from ABS Cat No 5206.0

Hours worked – non-farm; ABS Cat No 5206.0, special request

Real exchange rate – real trade-weighted exchange rate; RBA Statistical Table F15 Real Exchange Rate Measures

Terms of trade – ABS Cat No 5206.0

Appendix D: BVAR-DSGE Estimates

These are estimated using the Del Negro and Schorfheide (2004) approach (see Table D1).

Calibrated parameters have the same values as previously.

Table D1: BVAR-DSGE Estimation Results
(*continued next page*)

Australia						
Coefficient	**Description**	**Prior**			**Posterior**	
		Density	Mean	Std dev	Mean	90% HPD
h	Habits	B	0.5	0.25	0.19	0.01–0.36
σ	Intertemporal ES	N	1.0	0.4	1.12	0.66–1.46
λ^*	Elasticity foreign demand	G	1.5	0.5	0.38	0.18–0.57
η	Elasticity H-F goods	G	1.5	0.75	0.57	0.10–1.06
ρ_a	Technology	B	0.8	0.1	0.82	0.72–0.92
ρ_g	Preferences	B	0.8	0.1	0.81	0.70–0.92
$\rho_{cp,F}$	Cost-push imports	B	0.5	0.25	0.74	0.48–0.94
ρ_{rp}	Risk premium	B	0.8	0.1	0.75	0.60–0.91
ρ_L	Labour disutility	B	0.8	0.1	0.54	0.39–0.70
θ_i	Taylor rule, smoothing	B	0.5	0.25	0.86	0.81–0.91
α_H	Calvo domestic prices	B	0.5	0.1	0.71	0.57–0.85
α_F	Calvo import prices	B	0.5	0.1	0.53	0.37–0.70
α_W	Calvo wages	B	0.6	0.1	0.53	0.22–0.87
θ_π	Taylor rule, inflation	G	1.5	0.3	1.90	1.38–2.41
θ_{dy}	Taylor rule, growth	G	0.25	0.13	0.36	0.12–0.60
θ_y	Taylor rule, output	G	0.25	0.13	0.18	0.04–0.33
γ_H	Indexation, domestic	B	0.5	0.2	0.47	0.18–0.75
γ_F	Indexation, imports	B	0.5	0.2	0.37	0.08–0.65
γ_W	Indexation, wages	B	0.5	0.2	0.53	0.22–0.87
Standard deviations						
σ_a	Technology	IG	1	1	0.45	0.36–0.54
σ_g	Preferences	IG	1	1	0.98	0.58–1.35
$\sigma_{cp,H}$	Cost-push domestic	IG	0.25	1	0.25	0.18–0.31
$\sigma_{cp,F}$	Cost-push foreign	IG	1	1	1.81	0.74–3.06
σ_{rp}	Risk premium	IG	1	1	0.51	0.30–0.70
σ_n	Labour disutility	IG	5	1	5.33	3.81–6.83
σ_i	Monetary policy	IG	0.25	0.25	0.07	0.06–0.08

Table D1: BVAR-DSGE Estimation Results
(*continued*)

United States

Coefficient	**Description**	**Prior**			**Posterior**	
		Density	Mean	Std dev	Mean	90% HPD
h^*	Habits	B	0.5	0.1	0.53	0.38–0.70
σ^*	Intertemporal ES	N	1.0	0.4	1.04	0.65–1.42
ρ_{a^*}	Technology	B	0.8	0.1	0.83	0.74–0.93
ρ_{g^*}	Preferences	B	0.8	0.1	0.85	0.75–0.93
ρ_{L^*}	Labour disutility	B	0.8	0.1	0.56	0.42–0.71
θ_{i^*}	Taylor rule, smoothing	B	0.8	0.1	0.86	0.82–0.91
α_{H^*}	Calvo prices	B	0.6	0.1	0.81	0.75–0.87
α_{W^*}	Calvo wages	B	0.6	0.1	0.45	0.36–0.53
θ_{π^*}	Taylor rule, inflation	N	1.8	0.3	1.92	1.46–2.36
θ_{dy^*}	Taylor rule, growth	N	0.3	0.2	0.55	0.29–0.81
θ_{y^*}	Taylor rule, output	G	0.25	0.13	0.10	0.02–0.17
γ_{H^*}	Indexation, prices	B	0.5	0.2	0.35	0.10–0.60
γ_{W^*}	Indexation, wages	B	0.5	0.2	0.51	0.19–0.84
Standard deviations						
σ_a^*	Technology	IG	1	1	0.37	0.31–0.44
σ_g^*	Preferences	IG	1	1	1.17	0.67–1.62
σ_{cp}^*	Cost-push	IG	0.25	1	0.15	0.12–0.18
σ_n^*	Labour disutility	IG	5	1	5.34	3.58–6.88
σ_i^*	Monetary policy	IG	0.25	0.25	0.07	0.06–0.09

Notes: Prior distributions are B – Beta, N – Normal, G – Gamma, IG – inverse Gamma; HPD denotes highest probability density; ES denotes elasticity of substitution; H-F denotes home-foreign

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