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**Long-term Interest  
Rates, Risk Premia and  
Unconventional Monetary  
Policy**

*Callum Jones and  
Mariano Kulish*

RDP 2011-02

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# **LONG-TERM INTEREST RATES, RISK PREMIA AND UNCONVENTIONAL MONETARY POLICY**

Callum Jones and Mariano Kulish

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## **Abstract**

In a model where the risk premium on long-term debt is, in part, endogenously determined, we study two kinds of unconventional monetary policy: long-term nominal interest rates as operating instruments of monetary policy and announcements about the future path of the short-term rate. We find that both policies are consistent with unique equilibria, that long-term interest rate rules can perform better than conventional Taylor rules, and that, at the zero lower bound, announcements about the future path of the short-term rate can lower long-term interest rates through their impact on both expectations and the risk premium. With simulations, we show that long-term interest rate rules generate sensible dynamics both when in operation and when expected to be applied.

JEL Classification Numbers: E43, E52, E58

Keywords: unconventional monetary policy, Taylor rule, risk premia, term structure

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# LONG-TERM INTEREST RATES, RISK PREMIA AND UNCONVENTIONAL MONETARY POLICY

Callum Jones and Mariano Kulish

## 1. Introduction

In the recent downturn, central banks in the United States, the United Kingdom, Canada and the euro area pushed their policy rates close to their lower bound of zero, renewing interest in alternative policy instruments. These instruments, often termed unconventional monetary policies, involve both the expansion of the central bank's balance sheet through purchases of financial securities and announcements about future policy that explicitly aim to influence expectations. Both of these policies aim to lower borrowing costs and stimulate spending. As Dale (2010) and Gagnon *et al* (2010) emphasise, the financial crisis highlighted the importance of understanding alternative ways to conduct monetary policy.

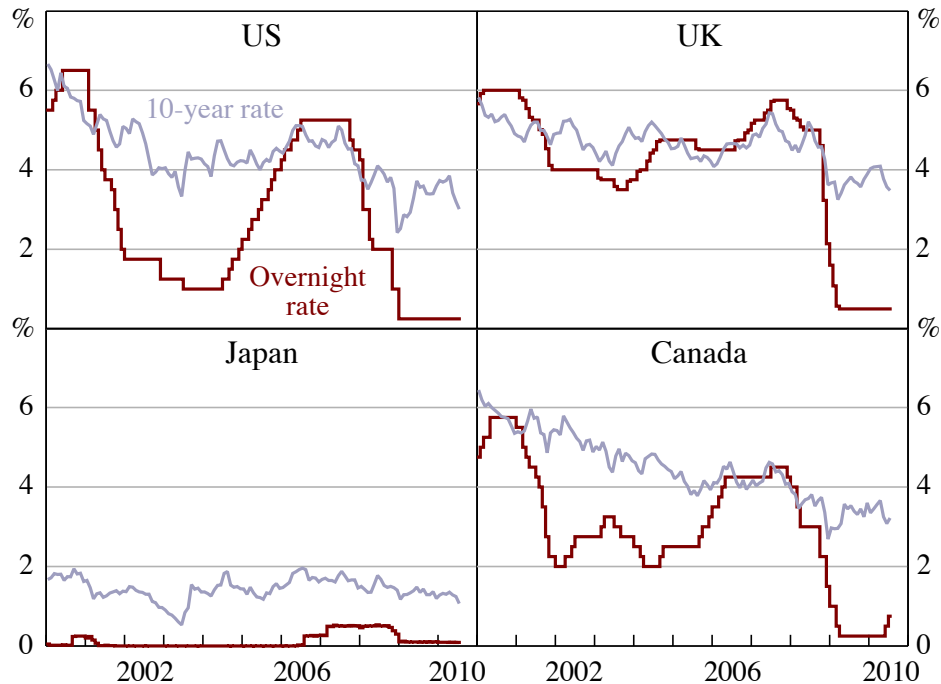
One possibility is for the central bank to purchase long-term securities in order to push down longer-term nominal interest rates. Indeed, the Bank of Japan, and more recently the Federal Reserve and the Bank of England, have pursued purchases of long-term assets.<sup>1</sup> Bernanke (2002) was one of the first to discuss this option,<sup>2</sup> while Clouse *et al* (2003) provided more detail. As Figure 1 shows, even when short rates have been close to zero in the recent episode, long rates have remained well above, suggesting that there may be greater capacity to stimulate the economy with long-term rates rather than short-term rates.

In this paper, we consider the more direct option of using a long-term interest rate as the policy instrument. Studying this possibility is more than just theoretically important. For instance, since late 1999 the Swiss National Bank has set policy by fixing a target range for the 3-month money market rate rather than setting a target for the conventional instrument of a very short-term interest rate. Jordan and Peytrignet (2007) argue that this choice gives the Swiss National Bank more flexibility to respond to financial market developments.

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1 For Japan see Ugai (2006), for the United Kingdom see Joyce *et al* (2010), and for the United States see Gagnon *et al* (2010)

2 See also Bernanke (2009).

**Figure 1: Interest Rates**

Sources: Thomson Reuters; central banks

Announcements about the path of the short rate are another way of influencing long-term rates. This too has recently been tried. The Bank of Canada, for example, announced on 21 April 2009 that it would hold the policy rate at  $\frac{1}{4}$  per cent until the end of the second quarter of 2010, while the Sveriges Riksbank announced on 2 July 2009 that it would keep its policy rate at  $\frac{1}{4}$  per cent ‘until Autumn 2010’. Also, the Federal Reserve has repeated that it intends to keep the federal funds rate low for an extended period of time.<sup>3</sup> While some central banks have previously given guidance about the direction or timing of future policy, these announcements have, at the least, been interpreted as an explicit attempt to influence expectations.

<sup>3</sup> See Board of Governors of the Federal Reserve System Press Release ‘FOMC statement’, 18 March 2009, Bank of Canada Press Release, 21 April 2009, and Sveriges Riksbank Press Release No 67, 2 July 2009.

Previous research suggests that long-term interest rate rules share the desirable properties of Taylor rules, can support unique equilibria, and their performance is comparable to more conventional Taylor rules.<sup>4</sup> However, previous studies do not contain a risk premium, or if there is one, it is exogenous. This raises important theoretical issues about the use of long-term interest rate rules. In particular, can long-term interest rate rules achieve a unique equilibrium if an endogenous risk premium prices long-term debt? And if so, how do these rules perform and what dynamics do they entail?

In this paper, we explore these questions in the context of a model in which the risk premium is endogenous and examine two kinds of unconventional monetary policy: long-term nominal interest rates as operating instruments of monetary policy and announcements about the future path of the short-term rate.

In the next section we discuss the model which is then used in Section 3 to analyse existence, uniqueness and multiplicity of the equilibrium under long-term interest rates rules. In Section 4, we study the dynamics associated with long-term interest rate rules and in Section 5 we find their optimal settings, which we compare to those of Taylor rules. Then, in Section 6, we analyse announcements about the future path of the short rate and the transition to a new rule. Section 7 concludes.

## 2. Model

In a standard log-linear New Keynesian model, long-term interest rates would be determined solely by the expected path of the short rate. However, in practice, long-term interest rates appear to deviate from the expected path of short-term rates. To take account of this, we are interested in the properties of long-term interest rate rules in a model with an explicit role for an endogenous risk premium, and so use the model developed by Andrés, López-Salido and Nelson (2004) in which there are endogenous deviations from the expectations hypothesis.

Andrés *et al* (2004) introduce an endogenous risk premium into a standard New Keynesian model by making households differ in their ability to purchase short-term and long-term bonds, together with some other frictions. *Unrestricted* households can hold both short-term and long-term securities whereas *restricted*

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<sup>4</sup> See McGough, Rudebusch and Williams (2005), Kulish (2007), and Gerlach-Kristen and Rudolf (2010).

households can only hold long-term securities. While this assumption may be somewhat unrealistic, it is useful in that it produces a tractable model with the realistic property that the risk premium is endogenous. This allows us to explore the simultaneous determination of interest rates and the risk premium when the central bank chooses a rule that sets the price of long-term debt.

The model generates two departures from the expectations hypothesis of the yield curve. First, it adds an exogenous risk premium shock. Second, it incorporates a portfolio balance term that gives a role for money in the yield curve equation. The supply side of the economy is standard, with firms operating in a monopolistically competitive environment and facing price rigidities as in Calvo (1983). For this reason, we do not discuss the supply side further, but discuss, for completeness, the less standard aspects of the model.

#### *Unrestricted households*

Unrestricted households make up a proportion,  $\lambda$ , of the population and have preferences over consumption,  $C_t^u$ , hours worked,  $N_t^u$ , and real money balances,  $M_t^u/P_t$ ; they have habits in consumption and face a cost of adjusting their holdings of real money balances. Their preferences are represented by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ a_t \left[ U \left( \frac{C_t^u}{(C_{t-1}^u)^h} \right) + V \left( \frac{M_t^u}{e_t P_t} \right) - \frac{(N_t^u)^{1+\varphi}}{1+\varphi} \right] - G(\cdot) \right\}, \quad (1)$$

where

$$U(\cdot) = \frac{1}{1-\sigma} \left( \frac{C_t^u}{(C_{t-1}^u)^h} \right)^{1-\sigma},$$

$$V(\cdot) = \frac{1}{1-\delta} \left( \frac{M_t^u}{e_t P_t} \right)^{1-\delta},$$

$$G(\cdot) = \frac{d}{2} \left\{ \exp \left[ c \left\{ \frac{M_t^u/P_t}{M_{t-1}^u/P_{t-1}} - 1 \right\} \right] + \exp \left[ -c \left\{ \frac{M_t^u/P_t}{M_{t-1}^u/P_{t-1}} - 1 \right\} \right] - 2 \right\},$$



and where,  $e_t$  is a stationary money demand shock,  $a_t$  is a stationary preference shock,  $\beta$  is the discount factor,  $\varphi$  is the inverse of the Frisch labour supply elasticity,  $\sigma$  is the coefficient of relative risk aversion, and  $\delta$ ,  $c$ , and  $d$  are positive parameters that jointly govern preferences over real money balances.

Each period, unrestricted households enter with money balances, short-term and long-term government debt left over from the previous period, and receive labour income,  $W_t N_t^u$ , dividends,  $D_t^u$ , and transfer payments from the government,  $T_t^u$ . These sources of funds are used to consume, to purchase short-term and long-term government bonds of maturity  $L$ ,  $B_t^u$  and  $B_{L,t}^u$ , at prices given by  $1/R_{1,t}$  and  $1/R_{L,t}$ , and, to hold real money balances to be carried to the next period. Their objective is to choose sequences,  $\{C_t^u, N_t^u, M_t^u, B_t^u, B_{L,t}^u\}_{t=0}^{\infty}$ , so as to maximise Equation (1) subject to a sequence of period budget constraints of the form:

$$\frac{M_{t-1}^u + B_{t-1}^u + B_{L,t-L}^u + W_t N_t^u + T_t^u + D_t^u}{P_t} = C_t^u + \frac{\frac{B_t^u}{R_{1,t}} + (1 + \zeta_t) \frac{B_{L,t}^u}{(R_{L,t})^L} + M_t^u}{P_t}. \quad (2)$$

In addition, short-term and long-term government bonds are imperfect substitutes, that is, both assets are held in positive amounts although their expected yields differ because unrestricted households face two frictions. The first is a stochastic transaction cost in the long-bond market which shifts the price of long-term bonds by  $1 + \zeta_t$ , so that households pay  $(1 + \zeta_t)/(R_{L,t})^L$  rather than  $1/(R_{L,t})^L$  for one unit of  $B_{L,t}^u$ . The second captures a liquidity risk in the market for long-term debt. Households which purchase a long-term government bond receive a return from that investment after  $L$  periods. Because there are no secondary markets for long-term government bonds in this model, by holding long bonds, households forego liquidity relative to an equivalent holding of short maturity assets. As explained by Andrés *et al* (2004), agents self-impose a reserve requirement on their long-term investments. Formally, the second friction is a utility cost specified in terms of households' relative holdings of money to long-term government bonds and is given by,

$$-\frac{v}{2} \left[ \frac{M_t^u}{B_{L,t}^u} \kappa - 1 \right]^2, \quad (3)$$

where  $\kappa$  is the inverse of unrestricted agents' steady-state money-to-long-term debt ratio and  $v > 0$  is a parameter that governs the magnitude of the cost.

### *Restricted households*

Restricted households can hold long-term government bonds but not short-term government bonds. Their preferences are like those of Equation (1), but their period budget constraint takes the form:

$$\frac{M_{t-1}^r + B_{L,t-L}^r + W_t N_t^r + T_t^r + D_t^r}{P_t} = C_t^r + \frac{\frac{B_{L,t}^r}{(R_{L,t})^L} + M_t^r}{P_t}.$$

Restricted agents do not face the other frictions. As explained by Andrés *et al* (2004), this assumption may be relaxed to a large extent, to obtain endogenous deviations from the expectations hypothesis that matter for aggregate demand. For this to be the case, agents must have different attitudes towards risk; restricted agents must regard long-term debt as a less risky investment than unrestricted agents. In any case, the assumption that a fraction of the population are not concerned about the price-risk of long-term debt can be motivated by appealing to those agents, like pension funds, that intend to hold the long-term debt to maturity.

### *Government*

The government does not spend and transfers all revenues to households. It finances these transfers through seigniorage and through the issuance of long-term and short-term government bonds. The government period budget constraint is:

$$\frac{\left( M_t + \frac{B_t}{R_{1,t}} + \frac{B_{L,t}}{(R_{L,t})^L} \right) - (M_{t-1} + B_{t-1} + B_{L,t-L})}{P_t} = \frac{T_t}{P_t}. \quad (4)$$

The supply of long-term government bonds follows an exogenous stationary process; the supply of short-term government bonds is sufficient to make up the short fall in government financing, after seigniorage and long-term bond issuance; and transfers are set according to the fiscal rule:

$$\frac{T_t}{P_t} = -\chi \frac{B_{t-1}}{P_{t-1}} + \varepsilon_t$$

where  $\chi \in (0, 1)$ .

### Monetary policy

We close the model in one of two ways. In one case, we assume that the central bank follows a policy rule in which it sets the short rate. This takes the form:

$$\hat{R}_{1,t} = \rho_R \hat{R}_{1,t-1} + \rho_\pi \pi_t + \rho_y y_t + \rho_\mu \mu_t + \varepsilon_{R,t}, \quad (5)$$

where  $\hat{R}_{1,t}$ ,  $\pi_t$  and  $y_t$  are the log deviations of the short rate, inflation and output from their steady-state values,  $\varepsilon_{R,t}$  is a stationary monetary policy shock, and  $\mu_t$  is the growth of the money supply. Alternatively, we assume instead that the central bank sets the long-term interest rate according to a policy rule of the form:

$$\hat{R}_{L,t} = \rho_R \hat{R}_{L,t-1} + \rho_\pi \pi_t + \rho_y y_t + \rho_\mu \mu_t + \varepsilon_{R,t}. \quad (6)$$

### Long-term interest rates

One can show that the nominal interest rate in period  $t$  associated with a zero-coupon bond that promises to pay one dollar at the end of period  $t + L - 1$  is determined by:

$$\hat{R}_{L,t} = \frac{1}{L} \sum_{i=0}^{L-1} \mathbb{E}_t \hat{R}_{1,t+i} + \frac{1}{L} \Phi_t, \quad (7)$$

where  $\Phi_t \equiv \zeta_t - \tau(m_t^u - b_{L,t}^u)$ , with  $m_t^u$  and  $b_{L,t}^u$  the log deviations of real money balances and long-term debt holdings from their steady-state values, and  $\tau > 0$  is a function of the structural parameters, in particular of the parameters that determine the magnitude of the financial frictions. Two terms govern the determination of  $\hat{R}_{L,t}$ . The first,  $\frac{1}{L} \sum_{i=0}^{L-1} \mathbb{E}_t \hat{R}_{1,t+i}$ , is the expectations hypothesis term, whereby the expected path of the short rate impacts on the long rate; if there were an increase in agents' expectations of future short-term rates, to avoid arbitrage opportunities, the long-rate must rise. The second is the risk premium,  $\Phi_t = \frac{1}{L} [\zeta_t - \tau(m_t^u - b_{L,t}^u)]$ , which embodies the two frictions that we discussed above:  $\zeta_t$  is the exogenous component of risk premium and  $\tau(m_t^u - b_{L,t}^u)$  is the endogenous one which depends on the relative stocks of the liquid and illiquid assets. If, for example,  $m_t^u$  falls, the loss of liquidity implies that the long-term interest rate must rise to induce agents

to hold long-term bonds. In what follows, the parameters are set to the values estimated by Andrés *et al* (2004). These are summarised in Table B1.<sup>5</sup>

### 3. Equilibrium Determinacy

A desirable property of a monetary policy rule is consistency with a unique equilibrium. Rules that fail to bring about a unique equilibrium are undesirable because they allow beliefs to turn into independent sources of business fluctuations. In other words, non-fundamental shocks may increase the volatility of equilibrium dynamics.<sup>6</sup> In general, the variables for which there may be large fluctuations due to indeterminacy include those that enter loss functions, that is those that matter for measures of an economy's welfare. So, any rule that achieves a unique equilibrium should be thought better than any rule that does not.

The log-linear equations that characterise the model's equilibrium can be written, following Sims (2002), as

$$\Gamma_0 \mathbf{y}_t = C + \Gamma_1 \mathbf{y}_{t-1} + \Psi \boldsymbol{\varepsilon}_t + \Pi \boldsymbol{\eta}_t \quad (8)$$

where  $\boldsymbol{\varepsilon}_t$  is a  $l \times 1$  vector of fundamental serially uncorrelated random disturbances, the  $k \times 1$  vector  $\boldsymbol{\eta}_t$  contains expectational errors, and the  $n \times 1$  vector  $\mathbf{y}_t$  contains the remaining variables including conditional expectations.<sup>7</sup> The matrices,  $C, \Gamma_0, \Gamma_1, \Psi$  and  $\Pi$  are of conformable dimensions. The number of generalized eigenvalues of  $\Gamma_0$  and  $\Gamma_1$  that are greater than one in absolute value is  $m$ . The values of the structural parameters that make it to the matrices  $\Gamma_0$  and  $\Gamma_1$  determine  $m$ . Cagliarini and Kulish (2008) show that

- if  $m = k$ , the solution to Equation (8) is unique;
- that if  $m < k$  there are infinitely many solutions that satisfy Equation (8);
- and that if  $m > k$  there is no stable solution that satisfies Equation (8).

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5 The results are robust to a wide range of parameter values. The MATLAB files are available on request.

6 See, for example, Lubik and Schorfheide (2004), Jääskelä and Kulish (2010) and the references therein.

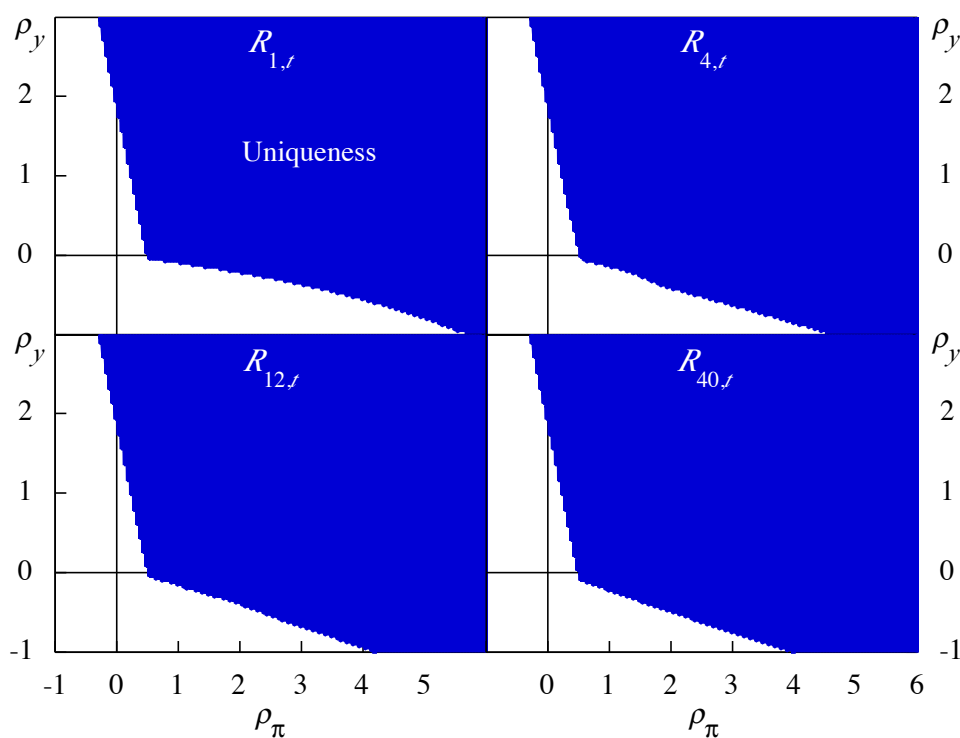
7 The expectational error for a variable  $x_t$  is  $\eta_t^x = x_t - \mathbf{E}_{t-1} x_t$ .

We use these conditions to characterise regions of existence, uniqueness and multiplicity of the equilibrium in the space of the structural parameters, in particular, in the space of the parameters of the monetary policy rule.

Figure 2 shows regions of the policy parameter space where the equilibrium is unique, for the Taylor rule and for long-term interest rate rules of maturities 4, 12 and 40.<sup>8</sup> The coefficients on inflation,  $\rho_\pi$ , and on output,  $\rho_y$ , vary; the remaining ones are fixed. The regions of uniqueness for long-term interest rate rules are large and as large as for the Taylor rule. The unshaded regions correspond to multiple equilibria or non-stationary equilibria. As in the conventional case, low responses to inflation lead to indeterminacy.

**Figure 2: Regions of Uniqueness**

$$\rho_R = 0.5 \text{ and } \rho_\mu = 0$$

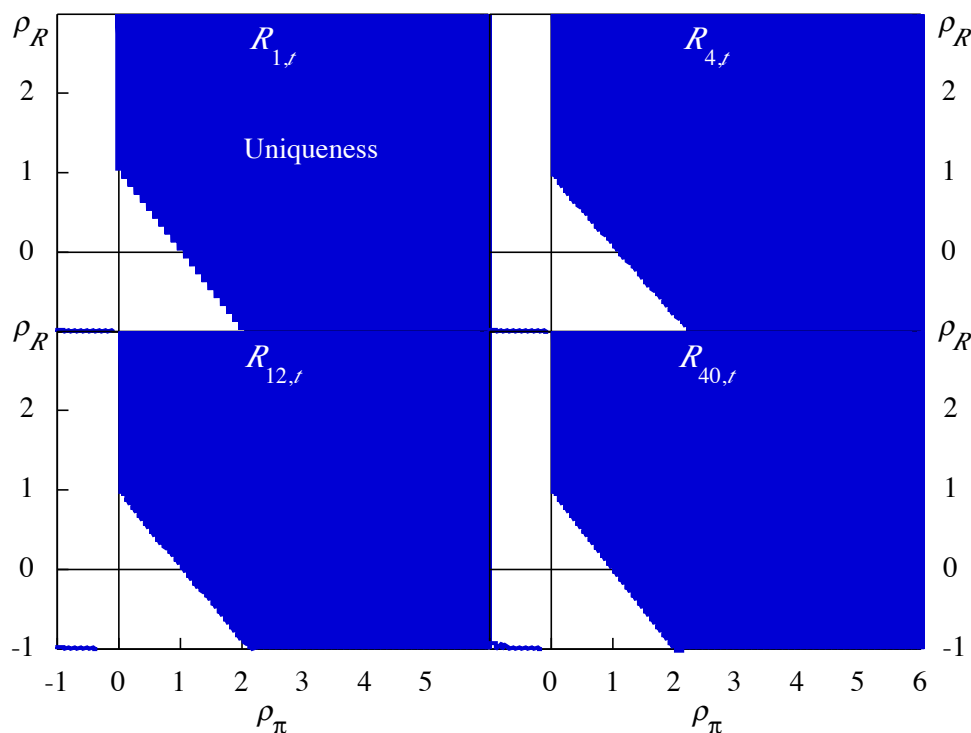


To explore equilibrium determinacy further, we compute regions of uniqueness in the space of  $\rho_R$  and  $\rho_\pi$ . Figure 3 shows that the Taylor principle holds for long-term interest rates. As the slope of the critical contour shows, uniqueness requires

<sup>8</sup> Under the Taylor rule, the friction applies at maturity 12. For the long-term interest rate rules, the friction applies at the set interest rate.

**Figure 3: Regions of Uniqueness**

$$\rho_y = 0 \text{ and } \rho_\mu = 0$$



$\rho_R + \rho_\pi > 1$ : that the long-run response of the interest rate to inflation exceeds unity.<sup>9</sup> Figure 3 shows that these regions are also large. Our analysis suggests that unique allocations are as feasible for long-term interest rate rules as they are for Taylor rules. The regions of uniqueness remain large for a wide range of other parameter values.

It may seem surprising that a long-term interest rate rule can support a unique equilibrium as well as a short-term interest rate rule can with the expectations hypothesis and an endogenous risk premium at work.<sup>10</sup> Imagine the central bank

<sup>9</sup> It is an *approximate* version of the Taylor principle, because as seen in Figure 2 the slope of contour is not exactly vertical. The condition  $\rho_R + \rho_\pi > 1$  would hold exactly if the other parameters in the rule were zero.

<sup>10</sup> McGough *et al* (2005) find that long-term interest rate rules often result in indeterminacy; more than our numerical analysis suggests. The main reason for this difference is that the long-term interest rate rules that we analyse allow for interest rate smoothing and for a response to output. Both of these, but especially the response to the lagged value of the interest rate instrument, significantly expand the regions of uniqueness.

wishes to set  $\hat{R}_{2,t}$ . Because Equation (7) requires that  $\hat{R}_{2,t} = \frac{1}{2}(\hat{R}_{1,t} + \mathbb{E}_t \hat{R}_{1,t+1} + \Phi_t)$ , it may appear that, for a value of  $\hat{R}_{2,t}$ , markets could find multiple ways in which to exhaust arbitrage opportunities, that is, multiple combinations of  $\hat{R}_{1,t}$ ,  $\mathbb{E}_t \hat{R}_{1,t+1}$  and  $\Phi_t$  that result in  $\hat{R}_{2,t}$ . This argument suggests that the ability of a long-term interest rate rule to achieve a unique equilibrium should be impaired. But, it is not.

Typically, we think of the expectations hypothesis as short rates determining long rates. But it is important to recognise that the expectations hypothesis works the other way: long rates can determine short rates too. To see this take Equation (7) for  $\hat{R}_{2,t}$  and rewrite it as a first-order, stochastic, difference equation in  $\hat{R}_{1,t}$

$$\hat{R}_{1,t} = 2\hat{R}_{2,t} - \mathbb{E}_t \hat{R}_{1,t+1} - \Phi_t.$$

Advance the equation one period and substitute the resulting expression back to obtain,

$$\hat{R}_{1,t} = 2\hat{R}_{2,t} - 2\mathbb{E}_t \hat{R}_{2,t+1} + \mathbb{E}_t \hat{R}_{1,t+2} + \mathbb{E}_t \Phi_{t+1} - \Phi_t.$$

Continue in this way to find the alternative expression for the short-term interest rate,

$$\hat{R}_{1,t} = 2\mathbb{E}_t \left[ \sum_{j=0}^{\infty} \hat{R}_{2,t+2j} - \sum_{j=0}^{\infty} \hat{R}_{2,t+2j+1} \right] - \mathbb{E}_t \left[ \sum_{j=0}^{\infty} \Phi_{t+2j} - \sum_{j=0}^{\infty} \Phi_{t+2j+1} \right].$$

The expression above shows that if a rule for  $\hat{R}_{2,t}$  supports a unique equilibrium, then it determines uniquely an expected path of the risk premium,  $\Phi_t$ , and an expected path of the long rate,  $\hat{R}_{2,t}$ . These paths simultaneously pin down the current level of the short rate,  $\hat{R}_{1,t}$ . This argument, generalised to an interest rate of an arbitrary maturity,  $\hat{R}_{L,t}$ , gives the expression below

$$\hat{R}_{1,t} = L\mathbb{E}_t \left[ \sum_{j=0}^{\infty} \hat{R}_{L,t+Lj} - \sum_{j=0}^{\infty} \hat{R}_{L,t+Lj+1} \right] - \mathbb{E}_t \left[ \sum_{j=0}^{\infty} \Phi_{t+Lj} - \sum_{j=0}^{\infty} \Phi_{t+Lj+1} \right]. \quad (9)$$

So, if the policy rule is consistent with a unique equilibrium, then there exists expected paths of the monetary policy instrument and of the risk premium, as given by Equation (9), that pin down interest rates of shorter and longer maturities.

The unique equilibrium of long-term interest rate rules is quite an important result. Fluctuations in risk premia are always found in the data.<sup>11</sup> So, imagine then, contrary to what has just been shown, that with an endogenous risk premium a long-term interest rate rule would always fail to achieve a unique equilibrium. This means that even if we were to obtain a unique outcome with the shortest of interest rates – a quarterly interest rate in a quarterly model and monthly interest rate in a monthly model – this unique outcome would not translate into a unique outcome at any higher frequency. Results from quarterly models or from monthly models would have no bearing on the real world, where monetary policy sets an overnight interest rate. But apart from the relief that the uniqueness of long-term interest rate rules may give to modellers, what’s perhaps as significant is the support that the result gives to long-term interest rates as candidate instruments of monetary policy.

Long-term interest rate rules support unique equilibria as well as Taylor rules. But what dynamics do they imply? This question is taken up next.

#### **4. Dynamics**

We compute impulse responses using the parameter values of Table B1. Figure 4 shows responses to a demand shock under the Taylor rule and under a long-term interest rate rule using  $\hat{R}_{12,t}$ , which at a quarterly frequency corresponds to a 3-year rate. The differences in the responses come from differences in the maturity of the interest rate of the policy rule.

Under both rules, output, inflation, and nominal interest rates rise for the first few periods. Indeed, the responses of all other variables are qualitatively similar and quantitatively close. The long-term interest rate rule gives rise to sensible dynamics. This is also true of the responses to other shocks.

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<sup>11</sup> See Cochrane and Piazzesi (2005) and the references therein.



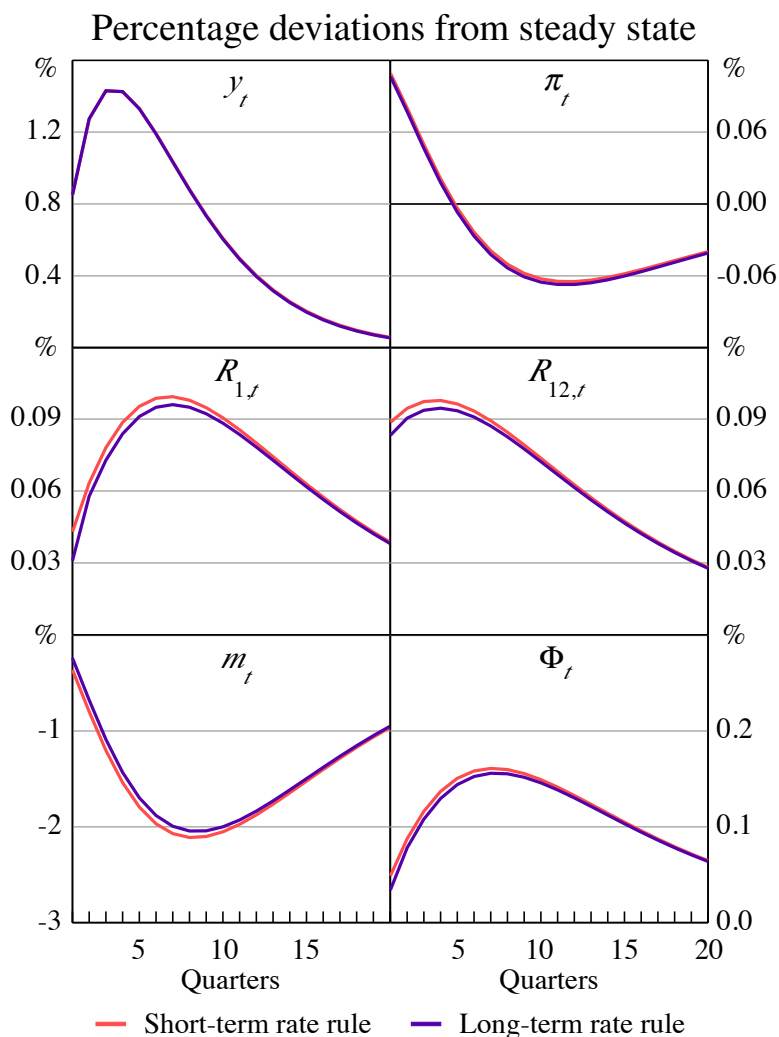
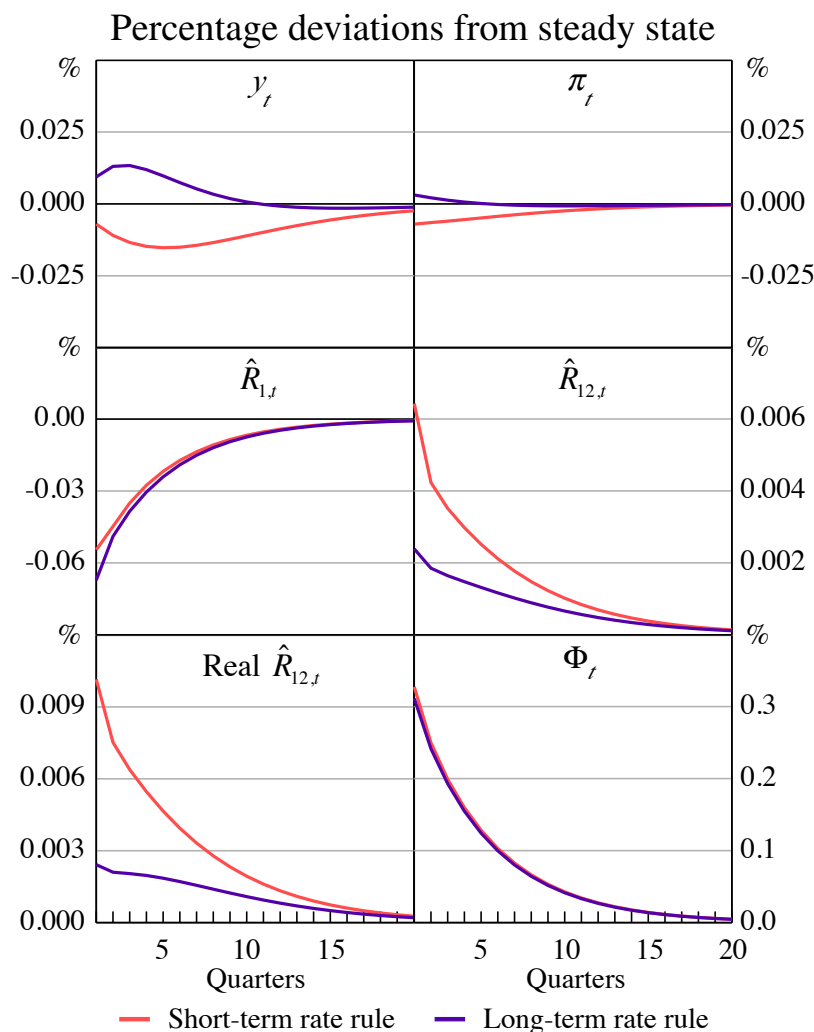
**Figure 4: Impulse Responses to Demand Shock**

Figure 5 shows responses to an exogenous risk premium shock under both rules. The responses are noticeably different. Under the Taylor rule, output and inflation both decline, whereas under the long-term interest rate rule, output and inflation rise. Because monetary policy sets  $\hat{R}_{12,t}$  but Equation (7) holds, the shock to the risk premium is absorbed by a lower sequence of short rates. This lower sequence is expansionary for unrestricted agents who can access short-term borrowing. As a result, output and inflation rise. In the case of the Taylor rule,  $\hat{R}_{12,t}$  rises by more, increasing the cost of borrowing for restricted households. As a result, output and inflation fall. In line with Jordan and Peytrignet (2007), financial shocks impact differently on the macroeconomy if policy is set with a longer-term interest rate rule.

**Figure 5: Impulse Responses to Exogenous Risk Premium Shock**

If the short-term interest rate were allowed to offset movements in the risk premium,  $\Phi_t$ , by the inclusion of the risk premium in the policy rule, Equation (5), the responses would be almost identical to those of a central bank that uses a long-term interest rate rule. However, this equivalence, of course, would break down if the zero lower bound were to prevent the short rate from offsetting increases in the risk premium.<sup>12</sup>

<sup>12</sup> In practice, in setting monetary policy central banks can take into account variables such as the risk premium. Battellino (2009) notes how the Reserve Bank of Australia has taken into account interest rate spreads, which capture risk premia, in setting interest rates in the recent episode.

Equation (7) holds regardless of the central bank's choice of policy rule. In particular, Figure 5 suggests that different rules give rise to different yield curve dynamics. To explore the impact of the maturity of the monetary policy instrument, Table 1 shows the standard deviations of  $\hat{R}_{1,t}$  and  $\hat{R}_{12,t}$ , of the expectation of future short rates, and of the risk premium implied by rules of different maturities. The parameter values of the policy rule  $\hat{R}_{L,t} = \rho_R \hat{R}_{L,t-1} + \rho_y y_t + \rho_\pi \pi_t + \rho_\mu \mu_t$  are fixed to the values in Table B1, that is,  $\rho_R = 0.75$ ,  $\rho_y = 0.09$ ,  $\rho_\pi = 0.49$  and  $\rho_\mu = 0.35$ , so differences come only from maturity.<sup>13</sup> The standard deviations of short and long rates fall as the maturity of the instrument increases. Consistent with this, the standard deviation of the sum of future expected short rates also falls. The volatility of the risk premium under the Taylor rule is about the same than when the instrument is  $R_{12}$ , and falls when instruments of longer maturity are used. This mirrors the volatility of money demand across the different policy settings.

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**Table 1: Second Moments**

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Instrument	$\sigma_{\hat{R}_1}$	$\sigma_{\hat{R}_{12}}$	$\sigma_\Sigma$	$\sigma_\Phi$
$\hat{R}_1$	100.0	88.5	84.7	16.1
$\hat{R}_{12}$	97.5	85.9	81.9	16.1
$\hat{R}_{40}$	95.1	82.5	79.2	15.9

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Notes: The friction is at  $R_{12,t}$ . Indexed to standard deviation of the short rate under the Taylor rule.  
 $\Sigma = \sum_{i=0}^{L-1} \mathbb{E}_t \hat{R}_{1,t+i}$ .

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Different rules generate different dynamics. So, how do the rules perform? The next section addresses this question.

## 5. Optimal Monetary Policy Rules

We assume that the objective of the monetary authority is to minimise a loss function which takes as arguments the variability of inflation, output and the short-term interest rate over the parameters of the policy rule. Formally, the central bank minimises

$$\sigma_\pi^2 + \omega_y \sigma_y^2 + \omega_{\hat{R}_1} \sigma_{\hat{R}_1}^2$$

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<sup>13</sup> The policy shock is set to zero for comparability across the rules.

over the parameters of the policy rule.<sup>14</sup> The parameters  $\omega_y$  and  $\omega_{\hat{R}_1}$  govern the relative concern for output and short-term interest rate variability. The terms  $\sigma_\pi^2$  and  $\omega_y \sigma_y^2$  in the loss function are standard. We include the variance of the short-term rate,  $\sigma_{\hat{R}_1}^2$  for two reasons. First, as real money balances enter households' utility functions, the central bank might wish to also attenuate fluctuations of the short-term rate so as to reduce variations of the opportunity costs of holding money. Second, as we are exploring the use of instruments of different maturities, it seems reasonable to penalise instruments which would require additional volatility of the short rate.

Table 2 evaluates the loss function and its components under a long-term interest rate rule for  $\hat{R}_{12,t}$  and the Taylor rule for a range of relative weights on output volatility of the monetary authority. For these preferences,  $\hat{R}_{12,t}$  performs slightly better than the  $\hat{R}_{1,t}$ . For some other preferences, however, long-term interest rate rules of different maturities do worse, though the differences are also never large. Also note that the variances of output, inflation and the short rate behave as expected across the central bank preferences: as the concern for output volatility increases, the variance of output falls and that of inflation and the short rate rises.<sup>15</sup>

**Table 2: Loss Function Evaluation,  $\omega_{\hat{R}_1} = 0.05$**

$\omega_y$	Instrument: $\hat{R}_{1,t}$				Instrument: $\hat{R}_{12,t}$			
	Loss	$\sigma_y^2$	$\sigma_\pi^2$	$\sigma_{\hat{R}_1}^2$	Loss	$\sigma_y^2$	$\sigma_\pi^2$	$\sigma_{\hat{R}_1}^2$
0.20	3.54	11.77	1.15	0.75	3.48	11.60	1.12	0.72
1.00	8.54	3.80	4.62	2.52	8.49	3.83	4.54	2.48
3.00	12.49	0.99	9.22	6.26	12.47	1.03	9.08	6.05

Note: All values are multiplied by 10 000

<sup>14</sup> This is equivalent to minimising  $\frac{1}{2} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \pi_t^2 + \omega_y y_t^2 + \omega_{\hat{R}_1} \hat{R}_{1,t}^2 \right) \right]$ .

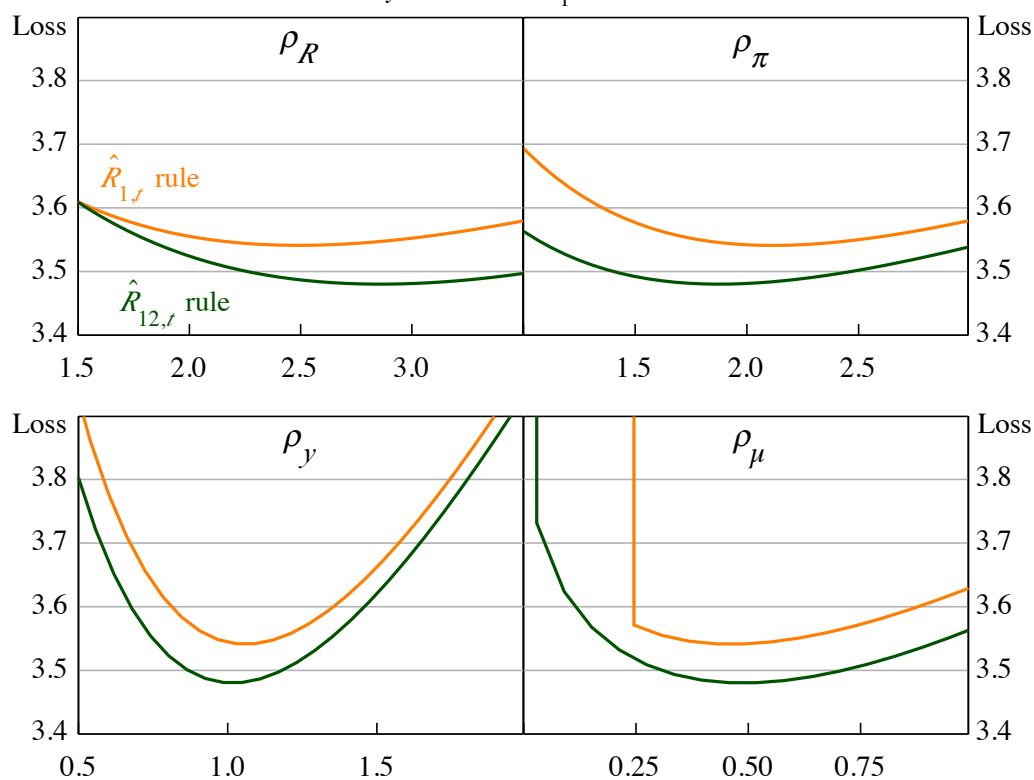
<sup>15</sup> When comparing the performance of long-term interest rate rules to each other, a choice must be made regarding the maturity of the frictions. One alternative is to fix the friction at some maturity and then evaluate different long-term interest rate rules. Another alternative is to move the friction with the maturity of the rule. We have done both exercises and found that the performance of long-term interest rates rules is about the same in both cases.

The settings of the optimised rules are similar. Figure 6 shows the loss function as we depart from the optimal value of one of the parameters in the policy rule – holding the other parameters at their optimal values. Thus, borrowing the parameter values from the Taylor rule seems not too costly.

For some preferences long-term interest rate rules perform as well as the Taylor rule, but for some preferences their performance is worse. Overall their performances are quite similar.

**Figure 6: Loss Over Parameters**

$$\omega_y = 0.2, \omega_{\hat{R}_1} = 0.05$$



Note: The loss is multiplied by 10 000

## 6. Announcements and Transitions

The analysis to this point assumes that the long-term interest rate rule has always been in operation. This is an unrealistic assumption for the types of policies we want to study. In practice, it is relevant to know how the economy would behave if the central bank announced a temporary deviation from a rule or the adoption of a different rule in the future. As Taylor (1993) stresses, the temporary deviation

from a rule and the transition towards a new rule are relevant practical concerns despite having received little academic attention. It is therefore important to know how the economy would behave if, in the case of long-term interest rate rules, the implementation of the new policy is announced in advance, or if, in the case of announcements about the future path of the short rate, the deviation from an established policy is temporary. Next, we study the economy's response to a temporary deviation from a rule at the zero lower bound and to a transition from a Taylor rule to a long-term interest rate rule.

Standard solutions for linear rational expectations models cannot capture temporary deviations or transitions if the reversion to an abandoned rule or the implementation of the new rule is known in advance. These announcements represent a foreseen structural change; standard solutions presuppose a constant structure. Cagliarini and Kulish (2008), however, extend the rational expectations solution to handle foreseen structural changes.<sup>16</sup> They show that if the structure to which an economy converges is consistent with a unique equilibrium, then so is the transition to it. So, if the policy rule to which an economy converges implies a unique equilibrium, the transition to that rule or a temporary deviation from that rule is also unique. At the zero lower bound, this result has an important implication. In general, a constant interest rate, by itself, generates indeterminacy. But if the central bank announces that it will keep the interest rate constant for a finite period and then revert to a rule that achieves a unique equilibrium, that path would be unique.

### *The zero lower bound*

Figure 7 shows two simulations in which the short rate approaches the zero lower bound. Interest rates are in per cent, rather than percentage deviations from steady-state. There are no more shocks from period five on. The first simulation – the baseline – shows how the economy returns to steady state if the policy rule remains unchanged. The second simulation considers the consequences of announcing in period 5 that the short-term interest rate will be held at zero for 8 quarters after which the central bank reverts to the abandoned rule.<sup>17</sup>

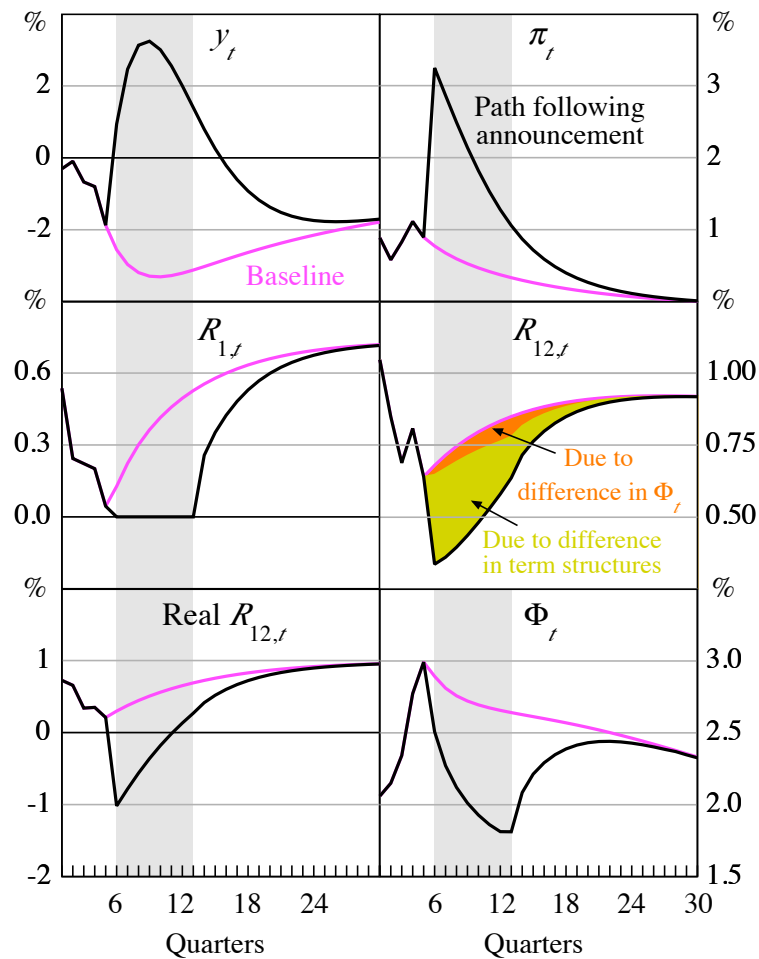
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16 See Appendix C for details.

17 We assume the announcement to be perfectly credible. For an analysis of imperfect credibility at the zero lower bound see Bodenstein, Hebden and Nunes (2010).

**Figure 7: Announcement**

$y_t$ ,  $\pi_t$  and  $\Phi_t$  in percentage deviations from steady state; interest rates in per cent



For the announcement to be stimulatory, the sequence of interest rates that is announced has to be lower than otherwise. If the interest rate would have been zero for 8 quarters regardless, the announcement would have no impact. The announcement matters precisely because it changes expectations about the future path of the short rate. The top panels show how the announcement of future lower short rates increases output and inflation relative to the baseline.

The middle right panel shows the response of the long-term nominal interest rate,  $R_{12,t}$ . The expectation that short-term rates will remain low for an extended period works, through the expectations hypothesis channel, to lower the long rate. Interestingly, the announcement also lowers the long rate through its impact on the risk premium. The reasoning lies in the response of the money supply. To implement a lower short-term nominal rate, the central bank has to expand the money supply, and in doing so, it increases liquidity and thereby decreases the premium required to hold long-term debt. Ugai (2006) argues that in Japan, a commitment to maintain zero interest rates is likely to impact on the risk premium between the short-term interest rate and the yield on long-term government bonds. In the model, this is true.

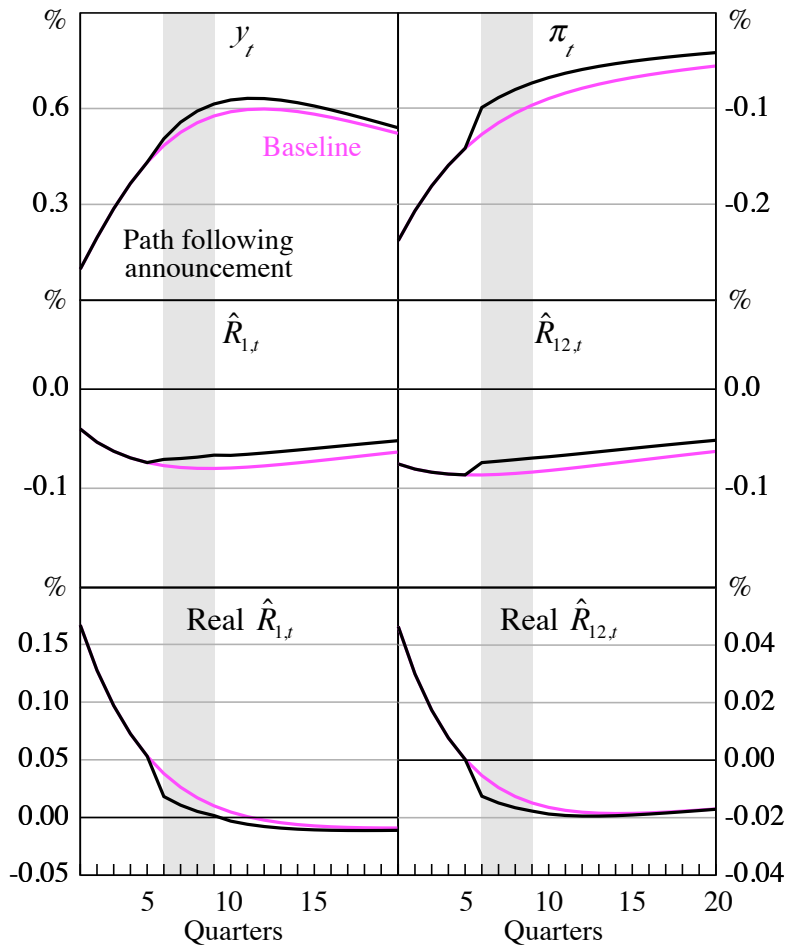
#### *The transition to a new rule*

If a central bank decides to adopt a long-term interest rate rule, an important consideration is how the economy reacts both to the change itself and to the announcement of the change.

Figure 8 shows a simulation in which the central bank announces that, in 4 quarters, it will use a rule for  $\hat{R}_{12,t}$  instead of the Taylor rule. At the same time, the central bank announces a more expansionary setting of the long-term interest rate rule ( $\rho_R = 0.75$ ,  $\rho_y = 0.045$ ,  $\rho_\pi = 0.49$  and  $\rho_\mu = 0.35$ ). The economy starts away from steady-state after being hit with a technology shock. The announcement happens in period 6. Figure 8 shows the dynamics of key variables over the period. Relative to the response that would prevail if the Taylor rule were not abandoned, the policy rule calls for a lower real long-term interest rate. Consistent with this, through the term structure relation, the real short rate is also lower. Output and inflation, as a result, are both stronger than otherwise.



**Figure 8: Transition from Taylor to Long-term Interest Rate**  
 Percentage deviations from steady state



## 7. Conclusions

This paper studies two kinds of monetary policies. One takes long-term nominal interest rates as operating instruments of monetary policy. The other considers credible announcements about the future path of short-term nominal interest rates. Within a general equilibrium model in which a component of the risk premium on long-term debt is endogenous, we show that long-term interest rates rules are consistent with unique rational expectations equilibria as much as conventional rules are.

This result is important both in theory and in practice. First, it implies that a unique equilibrium exists if a policy interest rate longer than one period is used in the model. This gives us confidence that results from models which use a policy

interest rate that matches the periodicity of the model, say quarterly, are still relevant when central banks in practice use a daily policy interest rate. Second, it implies that long-term interest rates are potential instruments for the conduct of monetary policy. In our framework, long-term interest rate rules give rise to sensible dynamics and depending on the preferences of the monetary authority, they can outperform Taylor rules.

It may seem surprising that a long-term interest rate rule does not necessarily give rise to multiple equilibria. The expectations hypothesis says short rates determine long rates, but it is right to think of long rates as determining short rates too. This is true if the pure expectations hypothesis holds or if more general versions of the expectations hypothesis hold; versions that include a risk premium between interest rates of different maturities.

The idea of monetary policy affecting long-term interest rates is not unprecedented. Friedman's (1968) description of the 'euthanasia of the rentier' shows that the central bank has been able to hold long-term interest rates low. Indeed, in many ways, setting a long rate seems less radical than the more conventional policy of setting an exchange rate. In a fixed exchange rate regime, the central bank also sets the price of an asset. But, the central bank's ability to maintain a given exchange rate with market forces that would otherwise depreciate the domestic currency is limited by its stock of foreign reserves. The central bank can buy foreign currency without bounds, but can sell foreign currency within bounds. Not surprisingly, fixed exchange rates often do not last for very long.

To set a long-term interest rate, however, the central bank could use the stock of government debt, of which, in principle, there could always be enough. It could also create its own instruments to set an interest rate of a chosen maturity.

The other policies we study are announcements about the future path of short-term interest rates. Credible announcements can be successful under two conditions: (i) if they entail a return to a monetary policy rule for which the equilibrium is unique, otherwise, every announcement leads to multiple equilibria; and (ii) if the announcement implies a path for the interest rate which is different than what the economy would have produced in any case. The credible promise of lower interest rates, and the actual implementation as the policy is carried out, reduces long-term interest rates through its impact both on expectations and on the risk premium. The first channel is a straightforward consequence of the expectations hypothesis, but

the second channel is a consequence of the additional liquidity that is needed to implement a sequence of lower interest rates. This additional liquidity lowers risk premia.

A full explanation of the monetary transmission mechanism, as King (1999) argues, involves understanding the determination of risk premia. We have made progress in this direction. But are the properties of long-term interest rate rules similar in other environments, like those set up by Alvarez, Atkeson and Kehoe (2007)? Are the properties of long-term interest rate rules the same if the zero lower bound on the entire yield curve binds? Do equilibria exist? These are, of course, challenging questions. But the efforts to address these will add to an expanded monetary policy toolkit.

## Appendix A: The Linearised Equations

The full set of linearised equations is given by:

$$\Lambda_t = \phi_0 \hat{a}_t + \phi_1 y_{t-1} - \phi_2 y_t + \beta \phi_1 \mathbb{E}_t y_{t+1} \quad (\text{A1})$$

$$\Lambda_t^u = L \left[ \hat{R}_{L,t} - \frac{1}{L} \mathbb{E}_t \sum_{j=0}^{L-1} \pi_{t+j+1} \right] + \mathbb{E}_t \Lambda_{t+L}^u - \zeta_t + \tau(m_t^u - b_{L,t}^u) \quad (\text{A2})$$

$$\Lambda_t^u = \hat{R}_t - \mathbb{E}_t \pi_{t+1} + \mathbb{E}_t \Lambda_{t+1}^u \quad (\text{A3})$$

$$\Lambda_t^r = L \left[ \hat{R}_{L,t} - \frac{1}{L} \mathbb{E}_t \sum_{j=0}^{L-1} \pi_{t+j+1} \right] + \mathbb{E}_t \Lambda_{t+L}^r \quad (\text{A4})$$

$$\Lambda_t = \lambda \Lambda_t^u + (1 - \lambda) \Lambda_t^r \quad (\text{A5})$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \tilde{\lambda} m c_t \quad (\text{A6})$$

$$m c_t = (\chi + \phi_2) y_t - \phi_1 y_{t-1} - \beta \phi_1 \mathbb{E}_t y_{t+1} - \phi_0 \hat{a}_t - (1 + \chi) \hat{z}_t \quad (\text{A7})$$

$$m_t^u = \mu_1 m_{t-1}^u + \mu_2 m_{t+1}^u + \mu_3 [\Lambda_t^u - \hat{a}_t] + \mu_4 \hat{R}_t + \mu_5 \hat{e}_t - \mu_6 \tau [m_t^u - b_{L,t}^u] \quad (\text{A8})$$

$$m_t^r = \mu_1 \hat{m}_{t-1}^r + \mu_2 \hat{m}_{t+1}^r + \mu_3 [\hat{\Lambda}_t^r - \hat{a}_t] + \mu_4 \hat{R}_t + \mu_5 \hat{e}_t \quad (\text{A9})$$

$$m_t = \lambda m_t^u + (1 - \lambda) m_t^r \quad (\text{A10})$$

$$\hat{R}_{L,t} = \rho_R \hat{R}_{L,t-1} + \rho_\pi \pi_t + \rho_y y_t + \rho_\mu \mu_t + \varepsilon_{R,t} \quad (\text{A11})$$

$$\mu_t = m_t - m_{t-1} + \pi_t \quad (\text{A12})$$

$$b_{L,t} = \lambda b_{L,t}^u + (1 - \lambda) b_{L,t}^r \quad (\text{A13})$$

$$b_{L,t}^u = \omega b_{L,t-1}^u + \varepsilon_{b_{L,t}^u} \quad (\text{A14})$$

$$b_{L,t}^r = \omega b_{L,t-1}^r + \varepsilon_{b_{L,t}^r} \quad (\text{A15})$$

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a_t} \quad (\text{A16})$$

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{e_t} \quad (\text{A17})$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z_t} \quad (\text{A18})$$

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \varepsilon_{\zeta_t} \quad (\text{A19})$$

All variables are in log deviations from steady state. Equation (A1) gives the evolution of the aggregate marginal utility of wealth,  $\Lambda_t$ , linking it to the preference shock  $\hat{a}_t$  and output  $y_t$ . Equations (A2) to (A4) give the restricted and unrestricted agents' intertemporal relationships. Equation (A2) is the unrestricted agents' first-order condition for long-term debt accumulation, where

$\hat{R}_{L,t} - \frac{1}{L} \mathbb{E}_t \sum_{j=0}^{L-1} \pi_{t+j+1}$  is the long-term real interest rate,  $\hat{R}_{L,t}$  is the nominal long-term interest rate and  $\pi_t$  is inflation, and  $\zeta_t + \tau(m_t^u - b_{L,t}^u)$  is the risk premium, where  $\zeta_t$  is the exogenous component of the premia,  $m_t$  is money demand and  $b_{L,t}$  long-term real bond holdings. Equation (A4) gives the restricted agents' first-order condition for long-term debt accumulation. Equation (A5) combines the restricted and unrestricted agents' Lagrange multipliers, weighted by  $\lambda$ , the proportion of unrestricted agents. Equations (A6) and (A7) give the supply-side relations, linking inflation to marginal costs  $mc_t$  and technology shocks  $\hat{z}_t$ . Equations (A8) to (A10) govern money demand relationships, where  $\hat{e}_t$  is a money demand shock. Equation (A10) aggregates across agents' money holdings. Equation (A11) gives the Taylor-type rule for a central bank targeting interest rates of maturity  $L$  with money growth  $\mu_t$  specified by Equation (A12). Equation (A13) aggregates across agents' long-term bond holdings. The exogenous processes are given by Equations (A14) to (A19).

## Appendix B: Calibration

**Table B1: Calibration of Model Parameters**

Parameter	Description	Value
$\beta$	Households' discount factor	0.991
$\delta$	Positive parameter relevant for households' money demand	4.36
$\sigma$	Coefficient for relative risk aversion	2
$h$	Degree of habit formation	0.9
$\delta_0$	Parameter governing the cost of portfolio rebalancing	1.82
$\lambda$	Proportion of unrestricted agents	0.29
$\tau$	Intensity of the endogenous friction	0.54
$\chi$	Supply-side parameter	1.36
$\tilde{\lambda}$	Slope of Phillips curve	0.014
$\rho_R$	Coefficient on $\hat{R}_{L,t-1}$ in policy rule	0.75
$\rho_y$	Coefficient on $y_t$ in policy rule	0.09
$\rho_\pi$	Coefficient on $\pi_t$ in policy rule	0.49
$\rho_\mu$	Coefficient on $\mu_t$ in policy rule	0.35
$\rho_a$	Persistence of preference shock	0.89
$\rho_e$	Persistence of money demand shock	0.99
$\rho_z$	Persistence of technology shock	0.97
$\rho_\zeta$	Persistence of exogenous risk premia shock	0.80
$\sigma_a$	Standard error of the preference shock innovation	0.039
$\sigma_e$	Standard error of money demand shock innovation	0.054
$\sigma_z$	Standard error of technology shock innovation	0.011
$\sigma_r$	Standard error of policy shock innovation	0.009
$\sigma_\zeta$	Standard error of exogenous risk premia shock innovation	0.004

Source: Andrés *et al* (2004)

## Appendix C: Anticipated Structural Changes Under Rational Expectations

Following Cagliarini and Kulish (2008), write the model in matrix form as follows

$$\tilde{\Gamma}_0 \mathbf{y}_t = \tilde{\Gamma}_1 \mathbf{y}_{t-1} + \tilde{C} + \tilde{\Psi} \boldsymbol{\varepsilon}_t \quad (\text{C1})$$

where the state vector is defined by

$$\mathbf{y}_t = \begin{pmatrix} \mathbf{y}_{1,t} \\ \mathbf{y}_{2,t} \\ \mathbb{E}_t \mathbf{z}_{t+1} \end{pmatrix}$$

and where  $\mathbf{y}_{1,t}$  is an  $(n_1 \times 1)$  vector of exogenous and some endogenous variables, and  $\mathbf{y}_{2,t}$  is an  $(n_2 \times 1)$  vector with those endogenous variables for which conditional expectations appear,  $\mathbf{z}_{t+1}$ ,  $(k \times 1)$ , contains leads of  $\mathbf{y}_{2,t}$ ; in the model above, however,  $\mathbf{z}_{t+1} = \mathbf{y}_{2,t+1}$  and  $k = n_2$ . The dimension of  $\mathbf{y}_t$  is  $n \times 1$ , where  $n = n_1 + n_2 + k$ . Also, we assume  $\boldsymbol{\varepsilon}_t$  to be an  $l \times 1$  vector of serially uncorrelated processes,  $\tilde{\Gamma}_0$  and  $\tilde{\Gamma}_1$  are  $(n_1 + n_2) \times n$  matrices,  $\tilde{C}$  is  $(n_1 + n_2) \times 1$  and  $\tilde{\Psi}$  is  $(n_1 + n_2) \times l$ .

$\boldsymbol{\eta}_t$  is the vector of expectations revisions given by,

$$\boldsymbol{\eta}_t = \mathbb{E}_t \mathbf{z}_t - \mathbb{E}_{t-1} \mathbf{z}_t \quad (\text{C2})$$

where  $\mathbb{E}_t \boldsymbol{\eta}_{t+j} = 0$  for  $j \geq 1$ .

Augment the system defined by Equation (C1) with the  $k$  equations from Equation (C2) to obtain Equation (8) reproduced below

$$\Gamma_0 \mathbf{y}_t = C + \Gamma_1 \mathbf{y}_{t-1} + \Psi \boldsymbol{\varepsilon}_t + \Pi \boldsymbol{\eta}_t.$$

A unique rational expectations solution takes the form:

$$\mathbf{y}_t = S_0 + S_1 \mathbf{y}_{t-1} + S_2 \boldsymbol{\varepsilon}_t.$$

Consider that at the beginning of forecast horizon, the monetary authority announces how the policy parameters will vary in the

future. An announcement of this form entails a form of structural change, from the perspective of the standard solution for rational expectations models. This induces a sequence of structures of the form,  $\{\tilde{C}_{t+1}, \tilde{\Gamma}_{0,t+1}, \tilde{\Gamma}_{1,t+1}, \tilde{\Psi}_{t+1}, \Pi, \{C_{t+k}, \Gamma_{0,t+k}, \Gamma_{1,t+k}, \Psi_{t+k}\}_{k=2}^T, (\bar{C}, \bar{\Gamma}_0, \bar{\Gamma}_1, \bar{\Psi}, \bar{\Pi})\}$ . Therefore, the system evolves as follows

$$\begin{aligned} \tilde{\Gamma}_{0,t+1}\mathbf{y}_{t+1} &= \tilde{C}_{t+1} + \tilde{\Gamma}_{1,t+1}\mathbf{y}_t + \tilde{\Psi}_{t+1}\boldsymbol{\varepsilon}_{t+1} & t+1 \\ \Gamma_{0,t+k}\mathbf{y}_{t+k} &= C_{t+k} + \Gamma_{1,t+k}\mathbf{y}_{t+k-1} + \Pi\boldsymbol{\eta}_{t+k} + \Psi_{t+k}\boldsymbol{\varepsilon}_{t+k} & 2 \leq k \leq T \\ \bar{\Gamma}_0\mathbf{y}_{t+k} &= \bar{C} + \bar{\Gamma}_1\mathbf{y}_{t+k-1} + \bar{\Pi}\boldsymbol{\eta}_{t+k} + \bar{\Psi}\boldsymbol{\varepsilon}_{t+k} & t \geq T+1. \end{aligned}$$

Under regularity conditions the solution for  $\mathbf{y}_{t+1}, \dots, \mathbf{y}_{t+T}$  satisfies

$$\begin{pmatrix} \tilde{\Gamma}_{0,t+1} & 0 & \dots & 0 \\ -\Gamma_{1,t+2} & \Gamma_{0,t+2} & & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & -\Gamma_{1,t+T} & \Gamma_{0,t+T} \\ 0 & \dots & 0 & \bar{Z}'_2 \end{pmatrix} \begin{pmatrix} \mathbf{y}_{t+1} \\ \vdots \\ \mathbf{y}_{t+T} \end{pmatrix} = \begin{pmatrix} \tilde{C}_{t+1} + \tilde{\Gamma}_{1,t+1}\mathbf{y}_t \\ C_{t+2} \\ \vdots \\ C_{t+T} \\ \bar{w}_{2,t+T} \end{pmatrix}.$$

After  $t+T$ , the standard solution for  $\{\bar{C}, \bar{\Gamma}_0, \bar{\Gamma}_1, \bar{\Psi}, \bar{\Pi}\}$  applies.



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