Reserve Bank of Australia

RESEARCH
DISCUSSION
PAPER

Direct Effects of Money on Aggregate Demand: Another Look at the Evidence

Stephen Elias and Mariano Kulish

RDP 2010-05

DIRECT EFFECTS OF MONEY ON AGGREGATE DEMAND: ANOTHER LOOK AT THE EVIDENCE

Stephen Elias and Mariano Kulish

Research Discussion Paper
2010-05

August 2010

Economic Research Department
Reserve Bank of Australia

We thank Peter Ireland, Christopher Kent and Tim Robinson for suggestions. We also thank Adrian Pagan for comments on an early draft. Responsibility for any remaining errors rests with us. The views expressed here are our own and do not necessarily reflect those of the Reserve Bank of Australia.

Authors: eliass or kulishm at domain rba.gov.au

Media Office: rbainfo@rba.gov.au

Abstract

Now that a number of central banks are faced with short-term nominal interest rates close to or at the zero lower bound, there is a renewed interest in the long-running debate about whether or not changes in the stock of money have direct effects. In particular, do changes in money have additional effects on aggregate demand outside of those induced by changes in short-term nominal interest rates? This paper revisits and reinterprets the empirical evidence based on single equation regressions which is quite mixed, with some results supporting and other results denying the existence of direct effects. We use a structural model with no direct effects of money to show that the finding of positive and statistically significant coefficients on real money growth can be misleading. The model generates data that, when used to estimate analogs of the empirical regressions, produce positive and statistically significant coefficients on real money growth, similar to those often found when using actual data. The problem is that single equation regressions leave out a set of variables, which in turn, gives rise to an omitted variables bias in the estimated coefficients on real money growth. Hence, they are an unreliable guide to calibrate monetary policies, in general, including at the zero lower bound.

JEL Classification Numbers: E40

Keywords: money, monetary base, direct effects, output gap

Table of Contents

DIRECT EFFECTS OF MONEY ON AGGREGATE DEMAND: ANOTHER LOOK AT THE EVIDENCE

Stephen Elias and Mariano Kulish

1. Introduction

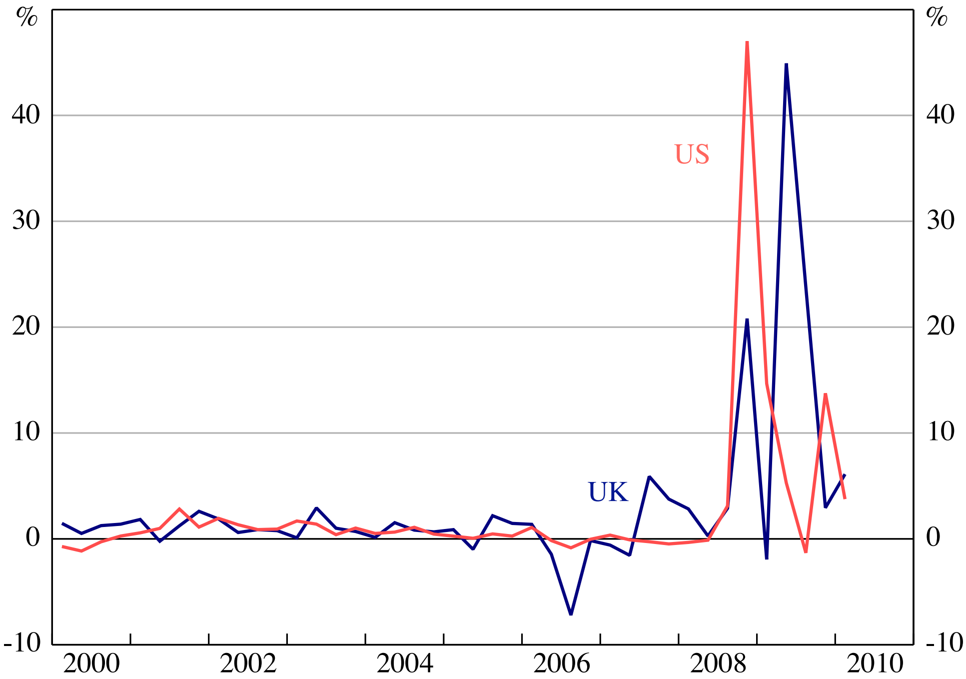
Money can be said to have direct effects if changes in the stock of money influence output and prices outside of its impact through short-term nominal interest rates. In general, it is an important issue whether or not money has such direct effects, but the issue is most prominent when the nominal interest rate approaches its zero lower bound, as it has in Japan, and more recently, in the euro area, the United Kingdom and the United States. With little or no room left to lower policy rates, central banks in these economies have moved to purchase a variety of assets with newly created money.[1] Figure 1 illustrates the recent increases in the real money base for the United Kingdom and the United States. Only if money has direct effects can this newly created money help to stimulate the economy.

In most recent formulations of the canonical New Keynesian model, money has no direct effects; the equilibrium paths of output, inflation and the nominal interest rate are fully described and determined by three equations: one for aggregate demand, one for aggregate supply and a monetary policy rule for the short-term nominal interest rate. In the background of this equilibrium lies a money market with a central bank ready to supply as much money as needed for that market to clear at the interest rate called for by the policy rule. It is possible to augment this system of three equations and track monetary aggregates, but adding such an equation – whatever it may be – does not give money any direct effect over output or prices. With the nominal interest rate in the system, money is redundant. This is not to say, however, that money growth does not determine inflation in the long-run. This would generally be the case both in models where money has

1 See Bernanke (2009), Borio and Disyatat (2009) and Dale (2010).

Figure 1: Real Money Base

Quarterly percentage change



Note: For data methodology and sources see Appendix A

only indirect effects as well as in models where money has direct effects.[2] The difference lies in the mechanism through which a change in the stock of money causes a change in prices.

Monetarists would consider that this New Keynesian description of the transmission mechanism is, at best, incomplete. In their view – exemplified in Meltzer (2001) – movements in short-term nominal interest rates are insufficient to capture all the power of monetary policy actions. These actions are felt in many financial markets – beyond the interbank market for short-term debt – and generate wealth effects via various asset price movements that affect, through spending, both output and prices.[3]

2 We say *generally* because in the models of Krugman (1998) and Svensson (1999), households become willing to hoard any additional money that authorities choose to supply after the nominal interest rate reaches its lower bound. In this situation, there is no well-defined equilibrium level of real money balances and policy-makers lose control of the price level. The relation which links the growth rate of the money supply to the growth rate of prices can be thought to break down in a liquidity trap.

3 See also Friedman (1956) and Brunner and Meltzer (1993).

The empirical evidence is mixed. Rudebusch and Svensson (1999, 2002) find little role for monetary aggregates in empirical aggregate demand specifications, while Favara and Giordani (2009) find that shocks to broad monetary aggregates have substantial and persistent effects on output. Hafer, Haslag and Jones (2007) find that money is not redundant, and Leeper and Roush (2003) find that whether money enters a model, and the way in which it does so, matters for inferences about policy impacts.

Nelson (2002) – complementing results in Koenig (1990) and Meltzer (2001) – shows that, after controlling for the short-term real interest rate, real base money growth is a significant determinant of total output in both the United Kingdom and the United States for the period 1960 to 1999. He then shows that an extended version of the New Keynesian model, in which the long-term nominal interest rate enters the money demand equation, can account for this finding. In particular, the extended model generates data which gives rise to positive and statistically significant coefficients of real money growth in regressions similar to those used on actual data.

As Nelson (2003) points out, however, this extension of the New Keynesian model does not introduce direct effects of money in the way that we describe them above. Money matters in Nelson's equations because it is an information variable, a proxy for other variables which are omitted. In this way, Nelson (2002) has redefined 'direct effects' in terms of the explanatory power for aggregate demand contained in the real money stock that is not captured in the short-term real interest rate. This definition is not particularly helpful, since money can be a function of many variables (and therefore contain information about them) but these many variables need not be a function of money. For base money expansions to stimulate the economy at the zero bound, money has to have structural direct effects and not informational ones.

The paper shows that the evidence both for and against structural direct effects, based on the statistical significance of the coefficients on real money growth in the regressions of Nelson (2002) and Rudebusch and Svensson (2002), is flawed. To do this, we follow Nelson (2002), take his extended model where money has no structural direct effects, generate data, and then estimate analogs of the empirical specifications. We also take another model – of Andrés, López-Salido and Nelson (2004) – where money has sizeable structural direct effects and do the same. We find that the model with no structural direct effects can give rise to informational direct effects and a model with structural direct effects can fail to do so. Our interpretation of the estimates is that the empirical specifications, by excluding a set of variables, introduce an omitted variables bias in the estimated coefficients on real money growth. These biases – although they could be thought to stand for informational effects – blur the structural relation between money and the rest of the economy. Although the regressions may at times reveal the existence of informational effects, they fail to uncover structural direct effects of money.

The rest of the paper is structured as follows. Section 2 updates and extends estimates of the specifications used by Nelson (2002) for the United States, the United Kingdom, Australia and Japan. Section 3 presents the extended model of Nelson (2002), which is used in Section 4 to conduct a Monte Carlo analysis. There, we generate data from the artificial economy, estimate empirical analogs of the specifications used in Section 2, and then decompose the bias of the real money growth coefficient. Section 5 concludes.

2. Empirical Analysis

We follow Nelson (2002) and estimate empirical aggregate demand specifications in which a measure of de-trended real output, $\tilde{y}_t$, is a function of its own lags, lags of a measure of the real interest rate, $\tilde{r}_t$, and lags of a measure of real money growth, $\Delta\tilde{m}_t$. Appendix A contains a full description of the data and its sources.

We update the empirical analysis of Nelson (2002) for the United States and the United Kingdom and then extend it to Australia and Japan. We do this for two reasons. First, we have 42 additional quarters of data. Second, and more importantly, the more recent data include some observations for which short-term nominal interest rates approach their zero lower bound. More observations always sharpen parameter estimates and extreme observations more so.

The regressions we report here, like those of Meltzer (2001) and Nelson (2002), are based on the growth rate of the real monetary base, although we have also considered a range of other measures of money, from narrow measures like currency to broader measures like M2.[4] As a measure of the output gap, we use

4 The results are broadly similar.

Table 1: Output Gap Regressions – United States

Dependent variable: $\tilde{y}_t$	**Sample period** 1961:Q2–1999:Q2		1961:Q2–2009:Q4	
Constant	–0.001	(0.001)	–0.000	(0.001)
$\tilde{y}_{t-1}$	1.130**	(0.080)	1.248**	(0.070)
$\tilde{y}_{t-2}$	–0.213**	(0.080)	–0.320**	(0.070)
$\tilde{r}_{t-1}$	–0.061*	(0.034)	–0.015	(0.031)
$\Delta\tilde{m}_{t-1}$	0.301**	(0.111)	–0.030*	(0.017)
$\Delta\tilde{m}_{t-2}$	0.071	(0.127)	0.013	(0.018)
$\Delta\tilde{m}_{t-3}$	–0.146	(0.124)	–0.005	(0.018)
$\Delta\tilde{m}_{t-4}$	0.141	(0.112)	0.017	(0.018)
Sum of real money growth coefficients	0.367	(0.098)	–0.005	(0.027)
R^2	0.904		0.903	
Long-run effect of money	4.422		–0.064	
F-statistic[(a)]	5.104		1.109	
p-value[(a)]	0.001		0.354	
Durbin-Watson statistic	2.065		2.134	

Notes: * and ** denote statistical significance at the 10 and 5 per cent levels, respectively. Standard errors are in parentheses.
(a) F-statistic for a test of the null hypothesis that the money coefficients are jointly insignificant and its p-value.

the estimate of the Congressional Budget Office for the United States, and we quadratically de-trend the log of real output for the other countries.

The first column of Table 1 estimates Nelson's specification over his sample period and the second column over the full sample 1961:Q1–2009:Q4 for the United States. Our results for the comparable sample are very close to his. For example, Nelson's estimate of the sum of real money growth coefficients is 0.33 while our estimate over that same sample period is 0.37. We estimate an associated long-run effect of 4.42 while Nelson's estimate is 3.05.[5] Over the full sample, however, the sum of real money growth coefficients becomes insignificant. Although a formal test rejects the exclusion of the money terms over the sample period 1961:Q1–1999:Q2 (p-value = 0.001), it fails to do so over the full sample (p-value = 0.354).

5 The long-run effect of money growth on output corresponds to the cumulative impact that a one percentage point increase in real money growth has on output over time.

Table 2: Output Gap Regressions – United Kingdom

Dependent variable: $\tilde{y}_t$	Sample period: 1977:Q1–1999:Q2		Sample period: 1977:Q1–2009:Q4	
Constant	–0.002	(0.001)	–0.001	(0.001)
$\tilde{y}_{t-1}$	0.699**	(0.111)	1.021**	(0.090)
$\tilde{y}_{t-2}$	0.494**	(0.161)	0.268*	(0.146)
$\tilde{y}_{t-3}$	0.012	(0.180)	–0.032	(0.149)
$\tilde{y}_{t-4}$	–0.250**	(0.125)	–0.311**	(0.100)
$\tilde{r}_{t-1}$	0.013	(0.115)	0.033	(0.010)
$\tilde{r}_{t-2}$	–0.004	(0.176)	0.072	(0.157)
$\tilde{r}_{t-3}$	0.098	(0.162)	–0.063	(0.146)
$\tilde{r}_{t-4}$	–0.095	(0.088)	–0.024	(0.077)
$\Delta\tilde{m}_{t-1}$	0.199**	(0.097)	0.006	(0.021)
$\Delta\tilde{m}_{t-2}$	0.139	(0.101)	0.041*	(0.022)
$\Delta\tilde{m}_{t-3}$	0.042	(0.096)	0.019	(0.038)
$\Delta\tilde{m}_{t-4}$	–0.132	(0.084)	–0.064	(0.045)
Sum of real money growth coefficients	0.248**	(0.108)	0.005	(0.034)
R^2	0.947		0.953	
Long-run effect of money	3.603		0.029	
F-statistic[(a)]	3.763		0.951	
p-value[(a)]	0.008		0.437	
Durbin-Watson statistic	2.157		2.047	

Notes: * and ** denote statistical significance at the 10 and 5 per cent levels, respectively. Standard errors are in parentheses.
(a) F-statistic for a test of the null hypothesis that the money coefficients are jointly insignificant and its p-value.

Table 2 contains results using Nelson's preferred specification for the United Kingdom.[6] These results are also in line with his over the sample period 1977:Q1–1999:Q2; we find that the sum of real money growth coefficients is positive and statistically significant at the 5 per cent level. As for the United States, a formal test rejects the exclusion of the money terms over the sample period 1977:Q1–1999:Q2, but fails to do so over the full sample.

6 Following the UK Money Market Reform on 18 May 2006, the Bank of England discontinued the series for M0, the Bank's main narrow money measure, and instead continued publishing series for 'reserve balances' at the Bank of England to accompany 'notes and coin' in circulation. To account for this, we compute the growth rate of real money using M0 prior to 2006:Q2, and then use the growth rate of the sum of 'reserve balances' and 'notes and coin'.

Table 3: Output Gap Regressions – Australia

Dependent variable: $\tilde{y}_t$	**Sample period** 1978:Q3–1999:Q2		1978:Q3–2010:Q1	
Constant	0.002	(0.002)	0.004**	(0.002)
$\tilde{y}_{t-1}$	1.045**	(0.117)	1.086**	(0.093)
$\tilde{y}_{t-2}$	–0.039	(0.171)	–0.078	(0.137)
$\tilde{y}_{t-3}$	–0.083	(0.171)	–0.094	(0.138)
$\tilde{y}_{t-4}$	–0.059	(0.114)	–0.007	(0.091)
$\tilde{r}_{t-1}$	–0.061	(0.222)	–0.125	(0.177)
$\tilde{r}_{t-2}$	0.014	(0.379)	0.029	(0.310)
$\tilde{r}_{t-3}$	0.097	(0.372)	0.134	(0.302)
$\tilde{r}_{t-4}$	–0.151	(0.217)	–0.148	(0.173)
$\Delta\tilde{m}_{t-1}$	–0.007	(0.039)	–0.005	(0.018)
$\Delta\tilde{m}_{t-2}$	–0.015	(0.039)	0.005	(0.018)
Sum of real money growth coefficients	–0.021	(0.047)	–0.000	(0.025)
R^2	0.891		0.930	
Long-run effect	–0.156		–0.003	
F-statistic[(a)]	0.111		0.086	
p-value[(a)]	0.895		0.918	
Durbin-Watson statistic	1.977		1.995	

Notes: * and ** denote statistical significance at the 10 and 5 per cent levels, respectively. Standard errors are in parentheses.
(a) F-statistic for a test of the null hypothesis that the money coefficients are jointly insignificant and its p-value.

Table 3 shows estimates of a similar specification for Australia. For both sample periods, individual real money growth terms and the sum of real money growth terms are insignificant, as are the long-run effects of real money growth. Formal tests fail to reject the exclusion of money terms.

Table 4 contains estimates with a similar specification for Japan.[7] We find that over the period 1972:Q1 to 1999:Q2, the sum of real money growth coefficients is positive (0.184) and statistically significant with an associated long-run effect of 9.82. Over the full sample, the sum of real money growth terms becomes

7 We followed Nelson (2002) in using quadratically de-trended real output. For Japan, however, this method results in a positive estimate of the output gap towards the end of the sample. Consequently, we considered different measures of potential output, such as HP-filtered output and a measure published by the OECD. The results are qualitatively the same.

Table 4: Output Gap Regressions – Japan

Dependent variable: $\tilde{y}_t$	**Sample period** 1972:Q1–1999:Q2		1972:Q1–2009:Q4	
Constant	–0.004**	(0.002)	–0.001	(0.001)
$\tilde{y}_{t-1}$	0.878**	(0.109)	1.025**	(0.085)
$\tilde{y}_{t-2}$	0.160	(0.146)	0.034	(0.127)
$\tilde{y}_{t-3}$	0.105	(0.145)	0.108	(0.129)
$\tilde{y}_{t-4}$	–0.161	(0.109)	–0.192**	(0.089)
$\tilde{r}_{t-1}$	0.303**	(0.125)	0.199*	(0.109)
$\tilde{r}_{t-2}$	–0.291	(0.209)	–0.225	(0.174)
$\tilde{r}_{t-3}$	–0.037	(0.210)	0.077	(0.173)
$\tilde{r}_{t-4}$	–0.089	(0.117)	–0.016	(0.103)
$\Delta\tilde{m}_{t-1}$	–0.040	(0.106)	–0.018	(0.046)
$\Delta\tilde{m}_{t-2}$	0.054	(0.131)	0.037	(0.057)
$\Delta\tilde{m}_{t-3}$	0.251*	(0.127)	0.040	(0.056)
$\Delta\tilde{m}_{t-4}$	–0.082	(0.103)	–0.015	(0.047)
Sum of real money growth coefficients	0.184*	(0.103)	0.044	(0.051)
R^2	0.962		0.954	
Long-run effect	9.820		1.840	
F-statistic[(a)]	1.925		0.581	
p-value[(a)]	0.112		0.678	
Durbin-Watson statistic	1.926		1.909	

Notes: * and ** denote statistical significance at the 10 and 5 per cent levels, respectively. Standard errors are in parentheses.
(a) F-statistic for a test of the null hypothesis that the money coefficients are jointly insignificant and its p-value.

insignificant and F-tests (equivalent to the ones conducted above) suggest that real money growth should not be included in the regression.

For the United States and the United Kingdom, the most recent data is potentially relevant to identify any direct effects of money because it contains observations for which the nominal interest rate stays more or less constant (close to the zero lower bound) but the real money base increases sharply. If we thought that these regressions were able to capture the structural relationship between money and aggregate demand, then the results from this section would weaken the evidence for direct effects of money on aggregate demand. However, as we show below, these regressions are not reliable. The estimates, whatever they may be, should not be interpreted as evidence for, nor evidence against, the existence of direct effects;

the coefficients in these regressions cannot be given a structural interpretation. This means, in particular, that the insignificant coefficients on the more recent data on real money growth for the United States and the United Kingdom do not imply that quantitative easing policies failed.

Because changes in policy often lead to changes in the correlations between variables, the instability of the estimated real money growth coefficients for Japan, the United Kingdom and the United States, is consistent with the idea that these regressions are misspecified.

3. An Extended New Keynesian Model

In this section we discuss a model of the economy with an explicit equation describing the money market equilibrium. We use a standard closed-economy model with sticky-prices but extended by Nelson (2002) to include a cost to adjusting real money balances. This cost makes money demand forward-looking because it increases the incentives for households to smooth out their holdings of real money balances over time. There are three types of agents in the model: optimising households, profit maximising firms and a central bank that controls the settings of the nominal interest rate. The details are in Nelson (2002), so we just present the log-linearised equations that characterise the economy's equilibrium. Variables are expressed in log-deviations from steady state.

$$
\begin{aligned}
\Lambda_t &= g_1 E_t c_{t+1} + g_2 c_t + g_3 c_{t-1} + g_4 a_t && (1)\\
\Lambda_t &= r_t + E_t \Lambda_{t+1} && (2)\\
r_t &= R_t - E_t \pi_{t+1} && (3)\\
\pi_t &= \beta E_t \pi_{t+1} - \alpha_\mu \mu_t && (4)\\
\mu_t &= \Lambda_t + y_t - n_t && (5)\\
y_t &= z_t + \alpha k_t + (1-\alpha) n_t && (6)\\
y_t &= s_c c_t + (1-s_c) x_t && (7)\\
\delta x_t &= k_{t+1} - (1-\delta) k_t && (8)\\
r_t &= \kappa_1 E_t (y_{t+1} - k_{t+1} - \kappa_2 x_t - \mu_{t+1}) + (1-\delta)\kappa_2 E_t x_{t+1} && (9)\\
R_t &= \rho_R R_{t-1} + \rho_\pi E_t \Big(\sum_{j=0}^{3} \pi_{t+1-j}\Big) + \rho_y y_t + \varepsilon_{R,t} && (10)\\
a_t &= \rho_a a_{t-1} + \varepsilon_{a,t} && (11)\\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}. && (12)\\
m_t &= -\frac{1}{\varepsilon_m}\phi_t - a_m \Delta m_t + \beta a_m E_t \Delta m_{t+1} - \frac{1}{R\varepsilon_m} R_t + \frac{1}{\varepsilon_m}\varepsilon_{m,t} && (13)
\end{aligned}
$$

Equation (1) defines the log marginal utility of consumption, Λ_t, as a function of consumption, c_t, and a demand shock a_t. Equation (2) is the Euler equation linking the real interest rate, r_t, with the marginal utilities of consumption over time. Equation (3) defines the real interest rate as the difference between the nominal interest rate, R_t, and expected inflation, $E_t\pi_{t+1}$. Equation (4) is the forward-looking Phillips curve implied by Calvo-price setting, where μ_t is the mark-up. Equation (5) comes from the labour market equilibrium condition, where n_t is hours worked and y_t is output. Equation (6) is the production function, where z_t is total factor productivity and k_t is the capital stock. Equation (7) is the goods market equilibrium condition. Equation (8) is law of motion of the capital stock, where x_t is investment.[8] Equation (9) is the household's first-order condition for capital accumulation. Equation (10) is the monetary policy rule. Equations (11) and (12) are the exogenous processes followed by the demand shock and total factor productivity. Equation (13) is the demand for real money balances, m_t.

8 See Neiss and Nelson (2003).

Finally, $\varepsilon_{a,t}, \varepsilon_{m,t}, \varepsilon_{R,t}$ and $\varepsilon_{z,t}$ are identically and independently distributed (iid) type shocks with mean zero and standard deviations $\sigma_a, \sigma_m, \sigma_R$ and σ_z.

Equations (1) to (13) govern the equilibrium dynamics of the 13 variables, $a_t, c_t, R_t, k_t, n_t, x_t, r_t, y_t, z_t, \Lambda_t, \pi_t, m_t$ and μ_t. It might not be immediately apparent upon visual inspection of Equations (1) to (13) that money has no structural direct effects in this economy. But this is indeed the case, because the solution for all other variables is the same as the one that would obtain after having removed m_t and Equation (13) from the system. Put differently, money has no direct effects in this economy because m_t and Equation (13) are unnecessary to determine the equilibrium dynamics of the remaining 12 variables.

This, however, would not be the case if the central bank were to respond with the interest rate to movements in real money balances, so that real money balances enter the policy rule. In this case, it would be impossible to remove m_t and Equation (13) and still arrive at the same solution for the remaining variables. Money would have direct effects but in an artificially induced way. Real money balances would matter, but only because of the central bank's choice of incorporating them in the policy rule. The evolution of real money balances would then become relevant for households to forecast the path of the nominal interest rate; although, even in this case, money's additional influence would not be truly independent of the nominal interest rate.

The calibration, summarised in Table 5, is that of Nelson (2002). The parameters, $g_1, g_2, g_3, g_4, \kappa_1, \kappa_2$ and a_m are, in turn, functions of deeper structural parameters.

The model can be written in matrix form and one of the available solution methods, like that of Uhlig (1995), can then be used to compute the rational expectations solution. If Y_t is the vector of jump variables, $(y_t, \Lambda_t, r_t, n_t, x_t, \mu_t)'$, X_t is the vector of state variables, $(a_t, k_t, z_t, c_t, R_t, \pi_t, \pi_{t-1}, m_t)'$, and ε_t is the vector of iid shocks, $(\varepsilon_{a,t}, \varepsilon_{m,t}, \varepsilon_{R,t}, \varepsilon_{z,t})'$, then the unique solution is an equilibrium law of motion of the form:

$$X_t = \mathbf{P}X_{t-1} + \mathbf{Q}\varepsilon_t \tag{14}$$

$$Y_t = \mathbf{R}X_{t-1} + \mathbf{S}\varepsilon_t \tag{15}$$

where $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ are the solution matrices.

Table 5: Model Calibration

Parameter	Description	Value
g_1	Function of structural parameters	15.41
g_2	Function of structural parameters	–32.75
g_3	Function of structural parameters	–15.53
g_4	Function of structural parameters	3.58
β	Household's discount factor	0.992
α_μ	Calvo price-setting parameter	0.086
α	Capital's share of income	0.36
s_c	Consumption-to-output ratio in steady-state	0.72
δ	Depreciation rate	0.025
κ_1	Function of structural parameters	0.046
κ_2	Function of structural parameters	0.19
ρ_R	Coefficient on the lagged short-rate in the policy rule	0.29
ρ_π	Coefficient on inflation in the policy rule	0.22
ρ_y	Coefficient on output in the policy rule	0.08
ρ_a	Persistence of demand shock	0.33
ρ_z	Persistence of technology shock	0.95
ε_m	Inverse of intertemporal elasticity of substitution in real money balances	5
a_m	Function of portfolio adjustment costs parameters	10
σ_a	Standard deviation of $\varepsilon_{a,t}$	0.01
σ_m	Standard deviation of $\varepsilon_{m,t}$	0.01
σ_R	Standard deviation of $\varepsilon_{R,t}$	0.002
σ_z	Standard deviation of $\varepsilon_{z,t}$	0.007

One way to verify that money has no structural direct effects in this model is by rewriting the solution as a vector autoregression for the augmented vector $(X_t', Y_t')'$. One would then notice that the coefficients on real money balances in the reduced-form equations for all variables other than real money balances are zero. In other words, for money to have direct effects, real money balances have to not only be a part of the state vector, X_t, as is the case here, but real money balances also have to matter for the rest of the system.

The solution expresses each endogenous variable as a linear combination of lags of the state variables and a linear combination of the structural shocks. In particular, the reduced-form equation for output, y_t, is the first row in Equation (15), which at the values shown in Table 5, is given by:

$$y_t = 0.02a_{t-1} - 0.34k_{t-1} + 0.20z_{t-1} - 0.25c_{t-1} - 0.43R_{t-1} \\ - 0.61\pi_{t-1} - 0.33\pi_{t-2} + \bar{\varepsilon}_t \tag{16}$$

where $\bar{\varepsilon}_t = \mathbf{S}_1'\boldsymbol{\varepsilon}_\mathbf{t}$ is the reduced-form shock in the output equation, a linear combination of the structural shocks and $\mathbf{S}_1'$ is the first row of $\mathbf{S}$. Note that real money balances do not enter Equation (16), so it's true coefficient is zero. And it is also the case that the money demand shock $\varepsilon_{m,t}$ does not enter the reduced-form shock, $\bar{\varepsilon}_t$.

4. Monte Carlo Analysis

Ideally, we would estimate Equation (16) using real data, which would allow us to identify whether there are direct effects of money. This is problematic, though, since many of the state variables are unobserved or at best observed imprecisely. Instead, we examine whether the omission of unobserved state variables biases the estimates of the coefficients to such an extent that it is no longer possible to identify direct effects of money.

To do this, we use the solution of the model to generate artificial data sets, and then use these to estimate a number of reduced-form specifications. Here we focus on Nelson's simplified analog of the empirical aggregate demand specifications of Section 2. Namely,

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 4r_t + \gamma_3 \Delta m_{t-1} + \varepsilon_t. \tag{17}$$

We construct 1 000 samples of 200 observations each, in line with the sample sizes of the empirical regressions of Section 2, and another 1 000 samples of 1 000 observations to capture the large sample properties of the estimators.[9]

If a model with structural direct effects produced data that when used to estimate the parameters of Equation (17) yielded a positive and statistically significant estimate of γ_3, but a model with no direct effects of money produced data that when used to estimate the parameters of Equation (17) yielded insignificant estimates of

9 We allow a burn-in period of 100 observations.

γ_3, then we could think that the empirical specifications of Section 2 are somewhat useful for identifying direct effects of money. But this is not the case.

Table 6 reports the average of the parameter estimates and the standard errors over the 1 000 samples for each of the sample sizes based on the model of Section 3 with no direct effects. The average estimate of γ_3, for a sample size of 200, would suggest that a 1 percentage point rise in the growth rate of the real money stock leads to a 0.25 percentage point rise in output in the next period, after accounting for the impact on the contemporaneous real interest rate. The average is statistically significant; in fact, not one of the 1 000 estimates comes close to zero, as Figure 2 shows.

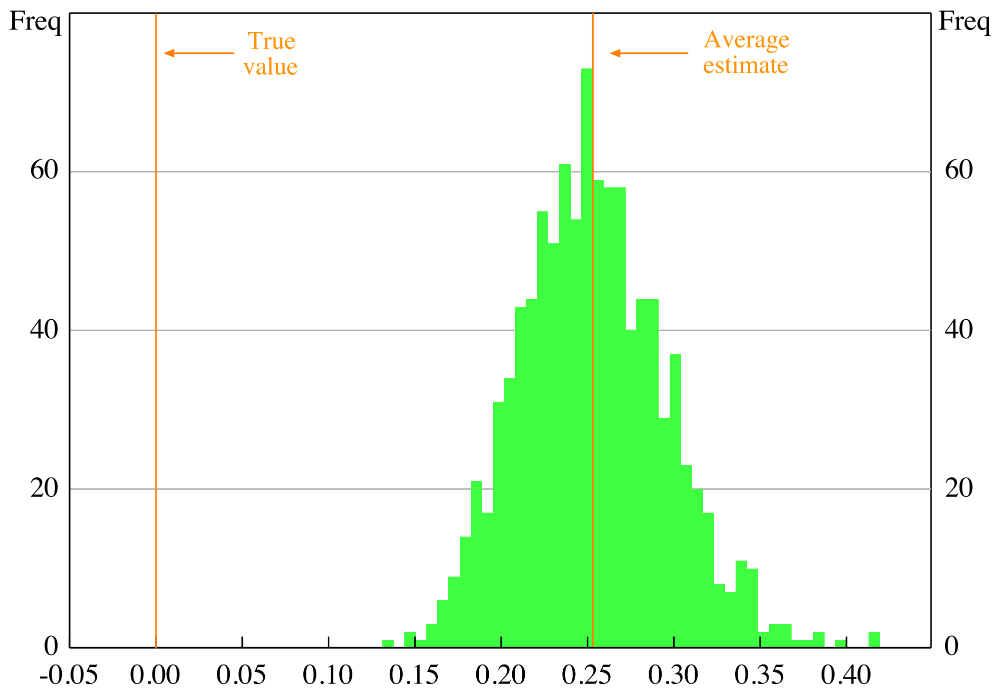
Table 6: Average Parameter Estimates

Dependent variable: y_t	$T = 200$		$T = 1\ 000$	
Constant	0.00	(0.0003)	0.00	(0.0001)
y_{t-1}	0.68**	(0.043)	0.72**	(0.018)
$4r_t$	–0.20**	(0.036)	–0.20**	(0.016)
Δm_{t-1}	0.25**	(0.051)	0.24**	(0.022)

Notes: * and ** denote statistical significance at the 10 and 5 per cent levels, respectively. Standard errors are in parentheses.

Figure 2: Distribution of γ_3 Estimates

$T = 200$



The model produces evidence for direct effects (in line with some of the results of Section 2), not because there are direct effects of money, but rather because the estimates are biased; the omission of key state variables from Equation (17) introduces a bias in the estimates of the remaining coefficients. Because we know the process that generated those data – that is, we know Equation (16) – we know that, in truth, γ_3 is zero.

Compared to Equation (16), Equation (17) excludes a set of variables and includes others, particularly Δm_{t-1}. In general, including spurious variables leads to inefficient estimates, but it is the omission of key variables that leads to biased estimates of the parameters that are left in the regression. To see this, suppose, following Greene (2000), that the correctly specified equation takes the form:

$$y_t = \mathbf{X}_{1,t}\beta_1 + \mathbf{X}_{2,t}\beta_2 + e_t \tag{18}$$

where in our case $\mathbf{X}_{1,t} = (1, y_{t-1}, r_t, \Delta m_{t-1})'$, and $\mathbf{X}_{2,t} = (a_{t-1}, k_{t-1}, z_{t-1}, c_{t-1}, R_{t-1}, \pi_{t-1}, \pi_{t-1})'$. If we collect the observations for output in $\mathbf{y}$ and the observations for the two sets of regressors in $\mathbf{X}_1$ and $\mathbf{X}_2$, and then regress $\mathbf{y}$ on $\mathbf{X}_1$ alone, the estimator of β_1 would be:

$$\hat{\beta}_1 = \beta_1 + (\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{X}_2\beta_2 + (\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{e} \tag{19}$$

This would be an unbiased estimator of β_1 only in the special case where the remaining terms in Equation (19) add to zero. However, the variables which are omitted, $\mathbf{X}_2$, are correlated with the ones which are included, $\mathbf{X}_1$, and β_2 is not zero, so the term $(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{X}_2\beta_2$ is not zero. And because r_t is simultaneously determined within the system of equations that describes the economy, the third term, $(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{e}$, cannot be expected to vanish either. Only by chance would the estimator not be biased.

We can decompose the average bias in γ_3 over the 1 000 regressions according to Equation (19) and show that the second term, $(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{X}_2\beta_2$, accounts for 66 per cent of the bias. An important factor underlying the positive estimate of γ_3, is the omission of the *lagged* capital stock from the regression. The coefficient estimate on real money growth that obtains from regressing the lagged capital stock on the regressors in Equation (17), $\mathbf{X}_1$, is –0.91; the coefficient on lagged capital in Equation (16), the reduced-form solution for output, is –0.34. Hence, a higher value of the lagged capital stock corresponds to a lower level of output and also to a lower level of real money balances (conditional on the real interest rate and lagged output). Other things equal, a higher value of the lagged capital stock corresponds to a lower level of current investment and correspondingly to a lower level of current output. So, excluding capital introduces a positive bias in γ_3.

In addition to the above analysis, we have also generated data from a model in which money has sizeable direct effects, a version of the model of Andrés *et al* (2004). The same Monte Carlo exercise using this model shows that the average estimate of γ_3 is statistically insignificant. We have also estimated other specifications, including ones that exactly match those used in Section 2. These results are not reported because they make essentially the same point. Namely, regressions like Equation (17) fail to uncover money's true structural role. A model without direct effects of money can give rise to data that produces positive and statistically significant coefficients on real money growth while a model with direct effects can produce insignificant ones.

5. Conclusion

The question of whether money has direct effects has become more important as policy rates have approached the zero lower bound in the euro area, Japan, the United Kingdom and the United States. Monetary authorities in these economies have turned to unconventional monetary policies which involve balance sheet expansions of one form or another.

Our contribution in this paper is to reinterpret some of the econometric evidence which on the surface often suggests that, after controlling for the short-term real interest rate, real base money growth can be a significant determinant of total output for a number of countries and sample periods. Our interpretation of these types of results is that they are likely to be biased.

We reach this conclusion by using a model that has no direct effects of money. Yet the model is capable of producing data which leads to positive and statistically significant coefficients on real money growth in a real output regression similar to those that are often found when using actual data. From the perspective of the structural model, it then becomes clear that the reduced-form regressions suffer from an omitted variable bias. In particular, the bias on real money growth is large enough to undermine the validity of any inference about the existence of direct effects of money.

We have also repeated the analysis with a model in which money has sizeable direct effects and have also found a bias, but this time operating in the opposite direction – that is, the estimate of the reduced-form coefficient on real money is not significantly different from zero. The empirical regressions also exclude key variables from this model, in which case the estimated coefficients on real money growth go to zero.

In short, the reduced-form regressions – even when they fit the data well – are misleading. They simply fail to uncover the true structural relationship between money and the rest of the economy. They lead to incorrect inferences on the existence of direct effects and they are an unreliable guide to calibrate monetary policies, in general, including at the zero lower bound.

Appendix A: Data Description and Sources

Australia

Real output: Chain volume GDP (ABS).

Price level: Trimmed mean CPI excluding interest and tax changes (RBA).

Real interest rate: Let $\tilde{r}_t$ be the real interest rate, and P_t be the price level. Then $\tilde{r}_t$ is calculated according to:

$$\tilde{r}_t = \sum_{i=0}^{3} \left[\left(1 + \frac{i_t}{100}\right)^{0.25} - 1 \right]_{t-i} - \frac{P_t - P_{t-4}}{P_{t-4}} \qquad \text{(A1)}$$

where i_t is the average of the cash rate in percentage terms over the quarter (RBA).

Money: Quarterly average of the money base series (RBA). For the results in Table 3, we subtract the log of the price level from the log of money, and then seasonally adjust using X12, before differencing to get real money growth.

Japan

Real output: Post 1980, GDP in constant 2000 Yen billions is used (Thomson Reuters). Prior to 1980, we splice the series using GDP data from the Economic and Social Research Institute, Cabinet Office, Government of Japan.

Price level: Seasonally adjusted CPI excluding fresh food (Thomson Reuters), adjusted for the effects of the value added tax increase in 1997.

Real interest rate: Calculated according to Equation (A1), where i_t is the average of the overnight call rate (Thomson Reuters).

Money: The average of the reserve requirement rate change adjusted and seasonally adjusted money base series (CEIC).

United Kingdom

Real output: Chain volume GDP (Office for National Statistics).

Price level: Retail price index excluding mortgage interest payments (Thomson Reuters).

Real interest rate: Calculated according to Equation (A1), with i_t being the 3-month rate on Treasury bills at the end of the period (Thomson Reuters).

Money: From May 2006, we use the sum of reserve balances and notes and coins. To get a value for April 2006, we splice the data using the growth rate for notes and coins. We then use the growth rate of M0 to splice the series back to the beginning of the sample (all series are from the Bank of England).

As with the Australian data, we subtract the log of prices from the log of money, and then seasonally adjust using X12, before differencing to get real money growth.

United States

Real output: We subtract potential from actual GDP, and divide by potential GDP (both from the Congressional Budget Office).

Price level: Seasonally adjusted CPI less food and energy (Thomson Reuters).

Real interest rate: Calculated according to Equation (A1), where i_t is the effective federal funds rate (Board of Governors of the Federal Reserve System).

Money: The St. Louis adjusted monetary base series (Federal Reserve Bank of St. Louis).

References

Andrés J, JD López-Salido and E Nelson (2004), 'Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium', *Journal of Money, Credit and Banking*, 36(4), pp 665–690.

Bernanke B (2009), 'The Crisis and the Policy Response', Speech delivered at the Stamp Memorial Lecture, London School of Economics, London, 13 January.

Borio C and P Disyatat (2009), 'Unconventional Monetary Policies: An Appraisal', Bank for International Settlements Working Paper No 292.

Brunner K and AH Meltzer (1993), *Money and the Economy: Issues in Monetary Analysis*, Raffaele Mattioli Lectures, Cambridge University Press, Cambridge.

Dale S (2010), 'QE - One Year On', Remarks at the Cambridge Centre for International Money and Finance and MMF Conference on 'New Instruments of Monetary Policy: The Challenges', Cambridge, 12 March.

Favara G and P Giordani (2009), 'Reconsidering the Role of Money for Output, Prices and Interest Rates', *Journal of Monetary Economics*, 56(3), pp 419–430.

Friedman M (1956), 'The Quantity Theory of Money: A Restatement', in M Friedman (ed), *Studies in the Quantity Theory of Money: With Essays by Milton Friedman ... [et al]*, The University of Chicago Press, Chicago, pp 3–21.

Greene WH (2000), *Econometric Analysis*, 4th edn, Prentice Hall, Upper Saddle River.

Hafer RW, JH Haslag and G Jones (2007), 'On Money and Output: Is Money Redundant?', *Journal of Monetary Economics*, 54(3), pp 945–954.

Koenig EF (1990), 'Real Money Balances and the Timing of Consumption: An Emperical Investigation', *The Quarterly Journal of Economics*, 105(2), pp 399–425.

Krugman PR (1998), 'It's Baaack: Japan's Slump and the Return of the Liquidity Trap', *Brookings Papers on Economic Activity*, 2, pp 137–187.

Leeper EM and JE Roush (2003), 'Putting "M" Back in Monetary Policy', *Journal of Money, Credit and Banking*, 35(6), Part 2: Recent Developments in Monetary Economics, pp 1217–1256.

Meltzer AH (2001), 'The Transmission Process', in Deutsche Bundesbank (ed), *The Monetary Transmission Process: Recent Developments and Lessons for Europe*, Palgrave, London, pp 112–130.

Neiss KS and E Nelson (2003), 'The Real-Interest-Rate Gap as an Inflation Indicator', *Macroeconomic Dynamics*, 7(2), pp 239–262.

Nelson E (2002), 'Direct Effects of Base Money on Aggregate Demand: Theory and Evidence', *Journal of Monetary Economics*, 49(4), pp 687–708.

Nelson E (2003), 'The Future of Monetary Aggregates in Monetary Policy Analysis', *Journal of Monetary Economics*, 50(5), pp 1029–1059.

Rudebusch GD and LEO Svensson (1999), 'Policy Rules for Inflation Targeting', in J Taylor (ed), *Monetary Policy Rules*, National Bureau of Economic Research Conference Report, The University of Chicago Press, Chicago, pp 203–246.

Rudebusch GD and LEO Svensson (2002), 'Eurosystem Monetary Targeting: Lessons from U.S. Data', *European Economic Review*, 46(3), pp 417–442.

Svensson LEO (1999), 'How Should Monetary Policy Be Conducted in an Era of Price Stability?', in *New Challenges for Monetary Policy*, A Symposium sponsored by the Federal Reserve Bank of Kansas City, Kansas City, pp 195–259.

Uhlig H (1995), 'A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily', Tilburg University, Center for Economic Research Discussion Paper No 9597.