Measuring Global Interest Rate Comovements with Implications for Monetary Policy Interdependence*

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Abstract

A general measure of the strength of U.S. and local interest rate comovement is developed to identify changes in monetary policy interdependence between January 1999 and May 2020. Entropy theory captures comovements through second-order comoments and higher-order comoments of coskewness, cokurtosis and covolatility. The sample contains monthly short-term shadow rates, with local rates for Australia, Canada, Europe, Japan, New Zealand, Switzerland, and the U.K. Monetary policy overall became more interdependent during the Global Financial Crisis but progressively more independent after adopting unconventional monetary policy by central banks. Measures using second-order comoments do not entirely capture changes in interest rate interdependence.

Keywords: Entropy; generalized exponential family; higher-order comomeC12; E43; E52; E58; F42nt decomposition; independence testing, zero-lower bound
JEL classifications: C12; E43; E52; E58; F42

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1 Introduction

Understanding the comovements of global interest rates is important for determining the ability of countries to conduct independent monetary policy (Frankel et al., 2004).[1] Identifying the strength of global interest rate comovements is also important for policy coordination (McKibbin, 1997; Corsetti and Pesenti, 2005; Liu and Pappa, 2008), avoiding sub-optimal monetary policy (Rey, 2016; Miranda-Agrippino and Rey, 2020), understanding global business cycle synchronization (Ductor and Leiva-Leon, 2016), financial integration of global markets (Bekaert and Mehl, 2019), the spread of contagion across international bond markets (Dungey et al., 2011), and portfolio diversification (Bekaert et al., 2017).

Aizenman et al. (2008) construct a monetary interdependence index based on the correlation between global interest rates with a positive correlation representing increasing monetary interdependence with local and global interest rates moving in the same direction, and a negative correlation representing increasing monetary independence as interest rates move in opposite directions. The monetary index MI proposed by Aizenman et al. (2008) is defined as MI=1-(ρ +1)/2 which ranges from $MI = 0$ with $\rho = 1$ (perfect monetary interdependence), to $MI = 1$ with $\rho = -1$ (perfect monetary independence). A broader measure of monetary interdependence is the monetary conditions index initially proposed by the Bank of Canada (see Freedman (1994) for a review of this index), which is a weighted average of the interest rate and the exchange rate. Another approach that captures global monetary interdependence is based on an extended Taylor rule where a country's interest rate is a function of domestic inflation and the output gap, as well as a measure of international interest rates (see Belke and Cui (2010); Beckmann et al. (2017); Gray (2013)).

[1] Monetary policy independence is not just a function of interest rates, but a range of factors as a result of the trilemma facing economies, including the exchange rate regime (Aizenman et al., 2016; Ligonnière, 2018; Obstfeld, 2015; Rohit and Dash, 2019; Hofmann and Takáts, 2015), and capital controls (Klein and Shambaugh, 2015).

A common feature of existing approaches to measuring monetary policy interdependence is that they are based on just one aspect of the relationship between local and global interest rates as captured by second-order moments between interest rates. What these approaches do not focus on is the role of interest rate risk as captured by the volatility in global interest rates. Interest rate models allowing for level effects (Chan et al. (1992); Longstaff and Schwartz (1992); Bali (2000)) specify interest rate volatility as a function of the level of the interest rate. Expanding this class of models to a multivariate setting would result in interest rate volatility being a function of both local and global interest rates, manifesting in coskewness between local and global interest rates. Multivariate models of interest rates which allow for time-varying volatility provide an a additional channel connecting interest rate comovements. A feature of this class of models is that interest rate volatility in one bond market can spillover onto other markets, which impacts the co-risks of interest rates resulting covolatility amongst interest rates. A natural extension of these channels is where interest rate risk is modelled in terms of skewness, resulting in cokurtotis between interest rates as the skewness of interest rates in another market would be related to the level of interest rates in another market.

Other forms of nonlinearities linking interest rates besides multivariate volatility models arise. The effects of the zero lower bound and the period of unconventional monetary policy (UMP) changes the stochastic relationships amongst interest rates. For a relationship between interest rates that is initially linear, operating in a period where the lower bound is binding can change the shape of the underlying distribution resulting in the occurrence of higher-order moment effects which would not have been present otherwise. By not correcting for this type of interest rate environment, can potentially bias estimates of monetary interdependence. The approach adopted in the empirical application of the paper is to use the shadow rates proposed by (Krippner, 2013, 2015). Another class of nonlinear models inducing higher-order comoment relationships amongst global interest rates are models based on nonlinear Taylor rules (Petersen, 2007).[2]

This paper aims to develop a broad measure of monetary interdependence that builds on previous approaches based on second-order moments by taking into account higher-order comoments of global interest rates, including coskewness, cokurtosis and covolatility. The approach uses entropy theory to weight second and higher-order comoments into a single index of monetary interdependence.[3] An important advantage is the ability to be able to identify the individual contributions of each channel to the overall index, as well as construct a general test of monetary independence which can be applied to individual countries and across sample periods.

The approach is applied to measuring interdependence between international and U.S. interest rates using monthly data from January 1999 to May 2020. To capture the effects of UMP on monetary interdependence, the empirical analysis is performed over two sub-periods consisting of a pre-UMP period (January 1999 to August 2008) and an UMP period (September 2008 to May 2020). In contrast to existing measures of global interest rate comovements which use overnight market rates or even long-term rates (Hofmann and Takáts, 2015; Obstfeld, 2015), Krippner shadow (short) rates are used to circumvent potential distortions when interest rates operate at or near the zero lower-bound such as during periods of UMP (Krippner, 2013, 2015).

The empirical results provide evidence of an increase in monetary policy interdependence during the Global Financial Crisis, progressively becoming more independent during the post-GFC period when central banks adopted UMP. The results also show that movements in monetary policy interdependence are not fully captured by conventional measures based on second-order comoments. Between the pre-UMP and UMP

[2]Simulation experiments highlighting the effects from adopting nonlinear Taylor rules on higher-order comoments are reported in the Appendix of this paper.

[3]For related work on the use of entropy theory in econometrics see Maasoumi and Racine (2002), Maasoumi and Racine (2008), and Granger et al. (2004), as well as Fry-McKibbin et al. (2021) who apply the framework to measuring financial interdependence in financial asset markets.

periods, there is an overall reduction in interdependence of just over 20 percent. Australia, Canada, Europe, New Zealand, and Switzerland show a decrease with respect to the U.S. In contrast, Japan and the U.K. reveal increased interdependence. Cokurtosis and covolatility are important linkages contributing to changes in monetary policy interdependence within each country for Australia, Canada and New Zealand during the UMP period, Europe in the UMP period, and the U.K. in pre-UMP and UMP periods. Second-order comoments are the dominant linkages for Europe pre-UMP, and for both periods in the case of Switzerland and Japan.

Replacing the shadow rates by either short-term money market rates or longer-term rates shows that the zero lower bound has a distorting effect on the degree of monetary interdependence amongst national rates. This is especially the case for Japanese monetary policy, which is no longer independent during the UMP period. The reverse occurs for Canada, where monetary policy becomes more independent of the U.S. This result is counter-intuitive given the strong economic relationships between these two countries.

The rest of the paper proceeds as follows. The interdependence index of monetary policy is derived in Section 2. Estimation methods and testing issues are also discussed. The data on global interest rates are discussed in Section 3, together with simulation results highlighting the effects of the lower bound on testing for monetary policy independence. Simulation results are presented in Section 4 of the effects of the zero lower bound on the higher-order comoment relationships amongst interest rates. Empirical evidence of the monetary policy interdependence index based on shadow rates are presented in Section 5, with the results compared with using more traditional interest rate measures based on short-term money market rates and longer-term rates. Concluding comments and suggestions for future research are provided in Section 6, with data definitions and additional empirical and simulation results given in the Appendices.

2 Theoretical Framework and Implementation

The theoretical framework for constructing an index of monetary interdependence is presented by specifying a sufficiently flexible model to allow for a range of channels in determining interest rate comovements, including second-order comoments, as well as higher-order comoments based on coskewness, cokurtosis and covolatility. The approach involves three steps. In the first step, the (negative) entropy of the joint distribution of interest rates is defined (Fry-McKibbin et al., 2021). The second step specifies the joint distribution using the generalized normal distribution, which has the convenient property that the interdependence index is expressed as a weighted average of the comoments. The third step involves constructing the measure corresponding to the special case of independence by restricting the parameters on the comoments in the joint distribution to zero. This independence measure is then used to rescale the interdependence index which assists in interpreting the index.

Let r_1 and r_2 represent two interest rates with joint probability distribution $f(r_1, r_2; \Theta)$, with unknown parameter vector Θ. Consider the following measure of certainty based on the (negative) entropy of the joint distribution

$$E[\log f(r_1, r_2; \Theta)] = \int \int \log(f(r_1, r_2; \Theta)) f(r_1, r_2; \Theta) \, dr_1 dr_2, \tag{1}$$

which involves weighting the natural logarithm of the joint distribution $f(r_1, r_2; \Theta)$ by the joint probabilities of the two interest rates. In the extreme case of independence equation (1) has a minimum, with increasing values representing increasing interdependence amongst interest rates.

To capture relative changes in the strength of comovements in interest rates over time, the measure in (1) is defined with respect to the case where interest rates are independent (Fry-McKibbin et al., 2021)

$$\psi_{1,2} = 1 - \frac{E\left[\log f\left(r_1, r_2; \Theta_1\right)\right]}{E\left[\log f\left(r_1, r_2; \Theta_0\right)\right]}, \tag{2}$$

where Θ_1 is the parameter vector of the distribution capturing joint movements in interest rates, and Θ_0 is the restricted set of parameters for the case where interest rates move independently of each other. A value of $\psi_{1,2} = 0$ corresponds to interest rate independence as by definition $E\left[\log f\left(r_1, r_2; \Theta_1\right)\right] = E\left[\log f\left(r_1, r_2; \Theta_1\right)\right]$. Interest rates are interdependent when $\psi_{1,2} > 0$, with higher values of $\psi_{1,2}$ representing increased interdependence. An aggregate interdependence index between r_1 and the interest rates for the $j = 2, ..., n$ countries is constructed by averaging across the bivariate interdependence measures as[4]

$$\Psi = \frac{1}{n-1} \sum_{j=2}^{n} \psi_{1,j}. \tag{3}$$

To implement the index in (2) a flexible class of distributions based on the exponential family (Lye and Martin, 1993; Cobb et al., 1983) is specified to capture potential interest rate comovements

$$f\left(r_1, r_2; \Theta\right) = \exp\left(h_t\left(\Theta\right) - \eta\left(\Theta\right)\right), \tag{4}$$

where

$$\begin{aligned} h_t\left(\Theta\right) = & -\frac{1}{2}\left(\frac{z_{1t}^2 + z_{2t}^2 - 2\theta_0 z_{1t} z_{2t}}{1-\theta_0^2}\right) + \theta_1 z_{1t} z_{2t}^2 + \theta_2 z_{1t}^2 z_{2t} \\ & + \theta_3 z_{1t} z_{2t}^3 + \theta_4 z_{1t}^3 z_{2t} + \theta_5 z_{1t}^2 z_{2t}^2 + \theta_6 z_{1t}^4 + \theta_7 z_{2t}^4, \end{aligned} \tag{5}$$

with $z_{it} = \left(r_{it} - E\left(\ r_{it}\right)\right) / \sqrt{var\left(r_{it}\right)}$, and $\eta\left(\Theta\right)$ is the normalizing constant

$$\eta\left(\Theta\right) = \log \int \int \exp\left(h\right) dr_1 dr_2, \tag{6}$$

[4]The aggregate measure is based on the result that aggregating individual (negative) entropy measures yield a total (negative) measure of entropy.

to ensure the probability density integrates to unity, $\int\int f(r_1, r_2; \Theta)\, dr_1 dr_2 = 1$. The form of h_t in (5) represents a generalized normal distribution.[5] The unknown parameters are $\Theta = (\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$. The first term on the right-hand side in (5) represents the bivariate normal distribution with parameter θ_0 which captures interdependence through the interaction term $z_{1t}z_{2t}$, as commonly measured by the correlation parameter. The remaining terms capture interdependence through the higher-order comoments, with the strength of these comoments determined by the parameters $\theta_i, i = 1, 2 \cdots, 5$. The parameters θ_1 and θ_2 control the strength of coskewness, θ_3 and θ_4 control the strength of cokurtosis and θ_5 controls the strength of covolatility. The remaining terms z_{1t}^4 and z_{2t}^4 represent fourth-order moments to allow the interest rate distributions to depart from normality, by setting $\theta_6 = \theta_7 = -1$. Under the assumption of independence the joint distribution decomposes into a product of the marginal distributions which occurs for the special case by restricting the parameters as

$$\theta_0 = \theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = 0. \tag{7}$$

2.1 Special Case of Bivariate Normality

A special case of the interdependence index in (2) is where $\theta_1 = \cdots = \theta_5 = 0$, together with the restrictions $\theta_6 = \theta_7 = 0$, so interdependence is now entirely encapsulated by the parameter θ_0. For this special case, the underlying distribution in (4) reduces to bivariate normality with θ_0 now representing the correlation parameter ρ, with the

[5]For alternative forms of this distribution, see Fry et al. (2010) and Fry-McKibbin et al. (2021). More generally, other subordinate distributions of the generalized exponential distribution can be specified to capture particular dependence characteristics inherent in the data.

index simplifying as[6]

$$\psi_{1,2}\left(\theta_1=\cdots=\theta_5=0\right)=1-\frac{1+\log\left(2\pi\right)+\log\left(\sigma_1^2\sigma_2^2\right)/2}{1+\log\left(2\pi\right)+\log\left(\sigma_1^2\sigma_2^2\right)/2+\log\left(1-\rho^2\right)/2}. \tag{8}$$

The index is characterized by a parabola in the correlation parameter ρ, which has a minimum of $\psi_{1,2}=0$ at $\rho=0$. As $\rho\rightarrow 1$, or $\rho\rightarrow -1$, $\psi_{1,2}$ approaches its maximum value of $\psi_{1,2}\rightarrow 1$.

2.2 Implementation

The unknown parameters Θ are estimated using maximum likelihood methods by defining the log-likelihood

$$\log L_T=\frac{1}{T}\sum_{t=1}^{T}h_t(\Theta)-\eta(\Theta), \tag{9}$$

with $h_t(\Theta)$ defined in (5) and $\eta(\Theta)$ in (6). As (9) is a nonlinear function of Θ, an iterative gradient algorithm is adopted to compute the maximum likelihood estimates. Confidence intervals for the normalized interdependence measure in (2) are based on standard errors computed using the delta method

$$se(\Psi(\widehat{\Theta}))=\sqrt{D'\Omega D}, \tag{10}$$

[6]The steps to derive the index in the case of bivariate normality is to note that an analytical expression for the normalizing constant is $\eta=\log\left(2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\right)$, in which case equation (12) is

$$\begin{aligned}
E\left[\log f\left(r_1,r_2;\Theta_1\right)\right] &= \int\int\left(-\log\left(2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\right)+\left(-\frac{z_1^2+z_2^2-2\rho z_1z_2}{2\left(1-\rho^2\right)}\right)\right)f\left(r_1,r_2;\theta\right)dr_1dr_2 \\
&= -\log\left(2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\right)-\frac{1}{2\left(1-\rho^2\right)}\int\int\left(z_1^2+z_2^2-2\rho z_1z_2\right)f\left(r_1,r_2;\theta\right)dr_1dr_2 \\
&= -\log\left(2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\right)-\frac{1}{2\left(1-\rho^2\right)}\left(2\left(1-\rho^2\right)\right) \\
&= -1-\log\left(2\pi\right)-\frac{1}{2}\log\left(\sigma_1^2\sigma_2^2\right)-\frac{1}{2}\log\left(1-\rho^2\right).
\end{aligned}$$

where $z_i=\left(r_i-Er_i\right)/\sqrt{var\left(r_i\right)}$, such that $E\left(z_1^2\right)=E\left(z_2^2\right)=1$, and for the bivariate normal distribution $E\left(z_1z_2\right)=\rho$. For the case of independence $\rho=0$, this expression reduces to $E\left[\log f\left(r_1,r_2;\Theta_0\right)\right]=-1-\log\left(2\pi\right)-\frac{1}{2}\log\left(\sigma_1^2\sigma_2^2\right)$.

where D is a $(N \times 1)$ vector of the derivatives of $\Psi_{1,2}$ with respect to the parameters Θ evaluated at $\widehat{\Theta}$, and Ω is the covariance matrix of $\widehat{\Theta}$ obtained directly from the maximum likelihood procedure. In the empirical application in Section 5, the derivatives in D are computed numerically.

3 Statistical Properties of Global Interest Rates

The data consist of monthly shadow interest rates for Australia, Canada, the Euro Area, Japan, New Zealand, Switzerland, the U.K. and the U.S.[7] All data begin January 1999, coinciding with the introduction of the euro and the year when Japan adopted a zero interest rate policy (Nakaso, 2017), and end May 2020 just after the beginning of the COVID crisis in March 2020, resulting in a sample size of $T = 257$. The adoption of monthly interest rates is chosen to match the frequency of the macroeconomics control variables used in the empirical analysis. Data definitions and sources are presented in Appendix A.

Short-term shadow rates are used over proxies for official policy rates in order to measure the stance of monetary policy better, particularly since the global financial crisis in 2008-09 when overnight market rates operated around the zero lower bound (Krippner, 2013, 2015).[8] Nonetheless, for comparative purposes overnight money market and long-term 10-year bond yields are also included in the empirical analysis. The latter is a complementary cross-check with observable data on the shadow rate results. While shadow rates can freely move below the lower bound to indicate unconventional monetary policy actions in addition to the near-zero and largely static short-maturity interest rates, they are necessarily estimated quantities so that they will vary with the

[7]The choice of countries is based on the availability of country shadow rates on Krippner's website.

[8]The Krippner shadow rates are adopted rather than those published by Wu and Xia (2016). Krippner (2020) highlights several anomalies for the Wu and Xia (2016) U.S. shadow rate series and the availability of the Wu and Xia (2016) shadow rates are limited to the United States, the Euro Area, and the United Kingdom.

model and data used for their estimation. The 10-year yields can also move freely during lower-bound periods due to policy rate settings and unconventional monetary policy actions, but are observable.

Figure 1 compares the shadow short rates and the conventional overnight short-term money market rates (short-rates) for each country. The two rates track each other very closely during periods where the money market rate is not constrained by the zero lower bound, but deviate from each other when overnight money market rates operate at or near the zero lower bound, or even at negative rates in the case of Europe, Japan and Switzerland. The Australian and New Zealand shadow rates are positive for most of the sample period, apart from the last three months in the case of Australia and the last two months in the case of New Zealand. At the other extreme, the Japanese shadow rate is mostly negative for the entire sample period. For the U.S. and the U.K., there are large deviations between shadow and overnight rates for extended periods during the UMP period, whereas for Canada, the divergences between the two rates are more moderate. Summary statistics on the shadow rates are given in Table 1 for the full sample period and two subperiods, with the breakpoint chosen as August 2008, the month before the collapse of Lehmann Brothers. The corresponding summary statistics for the short-term and long-term rates are respectively given in Tables D1 and D2 of Appendix D.

Figure 2 provides a comparison of the international shadow rates with the U.S. shadow rate. Tests of independence between country and U.S. shadow rates based on the Lagrange Multiplier testing framework of (Fry-McKibbin et al., 2021), are given in Table 2. The results are presented for the total period, the pre-UMP and the UMP periods.[9] The p-values reveal substantial evidence of interdependent shadow

[9]The choice of the U.S. as the base country is motivated by the empirical work of Frankel et al. (2004) and others who gauge the independence of country interest rates relative to the U.S. rate. See also Bekaert et al. (2020) who explore the importance of U.S. monetary policy decisions in driving global risk, uncertainty and asset prices.

Figure 1: Comparison of the short-term shadow rates (SR) and the overnight money market rates (MR), January 1999 to May 2020. The vertical line indicates the beginning of the unconventional monetary policy period.

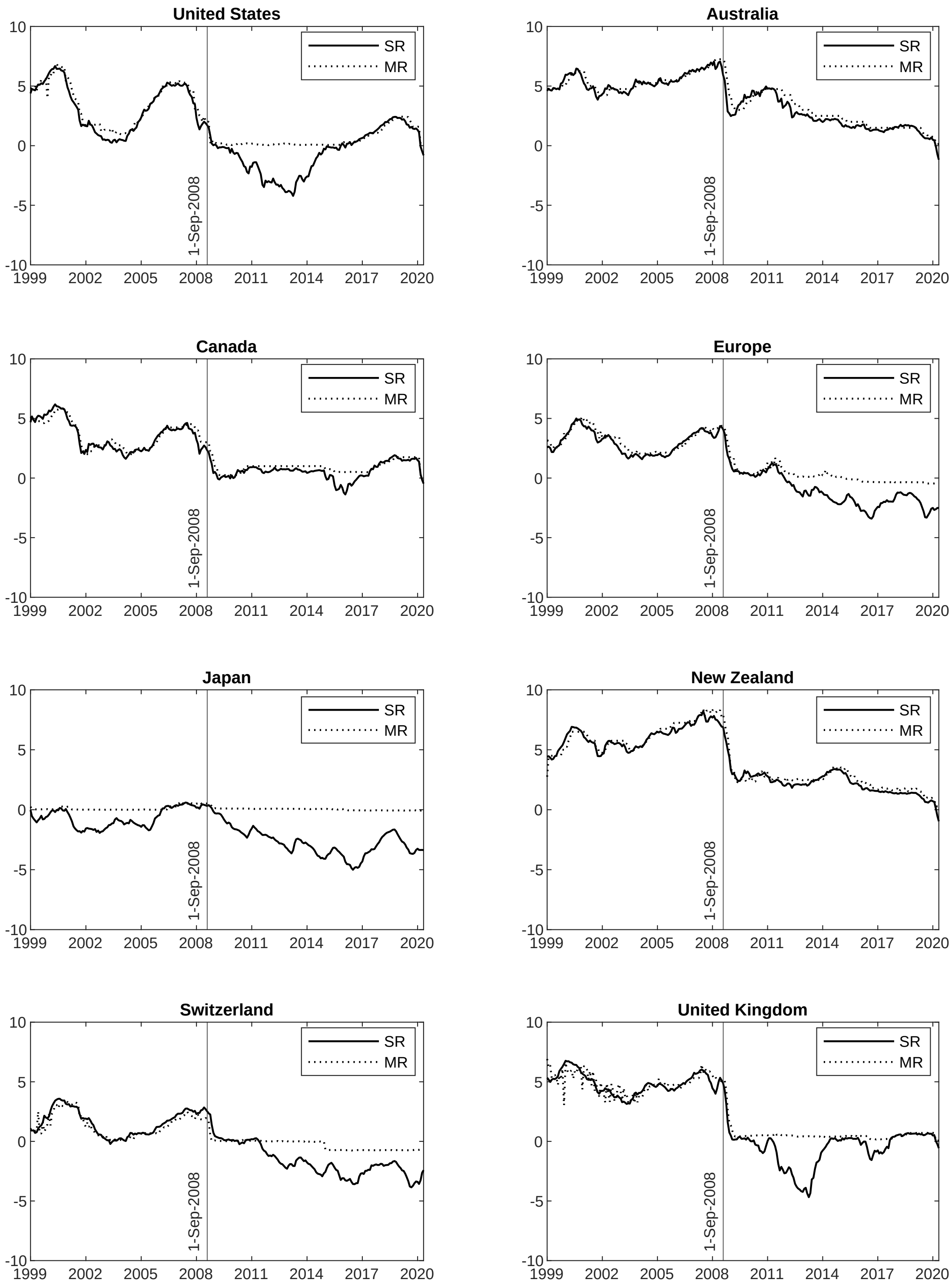


Table 1: Descriptive statistics of monthly shadow short rates for selected periods, January 1999 to May 2020.

Country	Mean	Min	Max	SD	Skew	Kurt	JB stat	P-value
Panel A. Full Sample (January 1999 to May 2020)								
United States	1.179	−4.213	6.682	2.683	0.104	2.335	5.193	0.075
Australia	3.734	−1.177	7.058	1.873	−0.180	1.861	15.275	0.000
Canada	1.933	−1.378	6.182	1.785	0.596	2.432	18.647	0.000
Europe	0.771	−3.407	4.999	2.383	−0.025	1.651	19.513	0.000
Japan	−1.780	−5.007	0.600	1.424	−0.198	2.140	9.605	0.008
New Zealand	3.952	−0.965	8.225	2.187	0.176	1.728	18.654	0.000
Switzerland	−0.175	−3.854	3.576	2.022	0.019	1.841	14.401	0.001
United Kingdom	1.910	−4.680	6.762	2.967	−0.131	1.861	14.633	0.001
Panel B. Pre-UMP (January 1999 to August 2008)								
United States	3.271	0.242	6.682	1.976	−0.022	1.611	9.335	0.009
Australia	5.420	3.857	7.058	0.768	0.319	2.153	5.432	0.066
Canada	3.573	1.613	6.182	1.254	0.416	1.917	9.012	0.011
Europe	3.046	1.580	4.999	0.946	0.136	1.812	7.176	0.028
Japan	−0.673	−1.933	0.600	0.781	0.059	1.594	9.626	0.008
New Zealand	6.103	4.184	8.225	1.001	0.003	2.080	4.094	0.129
Switzerland	1.616	−0.217	3.576	1.056	0.106	1.767	7.569	0.023
United Kingdom	4.879	3.141	6.762	0.876	0.207	2.495	2.061	0.357
Panel C. UMP (September 2008 to May 2020)								
United States	−0.543	−4.213	2.416	1.828	−0.251	2.027	7.042	0.030
Australia	2.347	−1.177	5.464	1.284	0.402	2.736	4.209	0.122
Canada	0.583	−1.378	2.141	0.692	−0.312	3.178	2.478	0.290
Europe	−1.100	−3.407	3.717	1.363	0.614	3.041	8.869	0.012
Japan	−2.691	−5.007	0.386	1.164	0.385	2.885	3.566	0.168
New Zealand	2.182	−0.965	6.183	0.977	0.485	5.760	50.268	0.000
Switzerland	−1.648	−3.854	2.353	1.315	0.604	2.791	8.832	0.012
United Kingdom	−0.533	−4.680	4.281	1.481	−0.885	4.310	28.484	0.000

rates with the U.S. for all countries except for Japan. The individual tests show that interdependence arises through the correlation channel and higher-order comoments. Similar results are found in Panel B for the first subperiod prior to the adoption of UMP in many countries, as well as the results given in Panel C for the UMP period, although the relative strength of various comoments may change depending upon the country and the subperiod.

Figure 2: Shadow short rates against the U.S.

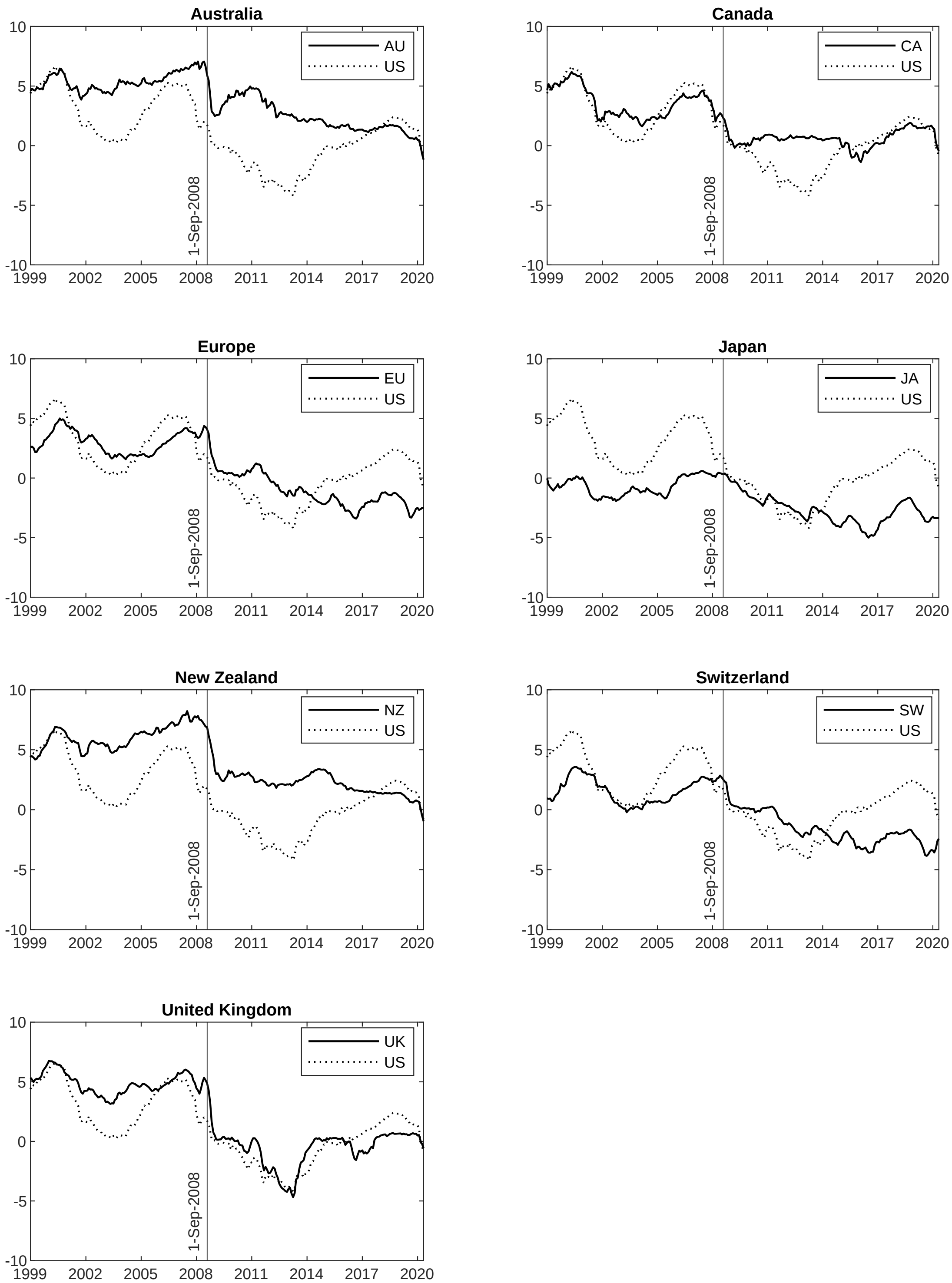


Table 2: Lagrange Multiplier tests of independence of global shadow rates (r_i) with the U.S. (r_{us}) for selected periods. Results are reported in terms of p-values.

Country	Joint	Correlation	Coskewness		Cokurtosis		Covolatility
		$r_{us}r_i$	$r_{us}r_i^2$	$r_{us}^2 r_i$	$r_{us}r_i^3$	$r_{us}^3 r_i$	$r_{us}^2 r_i^2$
Panel A. Full sample (January 1999 to May 2020)							
Australia	0.000	0.000	0.000	0.000	0.000	0.000	0.208
Canada	0.000	0.000	0.000	0.020	0.000	0.000	0.001
Europe	0.000	0.000	0.193	0.264	0.000	0.000	0.011
Japan	0.015	0.744	0.278	0.149	0.488	0.545	0.888
New Zeland	0.000	0.000	0.000	0.028	0.000	0.000	0.672
Switzerland	0.000	0.000	0.487	0.900	0.000	0.000	0.071
United Kingdom	0.000	0.000	0.806	0.063	0.000	0.000	0.012
Panel B. Pre-UMP (January 1999 to August 2008)							
Australia	0.000	0.000	0.269	0.872	0.005	0.000	0.352
Canada	0.000	0.000	0.078	0.307	0.000	0.000	0.045
Europe	0.000	0.000	0.688	0.884	0.000	0.000	0.199
Japan	0.112	0.907	0.303	0.251	0.700	0.977	0.831
New Zeland	0.000	0.000	0.458	0.167	0.040	0.000	0.177
Switzerland	0.000	0.000	0.788	0.747	0.000	0.000	0.053
United Kingdom	0.000	0.000	0.507	0.782	0.000	0.000	0.008
Panel C. UMP (September 2008 to May 2020)							
Australia	0.000	0.000	0.371	0.119	0.000	0.000	0.002
Canada	0.000	0.000	0.000	0.001	0.000	0.000	0.488
Europe	0.000	0.095	0.207	0.363	0.228	0.118	0.376
Japan	0.313	0.157	0.957	0.979	0.158	0.157	0.317
New Zeland	0.000	0.000	0.001	0.000	0.523	0.000	0.258
Switzerland	0.000	0.062	0.154	0.291	0.155	0.141	0.334
United Kingdom	0.000	0.000	0.348	0.724	0.262	0.000	0.193

4 Simulation Effects of the ZLB

Second-order comoments are commonly used to measure monetary interdependence and more generally, the strength of the interrelationships amongst global interest rates. Methods adopted include VARs and Granger causality tests (DeGennaro et al., 1994). An implicit assumption underlying these approaches is that the relationship between interest rates is assumed to be linear, which is a potentially a tenuous assumption given the adoption of UMP stemming from the GFC, resulting in nominal interest rates across countries operating near or at the lower bound (ZLB), or even below as in the case of Europe (Johannsen and Mertens, 2021).

To highlight some of the potential pitfalls from using the correlation parameter as a measure of interdependence at a time when interest rates are operating near the lower bound, a number of simulation experiments are presented. The data generating process (DGP) consists of two interest rates r_{1t}^* and r_{2t}^*, which evolve according to

$$\begin{aligned} r_{1t}^* &= 0.5882 + 0.7385\, z_{1t}, && (11) \\ r_{2t}^* &= 0.1594 + 0.7360\, z_{2t}, && (12) \\ z_{it} &\sim N(0,1), \qquad i = 1, 2, && (13) \\ E(z_{1t} z_{2t}) &= \rho, && (14) \end{aligned}$$

where ρ is the population correlation parameter measuring the strength of dependence between the two interest rates. The choice of the parameters in (11) and (12) is based on the daily overnight shadow rates for the U.S. (interest rate of country 1) and Europe (interest rate of country 2) from September 2008 to May 2020. The zero lower bound is imposed by restricting the interest rates to be zero whenever

$$r_{it} = \begin{cases} r_{it}^* & : \quad r_{it}^* > 0 \\ 0 & : \quad r_{it}^* \leq 0 \end{cases} \tag{15}$$

For the experiments where the interest rates are uncensored, $r_{it} = r_{it}^*$, resulting in the possibility of negative interest rates. Inspection of the parameter values shows that the interest rates have similar standard deviations, but quite different means. Given that r_{2t} has the lower mean, this suggests it would display a higher proportion of negative interest rates than r_{1t} when there is no censoring.

Six tests of independence are considered

$$
\begin{aligned}
TSTAT_1 &= \sqrt{T}\widehat{\rho} && (16)\\
TSTAT_2 &= \frac{\sqrt{T}CS_{12}}{\sqrt{4\widehat{\rho}+2}} && (17)\\
TSTAT_3 &= \frac{\sqrt{T}CS_{21}}{\sqrt{4\widehat{\rho}+2}} && (18)\\
TSTAT_4 &= \frac{\sqrt{T}\left(CK_{13}-3\widehat{\rho}\right)}{\sqrt{18\widehat{\rho}+6}} && (19)\\
TSTAT_5 &= \frac{\sqrt{T}\left(CK_{31}-3\widehat{\rho}\right)}{\sqrt{18\widehat{\rho}+6}} && (20)\\
TSTAT_6 &= \frac{\sqrt{T}(CV_{22}-(1+2\widehat{\rho}^2))}{\sqrt{4\widehat{\rho}^4+16\widehat{\rho}^2+4}}, && (21)
\end{aligned}
$$

where

$$
\begin{aligned}
\widehat{\rho} &= \frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t},\ CS_{12}=\frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t}^2,\ CS_{21}=\frac{1}{T}\sum_{t=1}^{T} z_{1t}^2 z_{2t},\\
CK_{13} &= \frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t}^3,\ CK_{31}=\frac{1}{T}\sum_{t=1}^{T} z_{1t}^3 z_{2t},\ CV_{22}=\frac{1}{T}\sum_{t=1}^{T} z_{1t}^2 z_{2t}^2.
\end{aligned}
$$

and $z_{it} = \left(r_{it} - \widehat{\mu}_i\right)/\widehat{\sigma}_i$, $i = 1, 2$, are the standardized residuals. Under the null hypothesis of independence all 6 tests statistics are distributed asymptotically as $N\left(0, 1\right)$. The first test given in equation (16) is the standard correlation test. The next two tests in equations (17) and (18) are the coskewness tests discussed in Fry et al. (2010). The tests in equations (19) and (20) are tests for cokurtosis, and in equation (21) is a test for covolatility, which are considered by (Fry-McKibbin and Hsiao, 2018).

Table 3 gives the probability of rejecting the null hypothesis of independence using (16) to (21), based on $T = 1000$ simulated interest rates from (11) to (14) with 50000 replications. The results are presented for three cases, with the first being the "No Censoring" case whereby $r_{it} = r_{it}^*$, so negative interest rates are allowed. The second case allows for "Partial Censoring" with the second interest rate r_{2t} being censored

Table 3

Probability of rejecting the null hypothesis of independence based on the test statistics in equations (16) to (21) with a nominal size of 5%, by simulating interest rates from equations (11) to (14). The sample size is $T = 1000$, and the number of replications is 50000.

Test	Statistic	Rejection Probability		
		No Censoring	Partial Censoring	Full Censoring
			$\rho = 0.0$	
Correlation	$TSTAT_1$	0.0516	0.0504	0.0512
Coskew(1,2)	$TSTAT_2$	0.0497	0.1273	0.1262
Coskew(2,1)	$TSTAT_3$	0.0509	0.0499	0.0450
Cokurt(1,3)	$TSTAT_4$	0.0506	0.3024	0.3040
Cokurt(3,1)	$TSTAT_5$	0.0492	0.0485	0.1007
Covol(2,2)	$TSTAT_6$	0.0467	0.1218	0.1100
			$\rho = 0.1$	
Correlation	$TSTAT_1$	0.8866	0.8076	0.7766
Coskew(1,2)	$TSTAT_2$	0.0487	0.5699	0.5522
Coskew(2,1)	$TSTAT_3$	0.0501	0.0526	0.2429
Cokurt(1,3)	$TSTAT_4$	0.0489	0.3445	0.3644
Cokurt(3,1)	$TSTAT_5$	0.0488	0.0502	0.1402
Covol(2,2)	$TSTAT_6$	0.0468	0.1236	0.2086
			$\rho = -0.1$	
Correlation	$TSTAT_1$	0.8846	0.8061	0.7686
Coskew(1,2)	$TSTAT_2$	0.0503	0.5724	0.5318
Coskew(2,1)	$TSTAT_3$	0.0504	0.0525	0.1467
Cokurt(1,3)	$TSTAT_4$	0.0503	0.3409	0.3146
Cokurt(3,1)	$TSTAT_5$	0.0489	0.0491	0.0778
Covol(2,2)	$TSTAT_6$	0.0488	0.1240	0.1723

according to (15), without any adjustment made to r_{1t}. This choice is adopted as the second interest rate is based on the European overnight shadow rates, which was negative for long periods over the sample used to determine the parameters of the DGP in equation (12). The third and final case is of "Full Censoring", where now r_{1t} and r_{2t} are censored according to (15) so that all interest rates are restricted to be non-negative.

The first block of results in Table 3 is based on independent interest rates by setting $\rho = 0$ in equation (14), with a nominal size of 5%. As the interest rates are independent under the null hypothesis, the rejection probabilities represent the test statistics' empirical size. As expected, the probability of rejecting the null hypothesis of independence without censoring are all around their nominal size of 5%, showing that the tests are correctly sized. Allowing for partial censoring causes the rejection probability to increase for the higher-order comoments of coskewness ($TSTAT_2$), cokurtosis ($TSTAT_4$) and covolatility ($TSTAT_6$). For the coskewness ($TSTAT_3$) and cokurtosis ($TSTAT_5$) comoments tests the probability of rejecting the null hypothesis remains at around 5%, as does the probability for the correlation test. In the case of full censoring, all higher-order comoment tests, except for the coskewness test statistic $TSTAT_3$, display increased probabilities of rejection. As with the partial censoring results, the probability of rejecting the null hypothesis of independence for the correlation test is still close to the size of the test of 5%.

The second block of results in Table 3 allows for positively dependent interest rates ($\rho = 0.1$) which is typical of interest rate comovements before the adoption of UMP, whereas the third block allows for negative dependence between interest rates ($\rho = -0.1$) which is more typical of interest rate comovements during the UMP period. As the interest rates are dependent in these experiments, the rejection probabilities represent the power of the tests. If there is no censoring, the probability of rejecting the null for the correlation test corresponds to power of nearly 90%. In contrast, the higher-order comoment test statistics all still have rejection probabilities at around 5%. This is expected as the true model is based on the assumption of normality, so any change in dependence amongst interest rates is solely captured through the correlation parameter and not the higher-order comoments. However, allowing for censoring, partial or full, causes the rejection probabilities to increase for nearly all higher-order comoment tests, revealing a change in dependence amongst the interest rates caused by the zero lower bound. In the case of the correlation test ($TSTAT_1$) the zero lower bound causes a slight reduction in the rejection probabilities suggesting a reduction in dependence.

The zero-lower-bound simulation experiments reported in Table 3 highlight two important features. First, the correlation parameter can be a misleading measure of interdependence when interest rates are affected by the zero lower bound. The correlation parameter is designed to model linear dependence and is not a sufficient statistic if nonlinearities arise due to a lower bound. Second, higher-order comoment tests based on coskewness, cokurtosis and covolatility can capture the nonlinearities linking interest rates brought about by the effects of imposing a zero lower bound on interest rates.

5 Empirical Results

An index of monetary interdependence expressed in equation (2) is constructed for the monthly shadow rates of the 7 countries discussed in Section 3. The index is estimated over two sample periods corresponding to the pre-UMP and UMP periods to demonstrate how monetary interdependence changes with UMP adoption. Empirical results of the monetary interdependence index are also presented over a rolling sample period to provide dynamic estimates of changes in monetary interdependence both globally and nationally.

Following Hofmann and Takáts (2015) and Obstfeld (2015), the effects of business cycles and common global financial conditions are captured by conditioning the interdependence results on a set of control variables. Five control variables are specified: domestic and U.S. real output growth rates, domestic and U.S. inflation rates, and a volatility measure of global financial conditions based on the change in the natural logarithm of the CBOE volatility index.[10] To condition the monetary index on the con-

[10]Other conditioning variables were also tried in the empirical analysis, such as a world business cycle measure based on a trade-weighted measure of world output covering a comprehensive group of countries obtained from the Netherlands Bureau for Economic and Policy Analysis. The empirical results from including world output in the set of conditioning variables did not change the qualitative results, so it was excluded from the final results.

trol variables, bivariate VARX models for each country and the U.S. shadow rates are estimated, together with lags of the 5 control variables. The residuals of the estimated VARX models are taken as the inputs in computing the monetary interdependence index. The empirical results reported use a lag of 3, although other lag choices generate similar qualitative results.

5.1 Global Monetary Interdependence Estimates

Table 4 gives estimates of the interdependence index for the pre-UMP and UMP periods. The index is expressed as a percentage relative to a value of zero (independence), based on the shadow rates of each country and the U.S., with standard errors given in parentheses based on (10). The parameter estimates are computed by maximizing the log-likelihood in (9) using an iterative gradient algorithm.[11]

The global interdependence index estimates are given in the first row of Table 4. The estimate of the index during the pre-UMP period is 22.6%, which is statistically significant from zero (independence), providing evidence of significant interdependence of monetary policies globally. As the monetary regime moves towards UMP period, the value of the index falls to 17.6%, just over one-fifth of the value of the index pre-UMP. Nonetheless, there is still evidence of monetary interdependence, with the estimate of 17.6% being statistically significant from zero.

Figure 3 presents time-varying estimates of the global monetary index using a 5-year moving average to estimate equation (2). For comparison the estimated index based on just the second-order comoment by setting $\theta_1 = \theta_2 = ... = 0.0$ in equation (5) is also presented. There is a high degree of volatility in the interdependence index during the pre-UMP period, with estimates ranging between 10% and 30%. Interdependence increases at the onset of the GFC and is maintained throughout the GFC and most

[11]The maximum likelihood parameter estimates are not reported here but are available from the authors upon request.

Table 4: Interdependence index, expressed as a percentage, between country and U.S. shadow rates based on (2) with the global index based on (3). A smaller (larger) value of the index represents greater independence (interdependence). The results are presented for the pre-UMP period (January 1999 to August 2008) and the UMP period (September 2008 to May 2020), with asymptotic standard errors given in parentheses based on (10).

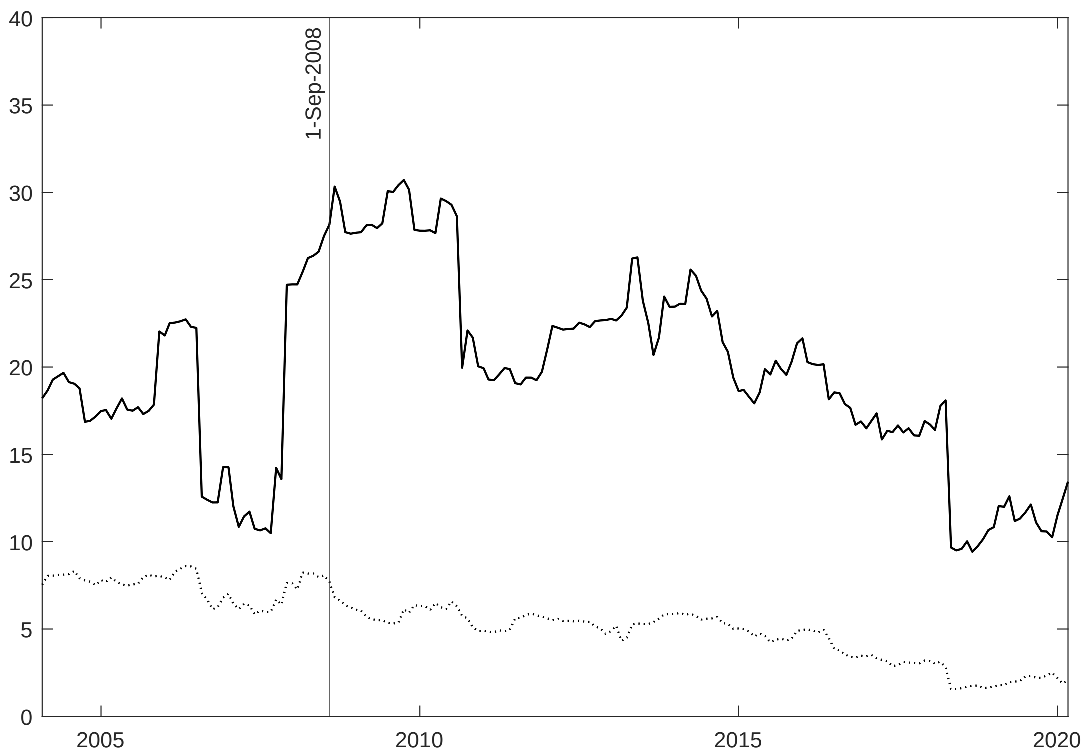
Country	Pre-UMP	UMP
Global	22.594	17.582
	(5.194)	(3.229)
Australia	21.035	13.030
	(11.159)	(4.093)
Canada	26.006	14.805
	(12.960)	(4.485)
Europe	37.860	19.648
	(22.733)	(11.788)
Japan	14.654	22.087
	(14.567)	(7.066)
New Zealand	19.945	9.412
	(15.334)	(6.439)
Switzerland	18.621	12.257
	(6.773)	(14.874)
United Kingdom	20.034	31.832
	(4.391)	(4.734)

of the European debt crisis up to 2011 . From thereon, the index trends downwards, with monetary policies over this period becoming less dependent. By the end of the sample in May of 2020, the level of the interdependence index practically reverts back to the same level as achieved just priot to the GFC.

5.2 Country Monetary Interdependence Estimates

A breakdown of the global interdependence index estimates in terms of countries relative to the U.S., is given in the second half of Table 4. All countries reveal significant monetary interdependence with the U.S. during the pre-UMP period, as all estimated

Figure 3: Interdependence index (solid line) against correlation (dashed lined) based on a rolling 5 year window.



indexes are statistically different from zero. Europe (37.9%) records the highest index value with the U.S., followed by Canada (26.0%), Australia (21.0%) and the U.K. (20.0%), with Japan recording the lowest estimate (14.6%). There appears to be substantial uniformity in the degree of interdependence of each country with the U.S., with most index estimates falling into a relatively narrow range.

The empirical results in Table 4 reveal widespread evidence of a reduction in the degree of interdependence during the UMP period, with most country indices falling except for Japan and the U.K., whose indices rise. For Europe, the index falls by about 50 percent and is not significantly different from independence in the latter period. Similar results occur for New Zealand and Switzerland, although they start from a lower base than Europe. Australia and Canada also experience reductions in interdependence with the U.S., although interdependence remains significant. For Japan, the increase in the index value from 14.6% to 22.1%, suggest that Japanese and U.S. shadow interest rates became more interdependent during the UMP period.

5.3 Decompositions

The contributions of second- and higher-order comoments to monetary interdependence of each country with respect to the U.S., are computed as

$$\begin{aligned}
C_{11} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\widehat{\theta}_0}{1-\widehat{\theta}_0^2}z_{1t}z_{2t} && \text{[Covariance(1,1)]} \\
S_{12} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_1 z_{1t}z_{2t}^2 && \text{[Coskewness(1,2)]} \\
S_{21} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_2 z_{1t}^2 z_{2t} && \text{[Coskewness(2,1)]} \\
K_{13} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_3 z_{1t}z_{2t}^3 && \text{[Cokurtosis(1,3)]} \\
K_{31} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_4 z_{1t}^3 z_{2t} && \text{[Cokurtosis(3,1)]} \\
V_{22} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_5 z_{1t}^2 z_{2t}^2, && \text{[Covolatility(2,2)]}
\end{aligned} \tag{22}$$

where $\{\widehat{\theta}_0, \widehat{\theta}_1, \widehat{\theta}_2, \widehat{\theta}_3, \widehat{\theta}_4, \widehat{\theta}_5\}$ are the maximum likelihood parameter estimates and $z_{it} = (r_{it} - \mu_i)/\sigma_i$, where μ_i and σ_i are additional parameters that need to be estimated along with the distributional parameters θ_i. Letting the total contribution of all comoments be given by

$$TOTAL = C_{11} + S_{12} + S_{21} + K_{13} + K_{31} + V_{22}, \tag{23}$$

the percentage contributions are given by dividing each comoment in (22) by (23) and multiplying by 100. A positive (negative) value corresponds to the comoment increasing (decreasing) interdependence amongst the shadow interest rates.

The results of the decompositions are given in Figure 4 for each country for the pre-UMP and UMP periods. Cokurtosis and covolatility are largest in magnitude in contributing to changes in interest rate interdependence within each country: Australia, Canada and New Zealand during the UMP period, Europe in the pre-UMP period, and the U.K. in both periods. In many cases the contributions of cokurtosis and covolatility tend to offset each other, particularly when those comoments dominate the correlation and coskewness comoments. Second-order comoments are the dominant linkages for Europe during the UMP period and for both periods in the case of Switzerland and Japan.

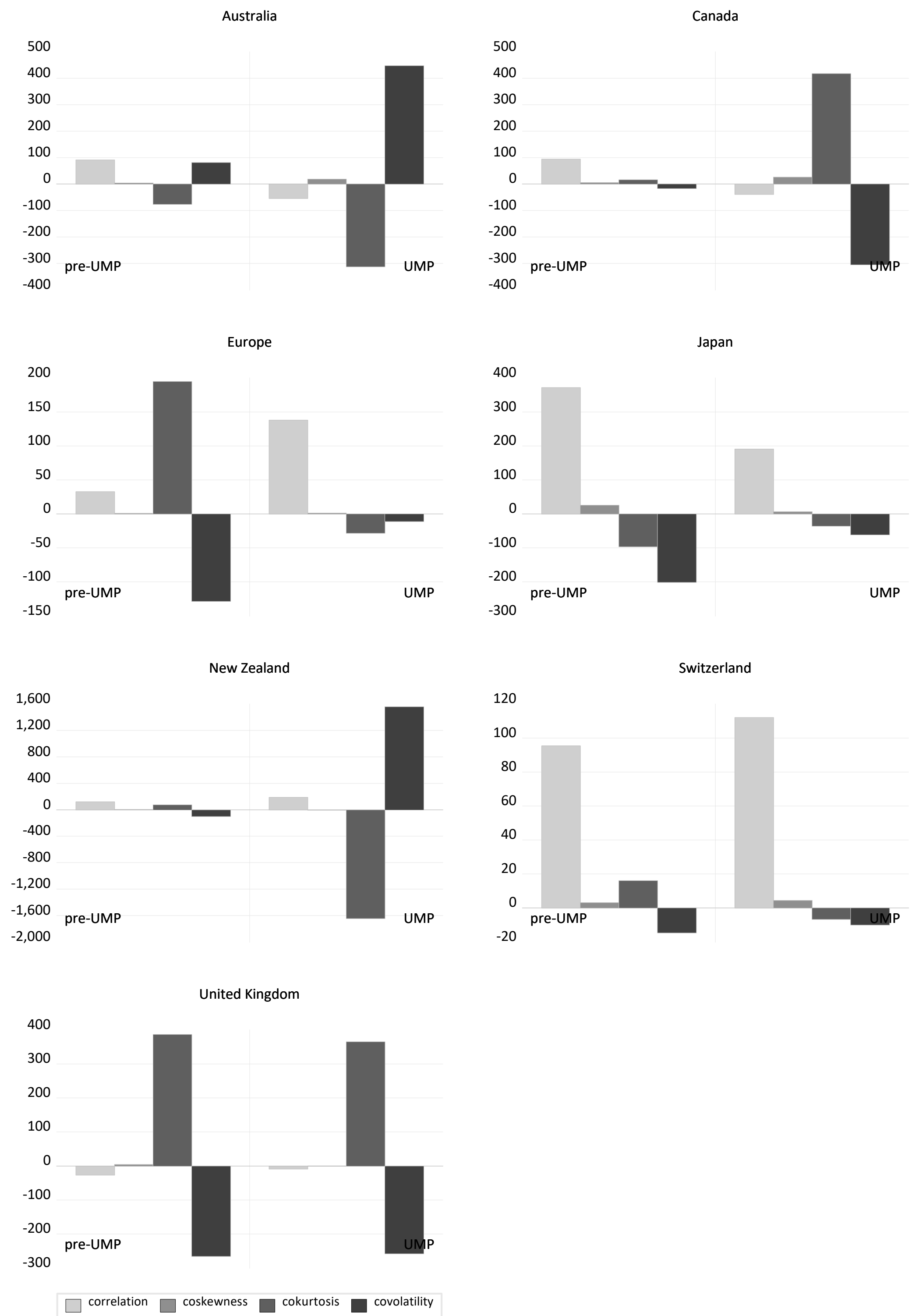
5.4 Interdependence Indexes Using Market Rates

5.4.1 Short-term Money Market Rates

The overnight money market interest rate is the traditional short-term measure of monetary policy commonly used in the monetary policy trilemma literature examining domestic monetary policy linkages with the U.S. or another base country (Ligonnière, 2018).[12] As Figure 1 shows, overnight rates are constrained by the zero lower bound

[12]The overnight money market interest rate is an endogenous measure of monetary policy that is commonly used in the trilemma literature, whereas the literature on monetary policy spillovers typi-

Figure 4: Interdependence index, expressed as a percentage, between country and U.S. shadow rates based on (2) with the aggregate index based on (3). The results are presented for the pre-UMP period (January 1999 to August 2008) and the UMP period (September 2008 to May 2020).



during the UMP period. Descriptive statistics of the money market rates presented in Table D1 of Appendix Appendix D, also reveal this point with all countries experiencing rates of less than 1% during the UMP period, while Europe, Japan and Switzerland even experience negative rates. There are also significant falls in the standard deviations of the rates in the UMP period compared to the pre-UMP period in Table D1 (see also Swanson and Williams, 2014).

To investigate the effects of the zero lower bound on interest rate interdependence, the index in equation (2) is recomputed by replacing the shadow rates of each country by their overnight market rates, with the results given in Table 5. Inspection of the results highlights the problems from using overnight money market rates to measure monetary policy interdependence.

The results suggest that monetary policy became more interdependent for most countries during the UMP period, with the global index more than doubling between the pre-UMP (4.3%) and UMP (10.3%) periods. The result of a high level of interdependence is opposite to the results presented in Table 4 for the shadow rates where monetary policy became more independent during the UMP period. The zero lower bound distorts the interdependence measure, giving the appearance that rates are more interdependent than they actually are. The case for Japan highlights the distorting effects of the lower bound, which suggest that Japanese monetary policy was no longer independent during the UMP period. In Canada's case, the opposite occurs with Canadian monetary policy becoming independent of the U.S. after the adoption of UMP in many countries, which is counter-intuitive given the strong economic relationships between the two countries.

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cally uses exogenous policy shocks. Obstfeld (2015) finds broad agreement with the two approaches, although there is less autonomy found if short-term interest rates are used as the policy instrument (for example, Bluedorn and Bowdler, 2010; Caceres et al., 2016).

Table 5: Interdependence index, expressed as a percentage, between global and U.S. overnight money market rates based on (2) with the global index based on (3). The results are given for the full sample period (January 1999 to May 2020), the pre-UMP period (January 1999 to August 2008) and the UMP period (September 2008 to May 2020), with asymptotic standard errors given in parentheses based on (10).

Country	Pre-UMP	UMP
Global	4.323	10.355
	(1.052)	(2.163)
Australia	1.032	17.538
	(2.343)	(8.893)
Canada	5.879	0.000
	(2.674)	(0.000)
Europe	4.915	1.019
	(1.662)	(2.363)
Japan	1.794	10.011
	(2.829)	(2.557)
New Zealand	4.461	16.446
	(1.726)	(6.257)
Switzerland	3.175	13.565
	(2.721)	(4.867)
United Kingdom	9.003	3.551
	(4.525)	(3.793)

5.4.2 Long-term rates

The results for international short-term money market rates highlight the distorting effects of the zero lower bound on the strength of interdependence between these rates. An alternative monetary policy measure is long-term bond yields, which reflect current and expected future short rates. Descriptive statistics on 10-year long-term bond yields are presented in Table D2.[13] During the pre-UMP period, long-rates for most countries except for Japan, operate above 1%. Interest rates above 1% are no longer the case in the UMP period, where long-term rates for all countries are less than 1%, and even negative for Japan and Switzerland.

[13]The 10-year bond rates for each month are computed as the average of the daily rates for each month.

Table 6: Interdependence index, expressed as a percentage, between global and U.S. long-term rates based on (2) with the global index based on (3). The results are given for the full sample period (January 1999 to May 2020), the pre-UMP period (January 1999 to August 2008) and the UMP period (September 2008 to May 2020), with asymptotic standard errors given in parentheses based on (10).

Country	Pre-UMP	UMP
Global	17.217	12.931
	(6.117)	(1.111)
Australia	24.051	17.675
	(3.260)	(3.468)
Canada	28.116	22.733
	(2.786)	(3.538)
Europe	17.762	6.474
	(39.245)	(2.225)
Japan	5.139	2.469
	(2.465)	(3.085)
New Zealand	21.877	17.201
	(2.706)	(2.572)
Switzerland	7.435	6.454
	(2.512)	(2.576)
United Kingdom	16.142	17.512
	(3.623)	(2.905)

Table 6 gives the interdependence index estimated using the long-term rates for each country relative to the U.S.. Comparison of these results with those for the shadow rates in Table 4 is qualitatively consistent, with interdependence declining during the UMP period. However, the zero lower bound nonetheless still distorts the interdependence measure. The results using shadow rates tend to show more considerable reductions in monetary policy interdependence during the UMP than for the shadow rates.

6 Conclusions and Extensions

This paper proposes a new and more general approach to identifying monetary interdependence based on international interest rates. A feature of the approach is the nesting of traditional interdependence measures by including higher-order comoments, including coskewness, cokurtosis and covolatility. To circumvent potential distorting effects of the zero lower bound as a result of the adoption of UMP, the empirical analysis used shadow rates instead of more conventional approaches based on short-term money market rates or long-term rates.

Using monthly data from January 1999 to May 2020, the empirical results revealed an increase in monetary policy interdependence before the introduction of UMP, with a reversal as central banks adopted UMP. Between the pre-UMP and UMP periods there was an overall reduction in the interdependence relative to the U.S. of just over 20 percent. Australia, Canada, Europe, New Zealand and Switzerland all contributed to this reduction, whereas the reverse occurred for Japan and the U.K. which were found to become even more interdependent with U.S. interest rates.

An important feature of the empirical results was that changes in monetary policy interdependence over time were not fully captured by conventional measures based on second-order comoments. Cokurtosis and covolatility were important linkages contributing to movements in monetary policy interdependence within each country for Australia, Canada and New Zealand during the UMP period, Europe in the UMP period, and the U.K. in pre-UMP and UMP periods. Nonetheless, second-order comoments were the dominant linkages for Europe pre-UMP and for both periods in the case of Switzerland and Japan. Redoing the empirical analysis by replacing the shadow rates with short-term money market rates or longer-term rates highlighted the distorting effects of the zero lower bound had measuring monetary interdependence, with shorter rates having a more significant distorting effect than longer rates. There was also evidence that longer-term rates exhibited greater interdependence than short-term shadow rates, a result which complemented previous findings that longer-term international rates showed higher correlations than short-term rates (Hofmann and Takáts, 2015; Obstfeld, 2015).

The empirical analysis presented in the paper suggests various extensions. First, the empirical analysis was conducted for countries with reasonably similar exchange rate systems. An important issue investigated in modelling international interest rates has been whether international interest rates are more independent for countries operating flexible exchange rates Frankel et al. (2004).[14] One way to proceed would be to widen the set of countries to include countries with less flexible exchange rate systems. An alternative approach would be to extend the dataset of the existing group of countries to cover periods where these countries did experience changes in their exchange rate regimes. Second, the empirical analysis conditioned the interest rates on lagged interest rates and a set of control variables and measured interdependence contemporaneously. A more dynamic analysis could be adopted following the work of Maasoumi and Racine (2002) by measuring interdependence between interest rates at different points in time. This point would especially be important given recent evidence of Bekaert et al. (2020) who provide evidence of causal feedback from countries to U.S. monetary policy.

[14] A caveat is that the Swiss Franc is classified as a de facto moving band see https://bit.ly/39lrIRC and Ilzetzki et al. (2019)

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Appendix A Variable Definitions and Sources

Table A1: Data Definitions for Interest Rates and Conditioning Variables

Country	**Definition**	**Retrieved**
	Panel A. Shadow Short Rates	
All	Shadow short rates computed and published by Leo Krippner. Shadow rates represent the price of the value of a hypothetical call option for holding cash. Data used is end of month.	https://www.ljkmfa.com/
	Panel B. Overnight Money Market Rates	
Australia	Interbank Overnight Cash Rate, end of month.	Reserve Bank of Australia
Canada	Overnight Money Market Financing Rate, end of month.	StatCan
Euro Area	Euro Interbank Offered Rate, end of month.	Deutsche Bundesbank
Japan	Call Rate, average Uncollateralized Overnight, end of month.	Bank of Japan
New Zealand	Overnight Interbank Cash Rate, end of month.	Reserve Bank of New Zealand
Switzerland	Swiss Average Rate Overnight, end of month.	Swiss National Bank
United Kingdom	Daily Sterling Overnight Index Average Rate, end of month.	Bank of England
United States	Effective Federal Funds Rate, end of month.	Federal Reserve Economic Data
	Panel C. Long-term Interest Rates	
All	10-year bond yields published by OECD Main Economic Indicators, monthly average.	Datastream
	Panel D. Industrial Production	
All	Real industrial production (excluding construction) indices, seasonally adjusted, published by OECD. In cases where data is only available in quarterly frequency (Australia, New Zealand and Switzerland), the series are transformed to a monthly frequency using cubic spline interpolation.	Datastream
	Panel E. Consumer Prices	
All	Consumer Price indices published by OECD. Data is seasonally adjusted using the X-12-arima method if it seasonally adjusted data is unavailable. When data is only available in quarterly frequency Australia and New Zealand), the series are transformed to a monthly frequency using cubic spline interpolation.	Datastream
	Panel F. Financial Market Volaility	
World	The CBOE VIX published by the Chicago Options Board.	Datastream
	Panel G. World Economic Activity	
World	The World Trade Monitor published by the Central Planning Bureau of the Netherlands's Bureau for Economic Policy Analysis.	Datastream

Table A2: Source codes for the data

	Money Market Rates	Long-rates	Industrial Production	Consumer Prices	VIX	World Economic Activity
Australia	FIRMMCRID	AUOIR080R	AUOPRI35G	AUQCP009E		
Canada	10-10-0139-02	CNOIR080R	CNOPRI35G	CNMCP009E		
Euro Area	BBK01.ST0304	EOIR080R	EKOPRI35G	EMCPHARMF		
Japan	FM01'STRDCLUCON	JPOIR080R	JPOPRI35G	JPOCP009E		
New Zealand	INM.DN.NZK	NZOIR080R	NZOPRI35G	NZOCP009F		
Switzerland	EPB@SNB.zimoma	SWOIR080R	SWOPRI35G	SWOCP009F		
United Kingdom	IUDSOIA	UKOIR080R	UKOPRI35G	UKOCP009F		
United States	FEDFUNDS	USOIR080R	USOPRI35G	USOCP009E		
World					CBOEVIX	WDCPBPWWG

Appendix B Approximation Properties and Non-linear Taylor Rules

An important component in the construction of the interdependence index is the bivariate generalized normal distribution. This choice of the distribution to capture interest rate interdependence has many advantages, including the ability to derive analytically tractable expressions linking the joint distribution and higher-order comoments. However, for the generalized normal distribution to be sufficiently flexible to capture nonlinear channels linking interest rates, the higher-order comoments need to represent an effective approximation to the underlying non-linear processes determining interest rate comovements.

To demonstrate the ability of the bivariate generalized normal distribution to approximate nonlinearities linking global interest rates, the following simulation experiment is performed based on a non-linear Taylor rule (Petersen, 2007). The nominal interest rate of a country is specified as a linear function of its inflation rate and output gap as is commonly adopted in standard Taylor rule models, as well as a non-linear function of an international nominal rate. The parameters of the DGP used in the simulations are based on Canada as the home country with interest rate $r_{ca,t}$, and the U.S. as the foreign country with interest rate $r_{us,t}$. Interest rates are annualized and expressed as a percentage. The non-linear Taylor rule is specified as

$$r_{ca,t} = \alpha_0 + \alpha_1 \pi_t + \alpha_2 gap_t + \phi g_t + u_t, \tag{24}$$

where π_t is the percentage inflation rate and gap_t is the percentage output gap, and u_t is a disturbance term with zero mean and constant variance. The term g_t represents the smooth transitional autoregressive model of Teräsvirta (1994)

$$g_t = \frac{1}{1 + \exp\left(-\lambda\left(r_{us,t} - c\right)\right)}, \tag{25}$$

which captures the nonlinear channel connecting Canadian and U.S. interest rates.

The parameter values of the DGP are based on monthly data from December 1999 to December 2007, before the UMP policy shift. The correlation matrix of the variables is

$$\begin{array}{cccc} r_{ca,t} & \pi_t & gap_t & r_{us,t} \end{array}$$
$$\begin{pmatrix} 1.000 & 0.289 & 0.601 & 0.888 \\ 0.289 & 1.000 & 0.245 & 0.119 \\ 0.601 & 0.245 & 1.000 & 0.661 \\ 0.888 & 0.119 & 0.661 & 1.000 \end{pmatrix}, \tag{26}$$

where $\pi_t = 100 \log\left(P_t / P_{t-12}\right)$ is the percentage inflation rate with P_t as the Canadian CPI, and $gap_t = 100\left(y_{ca,t} - y_{hp,t}\right)/y_{hp,t}$, where $y_{ca,t}$ is monthly industrial production in Canada and $y_{hp,t}$ is the corresponding Hodrick-Prescott filter. The U.S. and Canadian interest rates are highly correlated with a correlation of 0.888. Both interest rates are also highly correlated with the Canadian output gap with correlations of 0.601

Table B1: Simulation experiments of a nonlinear Taylor rule based on equations (24) and (25). Bivariate generalized normal distribution parameters $\theta_i, i = 0, 1, \cdots, 5$ in (5) are estimated by maximum likelihood with QMLE standard errors in parentheses.

DGP Parameters			Bivariate generalized Normal Parameters					
ϕ	λ	c	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5
0.0	20.0	3.5	0.344	0.502	0.696	-3.660	-0.170	2.888
			(0.101)	(0.517)	(0.612)	(2.013)	(2.272)	(1.697)
1.0	20.0	3.5	0.687	0.271	0.485	-8.214	1.778	4.131
			(0.080)	(0.703)	(0.668)	(2.024)	(1.559)	(1.723)
1.0	20.0	6.0	0.417	0.132	1.327	-5.481	1.772	1.331
			(0.102)	(0.851)	(0.872)	(2.144)	(1.056)	(2.127)
10.0	20.0	6.0	0.801	-4.771	6.917	-3.793	2.876	-14.61
			(0.073)	(1.034)	(1.370)	(2.233)	(1.385)	(3.054)
10.0	1.0	6.0	1.166	-5.089	5.794	-5.004	0.954	-1.930
			(0.054)	(1.339)	(1.464)	(3.128)	(1.597)	(2.393)

(Canada) and 0.661 (U.S.), with the latter correlation interpreted as a measure of interdependence between international business cycles. The DGP parameters in (24) are $\alpha_0 = 2.885, \alpha_1 = 0.226, \alpha_2 = 0.404$, which are based on estimating the linear Taylor rule by setting $\phi = 0$ in (24). The parameters λ, c in (25) as well as ϕ in (24) are chosen to allow for alternative nonlinear transmission mechanisms connecting the two interest rates.

The experiments are performed by simulating the nonlinear Taylor rule model in equations (24) and (25) to generate a simulated series for the Canadian interest rate for alternative values of the parameters $\{\phi, \lambda, c\}$ which control the strength of the nonlinear channel connecting the interest rates, with the length of the simulated time series matching the length of the sample period of the actual data. For each choice of parameters in the DGP Table B1 gives the results of the simulation experiments with QMLE standard errors associated with the bivariate generalized normal parameters estimates. The first experiment is the case of a linear Taylor rule by imposing the restriction $\phi = 0$. The parameter estimates on the higher-order comoments in the bivariate generalized normal distribution are all statistically insignificant at the 5% level, consistent with the base case of no nonlinearities linking the two rates. The occurrence of a positive and significant parameter estimate on θ_0 is a reflection of the indirect dependence between the Canadian and U.S. interest rates arising from the correlation between the U.S. interest rate and the output gap in Canada given in (26) caused by the interdependence of international business cycles.

The second experiment in Table B1 allows for the nonlinear term g_t in (24) to affect the Canadian simulated interest rate by setting $\phi = 1.0$. The bivariate generalized normal parameter estimates of θ_3 (cokurtosis) and θ_5 (covolatility) are now statistically significant, providing evidence the higher-order comoments can capture the nonlinear structure of the model. Changing the c parameter from $c = 3.5$ to $c = 6$ in the next experiment results in cokurtosis, with the only significant higher-order comoment parameter being θ_3. In the fourth experiment, the strength of the nonlinearity is increased by raising the parameter ϕ to $\phi = 10$, which now results in all higher-order comoments being significant, including coskewness, cokurtosis and covolatility. For the fifth and final experiment, the λ parameter changes to $\lambda = 1$, so there is a sluggish adjustment between regimes, resulting in only the two coskewness parameter estimates now being significant.

Appendix C Complete Decomposition of Comoment Contributions to Shadow Rate Interdependence

Table C1: Decomposition of comoment contributions to interdependence between global and U.S. short-term shadow rates for selected periods. Row sums of point estimates equal 100%, with standard errors in parentheses.

Country	Covariance	Coskewness			Cokurtosis			Covolatility
	C_{11}	$S_{12}+S_{21}$	S_{12}	S_{21}	$K_{13}+K_{31}$	K_{13}	K_{31}	V_{22}
Panel A. Full sample (January 1999 to May 2020)								
Australia	58.277	1.221	1.231	−0.009	204.386	76.878	127.508	−163.884
	(13.709)	(1.278)	(1.408)	(0.624)	(53.511)	(21.592)	(32.369)	(46.001)
Canada	76.542	1.480	1.060	0.420	90.645	35.400	55.245	−68.667
	(6.317)	(0.935)	(3.172)	(2.795)	(36.701)	(16.939)	(21.097)	(33.629)
Euro	80.492	0.003	−0.001	0.004	−12.315	−12.708	0.393	31.821
	(46.360)	(0.199)	(0.082)	(0.181)	(30.172)	(30.826)	(0.915)	(76.482)
Japan	29.614	16.723	−1.669	18.391	124.639	110.421	14.217	−70.975
	(33.075)	(19.282)	(18.876)	(28.353)	(290.430)	(180.173)	(123.455)	(277.402)
New Zealand	19.117	11.651	1.153	10.497	245.597	102.456	143.140	−176.364
	(40.829)	(9.224)	(0.925)	(8.338)	(364.523)	(161.727)	(204.559)	(325.653)
Switzerland	68.369	0.438	0.048	0.390	−9.934	−6.138	−3.796	41.127
	(6.174)	(1.287)	(0.195)	(1.143)	(2.190)	(13.756)	(13.346)	(4.406)
United Kingdom	70.349	1.088	1.176	−0.088	127.743	62.431	65.312	−99.180
	(6.328)	(0.879)	(0.939)	(0.071)	(30.947)	(13.907)	(17.378)	(29.523)
Panel B. Pre-UMP (January 1999 to August 2008)								
Australia	77.386	4.813	0.330	4.483	114.780	29.534	85.246	−96.979
	(5.980)	(2.623)	(0.220)	(2.447)	(29.841)	(11.775)	(19.850)	(28.053)
Canada	88.336	5.246	−3.084	8.330	44.383	13.257	31.126	−37.965
	(3.407)	(2.170)	(1.783)	(3.769)	(26.919	(12.475)	(15.820)	(25.571)
Euro	87.957	Appendix −1.423 9	2.792		40.301 6.853	33.448		- 29.627
	(3.256)	(1.139)	(1.686)	(2.559)	(27.838)	(17.228)	(11.618)	(26.259)
Japan	−187.754	1.388	0.088	1.300	1821.241	882.714	938.527	−1534.874
	(1080.429)	(8.799)	(5.543)	(12.822)	(9398.996)	(4466.891)	(4936.016)	(8332.266)
New Zealand	50.398	1.707	−0.141	1.848	233.930	99.011	134.919	−186.035
	(32.341)	(1.798)	(0.296)	(2.002)	(117.441)	(47.969)	(69.862)	(101.562)
Switzerland	82.961	4.078	−2.396	6.474	17.096	−9.686	26.781	−4.135
	(6.120)	(2.986)	(2.494)	(5.332)	(60.326)	(32.469)	(31.969)	(58.026)
United Kingdom	45.587	6.637	−0.477	7.114	191.245	88.353	102.893	−143.469
	(33.242)	(3.222)	(0.367)	(3.466)	(102.542)	(45.336)	(57.719)	(75.109)
Panel C. UMP (September 2008 to May 2020)								
Australia	12.290	12.227	14.193	−1.966	305.828	121.104	184.724	−230.345
	(44.544)	(7.989)	(8.807)	(1.443)	(149.930)	(64.544)	(85.841)	(114.305)
Canada	41.671	18.925	27.449	−8.524	143.925	56.615	87.310	−104.520
	(33.044)	(6.520)	(8.558)	(2.420)	(128.314)	(59.967)	(68.848)	(100.710)
Euro	65.598	4.983	−1.954	6.937	−2.279	10.713	−12.992	31.699
	(9.549)	(6.712)	(2.779)	(9.184)	(12.826)	(25.014)	(19.017)	(16.879)
Japan	41.965	25.804	0.953	24.851	25.901	47.930	−22.029	6.330
	(18.047)	(21.611)	(10.573)	(25.459)	(43.322)	(59.502)	(29.558)	(63.569)
New Zealand	20.873	7.172	−0.423	7.595	45.006	1.327	43.679	26.950
	(24.252)	(8.142)	(0.469)	(8.581)	(127.919)	(50.117)	(92.481)	(111.381)
Switzerland	45.379	14.762	−0.228	14.990	5.080	10.405	−5.325	34.780
	(8.276)	(13.624)	(0.277)	(13.871)	(7.639)	(10.679)	(4.297)	(17.746)
United Kingdom	80.230	0.945	1.654	−0.709	92.448	42.092	50.356	−73.623
Netherlands's	(0.988)	(1.875)	(1.105)		(29.574)(12.794)	(17.211)		(29.308)

Appendix D Descriptive Statistics of Overnight Money Market and Long-term rates

Table D1: Descriptive statistics of monthly overnight money market rates for selected periods, January 1999 to May 2020.

Country	Mean	Min	Max	SD	Skew	Kurt	JB stat	P-value
				Panel A. Full Sample (January 1999 to May 2020)				
United States	1.918	0.040	6.860	2.009	0.889	2.470	36.878	0.000
Australia	3.926	0.140	7.250	1.782	−0.193	1.910	14.319	0.001
Canada	2.169	0.233	5.800	1.593	0.699	2.275	26.578	0.000
Europe	1.576	−0.458	5.160	1.722	0.406	1.748	23.851	0.000
Japan	0.080	−0.076	0.715	0.160	2.062	6.804	337.108	0.000
New Zealand	4.137	0.270	8.400	2.184	0.345	1.792	20.706	0.000
Switzerland	0.459	−0.786	3.490	1.118	0.966	3.059	39.990	0.000
United Kingdom	2.508	0.067	6.913	2.282	0.336	1.354	33.846	0.000
				Panel B. Pre-UMP (January 1999 to August 2008)				
United States	3.569	0.940	6.860	1.816	0.039	1.586	9.695	0.008
Australia	5.464	4.230	7.250	0.767	0.626	2.642	8.197	0.017
Canada	3.655	1.994	5.800	1.139	0.252	1.819	7.965	0.019
Europe	3.242	2.050	5.160	0.931	0.264	1.850	7.735	0.021
Japan	0.135	0.000	0.715	0.205	1.369	3.389	36.946	0.000
New Zealand	6.243	2.750	8.400	1.182	0.020	2.370	1.927	0.381
Switzerland	1.393	0.053	3.490	0.985	0.460	2.071	8.267	0.016
United Kingdom	4.896	3.042	6.913	0.872	−0.094	2.419	1.801	0.406
				Panel C. UMP (September 2008 to May 2020)				
United States	0.560	0.040	2.450	0.747	1.429	3.528	49.641	0.000
Australia	2.661	0.140	7.000	1.326	0.534	2.691	7.264	0.026
Canada	0.946	0.233	2.998	0.493	0.859	4.327	27.674	0.000
Europe	0.205	−0.458	4.173	0.729	2.570	12.157	647.811	0.000
Japan	0.036	−0.076	0.544	0.089	1.667	10.352	382.862	0.000
New Zealand	2.405	0.270	7.250	0.948	1.697	10.432	392.218	0.000
Switzerland	−0.309	−0.786	1.666	0.416	0.622	4.468	21.770	0.000
United Kingdom	0.544	0.067	5.091	0.566	6.139	44.772	11,136.855	0.000

Table D2: Descriptive statistics of monthly long-term rates for selected periods, January 1999 to May 2020.

Country	Mean	Min	Max	SD	Skew	Kurt	JB stat	P-value
Panel A. Full Sample (January 1999 to May 2020)								
United States	3.489	0.660	6.660	1.329	0.248	2.121	10.903	0.004
Australia	4.472	0.860	7.180	1.557	−0.575	2.101	22.814	0.000
Canada	3.372	0.550	6.490	1.507	0.152	1.779	16.955	0.000
Europe	3.277	0.050	5.700	1.531	−0.539	1.996	23.235	0.000
Japan	0.967	−0.280	2.120	0.639	−0.393	1.847	20.845	0.000
New Zealand	4.867	0.640	7.480	1.612	−0.632	2.277	22.694	0.000
Switzerland	1.671	−0.980	4.190	1.401	−0.157	1.689	19.474	0.000
United Kingdom	3.385	0.270	5.880	1.551	−0.315	1.630	24.363	0.000
Panel B. Pre-UMP (January 1999 to August 2008)								
United States	4.742	3.330	6.660	0.729	0.623	2.758	7.798	0.020
Australia	5.775	4.800	7.180	0.438	0.502	3.039	4.874	0.087
Canada	4.823	3.510	6.490	0.727	0.217	2.030	5.459	0.065
Europe	4.440	3.160	5.700	0.631	0.115	2.099	4.179	0.124
Japan	1.484	0.530	2.120	0.295	−0.839	4.053	18.987	0.000
New Zealand	6.234	5.230	7.480	0.434	0.492	2.721	5.050	0.080
Switzerland	2.956	1.930	4.190	0.545	0.212	2.397	2.624	0.269
United Kingdom	4.837	4.080	5.880	0.384	0.261	2.635	1.960	0.375
Panel C. UMP (September 2008 to May 2020)								
United States	2.459	0.660	3.850	0.655	0.081	2.809	0.368	0.832
Australia	3.400	0.860	5.800	1.311	0.145	2.206	4.196	0.123
Canada	2.178	0.550	3.670	0.742	0.350	2.251	6.174	0.046
Europe	2.320	0.050	4.660	1.385	0.148	1.518	13.419	0.001
Japan	0.542	−0.280	1.490	0.521	0.213	1.660	11.613	0.003
New Zealand	3.743	0.640	6.020	1.335	−0.070	2.305	2.955	0.228
Switzerland	0.614	−0.980	2.760	0.926	0.502	2.130	10.384	0.006
United Kingdom	2.191	0.270	4.580	1.049	0.389	2.112	8.197	0.017