Gazing at r^*: A Hysteresis Perspective

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June 27, 2022

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Introduction

- The decline in real interest rates (r^*) over the last 30 years has attracted considerable attention

- Potential explanations for such a pattern include:
  - × Lower growth
  - × Demographic factors
  - × Increased inequality
  - × Shortage of safe assets

- However, many market commentators and some researchers (ex: Bianchi et al. (2022), Borio at al (2017)) suggest that monetary policy (MP) may have played a role
  - × The fall in r^* coincides with implementation of inflation targeting regimes.

Introduction

- A common/plausible/reasonable response:
  - × This decline is a long-run (LR) real phenomenon
  - × Since money is neutral in the LR, MP unlikely to be relevant

- May not be that clear cut

Introduction

- Even if money is neutral in the LR

  1. Monetary policy can affect:
     - the set of feasible neutral real interest rate r^*
     - their stability
     - their basin of attraction

  2. Key element for result: More than one r^* that equates LR asset demand to LR asset supply

  3. Are multiple r^* plausible? If savings decisions influenced by both retirement saving motives and intertemporal substitution motives,
     - Gives rise to C-shaped LR asset demands when elasticity of inter-temporal substitution < 1
     - Making multiple equilibrium likely

- A multiple SS equilibria story requires an important "within" component in change in wealth holding

Steps in presentation

1. Examine the "within" versus "between" group break down of increase in wealth-to-income ratio and savings rates over the period of decreasing interest rates.

2. Show how the combination of inter-temporal substitution and retirement motives offers an explanation of this pattern based on C-shaped LR asset demand

3. Introduce households with such asset-holding motives into GE
   - × Look at equilibrium implications for r^* under flexible prices
   - × Then allow prices to adjust along Phillips curve dynamics
   - × Look at both case with and without valuation effects in effective asset supply

Related literature

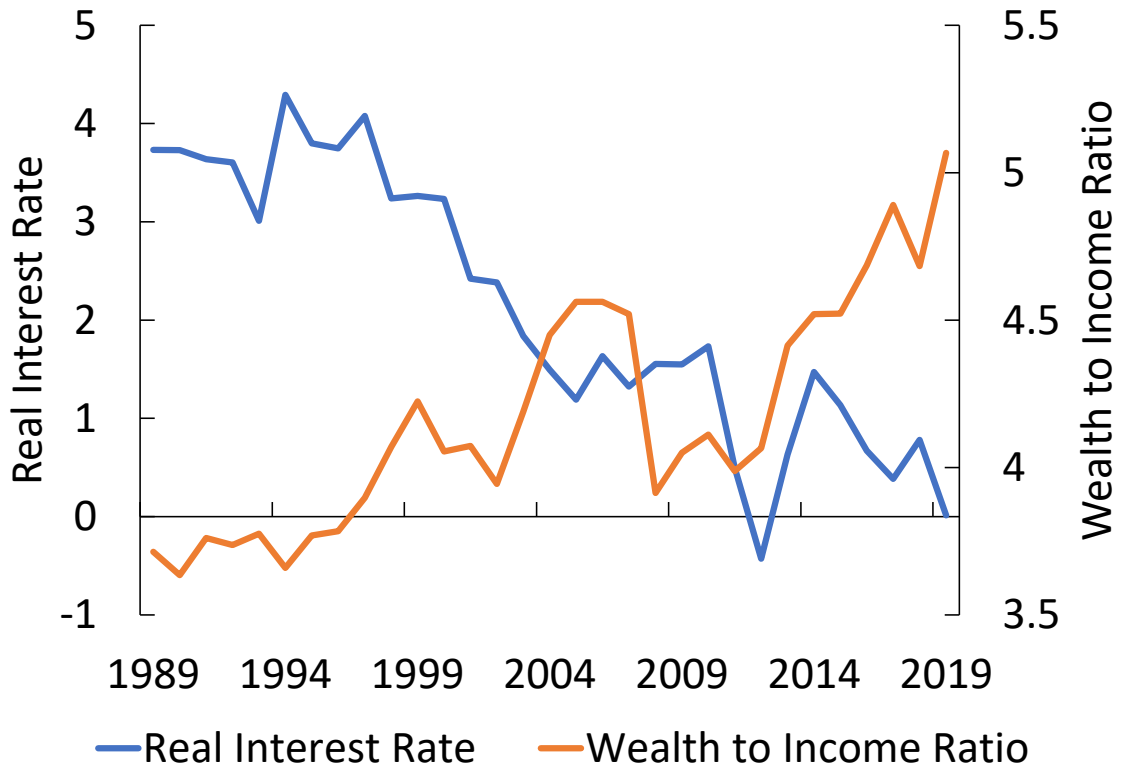
- **Large literature on sources of trend decline in real rates:**
  - × Demographics: Summers (2014); Eggertsson & Mehrotra (2014); Eichengreen (2015), Auclert, Malmberg, Martenet & Rognlie (2021)
  - × Productivity slowdown: Gordon (2017)
  - × Global saving glut and/or shortage of safe assets: Bernanke et al. (2005); Caballero, Farhi & Gourinchas (2008)
  - × Rise in inequality: , Mian, Straub & Sufi (2021a,b); Fagereng, Holm, Moll & Natvik (2019), Rachel & Smith (2017)
  - × Decline in desired investment: Rachel & Smith (2017)

- **Multiple equilibria with Taylor rule:** Benhabib, Schmitt-Grohé & Uribe (2001, 2002); Michaillat & Saez (2018)

- **OLG:** Gertler (1999); Blanchard(1985); Yaari (1965)

1. Decomposing changes in household wealth-to-income ratios in the US: 1989-2019

- Wealth-to-income ratios rose significantly over last 30 years
- Coincide with a period of declining interest rates



Last observation: 2019

Decomposition of change in total wealth-to-income ratios

Shift share decomposition: within & between

- SCF data: 1989-2019
- 30 household groups (5 age groups $\times$ 6 income groups)

Results

Definition	Total Change	Within (%)	Between (%)
Wealth (baseline)	2.82	61.6	38.4
Wealth less housing	2.65	61.4	38.6

- *Within* component accounts for about 60% of the change; *Between* around 40%

Within group savings

- Two potential interpretations of within group's importance for the increases in w/y:
  1. Increases in desired wealth holdings due to low expected returns on assets
  2. Increases in wealth due to unanticipated valuation effects
- To discriminate between these 2 possibilities, we look at changes in within group saving patterns over same period.
- Wealth-based synthetic saving approach
  - × approximates saving by each group by netting out valuation effects from their changes in wealth between 2 SCF waves

Within group saving (2)

- Small positive correlation between changes in saving rates and within group changes in w/y ratios:
  $Corr(\Delta s/y, \Delta w/y) = 0.16$
- Groups that faced greater increases in w/y ratios do not appear to systematically reverse this accumulation by decreasing their saving rates
- Support notion that increases in within group w/y ratios are likely reflecting changes in desired wealth holdings as opposed to excess wealth holdings

2. Why may households want to hold more assets when asset returns are lower?

- The economy is populated by a continuum of OLG (similar to Blanchard-Yaari, Gertler)
- Household can be in 3 states: active, retirement and dead
- Household starts in the active state and transits out of this state with instantaneous probability δ_1
  - × This shock can be seen as a health shock
- At this transition, with probability q the person retires; with probability $1 - q$, health shock is severe and the person dies
- If a person retires, he/she will die with probability $\delta_2 \geq \delta_1$

Retiree's decision problem

$$V_t \equiv \int_0^{\infty} e^{-(\delta_2+\rho)\tau} \frac{c_\tau^{1-\sigma_2}}{1-\sigma_2} d\tau,$$

subject to $\dot{a}_t = r_t a_t - c_t$

- Using the Euler equation of the retiree's problem, we have

$$V_t = V(a_t, \Gamma_t) = \frac{a_t^{1-\sigma_2}}{1-\sigma_2} [\Gamma_t]^{\sigma_2}$$

- Function of the whole future path of interest rates as captured by Γ.
- When interest rates are constant

$$V_t = V(a_t, r) = \frac{a_t^{1-\sigma_2}}{1-\sigma_2} \left[\frac{1}{\frac{\rho+\delta_2}{\sigma_2} - \frac{1-\sigma_2}{\sigma_2} r_t} \right]^{\sigma_2}$$

- **Lemma 1:** At fixed r, marginal value of assets to retiree, given by $V_{ar} < 0$, is falling in r when $\sigma_2 > 1$

Active household's decision problem

$$\int_0^{\infty} e^{-(\delta_1+\rho)t} \left[\frac{c_t^{1-\sigma_1}}{1-\sigma_1} + \delta_1 q V(a_t, \Gamma_t) \right] dt, \qquad \sigma_1 \leq \sigma_2$$

subject to

$$\dot{a}_t = y_t - c_t$$

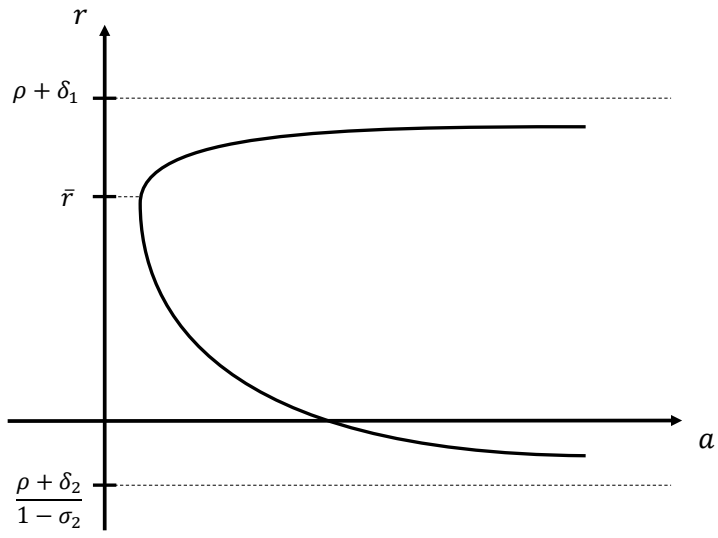
- $y_t = w_t + r_t a_t - T_t$: Total disposable income, w_t: Labor income, T_t: Lump sum taxes
- Euler equation of active households

$$\frac{\dot{c}_t}{c_t} = \underbrace{\frac{r_t - \rho - \delta_1}{\sigma_1}}_{\textit{Substitution effects}} + \underbrace{\frac{c_t^{\sigma_1}}{\sigma_1} \delta_1 q V_a(a_t, \Gamma_t)}_{\textit{Income effects}}$$

For fixed r, C-shaped LR asset demands

$$a^{a,ss}(y,r) = (\delta_1 q)^{\frac{1}{\sigma_2}} \left[\frac{\rho+\delta_2}{\sigma_2} - \frac{1-\sigma_2}{\sigma_2} r\right]^{-1} [\rho+\delta_1-r]^{\frac{-1}{\sigma_2}} y^{\frac{\sigma_1}{\sigma_2}}$$

- Two possibilities. When $\sigma_2 < 1$, then monotonically increasing in r
- When $\sigma_2 > 1$, C-shaped LR asset demand. (C-shaped asset to income ratio)

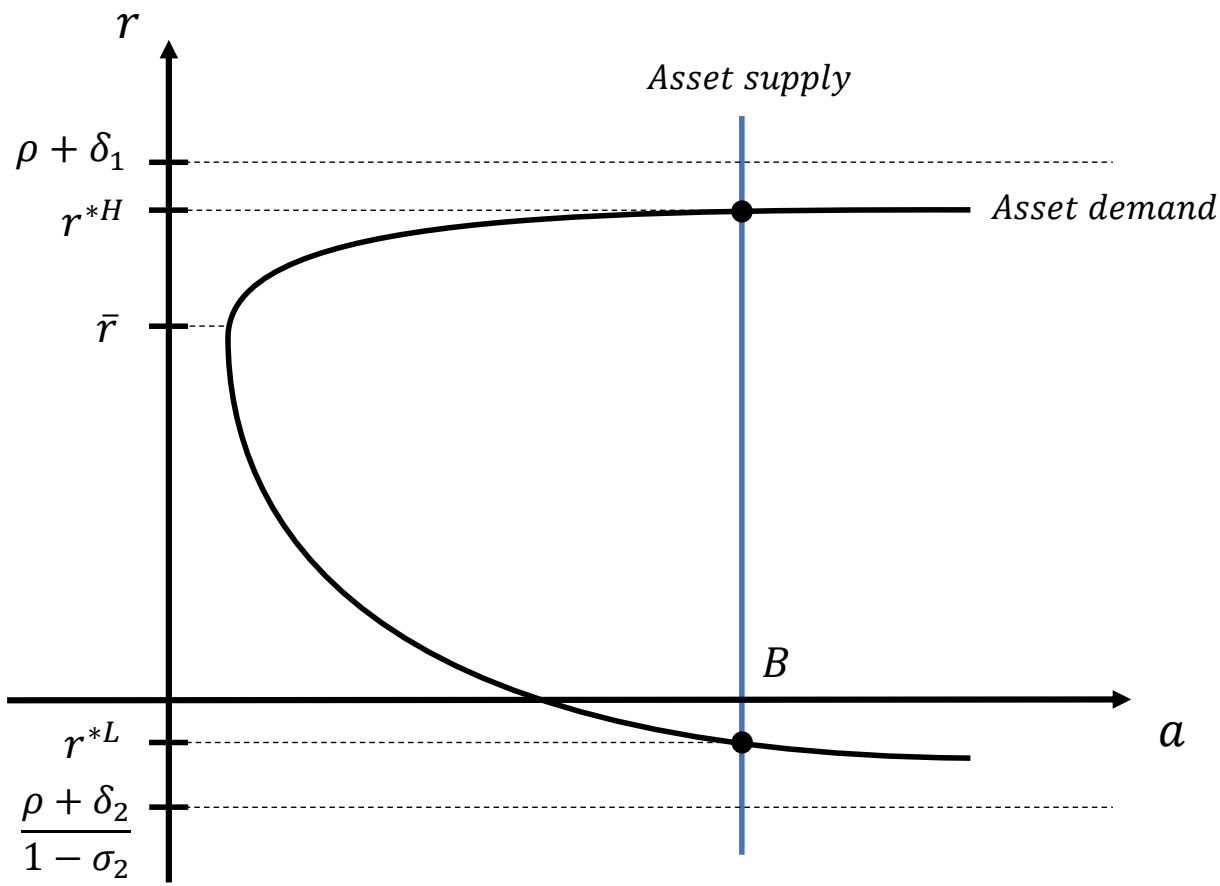


3. General equilibrium implications

- Embed OLG economy populated by active and retired households with such preferences in an economy with
  - × government that spends, taxes and issues bonds: $\phi T_{1t} = G + r_t B$
  - × central bank that sets interest rates
  - × focus on $\sigma_1 = \sigma_2 \equiv \sigma > 1$
- Start with flexible prices and short term bond: multiple SS equilibria
- Then move to nominal frictions and Phillips curve dynamics with a standard constrained Taylor rule
- Look at fiscal policy (and inflation shocks)
- Extend the model to include valuation effects (Lucas tree)

Asset market equilibrium with only ST bonds

- Unique equilibrium is impossible. At least two SS equilibrium real rates: $r^{*H} > \bar{r} > r^{*L}$



Introducing nominal rigidities and Phillips curve

- Nominal rigidities introduce in manner that gives rise to Vertical LR Phillips curve:
- $\dot{\pi}_t = \kappa(y_t - \bar{y})$
  - × Focus on case $\kappa > 0$, with π_t state variable
- MP follows a Taylor rule satisfying Taylor principle:

$$i_t = \max\left\{0, r^{*H} + \pi^T + \psi(\pi_t - \pi^T)\right\} \qquad \psi > 1$$

Introducing nominal rigidities and Phillips curve

- Equilibrium dynamics

$$\frac{\dot{c}_t}{c_t} = \frac{i_t - \pi_t - \rho - \delta_1}{\sigma} + \frac{c_t^{\sigma}}{\sigma}\delta_1 q^s V_a(B, \Gamma_t)$$

$$\dot{\pi}_t = \kappa(c_t + G - \bar{y}) \qquad \kappa > 0$$

$$i_t = \max\left\{0, r^{*H} + \pi^T + \psi(\pi_t - \pi^T)\right\} \qquad \psi > 1$$

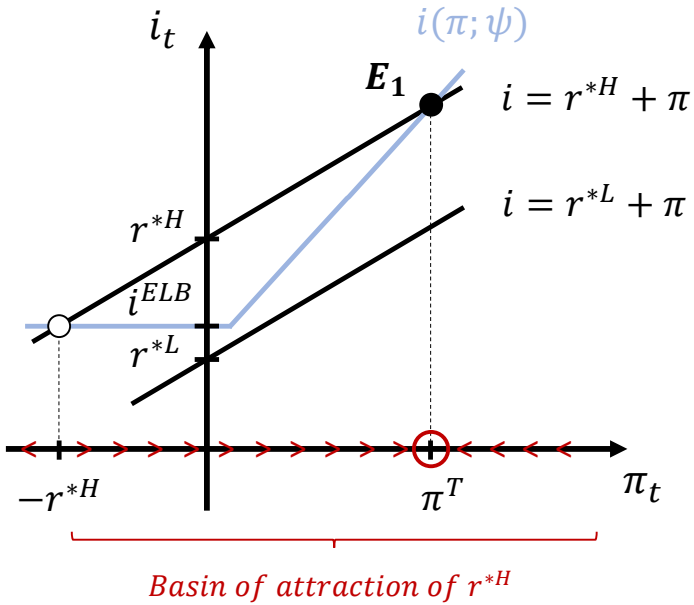
$$\dot{\Gamma}_t = -1 + \Gamma_t\left[\frac{\rho + \delta_2}{\sigma} - \frac{1-\sigma}{\sigma}(i_t - \pi_t)\right]$$

- Gives rise to cut off inflation: $\pi^{ELB} \equiv \frac{(\psi-1)\pi^T - r^{*H}}{\psi}$, rising in ψ

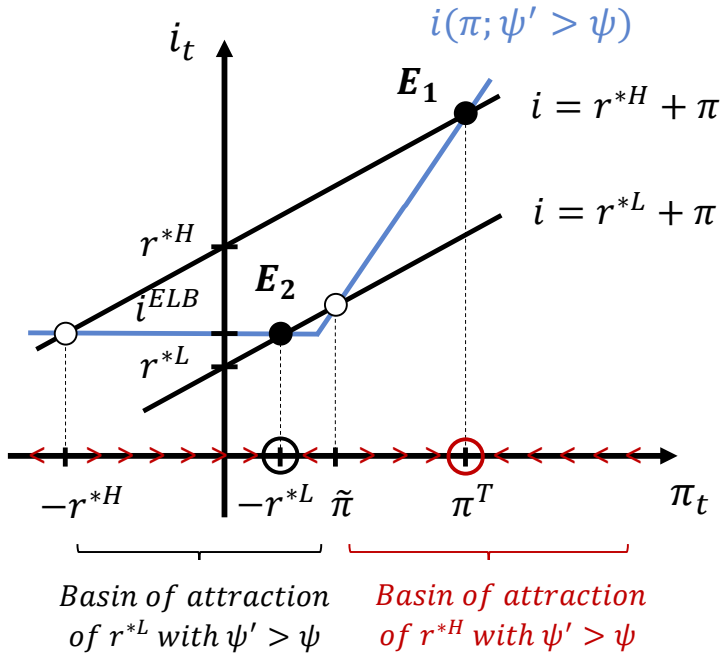
- ψ does not affect r^{*H} and r^{*L} (MP neutral in LR)

Two r^* and monetary policy

Panel A

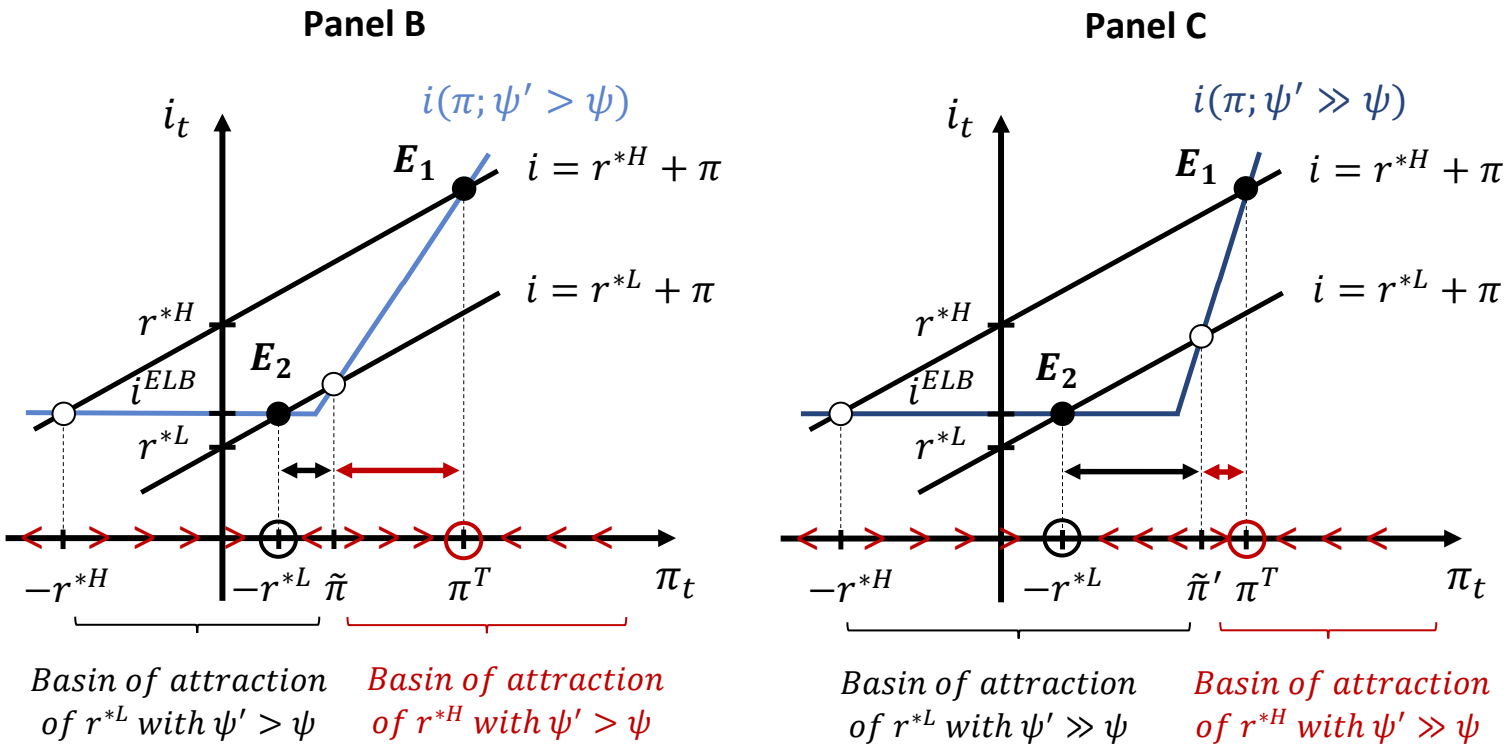


Panel B



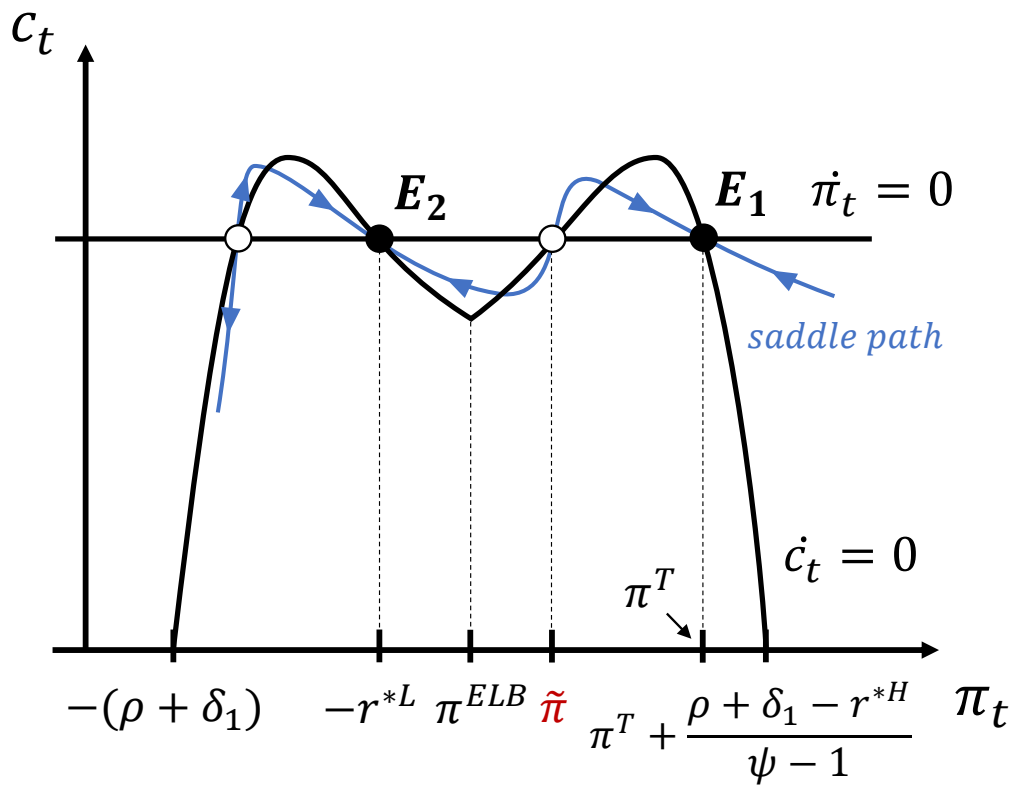
Fragility of high r^* with aggressive Taylor rule

- As $\psi \uparrow$, basin of attraction of r^{*H} eqm gets smaller ($\tilde{\pi} \uparrow$); basin of attraction r^{*L} eqm gets larger



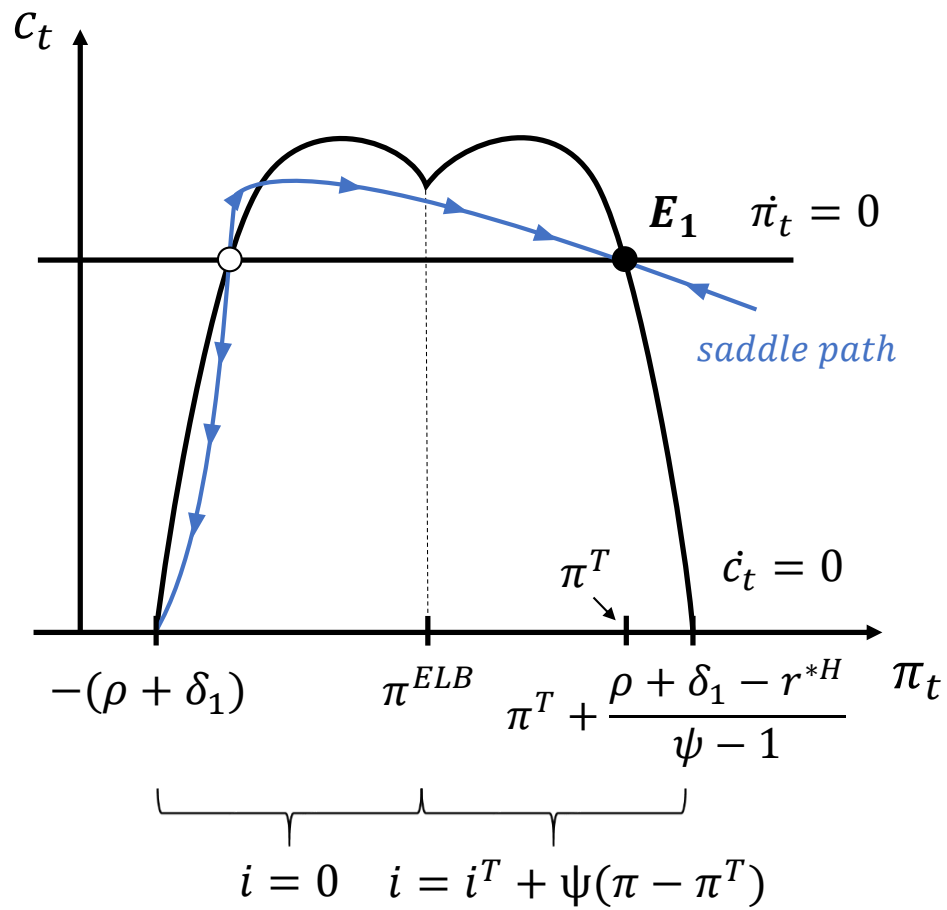
Inflation and consumption

- Aggressive Taylor rule: $\psi > \bar{\psi} > \mathbf{1}$
- Now two FE stable equilibria: E_1 and $\mathbf{E_2}$ $\Rightarrow$ Hysteresis
- $\tilde{\pi} = \pi^{\mathbf{T}} - \frac{\mathbf{r^{*H}} - \mathbf{r^{*L}}}{\psi - \mathbf{1}}$



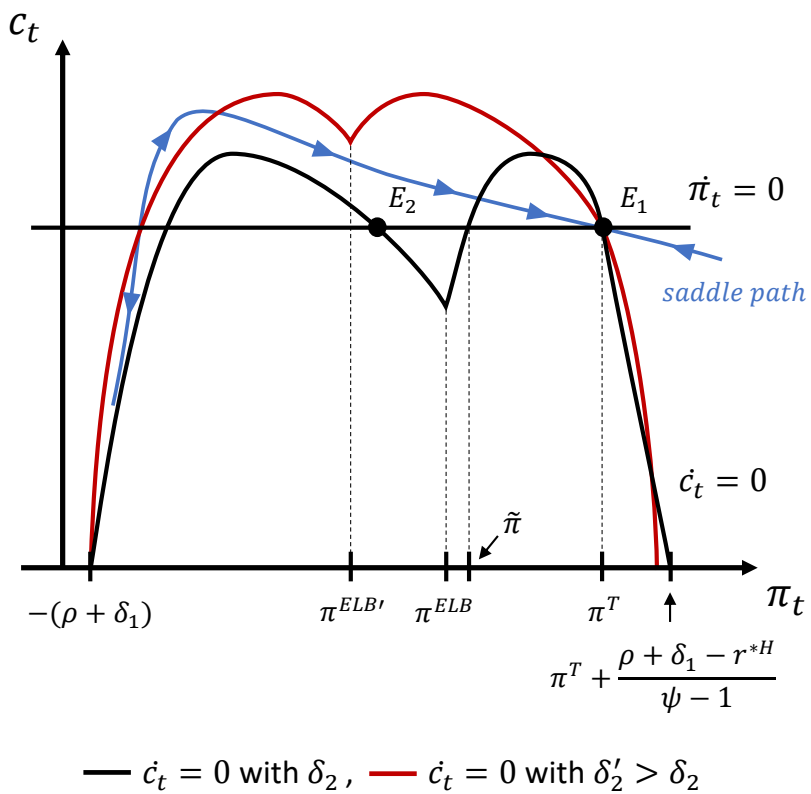
Inflation and output under minimal aggressive Taylor rule

- Not very aggressive Taylor rule: $\mathbf{1} < \psi < \bar{\psi} \equiv \frac{\mathbf{r^{*H}} + \pi^{\mathbf{T}}}{\mathbf{r^{*L}} + \pi^{\mathbf{T}}}$
- Only one full employment (FE) stable equilibrium: $\mathbf{E_1}$
- Similar to Benhabib, Schmitt-Grohé, & Uribe (2001, 2002)



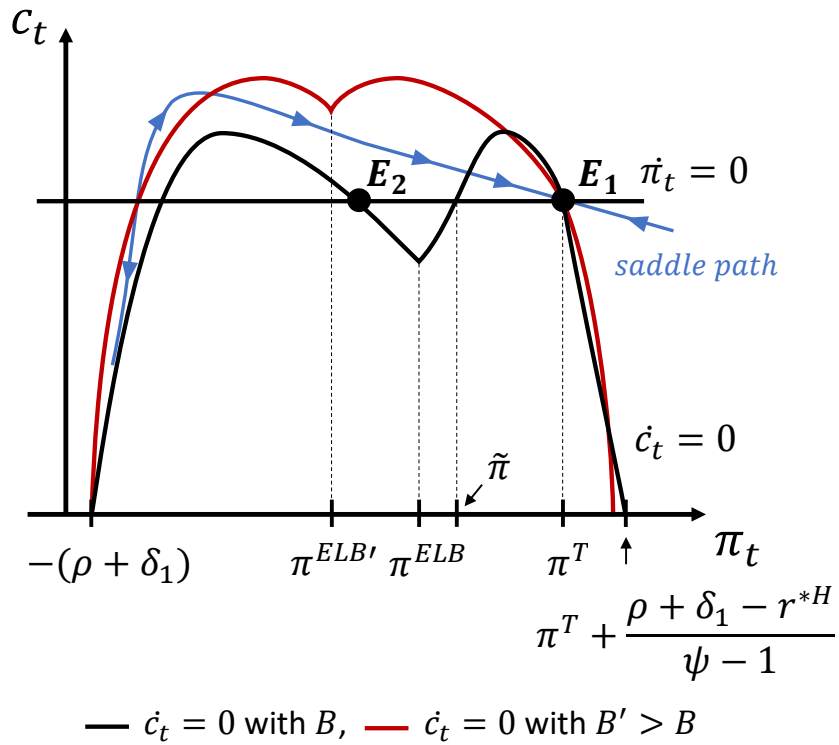
Complementarity between real factors and monetary policy

- Need sufficiently aggressive Taylor rule for existence of low-real-rate, low-inflation trap
- Complements real factors: For a given ψ, δ_2 needs to be sufficiently small for low-real-rate, low-inflation eqm.



Exiting low-real-rate: effects of expansionary fiscal policy

- Higher debt can make the low-real-rate, low-inflation eqm disappear, but LR inflation would jump



4. Extending to include valuation effects:Lucas trees:

- Steady state asset price: $z = \frac{f}{r+\omega}$
- Effective supply of asset with valuation effects: $\Omega = B + sz$
- Desired LR consumption-to-wealth ratio (same as previously)

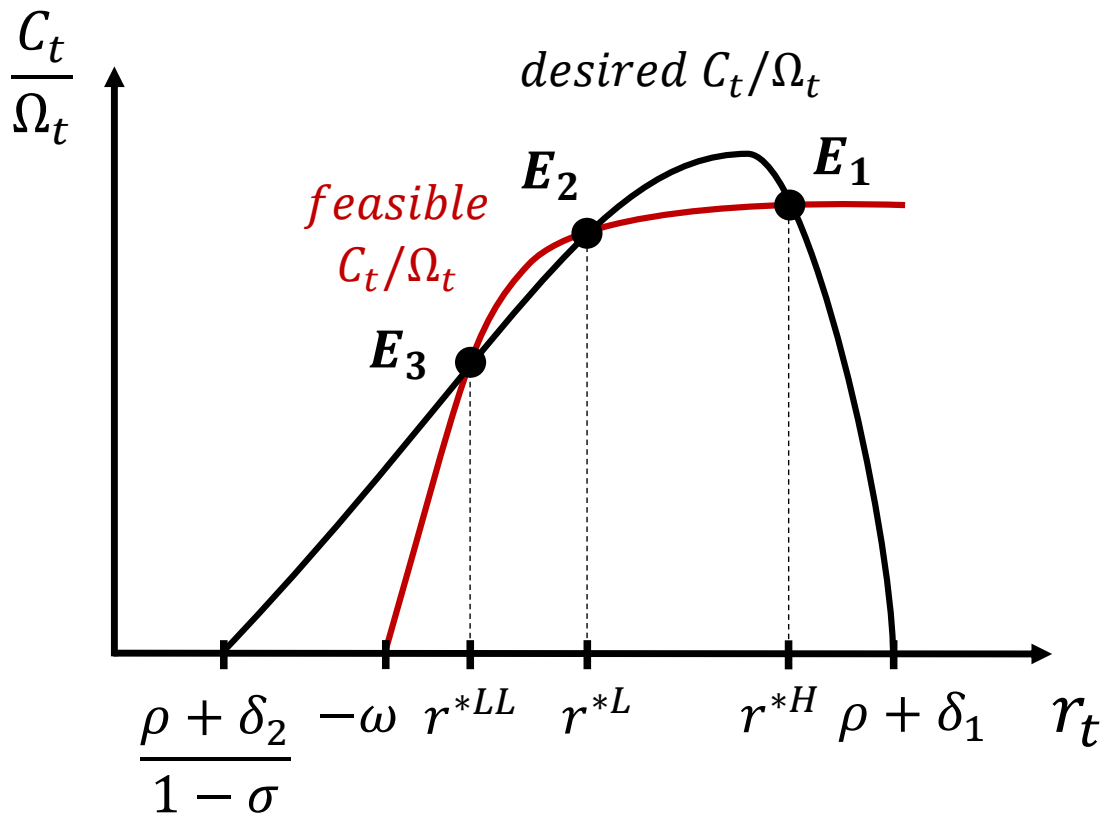
$$\frac{c}{\Omega} = (\delta_1 q^s)^{-1/\sigma} (\rho + \delta_1 - r)^{1/\sigma} \left[\frac{\rho + \delta_2}{\sigma} - \frac{1-\sigma}{\sigma} r \right]$$

- Feasible LR consumption-to-wealth ratio

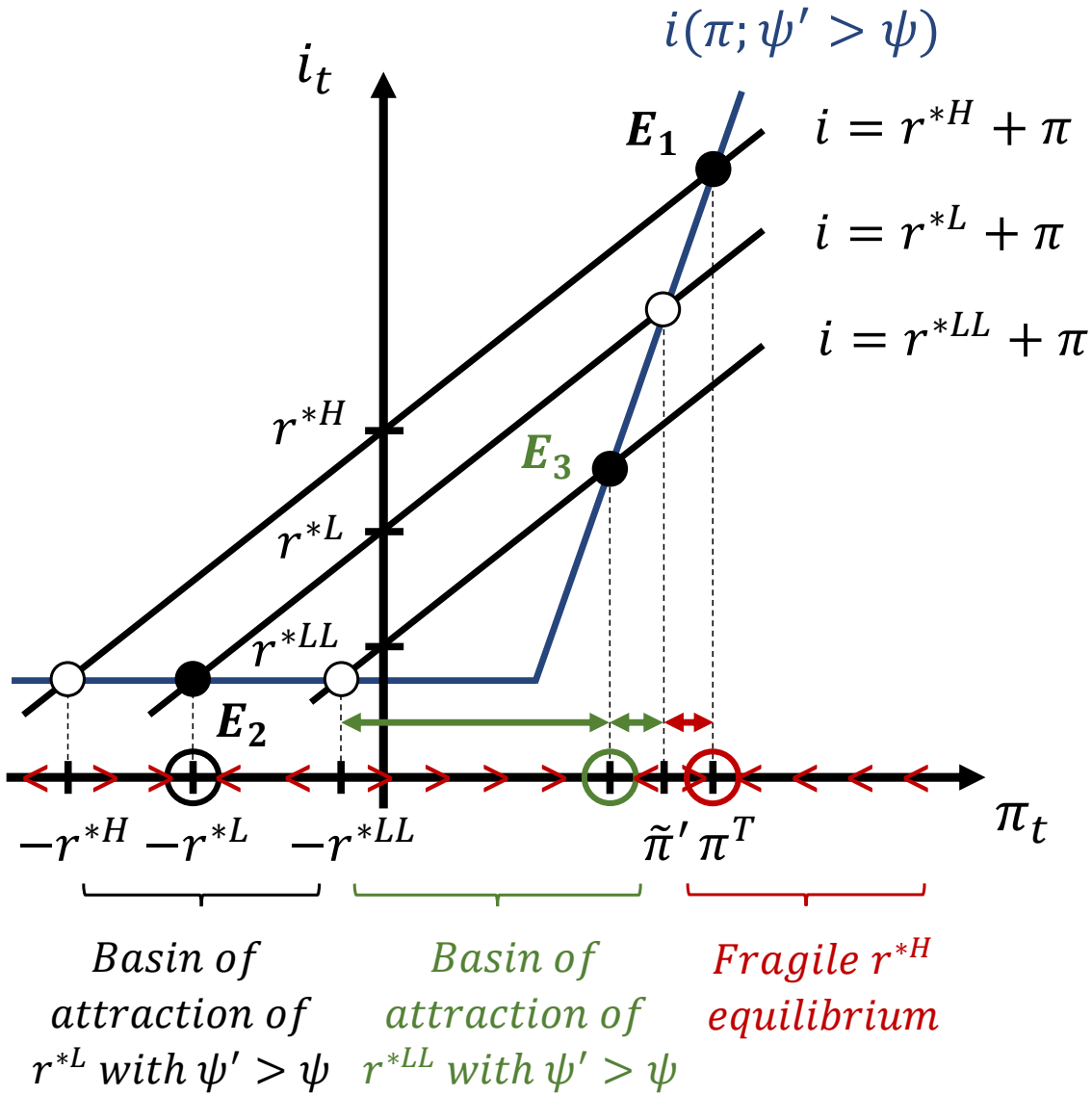
$$\frac{c}{\Omega} = \frac{\bar{y} + sf - G}{B + \frac{sf}{r+\omega}}$$

Extending to include valuation effects: Lucas trees:

- 3 potential r^* : $r^{*LL} < r^{*L} < r^{*H}$
- New equilibrium E_3: lowest r^{*LL} and low c/Ω. Possible because of valuation effects



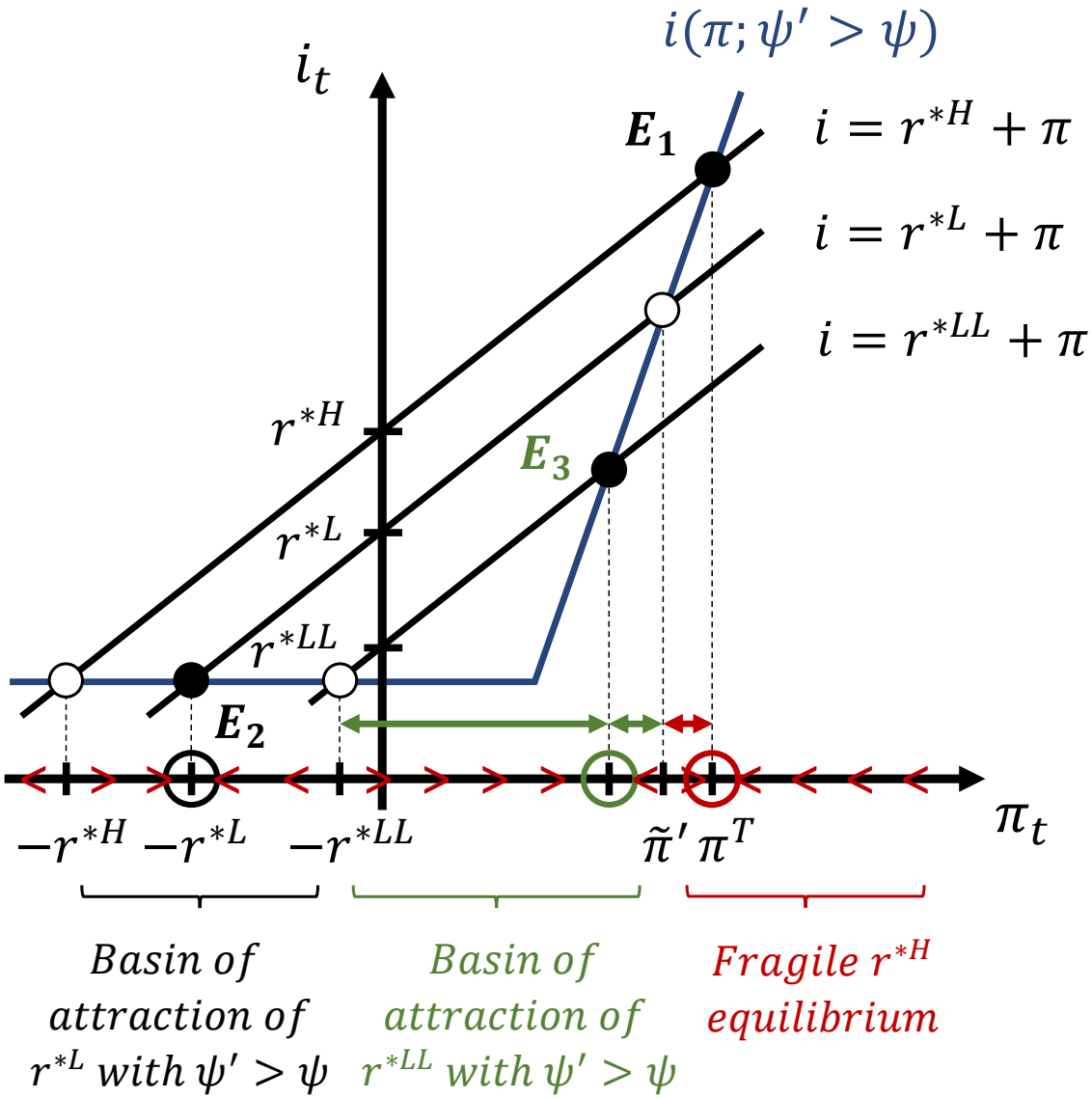
Inflation and interest rates with valuation effect



How an inflation shock could increase long term real rates

- Start at the low-real-rate, low-inflation equilibrium E_2
- Suppose there is unexpected shock to Phillips Curve equation that causes a discrete jump in inflation above π^T
- The central bank could increase nominal interest rates aggressively, causing real rates to rise too.
- This could place economy temporarily in recession in order to reduce inflation.
- As inflation declines and employment recovers, interest rates – both real and nominal – gradually fall.
- But, economy would not return to E_2. Instead, it would converge to SS eqm E_1 with high real rate (hysteresis).
- Hence, when economy is at E_2 and there is a large inflation shock, this can cause the LR real rate to rise from r^{*L} to r^{*H}.

The effects of an inflation shock



Conclusion

- When thinking of r^* we generally focus on slow moving forces such as demographics, productivity and income inequality.
- In such a case, we can debate whether the past trend could soon reverse itself, but this would be slow moving and unlikely to be affected much by the current crises.
- What this paper suggests: the economy could throw another curve ball.
  - × If there is more than one r^* –due to C-shaped demand for assets– then a reversal of the past trend for r^* could arise in a much more surprising and endogenous fashion.
  - × Increased debt could lead to a discontinuous jump in r^*
  - × A large inflation shock could move the economy away from a low r^* basin of attraction, to a high r^* basin of attraction.

Conclusion

- In terms of data, the observation of a substantial "within" component in increased asset holdings over the period places doubt on assets demands that are monotonically increasing in returns.
- We have presented one structure which is consistent with such "within" forces based on the competing roles of inter-temporal substitution and retirement motives in saving decisions, and he have explored implications.
- Other interpretations of such observation are certainly possible (ex: Mian, Straub and Sufi (2021)). Conjecture: most explanations will likely open the door to multiple r^* and the role of monetary policy in affecting LR outcomes.