OPTION PRICES AND IMPLIED VOLATILITIES: AN EMPIRICAL ANALYSIS

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ABSTRACT

This paper investigates the efficiency of Australian options markets using a version of the Black-Scholes model. Under the joint null hypothesis that the pricing model is valid, and that forecasts are efficient, the implied volatilities calculated from observed option prices should be efficient predictors of squared changes in the prices of the underlying instruments. This hypothesis is tested using weekly data on prices of Australian financial futures options, and over-the-counter currency options. The results indicate significant forecasting biases for each of the contracts studied. In each case, implied volatilities appear to overpredict changes in the true volatility of underlying prices. Although these conclusions are conditional on the validity of the pricing model used to calculate implied volatilities, our evidence suggests that biases in the Black-Scholes formula are unlikely to explain fully the apparent forecast biases.

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1. Introduction

Empirical analysis of option prices has focussed on two related but logically distinct questions. The first is concerned with discriminating between alternative pricing models. The widely used Black-Scholes model has the attraction of being both mathematically rigorous and relatively simple to use, since it specifies option values as a closed function of a small number of parameters which can be readily observed or estimated. Its validity, however, depends on a number of restrictive assumptions concerning the stochastic processes generating prices of the underlying assets. In particular, it assumes that asset prices follow diffusion processes with constant variances, and this assumption is thought to be unrealistic in many contexts. The Black-Scholes model has been generalised in a number of important directions to allow for a wider range of generating processes permitting, for example, price discontinuities and time-varying volatilities. A number of studies have focussed on the performance of such models relative to Black-Scholes in explaining observed option prices.

A second question concerns the accuracy with which market participants estimate the parameters needed to implement the option pricing formulas. Efficient markets theory hypothesises that the market's estimates of these parameters may be found to be statistically optimal, in the sense that they cannot be improved upon using any information available at the time the expectations are formed. This hypothesis is directly testable, conditional on assumptions about the appropriate pricing model. In the case of the Black-Scholes model, for example, the parameter of prime importance is the expected variance of the underlying asset price, and given the Black-Scholes assumptions, market estimates of this parameter can be inferred from observed option premiums. Forecast efficiency can thus be tested by comparing these implied volatilities with actual price volatilities observed over the subsequent life of the option.

These two empirical approaches are of course complementary, each assuming one part of the joint hypothesis in order to test the other. The present study falls within the second category, and is aimed specifically at testing the efficiency of volatility expectations implied in prices of Australian futures and currency options. We know of no earlier study which examines these particular options markets in Australia. For futures options, the study uses the Black-Scholes formula as modified by Black (1976) to obtain time series for implied volatilities; for currency options, the Garman-Kohlhagen (1983) version is used. The study derives testable implications relating these implied volatilities to subsequent price movements in the underlying instruments. In doing so, it follows an approach similar to that used in a number of earlier studies using data on U.S. stock options, for example, Schmalensee and Trippi (1978), Latane and Rendleman (1976) and Chiras and Manaster (1978), as well as more recent work by Shastri and Tandon (1986) on currency options. This work has generally found evidence against the hypothesis of forecast efficiency, although the issue remains unclear because of the conditional nature of the hypothesis tests. The present study aims to provide comparable evidence using data on Australian futures and currency options, and will also attempt to test the robustness of the statistical results by examining whether a hypothetical trading rule, aimed at exploiting apparent forecast inefficiencies, generates significant excess returns during the sample period.

Section 2 of the paper derives the tests of forecast efficiency to be used in the empirical work. Section 3 then discusses the data used and section 4 presents the main empirical results. Section 5 reports on an examination of within sample excess returns using a hedged trading strategy based on the estimated volatility predictions. Some conclusions are offered in section 6.

2. A Test of Forecast Efficiency

(a) Futures Options

The Black model for pricing options on futures contracts may be written in the following form:

$$c = c(f, x, t^*, \sigma) \qquad (1)$$

$$p = p(f, x, t^*, \sigma) \qquad (2)$$

where c is the price of a call option
p is the price of a put option
f is the futures price at time t
x is the option exercise price
σ is the standard deviation of the futures price
t^* is the time to expiry.[1]

The model assumes that both variances and interest rates are non-stochastic, and that the options cannot be exercised before expiry. Recent theoretical work by Ramaswamy and Sundaresan (1985), Schaeffer and Schwartz (1987) and Hull and White (1987) has begun to quantify the effects on option values when these assumptions are loosened; generally speaking the effects appear small when options are near the money or are relatively close to expiry. For example, the authors cited above compute pricing biases of the order of between zero and 1 per cent in Black-Scholes prices for at-the-money calls when the assumptions are violated. This may be compared with the size of discrepancies arising from likely errors in forecasting volatility. As an illustration, a 1 percentage point prediction error in estimating volatility on a one year option with true volatility of 0.1, would produce a mispricing of at-the-money calls of the order of 10 per cent under the Black-Scholes formula; such a prediction error would appear quite plausible when compared with observed historical variation in implied volatilities. These magnitudes, and casual observation, suggest that beliefs about volatility are likely to be much more important in determining actual option prices than beliefs about what is the appropriate pricing model, especially for options that are near the money. On this basis, we believe that an empirical focus on forecast accuracy rather than model accuracy is not unwarranted. It remains true, however, that the empirical results must be interpreted as being conditional on the assumption that Black-Scholes is a good approximation to the "true" formula.

All of the parameters of the Black model are readily observed in historical series apart from σ, which market participants are assumed to estimate. Given the option price, market estimates of σ can therefore be inferred by

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1. Interest rates are deleted from the formula on the basis that futures options are purchased on margin with no interest opportunity cost. The exact formulas used are:

$$c = fN(d_1) - xN(d_2)$$

$$p = -fN(-d_1) + xN(-d_2)$$

where $$d_1 = \frac{\log(f/x)}{\sigma\sqrt{t^*}} + \frac{1}{2}\sigma\sqrt{t^*}$$

$$d_2 = d_1 - \sigma\sqrt{t^*}$$

numerically solving equation (1) or (2). Cox and Rubinstein (1985) show that, provided conditional variances are non-stochastic, the implied values of σ^2 taken at any time t can be interpreted as annualised conditional variances of the log expiry price. We can therefore write:

$$(T - t)\sigma^2(t, T) = \text{var}_t(\log f_T) \tag{3}$$

where the notation $\sigma^2(t, T)$ denotes the implied variance observed at t for an option expiring at T, expressed as an annual magnitude. Defining $E_{t+i}(\log f_T) - E_{t+i-1}(\log f_T) \equiv \varepsilon_{t+i}$, we have

$$(T - t)\sigma^2(t, T) = E_t(\sum_i \varepsilon_{t+i})^2 = \sum_{i=1}^{T-t} E_t(\varepsilon^2_{t+i}). \tag{4}$$

The cross product terms in the above expression are eliminated by the rationality requirement that future revisions to forecasts are not correlated with current information at any point. Empirically, some measure of the innovation terms will be needed, and this paper uses the assumption that

$$E_{t+i}(\log f_T) = \log f_{t+i},$$

which implies that

$$\varepsilon_{t+i} = \log f_{t+i} - \log f_{t+i-1}.$$

A theoretical justification for the above assumption is provided by Samuelson (1965), and strong empirical support is provided in an earlier study, Edey and Elliott (1988), which uses the same data set as the present paper.

By leading equation (4) one period, we also have

$$(T-(t+1))\sigma^2(t+1, T) = \sum_{i=2}^{T-t} E_{t+1}(\varepsilon^2_{t+i}).$$

Taking expectations at time t:

$$E_t(T-(t+1))\sigma^2(t+1, T) = \sum_{i=2}^{T} E_t(\varepsilon^2_{t+i}).$$

This can be substituted into equation (4) to give

$$(T-t)\sigma^2(t, T) = E_t\{\varepsilon^2_{t+1} + \sum_{i=2}^{T} (\varepsilon^2_{t+i})\}$$

$$= E_t\{\varepsilon^2_{t+1} + (T-(t+1))\sigma^2(t+1,T)\}. \tag{5}$$

Equation (5) is a linear prediction equation relating current implied volatility to expected outcomes on observable variables realised in the next period. Standard efficient markets principles can be used to derive testable restrictions for this equation. In particular, if the current value of σ^2 is an efficient predictor, then significant predictive power should not be added by the inclusion of further information dated t or earlier.

This suggests a test of forecast efficiency using an equation of the form

$$(T-(t+1))\sigma^2(t+1, T) + \varepsilon^2_{t+1} = (T-t)\alpha + \beta(T-t)\sigma^2(t, T) + (T-t)\gamma Z_t + v_{t+1} \tag{6}$$

where Z_t is a vector representing information observable at t. The additional extraneous regressors are premultiplied by (T-t) to ensure that they always have the same order of magnitude as the quantity being forecast. In the tests reported in section 4, Z_t is taken to include current and past values of ε^2_t. Under the null hypothesis, $\alpha = \gamma = 0$, and $\beta = 1$.

The interpretation of equation (6) is fairly straightforward. Under the efficient markets hypothesis, the current estimate of volatility remaining over the life of the option should be an optimal predictor of realised volatility, expressed as the appropriately weighted sum of next period's estimate and the realised squared price innovation in the next period. Although equation (6) is similar in spirit to equations tested by other researchers in this context, the exact linear predictive relationship used here has not to our knowledge been previously noted.

As an aside we note that a possible problem with the proposed test is that if the σ^2 time series is nonstationary, the estimated β coefficient would be biased towards one, since the nonstationary magnitude appears on both sides of the regression equation. However the statistical results strongly suggest that the σ^2 series are stationary.

(b) Currency Options

Although options on currency futures are traded on the Sydney Futures Exchange, the most active currency options market in Australia is in over-the-counter options, traded mainly in the interbank market. There is an important difference in the expiry date conventions as between the two markets. In futures options, only a small number of standard expiry dates are used (coinciding with futures expiry dates), so that a time series of data can be used to obtain a series of observations pertaining to the same expiry date. In over-the-counter options, the main indicator rates are for standard periods of time ahead of the trading date; thus a time series of data will show a series of options with equal time to expiry. In this case the algebra for deriving a testable equation is somewhat simplified because there is no need to "accumulate" variances in the way used above. The equivalent of equation (3) in this case is

$$k\sigma^2(t,\ t+k) = E_t\Big(\sum_{i=1}^{k} \varepsilon_{t+i}\Big)^2 \tag{7a}$$

$$= E_t \sum_{i=1}^{k} (\varepsilon_{t+i}^2), \tag{7b}$$

leading to the regression equations

$$\Big(\sum_{i=1}^{k} \varepsilon_{t+i}\Big)^2 = \alpha + \beta k\sigma^2(t,\ t+k) + \gamma Z_t + v_{t+k} \tag{8a}$$

or

$$\Big(\sum_{i=1}^{k} \varepsilon_{t+i}^2\Big) = \alpha + \beta k\sigma^2(t,\ t+k) + \gamma Z_t + v_{t+k}\ . \tag{8b}$$

The difference between the two regression specifications is that the second imposes the zero restrictions on the expected value of the cross product terms, implied under the null hypothesis. This effectively removes one source of noise from the left hand side, and should result in improved efficiency in estimation. Equation (8b) is therefore used in the empirical work. To implement the equation, it is assumed that

$$\varepsilon_{t+k} = \log S_{t+k} - \log S_{t+k-1},$$

where S is the spot exchange rate, thus assuming that the log of the spot exchange rate follows a random walk. Implied volatilities are obtained using the Garman-Kohlhagen (1983) version of the Black-Scholes model, developed for pricing currency options. Because the error terms in equations (8a and 8b) contain overlapping forecast errors, the method of Hansen and Hodrick (1980) is used to correct the estimated standard errors for the resulting serial correlation.

3. Data Employed

Our data set on futures options comprises weekly observations on the three major contracts traded on the Sydney Futures Exchange (in 90-day bills, 10-year bonds, and the Share Price Index), with the data period running from the inception of trading in each option up to the end of May 1988, using Wednesday observations. At each date a single put and call option are selected, in each case choosing the option most heavily traded on that day. The reason for using this criterion is that most of the SFE options quoted at a point in time are not actively traded, and the study aims to derive its results only from those options which are most liquid. Also, volatilities derived from puts and calls are tested separately because there is some evidence (see for example Brenner, Courtadon and Subrahmanyam (1985)) of significantly different results for the two types of options. The approach used here thus differs slightly from some earlier work which uses weighted averages of implied volatilities taken from several options at each date.

The options used in the study are always written on the nearest maturing futures contract (or the following one, when the maturity date is very near) and have the same expiry date as their associated futures contracts. They thus range between zero and about 90 days to maturity. On one observation each 90 days, it is not possible to observe the relationship described in equation (6), since the consecutive weekly observations will be for options with differing maturity dates. "New contract" dummies at these points are therefore added to the equation for the purposes of estimation. These ensure that observations coinciding with contract changeovers have no influence on the statistical results. A helpful consequence of selecting only the most heavily traded options is that these are generally the available options that are nearest the money. Pricing biases arising from violation of the Black-Scholes assumptions should therefore be minimised in this data set, since it is generally thought that the Black-Scholes formula performs worst for options which are not close to the money.

The currency options data are weekly Wednesday quotes for at-the-money put and call options on the A/US exchange rate, of one month maturity. The data period is from December 1984 to December 1987.

4. Results

The implied volatilities calculated from the data exhibit considerable variation through time, as can be seen in Figures 1 to 4. A minimal requirement for this behaviour to be consistent with rational forecasting is the existence of conditional heteroskedasticity in the time series for future prices. A useful means of testing for this in an atheoretical context is to specify a generalised ARCH process of the form:

$$\varepsilon^2_{t+1} = a + \Sigma b_i \varepsilon^2_{t-i} + c\sigma^2_t$$

where the ε's represent price innovations, and are observable. The equation is atheoretical because it makes no attempt to match up the time horizon covered by the option implied volatilities with the timing of subsequent innovations. Nevertheless the test is offered as a preliminary indication of the information contained in option prices. If the b and c coefficients are jointly zero, the prima facie conclusion would be that variances are constant, and that any changes in implied volatilities would in reality be forecasting changes in variance which do not subsequently occur. In fact the results, summarised in Table 1, provide fairly robust evidence in favour of significant heteroskedasticity of the underlying prices, so that at least this weak form of efficiency is not rejected by the data. Also, the implied volatilities are usually significant. A reassuring aspect of this result is that the ß coefficient in the main regression equation (6), cannot then simply be interpreted as representing the ability of σ^2_t to predict σ^2_{t+1}. The implied volatility σ^2_t does contain at least some information useful for forecasting the expected value of ε^2_{t+1}.

The main sets of results involve direct testing of equations (6) and (8b), and are summarised in Tables 2, 3 and 4. The most striking feature of the results is that they consistently reject the efficient forecast hypothesis. For the interest rate and currency options, this rejection generally occurs through estimates of the ß coefficients being significantly less than one. The interpretation of this is that changes in the estimated volatilities overpredict subsequent movements in the true variance of prices. With a typical ß coefficient of around 0.7 for the interest rate options, the average extent of this overprediction is estimated to be about 40 per cent of the true changes in variance, or about 20 per cent when measured in terms of standard deviations. The apparent biases in currency options are even larger, with the ß coefficient estimated at around 0.5 or less.

TABLE 1

TESTS FOR CONDITIONAL HETEROSCEDASTICITY IN FUTURES PRICES

Dependent Variable (Squared change in futures price)	Constant	Implied σ^2	Lagged Dependents 1	2	3	4	R^2	dw
Bonds	.0002	0.49	0.13	-0.06	0.38	-0.13	0.20	2.00
	(0.71)	(1.97)	(1.28)	(-0.57)	(0.37)	(-1.31)		
	.0001	0.32	0.08	(-0.04)	0.10	(-0.14)	0.22	1.96
	(0.61)	(1.65)	(0.75)	(-0.36)	(0.98)	(-1.38)		
Bills	.00	1.49	0.07	-0.07	-0.04	0.17	0.29	1.92
	(-.08)	(3.61)	(0.75)	(-0.83)	(-0.37)	(1.36)		
	.00	1.37	-0.05	-0.23	-0.03	-0.11	0.37	1.86
	(1.39)	(4.11)	(-0.60)	(-2.67)	(-0.35)	(-1.26)		
SPI	.0004	0.896	-0.35	-0.29	0.10	0.13	0.34	2.34
	(.23)	(0.77)	(-2.91)	(-2.95)	(1.02)	(1.38)		
	-.002	3.34	-0.57	-0.46	-0.07	-0.11	0.35	2.31
	(-0.89)	(2.07)	(-3.58)	(-3.58)	(-0.66)	(-0.88)		

Notes: This table reports estimates of the equations

$$\varepsilon^2_{t+1} = a + \Sigma b_i \varepsilon^2_{t-i} + c\sigma^2_t + \upsilon_{t+1}$$

For each contract, the first row of results are estimated using call volatilities, while the second uses puts.

Figures in parentheses are t-statistics.

TABLE 2

TESTS OF THE EFFICIENT FORECAST CONDITION: CURRENCY OPTIONS

	Constant	ß	Lagged squared innovations 4	8	Rho(4)
Calls	0.307×10^{-3}	.454	-.261	-.320	-.023
	(2.69)	(8.61)	(2.69)	(4.19)	
Puts	0.278×10^{-3}	.337	-.213	-.261	.035
	(2.26)	(6.06)	(2.11)	(2.84)	

Notes: The table reports estimates of equation (9b). Figures in parentheses are t-statistics.

The above findings are fairly consistent across the interest rate and currency options data sets, but somewhat different results are obtained for the SPI contracts, and it is important to note that the SPI results are quite sensitive to the inclusion or exclusion of the sharemarket crash. Results for the SPI options are therefore reported on both a pre-crash and a full sample basis. It turns out that in both samples the joint null hypothesis is strongly rejected, but for different reasons. Using the pre-crash sample, the ß coefficient is estimated to be substantially less than one (the point estimates are around 0.3) which, on the face of it, suggests an even more significant element of market overreaction than appeared in the interest rate contracts. The interpretation of this result is, however, clouded by the difficulty in quantifying the effect of possible anticipations of a major market correction on the pricing of options in the pre-crash period. Expected volatilities may have been influenced by such anticipations, but there is no obvious way of testing for this because of the uniqueness of the period around the crash. In the full sample, the results are quite different. The estimated ß coefficient comes out much closer to one, and in the case of put options is insignificant from one. The joint hypothesis is nonetheless rejected in this case by the significant negative coefficients obtained on elements of the distributed lag on ε_t^2. This feature is also observed in the currency options results. The negative sign here indicates that σ_t^2 tends to overpredict the subsequent variance when a large absolute price change has been recently observed in the price of the underlying. This would be consistent with the view that market volatility estimates are excessively influenced on average by current and recent measured volatility in the price of the underlying. This result is equally true for puts and calls.

Once again, however, there are some difficulties of interpretation, because these results are dominated by the effects of the crash and its immediate aftermath. Following the crash, realised share price volatilities were actually slightly lower on average than they had been before it, yet the expected volatilities implied in both call and put prices remained historically very high (see Figure 3 where these volatilities are illustrated). Ex post, therefore, a forecasting rule which puts a negative weight on large recent absolute price movements is seen to produce estimates which are much closer to the true volatility. Given the unprecedented nature of this period, however, it seems likely that such a rule would take some time to be learned. Taken together, these considerations suggest that any

TABLE 3

TESTS OF THE EFFICIENT FORECAST CONDITION: FUTURES OPTIONS (CALLS)

	Constant	ß	Lagged squared innovations 1	2	3	4	R^2	dw
Bonds	0.5×10^{-4} (3.34)	0.6989 (9.20)	0.05 (1.33)	0.03 (0.85)	-0.05 (-1.50)	0.04 (1.39)	0.71	2.01
	0.8×10^{-5} (0.66)	1	0.02 (0.50)	0.01 (0.21)	-0.08 (-2.23)	0.03 (0.86)	0.66	2.18
Bills	0.5×10^{-6} (3.94)	0.6224 (8.20)	0.001 (0.06)	0.003 (0.14)	0.0005 (0.03)	0.03 (1.19)	0.72	1.91
	0.1×10^{-6} (1.01)	1	-0.009 (-0.40)	-0.0008 (-0.04)	-0.006 (-0.31)	0.19 (0.63)	0.66	2.15
SPI	0.6×10^{-3} (5.40)	0.8048 (12.49)	-0.17 (-23.03)	-0.13 (-21.00)	0.03 (5.01)	0.04 (7.79)	0.95	2.00
	0.4×10^{3} (4.38)	1	-0.19 (-29.46)	-0.14 (-22.68)	0.02 (3.92)	0.03 (6.93)	0.95	2.26

Notes: The table reports estimates of equation (6) for call options. Figures in parentheses are t-statistics.

TABLE 4

TESTS OF THE EFFICIENT FORECAST CONDITION: FUTURES OPTIONS (PUTS)

	Constant	ß	Lagged squared innovations 1	2	3	4	R^2	dw
Bonds	0.1×10^{-3} (3.96)	0.5268 (5.68)	0.05 (0.91)	0.002 (0.03)	-0.02 (-0.35)	0.009 (0.18)	0.61	2.17
	0.2×10^{-4} (1.08)	1	-0.05 (-0.88)	-0.06 (-1.00)	-0.08 (-1.34)	0.007 (0.12)	0.51	2.53
Bills	0.6×10^{-6} (4.87)	0.7486 (8.84)	-0.02 (-0.78)	-0.04 (-2.21)	-0.03 (-1.51)	-0.01 (-0.58)	0.61	1.85
	0.4×10^{-6} (3.76)	1	-0.03 (-1.79)	-0.05 (-3.07)	-0.04 (-2.12)	-0.02 (-1.04)	0.58	2.05
SPI	0.5×10^{-3} (4.76)	0.9290 (12.30)	-0.185 (-25.16)	-0.17 (-29.32)	0.03 (6.88)	-0.001 (-0.25)	0.97	1.91
	0.4×10^{-3} (6.30)	1	-0.19 (-41.50)	-0.17 (-38.18)	0.03 (8.27)	-0.004 (-1.33)	0.97	1.99

Notes: The table reports estimates of equation (6) for put options. Figures in parentheses are t-statistics.

conclusions concerning apparent biases in SPI options prices, need to be heavily qualified.

5. Further results on pricing discrepancies

It has been pointed out that results of the kind reported in section 4 are necessarily conditional on the assumed option pricing model; rejection of the joint null hypothesis might, therefore, indicate rejection of the Black-Scholes model rather than rejection of forecast efficiency. This section considers two additional pieces of evidence which may have a bearing on this question. The first uses the fact that when the Black-Scholes assumptions are violated, the bias in the formula will be related to the extent to which the option being priced is in or out of the money; the formula tends to overvalue deep in-the-money options and to undervalue those which are deep out-of-the-money (see for example, Merton (1976)). Thus, if the apparent pricing biases detected in section 4 are caused partially by deviations from the Black-Scholes assumptions, we would expect the empirical biases to be correlated with the option deltas, which can be used as a measure of the extent to which an option is in or out of the money.

Estimated pricing biases can be obtained by using fitted values for expected volatility from equations (6) and (8b) to obtain a time series of estimated option values, which can then be compared with observed prices. A comparison between the estimated volatilities and those implied in the option prices appears in Figures 1 to 4. Generally speaking, these imply price discrepancies which vary in a range between zero and about 30 per cent. (For the SPI options the calculations, though performed for the full sample, are based on the estimated pre-crash relationship.) The graphs also illustrate the general point that "fitted" volatilities tend to be less variable than those implied in actual option prices. The currency options case differs from the other three in that there is also an apparent unidirectional bias in levels, with implied volatilities always exceeding those predicted by the forecasting equation. However, the gap narrows through the sample period, perhaps reflecting an increasing competitiveness in the pricing of these options.

IMPLIED AND PREDICTED VOLATILITIES

Figure 1 :

10 YEAR BONDS CALL OPTIONS

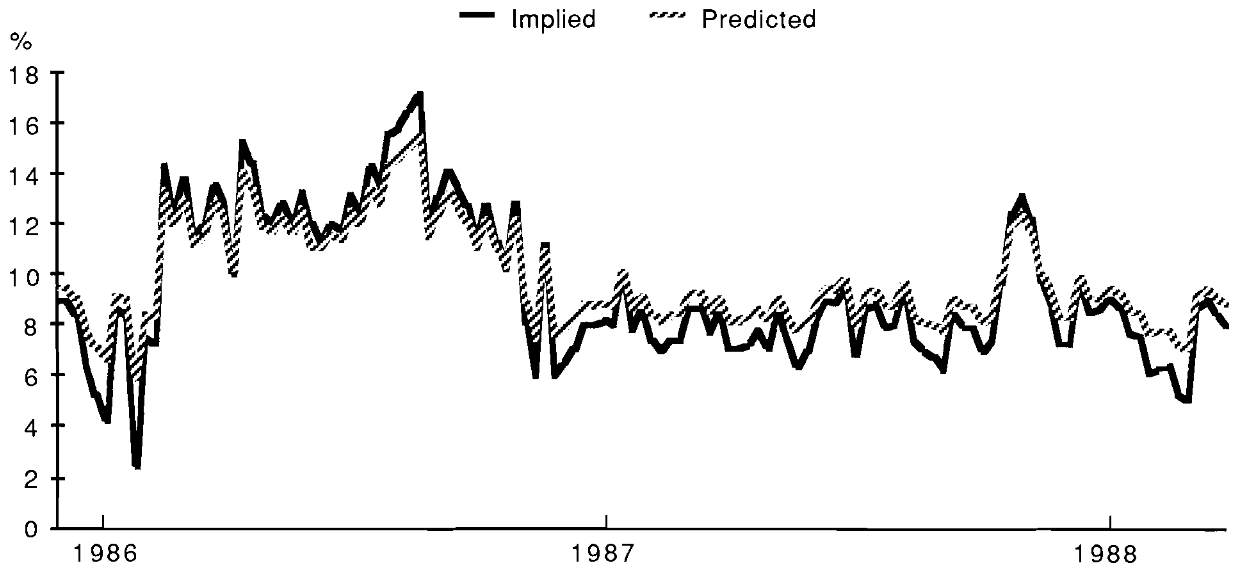
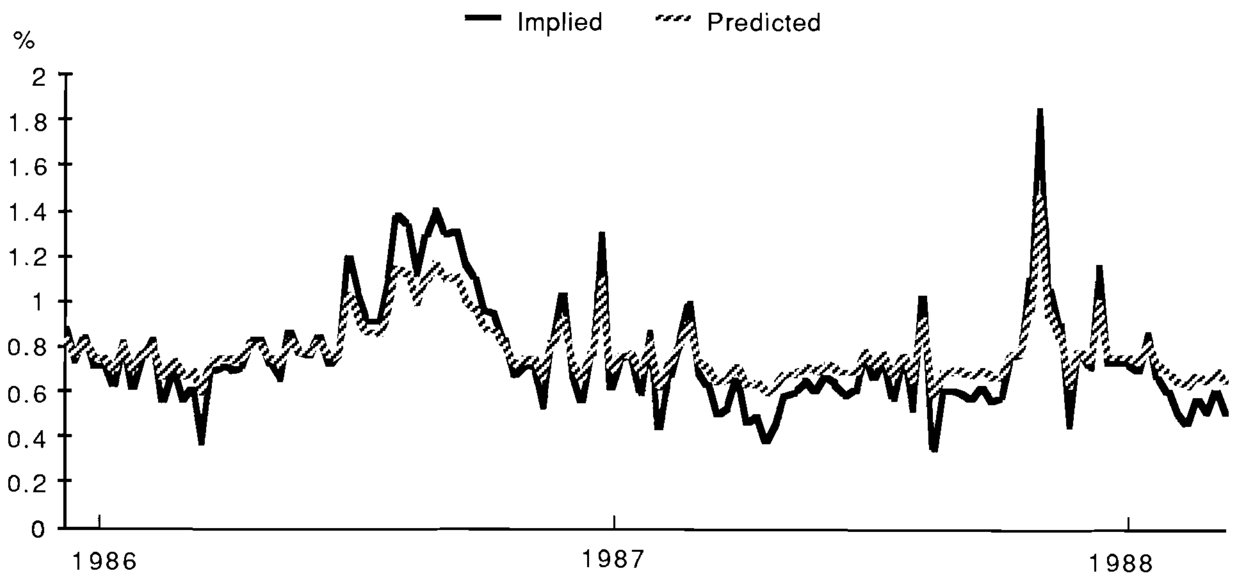


Figure 2 :

90 DAY BANK BILLS CALL OPTIONS



IMPLIED AND PREDICTED VOLATILITIES

Figure 3 :

SHARE PRICE INDEX CALL OPTIONS

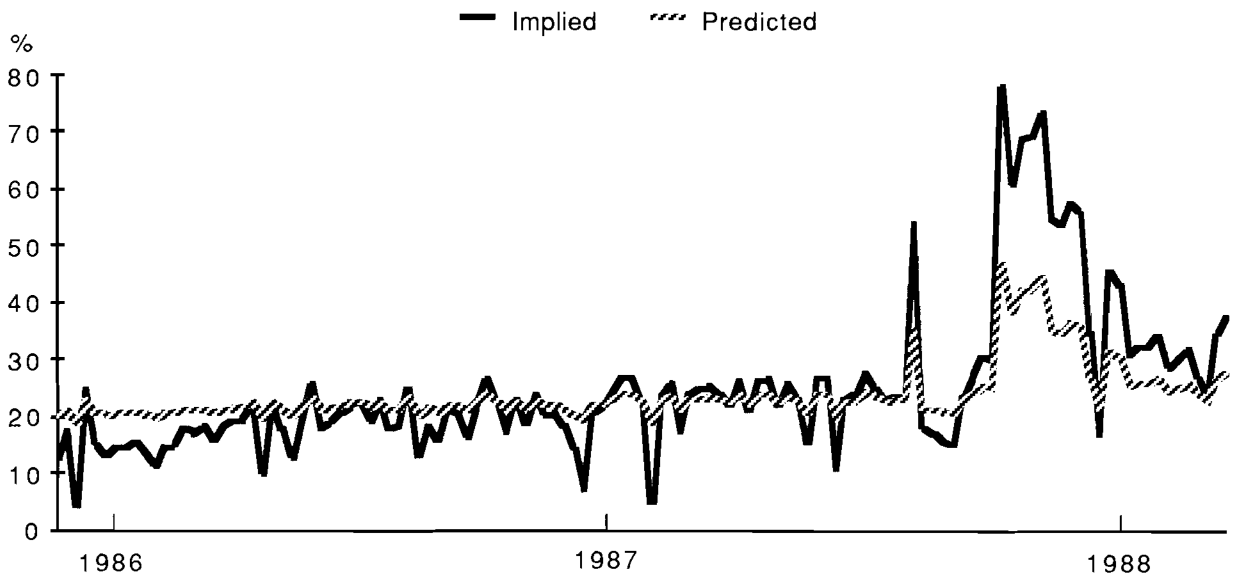
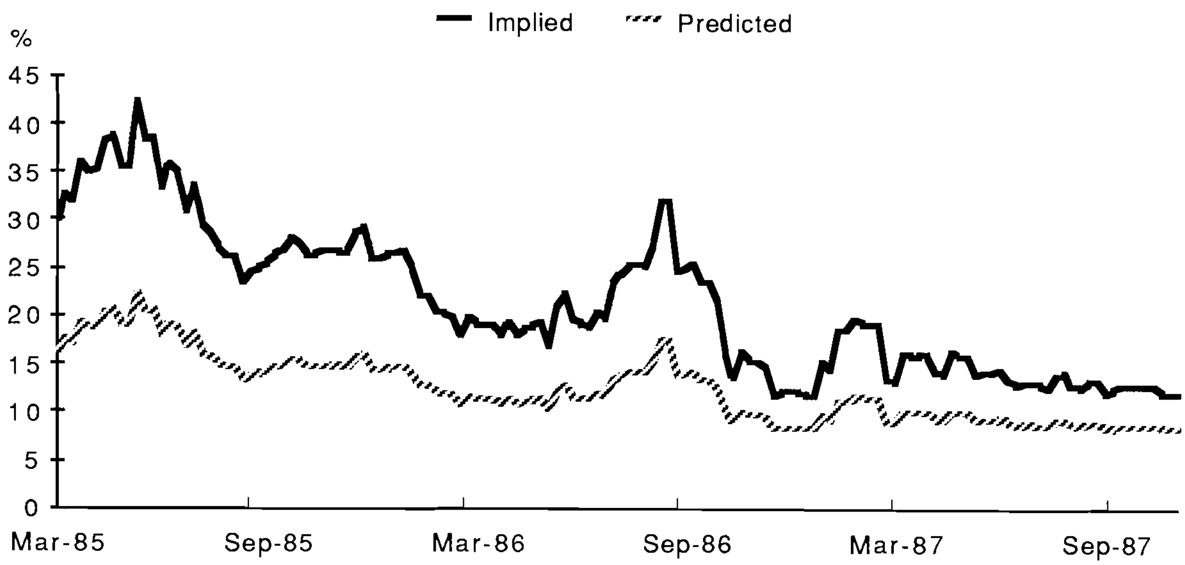


Figure 4 :

CURRENCY OPTIONS



The proposition that the pricing biases are related to the options' deltas is tested using simple linear regressions of the calculated biases against the relevant series of deltas. (This is not applicable to the case of currency options, since all the currency options in the data set are at-the-money.) Results for these regressions are reported in Table 5. They show quite clearly a lack of any significant correlation between these variables; estimated pricing biases appear not to be correlated with the extent to which options are close to the money. This provides one piece of evidence that our results are unlikely to be explained as being due to deviations from the Black-Scholes assumptions.

As a second piece of evidence, which has a direct bearing on market efficiency, it may be asked whether use of statistically efficient forecasts implied by use of equations (6) and (8b) would have led to significant excess returns within the sample period. (At a later date it is hoped to repeat this exercise using out of sample data.) To investigate this question the comparison between actual and estimated option prices described above was used to simulate the following hedged trading rule:

buy any option that is undervalued and hold to expiry;

sell any option that is overvalued, and hold the position to expiry;

in each case, delta hedge the initial position in the underlying instrument, but with no subsequent rebalancing.

The delta is evaluated using fitted volatilities.

The realised excess return on a hedged call transaction under this rule is then given by:

$$\Pi = D\,\{-c + \max\,(f_T - x,\ 0) - \delta(f_T - f_t)\} \tag{9}$$

where $D = 1$ for a buy
$\quad\;\; = -1$ for a sell

Put returns are defined analogously.

Within sample returns calculated in this way are summarised in Table 6.

TABLE 5

CORRELATIONS BETWEEN ESTIMATED PRICE BIASES AND OPTION DELTAS

Dependent variable Price bias on:	Constant	Option Delta	R^2	dw
Bonds calls	0.09	-0.006	0.50	1.05
	(4.19)	(-0.30)		
puts	0.16	0.04	0.15	0.95
	(4.62)	(1.13)		
Bills calls	0.12	0.06	0.12	1.38
	(4.01)	(2.11)		
puts	0.16	-0.02	0.24	1.04
	(7.18)	(-1.04)		
SPI calls	0.12	0.11	0.32	1.84
	(2.38)	(1.64)		
puts	0.07	0.04	0.26	0.92
	(2.38)	(1.31)		

Note: t - statistics in parentheses.

TABLE 6

PROPORTION OF OBSERVATIONS SHOWING WITHIN-SAMPLE EXCESS RETURNS

	Calls	Puts
Bonds	58	57
Bills	70	76
SPI	54	51
Currency	77	80

Note: The table shows the percentage of observations in the sample showing excess returns as defined by equation (10).

The most dramatic results are those for bill futures options, and currency options. In both these cases roughly three-quarters of the observations generate positive excess returns within sample. The currency option results are generated mainly by an implicit overpricing during the sample period; since forecast volatilities were always less than those implied in option prices, the trading rule given by equation (9) indicated a sell in every period. For bond and SPI options the figures for excess returns are smaller at around 60 per cent for bonds, while for the SPI, it is barely above the 50 per cent which one would expect to obtain randomly. This perhaps confirms some of the reservations which were expressed in the earlier discussion of the SPI results. For the remainder, however, the results provide some support for the view that expectations errors, rather than biases in the pricing formula, are responsible for the anomalies reported in section 4. In this regard, it is worth noting that the rule used to detect excess returns here was a particularly unsophisticated one. Outstanding options were not assumed to be delta hedged to maturity, and there was no attempt to weight the size of each trade according to the relative extent of each overvaluation or undervaluation. If anything, such a trading rule would be likely to understate the presence of excess returns in the sample.

6. Conclusions

The paper claims to have detected a tendency for both financial futures options and currency options to overpredict subsequent changes in the variance of the underlying prices. In currency options, the results also imply a consistent overstatement of the level of volatility, though this has tended to narrow steadily through the sample period. These anomalies were statistically significant in all the cases tested, but we recognise that the unusual circumstances surrounding recent share price movements make the results for SPI options difficult to interpret. For the remainder, however, the results clearly violate the joint hypothesis of Black-Scholes pricing and forecast efficiency. The fact that apparent pricing biases are not correlated with option deltas, and give rise to within-sample excess returns, points to forecast inefficiencies as an important element in explaining these results.

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