A WEEKLY MODEL OF THE FLOATING AUSTRALIAN DOLLAR

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ABSTRACT

In the first two years of its float, the Australian-U.S. dollar exchange rate has substantially depreciated and oscillated. This paper tests to see whether this exchange rate has, at least, followed a random walk with drift. Having established this benchmark, structural monetary models are constructed to see whether one can obtain better within-sample and/or out-of-sample results. Rational forecasts of exogenous variables are obtained using Muth's (1961) decomposition; interpolation is used to obtain weekly forecasts when the observation period is greater. It appears that the random walk can be beaten.

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1. Introduction

The Australian dollar became a market determined currency on 14 December 1983. With respect to the US dollar it depreciated about 5 per cent in its first year of floating and about 25 per cent in its second. The variance of the spot rate increased by nearly 40 per cent in its first year of floating, relative to the previous year of managed rates, and by a further 100 per cent in its second. The level and change in the logarithm of the spot rate are shown in Figure 1. The apparent drift in the level and the variability of the exchange rate need to be verified by empirical time series analysis. Once the univariate time series properties of the exchange rate have been established, a question worth addressing is whether a multivariate model can be found to encompass the univariate one.

The empirical exchange rate literature does not give much comfort to any particular exchange rate theory that has been postulated. Any success achieved usually turns out to be episodal and the particular model tends to do no better at out-of-sample forecasting than a random walk.[1] Mussa (1979) contended that flexible exchange rates, in common with other asset prices, generally behave largely like a random walk (with drift). A random walk is a sufficient condition for a series to be non-stationary. If this is true, inferences from the estimates of the parameters of a model of that series will need to account for that non-stationarity, or else transforms of the series (say, by differencing) are needed to obtain stationarity and the right to use classical distribution theory. Meese and Singleton (1982) establish the non-stationarity of the US dollar vis a vis the Swiss Franc, the Canadian dollar and the Deutschemark. This paper follows their lead, testing for the non-stationarity of the Australian dollar, but also taking into account the possible heteroscedasticity and non-normality of the error process, suggested by the facts in the first paragraph.

If a random walk with drift can be verified, a multivariate model can be postulated to try to find an explanation of the drift from the expected or actual values of other variables, and to try to reduce the noise process by

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1. Meese and Rogoff (1985) show that random walks have at least as much success as other theories.

substituting in the effects of unexpected values of these other variables. With such a short length of data accumulated since the float,[2] this exercise cannot be expected to be more than indicative of possible directions for future research.

Whenever an economic system underdoes a major regime switch, econometric modellers and forecasters have to wait patiently for enough data to accumulate so that they have sufficient degrees of freedom to be able to estimate the fundamentals of that system. Even though asset prices, such as the exchange rate, can be observed continuously, the essentially exogenous variables that are generally thought to be important influences on them are often only published monthly, or even quarterly and always with a substantial publication lag. Yet market determined asset prices are formed by market participants who have to continuously make conjectures about current and future fundamentals. All previously announced observations of fundamental variables contain information than can help to make these conjectures. It seems decidedly wasteful to throw out all but (say) quarterly information on all variables. If one can find a satisfactory method of modelling conjectures on a continuous, rather than discrete basis, one will not be constrained by the longest publication period amongst the fundamentals. In this paper, a method due to Muth (1961) is used to this end. A variable, or its rate of growth is assumed to be composed of unobservable permanent and transitory components. The permanent element is modelled as a random walk. All future forecasts are based on the current estimate of this element, and it is this feature which delivers the required property. Attempts are made to explain the exchange rate using these generated regressors. The obvious loss in efficiency from this procedure is hopefully more than balanced by the gain from using a larger sample.

Models with future price expectations that are determined rationally display the well-known problem of multiple solutions. This occurs because the future expectation is an additional endogenous variable in a system that has no extra equations. There are two strategies that one can adopt to solve the problem of non-uniqueness.

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2. Information about the exchange rate system can be gleaned from the study of arbitrage equations. Tease (1986) established inefficiency in the Australian foreign exchange market by testing whether the difference between the forward and the appropriate future spot rate is orthogonal to known information. Trevor and Donald (1986) use VAR estimation methods on daily data of the trade-weighted exchange rate index and international interest rates for Australia, U.S., Japan and West Germany. The Australian dollar appeared to be unaffected by Australian interest rates.

The first and least restrictive approach makes the weak rational expectations assumption that the actual expectational error of an asset price is only due to new information that arrived after the expectation was formed. This knowledge is known to the model builder, and can be used to eliminate expectational variables in the underlying model. The model to be estimated becomes a multivariate autoregressive moving average (ARMA) one with orders at least one higher than the original and can be estimated using a minimum distance procedure which is a good approximation to FIML with a small number of parameters. The estimated parameters then provide a unique solution. This solution can be analytically solved and the result may contain non-fundamental or bubble solutions. The second strategy is the standard method of finding a solution for the convergence of the series of future expectations. This latter problem is deterministic and is akin to that of finding the saddlepath in perfect foresight models.

In Section 2, the univariate time series properties of the exchange rate are investigated and a benchmark random walk model is established. The conclusions from this section are used to restrict the multivariate analysis in Section 3. Monetarist and Keynesian error correction models are set up to compete with the random walk benchmark. Section 4 offers some conclusions.

2. Univariate Time Series Modelling

(a) Methodology

Analysis of the univariate time series properties of exchange rate data provides a useful starting point, prior to econometric analysis. Finance theory suggests that 'speculative prices' ought to be represented by fairly simple time series processes, if financial markets are efficient. Naturally, these simple processes have important implications for the design of structural econometric models.[3] Accordingly, this section develops the time

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3. Zellner and Palm (1974) demonstrate the representational equivalence of a structural dynamic model, a multivariate ARIMA model and a set of univariate ARIMA equations. The univariate time series processes necessarily imply restrictions for multivariate and structural analysis, the presumption being that the general economic theory model encompasses the time series model.

series model to recover the lag structure, trend effects and unusual features of the exchange rate series.

The univariate models considered had the general form:

$$\phi(L)\ e_t = \zeta(t) + \eta(L)\ u_t \tag{1}$$

$$t = 1, \ldots, T; \qquad e_o = 0$$

where e_t is the logarithm of the spot exchange rate (as a deviation from its initial value) of domestic currency in terms of foreign currency, u_t is assumed to be a weakly (covariance) stationary, possibly heteroskedatic error process, $\zeta(t)$ is a polynomial function of time, and $\phi(L)$ and $\eta(L)$ are pth and qth order polynomials in the lag operator. If the characteristic function of $\phi(L)$ has d unit roots, then it can be factored to give $\phi(L) = \phi^*(L)\nabla^d$ where ∇ is a difference operator, and the order of $\phi^*(L)$ is $p = P - d$. The model would then be an ARIMA (p, d, q) incorporating a time polynomial.

On the basis of likelihood ratio testing of nested ARIMA models, the exchange rate series in common with most other economic series is of a low order in the polynomials. To begin, the following model will be considered, higher order autoregressive terms being statistically irrelevant

$$(1 - (\phi_1 + \phi_2)\ L + \phi_1\phi_2 L^2)\ e_t = \zeta_0 + \zeta_1 t + u_t \tag{2}$$

A key issue is the value of ϕ_1 and ϕ_2, the roots of $\phi(L)$. If ϕ_1 is unity, $\phi(L)$ factorises to $(1-\phi_2 L)\nabla$, and if ϕ_1 and ϕ_2 are unity we get ∇^2. Tests are undertaken for these hypotheses, and if accepted, estimation is redone using the appropriately differenced form.

If there is to be any credibility in the deduced time series properties, the selected model should be checked for the following criteria (at least).

Presence of Unit Roots

One of the critical issues in modelling the autoregressive part of univariate time series models is the test procedure for the presence of unit roots. This issue is of especial importance when examining 'speculative price' data. An efficient capital market will fully reflect all existing and publically available information, and one would expect the data to be consistent with a random walk perhaps with a time-dependent drift (for example, see Granger and Morgenstern (1976)). This would imply (at least) one root lying on the unit circle. However, when estimating autoregressive parameters, one normally presumes that the time series is weakly (covariance) stationary, with characteristic roots lying outside the unit circle. But under the null hypothesis of a unit root, the time series is not stationary and the asymptotic variance of the series is not finitely defined; hence classical t and F tests cannot be undertaken. Fuller (1976), Dickey and Fuller (1979) and Hasza and Fuller (1979) derive the appropriate test statistics and their distributions for the null hypothesis of one or two unit roots of an autoregressive process. The distribution of the autoregressive parameter estimates under the null is decidedly skewed to the left of unity, and one should not be surprised to obtain estimates that are significantly less than unity on classical t tests.

In the case of a single unit root, the form of the test statistic is identical to that for the Studentised t, and the method is a likelihood ratio test for the unit root (and if included, a zero mean and/or linear trend). In the regressions, the statistic is reported as τ_τ, and the distribution tables are obtained from Fuller (1976, page 373). For two unit roots, the form of the test statistic is similar to the F and two forms are reported: $\Phi_3(2)$ and $\Phi_3(4)$.[4] The first is a likelihood ratio test of the two unit roots alone, and the second is a joint one of the roots and of a zero mean and no linear trend. The tables for the distributions of these statistics can be found in Hasza and Fuller (1979, page 1116).

If the null hypothesis of a unit root can not be rejected, non-stationarity of the underlying process likewise can not be rejected. By ignoring the problem, one can generate frequently 'significant' but spurious regression outcomes (see Granger and Newbold (1977) for spurious regressions of one random walk on another). One procedure for dealing with this type of non-stationarity is to prefilter all the data by regressing all variables on a polynomial of time, and to use the residuals as the data. This strategy is effective for that express purpose, but dubious if one is also concerned with forecasting and structural explanation. A perceived trend in a sample may well turn out to be

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4. For example, $\Phi_3(4) = \sum_{t=1}^{T} (1,t,Le_t,\nabla e_{t-1})^1 \nabla^2 e_t / S^2$

where S^2 is the estimate of the error variance.

a transitory feature of a more complex dynamic process. One objective of multivariate and structural analysis is to explain the causes of this perceived trend - the prefiltering strategy precludes such an analysis.

Uncorrelated Innovations

The error process should be a pure innovation with respect to information available just prior to the derivation of its elements. An implication of this is that the error process must pass a test of the null hypothesis of no autocorrelation implying that the error process cannot be predicted from its own past. The Box-Pierce statistic, distributed χ^2_q where q is the maximal lag, is used in this regard and is reported as BP(q). Of course, one does not rule out the possibility of influence from the current and lagged values of the error processes derived from other variables.

Homoscedastic Innovations

The error process should be checked for homoscedasticity. If the null is rejected, the estimate of the variance-covariance matrix of estimates is inconsistent. Heteroscedasticity in asset price equations is an important and distinct possibility because an influential element in asset choices is relative risk. Risk premia are difficult to specify and, generally, time varying. Misspecification of risk premia would be expected to be detected as heteroscedasticity in the error process.

Two tests are used: the Engle (1982) ARCH test for the particular autoregressive form of heteroscedasticity, which involves regressing the T squared residuals on r of their lags, with TR^2 being distributed χ^2_r; and the less powerful White (1980) test for non-specific forms, which involves regressing the squared residuals on the products and cross-products of the k explanatory variables, with TR^2 being distributed $\chi^2_{k(k+1)/2}$. The test statistic is reported as WH(k(k+1)/2). The ARCH tests are undertaken because, if rejected, they may help to explain the existence of fat tails in the error distribution [see Engle (1982, p.992)]. The White correction for the variance-covariance matrix enables one to conduct valid inferences, provided the errors are serially uncorrelated.

Normality

The error process should be tested to see that it represents a random sample from a normal distribution. Otherwise, one could improve upon classical least squares methods. Typically, this is a difficult test to pass, and in many cases the test or the results are ignored. For samples less than fifty-one, the Shapiro-Wilk (1965) W statistic is computed, and for larger samples, the Kolmogorov D statistic is reported as KD (see Stephens (1974)).

Balanced Sample

The data sample used must be balanced in the sense that small subsets of observations must not have a substantial influence on the parameter estimates. Cook's (1979) D statistic is computed for each observation measuring the change in estimates resulting from the deletion of the observation. The statistic is distributed as an F(K,T-K). Such a test is invaluable for getting to know if there are any peculiar features in one's dataset which would require a deeper search into the causes. Such a test may indicate the need for the inclusion of dummy variables. The maximum D statistic across the sample is recorded in the tables.

Parameter Constancy

Tests for temporal stability of parameter estimates over the sample are essential if one is to accept the validity of constant parameter hypotheses. The Chow test provides the appropriate information on the assumption that the errors are homoscedatic. If heteroscedasticity is evident, the standard Chow test can seriously understate the Type 1 error. In that case, one can consult Schmidt and Sickles (1977) to get an approximate idea of the degree of the understatement.

(b) Empirical Results

Three samples were constructed for use in the regressions:

"1984-85" covered the period 14 December 1983 to 13 November 1985;
"1984" for 14 December 1983 to 21 November 1984; and
"1985" for 28 November 1984 to 13 November 1985.

The reason for the sub-sample breakdown was that at the beginning of 1985 monetary targetting was abandoned in favour of a 'checklist' approach. The announcement of the change came in February 1985, but the de facto switch probably occurred in the preceding months as it became apparent that monetary targets had become increasingly elusive.

The first set of regressions for the logarithm of the spot rate as in (1) are shown in Table 1.

The first regression (1a) for the two years of the float explains 98 per cent of the variance of the spot rate. On the basis of simple t tests, the parameters (apart from the constant) are significantly different from zero. Similarly, t tests for ϕ_1 and ϕ_2 being unity significantly reject that

Table 1
Univariate Time Series Properties: the Spot Rate

	T	Dependent Variable (Mean)	ζ_0	ζ_1	ϕ_1	ϕ_2	SSE	R^2	BP(18)	ARCH(8)	WH(10)	KD or W	D_{max}	τ_τ	$\Phi_3(2)$	$\Phi_3(4)$	CH(4,T-8)
1. 1984-85																	
a.	101	e_t (-0.126)	7.3-3 (4.7-3)	-4.2-4 (1.6-4)	.904 (.036)	.223 (.098)	3.2%	98%	20.36 [.31]	27.45 [<.01]	19.45 [≈.035]	.109 [<.01]	.227 [>.10]	-2.66 [>.10]	39.18 [<.01]	40.21 [<.01]	0.871 [>.5]
b.			(3.8-3)	(1.5-4)	(.038)	(.138)	-	-	-	-	-	-	-	-2.52 [>.10]	-	-	
2. 1984																	
a.	50	e_t (-.0192)	8.5-3 (4.2-3)	-4.4-4 (1.8-4)	.869 (.052)	.265 (.133)	.5%	95%	23.32 [.18]	5.14 [.73]	7.21 [>.5]	.969 [.40]	.236 [>.10]	-2.50 [>.10]	21.46 [<.01]	22.49 [>.01]	-
b.			(3.2-3)	(1.4-4)	(.045)	(.151)	-	-		-	-	-	-	-2.84 [>.10]	-	-	-
3. 1985																	
a.	51	e_t (-.18)	-.009 (.019)	-1.8-4 (3.1-4)	.897 (.053)	.186 (.142)	2.5%	92%	13.16 [.78]	9.86 [>.25]	19.50 [≈.035]	.948 [.06]	.180 [>.10]	-1.92 [>.10]	20.28 [<.01]	20.68 [<.01]	-
b.			(.017)	(2.9-4)	(.049)	(.159)	-		-		-	-	-	-2.10 [>.10]			-

Notes

1. Standard errors are reported below parameter estimates as (.).
2. Marginal significances are reported below appropriate statistics as [.]. These measure the strength of the evidence against the null.
3. SSE is the error sum of squares multiplied by 100.
4. All data analysis was undertaken using SAS software.

hypothesis. The estimate of ϕ_1, 0.904, is substantially below that of Meese and Singleton (1984)'s estimates for the US dollar against Swiss francs, Canadian dollars and Deutschemarks (respectively 0.999, 0.982, 1.008). Such a result may appear to suggest that the Australian experience has been somewhat different and does not lend support to the Mussa (1979) contention that the logarithms of spot rates approximately obey a random walk. Indeed it may seem to lend support for the speculative activities of financial traders based upon univariate "technical" analysis. However, such conclusions are spurious and invalid. The Meese and Singleton results were obtained using 285 observations, compared to 101 in 1a. Even if the true value of ϕ_1 were not unity, it is well known that ordinary least squares estimates of positive autoregressive parameters are biased downwards in small samples - White (1961) shows the bias in a first order autoregression to be $-2\phi_1/T$ which under the null would explain about 0.03 of the difference. But, as discussed above, the appropriate test for a unit root involves a distribution of the parameter estimate that is seriously skewed to the left of unity. Fuller (1976, p.370) shows that the probability of $\hat{\phi} < 1$ given $\phi = 1$ asymptotically approaches 0.6826. Applying the Fuller test, the statistic τ_t is seen to be unable to reject the unit root even at 10 per cent marginal significance (for 100 observations, the critical value of τ_τ at 10 per cent is -3.15 and at 90 per cent is -1.22). The classical t test acceptance of stationarity is evidently spurious and we can accept the null hypothesis of a single unit root conditional on the assumption of no heteroscedasticity.

Two tests involving two unit roots are undertaken. $\Phi_3(2)$ is an F-type statistic that jointly tests for $\phi_1 = \phi_2 = 1$, while $\Phi_3(4)$ jointly tests for $\phi_1 = \phi_2 = 1$, $\zeta_0 = \zeta_1 = 0$. The empirical percentiles for these two statistics for 100 observations at 5 per cent (1 per cent) are 9.58, (12.31) and 5.36, (6.74) respectively. Evidently, both null hypotheses are rejected.

From the Box-Pierce statistic, the marginal significance of 0.31 indicates that we can accept the null of no autocorrelation. This means that, if heteroscedasticity is present, White's (1979) correction for the variance-covariance matrix is appropriate for making inferences. On the ARCH test, heteroscedasticity is definitely present and may be consistent with an autoregressive form. The White test indicates heteroscedasticity at the 5 per cent significance level.

The application of the White correction, shown in line 1b, has one interesting effect. The single unit root test is unaffected, but the standard error of the second autoregressive parameter is increased by nearly 40 per cent. The implication is that, after heteroscedastic correction, the logarithm of the spot rate is, in fact, closely approximated by a random walk with drift.

Before accepting this conclusion, one needs to check the balance of the data. Cook's D statistics for each observation indicates a three week period (20 February 1985 - 6 March 1985) of unusual influence. This, of course, was the so called "MX Missile Crisis". However, the F test indicates that the crisis did not significantly "imbalance" the data set. Further tests (based on Belsley, Kuh and Welsch (1980)'s DFBETA statistics) indicate that ζ_1, ϕ_1 and ϕ_2 were the parameters most affected, but none were significantly influenced. Nevertheless, a dummy variable for the "MX Missile Crisis" was introduced, and it turned out to have a value -.019 with standard error 0.010. Other estimates were marginally reduced and all inferences remained intact. This may allow the conclusion that exchange rate crises, such as this, are merely crises of confidence that can be represented as statistical noise.

The test for normality of residuals unfortunately fails based on Kolmogorov's D statistic. This does suggest that least squares estimates could be improved upon by robust techniques. The residuals were also leptokurtic (a kurtosis coefficient of 1.48 being registered), which is often practically consistent with a distribution having a sharper peak and higher tails than the normal. We already know that the ARCH statistic was significant, and an autoregressive form of heteroscedasticity, will generally be associated with fat tails. Further the skewness coefficient had a value of 0.94 implying a positive overhang in spot rate innovations. This suggests that there may be unspecified exogenous variables which would have imparted forces for appreciation in the model. While the non-normality of the residuals is a cause for concern, it would be very surprising if robust estimates lead to a rejection of the single unit root hypothesis. This conclusion is supported by the results of the sub-sample regressions.

The usual parameter constancy test of Chow accepts the null that 1984 and 1985 data produced insignificantly dissimilar parameter estimates. Given the existence of heteroscedasticity, one needs to be sure that the inaccuracy of the assigned significance level (say, 5 per cent) is not too great. From the tables in Schmidt and Sickles (1977) (with equal sample sizes of about 50, and a ratio of estimated variances of $(.025/.005)^2 = .25$) the understatement will not be serious. However, there are some interesting differences arising in the sub-sample analysis - viz regressions 2a, 2b, 3a and 3b in Table 1. The unit root tests give identical results, but the ARCH tests and the normality tests are quite different. Admittedly, these differences may arise because of the power loss in decreased sample size. Nevertheless, it is worth noting that heteroscedasticity is rejected in 1984, but not in 1985. Similarly the residuals are acceptably normal in 1984, but not in 1985. This coincidence of effects is consistent with the notion of heteroscedasticity being associated with fat tails. Hence, even though parameter estimates are not significantly different between 1984 and 1985, the nature of the error process, the second moment in particular, was significantly different. The 1985 characteristics are also seen to predominate in the aggregate sample.

Before considering more fundamental reasons for this substantial difference between 1984 and 1985, it may be reasonable to think that it is the proven non-stationarity of the exchange rate process that is the source of the increasing variance. It is therefore instructive to consult Table 2 where a similar exercise is undertaken with first differences of the exchange rate as the regressand. First differences eliminate the time trend and imply a first order model. The following conclusions are obtained - now no unit root, heteroscedasticity in the combined sample (probably coming from 1985 rather 1984), non-normality in the combined sample only, no excessively influential observations, no autocorrelation and last but not least virtually no explanatory power coming from the model especially after the White adjustment for heteroscedasticity. Indeed, only the constant term shows any hint of significance; this reflects the linear time trend in Table 1. But note that the dependent variable is measured as a deviation from its initial observation (-7.74-3 for samples 1984-85, 1984 and -4.65-3 for 1985). Hence we can conclude that there is a significantly non-zero drift in the random walk model. For the full sample, the random walk model with drift is reported in line 1c where the implied drift is -2.8-3.

All in all, while first differencing certainly eliminates the unit root source of non-stationarity, it does not eradicate the heteroscedasticity and non-normality problem. Further, it appears to reduce the regressand to almost complete noise. Differencing is not necessarily the best solution to the non-stationarity issue. When one cannot reject the null of a unit root, it is not appropriate on classical principles to conclude that the value of the root must be fixed at unity in subsequent testing. The appropriate procedure is to

Table 2

Univariate Time Series Properties: Change in the Spot Rate

	T	Variable Mean	ζ_0	ϕ_1	SEE	R^2	BP(18)	ARCH(8)	WH(3)	KD or W	D max	τ_τ	CH
1. 1984-85													
a.	101	∇e_t -3.17-3	3.7-3 (1.9-3)	.19 (.099)	3.48%	3.45%	19.55 [.36]	26.75 [<.005]	14.76 [.056]	.088 [.056]	.43 [>.10]	-8.2 [<.01]	0.23 [<.01]
b.			(2.2-3)	(.15)								-5.4 [<.01]	
c.			4.96-3 (1.9-3)	0	3.71%	0	26.19 [.095]	17.92 [<.025]	-	.103 [<.01]	-	-	-
2. 1984													
a.	50	∇e_t -1.15-3	5.2-3 (1.9-3)	.23 (.14)	.64%	5.3%	22.9 [.20]	1.89 [>.95]	5.27 [>.1]	.96 [.27]	.32 [>.10]	-5.5 [<.01]	-
b.			(2.2-3)	(.16)								-4.81 [<.01]	
3. 1985													
a.	51	∇e_t -5.15-3	-4.6-4 (2.4-3)	.16 (.14)	2.81%	2.7%	14.5 [.70]	7.61 [>.25]	5.00 [>.1]	.098 [>.15]	.36 [>.10]	-6.0 [<.01]	-
b.			(3.3-3)	(.18)								-4.7 [<.01]	

See notes on Table 1.

undertake inference based on distributions conditional upon the non-stationarity induced by the existence of unit roots. Unfortunately, the appropriate distributions are pathologically dependent on the particular model and the theoretical developments are still few and far between. Given this limitation, differencing is a second-best strategy. If the purpose of the exercise is forecasting, and the true model involves a unit root, then the failure to difference will result in unwarranted (classical) confidence in the forecasts. If the true root is not unity, then differencing will give rise to forecasts that are too conservative. Overdifferencing is generally less dangerous than underdifferencing.

A final point on this issue, relevant to the next section, is that if the true model does not have a unit root, but one close by, then first differencing should produce a model with a first order moving average, the parameter of which should reflect the difference of the root from unity. A first order moving average was estimated in the equations for the change in the spot rate. For the two year sample, moving average parameter was estimated as -0.54 (0.39). The other parameters were only marginally affected, and so one is left with the conclusion that the analysis in the next section should allow for the possibility of a multivariate ARIMA (1, 1, 1) model.

3. Multivariate Modelling

From the previous section, it has been established that the exchange rate approximately obeys a random walk with drift, with a noise process that is uncorrelated, non-normal and heteroscedastic. The next stage of the analysis seeks to reduce the standard error of these residuals by introducing relevant explanatory variables. The objective of this exercise is to obtain some information about the possible fundamental variables driving exchange rates, amongst other asset prices. Given that data on these fundamentals is in short supply, a proper structural model cannot be specified and estimated. Instead, only excessively restricted multivariate models can be considered, the general form of which is

$$\phi(L)\, e_t = \alpha(L) Z_t + \beta(L)({}_t e_{t+1} - e_t) + \eta(L) u_t \tag{3}$$

where ${}_t e_{t+1} = E(e_{t+1} | I_t)$, $\phi(L)$, $\alpha(L)$, $\beta(L)$ and $\eta(L)$ are lag polynomials, and Z is a vector of weakly exogenous variables. This equation is a more general representation of the asset market approach to exchange rate modelling discussed by Mussa (1984). The pure monetary approach model, with absolute purchasing power parity and uncovered interest parity devolves into a special case of (3) with all the lag polynomials being of zero order and Z_t being a vector of domestic and foreign money supply and output. If ϕ was of the first order, the model would be consistent with a partial adjustment approach to monetary disequilibrium - see Woo (1985). Since the exchange rate process is not stationary, the appropriate model will have to explain the first difference. Accordingly, one would expect that $\phi(L)$ factorises to give $(1-L)\phi^*(L)$. In a first difference monetary approach model, applying relative purchasing power parity and uncovered interest parity, all of the lag polynomials factorise to include a first difference term - see Hartley (1983).

If the Z_t vector were to consist of money supplies and output only, (3) could still be a reduced form of many competing or complementary exchange rate theories. A sticky price model would have price changes dependent on the lagged exchange rate, money supply and output. With monetary equilibrium, this would reduce to (3) with all polynomials of the first order. A portfolio balance/current account model would attach wealth effects driven by the current account to the monetary model. If the current account was explained by current and lagged exchange rates (via a J-curve) and output, the first difference monetary approach model would be amended to give more complex polynomials in $\phi(L)$ and $\alpha(L)$. If wealth also affected the current account, the model would become even more complex.

Since the process governing the exchange rate in 1984 and 1985 appears to be of a low order, one may expect a simple theory (if any) to predominate. However, over such a small sample which even displayed an intra-sample regime switch, the power of the tests of any exchange rate theory must be very low. In the light of this, estimation in this section will only utilise the full data sample.

The presence of rational future expectations of the exchange rate in (3) will mean that the econometric solutions will be forward-looking and will require the current expectation of future values of the Z variables i.e. ${}_tZ_{t+j}$. This will become clear in Section 3b. The next section details the procedure used to form these expectations.

(a) Exogenous Variables and the Continuous Predictability Property

The term ${}_tZ_{t+i}$ for $i \geq 1$ represents expectations by agents in the market about current and future Z given information available at t. The most general way to obtain rational forecasts of the exogenous variables would be to use a vector autoregressive moving average model, and to simultaneously estimate the parameters of this vector process and of the exchange rate equation (for example see Woo (1985) and Hartley (1983); and Trevor and Donald (1986) for a VAR analysis for international asset prices). This procedure would be feasible if the data to be used conferred sufficient degrees of freedom on the estimation process. Except under very special conditions, the minimum available reporting periodicity amongst all the variables dictates the periodicity of the time series model. With the above two procedures, a quarterly (or perhaps with some major exclusion restrictions, a monthly) model of the exchange rate would be called for. This would be infeasible because of the short experience of the floating Australian dollar. Since one wishes to proceed with model estimation as soon as possible after a major regime switch, the following offers a neat and economically intuitive approach. The general idea is to obtain, say, quarterly forecasts and then undertake a weekly interpolation. Here, a simple interpolation is obtained using Muth's (1961) decomposition of the deseasonalised Z variables. One gets a continuous forecast which enables one to analyse weekly observations of the asset price, even though the periodicity of the explanatory variables may be higher.

Following Muth (1961) let Z be an ARIMA (0, 2, 1) process. Assume that the rate of growth of the explanatory variables are stationary in their means. Embodied in the observable rates of growth (μ_t) are two unobservable components - the permanent rate of growth (θ_t) and the transitory rate of growth (τ_t). The permanent rate of growth is modelled as a random walk. Summarising we have

$$\mu_t = Z_t - Z_{t-1} \tag{4a}$$

$$\mu_t = \theta_t + \tau_t \tag{4b}$$

$$\theta_t = \theta_{t-1} + \pi_t \tag{4c}$$

where π_t and τ_t are serially uncorrelated random variables having independent distributions with means of zero, and variances defined as σ_π^2 and σ_τ^2.[5]

If one wished to make a conditional estimate of a future value of the growth rate, ${}_t\mu_{t+k}$ for k>0, which minimised the variance of the forecast error, or

$$\min(\mu_{t+k} - {}_t\mu_{t+k})^2$$

then the optimal forecast has been shown by Muth (1961) to be

$${}_t\mu_{t+k} = \bar{\mu}_t = (1-\lambda) \sum_{j=1}^{\infty} \lambda^{j-1} \mu_{t+1-j} \tag{5a}$$

where[6]

$$\lambda = 1 + \frac{\sigma}{2} - \sqrt{\sigma^2 + 4\sigma} \tag{5b}$$

and $$\sigma = \frac{\sigma_\pi^2}{\sigma_\tau^2}$$

An important feature of (5) is that ${}_t\mu_{t+k}$ does not depend on k. This permits an extremely simple interpolation of forecasts with lower periodicity than the original data. We can redefine ${}_t\mu_{t+k}$ as $\bar{\mu}_t$. In these circumstances, the conditional expected growth rate (or the extracted permanent component) at any future date is equal to an exponentially weighted moving average of past observations of μ. Equivalently, it can be represented as an adaptive expectations process.

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5. The random walk model implies that the variances of θ_t, μ_t and Z_t are not independent of t. Initialising $\theta(0) = 0$, $\theta_t = \sum_{i=0} \pi_{t-i}$. Hence $\sigma_\theta^2(t) = \sigma_\pi^2 t$ This means that the variances grow linearly with time, and hence that the stochastic processes are not weakly stationary (order 2).

6. Muth (1961) assumed that there was a zero order moving average in (4b) and (4c). All the results would be unchanged if a moving average were permitted in either. σ in (4c) would have to be adjusted by a proportional factor of the form $(1+m_1^2+m_2^3 \ldots + m_k^2)$. The autocovariance function, below, would be lengthened accordingly thus maintaining identifiability.

$$\bar{\mu}_t - \bar{\mu}_{t-1} = (1-\lambda)(\mu_t - \bar{\mu}_{t-1}) \tag{6}$$

The weighting factor, λ, is seen to depend on the relative variance of the permanent to the transitory component. The weight attached to more recent observations increases with the relative variance; for relatively low σ_π^2, distant observations gain in importance enabling greater cancelling out of transitory effects.

Unfortunately, λ depends on apparently unobservable parameters, σ_π^2 and σ_τ^2. However an estimate of these can be retrieved from the autocovariance function of $(\mu_t - \mu_{t-1})$. To see this, add τ_t to both sides of (4b) and add and subtract τ_{t-1} to the RHS. This gives

$$(\theta_t + \tau_t) = (\theta_{t-1} + \tau_{t-1}) + (\tau_t - \tau_{t-1}) + \pi_t$$

or

$$(1-L)\mu_t = (1-L)\tau_t + \pi_t$$

where L is a lag operator. That is, μ_t is an ARIMA (0, 1, 1) process, and Z_t is an ARIMA (0, 2, 1) process.

The autocovariance function of $(1-L)\mu_t$ becomes

$$\begin{aligned} \text{cov}((1-L)\mu_t,\ (1-L)\mu_{t-j}) &= \text{cov}((1-L)\tau_t + \pi_t,\ (1-L)\tau_{t-j} + \pi_{t-j}) \\ &= 2\sigma_t^2 + \sigma_\pi^2 \quad \text{for } j = 0 \\ &\quad - \sigma_\tau^2 \quad \text{for } |j| = 1 \\ &\quad 0 \quad \text{for } |j| > 1 \end{aligned}$$

since π_t and τ_t are independent and serially uncorrelated.[7] Hence one can directly estimate, σ_τ^2 and σ_π^2 as

$$\hat{\sigma}_\tau^2(t) = -\frac{1}{N}\sum_{i=1}^{N} (\mu_{t+1-i} - \mu_{t-i})(\mu_{t-1} - \mu_{t-i-1}) \tag{7}$$

$$\hat{\sigma}_\pi^2(t) = -2\hat{\sigma}_\tau^2(t) + \frac{1}{N}\sum_{i=1}^{N} (\mu_{t+1-i} - \mu_{t-i})^2 \tag{8}$$

---

7. Muth (1961) demonstrated that a non-zero covariance between π and τ, $\sigma_{\pi\tau}$ would merely alter the definition of σ in (5) to $\sigma_\pi^2/(\sigma_t^2 + \sigma_{\pi\tau})$. Unfortunately, one would lose the identifiability property from the autocovariance function, and the procedure would be inoperative.

These estimates can then be used in (5b) to obtain $\hat{\lambda}_t$.[8] It is worth emphasising that the estimate of λ depends upon t. As data accumulates, the variance estimates are updated, and one may wish to interpret this as a learning process. The λ_t can be inserted in (5a) to get λ.[9] A time series of conditional expected growth rates is thus created which can be used to produce the expected levels of the exogenous variables, ${}_tZ_{t+i}$.

Remembering that Z is measured in logarithms, the expected levels are created by adding to the most recently announced observation Z_{T_j}, the associated expected growth rate, $\bar{\mu}_{T_j}$ multiplied by the amount of time j that has passed between t+i and the date to which the announcement applies, $T_j - k_j$ (k_j is the preparation lag of the data). That is,

$$ {}_tZ_{t+i} = Z_{T_j} + \bar{\mu}_{T_j}(t+i-T_j+k_j) \tag{9} $$

where $T_j \leq t < T_{j+1}$, $i > 0$

The growth rate and the time interval must be made dimensionally compatible. However, the virtue of (9) is that i can be chosen as a day, a week, a month, etc. Further, the logarithmic specification that generates (18) avoids the problem of taking the expected value of products associated with a non-logarithmic specification (see Cumby and van Wijnbergen (1983)).

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8. Cumby and van Wijnbergen (1983) obtain an identical estimate of $\hat{\lambda}$ by using the fact that the forecast error can not include information available at t. Although it is serially uncorrelated, they assume that the forecast error variance is constant over time. Any random walk process generates variances that are linearly dependent on time and the forecast error variance turns out to be a non-linear function of time, σ_π^2 and σ_τ^2. For large t, the difference between successive forecast errors is marginal provided σ_π^2 is small. Therefore series with a 'stable' permanent component would approximately satisfy the Cumby and van Wijnbergen assumption.

9. The infinite moving average has to be approximated by fixing a finite starting point, S periods in the past. To compensate for the approximation, thus ensuring that the sum of the weights is unity, (5a) was divided by $1-\hat{\lambda}_t^S$

For a weekly model of exchange rates, (9) can also be used to obtain current, but expected Z if the most recently announced observation occurred in any past period and with a non-zero reporting lag. This concept of imputing current values seems eminently reasonable. Asset prices do move on a daily basis, and often not in response to any apparent meaningful news. This may be due to noise, or perhaps bubbles as discussed in the next section, but it may be because of the imputed change in fundamentals. This imputation may be considered to be either market participants' conjectures or an estimate of an unobservable fundamental's effect on the flows in the foreign exchange market at that instant.

Exogenous Variable Forecasts

The Muth technique was applied to Australian M3 and nominal and real gross domestic product, and US M3 and nominal and real gross national product (all deseasonalised). The optimal forecasts were obtained using data going back to the beginning of 1979, sequentially adding on data as of the end of 1983. A summary of the results are shown in Table 3, where μ is the actual growth rate, λ is the estimated exponential parameter weight defined in (5b), $\bar{\mu}$ is the optimal forecast and $F\bar{\mu}$ is the forecast error. By comparing the root mean square error (RMSE) of μ to $F\bar{\mu}$, one can deduce the benefit from using the Muth forecast to that of a pure random walk for μ. Evidently, there is a gain, for all but real US GNP. In that latter case, λ was generally small, and so transitory shocks were relatively unimportant. Evidently, the application of the Muth technique to output appears inefficient. In contrast, Australian M3 was subject to some dramatic transitory shocks, especially in February to May 1985; for those months, λ had to be constrained to 0.999 which meant that the most recent observation was given almost equal weight as all past observations. Finally, all forecast errors had means that were not significantly different from zero, although Australian nominal GDP tended to be underestimated in 1985.

Table 3
Properties of the Exogenous Variable Forecasts

		Mean	Std Dev	Max	Min	RMSE
Australia						
M3 (monthly)	μ	.011	.009	–	–	.014
	$\underline{\lambda}$	.587	.053	.99*	.66	–
	μ	.012	.005	–	–	–
	$F\bar{\mu}$	−0.0008	.010	–	–	.010
Nominal GDP						
(quarterly)	μ	.032	.013	–	–	.034
	$\underline{\lambda}$	.625	.129	.80	.40	–
	μ	.029	.004	–	–	–
	$F\bar{\mu}$	.005	.018	–	–	.019
Real GDP						
(quarterly)	μ	.013	.014	–	–	.010
	$\underline{\lambda}$	.866	.191	.99*	.44	
	μ	.008	.004	–	–	
	$F\bar{\mu}$	−.006	.015	–	–	.016
USA						
M3 (monthly)	μ	.007	.004	–	–	.008
	$\underline{\lambda}$	.452	.028	.38	.30	–
	μ	.007	.002	–	–	–
	$F\bar{\mu}$	−.0003	.004	–	–	.004
Nominal GDP						
(quarterly)	μ	.010	.014	–	–	.018
	λ	.309	.163	.61	.20	–
	μ	.011	.011	–	–	–
	$F\bar{\mu}$	−.003	.018	–	–	.018
Real GDP						
(quarterly)	μ	.004	.016	–	–	.019
	$\underline{\lambda}$	.496	.145	.83	.33	
	μ	.005	.012	–	–	
	$F\bar{\mu}$	−.002	.018	–	–	.016

* Constrained value

(b) A Simple Model

Consider the simplest member of (3).

$$e_t = \alpha Z_t + \beta_t e_{t+1} + u_t \tag{10}$$

This model is used to demonstrate the econometric aspects of rational expectations used. In Section 2.3, more complex models are introduced.

The exchange rate at t depends on the expectation of its value at t+1. With rational expectations, the model then suffers from the well-known problem of multiple solutions. There are two strategies for solving the problem.

The first and least restrictive solution (see Chow (1983)) utilises the weak rationality assumption that the unexpected component of the exchange rate arises only on account of information that appeared after the expectation was formed. The second is the standard method of finding the convergent solution for the deterministic difference equation in future expectations. It is akin to the saddlepath solution of perfect foresight models. The first strategy allows the data to solve for the unique solution by estimation.

Weak Rationality

The assumption of weakly rationality implies that the conditional forecast error of the exchange rate does not depend on information available at t.

$$e_{t+j} - {}_te_{t+j} = \varepsilon_{t+j} \tag{11a}$$

where

$$\begin{aligned}\varepsilon_{t+j} &= R_0\, u_{t+j} + R_1\, u_{t+j-1} \cdots + R_j u_t \\ &\quad + K_0(Z_{t+j} - {}_tZ_{t+j}) + K_1(Z_{t+j-1} - {}_tZ_{t+j-1}) \\ &\quad \cdots\cdots + K_{j-1}(Z_{t+1} - {}_tZ_{t+1})\end{aligned} \tag{11b}$$

The forecast error is conjectured to depend on unexpected events that occur between t and t+j. By setting j=0, noting that current Z is observable and that u_t has a zero expected value, we get $e_t - {}_te_t = R_0u_t$. But from (11a), since ${}_te_t = \alpha Z_t + \beta_te_{t+1}$. Hence $R_0 = 1$. The remaining $R_1 \ldots R_j$, $K_0 \ldots$ K_{j-1} coefficients are free parameters which have to be estimated. To obtain an equation to estimate consider (10) dated at t+j, take expectations as of t, apply the law of iterated expectations, and replace the expected terms from (11). This leaves an equation in observables only (apart from the question of expected Z which was discussed in the previous section) with a jth order moving average error process

For j=0, this becomes

$$\begin{aligned}e_{t+1} &= \frac{1}{\beta}\, e_t - \frac{\alpha}{\beta}\, Z_t - K_0(Z_{t+1} - {}_tZ_{t+1}) \\ &\quad + u_{t+1} - (\tfrac{1}{\beta} - R_1)u_t\end{aligned} \tag{12}$$

This approach is extremely general in that it does not place any restrictions on the R_i and K_i parameters. The weak rational expectations assumption is essentially myopic, since long run convergence of future expectations is achieved only if all the free parameters have estimated values that are consistent with that property. In particular, it can be seen in (12) that if $R_1=0$, the AR and MA processes have a common root $(1/\beta)$ which allows cancellation down to a zero order equation.

Strong Rationality and Convergent Expectations

In stochastic systems, variables which depend upon their future expectations are the equivalent of non-predetermined variables in deterministic systems. For a unique solution to either system, the number of "unstable" eigenvalues must equal the numbers of these variables.[10]

Leading (10) and taking expectations yields a deterministic ordinary difference equation in expected future exchange rates:

$$ {}_te_{t+2} = \frac{1}{\beta}\, {}_te_{t+1} - \frac{\alpha}{\beta}\, {}_tZ_{t+1} $$

This can easily be solved to give the general solution combining particular and homogeneous elements. One obtains:

$$ {}_te_{t+1} = \alpha \sum_{i=0}^{\infty} \beta^{i}\, {}_tZ_{t+1+i} + d\beta^{-t-1} + \sum_{i=t-T}^{t} \beta^{i-t-1} S_i \qquad (13) $$

The first part of the solution is often referred to as the fundamentals and is the sum of discounted expected future Z's. The existence of the forward sum requires $\beta < 1$. The second term is known as a deterministic bubble, while the third is a stochastic bubble.[11] d is an arbitrary constant, while S_i is a serially uncorrelated random vector which has the critical feature that

$$ {}_tS_{t+j} = 0 \text{ for all } j > 0. $$

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10. For deterministic systems, see Blanchard and Kahn (1980); for a survey on stochastic system solutions, see Taylor (1985).

11. For example, see Blanchard and Watson (1982) and Diba and Grossman (1983). Deterministic bubbles have the unappealing properties that they do not contribute to the variance of the asset price, and that, if present, they must always have been there. Flood and Garber (1980) could not find a deterministic bubble in the German hyperinflation.

When one inserts the solution for the expected exchange rate (13) into (10), one gets

$$e_t = \alpha \sum_{i=0}^{\infty} \beta^i \, {}_tZ_{t+i} + d\beta^{-t} + \sum_{i=t-T}^{t} \beta^{i-t} s_i + u_t \tag{14}$$

The strong convergent solution for the exchange rate excludes bubbles of any form, so that d and s_i are always zero, or

$$e_t = \alpha \sum_{i=0}^{\infty} \beta^i_t Z_{t+i} + u_t \tag{15}$$

Computing (15) at t+1 and subtracting from it, $\frac{1}{\beta}$ of (15) at t gives the strong form as

$$e_{t+1} - \frac{1}{\beta} e_t = \alpha \sum_{i=0}^{\infty} \beta^i_{t+1} Z_{t+1+i} - \alpha \sum_{i=0}^{\infty} \beta^{i-1} {}_tZ_{t+i} + u_{t+1} - \frac{1}{\beta} u_t$$

$$= \frac{-\alpha}{\beta} Z_t + \alpha \sum_{i=0}^{\infty} \beta^i ({}_{t+1}Z_{t+1+i} - {}_tZ_{t+1+i}) + u_{t+1} - \frac{1}{\beta} u_t \tag{15'}$$

Comparing (15') to the weak form solution (12), one can see that the two are identical in <u>expectations</u> as of t (because the law of iterated expectations eliminates the forecast error terms). However, the solutions for the actual exchange rate differ because of the existence of K_0 and R_1 in (12) and because updated forecasts of all future Z between t and t+1 are relevant in (15'). Fortunately, because of the law of iterated projections, it turns out that (15') is a special case of (12) if $R_1 = 0$.

(c) <u>Higher Order Models</u>

The simple model of (10) is inappropriate for estimation because we know from Section 1 that a first difference model is needed to avoid non-stationarity. The drift and a possible first order ARMA process presents an opportunity to seek out a multivariate explanation. Two basic alternatives are pursued - Monetarist and Keynesian.

First consider the first differenced (∇), two country, monetary approach with a common interest elasticity parameter, stochastic purchasing power parity and uncovered interest parity (see Hartley (1983)).

$$\nabla m_t - \nabla m_t^* = \nabla p_t - \nabla p_t^* + \alpha_y \nabla y_t - \alpha_y^* \nabla y_t^* - \beta\,(\nabla i - \nabla i^*) + v_{1t}$$

$$\nabla p_t = \nabla p_t^* - \nabla e_t + v_{2t}$$

$$\nabla i_t = \nabla i_t^* - \nabla({}_t e_{t+1} - e_t) + v_{3t}$$

where m is the domestic money supply, y is real output, i is the interest rate, p is the price level and "*" variables are the foreign counterparts. The three equations reduce to:

$$\nabla e_t = -\frac{1}{1+\beta}\,[(\nabla m_t - \nabla m_t^*) - \alpha_y \nabla y_t + \alpha_y^* \nabla y_t^*] + \frac{\beta\nabla}{1+\beta}\,{}_t e_{t+1} + u_t \qquad (16)$$

Applying weak rationality to $\nabla_t e_{t+1}$, the equation for e turns out to be an ARIMA (1,1,2). Since the time series model seems to indicate, at most, an ARIMA (1,1,1) process it would seem that only the strongly convergent solution may be appropriate. Indeed, the weakly rational model failed to converge. Given the method of solving for the expected future value of exogenous variables discussed in Section 2.1, the convergent solution is easily computed[12] to be

$$\nabla e_t = (1+\beta\nabla)[\overline{\overline{\nabla m}}{}_t^* - \overline{\overline{\nabla m}}_t - \alpha_y^* \overline{\overline{\nabla y}}{}_t^* + \alpha_y \overline{\overline{\nabla y}}_t] + u_t \qquad (17)$$

where the $\overline{\overline{\nabla z}}_t$ are expected growth rates as of t.

It can be seen that in a stochastic steady state, exchange rate appreciation simply reflects relative expected money growth and output growth weighted by its elasticity. Outside of the steady state, a weighted average of current

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12. Taking expectations of (16) as of t-1, and writing the vector of explanatory variables and parameters inside the square brackets as ∇Z_t and α, we get

$$(1+\beta(1-\tfrac{1}{L}))_{t-1}\nabla e_t = \alpha_{t-1}\nabla Z_t \text{ which factorises to give}$$

$$_{t-1}\nabla e_t = \left(\frac{1}{1+\beta} + \frac{\beta L^{-1}}{(1+\beta)^2} + \frac{\beta^2 L^{-1}}{(1+\beta)^3} \ldots\right) \alpha_{t-1}\nabla Z_t$$

$$= \alpha_{t-1}\nabla Z_t = \alpha\bar{\mu}_{t-1}$$

because, from (5a), $L^{-j}{}_{t-1}\nabla Z_t = \bar{\mu}_{t-1}$. Hence (16) becomes

$$\nabla e_t = \alpha\nabla Z_t + \beta\alpha(\bar{\mu}_t - \bar{\mu}_{t-1}) = \alpha[\bar{\mu}_t + \beta(\bar{\mu}_t - \bar{\mu}_{t-1})] = (1+\beta\nabla)\alpha\bar{\mu}_t$$

and lagged values of these expected terms. This monetarist equation would be attempting to explain the drift as a time-varying phenomenon, and is evidently an ARIMA (0,1,0) process. An important feature is that an increase in the currently expected domestic (foreign) rate of growth of money leads to a larger depreciation (appreciation).

In Table 4, line 1, the OLS estimates of the parameters in equation (17) are presented.

The estimated sign of α_y is incorrect, all the parameter estimates are insignificant, the errors are autocorrelated and the sum of squared errors shows no improvement upon the random walk model - on an F test ($F(2,98) = 3.24$)), the marginal significance exceeds 5 per cent. Similar results are obtained if disequilibrium money markets are assumed, but with the additional problem that the estimated effect of lagged money implied unstable money markets. The conclusion is that the joint test of the first difference monetary model, convergent rational expectations, and the hypothesised process governing expectations of the money and real output variables must be rejected.

Having rejected the first differenced monetarist model, it would seem sensible to entertain the following alternative. Interest rates are now postulated to be determined in the money market, and the implied interest rate differential then drives the expected change of the exchange rate. Output and prices are predetermined and instantaneous purchasing power parity does not hold. Nominal output, Y_t and Y_t^*, is assumed to be the scale flow variable. Net wealth effects may also influence money demand; and these are assumed to be correlated with past exchange rates and/or money supplies and/or nominal output. In general, consider the following form of an inverted relative money demand equation

$$ {}_te_{t+1} - e_t = \frac{1}{\beta} [\alpha_0 + \alpha_e(L)e_t + \alpha_m(L)m_t - \alpha_y(L)Y_t - \alpha_m^*(L)m_t^* + \alpha_y^*(L)Y_t^*] + u_t \qquad (18) $$

The lag polynomial[13] $\alpha_e(L)$ which provides an error correction mechanism had a maximum order of 1. If at least one of the first elements in $\alpha_m(L)$, $\alpha_m^*(L)$, $\alpha_y(L)$ or $\alpha_y^*(L)$ are not restricted to unity, then β cannot be identified in (18). For the reason of parsimony, the lag polynomials in money and output of both countries were restricted to a zero-order. Applying weak rationality, (18) becomes

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13. The use of the non-stationary exchange rate level as a regressor is acceptable under fairly general conditions. Classical inference using OLS estimates is appropriate provided the unconditional mean of the first difference is zero. See West (1986).

Table 4

Multivariate Time Series Models: Change in the Spot Rate, ∇e_t

Parameters (Expected Sign)	α_0	α_e (-)	α_m (+)	α^*_m (+)	α_y (+)	α^*_y (+)	K_m (+)	K^*_m (+)	K_y (+)	K^*_y (+)	β (+)	R_1	S.E.E.	R^2	BP(18)	ARCH(8)	KD
17	-	-	-	-	-.02 (.22)	.35 (.43)	-	-	-	-	6.04 (7.25)	-	3.48%	2.3%	22.40 [.22]	18.36 [<.05]	.082 [.10]
19ai	-.82 (.05)	-	1	1	1	1	.55 (.66)	1.48 (1.79)	-.09 (.26)	.30 (.45)	12.4 (13.5)	.86 (.11)	3.51%	5.4%	20.07 [.33]	24.71 [<.01]	.117 [<.01]
19aii	-.10 (2.04)	-	.11 (.13)	.30 (.19)	-.07 (.2)	.12 (.24)	.60 (.68)	.24 (.46)	-.11 (.26)	.24 (.46)	-	.88 (.11)	3.44%	7.3%	20.62 [.30]	24.58 [<.01]	.090 [.04]
19aiii	-6.8 (14.3)	-	1	1	.81 (.39)	1.37 (1.63)	.55 (.67)	1.70 (1.83)	-.06 (.26)	.27 (.46)	7.15 (6.29)	.86 (.11)	3.49%	6.1%	21.52 [.25]	23.75 [<.01]	.095 [.02]
19bi	.01 (.08)	-.19 (.08)	1	1	1	1	.52 (.66)	1.45 (1.77)	-.03 (.25)	.40 (.44)	4.64 (2.36)	.90 (.11)	3.39%	9%	20.32 [.32]	20.99 [.025]	.095 [.025]
19bii	.37 (1.31)	-.23 (.03)	.07 (.08)	.88 (.15)	-.07 (.12)	.77 (.18)	.68 (.58)	1.80 (1.6)	-.12 (.22)	.82 (.40)	-	.78 (.11)	2.57%	31%	18.40 [.43]	14.67 [>.10]	.057 [>.15]
19biii	-11.44 (12.1)	-.60 (.44)	1	1	2.18 (1.11)	3.47 (2.22)	.46 (.66)	1.28 (1.80)	.001 (.25)	.49 (.45)	5.93 (3.92)	1.05 (.11)	3.25%	12%	19.39 [.37]	20.34 [<.01]	.073 [>.15]

1. See Notes on Table 1.
2. The sample size was 101 for all regressions.

$$\nabla e_{t+1} = \frac{1}{\beta}[\alpha_0 + \alpha_e e_{t-1} + \alpha_m m_t - \alpha_Y Y_t - \alpha_m^* m_t^* + \alpha_y^* Y_t^*]$$

$$+ K_m \, {}_tFm_{t+1} - K_y \, {}_tFY_t - K_m^* \, {}_tFm_{t+1}^* + K_y^* \, {}_tFY_{t+1}^*$$

$$+ u_{t+1} + (1-R_1)u_t \qquad (19)$$

where ${}_tFZ_{t+1} = Z_{t+1} - {}_tZ_{t+1}$ is the unexpected innovation in Z between t and t+1. The coefficient α_e is expected to be negative: one possible explanation for this error correction term is that a previous depreciation will worsen the current account,[14] hence raising net foreign debt, lowering relative money demand and interest rates and thus increasing the rate of appreciation.

In Table 4, six variations of equation (19) are estimated. In (19a), the error correction mechanism is absent, while in (19b) it is included. All parameters are expected to be positive except α_e (and α_o which can take any sign). The K parameters measure the effect of unexpected money and nominal output.

(19ai) includes no exchange rate lags and fixes the parameters on money and nominal output at unity, thus testing the homogeneity postulate. Only the constant term and the moving average term have significant coefficients. The equation obviously does no better than the random walk with drift, with the errors still displaying significant (autoregressive) heteroscedasticity and non-normality. The test of homogeneity comes from a comparison of (19ai) with (19aii) and (19aiii). With regard to the latter, the output parameters are freely estimated and are not significantly different from unity. The former provides unrestricted estimates, and on an F-test ($F(3,91)=.62$) there is no significant improvement over the restricted equations. Whilst homogeneity cannot be rejected, none of the (19a) equations is superior to the random walk and errors are not spherical.

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14. In the short period under consideration, one would not expect to be able to pick up the favourable side of the J-curve response to the current account. One of the principal problems in Australia in 1984/85 has been the failure to reach the turning point in the J-curve. This means that the estimated error correction equation (19) will not appear to have sensible long run properties. However, note that nominal income appears in (19): in the long run, real income takes on its natural rate, while relative prices reflect the exchange rate through purchasing power parity. Hence the long run properties will be sensible so long as $|\alpha_{e2}|$ is less than nominal income parameters.

On introducing the error correction term, the results change dramatically. (19bi) imposes full homogeneity and compared to (19ai), an improvement is accepted at a 10% significance level ($F(1,93)=3.29$). Compared to the random walk there is still no significant improvement ($F(7,93)=1.25$). The errors still display heteroscedasticity and non-normality.

Equation (19bii) gives unrestricted estimates of money and nominal output parameters and a significant improvement over all previous models is achieved. Compared to the random walk and to (19bi) we get $F(11,90)=3.63$ and $F(3,90)=9.57$ respectively which both have a marginal significance less than 1%. Normality, homoscedasticity and no serial correlation of the errors can not be rejected in (19bii). All the coefficients on US variables have the correct sign and are significant (except unexpected money). Unfortunately, the Australian variables do not perform well, especially output. In Figure 2, the predicted and actual rate of change of the exchange rate are presented. The model does remarkably well in picking up a high percentage of the turning points. When it fails to do so, it often achieves the second best alternative of foretelling. Towards the end of the sample, the model tends to under-estimate the scale of the changes. It is worth emphasising that current account announcements were not explicitly modelled (primarily because they were not amenable to the Muth technique). At least five of the unexplained dramatic changes in the exchange rate in 1985 coincided with current account announcements.

Equation (19biii) restricts only the coefficients on money to unity. Compared to the unrestricted model, these restrictions are rejected ($F(1,90)=23.81$) but compared to the full homogeneity model (19bi), this equation is not preferred ($F(2,91)=1.96$). Compared to the random walk this equation does no better ($F(9,91)=1.43$). Unfortunately, the error now displays heteroscedasticity, though normality cannot be rejected. All the parameters have the correct sign, and Australian output becomes significant.

The weak rationality moving average parameter, R_1, was generally significant. The lagged residual has an impact through $(1-R_1)$ on the current exchange rate that is not significantly different from zero; the marginal significance on a one-tailed test for the preferred equation, (19bii), is less than 10 per cent.

FIGURE 2
CHANGE IN $US-$A SPOT EXCHANGE RATE LOGARITHMS

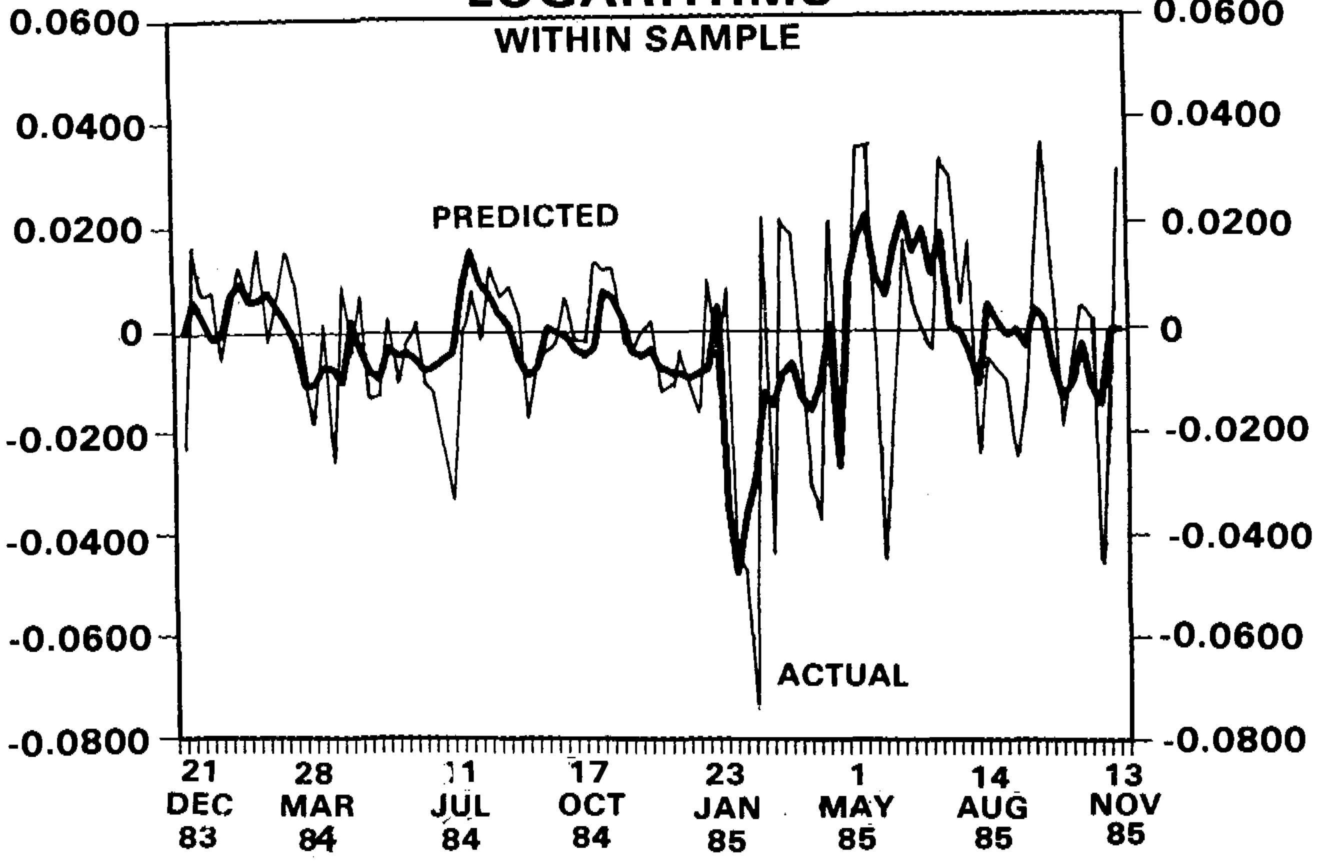
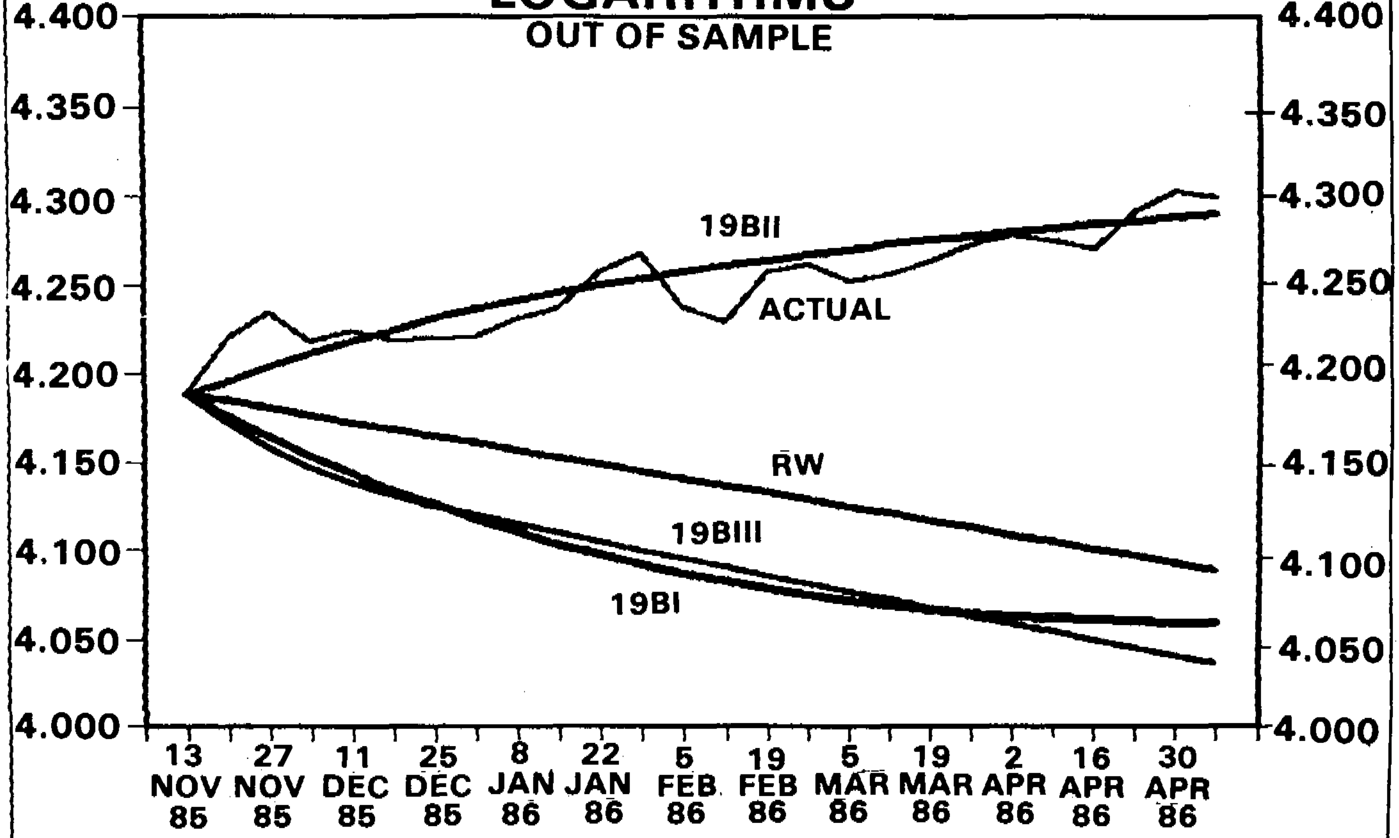


FIGURE 3
UNCONDITIONAL FORECASTS OF $US-$A LOGARITHMS



A key test of model capability is obtained with out of sample forecasting. In this regard, the actual exchange rate outcome from 20 November 1985 to 7 May 1986 was compared with the unconditional forecasts from the following four models: the random walk with drift (as estimated in Section 2) and the three error correction "Keynesian" models reported in Table 4 (i.e. homogeneity assumptions on money and income, unrestricted estimates and homogeneity on money alone).

Table 5
Unconditional Out of Sample Forecasts of e_t

	Root Mean Square Error	Mean Absolute Error
Random Walk with Drift	.125	.111
Full Homogeneity (19bi)	.164	.151
Unrestricted (19bii)	.167	.153
Partial homogeneity (19biii)	.014	.011

Table 5 and Figure 3 show that, the random walk model beats the model that best explained the actual sample (19bii). In fact, the latter does worst in the out-of-sample test. The partial homogeneity model (unit coefficients on the money variables) now does about ten times better than the random walk. Of the four models, only the partial homogeneity model predicted a strengthening of the exchange rate; at the end of the forecast interval, it reached 72.97 compared to the actual of 73.67. The other three models predicted a decline. This success of the partial homogeneity model was to be short-lived, because the exchange rate subsequently depreciated substantially. With conditional forecasting, one would expect the multivariate models to improve their performance.

4. Conclusions

The Australian dollar at first sign appeared not to be a random walk. This would mean that profits could be made by so-called "technical" analysis. When account was taken of the appropriate distribution under the null of unit roots and of the existence of heteroscedasticity (and the implied non-normality), a random walk with drift could not be rejected. This result bears out the conclusion of Lowe and Trevor (1986) that exchange rate forecasters did not do better than a random walk for one-step predictions.

There appeared to be significant differences between the first and second years of the float. While parameters were not significantly different, the exchange rate process in the first year was normal and not heteroscedastic, with an opposite result for the second year. This conclusion is consistent with the dramatically increased variability of the exchange rate in the second year. When monetary targetting was abandoned in 1985, evidently exchange rate targetting did not take its place; indeed, monetary policy was conducted on the basis of a checklist of key economic variables.

The first-differenced monetarist approach to flexible exchange rates did not (variance) encompass the univariate time series model, and the data evidence produced insignificant parameters. Dropping purchasing power parity, and using the monetary model in levels did give results that encompassed the univariate model. Higher (expected) U.S. money and lower output tended to significantly increase the rate of depreciation of the Australian dollar. The insignificant effects of Australian M3 can be attributed to the ever increasing difficulty in forecasting this variable over the sample. The process of de-intermediation and re-intermediation introduced a great deal of uncertainty (and thereby lack of faith) associated with recent observations of this variable. The exchange rate does not appear to have been significantly affected by Australian money and output, a result which is consistent with the Trevor and Donald (1986) conclusion that the trade-weighted exchange rate index appears independent of Australian interest rates.

The general conclusion is that there appears to be a structural model which will dominate the random walk. The structural results are conditioned by the very restrictive assumptions made about the generation of the expected future values of the predetermined variables. These restrictions were necessary to permit these early results. The results of this paper will give encouragement to structural exchange rate model builders when sufficient data has accumulated to undertake a less restrictive study. In particular, the simultaneous modelling of the current account and the exchange rate is bound to improve the explanation of the data generation process.

APPENDIX

Data Sources

e_t	Australian-U.S. Dollar Exchange Rate, Wednesdays, Commonwealth Bank, Sydney	
m_t	Australian M3, Monthly, Reserve Bank of Australia Bulletin	
m_t^*	U.S. M3, Monthly, Federal Reserve Board Bulletin	
Y_t	Australian Nominal GDP	Quarterly, OECD Main Economic Indicators
P_t	Australian GDP Deflator	Quarterly, OECD Main Economic Indicators
Y_t^*	U.S. Nominal GNP	Quarterly, OECD Main Economic Indicators
P_t^*	U.S. GNP deflator	Quarterly, OECD Main Economic Indicators

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