Heterogeneous Global Cycles *

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Abstract

Why do countries differ in terms of their exposure to fluctuations in the global supply of credit? We argue that frictions in global intermediation lead to an endogenous partitioning of economies into groups with low and high exposure to the global credit cycle. We show that investors with varying degree of information hold dissimilar portfolios, with low skilled investors sharply rebalancing their cross-country asset holdings across different aggregate states. The differential response of investors invites differential strategies of firms, jointly shaping heterogeneous global cycles. We connect the implications of our model to stylized facts on credit spreads, investment, safe asset supply, concentration of debt ownership, and the return on debt during various boom-bust episodes, both in the time series and in the cross-section. We demonstrate that a global savings glut not only exacerbates both booms and busts in high exposure countries, but also increases the exposure of some countries to credit cycles.

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1 Introduction

For decades before 2008, boom-bust cycles had been associated almost exclusively with emerging markets. The pattern a boom phase started by poorly regulated financial liberalization leading to a surge in foreign capital, large credit flows to the non-financial sector, build-up of debt at low interest rates, and rapidly increasing investment abruptly turning to a bust phase where interest rates spike and credit flies to safety, triggering a collapse in output — has been connected to a large catalog of structural weaknesses in Latin American, East Asian, and Eastern European economies.

However, the global financial crisis in 2008 and especially the Eurozone crisis in 2010 have dramatically exposed similar vulnerabilities in a group of advanced economies. This has in turn led to a shift in focus on the role of increasingly globalized financial intermediation and the implied changes in global capital supply.[1]

In this paper, we explore how fluctuations in global supply of capital lead to heterogeneous cycles across countries. We argue that frictions in global intermediation lead to a partitioning of economies into groups with low and high exposure to global credit cycles. We connect the implications of our model to stylized facts on credit spreads, investment, safe asset supply, concentration of debt ownership, and return on debt during various boom-bust episodes, both in the time series and in the cross-section. We further demonstrate that a global savings glut not only exacerbates both booms and busts in the high exposure region, but also increases the exposure of some countries to credit cycles.

We develop a model where firms across countries compete for capital from international investors. In our framework, firms operate a Holmström and Tirole (1998) technology. They allocate their endowment between investment and precautionary savings to manage the risk of future liquidity shocks. A firm that is hit by a liquidity shock has to either pay a maintenance cost, or abandon production. To pay the additional cost, firm can also access international capital markets for credit. However, a pledgeability constraint implies that it has to cover part of its financing needs from its own savings.

The key friction in our model is that international investors are heterogeneous in their skill to identify whether a firm's collateral is good or bad. Moreover, investor expertise is more important for identifying good firms in certain countries, which we refer to as opaque countries. While firms are heterogeneous in terms of their collateral quality within a country, all countries are identical in the composition of their firms.

Furthermore, we assume that investors' *prudence*, or the type of their information, varies with the aggregate state. In the high state investors are *bold*: they can identify some firms with bad collateral, whereas they cannot distinguish some others from firms with good collateral. Thus, they can avoid missing out on any good opportunities at the expense of extending loans to some bad firms

[1] For instance, see Caballero et al. (2017) and citations therein on the role of global scarcity of safe assets, Caballero and Simsek (2016) on fickle capital flows, and Avdjiev et al. (2016) on how globalization has pushed decisions on credit supply outside the boundaries of affected countries, which is a new phenomenon for advanced economies.

by mistake. On the contrary, in the low state they are *cautious*: they can identify all bad firms, but they also mistake some good firms for bad ones. This implies that they can avoid financing any firm that may not repay, even if doing so leads them to forgo some profitable investment opportunities. It then follows that investors lend to different firms in different aggregate states. The differential response of investors across countries invites differential strategies of firms, shaping heterogeneous global cycles.

In particular, as a rational response to their imperfect information, low skilled investors who are heavily invested in opaque countries during booms, re-balance away towards more transparent countries in busts. As a result, our model features boom-bust cycles with heterogeneous exposures across countries.[2] The most opaque countries form a high exposure group, which we also refer to as *periphery*. During booms, firms in these countries enjoy large credit inflows at low interest rates and high growth. However, during busts the firms in these countries can obtain new credit only at high rates, if at all, and their output and credit flows collapse. Instead, international capital floods a group of more transparent countries at low interest rates, their transparency effectively shielding them from negative exposure to the global cycle. We refer to this latter group of countries as low exposure or core economies.

Our model implies a qualitative difference in the functioning of credit markets between booms and busts. In booms, firms borrow at the same rate in core and periphery economies. In this state, credit quality is heterogeneous across investor portfolios, and highly skilled investors derive excess returns by extending credit to higher quality borrowers across all countries. In contrast, during busts there is a significant spread for borrowing between firms in core and periphery economies. In this state lenders are cautious, which implies the same credit quality across their portfolios. As such, highly skilled investors derive excess returns by lending at higher rates to good but opaque firms in periphery economies. This picture rationalizes the (sometimes puzzlingly) low premium on emerging market assets before the East-Asian and Russian Crises and assets in the south of Europe before the Eurozone Crisis (e.g. Kamin and von Kleist, 1999; Duffie et al., 2003; Gilchrist and Mojon, 2018).

The real investment and output in each country is determined by how firms trade off investment and liquidity risk management, which is in turn driven by the credit market conditions they face. This trade-off leads to risky investment decisions by firms in the periphery. Put differently, firms in high exposure countries *gamble*: they produce at a high scale iduring booms (when credit is cheap), at the expense of abandoning production in busts (when credit is expensive). Therefore, when investors are bold, both core and periphery economies enjoy a high output. However, when investors turn cautious, international credit markets become plagued by funding mismatch and the high exposure countries undergo a drastic output collapse. Thus, the two groups of countries

[2]Ivashina et al. (2015) and Gallagher et al. (2018) find that a group of money market funds stopped lending only to European banks, and not to other banks with similar risk in 2011. Ivashina et al. (2015) find evidence that this lead to significant disruption in the syndicated loan market. These facts are broadly consistent with our proposed mechanism.

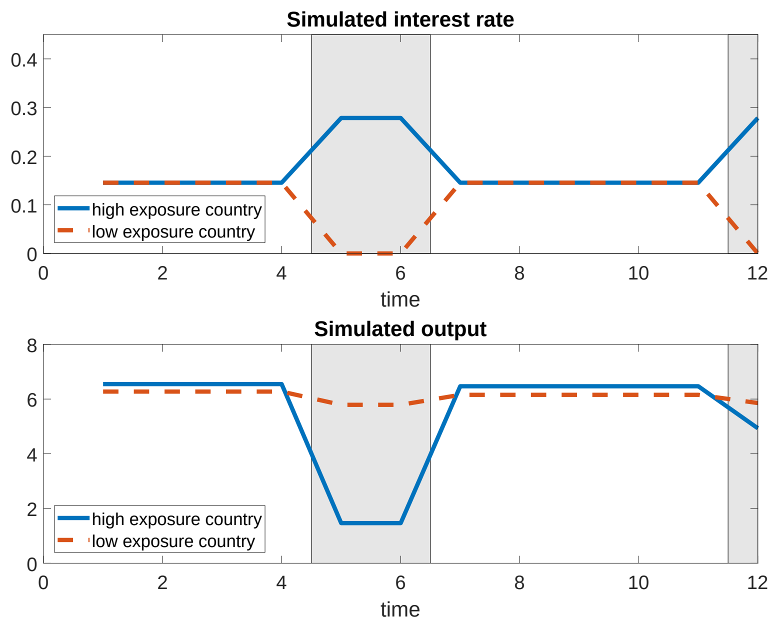


Figure 1: Simulated interest rate and output paths for a low (solid) and a high (dashed) transparency country. Shaded areas correspond to low aggregate states.

experience very different real outcomes during downturns.

Our model also suggests that most of the non-performing debt is issued in booms, in the periphery economies, and is financed by low skilled investors. This is consistent with the observed increase in the misallocation of capital during the pre-crisis years in the south of Europe (Reis, 2013; Gopinath et al., 2017).

The model provides further, yet-to-be-tested predictions about investor portfolio compositions throughout the cycle. It implies that ownership of debt is most concentrated during busts, especially in high exposure countries. In addition, the realized return on the sovereign bonds issued in booms (busts) in a given country is higher (lower) in low than high exposure countries.

Our framework emphasizes the roles of sophisticated and unsophisticated capital in international capital markets. We argue that this mechanism also sheds new light on the effects of global saving glut on investment cycles as well as safe asset determination. To highlight this, we analyze the effect of increased capital supply by low skilled investors. Consistent with the literature on rising global imbalances (for a review, see Caballero et al., 2017), this decreases the yield on bonds in a boom. It also increases the supply of safe assets as defined by He et al. (2016), but not nearly enough to satisfy the increasing demand for safe assets in busts. Not only do the high exposure countries experience an exacerbated boom-bust cycle, but some countries are also pushed from the moderate to the high exposure group during the bust.

To best illustrate how our model generates heterogeneous global cycles, we build a simple dynamic version of the model with consecutive generations of firms and investors. Figure 1 illustrates the simulated path for yields and output for a representative core and periphery economy. The output of the periphery economy is larger when investors are bold, but collapses sharply when investors turn cautious (shaded areas), and the yield at which its credit is traded spikes. The core economy experiences only a moderate drop in its output during the bust, and the yield on its bonds can even drop.

Finally, our paper provides a novel equilibrium framework to study investment decisions and equilibrium pricing outcomes in an asymmetric information environment where there is two-sided heterogeneity. We believe that this is a parsimonious model well-suited to explore a broader set of questions concerning the interaction among financial institutions and the spillover to the real economy.

Related Literature. Our paper is the first to show that frictions in the global supply of capital leads to an endogenous partitioning of countries into low and high exposure groups, creating heterogeneous global cycles. It is related to a large and diverse body of work that studies international output and credit cycles.

First, our paper contributes to the extensive literature started by Kiyotaki and Moore (1997), which generates boom and bust patterns from financial frictions. In this line of work, a collapse in the value of the collateral leads to a tightening of the credit constraint in recessions (e.g. Kiyotaki and Moore, 1997; Lorenzoni, 2008; Mendoza, 2010; Gorton and Ordonez, 2014). Our mechanism does not operate via tighter collateral constraints in recessions. Instead, in the face of an adverse prudence shock low skilled investors find it optimal to re-balance their portfolios towards firms in more transparent countries. As a consequence, on top of the time-series pattern, we can derive predictions about the cross-sectional differences of real outcomes across countries.

There is also a group of papers that connect flight-to-quality episodes to international risk-sharing.[3] For instance, Gourinchas et al. (2017) and Maggiori (2017) argues that since the US financial sector is less risk averse or less constrained than others, it takes a leveraged position in the global risky asset in booms and deleverages during busts. Given the two-country representative agent approach of these papers, they are better suited to capture the characteristics of capital flows between the US and the rest of the world. Instead, we focus on the detailed interaction between heterogeneous global financial institutions and local firms. As such, our approach is more useful to explore other dimensions of the data, such as the real effects of the heterogeneous re-balancing of asset managers, the time-series and cross-sectional differences in returns, the distribution of non-performing debt, and the concentration of debt ownership. Therefore, we think of our modeling approach as being complementary to this literature.

Another stream of literature studies why sudden stops are more frequent in emerging market countries. Aguiar and Gopinath (2007) and Rey and Martin (2006) point to technological differences, while Eichengreen and Hausmann (1999), Caballero and Krishnamurthy (2003), and Broner

[3]See Gourinchas and Rey (2014) for a detailed review

and Ventura (2016) point to differential incentives for saving in foreign versus domestic currency, as a consequence of differences in country fundamentals. In contrast, we propose a mechanism which implies that heterogeneous patterns can arise within advanced economies as well, where the technology, the level of human and physical capital, and the legal-economic-political system are similar.

Turning to the Eurozone crisis, a series of papers emphasize a wide range of mechanisms including less stringent credit constraint for large and inefficient firms (Reis, 2013; Gopinath et al., 2017), the political connections of some banks and firms (Cuñat and Garicano, 2009), compromised structural reforms (Fernandez-Villaverde et al., 2013), the role of downward wage rigidity (Schmitt-Grohé and Uribe, 2016), the role of perceived risk of a eurozone breakup (Battistini et al., 2014), the interaction between risk-shifting incentives of banks and sovereigns (Farhi and Tirole, 2016), coordination problems between monetary and fiscal policy (Aguiar et al., 2015), and the role of private debt expansion (Martin and Philippon, 2017). Our mechanism is complementary to these papers. Furthermore, while the loose financing conditions and the resulting ex-ante expansion of debt in periphery countries is exogenous in this stream of papers, our model is able to generate this pattern endogenously.

We are related to the literature which connects international capital flows to safe asset scarcity (He et al., 2016; Caballero et al., 2017; Farhi and Maggiori, 2017). Our model derives the equilibrium supply and demand for safe assets from informational frictions, a new element in this literature. As we demonstrate in Sections 4.3 and 5.3, this approach generates novel predictions.

There is also a group of finance papers which rationalize flight-to-quality in the financial markets using a Knightian-uncertainty shock (Caballero and Krishnamurthy, 2008), fund managers' incentives (Vayanos, 2004), or adverse-selection (Fishman and Parker, 2015). We add to this literature by explicitly modeling the interaction between flight-to-quality and real investment decisions, which contributes to the differences in credit cycles across countries.

Finally, we contribute to the large theoretical literature which studies trading under asymmetric information. The structure of the credit market in our model builds on Kurlat (2016), which we generalize in two directions. We first generalize this framework to allow for heterogeneous credit demand, and then embed the credit market into a macroeconomic environment to endogenize the credit demand curve. At the heart of our model is the feedback between the credit market and real investment, and firms' optimal resolution of the trade-off between investment and liquidity risk management across different states of the world. As such, both of these generalizations are crucial for our mechanism.

2 Model

Consider a three-period model, $t = 0, 1, 2$, with a single perishable good. There are two main groups of agents in the model. First, there are firms who invest and produce. They are located across a continuum of countries. Second, there are international investors who provide financing for firms. There is also a third group of agents, bankers, whose only role is to provide a frictionless saving technology to firms. All agents are risk neutral, and there is no discounting. Agents maximize the expected sum of consumption across all periods.

We start this section with a description of the components of the model, and then proceed to the agents' optimization problem and the equilibrium definition. Certain modeling choices are discussed in section 3.3.

2.1 Set-up

Shocks. There is an aggregate shock (θ) that determines the aggregate state, which is either $\theta = H$ (high) or $\theta = L$ (low). Let π_θ denote the probability of aggregate state θ. There is also an idiosyncratic liquidity shock at the firm level. Let ϕ denote the iid probability that a firm is hit by a liquidity shock. Both shocks are publicly observable. Shocks are sequentially realized at $t = 1$, with the aggregate shock being realized first.

Firms and Production Technology. There is a double continuum of firms, indexed by $j = (\omega, \tau)$. Firms invest and produce, and are subject to liquidity shocks.

$\omega \in [0, 1]$ denotes the *transparency* of the firm, where $\omega = 0$ is the most opaque and $\omega = 1$ is the most transparent firm. Firm transparency relative to the expertise of investors is the source of information friction in our model.

$\tau \in \{g, b\}$ denotes the *(pledgeability) type* of the firm, where g (b) is a good (bad) firm. λ $(1-\lambda)$ fraction of all firms are good (bad), and they are distributed iid across transparency classes. The type of the firm determines the fraction of its output that investors can seize.

Each firm is endowed with a technology akin to Holmström and Tirole (1998) and Lorenzoni (2008), and one unit of good. At $t = 0$, firm $j = (\omega, \tau)$ chooses the initial investment $I(\omega, \tau) \leq 1$, and saves the remainder of his endowment using the bankers. At $t = 1$, after the realization of the aggregate state θ, a fraction ϕ of firms are hit by a liquidity shock. The liquidity shock is observable and verifiable by all agents. Any such firm has to inject extra ξ per unit of initial investment that it wants to maintain. Any unit that does not receive the liquidity injection fully depreciates. Thus, a firm hit by the liquidity shock chooses to drive $i(\omega, \tau; \theta)$ unit(s) of investment to completion, where

$$i(\omega, \tau; \theta) \leq I(\omega, \tau),$$

and abandon the rest of its initial investment. The firm finances the liquidity injection (maintenance cost) from its savings and/or by issuing bonds to international investors.

At $t = 2$, each unit of completed investment produces ρ_τ units of good, where $\rho_g \geq \rho_b > \xi$. This implies that for a firm hit by a liquidity shock the production of $\rho_\tau i(\omega, \tau; \theta)$ units requires $I(\omega, \tau) + \xi i(\omega, \tau; \theta)$ total investment, while a firm not hit by the liquidity shock requires only $I(\omega, \tau)$

total investment.

In line with Holmström and Tirole (1998), we make the following assumption on the production technology.

Assumption 1 *Continuing with full scale, as well as abandoning production after a liquidity shock, are socially positive NPV for both good and bad firms,*

$$\rho_\tau > \max(1+\phi\xi, \frac{1}{1-\phi}), \qquad \tau = g, b.$$

Banks and Saving Technology. A state-contingent saving technology is available at actuarially fair terms to all firms through local banks. Bankers are competitive, deep-pocketed agents who do not have the expertise to seize any future income of firms. Thus, they cannot lend to firms. However, firms can save towards future aggregate or idiosyncratic states with bankers.

International Investors. There is a continuum of investors, indexed by their *skill* level, $s \in [0,1]$. Each investor is endowed with one unit of good in period $t = 1$, and can provide financing to (a selected subset of) firms who demand liquidity. Experts can seize exactly ξ per unit of maintained investment only from good firms. That is, the total credit a firm receives, $\ell(\omega, \tau; \theta)$, has to satisfy the pledgeability constraint

$$(1 + r(\omega, \tau; \theta))\, \ell(\omega, \tau; \theta) \leq \xi i(\omega, \tau; \theta). \tag{1}$$

Note that with perfect information all good firms could borrow at 0 interest rate, implying that investors could provide full financing for a liquidity shock to good firms. However, investors are subject to an information friction. They have imperfect and heterogeneous information about the firm type. Higher s investors have higher quality information, as we specify below. They use their expertise to lend to (potentially a subgroup of) firms hit by a liquidity shock in $t = 1$.

Let $w(s)$ denote the type-density of investors. We assume that $w(s)$ is continuous and strictly decreasing, $w'(s) < 0$, for $s \in [0,1]$, and $w(s) = 0$ otherwise. This assumption means that smart capital is in short supply.

Aggregate Shock and Information Friction. Each investor has a prior $1 - \lambda$ that a given firm is good. She searches for evidence about the true type of each firm that demands liquidity. The information of investor s about firm $j = (\omega, \tau)$ depends on her expertise level, s, transparency of the firm, ω, and the aggregate state, θ. Let $x(\tau; \omega, s, \theta)$ denote this information.

The type of evidence that investors search for is determined by the aggregate state. We call the aggregate shock *prudence shock*, as it determines how prudent the investors are. If the prudence shock is high, $\theta = H$, investors are *bold*. They search only for conclusive evidence whether a firm is bad. If an investor is sufficiently skilled relative to how opaque a bad firm is, she will find such evidence. Otherwise, the search is uninformative. Thus, we have

$$x(\tau;\omega,s,H)=\begin{cases} b & \text{if } s>1-\omega \text{ and } \tau=b \\ \varnothing & \text{otherwise} \end{cases} \tag{2}$$

where $x=b$ is evidence that a firm is bad, while $x=\varnothing$ implies that the investor did not find any evidence about the firm type. Thus, a bold investor of skill s identifies the bad firms that are sufficiently transparent, $\omega \geq 1-s$. However, when her signal is $x=\varnothing$, she is uncertain whether she did not find such evidence because the firm is good, or because the firm is too opaque relative to her skill. As such, bold investors only make false positive mistakes: they cannot differentiate relatively opaque bad firms from any of the good firms. One interpretation is that bold investors are interested in not missing out on any good firms, even at the expense of occasionally lending to bad firms by mistake.

In contrast, if the prudence shock is low, $\theta=L$, investors are *cautious*. They search only for conclusive evidence whether a firm is good. An investor finds such evidence only if she is sufficiently skilled and the firm is good. That is,

$$x(\tau;\omega,s,L)=\begin{cases} g & \text{if } s>1-\omega \text{ and } \tau=g \\ \varnothing & \text{otherwise} \end{cases} \tag{3}$$

Thus, a cautious investor of skill s identifies the good firms that are sufficiently transparent, $\omega \geq 1-s$. However, she does not find such evidence for good firms of lower transparency, or for any bad firms. As such, cautious investors only make false negative mistakes. One can interpret this as cautious investors being interested in not lending to bad firms even at the expense of occasionally missing out on the good ones.[4]

Liquidity Shock and Financing At $t=1$, a firm $j=(\omega,\tau)$ can obtain credit by issuing bonds on the international market to a subset of investors who are willing to lend to it. The firm receives one unit of financing per bond and promises to pay back $1+r(\omega,\tau;\theta)$ at date $t=2$. The repayment is subject to the pledgeability constraint (1). The interest rate $r(\omega,\tau;\theta)$ is determined in equilibrium.

Market Structure. At $t=1$, many markets open for issuing bonds. Each market m is defined by an interest rate $\tilde{r}(m)$ and it can be active or inactive in equilibrium. The set of all markets is denoted by M. A market is active if both firms and investors are present in that market.[5]

Firms can go to as many markets as they desire, and demand σ units of credit for the corre-

[4] See section 3.3 for a discussion.

[5] The structure of the credit market generalizes Kurlat (2016) by introducing quantities. We provide details on the methodological contribution in the Appendix.

sponding interest rate $\tilde{r}(m)$ in each market m, such that

$$\sigma(m, \omega, \tau; \theta) \leq \bar{L}, \tag{4}$$

where $\bar{L}$ is a common exogenous capacity limit for all markets. We assume that the capacity limit $\bar{L}$ corresponds to the maximum demand that any good firm would submit.[6]

Each investor s chooses (at most) one market m to lend in, the amount of credit she wants to provide δ, and an acceptance rule $\chi(\omega, \tau; \theta) \in \{0, 1\}$ she will use. The acceptance rule specifies which bonds the investor is willing to finance, and has to be measurable with respect to her collected evidence $x(\tau; \omega, s, \theta)$. Investors cannot observe, and thus cannot condition their decisions, on the total amount of credit that a given firm takes on.

If there are investors of multiple skills who offer credit at a given m, the transactions of the least selective investors, i.e., those with least informative evidence, clears first. The formal definition is provided in the Appendix.

Markets do not have to clear. In particular, firms understand that in each state θ, for each firm $j = (\omega, \tau)$, and each market m, there is an equilibrium measure $\eta(m, \omega, \tau; \theta)$ such that a firm j demanding $\sigma(m, \omega, \tau; \theta)$ credit at market m can raise only $\eta(m, \omega, \tau; \theta)\sigma(m, \omega, \tau; \theta)$. We call $\eta(m, \omega, \tau; \theta)$ the rationing function. As such, we define $\ell(\omega, \tau; \theta)$, the total amount of credit raised by firm j, and j's effective interest $r(\omega, \tau; \theta)$ as follows:

$$\ell(\omega, \tau; \theta) \equiv \int_M \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta), \tag{5}$$

$$r(\omega, \tau; \theta) \equiv \frac{\int_M \tilde{r}(m)\sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta)}{\ell(\omega, \tau; \theta)}. \tag{6}$$

Thus, an investor s who chooses market m with interest rate $\tilde{r}(m)$, finances a representative pool of firms (1) which demand credit in market m, (2) which satisfy investor s acceptance rule based on her evidence about the firms, and (3) whose demand is not exhausted by investors less selective than s.

Finally, equilibrium supply and demand determines the allocation function $A(\omega, \tau; \chi, m, \theta)$, the measure representing the fraction of bonds of firm $j = (\omega, \tau)$ financed by investor s, with acceptance rule χ, in market m, and aggregate state θ. Expert s choice of the amount of credit she wants to provide, δ, has to satisfy her budget constraint given the allocation function. That is, the total credit extended by an investor has to at most equal her endowment.

This market structure allows for many-to-many matching. A given firm might obtain credit from a group of heterogeneous investors (as described by the rationing function η), and a given investor might finance a pool of heterogeneous firms (as described by the allocation function A).

[6]This assumption is intuitive since markets should allow for orders that are consistent with the needs of all good firms, but should not increase the maximum order size to a range where only bad firms would submit.

Countries. Firms are distributed among a unit mass of countries, $c \in [0, 1]$. We assume that the distribution is iid with respect to the firm pledgeability type τ, but not with respect to the transparency ω. Let $\omega_c = \mathbb{E}_c[\omega]$ denote the implied transparency of country c, which is defined as the average transparency of the firms in that country.

To isolate our main mechanism, we assume that each country is populated with firms of a single transparency and no other firms. Thus, we can index the countries by ω_c up to a random permutation. Since all the firms in country ω_c have the same transparency level, to save on notation in the reminder of the paper we will use ω to index country transparency as well.

Furthermore, we assume that the mapping between transparency classes and country names is random, and investors have an uninformative prior about this mapping. Thus, from investor perspective, all countries are ex-ante identical.[7]

2.2 Equilibrium Definition

Firm and Expert Problems, and Timing. We next summarize the timeline of the model, along with the firm and investor optimization problems.

At $t = 0$, each firm chooses how much to invest and how much to save in order to insure the risk of liquidity shock. As long as the interest rate is not prohibitively high, the optimal decision for a firm $j = (\omega, \tau)$ who is hit by a liquidity shock is to issue the maximum number of bonds at interest rate $r(\omega, \tau; \theta)$ without violating the pledgeability constraint (1).[8] Thus, for a given continuation decision $i(\omega, \tau; \theta)$ in state θ,

$$\ell(\omega, \tau; \theta) = \frac{1}{1 + r(\omega, \tau; \theta)} \xi i(\omega, \tau; \theta), \tag{7}$$

where

$$i(\omega, \tau; \theta) \leq I(\omega, \tau). \tag{8}$$

The firm finances the rest of the liquidity needs from its own initial endowment, through state-contingent saving. Thus, the firm's ex-ante budget constraint can be written as

$$I(\omega, \tau) + \phi\xi \sum_{\theta} \pi_\theta \frac{r(\omega, \tau; \theta)}{1 + r(\omega, \tau; \theta)} i(\omega, \tau; \theta) = 1 \tag{9}$$

At $t = 1$, after the realization of the aggregate state and the idiosyncratic liquidity shock, each firm submits its demand for bond issuance to a subset of markets, taking each market's interest rate and rationing function as given.

[7]See section 3.3 for a discussion.

[8]See the Appendix for a more detailed argument.

The problem of a firm $j = (\omega, \tau)$ can be written as

$$\max_{I(\omega,\tau),\{i(\omega,\tau;\theta)\}_{\theta},\{\sigma(m,\omega,\tau;\theta)\}_{\theta,m}} \sum_{\theta} \pi_\theta \Big[(1-\phi)\rho_\tau I(\omega,\tau) + \phi\big(\rho_\tau - \mathbb{1}_{\tau=g}\xi\big) i(\omega,\tau;\theta)\Big] - 1 \tag{10}$$

subject to (4)-(9).

It is important to note that conditional on type τ, firms face a different problem only to the extent that they face a different interest rate $r(\omega, \tau; \theta)$. Therefore, heterogeneous decisions about initial and continued investment is driven by (expected) heterogeneous financing conditions in credit markets.

Experts arrive in $t = 1$, once both the aggregate and idiosyncratic shocks are realized. They have unit wealth and consume in $t = 1, 2$. Each investor picks a market, m, and submits her acceptance rule χ, and supply of credit δ. Thus, the problem of investor s in aggregate state θ can be written as

$$\max_{m,\chi,\delta} \delta \left[(1 + \tilde{r}(m;\theta)) \int_{\omega} dA(\omega, g; \chi, m, \theta) - \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m, \theta) \right] + 1 \tag{11}$$

s.t.

$$\chi \in X_s$$

$$\delta \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m, \theta) \leq 1$$

Since investors' objective function is linear in δ, they choose $\delta = 0$ if the net return on bond is negative; otherwise, δ is determined by their resource constraint.

We end this section with the formal definition of equilibrium.

Definition 1 *A global equilibrium is a set of the firm's investment plan $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_{\theta=H,L}$ and demand function for credit $\{\sigma(m, \omega, \tau; \theta)\}_{\theta=H,L}$, investor's choice of interest rate $\tilde{r}(m; \theta)$, and acceptance rule $\chi(s, \omega)$, along with a rationing function $\{\eta(m, \omega, \tau; \theta)\}_{\theta=H,L}$, allocation function $A(\omega, \tau; \chi, m, \theta)$, and interest rate schedule $\{\tilde{r}(m; \theta)\}_{\theta=H,L}$ and the corresponding $\{r(\omega, \tau; \theta)\}_{\theta=H,L}$, such that*

(i) firm investment plan and demand function solves the firm optimization problem (10)*, given the rationing function and the interest rate schedule;*

(ii) investor choice of interest rate and the corresponding acceptance rule maximizes the investor optimization problem (11)*, given the allocation function and the interest rate schedule;*

(iii) rationing functions, allocation functions, and interest rate schedules are consistent with investor and firm optimization.

3 Global Equilibrium

In this section, we characterize the global equilibrium. We start with the analysis of two simple benchmarks, and then move on to the full model. All the proofs are in Appendix C.

3.1 Benchmark

Here, we study two benchmarks: when international capital markets are completely shut down, and when there is abundant skilled capital. The key feature shared by the two benchmarks is that in equilibrium, there is no skill heterogeneity among investors who provide financing to firms. We show that ex-ante differences in ω among countries do not lead to any ex-post heterogeneity in either case. As such, countries with ex-ante identical production fundamentals are ex-post identical independent of their transparency. Moreover, the aggregate shock does not affect any of the outcomes.

Credit Market Shutdown. In this benchmark, firms are unable to raise any financing, and thus each firm is in autarky. Formally, assume $w(s) = 0, \forall\, s$.

Abundant Smart Money. In this case, the only constraint that good firms face in raising funding is the pledgeability constraint. Formally, we assume that investor has K units of wealth, $K \to \infty$. In particular, $w(1) \to \infty$, thus the most skilled investors are sufficiently wealthy to absorb the liquidity demand by all good firms.

The next lemma describes the equilibrium in the two benchmarks.

Lemma 1 [Benchmark]

(i) Credit market shutdown: $\forall\, \theta,\ \forall\, j = (\omega, \tau)$

(a) If $\xi > \frac{1}{1-\phi}$, *then* $I^A(\omega, \tau) = 1$ *and* $i^A(\omega, \tau; \theta) = 0$,

(b) Otherwise, $i^A(\omega, \tau; \theta) = I^A(\omega, \tau) = \frac{1}{1+\phi\xi}$.

Moreover, the total output is identical across countries and across states,

$$Y^A(\omega, \theta) = ((1-\lambda)\rho_g + \lambda\rho_b) \max\left(1-\phi, \frac{1}{1+\phi\xi}\right).$$

(ii) Abundant smart money: $\forall\, \theta, \forall\, \omega$

(a) $i^{FL}(\omega, g; \theta) = I^{FL}(\omega, g) \to 1$. *Moreover, good firms face zero interest rate,* $r(\omega, g; \theta) \to 0$.

(b) $I^{FL}(\omega, b) \to 1$ *and* $i^{FL}(\omega, b; \theta) = 0$.

The total output is identical across countries and across states,

$$Y^{FL}(\omega,\theta) = \rho_g\,(1-\lambda) + \rho_b\lambda(1-\phi)$$

While total output in each country is unsurprisingly smaller when the credit market is shut down, it is identical across countries and aggregate states in both cases. These benchmarks emphasize that in our model, all fluctuations across countries and for different aggregate prudence shockscome from the fact that credit is provided by investors who have scarce capital and imperfect information.

Moreover, Lemma 1 exhibits one of the recurring themes of our analysis: the trade-off between investment scale and liquidity risk management. Even when firms do not have access to capital markets, they can insure against future liquidity shocks by saving some of their own endowment using the bankers. The first part of the lemma shows that if the liquidity shock is large relative to the probability that it hits the firm, then it is too costly for the firm to insure against it. Thus, the firm forgoes risk management and enjoys high output when it is not hit by the shock, but has to liquidate when it faces one.

3.2 Simple Global Equilibrium

We now turn to equilibrium characterization for the full model, when there is heterogeneity in liquidity supply. In the Appendix, we discuss different variants of the equilibrium that arise depending on the choice of parameters. However, in order to highlight the main mechanism of the model we restrict focus on the simplest variant, and call it a "simple global equilibrium". To characterize the equilibrium, we proceed by backward induction. We start by analyzing the credit market outcome taking investment choices as given, and then characterize the equilibrium in real quantities.

3.2.1 Equilibrium Interest Rates and Credit Allocation

In this section, we characterize the credit market outcome for the two different aggregate states. The next two propositions describe the interest rates and the corresponding allocation of financing in the international credit market, given the ex-ante investment functions $\{I(\omega,\tau)\}_{\omega,\tau}$.

Proposition 1 [International Rates]

(i) If $\theta = H$, there is a single common prevailing interest rate, r_H. Moreover, there is a threshold skill level, $s_H \in [0,1]$, such that only investors who are more skilled than this threshold, $s \geq s_H$, participate in the credit market.

(ii) If $\theta = L$, there is an interest rate schedule, $r_L(\omega)$, at which good firms with transparency ω raise credit. This schedule is characterized by endogenous thresholds $0 \leq \omega_2 \leq \omega_3 \leq 1$, and a

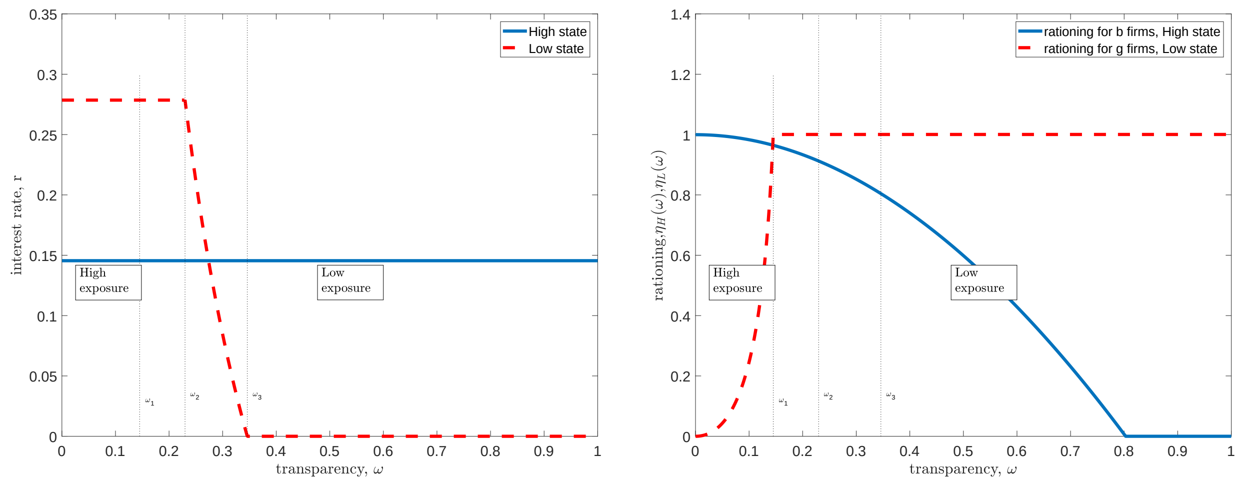


Figure 2: Interest rate schedule and the rationing function for bad and good firms of transparency ω, in high state (solid) and low state (dashed).

continuous, decreasing function $\hat{r}(\omega)$*, such that*

$$r_L(\omega) = \begin{cases} 0 & \text{if } \omega \in (\omega_3, 1] \\ \hat{r}(\omega) & \text{if } \omega \in (\omega_2, \omega_3] \\ \bar{r} & \text{if } \omega \in (0, \omega_2] \end{cases} \tag{12}$$

where $\hat{r}(\omega_2) = \bar{r}$ *and* $\hat{r}(\omega_3) = 0$*.*

Proposition 2 [International Credit Allocation]

(i) If $\theta = H$*, a good firm that is hit by the liquidity shock fully pledges its initial investment and obtains* $\ell(\omega, g; H) = \xi I(\omega, g)/(1 + r_H)$ *credit.*

Bad firms demand $\bar{L}$ *credit, but they are rationed. Specifically, there is a weakly decreasing function,* $\eta_H(\omega)$*, such that a bad firm with transparency* ω *obtains* $\ell(\omega, b; H) = \eta_H(\omega)\bar{L}$ *credit with* $\eta_H(0) = 1$ *and* $\eta_H(\omega) = 0$ *for all* $\omega > 1 - s_H$*.*

(ii) If $\theta = L$*, bad firms do not obtain any credit;* $\ell(\omega, b; L) = 0$*.*

There is a threshold $\omega_1 < \omega_2$ *such that good firms with transparency* $\omega \in [\omega_1, 1]$ *fully pledge their initial investment as collateral and obtain* $\ell(\omega, g; L) = \xi I(\omega, g)/(1 + r_L(\omega))$ *credit. In contrast, good firms with transparency* $\omega \in [0, \omega_1]$ *are only partially financed. Specifically, there is an increasing function* $\eta_L(\omega)$ *such that a firm with* $\omega \in [0, \omega_1]$ *obtains* $\ell(\omega, g; L) = \eta_L(\omega)\xi I(\omega_1, g)/(1 + r_L(\omega))$ *credit with* $\eta_L(0) = 0$ *and* $\eta_L(\omega_1) = 1$*.*

The above two propositions show that the credit market functions very differently across the two aggregate states. In the high aggregate state investors are bold, as illustrated in Figure 2. That is, although each investor recognizes all good firms, she can only identify bad firms which are sufficiently transparent relative to her skill. Proposition 1 shows that when investors are bold, there is a single prevailing interest rate r_H at which all bonds are issued. This is represented by the solid horizontal line on the left panel of Figure 2. Investors with sufficiently high expertise participate in the credit market. Each such investor distributes her endowment across firms that she cannot identify as bad. Low expertise investors $s \leq s_H$ remain inactive and do not extend any credit. We call s_H the marginal investor.

When investors are bold, all good firms are fully financed. On the other hand, bad firms are rationed and obtain credit only from those investors who mistake them to be good firms. The corresponding rationing function $\eta_H(\omega)$ is the solid curve on the right panel of Figure 2. It follows that the interest rate r_H and the marginal type s_H are pinned down by two conditions. First, the interest rate has to compensate the marginal investor for the defaults she experiences in her portfolio due to loans that are extended to bad firms. Second, the wealth of all participating investors has to be sufficient to cover the aggregate credit demand by good firms.

Good firms are willing to issue bonds at every market with a lower interest rate $r < r_H$ as well, and demand the maximum possible credit $\bar{L}$ at all such markets. Bad firms, on the other hand, demand $\bar{L}$ credit in every market since they know that they will not repay ex-post if they manage to obtain credit at any interest rate. Obviously, investors prefer to lend at interest rates higher than r_H. However, since good firms do not demand credit at those high interest rate, the pool of offered bonds only consist of bad firms, which in turn deters investors from lending. Good firms prefer to borrow at interest rates lower than r_H, but separation is not possible for high transparency firms because (i) all investors observe the same signal on all good firms, and hence good firms cannot be served at different markets, and (ii) for any interest rate the demand of bad firms is weakly higher than that of good firms, as their effective cost of credit is lower due to the fact that they are not paid back. Hence, bad firms follow good ones to any market. It thus follows that all firms that are able to issue bonds do so at the same interest rate r_H. Investors differ in the pool of firms which they finance, but not in the financing terms.

In the low aggregate state, investors are cautious. That is, although each investor recognizes all bad firms, she can only identify good firms which are sufficiently transparent relative to her skill. In equilibrium, investors only finance those good firms which they can recognize. It follows that investors with different skills finance not only a different set of firms, but they do so at different interest rates. This interest rate schedule $r_L(\omega)$, is the dashed curve on left panel of Figure 2. In particular, the most transparent good firms, $\omega \in (\omega_3, 1]$, which are recognized by almost all investors, are financed at 0 interest rate by investors with low skill. This is because there is a large supply of low skilled capital who can only identify the highly transparent good firms with limited demand. In contrast, opaque good firms $\omega \in [0, \omega_2]$ can only raise financing from highly-skilled investors, only at (endogenous) high interest rate $\bar{r}$. In fact, since the mass of highly skilled investors is small relative to the corresponding demand, the most opaque firms $\omega \in [0, \omega_1]$, are only partially financed even at this high level of interest rate. The corresponding rationing function $\eta_L(\omega)$ is the dashed curve on the right panel of Figure 2. Lastly, firms in the intermediate transparency group $\omega \in (\omega_2, \omega_3]$ are financed at intermediate interest rates by investors with intermediate skill.

In section 4, we argue that countries dominated by opaque firms $\omega \in [0, \omega_1]$ turn out to be the ones which become most exposed to cycles in equilibrium. Therefore, we refer to $[0, \omega_1]$ as the high exposure region, and to the countries within as high exposure or, inspired by the popular terminology in the Eurozone debt crisis, as periphery economies. On the other hand, countries dominated by transparent firms $\omega \in (\omega_3, 1]$ are the least cyclical in equilibrium. Hence, we refer to $(\omega_3, 1]$ as the low exposure region, and to the countries within as low exposure or core economies.

Figure 3 illustrates how thresholds $\omega_1, \omega_2, \omega_3$ are determined by the demand and supply of skilled and unskilled capital when $\theta = L$. The diagonal curve is $w(1-\omega)$, which represents the supply of capital by the marginal investor $s = 1 - \omega$, who can just distinguish good firms of transparency ω. Since all the good firms face the same interest rate when $\theta = H$, their demand for credit is different only to the extent that they face different interest rates when $\theta = L$. Let i_0 ($i_{\bar{r}}$) denote the maintained investment by a good firm that faces zero ($\bar{r} > 0$) interest rate when $\theta = L$, where $\bar{r}$ is the endogenous maximum interest rate that any good firm is willing to pay, which will be explained shortly. Note that $i_0 > i_{\bar{r}}$.

As such, the horizontal lines in Figure 3 represent the total credit demand for firms at two different interest rates. The red line (upper line) corresponds to the demand at zero interest rate, $\phi(1-\lambda)\xi i_0$. The blue dashed line (lower line) corresponds to the demand at interest rate $\bar{r}$, $\phi(1-\lambda)\frac{\xi}{1+\bar{r}} i_{\bar{r}}$. Then, threshold ω_3 is the lowest transparency level such that the wealth of the corresponding marginal investor, $s_3 = 1 - \omega_3$, is sufficient to cover the credit demand by ω_3 good firms at zero interest rate. Note that the total supply of capital of investors present at the zero interest rate market, area ABCE, is larger than the total demand from firms with transparency $\omega \in (\omega_3, 1]$, area ABCD. That is, investors with low skill "queue up" to finance the transparent good firms which they can identify confidently.

For the intermediate group $\omega \in (\omega_2, \omega_3]$, there is a feasible interest rate $\hat{r} \in [0, \bar{r}]$ per transparency level, at which the supply of credit by the marginal investor $s = 1 - \omega$ equals the demand by good firms with transparency ω. Essentially, in this region we have cash-in-the-market pricing, in the spirit of Allen and Gale (2005).

For the determination of the high exposure region $[0, \omega_1]$, we start with a description of $\bar{r}$, the endogenous maximum interest rate that any good firm is willing to borrow at. Such an interest rate exists because borrowing at a high interest rate requires a high co-payment, which comes at the cost of lower initial investment due to the budget constraint (9). If the interest rate exceeds $\bar{r}$, good firms prefer to abandon production after a liquidity shock and instead invest at a higher scale and continue only when they are not hit by the shock. It follows that every firm in $[0, \omega_2]$ region faces the same interest rate $\bar{r}$. Since price of credit is constant but the supply is lower for lower transparency firms, the allocation of credit should instead adjust. Recall that each investor active in a market distributes her endowment pro-rata across all the good firms she can identify.

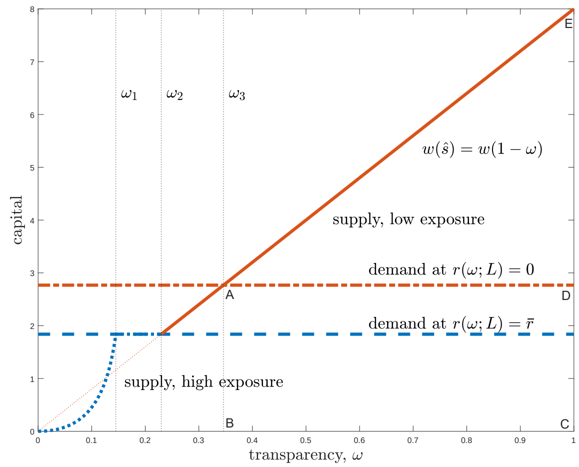


Figure 3: Supply and demand for investors' funds, and the determination of thresholds $\omega_1, \omega_2, \omega_3$.

Threshold ω_1 is the lowest transparency level such that the corresponding total supply of capital allocated to good firms with this level of transparency is exactly sufficient to cover their demand. For the most opaque good firms $\omega \in [0, \omega_1]$, the demand is higher than the corresponding supply curve at $\bar{r}$, and thus they are rationed. Put differently, good firms from most opaque countries gamble by making a large initial investment knowing that it will partially collapse if they need liquidity when investors are cautious.

Moreover, note that the maximum interest rate $\bar{r}$ in the low state is the level for which a firm is indifferent between two choices. The first option is a large initial investment which is partially or fully liquidated after a liquidity shock. The second option is to invest less initially and save towards the fraction of the maintenance cost which it cannot finance from credit, due to the pledgeability constraint. When the interest rate is high, constraint (9) implies that the sacrificed initial investment is sizable. In this sense, those firms who opt for the first option are gambling.

Bad firms submit the maximum demand $\bar{L}$ in every market with interest rate in $[0, \bar{r}]$ even though they are not served. This supports the equilibrium strategy of low skilled investors not to participate in high interest rate markets with opaque good firms that they cannot distinguish from bad ones. If they were to do so, they would end up financing a large fraction of bad firms.

It is worth emphasizing the qualitative difference in the functioning of credit markets between the high and low aggregate states. In the high state, there is no difference in financing terms across loans. Instead, highly skilled investors derive excess return by financing a pool of better quality firms than low skilled investors. In contrast, in the low state there is no difference in quality of extended credit across lenders, as only good firms are financed. Instead, highly skilled investors derive excess returns by charging high rates to opaque good firms. Moreover, note that investors with low skill hold a dispersed portfolio when they are bold, lending to both opaque and transparent firms. When they turn cautious, they re-balance their portfolio by shedding bonds of opaque firms, and financing transparent firms at low interest rates instead. From the perspective of firms, their ownership is dispersed in the high state, as investors of various skill levels finance them, but concentrated in the low state. We will revisit these observations when we derive empirical predictions in section 4.2.

3.2.2 Equilibrium in Real Investment

In this section, we characterize the equilibrium in the real economy. We analyze how a given firm $j = (\omega, \tau)$ chooses its investment plan, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, foreseeing the equilibrium in the credit market as we described in propositions 1 and 2. The first proposition describes the optimal investment plan for good firms.

Proposition 3 [Good Firm Investment]
In a simple global equilibrium, a good firm chooses

$$I(\omega, g) = \frac{1 + (1 - \eta_L(\omega)) \frac{\phi \xi \pi_L \frac{\bar{r}}{1+\bar{r}}}{1 + \phi \xi \pi_H \frac{r_H}{1+r_H}}}{1 + \phi \xi \left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)} \right)} \tag{13}$$

$$i(\omega, g; H) = I(\omega, g) \tag{14}$$

$$i(\omega, g; L) = \eta_L(\omega) I(\omega, g). \tag{15}$$

where $0 \leq \eta_L(\omega) \leq 1$ *is a weakly increasing function defined in* (23).

It is intuitive to analyze a firm's optimal investment and continuation decision backwards. First, consider a good firm hit by a liquidity shock at $t = 1$, which has already made the initial investment. (14) and (15) imply that all good firms fully maintain their initial investment when investors are bold. Moreover, most good firms also fully maintain their investment when investors are cautious, except those in the high exposure region. In this region, credit (partially) dries up even for good firms because of the scarcity of smart capital, as explained in section (3.2.1).

Furthermore, at $t = 0$ firms foresee this and choose their initial investment, $I(\omega, g)$, accordingly. Given a firm's future investment policy and the market conditions it expects to face, initial investment is determined by constraint (9). This constraint encapsulates a key yet simple trade-off. The fraction of the maintenance cost covered by the firm's saving limits the initial investment it can afford. The crucial observation is that a firm needs less savings for two different reasons: if it faces favorable credit conditions (low rates), or if it cannot raise financing and continue. Both of these channels lead to a high level of initial investment, though with very different future prospects. The former is maintained even in the face of shocks, while the latter can lead to a possible crash.

It follows that the scale of economic activity of good firms is non-monotonic across the exposure spectrum. Good firms in low exposure and high exposure regions invest more than firms in the intermediate regions, because they save little towards the state when investors are cautious; firms in the low exposure region obtain credit for maintenance at zero cost, while firms in the high exposure region abandon most of their production in state $\theta = L$ when they receive a liquidity shock.

Figure 4c shows total investment for each transparency level $I(\omega, g) + \phi\xi i(\omega, g, \theta)$ across aggregate prudence shocks. When investors are bold, good firms produce with full capacity. As such, total investment inherits the non-monotonic pattern of the initial investment, $I(\omega, g)$, across the exposure spectrum. When investors are cautious, there is a collapse in investment in the high exposure region as firms hit by the liquidity shock abandon production.

Bad firms' investment plan choice differs from that of good firms because they face different conditions in the market for credit. Bad firms understand that they are not be able to obtain any credit when investors are cautious and that they are rationed when investors are bold. The next proposition describes their optimal choice.

Proposition 4 [Bad Firm Investment] *In a simple core periphery equilibrium, bad firms choose the following investment plan:*

$$I\left(\omega, b\right) = 1 - \frac{r_H}{1 + r_H}\phi\pi_H\eta_H(\omega)\bar{L} \tag{16}$$

$$i(\omega, b; H) = \frac{\eta_H(\omega)\bar{L}}{\xi}$$

$$i(\omega, b; L) = 0.$$

where $0 \le \eta_H(\omega) \le 1$ *is a weakly decreasing function defined in* (24).

Bad firms' choice of their initial investment is also determined by the trade-off embedded in the financing constraint (9). Opaque bad firms can obtain more credit in the high state, which they do not plan to pay back. This is apparent in Figure 4d, which plots the total face-value of credit issued to bad firms in the high state.

3.2.3 Existence

We start by a set of sufficient conditions to ensure the existence of a simple global equilibrium.

Assumption 2 *Assume the parameters are such that*

(i) $\xi > \frac{1}{1-\phi}$

(ii) $\frac{\lambda}{1-\lambda} \le \frac{(\rho_g - \xi)}{\rho_g \xi(1-\phi) + (\rho_g - \xi)(\phi\pi_L\xi - 1)}$

(iii) $w(s)$ *is continuous, with* $w'(s) < 0$, $w(0) \ge \phi(1-\lambda)\xi$ *and* $\lim_{s\to 0} w(s) = 0$.

(iv) $\min\left\{\frac{(\rho_g - \xi)(1+\lambda\phi\xi\pi_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\xi}, \frac{\xi\phi(1-\lambda) - w(1-\omega)}{\xi\phi((1-\lambda)+w(1-\omega)\pi_L)}\right\} \le \frac{\lambda}{\lambda + (1-\lambda)\omega} \qquad \forall\omega$

Condition (i) ensures that without access to credit markets, firms choose to invest all of their initial endowment and do not use any part of it to manage liquidity risk. It also implies that without access to credit markets firms do want to invest (rather than consume right away), which requires $\rho_\tau > \frac{1}{1-\phi}$ and follows since $\forall\, \tau,\ \rho_\tau > \xi$. Condition (ii) ensures that the common interest rate is not prohibitively high when $\theta = H$, so that firms use international markets and part of their own endowment to manage liquidity risk, as opposed to investing all of their initial endowment. Condition (iii) ensures two properties of the wealth function. First, low-expertise investors have sufficient wealth so that some bonds are issued at zero interest rate. Second, expert capital is in short supply. Condition (iv) ensures that when investors are cautious, there is no equilibrium interest rate for which some investors are willing to buy up all the offered securities independent of their signal.

The next proposition spells out how the equilibrium objects $r_H, r_L(\omega), s_H, \omega_1, \omega_2, \omega_3, \eta_L(\omega)$ and $\eta_H(\omega)$ are constructed, and states that a simple global equilibrium exists.

Proposition 5 [Existence] *For parameters satisfying assumptions 1 and 2, there exists a simple global equilibrium. The equilibrium objects are pinned down by the fixed point x^* of the following equation.*

$$F(x) = \frac{\lambda(1 - s_H(x))\, D(0; x)}{\lambda(1 - s_H(x))\, D(0; x) + (1 - \lambda)\, \bar{D}(x)} \tag{17}$$

where $s_H(x)$ solves

$$\int_{s_H(x)}^{1} \frac{1}{\lambda(1-s)D(0;x) + (1-\lambda)\bar{D}(x)} w(s) ds = (1 - x)\phi, \tag{18}$$

if equation (18) *has a positive solution, and $s_H(x) = 0$ otherwise.*

Moreover

$$\bar{y}(x) = \frac{(\rho_g - \xi)(1 + \phi\xi\pi_H x)}{(\rho_g(1 - \phi) + \phi(\rho_g - \xi)\pi_H)\xi} \tag{19}$$

$$D(y; x) = \frac{\xi}{1 + \phi\xi(\pi_H x + \pi_L y)} \tag{20}$$

$$\bar{D}(x) = (1 - \omega_3(x))D(0; x) + \int_{\omega_2(x)}^{\omega_3(x)} D(y^C(\omega); x) d\omega + \left(\omega_2(x) + \frac{\phi\xi\pi_L\bar{y}(x)}{1 + \phi\xi\pi_H x}\int_0^{\omega_1(x)} (1 - \eta_L(\omega)) d\omega\right) D(\bar{y}(x); x). \tag{21}$$

where

$$y^C(\omega; x) \equiv \frac{\xi\phi(1 - \lambda) - w(1 - \omega)(1 + \phi\xi\pi_H x)}{\xi\phi((1 - \lambda) + w(1 - \omega)\pi_L)} \qquad \omega \in [\omega_2(x), \omega_3(x)]. \tag{22}$$

The rationing functions are given as follows

$$\eta_L(\omega) = \min\left(1, \int_{1-\omega}^{1} \frac{1}{\phi(1-\lambda)(1-\bar{y}(x))D(\bar{y}(x);x)(\omega_2(x)-(1-s)) - \int_{1-\omega_2(x)}^{1-\omega_1(x)} w(s)ds} w(s)ds\right) \tag{23}$$

$$\eta_H(\omega) = \min\left(1, \int_{s_H(x)}^{1-\omega} \frac{1}{\lambda(1-s)D(0;x)+(1-\lambda)\bar{D}(x)} \frac{w(s)}{\phi(1-x)} ds\right) \tag{24}$$

and $\omega_1(x), \omega_2(x), \omega_3(x)$ *are defined as follows.*

Let $\hat{\omega}_2(x)$ *and* $\hat{\omega}_3(x)$ *be the solution to the following two equations, respectively:*

$$w\left(1-\omega_2\right) - \phi\left(1-\lambda\right)\left(1-\bar{y}(x)\right) D\left(\bar{y}(x);x\right) = 0, \tag{25}$$

$$w\left(1-\omega_3\right) - \phi\left(1-\lambda\right) D\left(0;x\right) = 0. \tag{26}$$

Then

$$\omega_2(x) = \min\{\max\{\hat{\omega}_2(x), 0\}, 1\}, \tag{27}$$

$$\omega_3(x) = \min\{\max\{\hat{\omega}_3(x), 0\}, 1\}. \tag{28}$$

Moreover, let $\hat{\omega}_1$ *be the solution to*

$$1 = \int_{1-\omega_1}^{1} \frac{1}{\phi(1-\lambda)(1-\bar{y}(x))D(\bar{y}(x);x)\left(\omega_2(x)-(1-s)\right) - \int_{1-\omega_2(x)}^{1-\omega_1} w(s)ds} w(s)ds \tag{29}$$

$$\omega_1(x) = \min\{\max\{\hat{\omega}_1(x), 0\}, 1\}. \tag{30}$$

Finally, given the fixed point x^*, $\bar{r} = \frac{1}{1-\bar{y}(x^*)}$, *and interest rates* r_H *and* $r_L(\omega)$ *are given by*

$$r_H = \frac{1}{1-x^*} \tag{31}$$

$$\hat{r}(\omega) = \frac{1}{1-y^C(\omega;x^*)}. \tag{32}$$

and (12).

This proposition defines every equilibrium object as a function of a single equilibrium outcome, a transformation of the prevailing interest rate when $\theta = H$, which is itself the solution to a univariate fixed-point problem. The proof argues that Brouwer fixed-point theorem applies and hence, that a fixed point exists.

We end this section by discussing certain modeling assumptions. In the next section, we analyze the implications of our model for heterogeneous global cycles.

3.3 Comments

In this section, we remark on the interpretation of agents and markets, as well as some of the assumptions in our model.

The focus of our analysis is the capital flows that are channeled through global financial institutions towards local firms. In reality, multiple channels serve these flows with potentially several layers of intermediation. For instance, a large fraction of European firms finance themselves using loans from local banks. Banks in turn often fund these loans by selling commercial papers to money market funds (e.g. Ivashina et al., 2015; Gallagher et al., 2018). Larger firms can also raise capital on the corporate bond market directly from bond mutual funds and other asset managers (e.g. Gilchrist and Mojon, 2018). We expect the predictions of our model to hold in a variety of these contexts. However, the labels of the model should be changed accordingly. For instance, applied to the commercial paper market, local banks will play the role of firms and money market funds will play the role of international investors.

As our focus is on the interaction of international investors and local firms, our choice for modeling countries is decidedly simplistic: a country comprises a set of firms. For our mechanism to be relevant, we need two weak requirements related to the allocation of firms across countries. First, this allocation cannot be uniform in transparency, i.e., $\mathbb{E}_c[\omega]$ has to differ across countries. Second, investors' prior on the allocation has to be coarse. For simplicity, we push both these requirements to the extreme. We assume that each country is populated with firms of a single transparency, and furthermore, that investors have an uninformative prior about the mapping between country name and transparency level. To show that this latter assumption is stronger than needed, in section 7.2 we generalize our framework allowing for partially informative priors. For instance, in the European context, investors can recognize that it is harder to learn about Italian and Spanish firms compared to German ones, as long as their prior is uninformative about how Spanish and Italian firms compare to each other. In section 7.2, we show that all our results are robust to this generalization.

Intuitively, we think of the coarseness of prior information on transparency of a country, ω, as an assumption which captures the fact that boom-bust patterns are often preceded by major changes in the countries of interest, contributing to investors' uncertainty about ω. For example, the introduction of the European Monetary Union, or the major economic reforms preceding the fast growth of East Asian countries perhaps led investors to rely less on their existing knowledge of these markets.

Furthermore, to emphasize that our mechanism relies only on the informational frictions vis-a-vis the international capital supply (supply side), we suppress difference in production fundamentals (demand side) across countries. In particular, we assume that every country has the same composition of good and bad firms. We make this assumption solely for expositional purposes. However, we do not doubt that fundamental differences across emerging and developed countries, or core and periphery countries exist.

An additional advantage of the minimalist approach of our country representation is that it makes our analysis more general. In fact, our results characterize heterogeneous cycles in any context where categories of borrowers differ in terms of lenders' required skill to select those which are credit-worthy. We plan to explore other applications in future research.

Finally, an important simplifying assumption is that we treat the realization of the aggregate state as an exogenous "prudence shock" throughout the main body of the paper. Given the complex equilibrium interaction between financial and real variables across different countries, the exogenous treatment is necessary for tractability. In section 7.1, we provide some intuition on how prudence shocks can arise endogenously through two possible micro-foundations. In the first micro-foundation, an aggregate productivity shock triggers different prudence shocks, while in the second micro-foundation different prudence shocks arise due to changing sentiments. We fully explore endogenous prudence shocks in the companion paper, Farboodi and Kondor (2018), using a sufficiently stripped-down version of the current framework.[9]

4 Booms and Busts in Core and Periphery Economies

In this section, we examine the implications of the model in the cross-country exposure to credit cycles. We start by analyzing the real implications: investment, growth, debt and default; and then explore the implications related to the credit market. We conclude each piece with a corollary that summarizes our predictions.

4.1 The Real Economy: Investment, Growth, Debt and Default

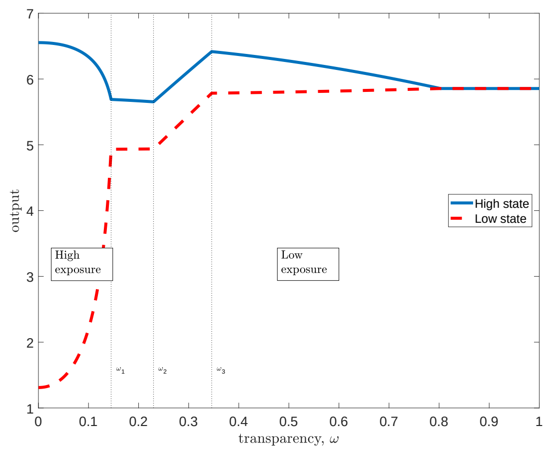
We argue that the high (low) aggregate state in our model closely resembles the boom (bust) phase of the global cycle.[10] To show this, we calculate the total output $Y(\omega,\theta)$ of each country in each regime by aggregating across all good and bad firms, either affected or unaffected by the liquidity shock.

$$Y(\omega,\theta) \equiv \begin{cases} \rho_g(1-\lambda)I(\omega,g) + \rho_b\lambda((1-\phi)I(\omega,b) + \phi i(\omega,b,H)) & \text{if } \theta = H \\ \rho_g(1-\lambda)((1-\phi)+\phi\eta_L(\omega))I(\omega,g) + \rho_b\lambda(1-\phi)I(\omega,b) & \text{if } \theta = L \end{cases}$$

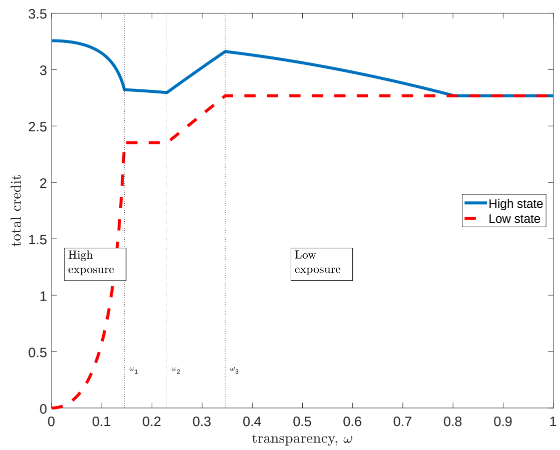
Similarly, we calculate the total face value of debt issued by good and bad firms in country ω

[9] See also Philippon (2006) and Bouvard and Lee (2016) for theory and evidence suggesting that investors are indeed less prudent when selecting investment projects during economic booms.

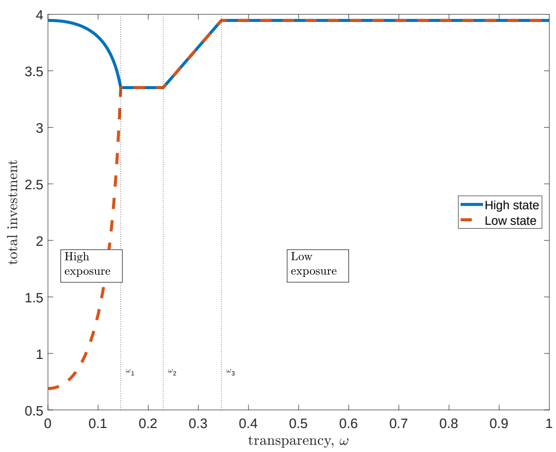
[10] In section 6, we explicitly consider a dynamic version of our set-up.



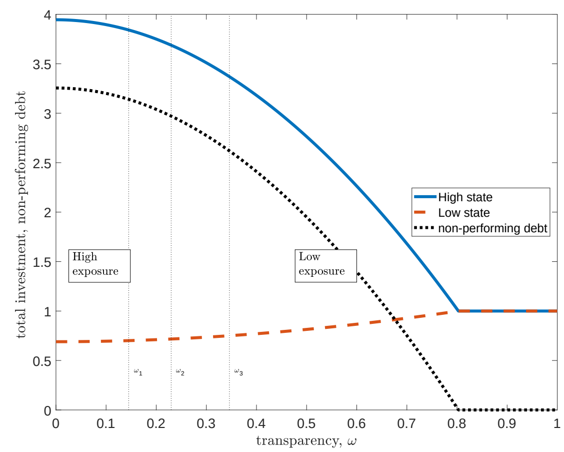
(a) Output, $Y(\omega, \cdot)$ per country in high (solid) and low (dashed) states



(b) Total credit, $C(\omega, \cdot)$ per country in high (solid) and low (dashed) states



(c) Good firms' total investment, $I(\omega, g) + \phi\xi i(\omega, g, \theta)$ in high and low states



(d) Bad firms' total investment, $I(\omega, b)+\phi\xi i(\omega, b, \theta)$, non-performing debt, $\phi\xi i(\omega, b, H)$.

Figure 4: Output, debt, investment.

in state θ.

$$C(\omega, \theta) \equiv \begin{cases} \phi\xi\left((1-\lambda)I(\omega, g) + \lambda i(\omega, b, H)\right) & \text{if } \theta = H \\ \phi\xi\left((1-\lambda)\eta_L(\omega)I(\omega, g)\right) & \text{if } \theta = L \end{cases}$$

Figure 4a and Figure 4b illustrate the total output and debt across countries in the two aggregate states, respectively. Note that in each country, production in the high aggregate state is higher than the low aggregate state. This is the key statistic that associates the former state with booms and the latter with busts.

Furthermore, the output dramatically collapses in periphery economies when investors turn cautious, while the drop is much less pronounced for other countries. Similarly, while periphery economies experience the largest credit dry-up during busts, they enjoy higher credit inflows compared to all other countries during booms.

Investment decisions by both good and bad firms contributes to the high cyclicality of output and credit in periphery economies. Note the endogenous positive correlation between the size of the boom and the collapse in these countries. The role of credit market conditions in firms' liquidity management is crucial in this relationship. All high exposure firms partially or fully abandon production when investors turn cautious, which leads to a collapse in output. The key is to recognize that the ex-post collapse implies that these firms save less for maintenance, which allows them to make a higher initial investment. This trade-off is embodied in the investment-saving constraint (9). Furthermore, when investors are bold, bad firms in the periphery receive more credit compared to those in other groups, since they are more opaque. Thus, they maintain a larger fraction of their production in the high state, which increases the output in the periphery even further during a boom and amplifies the positive correlation between the size of the boom and the collapse. These patterns are consistent with the stylized facts of sudden stop crises in emerging markets in general (Calvo et al., 2004), and with the experience of periphery countries during the European sovereign debt crisis (Lane, 2013; Martin and Philippon, 2017).

Figure 4d reveals the dynamics of non-performing debt issued during a boom across countries. Recall that in booms, more bad firms obtain credit in more opaque countries. As bad firms do not pay back by assumption, this also implies that a larger fraction of the credit initiated during booms in periphery economies will not be paid back. Moreover, as long as $\rho_g > \rho_b$, this is consistent with a larger dispersion of productivity in booms in the periphery as compared to the core. Our model also implies that within each country, the fraction of non-performing debt is higher for debt which is initiated during booms, compared to the debt that is initiated during busts, since bad firms are unable to raise financing when investors turn cautious. These predictions are consistent with the observed increase in the mis-allocation of capital during the pre-crisis years in the south of Europe (Reis, 2013; Gopinath et al., 2017).

The following Corollary summarizes the results.

Corollary 1 [Testable Predictions: Real Economy]

(i) Total output, total debt, and total investment by country

(a) More cyclical in countries within the high exposure region than in other countries.

(b) In booms, higher in countries within the high exposure region than in countries within the low exposure region.

(ii) Capital mis-allocation and non-performing debt

(a) In booms, productivity dispersion among firms obtaining credit is larger in countries within the high exposure region than in countries within the low exposure region.

(b) A larger fraction of debt turns to non-performing in countries within the high exposure region.

4.2 Credit Market: Price and Return on Debt, and Portfolio Compositions

In this section we analyze the predictions of the model for credit markets. Our model implies that while credit markets are integrated in booms, they become fragmented in recessions. That is, yields in periphery economies spike especially relative to borrowers in core countries, as the aggregate state changes from high to low. In relation to the European sovereign debt crisis, this fragmentation was observed not only in the market of sovereign bonds, but also on financial and non-financial corporate debt (Battistini et al., 2014; Farhi and Tirole, 2016; Gilchrist and Mojon, 2018), and bank credit (Darracq Paries et al., 2014).

For instance, Figure 5 illustrates that by 2010, non-financial firms active in the corporate bond market who were treated almost as equals before the Greek crisis, suddenly started facing very different market conditions depending on their country of origin. Whether an investment grade firm was French or Italian did not seem to matter before or even during the crisis in 20082009. By 2011, Italian firms we paying a much higher interest rate for credit than French firms. We connect this figure with the interest rate schedule on Figure 2. The shift from the high to the low aggregate state in our model corresponds to the fragmentation of the corporate bond market around 2010.

Furthermore, a direct testable implication of our assumed information structure is that the ownership of the portfolio of securities changes significantly across states. During a boom, credit from each country is held by a wide range of investors who have various skill levels. Hence, the concentration of ownership of bonds is low in each country, as even low skilled investors are willing to lend to firms in periphery economies. However, during busts, these low skilled investors stop lending to firms in high exposure countries, re-balancing their portfolio towards the core countries where they can confidently identify the good firms. In contrast, highly skilled investors re-balance toward periphery countries where they can earn high returns. This leads to a high concentration of ownership of credit in high exposure countries during busts.[11]

Indeed, in context of the Eurozone crisis, Ivashina et al. (2015) and Gallagher et al. (2018) find that in 2011 a group of US money market funds stopped lending only to European banks but not to other banks which had similar risk. In particular, Gallagher et al. (2018) finds that when these money market funds stopped financing firms in a European country, they did so irrespective of a firm's implied risk of default. These predictions are consistent with our mechanism when considering these funds as low skilled investors. Moreover, Ivashina et al. (2015) also find evidence that this process led to a significant disruption in the syndicated loan market, a possible channel for the real effects that our model predicts.

[11] (Battistini et al., 2014) documents a pattern consistent with our predictions on concentration of ownership in the context of sovereign lending in the Eurozone. However, we are not aware of any test of these predictions on corporate credit.

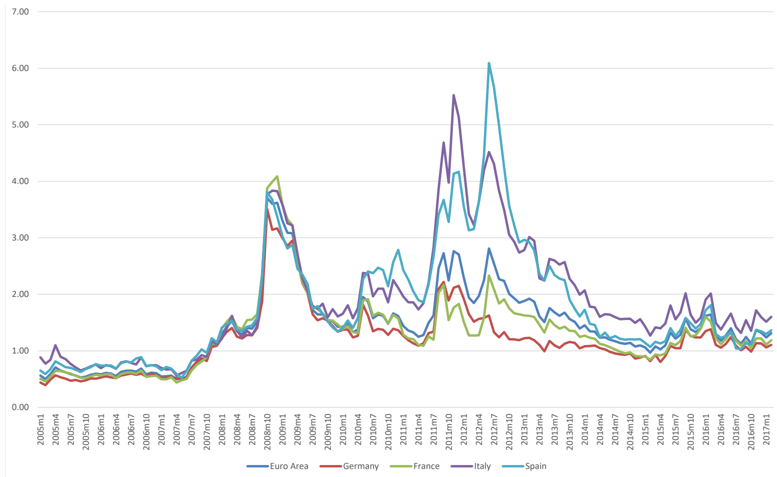


Figure 5: Eurozone non-financial corporate credit spreads compared to German treasuries of equal maturity as estimated by Gilchrist and Mojon (2018).

Examining the pattern of interest rates and non-performing debt, illustrated in Figures 2 and 4d, along with the heterogeneity in investors' rebalancing, we obtain predictions on the realized return across countries, investors, and states. While in booms yields are the same, more bonds issued in booms default in periphery countries. In contrast, the default rate among contracts issued in busts is zero in our model, but the yields for periphery bonds are higher. It follows that the realized returns on bonds are higher in periphery economies if they are issued during busts, while higher in core economies if issued during booms. Furthermore, although high skilled investors always outperform the low skilled ones, the reason is markedly different across states. In booms, despite lending at the same common interest rate as their peers, their realized return is high because of the low default rate in their portfolio of bonds. In contrast, in busts they hold bonds with higher than average nominal yield.

We summarize these observations as testable predictions in the following Corollary.

Corollary 2 [Testable Predictions: Credit Market]

(i) Credit markets are fragmented during busts, but integrated during booms. Nominal yields for comparable firms are

  (a) (close to) equal in different countries in booms,

  (b) higher in periphery than in core economies in busts.

(ii) Ownership of debt is more concentrated

  (a) in busts than booms in each country,

  (b) in high than in low exposure countries during busts.

(iii) Heterogeneity in investors' holding:

(a) In busts, skilled investors rebalance toward high exposure countries with high yields, while unskilled investors shed assets in high exposure countries and rebalance towards low exposure countries.

(b) Therefore, skilled investors' credit portfolio overperforms in booms due to lower default rate and in busts due to higher nominal yield.

(iv) Realized returns

(a) on bonds issued in booms is higher in countries within the high exposure region,

(b) on bonds issued in bust is higher in countries within the low exposure region.

To sum up, our model is consistent with the stylized picture that in booms, periphery countries experience a large credit boom, and build up more debt than core countries. However, a significant part of the debt issued in booms defaults later. Furthermore, in busts periphery countries experience a much larger contraction in credit, investment, and output, compared to core countries. Finally, while periphery countries raise credit at an interest rate similar to that of core countries during booms, during a bust the spread between core and periphery interest rates spikes, and the credit market becomes fragmented.

Let us emphasize that we generate these facts in a model where countries are ex-ante identical in their fundamentals. As such, the aggregate shock would not have any effect without investors being heterogeneously informed about firms in different countries. While we do not doubt that fundamental differences across European countries also contributed to their differential performance, we do not introduce such heterogeneity in order to make it very clear that the informational frictions affecting credit supply is the key to generating our predictions.

4.3 Corporate Credit, Sovereign Bonds and Safe Asset Determination

We have chosen not to explicitly model governments as decision makers; thus, there are no separate role for sovereign bonds and corporate credit in our framework. Nevertheless, it is reasonable to use the spread on the corporate bond portfolio of a given country in our model as a prediction for the corresponding sovereign spread, as sovereign bond and average corporate spreads move in tandem in the data. For instance, the average correlation between the non-financial corporate spreads depicted in Figure 2 and the sovereign bond spreads in the respective countries was 0.92 between 1999 and 2017 (ranging from 0.88 in Italy to 0.95 in Germany).

With this caveat in mind, our model has implications on the set of countries where safe assets, public or private, can be issued. We follow the definition of He et al. (2016) and Maggiori (2017), who define safe assets as those which are traded at a lower yield during bad times, often due to flight-to-quality episodes. We state our model prediction in the following Corollary.

Corollary 3 [Safe asset determination] *Safe assets are issued only in sufficiently transparent countries, with $\omega > \omega_3$. These countries have a low-exposure to credit cycles.*

Note that our framework provides a novel mechanism for safe asset determination. He et al. (2016) emphasizes the role of coordination of investors. Farhi and Maggiori (2017) focuses on the issuer's limited commitment not to default on the asset (or devalue the underlying currency). There is also a strand of literature (e.g. Caballero et al., 2008; Caballero and Farhi, 2013) where a given country is capable of issuing safe assets, and/or certain investors demand safe assets by assumption, and only the quantity is determined in equilibrium. In contrast, in our paper the set of issuers and the set of buyers are endogenously determined by the level of transparency, ω_3, at which the supply of sufficiently transparent assets just equals the demand of sufficiently unsophisticated investors, s_3.[12] In Section 5.3, we highlight how our mechanism leads to new predictions on the implications of excess global savings and the scarcity of safe assets.

5 Core, Periphery, and the Increase in Global Savings

The central insight of a simple global equilibrium is that firms' competition for the scarce capital of heterogeneously informed investors leads to heterogeneous boom and bust patterns. Even if all countries share the same production fundamentals, firms recognize the differences in the financing conditions they will face. Therefore, they choose different investment plans, leading to heterogeneous economic outcomes.

So far we have shown how in equilibrium, countries are endogenously partitioned into exposure groups along the cyclicality of their credit spreads. We have further argued that high exposure countries with counter-cyclical spreads have more volatile output, investment, and more non-performing credit after a boom, and a higher concentration of debt during a recession.

In this section, we shift our attention to the determinants of this partition. What economic forces dictate that a country subject to a given degree of information friction (vis-a-vis investors) belongs to one or another exposure group? How does a change in the size of one group spills over to the other groups and affect the characteristics of the boom-bust cycle?

5.1 The Demand for Skilled International Capital

In section 3.2.1 we used Figure 3 to describe how the partition of countries into high, low, and intermediate exposure groups is determined by the demand and supply for high and low expertise

[12]It is useful to recognize that in the existing literature the distinction between the concepts of "reserve currency" and "safe asset" is not always clear. A reserve currency is traditionally defined as an asset which serves three roles simultaneously: it is an international store of value, a unit of account, and a medium of exchange. Because of the second and third role, there can be only very few reserve currencies in the world almost by definition. While reserve currencies usually qualify, our definition of safe assets can also include a large set of other securities, potentially ranging from sovereign bonds and currencies of some small developed countries (e.g. Swedish sovereign bond, Swiss Franc), to highly rated commercial papers, or certain asset-backed securities.

capital. In this section, we will use the same intuition to explore the effect of changing parameters on the relative size of these groups, as well as the real output and investment.

Recall that in a simple global equilibrium, the low exposure group of countries are those with sufficiently high transparency firms, $\omega \in [\omega_3, 1]$. As Figure 3 illustrates, for a fixed type distribution of investors, $w(\cdot)$, ω_3 is determined using country demand curve at zero interest rate, $\phi(1-\lambda)\xi i_0$. Changes in parameters that leads to an upward shift of this demand curve implies a higher ω_3, that is, a shrinking low exposure group. Intuitively, larger demand by good firms implies that the marginal expert who absorbs the demand from core countries, $s_3 = 1 - \omega_3$, has to be richer. Since $w(s)$ is downward sloping, this implies that investor s_3 is less skilled, which in turn implies that ω_3 has to be more transparent.

Analogously, changes in parameters leading to an upward shift in a country's demand curve at $\bar{r}$, represented by $\phi(1-\lambda)\frac{\xi}{1+\bar{r}} i_{\bar{r}}$, implies a higher ω_1, and thus a larger high exposure group. The intuition here is slightly more subtle. When $\theta = L$, not only good firms in peripheral countries $\omega \in [0, \omega_1)$ borrow at interest rate $\bar{r}$, but some higher transparency firms $\omega \in [\omega_1, \omega_2)$ also only raise financing at the same high interest rate, although the latter group of firms are sufficiently transparent to get fully funded. In fact, ω_1 is determined precisely by the threshold where marginal demand surpasses the marginal residual supply, and firms from opaque countries which lie below the threshold start being rationed. When each firm demands more credit at $\bar{r}$, the higher transparency good firms exhaust the available credit supply even faster, implying a higher ω_1.

Since firm credit demand is an endogenous object, typically parameters affect this demand via multiple channels. Therefore, tracing back shifts in the demand curve to changes in the deep parameters of the model is not straightforward. Nevertheless, we can decompose the total effect into two parts. First, keeping the interest rate in the high state (r_H) fixed, changes in parameters have a direct effect on maintained investment $i(\omega, \tau, L)$, as described by equation (15). Second, there is an indirect effect through the spill-over across aggregate states. A change in credit demand in the low state affects initial investment $I(\tau, g)$ and, through the budget constraint (9), affects credit demand in the high state as well. This in turn changes the equilibrium interest rate in the high state, r_H, which then feeds back into the initial and maintained investment in both high and low states.

In the following proposition, we characterize the direct effect. In particular, we show that an increase in the probability or the size of the liquidity shock, ϕ and ξ, in the fraction of good firms, $1 - \lambda$, and in the probability of the low aggregate state, π_L, all increase the total credit demand of good firms at zero interest rate, and, consequently, shrink the set of core countries. Similarly, an increase in the size of the liquidity shock, in the fraction of good firms, and in the productivity of good firms increases the total credit demand of good firms at $\bar{r}$ interest rate, and, consequently, increases the set of peripheral countries. While we do not have analytical results on the indirect effect, the direct effect dominates in all our numerical simulations.

Proposition 6 *In a simple global equilibrium, keeping r_H fixed,* *(i) the set of low exposure countries shrinks if there is an increase in ξ, ϕ, $1-\lambda$, or π_L,*

$$\frac{\partial \omega_3}{\partial \xi}, \frac{\partial \omega_3}{\partial \phi}, \frac{\partial \omega_3}{\partial (1-\lambda)}, \frac{\partial \omega_3}{\partial \pi_L}|_{r_H \text{ fixed}} > 0.$$

(ii) The set of high exposure countries grows if there is an increase in ξ, $1-\lambda$ or ρ_g,

$$\frac{\partial \omega_1}{\partial \xi}, \frac{\partial \omega_1}{\partial (1-\lambda)}, \frac{\partial \omega_1}{\partial \rho_g}|_{r_H \text{ fixed}} > 0$$

Let us emphasize that in the context of our model, any intuition implying that better fundamentals in general correspond to an expansion of low exposure and/or a reduction in the high exposure set of countries is false. Neither is any intuition implying the opposite true. For instance, a larger fraction of good firms shrinks the low exposure region and expands the high exposure region, while a smaller idiosyncratic shock has the opposite effect. In contrast, increasing the productivity of good firms, i.e. a higher ρ_g, does not affect the set of low exposure countries, but increases the size of the high exposure group.

5.2 Excess Savings: Supply of Skilled Capital and the Indirect Effect

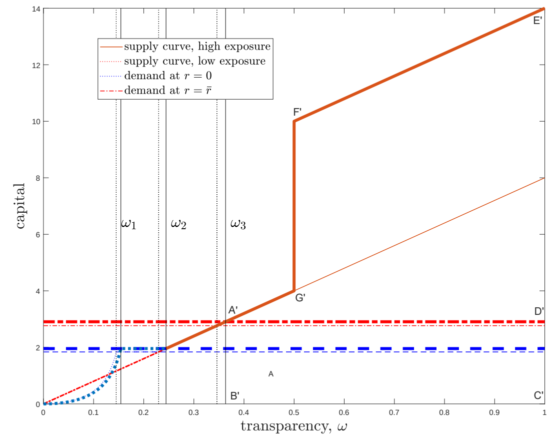
In this section, we examine the effect of an increase in the supply of capital by low skilled global investors. We interpret this exercise as a representation of the excess global savings phenomena, as described by Caballero et al. (2017). In particular, we consider the effect of substituting $w(s)$ with a $\tilde{w}(s)$, such that

$$\tilde{w}(s) = \begin{cases} w(s) + k & \text{if } s \leq \bar{s} \\ w(s) & \text{if } s > \bar{s} \end{cases}$$

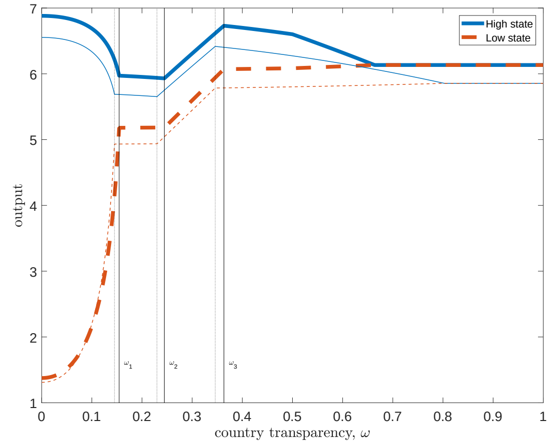
for some positive constant k. To make the analysis simple, we pick an $\bar{s}$ such that $\bar{s} < 1 - \omega_3$ associated with $w(s)$. This choice implies that the increase in the supply of global capital does not have a direct effect on demand and supply around thresholds ω_1 and ω_3. As such, all the results are driven by indirect effect.

Figure 6a illustrates this exercise. The key observation is that both thresholds ω_3 and ω_1 move to the right: the core shrinks and the periphery expands. The increase in the supply of capital decreases the common interest rate in the boom, r_H. This improvement in funding conditions during the boom allows firms to invest more, which increases their liquidity demand for low and high expertise capital alike, both in the boom and the bust. The upward shift in the demand curve when $\theta = L$ implies that both thresholds ω_1 and ω_3 shifts the right.

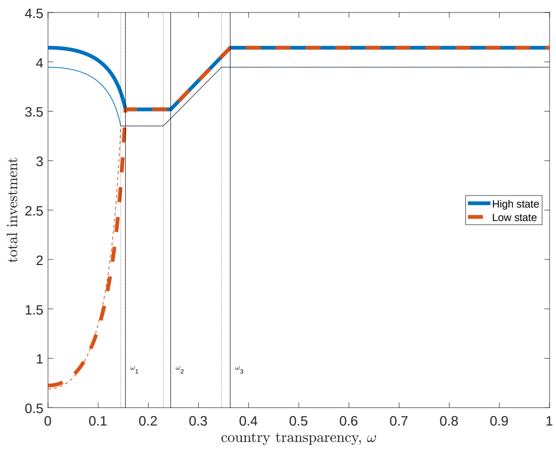
Moreover, this increase in the supply of capital affects the real economic outcome across countries in both states. The remaining panels of Figure 6 illustrate this by comparing the total investment by good and bad firms, and the total output after the increase in credit supply.



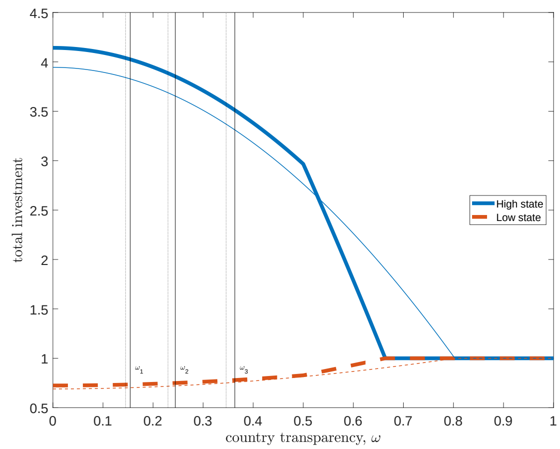
(a) The effect of increase in low s wealth on supply and demand for investors' funds in busts, and the determination of thresholds $\omega_1, \omega_2, \omega_3$.



(b) The effect of increase in low s wealth on output, $Y(\omega, \cdot)$ per country in high state (solid) and low state (dashed) states



(c) The effect of increase in low s wealth on good firms' investment, $I(\omega, g) + \phi\xi i(\omega, g, \theta)$ in booms (solid) and busts (dashed)



(d) The effect of increase in low s wealth on bad firms' investment, $I(\omega, b) + \phi\xi i(\omega, b, \theta)$, in booms (solid) and busts (dashed)

Figure 6: The effect of increase in low s wealth on output and investment

The higher availability of global supply of capital increases the investment of both good and bad firms during booms. Good firms increase investment, because excess saving decreases the interest rate, r_H. Bad firms in non-transparent countries increase investment even more, because the additional capital is channeled towards low skilled investors who cannot distinguish those firms from the good ones. The more pronounced boom implies a larger collapse and a larger volume of non-performing debt in the high exposure countries.

5.3 Excess savings and safe assets

The above discussion sheds new light on the problem of scarce safe assets described in Caballero et al. (2017). It is argued that during the last decades the supply of safe assets has not been able to keep up with the increasing demand, which has led to excessively low interest rates on these assets.

Under our interpretation of safe assets, introduced in Section 4.3, an increase in capital of low skill investors represents increasing demand for safe assets. In line with Caballero et al. (2017), this increasing demand results in a lower interest rate, r_H, during booms. Similar to the previous literature, our model predicts that this increases the supply of safe assets by each issuer who remains in the low exposure region, i.e., $(1-\lambda)\xi i_0|_{\omega\in[\omega_1,0]}$ goes up, but not nearly enough to offset the increase in demand. This is apparent by the increase in the area of the ADE triangle in Figure 3 to the A'D'E'F'G' area in Figure 6a, which corresponds to the increase in idle capital.

6 Simple Dynamics: Heterogeneous Global Cycles

To illustrate the heterogeneous global cycles implied by our model, in this section we introduce a simple dynamic version of our setting.

The dynamic model consists of consecutive generations of firms and investors, indexed by h. Each generation lives for a random number of periods, explained below, and once dead, it is replaced by a new generation. With some abuse of notation, let t_h denote date t of generation h lifetime, where we start at $t_h = 0$, $\forall$ h. The prudence of each generation of investors remains the same throughout their lifetime. Thus, each generation includes a group of firms, investors and an aggregate prudence shock.

In order to accommodate the generations' random life time, we introduce a new shock which we call *"generation shock."* Consider generation h. The first two periods of its lifetime, $t_h = 0, 1$, are identical to $t = 0, 1$ in the baseline model. However, to allow for dynamics, we adjust the date $t = 2$ in the static model. At the beginning of each period $t_h \geq 2$, the generation shock is realized. With probability ψ_θ, the realization is 1, in which case, the current generation dies at the end of the current period and is replaced by a new generation. With complementary probability $1-\psi_\theta$, the realization is 0, and the current generation survives to the next period.

Suppose that t_h is the last period of the existing generation's life. We assume that the next generation of (the managers of) the firms are not able to operate the previous generations' equipment. That is, firms' maintained units of investment pay a final ρ_τ cash-flow and cease operation. These firms repay their existing credit if $\tau = g$. All firms and investors in this generation consume their remaining wealth. That is, from the point of generation h, this period is similar to period $t = 2$ of the baseline model.

Furthermore, generation h of firms and investors are replaced by generation $h+1$ and $t_{h+1} = 0$. For the new generation of investors, we redraw their state of prudence, θ. For the new generation of firms, we redraw their type, $j = (\omega, \tau)$.

Alternatively, suppose the realization of the generation shock is 0, and generation h survives this period. That is, $t_h \geq 2$ is not the last period of their life. Then, each unit of the firms' existing project maintained in period $t_h - 1$ produces ρ_τ units, before being subject to an additional liquidity shock, with probability ϕ.

As we have assumed that $\rho_\tau > \xi$, the firms in a surviving generation do not need to borrow if they get a liquidity shock at any $t_h > 2$. Instead, they cover the cost of maintenance from the production of their existing units.[13] As a result, firms only require outside liquidity to insure against the potential liquidity shock at $t_h = 2$ when their project has not become productive yet. Once firms are past $t_h = 2$, they continue with their existing units, without any liquidation, until the generation dies.

Finally, we assume that in each period, firms can propose to their investors to delay repayment of credit for one more period. Investors do not discount the future and do not provide further credit after period $t_h = 1$, thus, they are indifferent to grant delay until the last period of the generation's life. We assume that the investors do so when they are indifferent.[14] As a result, firms repay their credit (or default) and reveal their pledgeability type only in their last period of operation.

Using these assumptions and with slight abuse of notation, we can rewrite the problem of a firm in generation h as

$$\max_{I(\omega,\tau),\{\sigma(m,\omega,\tau;\theta),i(\omega,\tau;\theta)\}_{\theta,m}} \sum_{t_h=0}^{\infty} \left((1-\psi_\theta)^{t_h} \sum_\theta \pi_\theta \Big[(1-\phi)\rho_\tau I(\omega,\tau) + \phi(\rho_\tau - \mathbb{1}_{\tau=g}\xi) i(\omega,\tau;\theta) \Big] \right) - 1 \quad (33)$$

subject to (4)-(9).

It is easy to see that (33) is almost identical to (10), and, consequently, the equilibrium in the dynamic version is almost identical to our baseline model.[15] Indeed, this version nests are baseline model with the choice of $\psi_\theta = 1$, $\theta = H, L$.

Figure 1 plots a simulated path of interest rate and output of the dynamic model, for a high and low exposure country, which illustrates the heterogeneous global cycles. The output of the high exposure country collapses sharply in low aggregate states (shaded areas), and its interest rate spikes. The low exposure country suffers only a moderate drop in output in the low aggregate state. The interest rate that this group faces can even drop.

[13] One can show that good firms weakly prefer to use their own cash for maintenance, as borrowing is costly. Because of this, bad firms would not get outside financing either.

[14] A simple intuition for this assumption is that if investors try to seize repayment, they have to write down the loss on their credit to bad firms. It is a weak and realistic assumption that investors prefer to delay the realization of their losses as long as it does not effect their expected cash flows.

[15] The steps of the proof of the baseline case in the Appendix go through with trivial modifications. The only caveat is that if $\psi_H \neq \psi_L$, then (19) is replaced with $\bar{y}(x) = \frac{\psi_H}{\psi_L} \frac{(\rho_g - \xi)(1+\phi\xi\pi_H x)}{\left((1-\phi)\rho_g\left(\pi_L \frac{\psi_H}{\psi_L} + \pi_H\right) + \phi\left(\rho_g - \xi\right)\pi_H\right)\xi}$.

7 Generalizations

In this section, we generalize two aspects of our framework. In the first part, we argue that the prudence shock might be triggered endogenously by an aggregate productivity shock or a sentiment shock. In the second part, we partially relax the assumption that investors' prior is uninformative on the implication of a firm's country of origin on its opaqueness.

7.1 Bold or Cautious Experts? Enodogenous Information

In our baseline, we modeled the prudence shock as exogenous. In this section, we provide a simple micro-foundation to show that with endogenous information acquisition, standard aggregate shocks commonly associated with recessions can turn investors from bold to cautious.

Let θ denote the aggregate shock, to be specified below. Consider that an investor of skill s receives her signals from her analyst, of type s. Investor's financing decision has to be measurable with respect to the signal she receives, as in the baseline model. At $t = 1$, after the realization of the aggregate shock θ, the analyst can choose to be bold or cautious, where the interpretation of bold and cautious is the same as in the baseline model. The analyst is compensated on the basis of the number of his correct and incorrect assessments of firms, independent of his prudence, as specified in Table 1.

Bold	$\varnothing$	b
g	$a_{g,\theta}$	$-c_{g,\theta}$
b	$-c_{b,\theta}$	$a_{b,\theta}$

Cautious	g	$\varnothing$
g	$a_{g,\theta}$	$-c_{g,\theta}$
b	$-c_{b,\theta}$	$a_{b,\theta}$

Table 1: Pay-off of an analyst as a function of his report and the true type of the given firm.

When the a firm is good, the analyst's bonus if he communicated a favorable opinion (i.e., he states that there is evidence that the firm is good or states that there is no evidence for the contrary) is $a_{g,\theta}$, while his penalty is $c_{g,\theta}$ otherwise. If the firm's type is b, the bonus if he communicated an unfavorable opinion is $a_{b,\theta}$, while the penalty for contrary is $c_{b,\theta}$. The pay-offs are arbitrary as long as $a_{g,\theta}, a_{b,\theta} > 0$ and $c_{g,\theta}, c_{b,\theta} \geq 0$.

To see the optimal decision of the analyst, we compare his expected pay-off of being bold versus cautious. Analyst s chooses to be bold if and only if

$$\lambda_\theta \left[s a_{b,\theta} + (1-s)(-c_{b,\theta}) \right] + (1-\lambda_\theta) a_{g,\theta} > (1-\lambda_\theta) \left[s a_{g,\theta} + (1-s)(-c_{g,\theta}) \right] + \lambda_\theta a_{b,\theta}$$

which simplifies to

$$\frac{1}{1 + \frac{a_{b,\theta}+c_{b,\theta}}{a_{g,\theta}+c_{g,\theta}}} > \lambda_\theta. \tag{34}$$

That is, the analyst chooses to be bold when the fraction of bad firms in the economy are relatively small. This is intuitive. For instance, if almost all firms are good, the marginal value of conclusive evidence to identify the few bad firms is higher. Condition (34) suggests two simple ways to microfind the idea that investors turn to cautious in the low aggregate state.

The first microfoundation exhibits that an aggregate productivity shock triggers the prudence shock.

Microfoundation 1 *Assume that the aggregate shock is a productivity shock. In particular, θ determines the fraction of bad firms such that $\lambda_H < \lambda_L$. That is, there are more bad firms in the low state. Furthermore, suppose that the bonus and penalty depends only on the fraction of firms the analyst was correct or incorrect about, and not on the firm type or the aggregate state $a_{\tau,\theta} = a$ and $c_{\tau,\theta} = c$, $\forall\, \theta, \forall\, \tau$.*

If $\lambda_H < \frac{1}{2} < \lambda_L$, the aggregate productivity shock triggers the prudence shock.[16]

The second microfoundation shows that a particular type of sentiment shock triggers the prudence shock.

Microfoundation 2 *Assume that the aggregate shock is a sentiment shock. In particular, the analyst is penalized more to miss out on a good firm in the high state than in the low state. More specifically, consider that the bonus for correct recommendations is identical across aggregate states, normalized to 1: $a_{g,\theta} = a_{b,\theta} = 1$, $\forall\, \theta$. However, $c_{g,H} > c_{g,L}$.* [17]

If $\frac{1}{1+\frac{1+c_{b,H}}{1+c_{g,H}}} > \lambda > \frac{1}{1+\frac{1+c_{b,L}}{1+c_{g,L}}}$, this sentiment shock triggers the prudence shock.

Endogenizing $a_{g,\theta}, a_{b,\theta}, c_{g,\theta}$ and $c_{b,\theta}$, and connecting these parameters to returns of investors in our model would be a natural next step. However, in our current set up with a continuum of firm and investor types, this would lead to an analytically intractable model. Instead, in a companion work in progress, Farboodi and Kondor (2018), we implement this idea using a stripped-down version of our current framework.

Finally, we refer the reader to Philippon (2006) and Bouvard and Lee (2016). Both papers argue that the investors selecting the risky projects choose to be less cautious in booms than in recessions. The idea is that the due diligence of these projects takes time, and the opportunity cost is higher in booms due to more positive NPV projects present. Bouvard and Lee (2016) shows that in equilibrium competition drives the resources spent on due diligence efficiently low, especially in booms. Philippon (2006) presents evidence that supports this point.

[16] See Appendix D.1 for the generalization of relevant model equations with state dependent λ_θ.

[17] The idea that managers are particularly averse to false negative mistakes in booms is consistent with the infamous quote of Charles Prince, the CEO of Citibank in 2007, at the onset of the financial crisis in the context of the role of Citibank as the major private equity deal provider.

> When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing. (Financial Times, July 9, 2007)

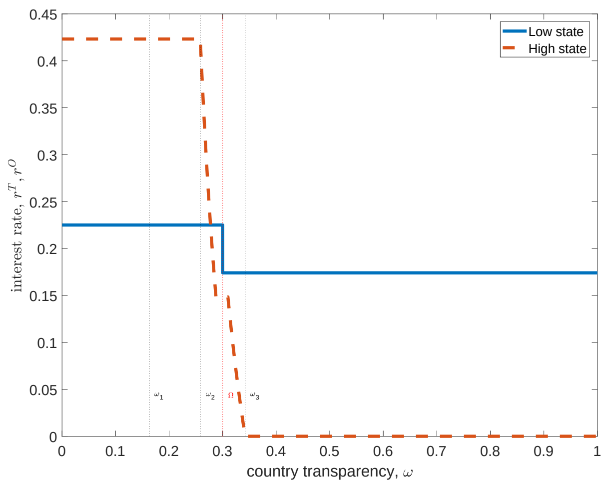


Figure 7: Interest rates for transparent and opaque countries in the high (solid) and low (dashed) aggregate state.

7.2 Partitioned Transparency Groups

In the baseline model, we assume that investors have an uninformative prior about ω, the average transparency of firms in a given country. That is, if an investor does not find conclusive evidence on a firm, the country of origin does not help her do any further inference.

In this section, we weaken this assumption. In particular, suppose that a public signal partitions countries into a transparent and an opaque group. That is, observing the public signal, each investor knows that the transparency, ω of the given country is $\omega > \Omega$ or $\omega < \Omega$, where Ω is an arbitrary cut-off. Intuitively, investors understand that a firm from a southern country in Europe tends to be more opaque than a northern country firm, but they have no information on how firms in different south European countries compare to each other.

Figure 7 illustrates the effect of this treatment on the equilibrium interest rate schedules. Compared to the corresponding figure for the baseline case, the left panel of Figure 2, it is clear that the qualitative difference is small. The main effect of the extra signal is the partial separation in the high aggregate state. With the public signal, investors have an additional choice. They can choose to accept only firms from the transparent group to lend to. For less skilled investors, this implies a portfolio with less bad firms, as their mistakes are concentrated in opaque countries. Therefore, in equilibrium, less skilled investors lend to firms from the transparent group only, albeit at a lower interest rate. On the other hand, more skilled investors lend to firms from the opaque group but for higher interest rate. The marginal investor who is just indifferent between these two choices is determined in equilibrium.[18]

While it is an intuitive assumption that investors have some prior knowledge on the average

[18] The public signal also introduces a small bunching region around Ω in the low aggregate state interest rate schedule. As we explain in Appendix B.1, this comes from the requirement that the interest rate schedule has to be weakly monotonically decreasing in ω, and is obtained by an ironing procedure.

transparency of firms in different countries, we assume this away in the baseline model because of two main reasons. First, we believe the additional analytical complexity does not justify the additional insight. Second, one of the main focuses of our analysis is how investors endogenously classify countries into low and high exposure groups in equilibrium. As this extension illustrates, a public signal on ω classify countries exogenously, and obscures our analysis.

8 Conclusion

We argue that in the presence of heterogeneous information frictions between international investors and firms, financial liberalization impacts countries heterogeneously, even if these countries have similar fundamentals. Our main premise is that identifying good lending opportunities requires skill, and investor skill is particularly important in certain countries, which we refer to as opaque. As such, countries are subject to varying degrees of information frictions vis-a-vis investors.

We show that countries subject to the most severe frictions become highly exposed to aggregate shocks to investors' information, which leads to counter-cyclical credit flows and output. Countries that are less exposed to these information frictions experience a much smaller drop in output in a bust, and benefit from low spreads and large capital-inflows. We further illustrate that the key to both the existence of the credit cycle and the heterogeneous exposure spectrum across countries is the scarcity of skilled investor capital.

Our framework provides a wealth of predictions about credit spreads, investment, safe asset supply, concentration of debt ownership, and the return on debt during the boom-bust cycle, both in the time series and in the cross-section. These implications provide a useful guidance for future empirical work. Moreover, we plan to examine the normative implications of our framework and the consequences of different policy interventions on the structure of the global equilibrium. Furthermore, our analysis can be extended in multiple directions. It would be interesting to explicitly model dynamics and study the spillovers in investment and wealth over time. Another fruitful avenue is to investigate the endogenous determination of investors' information regime.

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Online Appendices

The appendix is organized as follows: Appendix A introduces the required formalization to solve for equilibrium. Appendix B constructs the equilibrium. Since we solve for the equilibrium by backward induction, this appendix is divided into two sections. Section B.2 solves for credit market equilibrium at $t = 1$, using the formalization provided in appendix A. Subsection B.1 then solves for the date $t = 0$ equilibrium. Appendix C provides the proofs for the results in the text, using the equilibrium structure of appendix B. Appendix D discusses some extensions.

A Credit Market Formalization

In this section we build on Kurlat (2016) to provide a formal description of the credit markets. The concepts and definitions follow Kurlat (2016) closely, however the proofs have to be generalized in a few key dimensions.

(i) In our model firms have heterogeneous liquidity needs in the international markets, while in Kurlat (2016) there is homogeneous scale. We generalize the proofs in Kurlat (2016) to accommodate heterogeneous scale. This generalization requires additional variables that are absent from Kurlat (2016).

(ii) In our model, there is a maximum interest rate that any firm would accept. In Kurlat (2016), there is no minimum acceptable price.

(iii) Since interest rates cannot fully adjust everywhere in our model, credit quantity has to adjust. Thus we have credit rationing when $\theta = L$ as well. There is no rationing in Kurlat (2016) in the corresponding information regime, false negative.

We also adjust Kurlat (2016) properly to work with interest rates payable at $t = 2$ as opposed to prices payable at $t = 0$.

We break the firm problem into two sub-problems. Firm j first chooses its initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, and then chooses how to raise the required liquidity on the international markets. In this section we formulate the liquidity raising plan on the international credit markets, taking the maximum amount that the firm can raise in total as given. We then solve this problem in section B.1. In section B.2 we relate the maximum liquidity that the firm can raise to its pledgeability constraint, and solve the firm's initial and maintained investment problem given the solution to liquidity raising subproblem.

A.1 International Credit Markets

There are many markets at $t = 1$, m, open simultaneously, where firms can demand credit. M denotes the set of all markets. Each market in aggregate state θ is defined by two features. The first feature is the market interest rate, $\tilde{r}(m;\theta)$, paid by firms to international investor in exchange for bonds. If in market m only firms from a single transparency ω are serviced, we use $r_\theta(\omega) = \tilde{r}(m;\theta)$ to denote the interest rate associated with that market m.

The second is a clearing algorithm. A clearing algorithm is a rule that determines which bonds are bought, as a function of demand and supply in a market. Since investors have different information sets, different clearing algorithms result in different allocations and we need to specify what algorithm will be used. We use an adjusted version of the algorithms proposed by Kurlat (2016).

Definition A.2 [LRF Clearing Algorithm] *A clearing algorithm is a total order on X, which determines which acceptance rule is executed first. ζ is a less-restrictive-first (LRF) algorithm if it orders nested acceptance rules according to $\chi_h <_\zeta \chi_{h'}$ if $\chi_{h'}$ is nested in χ_h; i.e. the less restrictive acceptance rule first.*

Thus, acceptance rules of the form $\chi_h(\omega,\tau) = \mathbb{1}\left(\tau \in T_1 \,||\, (\tau \in T_2 \,\&\, \omega \leq 1-h)\right)$ are ordered according to $\chi_h <_\zeta \chi_{h'}$ if $h < h'$, when ζ is an LRF clearing algorithm. Given the signal structure of investors when $\theta = H$, the less restrictive acceptance rule is also the less accurate.

Definition A.3 [NMR Clearing Algorithm] *ζ is a nonselective-then-more-restrictive-first (NMR) algorithm if it orders nested acceptance rules according to χ_h first if it imposes no restriction, and among acceptance rules with restrictions, the more restrictive acceptance rule first; i.e. $\chi_h <_\zeta \chi_{h'}$ if χ_h is nested in $\chi_{h'}$.*

Let $T_0 = \{g, b\}$. If ζ is an NMR clearing algorithm, acceptance rules of the form $\chi_h(\omega,\tau) = \mathbb{1}\left(\tau \in T \,\&\, \omega \geq 1-h\right)$ are ordered according to

(i) $\chi_{1,T_0} <_\zeta \chi_{h,T}$, for all $h < 1$ and T a subset of $\{g, b\}$;

(ii) $\chi_{h,T} <_\omega \chi_{h',T}$ if $h < h'$, for all $h, h' < 1$;

Given the signal structure of investors when $\theta = L$, the more restrictive acceptance rule is the less accurate.

Kurlat (2016) proves that in the presence of markets with different clearing algorithms, there exist an equilibrium where investors self-select into markets using LRF algorithm when the information structure is akin to ours in $\theta = H$, and markets using NMR algorithm when the information structure is that of $\theta = H$. For simplicity, we will directly assume that the clearing algorithm is LRF when $\theta = H$ and NMR when $\theta = L$. These algorithms guarantee that each investor receives a representative sample of the overall supply of bonds he is willing to accept, in the market where he participates.

A.2 Firm Problem

Assumption A.3 [Maximum Market Demand] *There is a maximum amount of credit each firm j can demand in each market m, denoted by $\bar{L}$. We require $\bar{L} \geq \max_\omega \ell(\omega, g; \theta)$.*

Definition A.4 [Rationing Function] *A rationing function η assigns a measure $\eta(.,\omega,\tau;\theta)$ on M to each bond issued by firm $j=(\omega,\tau)$.*

Let $M_0 \subset M$ denote a set of markets. Then $\eta(M_0,\omega,\tau;\theta)$ is the number of bonds firm (ω,τ) issues if he submits one unit of credit demand to each market $m \in M_0$ in aggregate state θ. The firm receives one unit per bond issued, and $r(\omega,\tau;\theta)$ denote the average interest rate firm $j=(\omega,\tau)$ has to pay back if aggregate state is θ.

Firm Optimization in International Markets. The firm participates in the international markets in each state θ if he is hit by the liquidity shock. We will closely map the problem of the firm in international markets to the seller problem of Kurlat (2016), in order to raise liquidity required to maintain investment. In order to do so, we introduce the following auxiliary variable, $\hat{y}$.

Definition A.5 [Total International Demand] *$\hat{y}\left(\omega,\tau;\theta,r_H,r_L\right)$ is the total credit firm j with (ω,τ) can raise on the international markets when aggregate state is θ, and the firm faces interest rate r_H (r_L) in state H (L), respectively. We assume $\hat{y}(\omega,\tau;\theta,r_H,r_L) \leq \bar{L}$, and $\hat{y}(\omega,\tau;\theta,r_H,r_L)$ is continuous and weakly decreasing in r_θ, $\forall\theta$.*

Here we define the firm's problem on the international credit market as an independent problem, which takes one state variable, $\hat{y}(\omega,\tau;\theta,r_H,r_L)$. Later in section B.2 we relate $\hat{y}\left(\omega,\tau;\theta,r_H,r_L\right)$ to the firm's pledgeability constraint, and the technological constraint $\bar{L}$. We also show that $y(\omega,\tau;\theta,r_H,r_L)$ in problem (A.1) maps to $\ell(\omega,\tau;\theta)$ as defined in equation (5). Moreover, in section B.2 we verify that in equilibrium, $\hat{y}(\omega,\tau;\theta,r_H,r_L)$ is weakly decreasing in r_H and r_L.

$$V_{\omega,\tau}\left(\hat{y}\left(.;\theta,r_H,r_L\right)\right) \equiv \max_{\{\sigma(m,\omega,\tau;\theta)\}_m} \left(1+r(\omega,\tau;\theta)\right)\left(\frac{\rho_\tau}{\xi} - \mathbb{1}_{\tau=g}\right) y(\omega,\tau;\theta,r_H,r_L) \tag{A.1}$$

$$\begin{aligned}
\text{s.t.}\quad & \\
& y(\omega,\tau;\theta,r_H,r_L) = \int_M \sigma(m,\omega,\tau;\theta)d\eta(m,\omega,\tau;\theta) \\
& y(\omega,\tau;\theta,r_H,r_L) \leq \hat{y}(\omega,\tau;\theta,r_H,r_L) \\
& 0 \leq \sigma(m,\omega,\tau;\theta) \leq \bar{L} \\
& r(\omega,\tau;\theta) = \frac{\int_M \tilde{r}(m;\theta)\sigma(m,\omega,\tau;\theta)d\eta(m,\omega,\tau;\theta)}{\int_M \sigma(m,\omega,\tau;\theta)d\eta(m,\omega,\tau;\theta)}
\end{aligned} \tag{A.2}$$

To any unit of bonds that the firm issues to international investors, he adds $r(\omega,\tau;\theta)$ units of what he has saved using the bankers. He then injects this as the required liquidity to maintain investment. Thus by issuing $y(\omega,\tau;\theta,r_H,r_L)$ bonds, the firm continues at scale $\frac{1+r(\omega,\tau;\theta)}{\xi}y(\omega,\tau;\theta,r_H,r_L)$, which pays off ρ_τ at date $t=2$. Good firms then have to pay back $1+r(\omega,\tau;\theta)$ per unit bond issued, which leads to the objective (A.1).

Similar to Kurlat (2016), the choice of $\sigma(m,\omega,\tau;\theta)$ for any single market m such that $\eta(m,\omega,\tau;\theta) = 0$ has no effect on the funding obtained by the firm. Formally, this implies that program (A.1) has multiple solutions. We follow Kurlat (2016) and assume that when this is the case, the solution has to be robust to small positive $\eta(m,\omega,\tau;\theta)$, meaning that the firm must attempt to sell an asset in all the markets where if he could he would want to, and must not attempt to sell an asset in any market where if he could he would not want to.

Definition A.6 [Robust Program] *A solution to program 10 is robust if for each θ and every (m_0,ω_0,τ_0) such that $\eta(m_0,\omega_0,\tau_0;\theta) = 0$ there exists a sequence of strictly positive real numbers $\{z_n\}_{n=1}^{\infty}$ and a sequence of credit demand, and bond issuance decisions $\{\sigma^n(m,\omega,\tau;\theta)\}_m$, and $y^n(\omega,\tau;\theta,r_H,r_L)$, such that defining*

$$\eta^n(M_0,\omega_0,\tau_0;\theta) = \eta(M_0,\omega_0,\tau_0;\theta) + z_n\mathbb{I}(m_0 \in M_0)\mathbb{I}(j = j_0)$$

(i) In aggregate state θ, $\{\sigma^n(m,\omega,\tau;\theta)\}_m$ solve program

$$\max_{\{\sigma(m,\omega,\tau;\theta)\}_m} \quad (1 + r(\omega,\tau;\theta))\left(\frac{\rho_\tau}{\xi} - \mathbb{1}_{\tau=g}\right) y(\omega,\tau;\theta,r_H,r_L) \tag{A.3}$$
$$s.t.$$
$$y(\omega,\tau;\theta,r_H,r_L) = \int_M \sigma(m,\omega,\tau;\theta)d\eta^n(m,\omega,\tau;\theta)$$
$$y(\omega,\tau;\theta,r_H,r_L) \le \hat{y}(\omega,\tau;\theta,r_H,r_L)$$
$$0 \le \sigma(m,\omega,\tau;\theta) \le \bar{L}$$

(ii) $z_n \to 0$

(iii) $\sigma^n \to \sigma$, $y^n \to y$; $\forall(\omega,\tau), m$

Why don't that cross-sectional differences across $\sigma(m,\omega,\tau;\theta)$, at the same market m_0, reveal the firm $j = (\omega,\tau)$? We assume that $\sigma(m,\omega,\tau;\theta)$ is divisible, and firms submit unit by unit. The investment that can potentially serve as collateral, if $\tau = g$, is verified and "marked", to avoid double promising.

A.3 Expert Problem

There is a distribution $w(s)$ of investors of expertise s, each endowed with one unit of wealth. Experts consume at dates $t = 1, 2$ and participate in international markets at $t = 1$. Since this is after realization of aggregate shock θ, we will suppress the dependence of their decisions on θ.

Definition A.7 [Acceptance Rule] *An acceptance rule is a function $\chi : \{R, U\} \times [0,1] \to \{0,1\}$.*

Definition A.8 [Feasibility] *An acceptance rule χ is feasible for investor s if it is measurable with respect to his information set, i.e. if*

$$\chi(\omega,\tau) = \chi(\omega',\tau') \quad \textit{whenever} \quad x(\omega,\tau,s) = x(\omega',\tau',s).$$

Let X denote the set of all possible acceptance rules, and X_s the set of acceptance rules that are feasible for investor s.

Definition A.9 [Allocation Function] *An allocation function A assigns a measure $A(.;\chi,m,\theta)$ on $[0,1]$ to each acceptance rule-market pair $(\chi,m) \in X \times M$.*

If $I_0 \subseteq \{R,U\} \times [0,1]$, $A(I_0;\chi,m,\theta)$ represents the fraction of bonds issued by firms j with $(\omega,\tau) \in I_0$ that an investor will obtain if he demands to buy one unit in market m and imposes acceptance rule χ.

In each aggregate state θ, investor s chooses the market he participates in, m, how many bonds he intends to finance δ, and a feasible acceptance rule χ to maximize

$$\max_{m,\chi,\delta} \sum_{t=1}^{2} c_t$$

s.t.

$$\chi \in X_s \tag{A.4}$$

$$\delta \int_{(\omega,\tau)} dA(\omega,\tau;\chi,m,\theta) \leq 1 \tag{A.5}$$

$$c_1 = 1 - \delta \int_{(\omega,\tau)} dA(\omega,\tau;\chi,m;\theta) \tag{A.6}$$

$$c_2 = \big(1 + \tilde{r}(m;\theta)\big)\delta \int_{\omega} dA(\omega,g;\chi,m,\theta) \tag{A.7}$$

Constraint (A.4) restrict the investor to using feasible rules. Constraint (A.5) says that each investor can only provide credit from her own wealth. Constraint (A.6) says the investor consumes her leftover endowment at $t = 1$, while (A.7) says that at $t = 2$ she is paid back by good firms and consume. Substitute the consumption into investor utility function and simplify to get the objective function (11) in the text.

B Construction of Equilibrium

We proceed by backward induction. At $t = 1$, we take function $\hat{y}(\omega, \tau; \theta, r_H, r_L)$, satisfying the properties of definition A.5, as given. We construct a more general version of the equilibrium compared to the one used in the main text. Throughout, we point out how the simplified version of certain expressions look like, given the date $t = 0$ structure of the problem. In section B.2 we connect $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ to $\ell(\omega, \tau; \theta)$, and show that the properties are satisfied in equilibrium.

B.1 t = 1: International Credit Market Equilibrium

At $t = 1$, given the maximum level of liquidity that a firm can raise on the credit markets, $\hat{y}(\omega, \tau, \theta, r_H, r_L)$, and under certain parametric assumptions, the equilibrium in international markets is such that firms maximize problem (A.1), international investors maximize problem (11), and active markets clear. The equilibrium in credit markets is as follows.

We start with the firm problem at $t = 1$ in aggregate state θ. Since we solve for the credit market equilibrium state-by state, to save on notation we often suppress the dependence on r_H, and r_L, and sometimes the dependence on θ, unless useful to clarify the context. So $\hat{y}(\omega, \tau) \equiv \hat{y}(\omega, \tau; \theta, r_H, r_L)$, $\sigma(m, \omega, \tau) \equiv \sigma(m, \omega, \tau; \theta)$, $y(\omega, \tau) \equiv y(\omega, \tau, \theta, r_H, r_L)$, and $\eta(m, \omega, \tau) \equiv \eta(m, \omega, \tau; \theta)$. We will also use, whenever helpful to ease the notation, $\eta_H(\omega) = \eta(m_H, \omega, b; H)$, and $\eta_L(\omega) = \eta(\bar{m}, \omega, g; L)$.

B.1.1 t = 1; Bold Experts, θ = H.

Equilibrium Description. The equilibrium consists of a pair (r_H, s_H), firm and investor optimization, an allocation function, and a rationing function. Market m_H is the market defined by interest rate r_H and an LRF algorithm. The equilibrium is described as follows.

(i) Pair (r_H, s_H) is the solution to the pair of equations

$$r = \frac{\lambda \int_0^{1-s} \hat{y}(\omega, b; H) d\omega}{(1-\lambda) \int_0^1 \hat{y}(\omega, g; H) d\omega} \tag{B.1}$$

$$\phi = \int_s^1 \frac{1}{\lambda \int_0^{1-s'} \hat{y}(\omega, b; H) d\omega + (1-\lambda) \int_0^1 \hat{y}(\omega, g; H) d\omega} w(s') ds' \tag{B.2}$$

(ii) Firm decision

- Good firm

$$\sigma(m, \omega, g; H) = \begin{cases} \min\left\{\bar{L}, \hat{y}(\omega, g; H)\right\} = \hat{y}(\omega, g; H) & \text{if } \tilde{r}(m) = r_H \\ \bar{L} & \text{if } \tilde{r}(m) < r_H \\ 0 & \text{otherwise} \end{cases}$$

where the first line in σ follows from definition (A.3) along with construction of $\hat{y}(\omega, g)$.

- Bad firm

$$\sigma(m, \omega, b; H) = \min\left\{\bar{L}, \hat{y}(\omega, b; H)\right\} = \hat{y}(\omega, b; H) \qquad \forall\, m$$

(iii) Expert decision

- $s < s_H$

$$\begin{aligned} \delta_s &= 0 \\ m_s &= m_H \\ \chi_s(\omega, \tau) &= \mathbb{I}\left(\tau = g \,||\, \left(\tau = b \,\&\, \omega \leq 1 - s\right)\right) \end{aligned}$$

- $s \geq s_H$

$$\begin{aligned} \delta_s &= 1 \\ m_s &= m_H \\ \chi_s(\omega, \tau) &= \mathbb{I}\left(\tau = g \,||\, \left(\tau = b \,\&\, \omega \leq 1 - s\right)\right) \end{aligned}$$

(iv) Allocation function

- For market m_H and $\chi(\omega, \tau) = \mathbb{1}\left(\tau = g \,||\, (\tau = b \,\&\, \omega \leq 1 - h)\right)$ for some $h \in [0, 1]$

$$a(\omega, \tau; \chi, m_H) = \frac{1}{\hat{y}(\omega, \tau; H)} \frac{\left(\mathbb{I}(\tau = g) + \mathbb{I}(\tau = b \,\&\, \omega \leq 1 - h)\right) \sigma(m, \omega, \tau; H)}{(1 - \lambda) \int_0^1 \sigma(m_H, \omega', g; H) d\omega' + \lambda \int_0^{1-h} \sigma(m_H, \omega', b; H) d\omega'} \tag{B.3}$$

- For market m_H and any other acceptance rule

$$a(\omega, \tau; \chi, m_H) = \begin{cases} \frac{\chi(\omega,\tau)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'} \int_{\omega'} \chi(\omega',\tau') \hat{\sigma}(m_H,\omega',\tau';H)[1-\eta(m_H,\omega',\tau';H)] d\omega'} & \text{if } \chi(\omega', \tau') \notin X_s \;\&\; \sum_{\tau'} \int_{\omega'} \chi(\omega', \tau') \sigma(m_H, \omega', \tau')[1 - \eta(m_H, \omega', \tau')] d\omega' > 0 \\ \frac{\chi(\omega,\tau)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'} \sum_{\omega'} \chi(\omega',\tau') \sigma(m_H,\omega',\tau';H)[1-\eta(m_H,\omega',\tau';H)]} & \text{if } \chi(\omega', \tau') \notin X_s \;\&\; \sum_{\tau'} \int_{\omega'} \chi(\omega', \tau') \sigma(m_H, \omega', \tau')[1 - \eta(m_H, \omega', \tau')] d\omega' = 0, \\ & \text{but } \sum_{\tau'} \sum_{\omega'} \chi(\omega', \tau') \sigma(m_H, \omega', \tau')[1 - \eta(m_H, \omega', \tau')] d\omega' > 0 \\ 0 & \text{otherwise} \end{cases} \tag{B.4}$$

where $\eta(m_H, \omega, \tau; H)$ is defined below.

- For any other market

$$a(\omega,\tau;\chi,m)=\begin{cases}\frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';H)S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau')S(m,\omega',\tau')d\omega'>0\\ \frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';H)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau')S(m,\omega',\tau')d\omega'=0,\\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';H)S(m,\omega',\tau')d\omega'>0\\ 0 & \text{otherwise}\end{cases}\tag{B.5}$$

where

$$S(m,\omega,\tau)=\begin{cases}1 & \text{if } \begin{cases}\tau=b\\ \text{or}\\ \tau=g \,\&\, \tilde{r}(m)\in(0,r_H]\\ \text{or}\\ \tilde{r}(m)\le 0\end{cases}\\ 0 & \text{if } \tau=g,\ \tilde{r}(m)>r_H\end{cases}$$

(v) Rationing function

$$\eta(M_0,\omega,\tau;H)=\begin{cases}1 & \text{if } m_H\in M_0 \text{ and } \tau=g\\ \int_{s_H}^{1-\omega}\frac{1}{\phi\lambda\int_0^{1-s}\hat{y}(\omega',b;H)d\omega'+\phi(1-\lambda)\int_0^1\hat{y}(\omega',g;H)d\omega'}w(s)ds & \text{if } m_H\in M_0 \text{ and } \tau=b \text{ and } \omega\le 1-s_H\\ 0 & \text{otherwise}\end{cases}\tag{B.6}$$

Proof.

(i) $(\mathbf{r_H},\mathbf{s_H})$. There is a single market m_H, with $\tilde{r}(m_H)=r_H$, where all trades take place. In this market, firms try to issue as many bonds as they can. Total supply is therefore $(1-\lambda)\int_0^1\hat{y}(\omega',g;H)d\omega'$ good bonds and $\lambda\int_0^1\hat{y}(\omega',b;H)d\omega'$ bad bonds. Supply decisions in markets $m\neq m_H$ have no effect on firm utility since $\eta(m,\omega,\tau;H)=0$, so they are determined in equilibrium by the robustness requirement.

Buying from markets with interest rate other than r_H is not optimal for investors. At interest rates above r_H, the supply includes only bad firms, so investors prefer to stay away, whereas at interest rate below r_H, the supply of bonds is exactly the same as at interest rate r_H but the interest rate is lower. This does not settle the question of whether an investor chooses to buy at all. Expert optimization below then shows that investor with $s=s^*$ faces terms of trade of $v(s^*)=1$ in market m_H, and is indifferent between buying and not buying. This results in equation (B.1).

All investors with $s>s^*$ thus spend all of their wealth buying in market m_H and those with $s<s^*$ choose not to buy at all. The fraction of bonds by firm $j=(\omega,\tau)$ that can be issued in market m_H is given by the ratio of the total allocation of that bond across investors, to the

supply of that bond. Noticing that only firms hit by liquidity shock issue bonds, and adding across investors and imposing that all good bonds are issued results in (B.2).

Next define the monotone transformation $q_H = \frac{r_H}{1+r_H}$. (B.1) can be represented as

$$q = \frac{\lambda \int_0^{1-s} \hat{y}(\omega, b; H) d\omega}{\lambda \int_0^{1-s} \hat{y}(\omega, b; H) d\omega + (1-\lambda) \int_0^1 \hat{y}(\omega, g; H) d\omega} \tag{B.7}$$

Let $H(r_H) = \hat{y}(\omega, b; H, r_H)$ and $G(r_H) = \int_0^1 \hat{y}(\omega, g; H, r_H) d\omega$, noting that $\hat{y}(\omega, g; H)$ depends on q_H (through r_H). We have assumed $\hat{y}$ is continuous in r_H (Which we will verify in section B.2), and r_H is by construction continuous in q_H. It follows that from lemma C.1, there exists a solution to pair of equations (B.7) and (B.2) such that $q_H \geq 0$ and $0 \leq s_H \leq 1$, which in turn implies there exists a solution to pair of equations (B.1) and (B.2) such that $r_H \geq 0$ and $0 \leq s_H \leq 1$.

(ii) **Firm optimization.** Taking the equilibrium market structure, rationing function and allocation function as given, $y(\omega, \tau) = \sigma(m_H, \omega, \tau)\eta(m_H, \omega, \tau)$. Since $\frac{\rho_\tau}{\xi} - \mathbb{1}_{\tau=g} > 0$, firm j's optimal choice of $\sigma(m_H, \omega, \tau)$ is determined by the constraints. For a good firm, $\eta(m_H, \omega, g) = 1$ from rationing function (B.6), which implies $y(\omega, g) = \sigma(m_H, \omega, g)$. As such, condition (A.2) is the binding constraint which in turn implies $y(\omega, g) = \sigma(m_H, \omega, g) = \hat{y}(\omega, g)$, when $\theta = H$.

For a bad firm $\eta(m_H, \omega, b) = \int_{s_H}^{1-\omega} \frac{1}{\lambda \int_0^{1-s} \hat{y}(\omega', b; H) d\omega' + (1-\lambda) \int_0^1 \hat{y}(\omega', g; H) d\omega'} \frac{w(s)}{\phi} ds$ from rationing function (B.6). From equation (B.2), $\eta(m_H, L, 0) = 1$, so $\eta(m_H, \omega, b) < 1$, $\forall\, 1 - s_H < \omega \leq 1$, thus $y(\omega, b) \leq \sigma(m_H, \omega, b)$. Since $\sigma(m_H, \omega, b) = \hat{y}(\omega, b) = \bar{L}$, constrain (A.2) is satisfied, which in turn implies $y(\omega, b) = \eta(m_H, \omega, b)\hat{y}(\omega, b)$.

Put together, the rationing function (B.6) implies that in market m^H, all good firms will be able to issue as many bonds as they demand. A bad firm with transparency ω will be able to sell a fraction $\eta(m_H, \omega, b) < 1$ of bonds he demands. No other bond can be issued. Put together this implies

$$y(\omega, \tau) = \begin{cases} \hat{y}(\omega, \tau) & \text{if } \tau = g \\ \eta(m_H, \omega, \tau)\hat{y}(\omega, \tau) = \eta_H(\omega)\hat{y}(\omega, \tau) & \text{if } \tau = b \end{cases} \tag{B.8}$$

Off equilibrium, in all cheaper markets (lower interest rate), all good firms submit $\bar{L}$. In all more expensive markets, they submit zero demand. All bad firms submit the maximum that they can submit, $\bar{L}$, on every other market. These decisions satisfy the robust program (A.3). Note that the equilibrium $\sigma(m, \omega, \tau)$ satisfy the form of lemma B.1.

(iii) **Expert optimization.**

Choosing any feasible acceptance rule other than $\chi(\omega, \tau) = x(\omega, \tau, s)$ in market m_H would, according to (B.3) and (B.4), result in a lower fraction of good assets, so choosing $\chi(\omega, \tau) =$

$\mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \leq 1 - s))$ is optimal.

Define the terms of trade that an investor obtains in market m with acceptance rule χ as

$$\upsilon(m,\chi) = \begin{cases} \frac{(1+\tilde{r}(m)) \int_\omega \mathbb{1}[\tau=g]\sigma(m,\omega,g;\theta)dA(\omega,g;\chi,m,\theta)}{\int_{\omega,\tau} \sigma(m,\omega,\tau;\theta)dA(\omega,\tau;\chi,m,\theta)} & \text{if } A(\{g,b\},[0,1];\chi,m) > 0 \\ 0 & \text{otherwise} \end{cases}$$

which is his expected repayment per unit of bond he finances, i.e. the principal and interest rate he receives at $t = 2$. Let

$$\upsilon^{max}(s) \equiv \max_{m \in M, \chi \in X_s} \upsilon(m,\chi)$$

be the best term of trade that investor s can achieve, and let $M^{max}(s)$ be the set of markets where investor s can obtain terms of trade υ^{max} with a feasible acceptance rule.

Necessary and sufficient condition for investor optimization are that investors for whom $\upsilon^{max} < 1$ choose not to finance any bonds, investors for whom $\upsilon^{max} > 1$ spend their entire endowment in a market $m \in M^{max}(s)$, and investors for whom $\upsilon^{max} = 1$ choose a market $m \in M^{max}(s)$. Using equation (B.3), an investor s that uses acceptance rule $\chi(\omega,\tau) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega < 1 - s))$ in market m obtains terms of trade

$$\upsilon(m,\chi) = \begin{cases} \frac{(1+\tilde{r}(m))(1-\lambda)\int_0^1 \hat{y}(\omega,g;H)d\omega}{\lambda \int_0^{1-s} \hat{y}(\omega,b;H)d\omega + (1-\lambda)\int_0^1 \hat{y}(\omega,g;H)d\omega} & \tilde{r}(m) \leq r_H \\ 0 & \text{otherwise} \end{cases}$$

Thus for all investors

$$\upsilon^{max}(s) = \frac{(1+r_H)(1-\lambda)\int_0^1 \hat{y}(\omega,g;H)d\omega}{\lambda \int_0^{1-s} \hat{y}(\omega,b;H)d\omega + (1-\lambda)\int_0^1 \hat{y}(\omega,g;H)d\omega}$$

and the maximum is attained in any market where the interest rate is r_H, including m_H. Rewrite

$$\upsilon^{max}(s) = (1+r_H)J(s)$$
$$J(s) = \frac{(1-\lambda)\int_0^1 \hat{y}(\omega,g;H)d\omega}{\lambda \int_0^{1-s} \hat{y}(\omega,b;H)d\omega + (1-\lambda)\int_0^1 \hat{y}(\omega,g;H)d\omega},$$

Note that from equation (B.1), $J(s_H) = \frac{1}{1+r}$, so $\upsilon^{max}(s_H) = 1$. Moreover, $J'(s) > 0$. This implies that investors $s < s_H$ have $\upsilon^{max}(s) < 1$, so not financing any bonds is optimal for them. Experts of types $s \geq s_H$ have $\upsilon^{max}(s) \geq 1$, so financing bonds such that they spend their entire wealth in market m_H at $t = 2$ is optimal for them too.

(iv) **Allocation function.**

In all markets except m_H (off equilibrium path), there are no investors, so for any clearing algorithm the residual set of bonds any investor faces is just the original set of bonds demanded by firm on that market. In this case, (B.5) follows from Appendix B of Kurlat (2016), equation (65).

For market m_H, the LRF algorithm implies that an investor who imposes $\chi(\omega,\tau) = \mathbb{1}\left(\tau = g \mid\mid (\tau = b \;\&\; \omega \leq 1-h)\right)$ faces a residual firm demand of acceptable bonds that is proportional to the original firm demand. Therefore, the measure of assets he will obtain is the same as if he traded first. Therefore (B.3) follows from Appendix B of Kurlat (2016), equation (65).

For market m_H and rules that are not of the form $\chi(\omega,\tau) = \mathbb{1}\left(\tau = g \mid\mid (\tau = b \;\&\; \omega \leq 1-h)\right)$, (off equilibrium path), their trades clear after all other investors, so the bond financing demand they face only includes bonds demanded by bad firms. Therefor (B.4) follows from Appendix B of Kurlat (2016), equation (65).

(v) **Rationing function.**

(B.6) follows from B.2 using Appendix B of Kurlat (2016), equation (67). It is the fraction of bonds that the firm is able to issue , out of the total bonds offered (i.e. a number between zero and one).

■

B.1.2 t = 1; Cautious Experts, $\theta = \mathbf{L}$.

Equilibrium Description. The equilibrium consists of an interest rate schedule $0 \leq r_L(\omega) \leq \bar{r}$, cut-offs $\omega_1 < \omega_2 < \omega_3$, firm and investor optimization, an allocation function, and a rationing function. For any $\omega \in [0,1]$, let $m(\omega)$ denote the market where the price is $r_L(\omega)$, where $r_L(\omega)$ is found by the procedure described in the proof below, and the clearing algorithm is NMR. Because of bunching, $m(\omega)$ could mean the same market for different ω. For any $\Omega_0 \subseteq [0,1]$, let the set of markets $M(\Omega_0)$ be $M(\Omega_0) = \{m(\omega) : \omega \in \Omega_0\}$. The set of active markets is $M([0,1])$.

The equilibrium is described as follows.

(i) Premium schedule $0 \leq r_L(\omega) \leq \bar{r}$ such that the interest rate falls into one of the cash-in-the-market, bunching, bunching-with-scarcity, or non-selective regions as described below.

(ii) Firm decision

- Good firm

$$\sigma(m,\omega,g;L)=\begin{cases}\min\{\bar{L},\hat{y}(\omega,g;L)\}=\hat{y}(\omega,g;L) & \text{if } \tilde{r}(m)=r_L(\omega),\omega\geq\omega_2\\ \hat{y}(\omega_2,g;L) & \text{if } \tilde{r}(m)=r_L(\omega),\omega<\omega_2\\ \bar{L} & \text{if } \tilde{r}(m)<r_L(\omega)\\ 0 & \text{otherwise}\end{cases}$$

$$y(\omega,g;L)=\int_{M([\omega,1])}\sigma(m,\omega,g;L)d\eta(m,\omega,g;L)$$

where the first line in σ follows from definition (A.3) along with construction of $\hat{y}(\omega,g;L)$.

- Bad firm

$$\sigma(m,\omega,b;L)=\begin{cases}\min\{\bar{L},\hat{y}(\omega,b;L)\}=\hat{y}(\omega,b;L) & \text{if } m \text{ falls into the non-selective region}\\ \bar{L} & \text{otherwise}\end{cases}$$

$$y(\omega,b;L)=\int_{M([0,1])}\sigma(m,\omega,b;L)d\eta(m,0,b;L)$$

where the rationing functions $\eta(m,\omega,\tau;L)$ are defined in (B.14) and (B.15).

Selling decisions follow the reservation interest rate strategy. A good firm (ω,g) raises total liquidity equal to all the bonds they are able to sell on all $M([\omega,1])$ markets. A bad firm (ω,b) tries to sell in all markets $M([0,1])$. Since in equilibrium all bad assets sell at the same ratio, $\eta(m,\omega,b;L)=\eta(m,0,b;L)$, $\forall\, m,\forall\,\omega\in[0,1]$.

(iii) Expert decision:

Let ω_3 denote the lowest-ω transparency whose firm face a zero interest rate when investors are cautious, $r_L(\omega)=0$. Define s_N by

$$\int_{s_N}^{\hat{s}(\omega_3)}w(s)ds=\phi(1-\lambda)\int_{\omega_3}^{1}\hat{y}(\omega,g;L)d\omega.$$

Thus the aggregate wealth of investors in the interval $[s_N,\hat{s}(\omega_3)]$ is just sufficient to finance all the bonds offered by good firms with transparency $\omega>\omega_3$ at interest rate 0, and each of these investors can identify some good bond in this interval. Investors with lower degree of expertise than s_N either buy non-selectively or do not buy at all.

Define the function $\tilde{s}(\omega)$ as the solution to the following differential equation

$$\tilde{s}'(\omega)=-\frac{1}{w\left(\tilde{s}(\omega)\right)}\phi\left[\lambda\int_0^1\hat{y}(\omega',b;L)d\omega'+(1-\lambda)\int_0^{\omega}\hat{y}(\omega',g;L)d\omega'\right]\varepsilon'(\omega)\tag{B.9}$$

with boundary condition $\tilde{s}(1)=s_N$. Finally, let $s_0=\tilde{s}(0)$ and define $\tilde{\omega}(s)$ for $s\in[s_0,s_N]$ by

$$\tilde{\omega}(s)=\min\{\omega:\tilde{s}(\omega)=b\}$$

(a) for $s \geq s_N$

$$\delta_s = 1$$
$$m_s = m(1-s)$$
$$\chi_s(\omega,\tau) = \mathbb{I}(\tau = g \ \& \ \omega \geq 1-s)$$

(b) $s \in [s_0, s_N)$

$$\delta_s = 1$$
$$m_s = m(\tilde{\omega}(s))$$
$$\chi_s(\omega,\tau) = 1$$

(c) $s < s_0$

$$\delta_s = 0$$
$$m_s = m(1)$$
$$\chi_s(\omega,\tau) = 1$$

Experts $s \geq s_N$ spend their entire endowment financing bonds in market $m(1-s)$, i.e. the market for the lowest transparency ω firms (most opaque), for which they can observe a good signal, and they use the selective acceptance rule $\mathbb{I}(\tau = g \ \& \ \omega \geq 1-s)$, which only accepts good assets. Some of these investor are in cash-in-the-market region, some in bunching, and some in bunching-with-scarcity. Experts $s \in [s_0, s_N)$ are nonselective. The function $\tilde{\omega}(s)$ assigns each one to a market: in market $m(\omega)$, nonselective investors bring down the un-financed reminder fraction by $\varepsilon'(\omega)$, which requires buying $\varepsilon'(\omega)\phi(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'$ good assets and $\varepsilon'(\omega)\phi\lambda\int_0^1 \hat{y}(\omega', b; L)d\omega'$ bad assets. If investor $\tilde{s}(\omega)$ is the nonselective investor that buys in market $m(\omega)$ then the total nonselective wealth available in that market is $-w\left(\tilde{s}(\omega)\right)\tilde{s}'(\omega)$, so market clearing implies (B.9). Inverting this function results in investor s choosing market $m\left(\tilde{\omega}(s)\right)$. Experts $s < s_0$ don't finance (buy) anything. Since they are indifferent between buying and not buying, many other patterns of demand among non-selective investors are possible.

(iv) Allocation function

- For markets $m(\omega) \in M([0,1])$ where ω falls in either a cash-in-the-market or a nonselective region

$$a(\omega,\tau;\chi,m)=\begin{cases}\frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\sigma(m,\omega',\tau';L)\chi(\omega',\tau')S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')d\omega'>0\\ \frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')d\omega'=0,\\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')d\omega'>0\\ 0 & \text{otherwise}\end{cases} \tag{B.10}$$

where

$$S(m,\omega,\tau)=\begin{cases}1 \quad \text{if} \begin{cases}\tau=b\\ \text{or}\\ \tau=g \;\&\; \tilde{r}(m)\in(0,r_L(\omega)]\\ \text{or}\\ \tilde{r}(m)\leq 0\end{cases}\\ 0 \quad \text{if } \tau=g,\ \tilde{r}(m)>r_L(\omega)\end{cases}$$

- For market $m(\omega)$ where ω falls in $[\omega^L,\omega^H]$ which is either a bunching, or bunching-with-scarcity region ($[\omega^L,\omega^H]=[0,\bar{\omega}]$); and χ is of the form $\mathbb{I}\,(\tau=g \;\&\; \omega\geq 1-h)$:

$$a(\omega,\tau;\chi,m)=\begin{cases}\frac{\chi(\omega,\tau)S^h(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\sigma(m,\omega',\tau';L)\chi(\omega',\tau')S^h(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')d\omega'>0\\ \frac{\chi(\omega,\tau)S^h(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')d\omega'=0,\\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')d\omega'>0\\ 0 & \text{otherwise}\end{cases} \tag{B.11}$$

where $S^h(m,\omega,\tau)$ is the solution to differential equation

$$\frac{dS^h(m,\omega,\tau)}{dh}=\begin{cases}-w(h)\frac{S^h(m,\omega,\tau)\mathbb{I}[1-h\leq s\leq\omega^H]}{\int_g^{\omega^H}\sigma(m,\omega',\tau';L)S^h(m,\omega,g')d\omega'} & \text{if } \tau=g \text{ and } 1-h\in[\omega^L,\omega^H]\\ 0 & \text{otherwise}\end{cases} \tag{B.12}$$

If $m(\omega)$ is in a bunching-with-scarcity region, $\sigma(m,\omega,g;L)=\sigma(m,\bar{\omega},g;L)$, $\forall\ \omega$. The terminal condition is

$$S^0(m,\omega,\tau)=\begin{cases}1 & \text{if } \tau=b \text{ or } (\tau=g \text{ and } \omega\in[0,\omega^H])\\ 0 & \text{otherwise}\end{cases} \tag{B.13}$$

Except for bunching and bunching-with-scarcity markets, the clearing algorithm implies that all investors draw bonds from a sample that is proportional to the original supply. This results in (B.10). In bunching markets, investor s imposes acceptance rule of the form $\chi_s(\omega,\tau)=\mathbb{I}\,(\tau=g \;\&\; \omega\geq 1-h)$ with $h=s$; therefore when he buys his bond portfolio, the demand for bonds from good firms in transparency ω falls in proportion to his wealth, $w(s)$, times the ratio between the demand for credit by good firms with transparency ω and all the other bonds acceptable by investor s. This results in differential equation (B.12) which characterizes how the demand for bonds fall as the clearing algorithm progresses.

(v) Rationing function

- Firm (ω, τ), $\omega \geq \omega_1$

$$\eta(M([l,1]), \omega, \tau; L) = \begin{cases} 1 - \varepsilon(l) & \omega < l \text{ or } (\omega = l \text{ and } \tau = b) \\ 1 & \omega \geq l \text{ and } \tau = g \\ 0 & \text{otherwise} \end{cases} \tag{B.14}$$

- Firm (ω, τ), $\omega < \omega_1$

$$\eta(M([l,1]), \omega, \tau; L) = \begin{cases} 1 - \varepsilon(l) & \omega < l \\ \int_{1-\omega}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L)} w(s) ds & \omega \geq l \text{ and } \tau = g \\ 0 & \text{otherwise} \end{cases} \tag{B.15}$$

where $R_D(.)$ and $\tilde{\omega}$ are defined in equations (B.20) and (B.21), respectively. The rationing function is separately defined for firms with transparency $\omega \geq \omega_1$ and $\omega < \omega_1$. It says that when $\omega \geq \omega_1$, a good firm with transparency $\omega < l$ who offers a bond at every market with interest rate $r(m) \in [0, r_L(l)]$, $(r_L(l) < \bar{r})$ will be able to sell a fraction $1 - \varepsilon(l)$ (so the unsold *fraction* of the good assets is $\varepsilon(l)$), and $\varepsilon(\omega)$ fraction can be sold in market $m(\omega)$.

When $\omega < \omega_1$, a good firm with transparency $\omega < l$ who offers a bond at every market with interest rate $r(m) \in [0, r_L(l)]$, $(r_L(l) < \bar{r})$ will be able to sell a fraction $1 - \varepsilon(l)$, and then $\varepsilon(\omega)\eta_L(\omega)$ fraction can be sold in market $m(\omega)$ which has interest rate $\bar{r}$, where $\eta_L(\omega) = \eta(m(\omega), \omega, g; L)$ is defined in (B.33).

Condition ($l = \omega$ and $\tau = b$) in (B.14) handles bad firms with transparency ω who sells on the same non-selective pricing market where the good firms from the same transparency sell all the reminder of their bonds.

Proof. From Lemma B.1 below, each firm decision is expressed in terms of a reservation interest rate $r^L(\omega, \tau)$. The idea is to show the following statements: all bad firms are identified under equilibrium acceptance rule, so $r^L(\omega, b) = 0$. However, unlike $\theta = H$, $r^L(\omega, g)$ is different for good firms of different transparencies. Finding $r^L(\omega, g)$ is equivalent to finding the highest interest rate at which bonds of a good firm with transparency ω trades.

Moreover, unlike $\theta = H$, firm (ω, g) might be able to sell some bonds at interest rate below $r^L(\omega, g)$, so the equilibrium must characterize $r^L(\omega, g)$ and any other prices at which bonds of firm (ω, g) are sold.

(i) **Premium schedule $\mathbf{0} \leq \mathbf{r_L}(\omega) \leq \bar{\mathbf{r}}$**

Let $r_L(\omega) = r^L(\omega, g)$. $r_L(\omega)$ falls into three possible classes: a "cash-in-the-market" interest rate, a"bunching" interest rate, a"bunching-with-scarcity" interest rate, or a "non-selective" interest rate.

Cash-in-the-market. The cash in the market interest rate $r^C(\omega)$ for the bond issued by the good firm of transparency ω is determined by equating demand and supply in the corresponding market.[19] The total amount of liquidity demanded by firm $j = (\omega, g)$ at interest rate $r^C(\omega)$ should be equal to total wealth of investor $\hat{s}(\omega)$ which is the financier in that market.

$$\varepsilon(\omega)\phi(1-\lambda)\hat{y}(\omega, g; L; r^C(\omega)) = w\left(\hat{s}(\omega)\right) \tag{B.16}$$

As long as $r^C(\omega)$ is a strictly decreasing function and in the correct range, the equilibrium would be a cash-in-the-market pricing equilibrium. Each good firm of transparency ω demands bonds in all markets where $r(m) \leq r^C(\omega)$, and no market with a higher interest rate, while bad firms demand maximum bonds on every (active) market. Given the prudence shocks, each investor imposes $\chi_s(\omega, \tau) = \mathbb{I}(\tau = g \;\&\; \omega \geq 1 - s)$, i.e. he finances bond in the most profitable (highest interest rate) market for which he observes $x(g; \omega, s, L) = g$. Now consider a market with $r = r^C(\omega)$. Firms with transparency $\omega' \leq \omega$ demand credit in that market, but no firm with transparency $\omega' > \omega$ demand in this market because they have been able to issue all the bonds that they want at lower interest rate. Investor $s = \hat{s}(\omega)$ is able to recognize good assets in this markets, but investors $s < \hat{s}(\omega)$ are not. Moreover, if $r^C(\omega)$ is strictly decreasing, this is the highest interest rate where $\hat{s}(\omega)$ can detect good firms, so he will spend his entire wealth financing bonds demanded on this market. Then equation (B.16) implies all the bonds demanded by firm $j = (\omega, g)$ are financed at this market, and there will be non of them for sale at interest rate higher than $r^C(\omega)$.

Bunching. If $r^C(\omega)$ turns out to be upward sloping in any range, the logic of cash-in-the-market pricing breaks down because it implies the good firm with a higher transparency is paying a higher interest rate to issue bonds, $\omega > \omega'$ and $r^C(\omega) > r^C(\omega')$. The investor who is financing the firm from lower transparency, ω', can also identify the firm from a higher transparency, ω, so he is better off financing the more transparent firm and collect a higher interest rate $r^C(\omega) > r^C(\omega')$, so there will be no financier for the less transparent firm ω'; a contradiction. In this region, there will be "bunching" of all the firms $[\omega', \omega]$ at a single price, i.e. an ironing procedure that restores a weakly monotone function. The clearing algorithm

[19] The full notation would be $r^C(\omega, \tau)$, but since when $\theta = L$ only $\tau = g$ firms have a reservation interest rate, we suppress the dependence on τ.

is such that the lower s investor picks the bonds that he finances first in a bunching market.

Since $w(.)$ function is decreasing in s (increasing in ω), and $\hat{y}(.)$ function is decreasing in r^C, for small enough s (high enough ω), and appropriate set of parameters, equation (B.16) requires $r^C(\omega) < 0$. Let $\hat{\omega} = \min\ \omega$ such that $r^C(\omega) \leq 0$, then as long as $r^C(\omega)$ is decreasing, or ironed as explained above, $\forall\ \omega'$ s.t. $\hat{\omega} < \omega' \leq 1$, $r^C(\omega') < 0$. Thus the requirement that there is a zero lower bound on the interest rate (no negative interest rate), implies there is a range of transparencies at the top, $\omega \geq \hat{\omega}$, whose good firms face zero interest rate in issuing bonds. Investors with $s \leq 1 - \hat{\omega}$ have idle wealth that is not financing any bonds, as there is not enough credit demand from good firms that they can recognize. In order for $\hat{\omega} < 1$ it must be that

$$w(0) > \phi(1 - \lambda)\hat{y}(1, g; L), \tag{B.17}$$

where we have used that $r_L(1) = 0$. Later in proof of Proposition 5 we make the appropriate parametric assumption to ensure this condition holds.

Nonselective pricing. Consider a market m with interest rate $r = \tilde{r}(m)$, where good firms with transparency ω submit credit demand in that market. That implies all the good firms with transparency $\omega' < \omega$ also submit demand in market m, as well as all the bad firms with any level of transparency. An investor can choose to impose $\chi_s(\omega, \tau) = 1$ in market m and buy a representative sample of the pool.

The terms of trade that he will get is

$$\begin{aligned} v^N(r) &= \frac{(1+r)(1-\lambda)[g \text{ Supply at interest rate q in FN}]}{(\lambda[b \text{ Supply at interest rate q in FN}] + (1-\lambda)[g \text{ Supply at interest rate q in FN}])} \\ &= \frac{(1+r)(1-\lambda)\int_0^{\omega} \hat{y}(\omega, g; L)d\omega'}{\left(\lambda \int_0^1 \hat{y}(\omega, b; L)d\omega' + (1-\lambda)\int_0^{\omega} \hat{y}(\omega, g; L)d\omega'\right)} \end{aligned}$$

As long as $\omega_3 < 1$, there are (low expertise) international investors who finance bonds issued by good firms with transparency $\omega > \omega_3$. The interest rate for these bonds is zero, so these investors make zero profits and are indifferent between financing and not financing bonds. Alternatively, if they trade non-selectively at a market at interest rate r, they can get the above terms of trade. As a result if $v^N(r) > 1$ these investors are better off trading at interest rate r nonselectively, which in turn implies no good bond from transparency ω can be offered at a interest rate above $r^{NS}(\omega)$. In other words, $\upsilon_N(r) \leq 1$ implies $r \leq r^{NS}(\omega)$.

$$\frac{(1+r)(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'}{\left(\lambda\int_0^1 \hat{y}(\omega, b; L)d\omega' + (1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'\right)} \leq 1$$

$$\frac{1}{1+r} \geq \frac{(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'}{\left(\lambda\int_0^1 \hat{y}(\omega, b; L)d\omega' + (1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'\right)} = \frac{1}{1+\frac{\lambda\int_0^1 \hat{y}(\omega,b;L)d\omega'}{(1-\lambda)\int_0^\omega \hat{y}(\omega',g;L)d\omega'}}$$

$$r \leq \frac{\lambda\int_0^1 \hat{y}(\omega, b; L)d\omega'}{(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'}$$

so

$$r^{NS}(\omega) \equiv \frac{\lambda\int_0^1 \hat{y}(\omega, b; L)d\omega'}{(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'} \tag{B.18}$$

When this upper bound interest rate is operative, bonds are finances in markets where both selective and non-selective buyers are active. In the markets where the interest rate is $r^{NS}(\omega)$, non-selective buyers will buy just enough assets (distributed pro-rate among the assets offered) such that the interest rate $r^C(\omega)$ is pushed down such that marginal investor $\hat{s}(\omega)$ can charge exactly interest rate $r^{NS}(\omega)$:

$$\varepsilon(\omega)\phi(1-\lambda)\hat{y}(\omega, g; L; r^{NS}(\omega)) = w\left(\hat{s}(\omega)\right) \tag{B.19}$$

In other words, if $\hat{s}(\omega) = 1 - s$ international investors are poor, that requires a high interest rate to push the demand of firms (ω, g) down so that equation (B.19) is satisfied. At this high interest rate, investors financing high ω good firms will enter this market and be non-selective financiers. This takes some bonds off of the market, which in turn implies a lower interest rate.

Bunching-with-scarcity. If there is a maximum interest rate $\bar{r}$ that firms are willing to pay to get bonds from investors, and if the wealth of smart investors, in the sense precisely defined below, is in short supply, then there will be a bunching region where some good firms will be rationed.

At any interest rate $r > \bar{r}$, good firms have zero demand for bonds, and (with linear objective function) at $r = \bar{r}$ they are indifferent between all levels of bond issued. So if the interest rate hits $\bar{r}$ in any market, it cannot increase any further than that.

Let $\bar{m}$ denote the market with interest rate $\bar{r}$, $\tilde{r}(\bar{m}) = \bar{r}$, and let $\bar{\omega}$ denote the highest transparency level whose good firms demand credit on market $\bar{m}$. Firms $(\bar{\omega}, g)$ submit $\sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$ on market $\bar{m}$ and by definition their demand is exactly fully satisfied at interest rate $\bar{r}$. Good firms with transparency $\omega < \bar{\omega}$ also demand credit on this market. Since these firms are indifferent about how many bonds they raise on market $\bar{m}$

(given the linearity of $t = 0$ objective function), we assume that all of them submit $\hat{y}(\bar{\omega}, g; L)$: $\forall\, \omega < \bar{\omega},\ \sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$; and how many bond they raise is determined by rationing explained next.[20]

Bad firms with any transparency level also demand credit on market $\bar{m}$, but none is able to issue any bonds in this market. Thus the demand submitted on market $\bar{m}$ is given by

$$\sigma(\bar{m}, \omega, \tau; L) = \begin{cases} \hat{y}(\omega, b; L) & \text{if } \tau = b \\ \hat{y}(\bar{\omega}, g; L) & \text{if } \tau = g \text{ and } \omega < \bar{\omega} \\ 0 & \text{otherwise} \end{cases}$$

As such, if

$$\bar{\omega} \times \hat{y}(\bar{\omega}, g; L) > \int_{1-\bar{\omega}}^{1} w(s)ds,$$

then the wealth of investor who are able to recognize good firms from some transparency in $(0, \bar{\omega})$ is collectively in short supply, and some of the good firm demand is rationed at maximum interest rate $\bar{r}$. Next we determine the range of transparencies whose good firm demand for bonds is fully satisfied at interest rate $\bar{r}$. In order to do so, introduce the following function.

$\mathbf{R_D}(\omega', \omega, \mathbf{r}, \varepsilon; \mathbf{x})$**.** For $\omega' < \omega$, and interest rate r, let

$$R_D(\omega', \omega, r, \varepsilon; x) \equiv \varepsilon\phi(1-\lambda) \int_{\omega'}^{\omega} \hat{y}(z, g; L; r)dz - \int_{\hat{s}(\omega)}^{\hat{s}(\omega')} w(s)ds. \tag{B.20}$$

where x is a parameter.

$R_D(\omega', \omega, r, \varepsilon; x)$ Measures the excess residual demand (ε) by good firms with transparency in the interval (ω', ω), at interest rate r, which is not met by the cumulative wealth of the investors who are able to identify some good firm in this interval but no good firms with transparency $\omega'' < \omega'$, i.e. $2 - \omega \leq s \leq 1 - \omega'$.

For $\omega = \bar{\omega}$ and $\varepsilon(\bar{\omega}) = 1$, we have

$$R_D(\omega', \bar{\omega}, \bar{r}, 1; x) = \phi(1-\lambda)(\bar{\omega} - \omega')\hat{y}(\bar{\omega}, g; L; \bar{r}) - \int_{1-\bar{\omega}}^{1-\omega'} w(s)ds$$

[20] This is slightly stronger than what we actually need to simplify the equilibrium derivation. The precise assumption we need is that when $\theta = L$, on the market where interest rate is $\bar{r}$, no good firm submits a total international demand which is higher than the total international demand submitted by the highest-transparency good firm. The latter firm is $j = (\omega_2, g)$, and even absent this assumption, $\hat{y}(\omega, g; L) = \hat{y}(\omega_2, g; L)$ for $\omega_1 \leq \omega < \omega_2$. So what we need is $\hat{y}(\omega, g; L) = \hat{y}(\omega_1, g; L)$ for $\omega_1 \leq \omega < \omega_1$, weaker than what specified here.

In the special case where $\hat{y}(\omega, \tau; L; \bar{r})$ is determined given the specific structure of $t = 0$, the above equation simplifies to

$$R_D(\omega', \bar{\omega}, \bar{r}, 1; x) = \phi(1-\lambda)(\bar{\omega} - \omega')\frac{D(\bar{r}; x)}{1+\bar{r}} - \int_{1-\bar{\omega}}^{1-\omega'} w(s)ds$$

which is the excess residual demand of the good firms in $(\omega', \bar{\omega})$ which should be absorbed by investors with expertise $s > 1 - \omega'$. Recall that in markets where there is bunching, the clearing algorithm used lets lower-s investors, who impose more restrictive acceptance rules, trade before higher-s investors.

Moreover, note that $R_D(\omega', \bar{\omega}, \bar{r}, 1; x) > 0$, $\forall\ \omega' < \bar{\omega}$. The reason is the following. By the logic of cash-in-the-market pricing, $\bar{r}$ is the interest rate at which demand of good firms of transparency $\bar{\omega}$ is exactly absorbed by wealth of the marginal investor $s(\bar{\omega})$. Consider a good firm with transparency ω' right below $\bar{\omega}$. Let $\tilde{r}'$ denote the hypothetical interest rate which clears the market for such good firm $\omega' < \bar{\omega}$, if this firm was still in a cash-in-the-market pricing. Again, using the logic of cash-in-the-market pricing, and the downward sloping wealth distribution of investors, it must be that $\tilde{r}' > \bar{r}$ as $\omega' < \bar{\omega}$. However, since $\bar{r}$ is the maximum interest rate any good firm accept, good firm $\omega' < \bar{\omega}$ faces a lower interest rate compared to what would clear his demand using only the wealth of his marginal investors, $s(\omega')$. Applying the same logic backward which would lead to a positive excess demand by good firms with transparency level in $(\omega', \bar{\omega})$ compared to what can be absorbed by their marginal investors collectively.

Let $\tilde{\omega} \in (0, \bar{\omega})$ be the lowest transparency where the demand of good firms is fully absorbed by all the investors active in market $\bar{m}$.

$$R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) = \int_{1-\tilde{\omega}}^{1} \frac{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x)}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})} w(s)ds$$

which implies $\tilde{\omega}$ is the solution to

$$1 = \int_{1-\tilde{\omega}}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})} w(s)ds \tag{B.21}$$

In Proposition 5 we argue that under our assumptions, $\tilde{\omega} > 0$.

For a good firm from any transparency $\omega < \tilde{\omega}$, none of his offered bonds can be bought by investors of expertise $s < 1 - \tilde{\omega}$, since those investors cannot identify him as good. Thus he can only sell what can be absorbed by the residual wealth of the subset of investors $s > 1 - \tilde{\omega}$ who can identify him, $s > 1 - \omega > 1 - \tilde{\omega}$.

For $s > 1 - \tilde{\omega}$, let

$$\zeta(s) = \frac{(\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})}$$

$\zeta(s)$ captures how much of the portfolio held by investor $s > 1-\tilde{\omega}$ is bonds issued "collectively" by good firms with transparency $\omega < \tilde{\omega}$ that s can identify. The measure of those good firms is $(\tilde{\omega} - (1-s))\,\phi(1-\lambda)$. Thus for an individual firm of transparency $\omega < \tilde{\omega}$, aggregating over holdings of his bonds, by all the investors $s > 1-\omega$, we find how much $j = (\omega, g)$ can issue.

let $\eta_L(\omega) = \eta(\bar{m}, \omega, g; L)$ denote the rationing function in this market. The above argument implies

$$\begin{aligned}\eta(\bar{m}, \omega, g; L) = \eta_L(\omega) &= \frac{1}{\hat{y}(\bar{\omega}, g; L; \bar{r})} \int_{1-\omega}^{1} \frac{1}{(\tilde{\omega} - (1-s))\,\phi(1-\lambda)} \zeta(s) w(s) ds \\ &= \int_{1-\omega}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})} ds\end{aligned}$$

and for good firms with transparency $\tilde{\omega} \le \omega < \bar{\omega}$, $\eta_L(\omega) = 1$.

Interest Rate Regimes. Next, we need to determine what range of bonds has each kind of interest rate. In order to do so, introduce the following function.

$\mathbf{E}(\omega, \mathbf{r}, \varepsilon; \mathbf{x})$. Define

$$E(\omega, r, \varepsilon; x) \equiv \max_{\omega' \in [0,\omega]} \int_{\hat{s}(\omega)}^{\hat{s}(\omega')} w(s) ds - \varepsilon\phi \left(\lambda \int_{\omega'}^{\omega} \hat{y}(z, b; L; r) dz + (1-\lambda) \int_{\omega'}^{\omega} \hat{y}(z, g; L; r) dz \right)$$

For a bond issued by good firm of transparency ω, interest rate r and remaining firm demand for bonds issuance ε, $E(\omega, r, \varepsilon; x)$ measures the maximum over $\omega' < \omega$ of the difference between the endowment of all investors who can recognize ω is good but cannot recognize that ω' is good, and how much is needed to finance ε units of all the bonds in $[\omega', \omega]$ which firms demand if they all face interest rate r. A bond interest rate can only be determined by cash-in-the-market if $E(\omega, r^C(\omega), \varepsilon(\omega); x) = 0$. A strictly positive value would mean that there exists a range of investors $[\hat{s}(\omega), \hat{s}(\omega')]$ for some $\omega' < \omega$, all of whom can identify some bond in the range $[\omega', \omega]$ as a good bond (but not any bonds offered by firms with transparency lower than ω') and whose collective endowment exceeds what is necessary to finance all the bonds demanded by firms in $[\omega', \omega]$ facing a interest rate $r^C(\omega)$. Since these investors will want to spend their entire endowment financing bonds, it must be that some bond in the range $[\omega', \omega]$ must face a interest rate lower than $r^C(\omega)$. This is because $\frac{\partial D}{\partial r} < 0$, and a lower interest rate would push the firm demand up and bring demand closer to supply. But then monotonicity implies that the interest rate faced by a good firm with transparency ω must be lower than

$r^C(\omega)$, a contradiction.

Next, suppose one knows that $\breve{\omega}$ is the upper limit of one type of region. In a similar manner to (Kurlat, 2016), the following procedure finds the lower end of that region, the type of region immediately below and the prices within the region.

1. For a cash-in-the-market region, the lower end is

$$\sup\{\omega < \breve{\omega} : r^{NS}(\omega) < r^C(\omega) \text{ or } E(\omega, r^C(\omega), \varepsilon(\omega); r_H) > 0 \text{ or } r^C(\omega) > \bar{r}\} \quad \text{(B.22)}$$

   and the region to the left is a nonselective region (first condition) or a bunching region (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^C(\omega)$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

2. For a bunching region, the lower end is

$$\max\{\omega < \breve{\omega} : E(\omega, r^C(\bar{\omega}), \varepsilon(\bar{\omega}); r_H) = 0\} \quad \text{(B.23)}$$

   and the region to the left is a cash-in-the-market region. Within the region, $r_L(\omega) = r_L(\breve{\omega})$ and $r(\omega) = r(\breve{\omega})$.

3. For a non-selective region, the lower end is

$$\sup\left\{\omega < \breve{\omega} : \frac{w\left(\hat{s}(\omega)\right)}{\phi(1-\lambda)\hat{y}(\omega, g; L, r^{NS}(\omega))} > r(\omega') \text{ for some } \omega' \in (\omega, \breve{\omega}) \right.$$
$$\left. \text{or } E(\omega, r^C(\breve{\omega}), \varepsilon(\breve{\omega}); r_H) > 0 \text{ or } r^{NS}(\omega) \geq \bar{r} \right\} \quad \text{(B.24)}$$

   and the region to the left is a cash-in-the-market region (former condition), (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^{NS}(\omega)$ and $\varepsilon(\omega) = \frac{w(\hat{s}(\omega))}{\phi(1-\lambda)\hat{y}(\omega,g;L,r^{NS}(\omega))}$.

(a) For a bunching-with-scarcity-region, the lower end is 0. Within the region, $r_L(\omega) = \bar{r}$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

The first region is a bunching region with $\breve{\omega} = 1$, $r_L(\breve{\omega}) = 0$, and $\varepsilon(\breve{\omega}) = \phi(1-\lambda)\bar{L}$. Either one of the sets defined by B.22, B.23, B.24 is empty, in which case that region extends up to 0; or the bunching-with-scarcity region is hit and it extends all the way to zero.

(ii) **Firm optimization.**

Let ω_2 denote the index of the highest ω firm who faces a interest rate $\bar{r}$

$$w(1-\omega_2) = \phi(1-\lambda)\hat{y}(\omega_2, g; L),$$

and let ω_1 denote the index of the lowest ω transparency whose good firms do not face rationing in the bunching-with-scarcity region, defined as the solution to equation (B.21).

For any good firm with transparency $\omega > \omega_2$, since $r(\omega) > 0$, the rationing function (B.14) implies that in order to issue all of the firm bonds, the reservation interest rate should be $r_L(\omega)$. Good firm with transparency $\omega < \omega_2$ are indifferent between raising any number of bonds, so the issuance decision is optimal. For any bad firm, the rationing function implies that reservation interest rate is $\bar{r}$. Therefor credit issuance decisions are optimal for all firms and the total number of bonds they issue follows directly.

(iii) **Expert optimization.**

For $s \in [s_N, 1]$, each investor chooses the highest interest rate market on which there is a transparency ω such that $x(g;\omega,s,L) = g$ and $S(m,\omega,g) > 0$. Since $\tilde{r}(m) \geq 0$, this is optimal. For $s \in [s_0, s_N)$, investors only place weight on markets where nonselective pricing prevails. Equation (B.18) implies they are indifferent between financing bonds and staying out, since the highest interest rate market where there is a ω such that $x(b,\omega,g) = 1$ and $S(m,\omega,g) > 0$ has $\tilde{r}(m) = 0$, there is no other market in which they would strictly prefer to trade. For $s < s_0$, the same logic implies that no trading is optimal.

(iv) **Allocation function.**

For any market $m(\omega)$ where ω falls in either a cash-in-the-market or a nonselective range, the NMR algorithm implies that all the investors face a residual supply proportional to the original supply, so equation (B.10) follows from Appendix B of Kurlat (2016), equation (65).

For markets $m(\omega)$ where ω falls in a bunching or bunching-with-scarcity region as described above and χ is of the form $\chi(\omega,\tau) = \mathbb{I}(\tau = g \;\&\; \omega > 1-h)$, then the differential equations (B.12) follows from Appendix B of Kurlat (2016), equation (66), along with equation (21). Then (B.11) follows from applying the NMR algorithm.

(v) **Rationing function.**

Follows from applying equation (67) in Appendix B of Kurlat (2016).

■

Lemma B.1 *Every solution to robust program A.3 satisfies*

$$\begin{array}{ll} \sigma(m,\omega,\tau;\theta) \geq \hat{y}\left(\omega,\tau;\theta,r_H(\omega),r_L(\omega)\right) & \text{if } \tilde{r}(m;\theta) < r^R(\omega,\tau;\theta) \\ \sigma(m,\omega,\tau;\theta) = 0 & \text{if } \tilde{r}(m;\theta) > r^R(\omega,\tau;\theta) \end{array}$$

for some reservation interest rate, $r^R(\omega,\tau;\theta)$.

Furthermore, if $\tilde{r}(m;\theta) < r^R(\omega,\tau;\theta)$, $\frac{d\sigma(m,\omega,\tau;\theta)}{d\tilde{r}(m;\theta)} \leq 0$.

Proof. We start with the first part of the proposition. For simplicity, let j denote the firm (ω, τ), $\hat{y}(\omega, \tau) \equiv \hat{y}(\omega, \tau; \theta, r_H, r_L(\omega))$, $\sigma(m, j) \equiv \sigma(m, \omega, \tau; \theta)$, and $\eta(m, j) \equiv \eta(m, \omega, \tau; \theta)$. Also, we suppress the dependence of interest rate on prudence shock $\theta = H, L$ and write $\tilde{r}(m)$. Each individual firm is small and takes the prices as given, and does not affect the schedule of prices either.

Assume the contrary. This implies that there are two markets, m and m' with $\tilde{r}(m') < \tilde{r}(m)$ such that, for some j, the firm chooses $\sigma(m, j) > 0$ and $\sigma(m', j) < \hat{y}(\omega, \tau)$. There are four possible cases:

(i) $\eta(m; i) > 0$ and $\eta(m', j) > 0$. Then the firm can increase his utility by choosing demand $\tilde{\sigma}$ with $\tilde{\sigma}(m', j) = \sigma(i, m') + \epsilon$ and $\tilde{\sigma}(m, j) = \sigma(m', j) - \epsilon \frac{\eta(m', j)}{\eta(m, j)})$ for some positive ϵ.

(ii) $\eta(m, j) > 0$ and $\eta(m', j) = 0$. Consider a sequence such that $\eta^n(m', j) > 0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^n(m, j) = 0$ or $\sigma^n(m', j) \geq \hat{y}(\omega, \tau)$ (or both). Therefore either the condition that $\sigma^n(m, j) \to \sigma(m, j)$ or the condition that $\sigma^n(m', j) \to \sigma(j, m')$ in a robust solution is violated.

(iii) $\eta(m, j) = 0$ and $\eta(m', j) > 0$. Consider a sequence such that $\eta^n(m', j) > 0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^n(m, j) = 0$ or $\sigma^n(m', j) \geq \hat{y}(\omega, \tau)$ (or both). Therefore either the condition that $\sigma^n(m, j) \to \sigma(m, j)$ or the condition that $\sigma^n(m', j) \to \sigma(m', j)$ in a robust solution is violated.

(iv) $\eta(m, j) = \eta(m', j) = 0$. Consider a sequence such that $\eta^n(m', j) > 0$ and suppose that there is a sequence of solutions to robust firm problem which satisfies $\sigma^n(m', j) \to \sigma(m', j) < \hat{y}(\omega, \tau)$. This implies that for any sequence such that $\eta^n(m, j) > 0$ and for any n, the solution to robust firm problem must have $\sigma^n(m, j) = 0$. Therefore the condition that $\sigma^n(m, j) \to \sigma(m, j)$ in a robust solution is violated.

■

Lemma B.2 *In any equilibrium $r_L(\omega)$ is non-increasing in ω everywhere.*

Proof.

Assume the contrary. Then when $\theta = L$, there exists bonds offered by good firms with transparency ω, ω' with $\omega' > \omega$ such that $r_L(\omega') > r_L(\omega)$.For this to be consistent with firm optimization, it must be that

$$\eta(M_0, \omega', g; L) < \eta(M_0, \omega, g; L) = 1,$$

where the inequality follows from firm optimization, and the equality from definition of $r_L(\omega)$ and M_0, where $M_0 = \{m : \tilde{r}(m) \leq r_L(\omega)\}$. But investor optimization and the signal structure when $\theta = L$ requires that investors only use rules of the form $\chi(\omega, \tau) = \mathbb{1}(\tau = g \; \& \; \omega \geq 1 - h)$. This implies that for any $M_0 \subset M$,

$$\eta(M_0, \omega', g; L) \geq \eta(M_0, \omega, g; L),$$

a contradiction. ■

B.2 t = 0: Real Investment

At $t = 0$, firms anticipate the date $t = 1$ continuation value and choose the initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, to maximize their expected utility as defined in program (10).

Throughout this section, for brevity we will use $q_H = \frac{r_H}{1+r_H}$, $r_L(\omega) = r(\omega, g; L)$, $q_L(\omega) = \frac{r_L(\omega)}{1+r_L(\omega)}$, and $\bar{q} = \frac{\bar{r}}{1+\bar{r}}$.

We start by constructing $\hat{y}(\omega, \tau; \theta)$, i.e. the maximum liquidity that a firm can raise on the international markets. Maintaining $i(\omega, \tau; \theta)$ units allows a good firm to issue up to $\ell(\omega, \tau; \theta) = \frac{1}{1+r(\omega,\tau;\theta)} \xi i(\omega, \tau; \theta)$ bonds, with unit face value each, without violating the pledgeability constraint.

Bad firms value each unit of continued investment more than good firms since investors cannot seize anything from their output. Moreover, (1) they do not need any liquidity if $\theta = L$, since they cannot continue if hit by a liquidity shock, and (2) they face the same financing condition as good firms if $\theta = H$ but can only partially continue. It follows that bad firms save less liquidity, and every bad firm have enough collateral (initial scale) to issue up to $\bar{L}$. See section "Firm problem given the optimal choice of issuance" in B.2.2 for more detail.

Putting this together we have[21]

$$\hat{y}(\omega, \tau; \theta) = \begin{cases} \ell(\omega, \tau; \theta) & \tau = g;\ \theta = H \text{ or } (\theta = L \text{ and } \omega \geq \omega_2) \\ \ell(\omega_2, \tau; \theta) & \tau = g;\ \theta = L \text{ and } \omega < \omega_2 \\ \bar{L} & \tau = b;\ \forall\, \theta, \forall\, \omega \end{cases} \tag{B.25}$$

Substituting the above $\hat{y}$ into the $t = 1$ credit market equilibria considerable simplifies the expressions. In particular, when $\theta = H$, $\lambda \int_0^{1-s} \hat{y}(\omega, b; H) d\omega$ reduces to $\lambda(1 - s)\bar{L}$. Similarly, when $\theta = L$, $\lambda \int_0^1 \hat{y}(\omega, b; H) d\omega$ reduces to $\lambda \bar{L}$.

Remark. Recall that in section B.2 we assumed $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is decreasing in the (common) interest rate when $\theta = H$, and in the firm specific r_L. With the above mapping, we need to verify that the equilibrium $\ell(\omega, \tau; \theta)$ is in fact downward sloping in r_H, r_L, which we will do in this section.

Next, using the $\hat{y}$ defined in (B.25), we specialize the investor wealth function to satisfy the sufficient condition 2.(iv). This specification implies a particular form of equilibrium when $\theta = L$,

[21] We will show that when $\theta = L$, good firms with transparency $\omega < \omega_2$ are indifferent in the scale at which they continue. Thus the above $\hat{y}$ is an equilibrium. We pick this tie-breaking rule because it simplifies the exposition. For more detail see section B.1.2, Bunching-with-scarcity.

which we use along with the equilibrium in $\theta = H$ to derive firm's optimal investment decision at $t = 0$.

B.2.1 Specializing the Expert Wealth Function

Consider $r^C(\omega)$ and $r^{NS}(\omega)$ defined in equation (B.16) and (B.18), respectively. We make two assumptions to restrict the wealth function.

First, we have assumed that wealth function is monotonically decreasing, $w'(s) < 0$, so $r^C(\omega)$ does not become non-monotone. Thus bunching region can only emerge above some threshold, $\underline{\omega} < \omega \leq 1$, and bunching-with-scarcity only below some threshold, $0 \leq \omega < \bar{\omega}$.

Second, in what follows, we derive a parametric assumption to ensure that non-selective region does not emerge. Non-selective interest rate schedule is an upper bound on the prevailing interest rate in each market. Thus a sufficient condition for this upper bound to never be active, i.e. for the non-selective pricing region not to emerge, is to have $r^C(\omega) \leq r^{NS}(\omega)$ for markets where $0 < \tilde{r}(m) < \bar{r}$.

$$r^C(\omega) = \ell^{-1}\left(\frac{w\left(\hat{s}(\omega)\right)}{\phi(1-\lambda)}\right) \leq \frac{\lambda \bar{L}}{(1-\lambda)\int_0^{\omega} \ell(\omega, g; L, r_H, r^C(\omega))d\omega'},$$

where $\ell^{-1}(.)$ denotes the inverse of function $\ell(\omega, g; L; \{r_H, r^C(\omega)\})$ with respect to $r^C(\omega)$, and $\{r_H, r^C(\omega)\}$ indicates the dependence of demand function on (H, L) interest rate explicitly.

Moreover, we have used that $\hat{y}(\omega, g; L; \{r_H, r^C(\omega)\}) = \ell(\omega, g; L; \{r_H, r^C(\omega)\})$ for $r^C(\omega) < \bar{r}$, and that $\varepsilon(\omega) = 1$ when there is no non-selective region in equilibrium. Note that $\hat{y}(\omega', g; L) = \bar{L}$ ($\forall\ \omega'$) minimizes the right hand side on the above equation, which yields the following sufficient condition

$$r^C(\omega) = \ell^{-1}\left(\frac{w\left(\hat{s}(\omega)\right)}{\phi(1-\lambda)}\right) \leq \frac{\lambda}{(1-\lambda)\omega}. \tag{B.26}$$

In the proof of Proposition 5 we derive a sufficient condition on primitives to ensure that (B.26) holds.

Under Assumption 2, there is no non-selective region when $\theta = L$, $\omega_1 > 0$ and $\omega_3 < 1$. The equilibrium pricing regions are thus characterized by three thresholds $\omega_1 < \omega_2 < \omega_3$ such that

(i) Good firms with transparency $0 \leq \omega < \omega_1$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (19), and $\eta(\bar{m}, \omega, g; L) < 1$.

(ii) Good firms with transparency $\omega_1 \leq \omega < \omega_2$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (19), and $\eta(\bar{m}, \omega, g; L) = 1$.

(iii) Good firms with transparency $\omega_2 \leq \omega < \omega_3$ are in cash-in-the-market pricing region.

(iv) Good firms with transparency $\omega_3 \leq \omega < 1$ are in bunching region and face zero interest rate.

(v) No bad firm issues any bonds in any market.

In this equilibrium

$$y(\omega,\tau) = \begin{cases} \hat{y}(\omega,\tau) & \text{if } \tau = g \text{ and } \omega \geq \omega_1 \\ \eta(m(\bar{r}),\omega,\tau)\hat{y}(\omega,\tau) = \eta_L(\omega)\hat{y}(\omega,\tau) & \text{if } \tau = g \text{ and } \omega < \omega_1 \\ 0 & \text{if } \tau = b \end{cases} \tag{B.27}$$

B.2.2 Firm Optimal Decision

Consider the firm problem (10). Each firm j takes his optimal behavior at $t = 1$ as given, which along with $t = 1$ prices in different prudence shocks, the allocation function and the rationing function fully describes firm j continuation payoff. Firm j then chooses his business plan to maximize his expected utility given this continuation payoff.

Derivation of firm optimal choice of bond issuance, equations (7) and (9). A firm hit by liquidity shock has three possible options, at $t = 0$, in how to manage a liquidity shock in each aggregate state at $t = 1$. First, the firm can choose not to insure against the liquidity risk and abandon investment if a liquidity shock happen. This would lead to the highest ex-ante level of investment, $I(\omega,\tau)$. Second, the firm can choose to save enough out of his own endowment, through the banker, such that he has sufficient liquidity at $t = 1$ and does not need to raise any extra financing on the international markets. This option leads to the lowest ex-ante level of investment. Third, the firm can choose to save a lower amount from his initial endowment and borrow the rest from international investors. This leads to an intermediate level of ex-ante investment.

From the linearity of the firm problem, the firm chooses one option and does the same thing for all units of investment. Moreover, Assumption 2.(i) implies the first option dominates second. Then Assumption 2.(iii) implies that borrowing on the international markets are sufficiently cheap that the third option dominates the first one, which in turn leads to firm's optimal liquidity choice, equation (7). Conditions (i) and (iii) of assumption (2) are derived in proof of Proposition 5.

Alternatively, a good firm who is not hit by a liquidity shock is indifferent between issuing bonds or not if $r(\omega,\tau;\theta) = 0$, and otherwise prefers not to issue. Thus these firms do not participate in the international markets. It follows that, if a bad firm not hit by a liquidity shock tries to issue bonds, his type is revealed and he does not succeed in raising funding, and it will not participate either.

As such, only firms hit by liquidity shock attempt to raise funding from international investors at $t = 1$, which in turn implies the ex-ante budget constraint 9.

Firm problem given the optimal choice of issuance. Since problem (10) is linear, equations (4)-(9) determine the optimal firm choices, $i(\omega,\tau;\theta)\ \forall\ \theta$ whenever they are non-zero. Plugging these solutions into (9) determines $I(\omega,\tau)$.

The rest of the argument follows from a parallel logic to (Holmström and Tirole, 1998), (Holmström and Tirole, 2011). Conjecture $i(\omega, \tau; H) = I(\omega, \tau)$, and let $0 \leq x \leq 1$ denote the scale of investment for firm $j = (\omega, \tau)$ when $\theta = L$.

Use the $t = 2$ interest rate along with equation (9) to get $I(\omega, \tau)$. Substitute $I(\omega, \tau)$ in the objective function (10).

Good firms. Consider a good firm $j = (\omega, g)$. The objective function of the good firm boils down to

$$\Pi(x) = \frac{\phi(\rho_g - \xi)(\pi_H + \pi_L x) + (1-\phi)\rho_g}{1 + \phi\xi(\pi_H q_H + \pi_L q_L(\omega) x)} - 1$$

Thus he optimal investment is determined by

$$\Pi'(x) = \frac{\pi_L \phi \Big(\rho_g - \xi - \pi_H \phi \xi^2 (q_H - q_L(\omega)) - \rho_g \xi \big(q_L(\omega)\,(1 - \pi_L \phi) - \pi_H q_H \phi\big)\Big)}{\left(1 + \phi\xi(\pi_H q_H + \pi_L q_L(\omega) x)\right)^2}$$

As such, $\Pi'(x) > 0 (< 0)$ implies $x = 1 (x = 0)$, and if $\Pi'(x) = 0$ good firm j is indifferent between any level of continuation when $\theta = L$ and the firm has a liquidity shock . This implies

$$q_L(\omega) < \bar{q} = \frac{(\rho_g - \xi)(1 + \phi \pi_H r_H \xi)}{\xi((1-\phi)\rho_g + \phi\pi_H(\rho_g - \xi))}. \tag{B.28}$$

Substitute $\frac{r_L(\omega)}{1+r_L(\omega)}$ for $q_L(\omega)$ to get equation (19).

Next, we need to make sure that our conjecture for continuation at full scale in high state regime, $i(\omega, \tau; H) = I(\omega, \tau)$, is correct for a good firm. For this conjecture to hold, it must be that $r_H < \bar{r}_H$ such that every good firm j prefers to submit liquidity demand to international markets when $\theta = H$. Using Assumption 2.(i), the alternative is to set $i(\omega, \tau; H) = 0$, do not do any liquidity risk management and abandon production if hit by a liquidity shock in state $\theta = H$, and instead increase $I(\omega, \tau)$. Since firms with transparency $\omega = 1$ are those who face the lowest interest rate in $\theta = L$, such deviation is most profitable for them. Thus it is sufficient to ensure that they do not want to deviate. Thus $\bar{r}_H$ solves

$$\rho_g(1-\phi) + (\rho_\tau - \xi)\phi\pi_L = \frac{\rho_g(1-\phi) + (\rho_g - \xi)\phi}{1 + \phi\pi_H\xi\frac{\bar{r}_H}{1+\bar{r}_H}}$$

Thus if

$$r_H < \bar{r}_H = \frac{(\rho_g - \xi)}{\rho_g \xi(1-\phi) + (\rho_g - \xi)(\phi\pi_L\xi - 1)} \tag{B.29}$$

all good firms prefer to do liquidity management using a combination of own saving and international markets.

Next consider the most transparent good firm, $j_{1,g} = (1, g)$. When $\theta = L$ this firm faces zero interest rate, and thus does not need to hold any precautionary liquidity against this state. Moreover, as long as $r_H < \bar{r}_H$, every good firm (including $j_{1,g}$) prefers to do liquidity management against the liquidity shock in $\theta = H$ state, and saves $\pi_H \phi \frac{r_H}{1+r_H}$ per unit of initial investment towards this state. It follows that $j_{1,g}$ faces the lowest possible interest rate in both states of the world, and thus has the highest investment level among all good firms, $I(1,g)$. As explained at the end of the proof, we have chosen $\bar{L} \equiv I(1,g)$.

Bad firms. Consider any bad firm. Assumption 2.(i) implies firms either do liquidity management using international markets, or do not do any liquidity management. When $\theta = L$ a bad firms hit by a liquidity shock is not able to raise any international financing, so he has to liquidate investment, which means for a bad firm $x = 0$. Thus no bad firms save any liquidity against $\theta = L$ aggregate state. Next, consider the least transparent bad firm, $j_{0,b} = (0,b)$. When $\theta = H$, $\eta_H(0) = 1$, thus $j_{0,b}$ is not rationed, and is treated as a good firm. Thus he needs to save $\pi_H \phi \frac{r_H}{1+r_H}$, per unit of initial investment, to be able to continue at full scale. It follows that $j_{0,b}$ saves the same amount of liquidity as $j_{1,g}$, and thus chooses the same level of investment $\bar{L}$.

Every other bad firm, $\omega > 0$ is rationed when $\theta = H$, thus they hold lower liquidity, compared to $j_{0,b}$, against this state of the world. This in turn implies they choose a higher level of initial investment: $I(\omega, b) > I(0,b)$, $\forall\ \omega > 0$. Thus every bad firm has enough collateral to borrow up to $\bar{L}$. Furthermore, bad firms face the same interest rate r_H as good firms when $\theta = H$, and moreover they do not pay back, so if good firms participate in the international markets when $\theta = H$, it is optimal for bad firms to do so as well. It follows that $\hat{y}(\omega, b; \theta, r_H, r_L)$ as defined in equation (B.25) is optimal.

Thus at $t = 0$, similar to all bad firms, firm $\hat{j}$ does not save any precautionary liquidity for $\theta = L$ state. Bad firms also face the same interest rate r_H when $\theta = H$, and furthermore they do not pay back, so if firm j participates in the international markets when $\theta = H$, all bad firms will do so as well.

Firm investment at $\mathbf{t = 0}$. Next we characterize the $t = 0$ firm investment. Substitute back the optimal continuation decision, and corresponding date $t = 2$ prices into equation (9) to get the optimal investment decision

$$I(\omega, g) = \begin{cases} \frac{1}{1+\xi(\pi_H q_H + \pi_L q_L(\omega))} & \text{if } \omega > \omega_2 \\ \frac{1}{1+\xi(\pi_H q_H + \pi_L \bar{q})} & \text{if } \omega_1 < \omega \leq \omega_2 \\ \frac{1+(1-\eta_L(\omega))\frac{\phi\xi\pi_L\bar{q}}{1+\phi\xi\pi_H q_H}}{1+\phi\xi(\pi_H q_H + \pi_L \bar{q})} & \text{if } \omega \leq \omega_1 \end{cases} \tag{B.30}$$

where $\eta_L(\omega) = \eta\left(m_(\bar{r}), \omega, g; L\right)$, ω_1 is defined by (30); and the investment for $\omega \leq \omega_1$ follows from substituting the continuation decision corresponding to each aggregate state in the date $t = 0$

budget constraint:

$$I(\omega,g)=\frac{1-\phi\pi_L\bar{q}D(\bar{r};r_H)\eta_L(\omega)}{1+\phi\xi\pi_Hq_H}=\frac{1-\frac{\phi\xi\pi_L\bar{q}}{1+\phi\xi(\pi_Hq_H+\pi_L\bar{q})}\eta_L(\omega)}{1+\phi\xi\pi_Hq_H}$$
$$=\frac{\frac{1+\phi\xi\pi_Hq_H+\phi\xi\pi_L\bar{q}(1-\eta_L(\omega))}{1+\xi(\pi_Hq_H+\pi_L\bar{q})}}{1+\phi\xi\pi_Hq_H}=\frac{1+(1-\eta_L(\omega))\frac{\phi\xi\pi_L\bar{r}}{1+\phi\xi\pi_Hq_H}}{1+\phi\xi\left(\pi_Hq_H+\pi_L\bar{q}\right)}.$$

Moreover,

$$I(\omega,b)=1-q_H\xi\phi\pi_H\eta_H(\omega)\bar{L} \tag{B.31}$$

where $\eta_H(\omega)=\eta(m_H,\omega,b;H)$.

Next we verify that for good firms who do payback the international experts, the liquidity a firm raises at $t=1$ on the international market, $y(\omega,\tau;\theta)$ in problem (A.1), is equal to its liquidity need, $\ell(\omega,\tau;\theta)$ associated with optimal investment decision (B.30). This is immediate from comparing equations (B.8), (B.27) and (B.25). It follows that firm j's realized issuance of bonds on the international market, $\ell(\omega,\tau;\theta)$, is given by:

(i) Good firms, $\tau=g$

$$\ell\left(\omega,g;\theta\right)=\int_{M_j}\xi I(\omega,\tau)d\eta\left(m,\omega,g;\theta\right) \tag{B.32}$$

where $\eta\left(m,\omega,g;\theta\right)$ is given by

$$\eta\left(m,\omega,g;\theta\right)=\begin{cases}\int_{1-\omega}^{1}\frac{1+\bar{r}}{\phi(1-\lambda)D(\bar{r};r_H)(\omega_2-(1-s))-\int_{1-\omega_2}^{1-\omega_1}w(s)ds}w(s)ds, & \tilde{r}(m)=\bar{r}\ \&\ \omega<\omega_1\ \&\ \theta=L\\ 1 & \Big(\tilde{r}(m)<\bar{r}\text{ or }\big(\tilde{r}(m)=\bar{r}\ \&\ \omega\geq\omega_1\big)\Big)\ \&\ \theta=L\\ & \text{or }\tilde{r}(m)=r_H\ \&\ \theta=H\\ 0 & \text{otherwise}\end{cases} \tag{B.33}$$

(ii) Bad firms, $\tau=b$

$$\ell\left(\omega,\tau;\theta\right)=\bar{L}\int_{m\in M_j}\eta(m,\omega,\tau;\theta)dm \tag{B.34}$$

where M_j, $j=(\omega,b)$ is the set of markets firm $j=(\omega,b)$ can sell bonds in, and

$$\eta(m,\omega,b;\theta)=\begin{cases}\int_{s_H}^{1-\omega}\frac{1}{\lambda(1-s)D(0;r_H)+(1-\lambda)\bar{D}(r_H)}\frac{w(s)}{\phi(1-r_H)}ds & \tilde{r}(m)=r_H\ \&\ \omega\leq 1-s_H\ \&\ \theta=H\\ 0 & \text{otherwise}\end{cases} \tag{B.35}$$

To complete the proof we need to verify that there is a fixed point to the joint $t = 0, 1$ problem, i.e. date $t = 0$ optimal outcomes do constitute an equilibrium in the international markets at $t = 1$. We do this in Proposition 5.

Maximum liquidity demand on international markets. By construction of the model, firms can always submit excess demand $\hat{y}$ on (different) markets at $t = 1$ to undo the rationing by investors. To avoid this, we need to impose an exogenous upper bound on how much demand for bond issuance firm can submit. We choose $\bar{L} \equiv I(1, g) = D(0; r_H)$ for convenience as in this case good firms are not constrained by this limit in equilibrium, while no bad firms can undo rationing by submitting more than what they need.

C Proofs of Propositions in the Main Text

Proof of Lemma 1. The statements come from direct substitution of $I^A(\omega,\tau)$ and $i^A(\omega,\tau;\theta)$ into the definition of country output

$$Y^A(\omega,\tau;\theta) \equiv \rho_g(1-\lambda)\left((1-\phi)I^A(\omega,g)+\phi i^A(\omega,g;\theta)\right)+\rho_b\lambda\left((1-\phi)I^A(\omega,b)+\phi i^A(\omega,b;\theta)\right)$$

■

Proof of Proposition 1.

The firm problem at date $t = 1$ is defined in (A.1).

(i) The general form of equilibrium for $\theta = H$ is characterized in section B.1.1. (r_H, s_H) are given by equations (B.1) and (B.1), respectively, using $\hat{y}$ defined in (B.25).

(ii) The general form of equilibrium for $\theta = L$ is characterized in section B.1.2. The form in (12) is then derived by specializing the wealth function in section B.2.1, which also uses $\hat{y}$ defined in (B.25) as well as Assumption 2.

■

Proof of Proposition 2.

The firm problem at date $t = 1$ is defined in (A.1).

(i) The general form of equilibrium for $\theta = H$ is characterized in section B.1.1. Section B.2.2 shows that the optimal continuation decision is determined by the constraint. It follows that the equilibrium amount that the firm raises is given by $y(\omega,\tau;H,r_H,r_L)$ in program (A.1), using $\hat{y}$ defined in (B.25) and equations (7) and (9) with the optimal $i(\omega,\tau;H)$.

(ii) The general form of equilibrium for $\theta = L$ is characterized in section B.1.2, and specialized in section B.2.1 by specializing the wealth function under Assumption 2. Section B.2.2 shows that the optimal continuation decision is determined by the constraint. It follows that the equilibrium amount that the firm raises is given by $y(\omega,\tau;L,r_H,r_L)$ in program (A.1), using $\hat{y}$ defined in (B.25) and equations (7) and (9) with the optimal $i(\omega,\tau;L)$.

■

Proof of Proposition 3.

The derivation of optimal firm ex-ante investment, as well as the optimal continuation decision, is provided in section B.2.2. (13) follows from (B.30), where the rationing function is defined in (B.33). ■

Proof of Proposition 4.

The derivation of optimal firm ex-ante investment, as well as the optimal continuation decision, is provided in section B.2.2. (16) follows from (B.31), where the rationing function is defined in (B.35). ■

Lemma C.1 *Assume $G(x)$ and $H(x,z)$ are continuous in x. Equation* (C.1) *has a fixed point $x \in [0,1]$,*

$$F(x) = \frac{\lambda \int_0^{1-s_H(x)} H(x,z)dz}{\lambda \int_0^{1-s_H(x)} H(x,z)dz + (1-\lambda)G(x)}; \tag{C.1}$$

where $s_H(x)$ is the solution to

$$\int_{s_H(x)}^{1} \frac{1}{\lambda \int_0^{1-s} H(x,z)dz + (1-\lambda)G(x)} w(s)ds = \phi(1-x), \tag{C.2}$$

if equation (C.2) *has a solution, and $s_H(x) = 0$ otherwise.*

Proof of Lemma C.1.

First note that if equation (C.2) has a solution in $s_H(x)$, it will be $s_H(x) \in [0,1]$. The reason is that $w(s) = 0$ for $s > 1$ and $s < 0$, so moving s_H outside the $[0,1]$ interval does not change the left hand side of equation (C.2).

Case 1 [Equation (C.2) **holds with equality,** $s_H \in [0,1]$**].** Consider the case where s_H is interior. Consider the self-map on $F : [0,1] \mapsto [0,1]$. We use Brouwer's fixed-point theorem to prove existence of a fixed point. $[0,1]$ is a compact convex set. We need to show is that $F(x)$ is a continuous function, and maps $[0,1]$ to itself, which is immediate since the ratio in $F(x)$ is positive and (weakly) smaller than one.

Next we move to proving continuity. $G(x)$ is continuous in x. $H(x,z)$ is also continuous in x, and so is $\int H(x,z)dz$. Thus if a solution $s_H(x)$ to equation (C.2) exists, it is also continuous.

This implies that if a solution to equation (C.2) exists, then everything on the right hand side of equation (C.1) is continuous, so $F(x)$ is a continuous map from $[0,1]$ to $[0,1]$, which implies by Brouwer's theorem a fix point exists.

Case 2 [Equation (C.2) **only holds with inequality, thus** $s_H = 0$**].** Then equation (C.1) becomes one equation in one unknown in x, which with the same argument as the previous case has a fixed point. ■

Proof of Proposition 5.

We start with explaining the mapping between the equations in the statement of the proposition to the solution developed in sections B.2 and B.1. To simplify the formulas, the proposition is stated in terms of premia rather than interest rates, using the monotone transformation

$$q = \frac{r}{1+r}. \tag{C.3}$$

Equation (20) writes the general form of required maintenance cost of a good firm who faces premia x when $\theta = H$, and y when $\theta = L$, or interest rates $\frac{x}{1-x}$ and $\frac{y}{1-y}$, respectively. It uses the equilibrium firm ex-ante investment, defined by equations (B.30) and (B.31), and optimal continuation scale. Substitute in equation (7) to get firm liquidity demand in international markets: $\ell(\omega, g; \theta, r_H) = D(r_L(\omega); r_H)$. Using this demand functions, $\hat{y}$ at $t = 1$ is defined in (B.25). It is straight forward to verify that using (B.25) to solve for the firm problem at $t = 1$ (section B.1), $y(\omega, \tau; \theta) = \ell(\omega, \tau; \theta)$.

Under the appropriate sufficient conditions on the parameters (see the end of this proposition), firms choose to participate in international markets when $\theta = H$, with the equilibrium described in B.1.1, and when $\theta = L$ with the equilibrium described in sections B.1.2 and B.2.1,. Under this equilibrium structure, equation (21) aggregates the total required maintenance across the pricing regions when $\theta = L$.

Equation (19) rewrites the maximum premium $\bar{q}$ in $\theta = L$, defined in equation (B.28), when the common premium in high state is x. The threshold transparencies ω_1, ω_2 and ω_3 are defined in equations (30), (27), and (28), respectively, and $\omega_1 = \tilde{\omega}$ and $\omega_2 = \bar{\omega}$ in the $\theta = L$ equilibrium in section B.1.2. This leads to the rationing function in equation (B.33). Equation (26) determines the threshold where bunching region ends, at zero interest rate, given liquidity demand function (20). Equation (25) determines the threshold where bunching-with-scarcity region starts, at premium $\bar{q}$ (interest rate $\bar{r}$). Equation (29) determines the threshold where rationing starts in bunching-with-scarcity region, given the liquidity demand

Finally, equations (17) and (18) jointly determine the pooling premium and marginal investor when $\theta = H$, at the above liquidity demand levels.

We use lemma C.1 to prove existence of equilibrium. Let $G(x) = \bar{D}(x)$ and $H(x) = D(0; x)$. As such we need to show both functions are continuous.

$\bar{y}(x)$ is continuous. $D(y; x)$ is continuous in x for any $x, y > 0$ since $1 + \phi\xi(\pi_H x + \pi_L y) > 0$. Thus $D(0; x)$ and $D(\bar{y}(x); x)$ are also continuous.

Now turn to $\omega_1(x)$, $\omega_2(x)$ and $\omega_3(x)$. $w(.)$ and $D(y; x)$ are continuous in x. $D(y, x)$ is constant in ω and $w(.)$ is increasing in ω, so equations (25) and (26) have a unique solution in ω, so $\hat{\omega}_3(x)$ and $\hat{\omega}_2(x)$ exist, are unique, and continuous.

Next, $D(0, x)$ is decreasing in x. Moreover

$$D(\bar{y}(x); x) - (1 - \bar{y}(x))D(\bar{y}(x); x) = \bar{y}(x)D(\bar{y}(x); x) = \frac{\rho_H - \xi}{\rho_g - \phi\xi},$$

$$\frac{d\Big((1 - \bar{y}(x))D(\bar{y}(x); x)\Big)}{dx} = \frac{dD(\bar{y}(x); x)}{dx} = \frac{\xi^2\pi_H\phi(\rho_g(1-\phi) + \phi\pi_H(\rho_g - \xi))}{(\rho_g - \phi\xi)(1 + \phi\xi\pi_H x)^2} < 0.$$

Thus both $\hat{\omega}_2(x)$ and $\hat{\omega}_3(x)$ are monotonically decreasing in x. Since $\hat{\omega}_2(x)$ and $\hat{\omega}_3(x)$ are continuous, (27) and (28) imply that $\omega_2(x)$ and $\omega_3(x)$ are also continuous and weakly decreasing in x.

Next consider the right hand side of (29). $\omega_2(x)$ is continuous. Moreover, (29) is the simplified version of (B.21). We have already shown that $R_D(\omega_1, \omega_2(x), \bar{y}(x), 1; x) > 0$, and $\hat{y}(\omega_2(x), g; L) = D(\bar{y}(x); x)(1 - x) > 0$, thus the denominator is positive. Each term is also continuous in x, which in turn implies the right hand side is continuous in x. Thus $\hat{\omega}_1(x)$ is continuous as well, and using equation (30), $\omega_1(x)$ is also continuous.

Finally, continuity of $\omega_i(x)$ $i = 1, 2, 3$, along with continuity of $w(.)$, $\eta(.)$ and $D(y; x)$ (in x) implies $\bar{D}(x)$ is continuous. So by lemma C.1, the fixed point exists.

Parametric Assumptions.

Optimal Firm Decision without Access to International Market [Assumption 2.(i)]. Assume the firm does not have access to international investors. So the firm can do one of the two things. The first option is to invest all of his initial endowment. Then the firm continues with a high scale, $I_I = 1$, if not hit by a liquidity shock, and terminate the project if hit. Thus the payoff is $\Pi_I = \rho_\tau(1 - \phi)I_1 = \rho_\tau(1 - \phi)$. Alternatively, the firm can save enough of his own endowment using bankers to insure against the liquidity shock in either or both aggregate states. Since the aggregate state is only relevant in the interaction with the international investors, if the firm choose to insure against liquidity shock from own endowment, it will be for both aggregate states. The firm investment scale is given by $I_S = \frac{1}{1+\phi\xi}$, and his expected payoff is $\Pi_I = \rho_\tau I_2$. Thus for $\Pi_I > \Pi_S$ we need

$$1 - \phi > \frac{1}{1 + \phi\xi} \Rightarrow \phi < \frac{\phi\xi}{1 + \phi\xi} \Rightarrow \xi > \frac{1}{1 - \phi},$$

which is Assumption 2.(i). Under this assumptions when firms can access the international credit market, we only need to compare borrowing on the international markets with investing all of their endowment. This is the next parametric restriction that we consider.

Sufficient Condition for Inequality (B.29) [**Assumption 2.(ii)**]. Let $q_H = \frac{r_H}{1+r_H}$. From equation (17)

$$q_H = \frac{r_H}{1+r_H} = \frac{\lambda\,(1-s_H(r_H))}{\lambda\,(1-s_H(r_H)) + (1-\lambda)\,\frac{\bar{D}(r_H)}{D(0;r_H)}} \leq \frac{\lambda}{\lambda + (1-\lambda)\frac{\bar{D}(r_H)}{D(0;r_H)}},$$

which in turn implies

$$r_H \leq \frac{\lambda}{(1-\lambda)\frac{\bar{D}(r_H)}{D(0;r_H)}} = \frac{\lambda}{1-\lambda}\frac{D(0;r_H)}{\bar{D}(r_H)}.$$

So to find an upper bound on r_H, it is sufficient to find an upper bound on $\frac{D(0;r_H)}{\bar{D}(r_H)}$. Note that $D(0;r_H)$ is the maintenance cost of the firms with the highest transparency. Moreover, $\bar{L} = \frac{D(0;r_H)}{1+r_H}$ is by construction have the highest liquidity demand submitted by any firm to the international markets, which in turn implies $D(0;r_H)$ is the highest maintenance cost for any good firm from any transparency. Thus $\bar{D}(r_H) \leq D(0;r_H)$, which in turn implies

$$r_H \leq \frac{\lambda}{1-\lambda} \Rightarrow q_H \leq \lambda$$

In Assumption 2.(ii) we assume $\frac{\lambda}{1-\lambda} \leq \bar{r}_H$, where $\bar{r}_H$ is defined in equation (B.29). This in turn insures that $r_H \leq \bar{r}_H$. Moreover, one can substitute λ for x in (19) to get an upper bound on $\bar{q}$.

Sufficient Condition for Inequality (B.17) [**Assumption 2.(iii), part 1**]. Using (B.25), we can write condition (B.17) as

$$w(0) > \phi(1-\lambda)\ell(1, g; L) = \phi(1-\lambda)D(0;r_H)$$

Note that in $\hat{\omega} = \omega_3(r_H)$. A sufficient condition for the above inequality to hold is

$$w(0) \geq \phi(1-\lambda)\xi, \tag{C.4}$$

which ensure that $\omega_3 < 1$, and constitutes the first part of Assumption 2.(iii).

Sufficient Condition for Strictly Positive Solution to Equation (B.21) [**Assumption 2.(iii), part 2**]. The second part of 2.(iii), $\lim_{s\to 0} w(s) = 0$, directly ensures insures that $\tilde{\omega}$ that solves equation (B.21) is strictly positive, i.e. $\omega_1 > 0$.

Sufficient Condition for Inequality (B.26) [**Assumption 2.(iv)**]. The only set of markets we need to consider are those with cash in the market pricing. Let $q^C(\omega) = \frac{r^C(\omega)}{1+r^C(\omega)}$ and $\bar{q}(r_H) =$

$\frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}$. From (B.26)

$$q^C(\omega) \leq \frac{\lambda}{\lambda + (1-\lambda)\omega}$$

Start by noting that $\bar{q}(r_H)$ is the maximum $q^C(\omega)$ can achieve, so a sufficient condition for inequality (B.26) is

$$\min\{\bar{q}(r_H), q^C(\omega)\} \leq \frac{\lambda}{\lambda + (1-\lambda)\omega}.$$

Next from (19)

$$\bar{q}(r_H) = \frac{(\rho_g - \xi)(1 + \phi\xi\pi_H q_H)}{(\rho_g(1-\phi) + \phi(\rho_g - \xi)\pi_H)\,\xi} \leq \frac{(\rho_g - \xi)\,(1 + \lambda\phi\xi\pi_H)}{(\rho_g\,(1-\phi) + \phi\,(\rho_g - \xi)\,\pi_H)\,\xi}$$

where the inequality used part (ii) to replace q_H with it's maximum, λ. Next, from (22)

$$q^C(\omega) = \frac{\xi\phi(1-\lambda) - w(1-\omega)(1 + \phi\xi\pi_H x)}{\xi\phi((1-\lambda) + w(1-\omega)\pi_L)} \leq \frac{\xi\phi(1-\lambda) - w(1-\omega)}{\xi\phi((1-\lambda) + w(1-\omega)\pi_L)}$$

where the inequality just uses $q_H \geq 0$. Substitute both back to get a sufficient condition

$$\min\left\{\frac{(\rho_g - \xi)\,(1 + \lambda\phi\xi\pi_H)}{(\rho_g\,(1-\phi) + \phi\,(\rho_g - \xi)\,\pi_H)\,\xi}, \frac{\xi\phi(1-\lambda) - w(1-\omega)}{\xi\phi((1-\lambda) + w(1-\omega)\pi_L)}\right\} \leq \frac{\lambda}{\lambda + (1-\lambda)\omega}.$$

which is Assumption 2.(iv).

■

Proof of Proposition 6. Using (26), the size of the low exposure group is determined by

$$w(1-\omega_3) = \phi\,(1-\lambda)\,\xi i\,(\omega, g, L)\,|_{\omega\in[\omega_3,1]}. \tag{C.5}$$

The direct effects come from simple differentiation using equations (13) and (15) and noting that $\eta_H(\omega) = \eta_L(\omega) = 1$ in the low exposure region. The size of the group of high exposure countries is defined implicitly in (29). Let $Z_1 = \phi\,(1-\lambda)\,\frac{\xi}{1+\bar{r}} i\,(\tau_j = \omega, g, L)\,|_{\omega\in[\omega_1,\omega_2]}$, the amount an unrationed representative good firm borrow facing the maximum interest rate $\bar{r}$. In the left panel of Figure 3, we plot the supply of capital of a $k \leq \omega_1$ firm, $\eta_L(\omega)Z_1$ as the dashed curve, which, using the definition of ω_2 in (25), we can rewrite as

$$Z_1 \int_{1-\omega}^{1} \frac{1}{Z_1\,(1 - w^{-1}\,(Z_1) - (1-s)) - \int_{w^{-1}(Z_1)}^{1-\omega_1} w(s)ds} w(s)ds. \tag{C.6}$$

By definition, ω_1 is determined by the point where this curve is equal to the demand Z_1, the dashed line, as this is the least transparent country where firms demand for credit is fully met. While a change in Z_1 moves both curves, using the implicit function theorem, we can verify that $\frac{\partial\omega_1}{\partial(Z_1)} > 0$.

The direct effects then come from simple differentiation using equations (13) and (15) and noting that $\eta_H(\omega) = \eta_L(\omega) = 1$ in the region $\omega \in [\omega_1, \omega_2]$. ■

D Extensions

D.1 Aggregate Productivity Shock and State Dependent Fraction of Good Firms

In this section, we briefly discuss how the equilibrium objects change under the generalization that the fraction of good firms, λ, is state dependent.

As we assume that each firm knows its type already in period 0, in this version we modify the timing of the realization of the aggregate state. We assume that the aggregate state is realized in period 0, determining the fraction of bad firms, λ_θ, but firms do not observe this until period 1.

Then, going through the same derivation as before, we replace each λ in each expression by λ_H or λ_L depending on whether that expression is determined by the fraction of bad firms in the high or the low state. For instance, expressions 17-18 change to

$$F(x) = \frac{\lambda_H (1 - s_H(x)) D(0; x)}{\lambda_H (1 - s_H(x)) D(0; x) + (1 - \lambda_H) \bar{D}(x)} \tag{D.7}$$

and

$$\int_{s_H(x)}^{1} \frac{1}{\lambda_H (1 - s) D(0; x) + (1 - \lambda_H) \bar{D}(x)} w(s) ds = (1 - x)\phi, \tag{D.8}$$

as D.7 is determined by the fraction of good and bad firms in an investor portfolio with skill $s_H(x)$ in the high state, and the fraction in the integrand of the left hand side of D.8 is determined by the fraction of wealth an investor with skill s is allocated towards good firms in the high state.

In contrast, expression (22) is determined by the market clearing condition for firms with transparency $\omega \in [\omega_2(x), \omega_3(x)]$ in the low state. Therefore, it changes to

$$y^C(\omega) \equiv \frac{\xi\phi(1 - \lambda_L) - w(1 - \omega)(1 + \phi\xi\pi_H x)}{\xi\phi((1 - \lambda_L) + w(1 - \omega)\pi_L)} \qquad \omega \in [\omega_2(x), \omega_3(x)]\,. \tag{D.9}$$