NEW CLASSICAL MODELS AND UNOBSERVED AGGREGATES

Robert G. Trevor*

Reserve Bank of Australia

Research Discussion Paper

8601

February 1986

* I am grateful to Alan Blinder, Jeffrey Carmichael and John Taylor for helpful comments on earlier versions of the paper. The views expressed herein and any remaining errors are my own and should not be attributed to any of the above or to my employer.

ABSTRACT

The New Classical macroeconomic models seek to explain observed cyclical fluctuations in real activity by agents' reactions to nominal demand disturbances, about which they have incomplete information. While these models are driven by incomplete information about stochastic shocks, it is invariably assumed that agents have comprehensive information about the structure of the model, the associated probability distributions and the past values of all relevant variables. This paper analyses a simple New Classical model where agents cannot observe any lagged values of the true aggregate of an important variable - the money stock - but can see (lagged values of) an imperfectly measured estimate of this aggregate. Agents filter this noisy signal and all other available information to produce optimal estimates (i.e., rational expectations) of the current and lagged aggregate money stocks. Analytically tractable expressions are obtained from the stationary solution to the inherent recursive Kalman filtering problem. It is found that, under fairly general conditions, this filtering process induces serially correlated errors into the agents' expectations of the money stock, even though their expectations are rational. This serial dependence feeds through to generate a persistent response of real activity to demand shocks. Furthermore, it is shown that the correlation of unanticipated movements in the measured money stock with movements in real activity, may not be indicative of the relationship between activity and the true money stock. These results suggest that incomplete information about macroeconomic variables may explain some of the observed business cycle persistence and some of the instances of a lack of a sizeable measured correlation between real output and money supply innovations.

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1. Introduction

In the decade since Lucas's seminal work (Lucas (1972), Lucas (1973)), the themes of rational expectations and New Classical macroeconomics have occupied much attention in the literature. Many variants of Lucas's basic model have been generated; mainly in attempts to explain observed cyclical fluctuations in real variables by private agents' reactions to nominal demand disturbances, about which they have incomplete information. The maintained hypothesis of these models is that agents behave as if they know the model's structure (the equations, parameters and sufficient statistics for the distributions of the exogenous stochastic terms), and at least some lagged values of all relevant macroeconomic variables. Much of the debate generated by this literature, both at the theoretical and empirical level, has centered on the two main hypotheses derived from these New Classical models - the neutrality of fully anticipated monetary policy with respect to output, and the positive correlation between output fluctuations and unanticipated fluctuations in the money stock. An associated issue that has also received considerable attention, is the consistency of the theoretical models with observed business cycle persistence - the tendency of current demand shocks to affect future levels of output.

While these models are driven by the assumption of incomplete information, the information set assumed to be at the agents' disposal is unrealistically encompassing. In particular, the assumption that agents know at least some lagged values of all "appropriate" aggregates (that is the sum over all agents of variables that appear in the model) is very restrictive. In practice such conceptual definitions are rarely matched by observable measured aggregates. It is also well known that statistical measurement errors can be considerable. One indicator of divergence between measured and conceptual aggregates is the proliferation of measured aggregates that are often available. This is the case for a number of macroeconomic variables; it is especially true in the case of the money supply.[1]

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1. One approach to the problem of the optimal level of aggregation of the money supply (due to Barnett) attempts to sum value, rather than physical, units together via a Divisia index weighting scheme. For recent developments in this debate over the relationship between the various observed aggregates and the economic concept of "the money supply", see the papers by Barnett (1982), Cagan (1982), Fellner (1982), the comments in Goldfeld (1982) and Hamburger (1982), and the references contained therein.

This paper analyses the effects, in a New Classical model, of relaxing the informational assumption about past values of true aggregates. In particular, to examine the importance of the informational assumption, I assume that agents do not know (any of the) past values of the true money supply. The approach takes the simplest representative New Classical model (with no apparent source of persistence) and introduces an imperfectly measured monetary aggregate. The authorities are assumed to directly control the monetary base, which in turn influences both the true and measured monetary aggregates. Both the measured monetary aggregate and the monetary base are assumed to be observable (published) variables. Agents are assumed to have rational expectations of the unobservable monetary aggregate and its lagged values.

I show that, if the reduced form of the unobservable true monetary aggregate contains any of its own lagged values, the agents' optimal projection (i.e., rational expectation) of this aggregate introduces persistence into the output equation. That is, output responds to lagged, as well as contemporaneous, demand shocks. This result is due entirely to the assumption that the true aggregate is not observed. The partial information flow that agents observe allows them continually to update their estimates of past values of the true aggregate by a filtering process. The presence of a lagged value in the reduced form of the process generating the true money supply introduces serial dependence into the optimal filter. The difference between the true money supply and the agents' rational expectation of it (the expectational error) is then serially correlated. This leads to persistence in the output equation.

A second result of the analysis, under the maintained hypothesis of a New Classical model, is that empirical tests of the neutrality of <u>fully anticipated</u> monetary movements are unaffected[2] by the use of the imperfectly measured monetary aggregate. However, the measurement errors are important when testing the non-neutrality of <u>unanticipated</u> monetary shocks. In particular, for some parameterisations of the measurement error, tests based on the observed monetary data will fail to reject the false (null) hypothesis of zero correlation.

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2. That is, unaffected with respect to population statistics. The sample questions of efficiency and power are not considered here.

The remainder of the paper is split into six sections. The first presents the simple New Classical model that provides the framework for the analysis, formalises the relationships among the various monetary aggregates and derives some partial solutions. Section 3 sets up and solves the agents' filtering problem. This solution is used to solve the model in the following section. Tests of the New Classical hypotheses are then examined in Section 5. The current model is compared to those of Brunner, Cukierman and Meltzer (1980) and Boschen and Grossman (1982) in the next section. The final section presents some conclusions.

2. The Model

Although there is no one model that is representative of all variants of the New Classical theory, the central ideas can be captured in a very simple formulation of the demand and supply sides of the economy. In particular, I assume that the supply of aggregate output is determined by a log-linear (Lucas) supply function,

$$(1) \quad y_t^s = \alpha(p_t - {}_{t-1}p_t) + \varepsilon_t^s$$

where y_t^s is the log of the (proportionate) deviation of current aggregate supply from its natural rate, p_t is the log of the nominal price of a bundle of goods, ${}_{t-1}p_t = E_{t-1}p_t$ is the subjective expectation held by producers of (the log of) this price (based on information available at the end of period t-1) and ε_t^s is a random disturbance to supply which is independently and identically distributed $N(0,\sigma_s^2)$.

The hypothesis of rational expectations is an important component of all New Classical models. This hypothesis amounts to assuming that the subjective expectations operator ${}_{t-1}(.) = E_{t-1}(.)$ is equal to the true statistical expectation, conditional on the information set I_{t-1}, that is

$${}_{t-1}(.) = E_{t-1}(.) = E(.|I_{t-1})$$

Specific assumptions about I_{t-1} will be addressed later.

The demand for aggregate output is assumed to be proportional to the real money stock via the Quantity Theory equation,

$$(2) \quad y_t^d = M_t - p_t + \varepsilon_t^d$$

where y_t^d is the log of the (proportionate) deviation of current aggregate demand from its natural rate, M_t is the log of the true, unobservable, monetary aggregate and ϵ_t^d is a random velocity shock to demand which is assumed to be an $N(0,\sigma_d^2)$, independently and identically distributed variate.[3]

The specification of activity in the economy is closed by assuming that prices adjust so that all markets clear each period,

$$(3) \quad y_t = y_t^s = y_t^d$$

where y_t is the observed log of the (proportionate) deviation of real output from its natural rate.

On the nominal side of the model I distinguish between two aggregate money supply indexes. In addition to the unobservable true aggregate, M_t, there is $\hat{M}_t$, (the log of) an observable aggregate that measures the true one imperfectly.[4] Both these aggregates are influenced by the "monetary base", which the authorities are assumed to be able to control directly. B_t is (the log of) this observable variable. I make these distinctions for two reasons. First, in the New Classical spirit, the authorities cannot be assumed to have an informational advantage over private agents by controlling the true (unobservable) aggregate. Second, because of the central role of monetary information in this model, I do not wish to allow the authorities direct control over the measured aggregate without any control over the true one.

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3. This specification of activity clearly abstracts from a number of features found in some of the New Classical models. However, generalisations of this model to include intertemporal considerations such as real interest rate effects (as in Barro (1976)) or disaggregated activity (via the Phelps island paradigm as in Lucas (1975)) would only serve to complicate the algebra without adding to the particular information restriction being considered.

4. Hence this aggregate ($\hat{M}_t$) shall be referred to as the "measured" aggregate.

For what follows it will be convenient to define the following identities,

(4) $M_t = B_t + z_t$

(5) $\hat{M}_t = B_t + \hat{z}_t$

where B_t is (the log of) the monetary base, and z_t and $\hat{z}_t$ are respectively the true and measured "multipliers".

While this is a convenient terminology, and one that I shall use throughout the discussion, it should not necessarily be given a behavioral interpretation. In particular, there is no presumption of simple sum aggregation, or that the controlled variable, B_t, is necessarily narrower in definition than either of the two aggregates, M_t and $\hat{M}_t$.[5]

The monetary authorities are assumed to control the monetary base (B_t) via some general reaction function,

(6) $B_t = \theta B_{t-1} + \lambda y_{t-1} + \pi p_{t-1} + \rho \hat{z}_{t-1} + \tau_{t-1} z_{t-1} + \eta_t$

where η_t is an independently and identically distributed $N(0,\sigma_\eta^2)$ term.[6]

The true multiplier is assumed to be determined by the reduced form,

(7) $z_t = \varphi z_{t-1} + \chi y_{t-1} + \psi p_{t-1} + \omega B_{t-1} + \nu_t$

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5. Some readers may prefer to think of B_t as a "controlled" variable and z_t and $\hat{z}_t$ as the (logs of the) proportionate deviations of the true and measured aggregates from this controlled one, respectively.

6. The solution of this model will exhibit "policy neutrality" because of its New Classical structure. To this extent, one choice of (lagged) deterministic policy rule is as good as any other. A general formulation is used here to demonstrate the robust nature of the results.

where ν_t is independently and identically distributed as a $N(0,\sigma_\nu^2)$ variate.[7]

Contemporaneous variables have been explicitly excluded from the right-hand side of equations (6) and (7). This prevents the authorities from having any informational advantage over private agents. It also reflects the usual New Classical assumption that the monetary aggregate (that is important for private behavior) is independent of other contemporaneous variables.[8] Lagged variables are included because New Classical models almost always assume that the money supply process is dynamic - the simplest process would have $\theta = \varphi \neq 0$ and the other parameters set to zero.

The measured multiplier is assumed to be related to the true one by a generalised errors-in-variables equation,

$$(8) \quad \hat{z}_t = z_t + \beta y_t + \gamma p_t + \delta B_t + \mu_t$$

where μ_t is an independently and identically distributed $N(0,\sigma_\mu^2)$ error. This specification of the measurement error, $\hat{M}_t - M_t$, allows for both the conceptual error in the definition of the measured aggregate and the statistical errors inherent in its measurement. It is best thought of as a convenient parameterisation of a number of general possibilities, that is also analytically tractable.[9]

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7. More generally z_t could be thought of as being generated by a polynomial distributed lag model. All that is required for the results to follow is that the lag function $\varphi(.)$ have at least one nonzero parameter - that is, that there is at least one lagged value of z_t appearing in its own reduced form. An alternative way of expressing this requirement is that $\partial M_t/\partial M_{t-1} \neq 0$ for some value of i greater than zero.

8. This assumption is made so that the model stays within the New Classical framework. It does not affect the qualitative nature of the results in this paper.

9. In the case of $\beta = \gamma = \delta = 0$, the specification collapses to a more standard errors-in-variables situation of a white noise measurement error. When $\beta = \gamma = 0$, the measurement error also depends on lagged variables via the monetary base, but not on the contemporaneous values of other variables. Other restrictions on the values of these parameters can make the measurement error depend only on anticipated movements (apart from μ_t), or only on unanticipated movements or on some linear combination thereof.

In empirical applications one may wish to use less general parameterisations of equations (6), (7) and (8). For current purposes, it is important to demonstrate how such restrictions affect the results.

I assume that the information set, I_{t-1}, includes knowledge of the structure of the model (equations (1) through (8)), the values of the parameter set, and the sufficient statistics for the joint distribution of the stochastic disturbance terms. In addition, it is assumed to contain:

$$< y_i, p_i, B_i, \hat{M}_i, \hat{z}_i, \varepsilon^s_i, n_i >$$

where <.> indicates sequences of observations in the interval $(-\infty, t-1]$. It will not contain any variables dated at t and later. That is, I assume that agents can directly observe all past values of aggregate output, prices, the monetary base and the measured money supply. From this they can deduce past values of the measured money multiplier and two of the stochastic shocks. They cannot observe any past values of the true money supply, hence I_{t-1} will not contain:

$$< M_i, z_i, \varepsilon^d_i, \nu_i, \mu_i >$$

Notice that all observable variables (including the measured money supply) are revealed at the end of the period to which they pertain, as in most New Classical models. There is no notion of preliminary monetary information that is available contemporaneously as in Barro and Hercowitz (1980), Boschen and Grossman (1982) or King (1981). The analysis here will focus on permanent measurement errors and the role of anticipated/unanticipated money and not the temporary errors and perceived/unperceived money with which these authors are concerned. This other body of literature analyses models where the New Classical information set has been expanded to include contemporaneous monetary data. The current paper considers the effects of removing information (lagged values of the true money supply) from agents' information sets.

Substituting for supply and demand (from (1) and (2)) into the equilibrium condition (3) yields

$$\alpha(p_t - {}_{t-1}p_t) + \varepsilon^s_t = M_t - p_t + \varepsilon^d_t$$

from which

$$ {}_{t-1}p_t = {}_{t-1}M_t $$

can be deduced by taking $E_{t-1}(.)$ of both sides. Hence, by substituting back for ${}_{t-1}p_t$, we get

$$ (9) \quad p_t = {}_{t-1}M_t + (1/(1+\alpha))(M_t - {}_{t-1}M_t) + (1/(1+\alpha))\varepsilon_t^d - (1/(1+\alpha))\varepsilon_t^s $$

and therefore,

$$ (10) \quad y_t = (\alpha/(1+\alpha))(M_t - {}_{t-1}M_t) + (\alpha/(1+\alpha))\varepsilon_t^d + (1/(1+\alpha))\varepsilon_t^s $$

In order to solve these equations, an expression for the conditional expectation of the true money supply (${}_{t-1}M_t$) is required. In standard New Classical models all lagged variables are elements of the information set, so a simple application of the conditional expectations operator to both sides of equations (6) and (7) would yield the solution. This solution would have the property that the agents' expectational error, and hence output, are white noise disturbance terms. However, in the current model, no lagged values of the true monetary aggregate (or the true multiplier) are in the information set. Thus expressions for the conditional expectations of these unobserved lagged variables (${}_{t-1}M_{t-1}$ and ${}_{t-1}z_{t-1}$) are required before the solution of (9) and (10) can be found. It is to this problem (or at least the more general problem of finding ${}_tz_t$) that I now turn.

3. An Application of the Kalman Filter

At the end of period t, agents can update their information sets by observations on current output, prices, the measured money supply and the monetary base. (This information can also be used at the beginning of the next period to generate expectations for period t+1.) Agents cannot observe the true money supply (or the true money multiplier), but they can use this new information to update the expectations that they held at the beginning of the period, generating end of period estimates of the unobservable variables. The Kalman filter provides such an updating process. In this model it corresponds to the rational expectations solution. It is more easily derived by moving to a more compact notation.

Rewriting equation (7) in terms of the vector of information that becomes available at the end of period t, x_t, gives

(11) $z_t = \varphi z_{t-1} + \Phi x_{t-1} + v_t$

where $\Phi = [\chi, \psi, 0, \omega]$ and $x_{t-1} = [y_{t-1}, p_{t-1}, \hat{z}_{t-1}, B_{t-1}]'$

After substituting out the identities (4) and (5), equations (10), (9), (8) and (6) may be stacked,

(12) $x_t = \Gamma_1 z_t + \Gamma_{2\,t-1}z_t + \Gamma_{3\,t-1}z_{t-1} + \Gamma_4 x_{t-1} + \Delta_1\xi^o_t + \Delta_2\xi_t$

where the coefficient matrices are given in the Appendix and $\xi^o_t = [\eta_t, \epsilon^s_t]'$ and $\xi_t = [\epsilon^d_t, \mu_t]'$.

In this compact notation, it is assumed that $[v_t, \xi_t, \xi^o_t]'$ is independently and identically distributed $N(0,\Sigma)$ where $S = \mathrm{diag}(\sigma^2_v, \Omega, \Omega_0)$, $\Omega = \mathrm{diag}(\sigma^2_d, \sigma^2_\mu)$ and $\Omega_0 = \mathrm{diag}(\sigma^2_\eta, \sigma^2_s)$. The sequence $< x_i, \xi^o_i >$ is assumed to be contained in I_{t-1}, but no elements of the sequence $< z_i, v_i, \xi_i >$ are contained in I_{t-1}. Given these assumptions, it may be shown that

$$E(z_t|I_t) = E(z_t|I_{t-1}) + E\{(z_t - E(z_t|I_{t-1}))|(x_t - E(x_t|I_{t-1}))\}$$

where x_t is the vector of information that becomes available at the end of period t. This "recursive projection" formula (Sargent (1979, p.208)) reduces to

(13) ${}_t z_t = {}_{t-1}z_t + \kappa_t(x_t - {}_{t-1}x_t)$

where κ_t is the vector of Kalman filter coefficients given by[10]

(14) $\kappa_t = \mathrm{cov}\{(z_t - {}_{t-1}z_t),(x_t - {}_{t-1}x_t)\} \times \{\mathrm{var}(x_t - {}_{t-1}x_t)\}^{-1}$

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10. See Chow (1975) for a presentation of the theory of Kalman filtering.

The recursive set of equations for κ_t are

$$(15)\quad \kappa_t = {}_{t-1}\sigma_t^2 \, \Gamma_1' \, \{\Gamma_1 \, {}_{t-1}\sigma_t^2 \Gamma_1' + \Delta_1\Omega_0\Delta_1' + \Delta_2\Omega\Delta_2'\}^{-1}$$

$$(16)\quad {}_{t-1}\sigma_t^2 = \varphi \, {}_{t-1}\sigma_{t-1}^2 \varphi' + \sigma_v^2$$

$$(17)\quad {}_t\sigma_t^2 = (1 - \kappa_t\Gamma_1) \, {}_{t-1}\sigma_t^2$$

where ${}_{t-1}\sigma_t^2 = \text{var}(z_t - {}_{t-1}z_t)$ and ${}_t\sigma_t^2 = \text{var}(z_t - {}_t z_t)$. These results are derived in the Appendix.

Equations (13), (15), (16) and (17) define the resursive set of equations that optimally solve the agents' projection problem. If the model is covariance stationary, then this filter will approach a stationary solution. Assuming that the model's parameters satisfy stationarity conditions,[11] these equations may be solved to get a stationary solution for the Kalman filter weights.

Lemma: The stationary solution for κ_t is

$$(18)\quad \kappa = \sigma^2[(1/\sigma_d^2 - \beta/\sigma_\mu^2),\ (1/\sigma_d^2 - \gamma/\sigma_\mu^2),\ 1/\sigma_\mu^2,\ (-1/\sigma_d^2 - \delta/\sigma_\mu^2)]$$

where σ^2 is the stationary value for ${}_t\sigma_t^2$.

Proof: See the Appendix.

The vector of coefficients given in equation (18) contains the weights used to filter the unanticipated information that arrives at the end of period t, in order to calculate the best guess for the unobserved value of the true multiplier for the period just ended. Thus the agents' measure of the true money supply uses all available information, not just the measured aggregate, because they know that the unobservable measurement error is correlated with other observable variables. This projection in terms of observed variables may be transformed by using equation (12), into a relationship involving (observed linear combinations of) unobserved variables,

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11. The exact form of these stationarity restrictions will be seen later.

(19) $${}_{t}z_t = {}_{t-1}z_t + ((\sigma/\sigma_\mu)^2 + (\sigma/\sigma_d)^2)\,(z_t - {}_{t-1}z_t) + ((\sigma/\sigma_d)^2)\varepsilon_t^d + ((\sigma/\sigma_\mu)^2)\mu_t$$

The information that arrives at the end of period t reveals the sum $z_t + \varepsilon_t^d$, from equations (2) and (4), and the sum $z_t + \mu_t$, from equations (8) and (4). Equation (19) shows that this information, appropriately weighted by variance ratios, is all that agents require to optimally update the expectations that they held at the beginning of the period.

4. Persistence of Expectational Errors and Demand Shocks

The solutions to the equations at the end of Section 2 required expressions for ${}_{t-1}B_t$ and ${}_{t-1}z_t$. These can now be obtained by applying the expectations operator to equations (6) and (7) and using equation (19) to substitute out for ${}_{t-1}z_{t-1}$. The model may then be represented by the block-recursive, vector-autoregressive process

(20) $$\begin{bmatrix} y_t \\ p_t \\ {}_{t-1}\hat{z}_t \\ (\hat{z}_t - {}_{t-1}\hat{z}_t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_1 & a_1 \\ 1 & 1 & a_2 & a_2 \\ a_3 & a_4 & 0 & 0 \\ 0 & 0 & a_5 & a_6 \end{bmatrix} \begin{bmatrix} {}_{t-1}B_t \\ {}_{t-1}z_t \\ (B_t - {}_{t-1}B_t) \\ (z_t - {}_{t-1}z_t) \end{bmatrix} + \begin{bmatrix} a_1 & a_2 & 0 \\ a_2 & -a_2 & 0 \\ 0 & 0 & 0 \\ a_7 & a_8 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^d \\ \varepsilon_t^s \\ \mu_t \end{bmatrix}$$

and

(21) $$\begin{bmatrix} {}_{t-1}B_t \\ {}_{t-1}z_t \\ (B_t - {}_{t-1}B_t) \\ (z_t - {}_{t-1}z_t) \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ f_1 & f_2 & f_3 & f_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_8 \end{bmatrix} \begin{bmatrix} {}_{t-2}B_{t-1} \\ {}_{t-2}z_{t-1} \\ (B_{t-1} - {}_{t-2}B_{t-1}) \\ (z_{t-1} - {}_{t-2}z_{t-1}) \end{bmatrix} + \begin{bmatrix} c_5 & c_6 & c_7 \\ f_5 & f_6 & f_7 \\ 0 & 0 & 0 \\ -f_9 & 0 & -f_7 \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1}^d \\ \varepsilon_{t-1}^s \\ \mu_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}$$

where the elements of the coefficient matrices are given in the Appendix.

The conditions required for covariance stationarity of this system of equations are that the roots of the quadratic

$$0 = (c_1-r)(f_2-r) - c_2f_1$$

lie within the unit circle,[12] with the additional restriction that

$$|\varphi(1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2))| < 1$$

that is,

$$|\varphi({}_t\sigma_t^2)/({}_{t-1}\sigma_t^2)| < 1$$

It was the assumption of these three conditions that allowed a stationary solution to the Kalman filtering problem to be calculated. These stationarity assumptions are maintained for the remainder of the analysis.

The solutions for the left-hand side variables of (21), and hence (20), will generally be infinite moving averages of terms in $[\varepsilon^d_{t-1}, \varepsilon^s_{t-1}, \mu_{t-1}]'$ and $[\eta_t, \nu_t]'$. The first result of the model follows from this property. The unanticipated multiplier is not a white noise disturbance, as it would be in a standard New Classical model, but the infinite moving average process

$$(22)\quad (z_t - {}_{t-1}z_t) = \Sigma_i\varphi^i \{1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2)\}^i \times \{\nu_{t-i} - \varphi((\sigma/\sigma_\mu)^2)\mu_{t-1-i} - \varphi((\sigma/\sigma_d)^2)\varepsilon^d_{t-1-i}\}$$

where Σ_i is the sum from zero to infinity. Therefore, the error in the rational expectation of the true money stock is

$$(23)\quad (M_t - {}_{t-1}M_t) = \eta_t + \Sigma_i\varphi^i \{1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2)\}^i \times \{\nu_{t-i} - \varphi((\sigma/\sigma_\mu)^2)\mu_{t-1-i} - \varphi((\sigma/\sigma_d)^2)\varepsilon^d_{t-1-i}\}$$

Both of these expectational errors exhibit serial correlation. Yet it is traditionally thought that the assumption of rational expectations, and hence the application of the "orthogonality principle",[13] ensures white noise

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12. In the special case where the authorities adjust the monetary base only in response to lagged values of itself and output ($\pi=\rho=\tau=0$), these two conditions collapse to requiring that $|\theta|$ and $|\varphi+\psi|$ both be less than unity.

13. This is the term given by Sargent (1979, p.204) to the fact that the projection (rational expectation) error is orthogonal to every element of the information set, and therefore to the projection itself. Intuitively, if this were not true, the projection would not have utilised all the relevant information assumed available to the agents and in this sense would not be "rational".

expectational errors. However, this assumption only rules out serially correlated expectational errors when the lagged values of these errors are included in the information set. In this model M_{t-1} and z_{t-1} are not in the agents' information set; hence nor are the lagged values of these expectational errors. In such a situation this analysis shows that the assumption of rational expectations does not necessarily prevent the occurrence of serially correlated expectational errors.

Combining equations (20) and (22) gives the second result,

$$(23)\quad y_t = (1/(1+\alpha))\{\epsilon_t^s + \alpha\epsilon_t^d + \alpha\eta_t + \alpha\Sigma_i\varphi^i\ \{1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2)\}^i$$

$$\times\ \{\nu_{t-i} - \varphi((\sigma/\sigma_\mu)^2)\mu_{t-1-i} - \varphi((\sigma/\sigma_d)^2)\epsilon_{t-1-i}^d\}\}$$

That is, output contains an infinite moving average of demand disturbances, with geometrically declining weights. This persistence of demand shocks is due entirely to the unobservable nature of the true monetary aggregate. Agents only see partial information, from which they must filter out the "signal" from the "noise", to generate optimal guesses about the unobserved variables. The persistence does not depend on the parameterisation of the measurement error in equation (8) - that is part of the information that can be filtered out. It does depend on the appearance of some lagged value of the true multiplier in its own reduced form, equation (7).[14]

This result is for the stationary solution of the Kalman filtering problem. Outside of this stationary state, the weights in the moving average will vary each period as the filtering error variances change (equations (15), (16) and (17)). In such a situation, output will respond to demand shocks with long and variable lags. This confirms a conjecture of Lucas (1977):[15]

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14. This persistence result depends on some direct intertemporal linkage of the unobserved variables. In this model, such a linkage exists when φ is non-zero. In an expanded model where the aggregate price index is also measured imperfectly, the linkage could be provided by the parameter ψ in equation (7).

15. Emphasis in original.

These remarks do not, of course, explain <u>why</u> monetary effects work with long and variable lags. On this question little is known. It seems likely that the answer lies in the observation that a monetary expansion can occur in a variety of ways, depending on the way the money is "injected" into the system, with different price response implications depending on which way is selected. This would suggest that one should describe the monetary "state" of the economy as being determined by some <u>unobservable</u> monetary aggregate, loosely related to observed aggregates over short periods but closely related secularly.

5. Tests of the New Classical Hypotheses

The orthogonality principle and the assumption of covariance stationarity, may be used to show that[16]

$$\text{cov}\{{}_{t-1}B_t, (B_t - {}_{t-1}B_t)\} = 0 \qquad \text{cov}\{{}_{t-1}B_t, (z_t - {}_{t-1}z_t)\} = 0$$

$$\text{cov}\{{}_{t-1}z_t, (B_t - {}_{t-1}B_t)\} = 0 \qquad \text{cov}\{{}_{t-1}z_t, (z_t - {}_{t-1}z_t)\} = 0$$

$$(25) \quad \text{var}\{B_t - {}_{t-1}B_t\} = \sigma_\eta^2 \qquad \text{var}\{z_t - {}_{t-1}z_t\} = \varphi^2\sigma^2 + \sigma_\nu^2$$

$$\text{cov}\{(B_t - {}_{t-1}B_t), (z_t - {}_{t-1}z_t)\} = 0$$

The first hypothesis to arise from the New Classical models is that anticipated movements in the money supply are neutral with respect to output. Using the equation for output in (20) and the covariances (25), it is found that

$$(26) \quad \text{cov}\{y_t, E_{t-1}(M_t - M_{t-1})\} = -(\alpha/(1+\alpha))\varphi\{((\sigma/\sigma_\mu)^2)+((\sigma/\sigma_d)^2)\}$$

$$\times \{(1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2))(\varphi^2\sigma^2+\sigma_\nu^2) - \sigma^2\}$$

$$= 0$$

If an econometrician were to test this hypothesis using the measured money stock, the result would be[17]

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16. I make use of the result that σ^2 solves a particular quadratic equation given in the Appendix.

17. More correctly, tests should be based on partial covariances. However, in the cases considered here, the orthogonality principle ensures that the partial and simple covariances are the same.

$$(27)\quad cov\{y_t, E_{t-1}(\hat{M}_t - \hat{M}_{t-1})\} = -(\alpha/(1+\alpha))\varphi(1 + (\alpha\beta+\gamma)/(1+\alpha))$$

$$\times \{(1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2))(\varphi^2\sigma^2+\sigma_\nu^2) - \sigma^2\}$$

$$= 0$$

Equations (26) and (27) both use the result, shown in the Appendix, that σ^2 is the root of the quadratic in curly brackets.

Thus, in this model, the unobservable nature of the true money supply makes no difference to the testing of the neutrality hypothesis; anticipated movements in both the true aggregate and the measured one are neutral for output.[18]

The second New Classical hypothesis is that unanticipated movements in the money supply are positively correlated with fluctuations in output. For this model,

$$(28)\quad cov\{y_t,((M_t-M_{t-1}) - E_{t-1}(M_t-M_{t-1}))\} = (\alpha/(1+\alpha))(\sigma_\eta^2 + \varphi(\varphi-1)\sigma^2 + \sigma_\nu^2)$$

which is not necessarily positive but depends on the parameter φ, because the lagged value of the true money supply is not in agents' information sets. Testing this hypothesis with the measured money supply would give

$$(29)\quad cov\{y_t,((\hat{M}_t-\hat{M}_{t-1}) - E_{t-1}(\hat{M}_t-\hat{M}_{t-1}))\} = (1/(1+\alpha))((\beta-\gamma)/(1+\alpha))\sigma_s^2$$

$$+ (\alpha/(1+\alpha))\{((\alpha\beta+\gamma)/(1+\alpha))(\sigma_\eta^2 +\varphi^2\sigma^2+\sigma_\nu^2 +\sigma_d^2) + (1+\delta)\sigma_\eta^2 +\varphi^2\sigma^2+\sigma_\nu^2\}$$

where the sign depends on the parameterisation of the measurement error in equation (8). A sufficient condition for unanticipated fluctuations in the measured money supply to be positively correlated with output is that the measurement error depend positively on output ($\beta>0$), or on the monetary base ($\delta>0$), but not on prices ($\gamma=0$).

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18. Abel and Mishkin (1983) analyse tests of neutrality based on incorrectly specified information sets. They show, more generally, that exclusion of relevant variables (such as M_t) and inclusion of irrelevant variables (such as $\hat{M}_t$) will not of itself lead to the rejection of the null hypothesis of neutrality. These results follow from the orthogonality principle.

From a statistical point of view, if one is testing the null hypothesis of zero correlation against a two-sided alternative, the sign is not of major importance. However, it is likely that in the past some researchers may have "pre-tested" out results of a negative correlation between unanticipated fluctuations in the observed money supply and movements in output. Moreover, to the extent that certain parameterisations of the measurement error can produce measured correlations (29) much closer to zero than the true one (28), the use of the measured aggregate could lead to a failure to reject the (false) null hypothesis of zero correlation. This could explain some of the instances of a lack of a sizeable measured correlation between real output and money supply innovations that are found in the literature.[19]

6. Comparison with other Models

It was shown above that the assumption that agents could not observe any values of the true money supply produced persistence of demand shocks in the solution for the deviation of real output around its natural rate. This source of persistence is different from most other explanations found in the literature which rely on the adjustment of capital (Lucas (1975)) or labor (Sargent (1978)) to shocks (leading to persistence in the natural rate of output); or the adjustment of inventories (Blinder and Fischer (1981)) or staggered wage contracts (Taylor (1980)) to shocks (leading to persistence in the deviation of output around its natural rate). It does, however, have some similarities with the explanation put forward by Brunner, Cukierman and Meltzer (1980). (Hereafter referred to as "BCM").

In BCM's model, persistence in prices and unemployment is explained by postulating a particular structure for the stochastic demand and supply shocks that perturb the economy. It is assumed that each of these disturbance terms consists of a permanent (random walk) and a transitory (white noise) component. Agents have complete knowledge of the past values of all variables

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19. See, for example, the results reported in Mishkin (1982) and Mishkin (1983) when the lag length restrictions are relaxed.

and of the total shocks. They cannot, however, distinguish between the permanent and transitory components of these shocks. Under rational expectations, the solution to this filtering problem induces a persistent response of unemployment and prices. This property of their model rests on the demonstration by Muth (1960) that adaptive expectations are rational when the variable concerned is the sum of a random walk and white noise, with the individual components unobserved. BCM have subsequently used these results in Brunner, Cukierman and Meltzer (1983).

The current model can be shown to have some similarities to this basic mechanism of BCM's model. Observe that the equations of this model can be partitioned into two (simultaneous) blocks; one that contains only observable variables and disturbance terms (equations (1), (3), (5), and (6))), and one that also contains unobservable variables and disturbance terms (equations (2), (4), (7), and (8)). Rewriting equations (2) and (8) after substitution with the identity (4), yields

$$y_t = B_t - p_t + \iota_t$$

$$\hat{z}_t = \beta y_t + \gamma p_t + \delta B_t + \zeta_t$$

where ι_t and ζ_t are defined by

$$\iota_t = z_t + \varepsilon_t^d$$

$$\zeta_t = z_t + \mu_t$$

Let $\chi = \psi = \omega = 0$ and equation (7) becomes

$$z_t = \varphi z_{t-1} + \nu_t$$

This block of the model may now be seen to have a BCM (or at least a Muth (1960)) mechanism at work as $\varphi \to 1$.[20] Since agents know the lagged values of y_t, p_t, $\hat{z}_t$ and B_t, they can deduce the lagged values of ι_t and

20. When $\varphi = 1$, this version of the model is no longer covariance stationary and the previous results cannot be applied. The solution could be calculated by modifying the method outlined in Muth (1960) to utilise the information on z_t contained in both ι_t and ζ_t.

ζ_t. These "shocks" to demand and the measured multiplier consist of an unobservable, permanent (random walk) component, z_t, and unobservable, transitory (white noise) components, ϵ_t^d and μ_t.

Although the algebra of these two models are similar, their economic motivations are very different. The BCM result obtains in a general model with very specific assumptions about the intertemporal structure of the stochastic disturbance terms. The results in this essay do not depend on assumptions about the structure of the stochastic shocks that perturb the economy. They depend only on the existence of an unobserved (or imperfectly measured) aggregate variable, whose true values are partly determined by at least one of its own lagged values. It is a central tenet of this paper that this relaxation of the usual New Classical assumptions is one that is likely to be encounted in practice.

The recent paper of Boschen and Grossman (1982) (hereafter referred to as BG) also bears some relevance to the current analysis. Their paper presents a New Classical model in which the information set is assumed to include published official preliminary estimates of the <u>contemporaneous</u> money supply. These preliminary estimates are subsequently revised by the authorities as the true values become available. On the basis of some empirical observations, BG assume that the measurement error in the current estimate of the contemporaneous money stock is positively correlated with today's measurement error in last period's money stock. They also assume that each of these errors is serially independent and uncorrelated with the other disturbance terms in the model.

In their model BG <u>assume</u> that the authority's published estimates are always the rational expectations of the appropriate money stocks, given the relevant information sets. Using this, they show that the correlation in the revisions of these official estimates induces persistence into the output equation. Unfortunately, Mankiw, Runkle and Shapiro (1984) have shown, using BG's data, that these revisions are partly forecastable. Hence, the published estimates can not be rational expectations.

The current model, on the other hand, does not assume that the measured money stock is the optimal predictor of the true one, nor is there any contemporaneous information available to agents. Moreover, the analysis is concerned with measurement errors that involve a (permanent) conceptual component rather than just a (temporary) statistical one. These points aside, there is a similarity in the mechanisms generating persistence of demand shocks in the solutions for output. However, in the BG case this mechanism is implicit in the assumption of positive covariance mentioned above, while in the current model such covariances are determined by the solution. To formalise a comparison, the current model may be re-interpreted in the BG paradigm. That is, assume that the authorities publish ${}_{t-1}M_t$ and ${}_{t-1}M_{t-1}$ during period t as their preliminary estimate of the contemporaneous money stock and their revised estimate of last period's money stock, respectively.[21] Now consider the solution for the error in today's estimate of the contemporaneous money stock,

$$(M_t - {}_{t-1}M_t) = \varphi\{(1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2)(z_{t-1} - {}_{t-2}z_{t-1})$$

$$- ((\sigma/\sigma_\mu)^2)\mu_{t-1}-((\sigma/\sigma_d)^2)\varepsilon^d_{t-1}\} + \eta_t + \upsilon_t$$

Application of equation (19) gives the error in today's estimate of last period's money stock as

$$(M_{t-1} - {}_{t-1}M_{t-1}) = (1-((\sigma/\sigma_\mu)^2) - ((\sigma/\sigma_d)^2)(z_{t-1} - {}_{t-2}z_{t-1})$$

$$- ((\sigma/\sigma_\mu)^2)\mu_{t-1} - ((\sigma/\sigma_d)^2)\varepsilon^d_{t-1}$$

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21. Note that in my notation the information set available during period t is I_{t-1}, not I_t as in BG's notation. The re-interpretation of the model allows BG's perceived/unperceived money distinction to be addressed. It is not, however, a very good interpretation because it implies an inconsistent monetary authority - why publish both M_t and ${}_tM_t$ at the end of period t, when ${}_tM_t$ is the optimal measure?

Provided $\varphi > 0$, these two errors will be positively correlated (the unconditional covariance is $\varphi\sigma^2$) as BG assume in their model. To this extent, these results endogenise BG's assumptions. However, the same two equations also predict that these errors will be correlated with (lagged values of) the demand shocks and will not be serially independent. This suggests that BG's assumptions are mutually inconsistent.

The "persistence" of demand shocks in the solution for output in the current model and the BG model, and in the solution for unemployment in the BCM model, is of a special character. In each case, the relevant variable contains a moving average of demand disturbances and yet is serially uncorrelated in the usual unconditional sense.[22] To illustrate this property, consider the case of the current model.

Recall from equation (23) that the solution for the error in the agents' expectation of the money stock is an infinite moving average of demand disturbances. As mentioned before, these errors are serially correlated in the usual sense[23] - indeed they are generated by a first order autoregressive process. From equation (10), output is just the sum of this expectational error and two white noise disturbance terms. However, even though output is a function of this same infinite moving average term (equation (24)), the (unconditional) covariance of output and lagged output (and hence, the serial correlation coefficient) is zero. Mathematically, this is because the weights of the moving average are terms in σ^2, which solves a particular quadratic equation given in the Appendix.[24] Intuitively, since output is an observed variable its unanticipated values must be serially independent (in the unconditional sense) by the orthogonality principle. Given the Lucas supply specification (of equation (1)), output must also be serially independent in the usual sense.[25]

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22. This abstracts from the exogenous source of persistence (a(L)) in the BG model and from the supply shocks in the BCM model.

23. It may be shown that $\text{cov}\{(M_t - {}_{t-1}M_t),(M_{t-1} - {}_{t-2}M_{t-1})\} = \varphi\sigma^2$. A similar result holds for the unobservable permanent shocks in the BCM model.

24. In the BCM model λ has this property.

25. Since output and unanticipated output are equivalent. This result also holds for output in the BG model (abstracting from the exogenous a(L) term) and for unemployment in the BCM model.

Nevertheless, demand shocks do persist in the solution for output.[26] Consider an experiment where drawings are made from the distributions of the stochastic shocks so that a "sample" $< y_i, p_i, B_i, \hat{M}_i >$ from t_o to T is is generated. Now generate a second sample using the same sequence of drawings for the stochastic shocks as before, except that for some i=t* (between t_o and and T) replace either ν_i, μ_i or ε^d_i by a new drawing from the relevant population. A comparison of the resulting sequences for output in these two samples will show that the effects of a demand shock (i.e. a different drawing) will persist for many periods beyond t*.

BCM note that their model has this property and suggest that the relevant serial correlation coefficient is one that is conditional on the value of some past shock. In the current model, conditional covariances such as $\mathrm{cov}\{y_t, y_{t-1} | \varepsilon^d_{t-2}\}$ are non-zero.[27] Perhaps a more relevant measure is the serial correlation of the deviations of output around its complete information level (the level that would pertain with a zero measurement error). In the current model these deviations have a first order serial correlation coefficient of $\varphi^2[1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2)]^2$.

7. Conclusions

In the context of a New Classical model with an imperfectly measured monetary aggregate, it has been shown that a flow of partial information can induce persistence into the agents' expectational errors and hence into output. The partial information flow allows agents continually to improve their best guesses about the unobservable true monetary aggregate, but (even over time) never fully reveals the true values. The only requirement is that the unobservable true aggregate depends (directly) on at least one of its own lagged values.

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26. Current demand shocks "Granger-cause" future output. This is analogous to the bivariate transfer function model in time series analysis (where the leads and lags between the white noise residuals of two ARIMA models are modelled). See, for example, Granger and Newbold (1977). Sargent (1979, p. 256) points out that serial correlation and business cycle persistence are not analagous. Indeed, output displays very little serial correlation of an order that could be associated with business cycles. This distinction clearly deserves more attention in the literature.

27. Note, however, that in their paper BCM neglect to subtract the conditional means when doing the covariance calculation. The correct conditional covariance will not depend on the value of the conditioning shock. Their discussion about the size of this shock relative to its variance is, therefore, incorrect. See Brunner, Cukierman and Meltzer (1980, section 5.2).

The resulting serial correlation in agents' expectational errors is perfectly consistent with rational expectations. Indeed, there is a sense in which the model's mechanism is a variant of Muth's (1960) result that adaptive expectations may be rational. This was shown by a comparison of the model with that of Brunner, Cukierman and Meltzer (1980).

Given the New Classical assumptions, the use of an imperfectly measured monetary aggregate was shown to not (of itself) lead to the rejection of the (null) hypothesis of neutrality of anticipated monetary disturbances for output. However, the use of these data to test the correlation between output and unanticipated monetary disturbances, was shown to lead to a failure to reject the false (null) hypothesis of zero correlation for some parameterisations of the measurement error.

While these results have been derived under the specific assumption that the true monetary aggregate is unobservable, they may be relevant for other situations. For example, it would not be unrealistic to treat the true aggregate price index as an unobservable variable. Under certain conditions, this assumption could produce similar results to those derived above.

This analysis illustrates the need to examine the restrictive informational assumptions that are made in the rational expectations literature. Further research in this area is particularly important because normally only joint hypotheses of New Classical models can be subjected to empirical testing. The controversial nature of such tests can be mitigated to some extent if the effects of different informational assumptions have been clarified.

APPENDIX

The coefficient matrices of the compacted model given in equation (12) are

$$\Gamma_1 = (1/(1+\alpha))[\alpha, 1, (1+\alpha+\alpha\beta+\gamma), 0]'$$

$$\Gamma_2 = (1/(1+\alpha))[-\alpha, \alpha, \alpha(\gamma-\beta), 0]'$$

$$\Gamma_3 = [0, \tau, \tau(\gamma+\delta), \tau]'$$

$$\Gamma_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda & \pi & \rho & \theta \\ \lambda(\gamma+\delta) & \pi(\gamma+\delta) & \rho(\gamma+\delta) & \theta(\gamma+\delta) \\ \lambda & \pi & \rho & \theta \end{bmatrix}$$

$$\Delta_1 = (1/(1+\alpha)) \begin{bmatrix} \alpha & 1 \\ 1 & -1 \\ \delta+\alpha\delta+\alpha\beta+\gamma & \beta-\gamma \\ 1+\alpha & 0 \end{bmatrix}$$

$$\Delta_2 = (1/(1+\alpha)) \begin{bmatrix} \alpha & 0 \\ 1 & 0 \\ \alpha\beta+\gamma & 1 \\ 0 & 0 \end{bmatrix}$$

To solve for the recursive set of equations that determine the vector of Kalman filter weights, use equations (11) and (12) to give

(A1) $(z_t - {}_{t-1}z_t) = \varphi(z_{t-1} - {}_{t-1}z_{t-1}) + \upsilon_t$

and

(A2) $(x_t - {}_{t-1}x_t) = \Gamma_1(z_t - {}_{t-1}z_t) + \Delta_1\xi_t^0 + \Delta_2\xi_t$

Using these two equations to evaluate the variance and covariance terms needed in (14) yields

$$\text{cov}\{(z_t - {}_{t-1}z_t), (x_t - {}_{t-1}x_t)\} = \text{var}(z_t - {}_{t-1}z_t)\Gamma_1'$$

and

$$\text{var}(x_t - {}_{t-1}x_t) = \Gamma_1(\text{var}(z_t - {}_{t-1}z_t))\Gamma_1' + \Delta_1\text{var}(\xi_t^0)\Delta_1' + \Delta_2\text{var}(\xi_t)\Delta_2'$$

Denoting $\mathrm{var}(z_t - {}_{t-1}z_t) = {}_{t-1}\sigma_t^2$, and substituting these expressions into (14) gives

$$(15)\quad \kappa_t = {}_{t-1}\sigma_t^2\Gamma_1'\{\Gamma_{1\,t-1}\sigma_t^2\Gamma_1' + \Delta_1\Omega_0\Delta_1' + \Delta_2\Omega\Delta_2'\}^{-1}$$

as the expression for Kalman coefficients. Taking the variance of both sides of equation (A1) results in

$$\mathrm{var}(z_t - {}_{t-1}z_t) = \varphi\mathrm{var}(z_{t-1} - {}_{t-1}z_{t-1})\varphi' + \mathrm{var}(\nu_t)$$

or, more compactly

$$(16)\quad {}_{t-1}\sigma_t^2 = \varphi_{t-1}\sigma_{t-1}^2\,\varphi' + \sigma_\nu^2$$

Using the filtering formula (13) to substitute out for ${}_tz_t$ shows that

$$(z_t - {}_tz_t) = (z_t - {}_{t-1}z_t) - \kappa_t(x_t - {}_{t-1}x_t)$$

Taking variances of both sides and noting the definition in (14) gives

$$\mathrm{var}(z_t - {}_tz_t) = \mathrm{var}(z_t - {}_{t-1}z_t) - \kappa_t\mathrm{cov}\{(x_t - {}_{t-1}x_t),(z_t - {}_{t-1}z_t)\}$$

or in compact notation

$$(17)\quad {}_t\sigma_t^2 = (1 - \kappa_t\Gamma_1)_{t-1}\sigma_t^2$$

In order to derive the covariance stationary solution to the recursive Kalman filter given by equations (13), (15), (16) and (17), combine equations (15) and (17) to get,

$${}_t\sigma_t^2 = {}_{t-1}\sigma_t^2 - {}_{t-1}\sigma_t^2\Gamma_1'(\Gamma_{1\,t-1}\sigma_t^2\Gamma_1' + \Delta_1\Omega_0\Delta_1' + \Delta_2\Omega\Delta_2')^{-1}\Gamma_{1\,t-1}\sigma_t^2$$

and then use (16) to yield,

(A3) $\varphi_{t-1}\sigma^2_{t-1}\varphi' + \sigma^2_\nu - {}_t\sigma^2_t = (\varphi_{t-1}\sigma^2_{t-1}\varphi' + \sigma^2_\nu) \times$

$$\Gamma_1'\{\Gamma_1(\varphi_{t-1}\sigma^2_{t-1}\varphi'+\sigma^2_\nu)\Gamma_1'+\Delta_1\Omega_0\Delta_1'+\Delta_2\Omega\Delta_2'\}^{-1}\Gamma_1(\varphi_{t-1}\sigma^2_{t-1}\varphi'+\sigma^2_\nu)$$

The stationary solution for ${}_t\sigma^2_t$ is then the solution to the quadratic in σ^2,

(A4) $\varphi\sigma^2\varphi' + \sigma^2_\nu - \sigma^2 = (\varphi\sigma^2\varphi' + \sigma^2_\nu)\Gamma_1' \times$

$$\{\Gamma_1(\varphi\sigma^2\varphi' + \sigma^2_\nu)\Gamma_1' + \Delta_1\Omega_0\Delta_1' + \Delta_2\Omega\Delta_2'\}^{-1}\Gamma_1(\varphi\sigma^2\varphi' + \sigma^2_\nu)$$

Denote the matrix $\{\Gamma_1(\varphi\sigma^2\varphi' + \sigma^2_\nu)\Gamma_1' + \Delta_1\Omega_0\Delta_1' + \Delta_2\Omega\Delta_2'\}$ by V. This is the symmetric matrix $\mathrm{var}(x_t - {}_{t-1}x_t)$ and by using the definitions in equation (12) its lower triangular elements are found to be $(1+\alpha)^{-2}$ times the following entries,

$$V_{11} = \alpha^2 W + \alpha^2\sigma^2_\eta + \sigma^2_s + \alpha^2\sigma^2_d$$

$$V_{21} = \alpha W + \alpha\sigma^2_\eta - \sigma^2_s + \alpha\sigma^2_d$$

$$V_{22} = W + \sigma^2_\eta + \sigma^2_s + \sigma^2_d$$

$$V_{31} = \alpha(1+\alpha+\alpha\beta+\gamma)W + (\beta-\gamma)\sigma^2_s + \alpha(\delta+\alpha\delta+\alpha\beta+\gamma)\sigma^2_\eta + \alpha((\alpha\beta+\gamma))\sigma^2_d$$

$$V_{32} = (1+\alpha+\alpha\beta+\gamma)W - (\beta-\gamma)\sigma^2_s + (\delta+\alpha\delta+\alpha\beta+\gamma)\sigma^2_\eta + ((\alpha\beta+\gamma))\sigma^2_d$$

$$V_{33} = (1+\alpha+\alpha\beta+\gamma)^2 W + (\beta-\gamma)^2\sigma^2_s + (\delta+\alpha\delta+\alpha\beta+\gamma)^2\sigma^2_\eta + ((\alpha\beta+\gamma))^2\sigma^2_d + (1+\alpha)^2\sigma^2_\mu$$

$$V_{41} = \alpha(1+\alpha)\sigma^2_\eta$$

$$V_{42} = (1+\alpha)\sigma^2_\eta$$

$$V_{43} = (1+\alpha)(\delta+\alpha\delta+\alpha\beta+\gamma)\sigma^2_\eta$$

$$V_{44} = (1+\alpha)^2\sigma^2_\eta$$

where $W = \varphi^2\sigma^2 + \sigma_\upsilon^2$. The inverse of V may be found by a sequence of elementary row operations. Its elements are $\{\sigma_d^2\sigma_\mu^2 + W(\sigma_d^2 + \sigma_\mu^2)\}^{-1}$ times the entries,

$$V^{11} = (1/\sigma_s^2)\{\sigma_d^2\sigma_\mu^2 + W(\sigma_d^2 + \sigma_\mu^2)\} + \sigma_\mu^2 + \beta^2\sigma_d^2 + (1+\beta)^2 W$$

$$V^{21} = -(\alpha/\sigma_s^2)\{\sigma_d^2\sigma_\mu^2 + W(\sigma_d^2 + \sigma_\mu^2)\} + \sigma_\mu^2 + \beta\gamma\sigma_d^2 + (1+\beta)(1+\gamma)W$$

$$V^{22} = (\alpha^2/\sigma_s^2)\{\sigma_d^2\sigma_\mu^2 + W(\sigma_d^2 + \sigma_\mu^2)\} + \sigma_\mu^2 + \gamma^2\sigma_d^2 + (1+\gamma)^2 W$$

$$V^{31} = -\beta\sigma_d^2 - (1+\beta)W$$

$$V^{32} = -\gamma\sigma_d^2 - (1+\gamma)W$$

$$V^{33} = \sigma_d^2 + W$$

$$V^{41} = -\sigma_\mu^2 + \beta\delta\sigma_d^2 - (1+\beta)(1-\delta)W$$

$$V^{42} = -\sigma_\mu^2 + \gamma\delta\sigma_d^2 - (1+\gamma)(1-\delta)W$$

$$V^{43} = -\delta\sigma_d^2 + (1-\delta)W$$

$$V^{44} = (1/\sigma_\eta^2)\{\sigma_d^2\sigma_\mu^2 + W(\sigma_d^2 + \sigma_\mu^2)\} + \sigma_\mu^2 + \delta^2\sigma_d^2 + (1-\delta)^2 W$$

Substituting back into equation (A4) gives,

$$\text{(A5)} \quad (\varphi^2-1)\sigma^2 + \sigma_\upsilon^2 = \{(\varphi^2\sigma^2 + \sigma_\upsilon^2)^2(\sigma_\mu^2 + \sigma_d^2)\} \times \{\sigma_d^2\sigma_\mu^2 + (\varphi^2\sigma^2 + \sigma_\upsilon^2)(\sigma_\mu^2 + \sigma_d^2)\}^{-1}$$

which is the quadratic in σ^2,

$$\text{(A6)} \quad 0 = \varphi^2(\sigma_\mu^2 + \sigma_d^2)(\sigma^2)^2 - \{(\varphi^2-1)\sigma_d^2\sigma_\mu^2 - \sigma_\upsilon^2(\sigma_\mu^2 + \sigma_d^2)\}\sigma^2 - \sigma_\upsilon^2(\sigma_d^2\sigma_\mu^2)$$

The stationary solution for σ^2 is then the positive root of the solution[28]

$$\text{(A7)} \quad \sigma^2 = \{2\varphi^2(\sigma_\mu^2 + \sigma_d^2)\}^{-1} \{(\varphi^2 - 1)\sigma_d^2\sigma_\mu^2 - \sigma_\upsilon^2(\sigma_\mu^2 + \sigma_d^2)$$

$$\pm\sqrt{[(\varphi+1)^2\sigma_d^2\sigma_\mu^2 + \sigma_\upsilon^2(\sigma_\mu^2 + \sigma_d^2)][(\varphi-1)^2\sigma_d^2\sigma_\mu^2 + \sigma_\upsilon^2(\sigma_\mu^2 + \sigma_d^2)]}\}$$

Given this stationary solution for σ^2, the stationary solution for the Kalman weights may be found by substituting into equation (15)

$$\kappa_t = \{\varphi^2\sigma^2 + \sigma_\nu^2\}[\sigma_d^2\sigma_\mu^2 + (\varphi^2\sigma^2 + \sigma_\nu^2)(\sigma_\mu^2 + \sigma_d^2)]^{-1}$$

$$\times\ [(\sigma_\mu^2 - \beta\sigma_d^2),\ (\sigma_\mu^2 - \gamma\sigma_d^2),\ \sigma_d^2,\ (-\sigma_\mu^2 - \delta\sigma_d^2)]$$

Judicious use of equations (A5) and (A6) reduces this to

$$\text{(A8)} \quad \kappa_t = \sigma^2\{\sigma_d^2\sigma_\mu^2\}^{-1}[(\sigma_\mu^2 - \beta\sigma_d^2),(\sigma_\mu^2 - \gamma\sigma_d^2),\sigma_d^2,(-\sigma_\mu^2 - \delta\sigma_d^2)]$$

which proves the Lemma in the text. Substituting these weights into equation (13) and using equation (A2) to substitute out the terms in y_t, p_t, $\hat{z}_t$ and B_t gives equation (19).

The elements of the coefficients matrices for equations (20) and (21) are

$a_1 = \alpha/(1+\alpha)$ $\qquad$ $a_2 = 1/(1+\alpha)$

$a_3 = \delta + \gamma$ $\qquad$ $a_4 = 1 + \gamma$

$a_5 = \delta + (\alpha\beta+\gamma)/(1+\alpha)$ $\qquad$ $a_6 = 1 + (\alpha\beta+\gamma)/(1+\alpha)$

$a_7 = (\alpha\beta+\gamma)/(1+\alpha)$ $\qquad$ $a_8 = (\beta-\gamma)/(1+\alpha)$

$c_1 = \theta + \pi + \rho(\delta+\gamma)$ $\qquad$ $f_1 = \omega + \psi$

$c_2 = \tau + \pi + \rho(1+\gamma)$ $\qquad$ $f_2 = \varphi + \psi$

$c_3 = \theta + \lambda(\alpha/(1+\alpha)) + \pi(1/(1+\alpha))$ $\qquad$ $f_3 = \omega + \chi(\alpha/(1+\alpha)) + \psi(1/(1+\alpha))$

$\quad + \rho(\delta+\beta(\alpha/(1+\alpha)) + \gamma(1/(1+\alpha)))$

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28. From the coefficients of the quadratic in A(6), the product of the roots must be negative. This solution thus yields a unique, positive (real) value for σ^2.

$c_4 = \tau + \lambda(\alpha/(1+\alpha)) + \pi(1/(1+\alpha))$
$+ \rho(1+\beta(\alpha/(1+\alpha)) + \gamma(1/(1+\alpha)))$
$- \tau(1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2))$

$\varepsilon_4 = \varphi + \chi(\alpha/(1+\alpha)) + \psi(1/(1+\alpha))$
$- \varphi(1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2))$

$c_5 = \lambda(\alpha/(1+\alpha)) + \pi(1/1+\alpha))$
$+ \rho(\beta(\alpha/(1+\alpha)) + \gamma(1/(1+\alpha)))$
$+ \tau((\sigma/\sigma_d)^2)$

$\varepsilon_5 = \chi(\alpha/(1+\alpha)) + \psi(1/(1+\alpha))$
$+ \varphi((\sigma/\sigma_d)^2)$

$c_6 = (\lambda-\pi)(1/(1+\alpha))$
$+ \rho(\beta-\gamma)(1/(1+\alpha))$

$\varepsilon_6 = (\chi-\psi)(1/(1+\alpha))$

$c_7 = \rho + \tau((\sigma/\sigma_\mu)^2)$

$\varepsilon_7 = \varphi((\sigma/\sigma_\mu)^2)$

$\varepsilon_8 = \varphi(1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2))$

$\varepsilon_9 = \varphi((\sigma/\sigma_d)^2)$

It only remains to show that σ^2 solves the quadratic in equations (26) and (27). This quadratic is

$$\{1-((\sigma/\sigma_\mu)^2)-((\sigma/\sigma_d)^2)\}(\varphi^2\sigma^2 + \sigma_v^2) - \sigma^2$$
$$= \{1/(\sigma_\mu^2\sigma_d^2)\}\{(\sigma_d^2\sigma_\mu^2 - \sigma^2(\sigma_\mu^2 + \sigma_d^2))(\varphi^2\sigma^2 + \sigma_v^2) - \sigma^2(\sigma_d^2\sigma_\mu^2)\}$$
$$= 0$$

by equation (A6).

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