



Reserve Bank of Australia

RESEARCH  
DISCUSSION  
PAPER

---

**Assessing Some Models  
of the Impact of Financial  
Stress upon Business  
Cycles**

*Adrian Pagan and  
Tim Robinson*

RDP 2011-04

---

# **ASSESSING SOME MODELS OF THE IMPACT OF FINANCIAL STRESS UPON BUSINESS CYCLES**

Adrian Pagan\* and Tim Robinson\*\*

Research Discussion Paper  
2011-04

December 2011

\*University of Sydney

\*\*Economic Research Department, Reserve Bank of Australia

The views expressed herein are those of the authors and are not necessarily those of the Reserve Bank of Australia. Mardi Dungey and Jonathan Kearns gave us useful comments on an earlier version, while Richard Swain drew our attention to some issues with the simulations.

Authors: [adrian.pagan at domain sydney.edu.au](mailto:adrian.pagan@sydney.edu.au) and [robinsont at domain  
rba.gov.au](mailto:robinsont@rba.gov.au)

Media Office: [rbainfo@rba.gov.au](mailto:rbainfo@rba.gov.au)

## Abstract

In the wake of the global financial crisis a considerable amount of research has focused on integrating financial factors into macroeconomic models. Two common approaches for doing so include the financial accelerator and collateralised lending, examples of which are Gilchrist, Ortiz and Zakrajšek (2009) and Iacoviello (2005). This paper proposes that two useful ways to evaluate such models are by focusing on their implications for business cycle characteristics and whether the models can match several stylised facts about the impact of financial conditions. One of these facts is that credit crises produce long-duration recessions. We find that while in the Gilchrist *et al* (2009) model financial factors can impact on particular cycles, they do little change to the average cycle characteristics. Some, but not all, of the stylised facts are captured by the model.

JEL Classification Numbers: E13, E32, E44, E51

Keywords: financial crises, business cycles

## Table of Contents

1.	Introduction	1
2.	Model Designs	2
2.1	The Core Macroeconomic Model	2
2.2	Expenditure Components Influenced by Financial Factors	3
2.2.1	Fixed investment	4
2.2.2	Residential investment	5
2.2.3	Consumption	6
2.3	The Supply of Credit	7
3.	Evidence of Financial Effects on Aggregate Activity	9
4.	Two Models with Financial-Real Linkages	10
4.1	The Gilchrist <i>et al</i> (2009) (GOZ) Augmentation of the SW Model	10
4.2	The Iacoviello (2005) Model	11
5.	Business Cycle Characteristics of the Models	12
5.1	The GOZ Model	12
5.2	The Iacoviello Model	13
6.	Correspondence of GOZ Model Business Outcomes with Stylised Facts	14
6.1	Credit and Output Growth Following a Financial Crisis	15
6.2	Recessions and Financial Crises	16
6.3	The Duration of a Recession and the Size of the External Finance Premium	18
6.4	Credit Growth and Recession Prediction	19
6.4.1	Relative performance of investment prediction	21
7.	Correspondence of Iacoviello Model Growth Cycle Outcomes with Stylised Facts	23

8. Conclusion	25
Appendix A: Derivation of Debt Growth Equation	26
References	27

# ASSESSING SOME MODELS OF THE IMPACT OF FINANCIAL STRESS UPON BUSINESS CYCLES

Adrian Pagan and Tim Robinson

## 1. Introduction

The global financial crisis (GFC) has led to a consideration of how one models the connections between financial stress and the business cycle. These interlinkages currently are the focus of a great deal of research and many models and analyses have emerged that aim to elucidate these relationships, for example Christensen and Dib (2008), Greenlaw *et al* (2008), Beneš *et al* (2009), Gilchrist *et al* (2009), Liu, Wang and Zha (2009) and Gertler and Kiyotaki (2010). These papers deal with a number of issues such as credit availability, collateral and the role of ‘animal spirits’ in initializing and propagating cycles. Questions that naturally arise concern: the size of the financial effects upon cycles in activity, how models can be designed with financial-real linkages and whether such models might be usefully employed to predict recessions. This paper aims to provide a framework to look at such questions and more generally to assess these models.

In Section 2 of the paper we review some of the major contributions to the literature that have addressed extended financial-real linkages over and above those coming from interest rates and monetary factors, which have always been present in conventional models. We outline the various strategies that have been employed, and discuss whether they influence the price or quantity of credit available to agents. We subsequently select two examples of the different strategies which seem to have enjoyed some success – Gilchrist *et al* (2009) (termed GOZ hereafter) and Iacoviello (2005) – and ask what characteristics of the business or growth cycles are altered by the financial factors in these models. Neither paper directly addresses this issue. Both papers report impulse responses with and without financial factors. In addition GOZ present a decomposition of the level and volatility of the transitory component of GDP into contributions from various identified shocks. One difficulty with the latter is that in this decomposition, output must be fully explained by the model shocks, that is, there is no residual term. Consequently, it is always unclear in such decompositions whether a shock accords with its label or is simply a residual. Our approach is to additionally assess

the models on their ability to capture the length, duration and other important characteristics of the business or growth cycles.

For this purpose we assemble some ‘stylised facts’ relating to recessions and the role of credit. These are drawn from a number of sources, but principally from the work of the International Monetary Fund (IMF) reported in the *World Economic Outlook*, particularly the April 2009 release. Because many of the measures the IMF use emphasise turning points in economic activity, we need a method to locate those. Therefore we adopt the method set out in Harding and Pagan (2002) for locating turning points, which has close connections with the NBER dating of business cycles. We find that, while at least one of the models can replicate some of the features set out in the IMF work, it is clear that neither paper is able to fully capture the effects of financial fragility upon recessions.

## **2. Model Designs**

It is useful to think about models to handle financial conditions in two stages. First, a ‘core’ model needs to be specified that details how expenditures are determined. Second, this is augmented with a sub-model involving the financial sector and showing how the latter impacts upon financial variables in the core model. This augmentation generally involves the introduction of a financial intermediary (FI) which responds to the demand for credit by supplying it. The FI may be introduced explicitly and a detailed description is given of its operation, although often only a simple summary of what governs the demand for and supply of credit is provided.

### **2.1 The Core Macroeconomic Model**

The selection of a core model will always be controversial and a range of possibilities exist, which are described in Fukač and Pagan (2010). Some, for example Muellbauer (2010), seem to have a preference for what have been termed second generation models in Fukač and Pagan (2010). These provide a set of equations describing macroeconomic outcomes which are guided by theory to varying degrees, rather than being explicitly based on, say, optimizing behaviour by agents. This greater flexibility may allow important features of the economy or the data to be incorporated more easily.

Alternatively, fourth generation or Dynamic Stochastic General Equilibrium (DSGE) models place more weight on being consistent with theory. A popular core model for quite a few macroeconomic investigations has been the DSGE model of Smets and Wouters (2007) (termed SW hereafter). This model describes the determination of consumption, investment, wages, inflation, monetary policy and the supply side, and is derived from optimizing behaviour by agents. There are, however, clearly missing items in the model that are likely to be important to macroeconomic outcomes, for example government expenditure is strictly exogenous. Each of the structural equations for consumption, investment, the price of capital, inflation and wages have effects from expectations about the future as well as past events ( $z_{t-1}$ ) and other model variables ( $w_t$ ), that is, they have the structure

$$z_t = \phi_1 z_{t-1} + \phi_2 E_t z_{t+1} + \phi_3 w_t.$$

In some cases  $\phi_1 + \phi_2 = 1$ . Identities are also present and supply is constrained by a production function.

## 2.2 Expenditure Components Influenced by Financial Factors

To augment the core model it is useful to ask what items of expenditure the financial sector would impact upon. Four broad areas are suggested.

1. Fixed investment by firms.
2. Residential investment by households.
3. Consumer durable expenditures by households.
4. Consumption of goods and services by households.

We review work on how financial conditions have been introduced so as to have an impact upon the expenditures above. Mostly, core models do not specifically distinguish these categories, dealing only with aggregate investment and consumption, so we also briefly discuss how some have been introduced into macroeconomic models. Moreover, the four types of expenditures listed are not an exhaustive list of those that financial conditions can have an impact upon. For example, the availability of credit for international trade was an issue during the GFC. Similarly, there is extensive use of credit for the financing of inventories, and in Australia during the GFC car dealers had difficulty in obtaining credit to hold



the vehicles on display in their car yards. Even in more normal times, inventories need to be financed for the period of time between delivery and sale. Trade credit is also needed in order to pay for raw materials and even labour. But there has been less empirical work on these latter elements than on the four areas listed above.

### 2.2.1 *Fixed investment*

Fixed investment is the area where introducing financial effects is by far the best developed, typically using the financial accelerator mechanism. A notable example of this approach is Bernanke, Gertler and Gilchrist (1999). It implicitly involves an FI, which can be thought of as taking deposits from the household sector and then lending to the business sector (usually referred to as entrepreneurs) that is in need of credit to finance fixed investment. The credit-using agents are taken to be less patient than the lending agents.

In a model with a financial accelerator, credit comes at a cost that is a premium over internal financing, namely the *external finance premium*. This premium results from the fact that there is asymmetric information between the borrower and the FI, typically about the realised return on capital. The external finance premium therefore governs the amount of credit that can be obtained, and so it is necessary to model its determinants. Mostly the premium is simply taken to be increasing in the degree of leverage.<sup>1</sup> Therefore, increasing amounts of credit are costly, and this impacts on real and nominal quantities. Shocks which impact on the net worth (leverage) of the firm will alter the external finance premium, potentially amplifying the impact of the shock. Because the emphasis in this extension is on variations in the price of credit reflecting the credit-worthiness of the borrower, the external premium equations are often augmented with a shock that is intended to capture unrelated variations in supply, that is, the equation is more of a reduced-form than a structural equation. This is the strategy used in Gilchrist *et al* (2009).

Because there are no directly observed series on the external finance premium, empirical work either requires a proxy to be constructed or the external finance premium to be left unobservable. Gilchrist *et al* (2009) take data on the spreads between medium risk long-maturity US corporate bonds and the 10-year Treasury

---

<sup>1</sup> This can be derived from a costly state verification problem between the lender and borrower; see, for example, Bernanke *et al* (1999).

yield to be a measure of this premium. They also utilise data on the leverage ratio of US firms, and this series helps to estimate the elasticity of the external finance premium to the leverage ratio.

### 2.2.2 Residential investment

The events preceding the GFC led to an interest in the role of housing investment in the business cycle. Indeed some see it as the key to the business cycle, such as Leamer (2007). But inspection of the cycle data has to cast doubt on such a position. The duration of the US residential investment cycle is quite short, on average around 12 quarters, which is around half the length of the business cycle. This reflects that the average growth rate in residential investment is around half that of GDP, while the volatility is about five times as high. Thus negative growth in residential investment occurs relatively frequently, and often results in a turning point in the series. These differences, combined with residential investment's small share of output mean that, even if one had the knowledge that residential investment was in a contraction, the probability of predicting an NBER-defined recession would just rise to 0.27 from its unconditional probability of 0.16.<sup>2</sup> Thus it is hard to subscribe to Leamer's viewpoint that *housing is the business cycle*. This is not to deny that it has a role, but it is not an exclusive one.<sup>3</sup>

A number of papers have developed macroeconomic models including residential investment. Iacoviello and Neri (2010) augment a standard macroeconomic model by adding a second production sector, building upon the work of Davis and Heathcote (2005). In Iacoviello and Neri, the first sector produces consumption, intermediate and investment goods with capital and labour, while the other creates new houses using capital, labour, intermediate goods and land. Land, which is assumed to be in fixed supply, together with adjustment costs to capital, creates some sluggishness in the supply of housing. Iacoviello and Neri (2010) also allow

---

2 Using the sample 1947:Q1–2009:Q4. Data is from the Bureau of Economic Analysis 'National Income and Product Account' tables 1.1.6 and 1.1.6B–D.

3 It should be noted that Leamer considerably transformed the data before reaching his conclusion. This included smoothing the residential investment data, which eliminates some of its peaks and troughs and probably made it more closely resemble those of GDP, and eliminating the difference in the growth rates of the two series. The latter has a very strong effect as the average growth rate is a key determinant of cycle characteristics – see Harding and Pagan (2002).

for different trend rates in technology growth across sectors. They demonstrate that their model can capture some observed correlations in the data (such as a strong correlation between detrended residential investment and output) and that housing-specific shocks, namely technology and preference shocks, account for around one-quarter of the fluctuations in residential investment and house prices. Iacoviello and Neri acknowledge that the housing preference shocks may be capturing other unmodelled factors shifting housing demand rather than changes in preferences.

Apart from the sectoral disaggregation of production, the Iacoviello and Neri (2010) model features the idea that housing could serve as a collateral asset to finance either investment or consumption. This was introduced in Iacoviello (2005), and we discuss further in Section 2.3 on the supply of credit. Alternatively Aoki, Proudman and Vlieghe (2004) introduce a Bernanke *et al* (1999) style financial accelerator mechanism by making the rate at which homeowners can borrow depend on their net worth. However, production of new housing in Aoki *et al* (2004) is very simply modelled, with the only inputs being the existing housing stock and the consumption good.

Beneš *et al* (2009) is notable for augmenting a core model to capture housing investment in an open, rather than closed, economy.<sup>4</sup> Credit is required by the household sector to purchase housing and the FI raises funds in a foreign market. These are then loaned out to the domestic market. Consequently, the external premium reflects the difference between the domestic and foreign interest rates.

### 2.2.3 Consumption

In the core macroeconomic model, the consumption Euler equation (with habit persistence) takes the form (after log-linearization)

$$\tilde{c}_t = \alpha E_t \tilde{c}_{t+1} + (1 - \alpha) \tilde{c}_{t-1} + \theta \tilde{r}_t \quad (1)$$

where  $r_t$  is the real interest rate and tildes denote log departures from a steady state position. Preference shocks may also appear in the structural equation.

---

4 Their core model is not strictly the Smets and Wouters one, but the principles underlying it are the same.

Equation (1) can be written as

$$\Delta\tilde{c}_t = \frac{\alpha}{1-\alpha} E_t \Delta\tilde{c}_{t+1} + \frac{\theta}{1-\alpha} \tilde{r}_t. \quad (2)$$

The term  $E_t \Delta\tilde{c}_{t+1}$  varies with the model, and in general there will be a large number of influences on expected future consumption growth. When the model is extended to incorporate financial influences the number of factors would grow. The introduction of borrowing constraints, which is described below, modifies the Euler equation for credit-constrained households further; for example, nominal, rather than just real interest rates, become important.

Alternative approaches to assessing the impact of credit conditions on consumption, rather than using a general equilibrium model, do exist. One example is Aron *et al* (2010), who focus on a ‘solved-out’ consumption function, rather than the Euler equation; the  $E_t \Delta\tilde{c}_{t+1}$  term in Equation (2) does not appear but is replaced by a number of factors involving liquid assets, housing wealth etc. The coefficients on some of these terms are made functions of a credit conditions index that is constructed differently for each of the countries they examine, but which essentially extracts a common factor from many series that contain some information on the tightness of credit. Of course the credit conditions index used by Aron *et al* is generally constructed from information that is not in the augmented model, but series encapsulating that information might be employed when estimating it by adding them to the observation equations relating to the unknown external finance premium. If a number of series representing credit conditions are added a common factor among them would then be extracted. Another difference is that the core model described above is linear in logs, and so there would be no interaction terms with whatever is used to represent financial stress in the model. Again this might be emulated by performing a second-order approximation of the core model, as that will produce interaction terms involving covariances.

### 2.3 The Supply of Credit

In addition to the factors discussed above that influence the demand for credit, and its price, there may also be constraints on the quantity of available credit. There is no doubt that credit rationing was a primary financial mechanism in the models of the 1960s and 1970s, as it reflected the regulated financial markets

then in operation. Since that time, however, the amount of credit supplied by financial intermediaries has been more endogenously determined. Nevertheless, some constraints still operate, reflecting asymmetric information. In particular, it is often assumed that credit is only supplied if there is an adequate amount of collateral posted by the borrower. Consequently the value of the collateral asset determines the quantity of credit available, whereas the financial accelerator mechanism alters the price of credit. The collateral could be any asset which serves that purpose, such as the capital stock, but often a new asset is introduced that may be demanded by either entrepreneurs/firms, households, or both. Sometimes this asset is referred to as ‘housing’, ‘land’ or ‘real estate’, since the main component of the value of a house is generally the land value, but often most of its attributes are the same as a durable consumption good. Households consume housing services and entrepreneurs may use the asset in production, and so it may have a role in producing output as well as facilitating the acquisition of credit.

Agents in these models typically are split into those who are subject to a borrowing constraint and those who aren't. The latter are taken to be patient and so have a lower discount rate than the former. The borrowing constraint operates through a loan-to-value (of collateral) ratio based on the assumption that, in the event of a loan failure, only part of the loan is recouped. It is also usually assumed that the maximum possible amount is borrowed.<sup>5</sup> Consequently, as the price of the collateral asset rises, greater quantities of credit can be raised. Iacoviello (2005) estimates a model based on these ideas. The loan-to-value ratio need not be fixed (although it is in many models, such as Iacoviello (2005)) and could be allowed to vary in a stochastic way, as in Gerali *et al* (2010). However, ideally this would be endogenous, which requires a description of how the ratio would be set by a lending institution. Gerali *et al* (2010) extend the Iacoviello (2005) model to include an explicit FI sector, which allows different interest rates to different agents and exogenous shocks to bank capital. Introducing into these models shocks originating in the financial sector seems to be an important extension.

Two other ways of modelling the supply of credit by FIs should be mentioned. Gertler and Kiyotaki (2010) have many financial intermediaries which are aggregated. This serves to provide both a retail market for funds and a wholesale (inter-bank) market. Because one can observe data on the inter-bank market such

---

<sup>5</sup> This is done to allow log-linearisation of the model.

extensions in the future may be helpful for empirical work. Another important feature that might need to be captured in models was pointed out by Greenlaw *et al* (2008). They effectively observed that the credit supplied by financial institutions would likely vary with the value at risk (VaR) of their portfolio, as that has become the standard method of determining the limit on the amount of loans that can be supplied. Because the VaR is based on the probability of returns being less than a given value this will rise in a recession, and so the ‘credit multiplier’ would be smaller.

### **3. Evidence of Financial Effects on Aggregate Activity**

What is the evidence concerning the impact of financial factors upon the aggregate level of activity? Here we exclude questions relating to the impact of the short-term interest rate as these generally appear in the core model. Instead we ask about the evidence on the impact of credit conditions upon aggregate activity. A good summary of this evidence has been provided in sources such as IMF (2009), and here we select four conclusions from that document. There are more ‘stylised facts’ but these seem to be a useful starting point.

1. In the first two years of an expansion after a financial crisis, real credit grows quite weakly, at a slower rate than output.<sup>6</sup>
2. The probability of the economy entering a recession increases markedly once the external finance premium exceeds some ‘crisis level’.
3. The probability that an economy will stay in a recession beyond a certain number of quarters is higher when the onset of a recession was accompanied by a financial crisis. A crude interpretation of this would be that recessions with a financial crisis are of longer duration.
4. A measure of financial stress can help predict output growth. Moreover, real investment growth shows even greater predictability. The latter seems to imply that the effects of financial factors will be greater on investment than on aggregate economic activity. In particular, the cycle in investment expenditure should be more closely related to credit conditions.

---

<sup>6</sup> Strictly speaking, in IMF (2009) this is for the manufacturing sector rather than aggregate output.

To look at these outcomes in the context of a model, a measure of financial stress is needed. In the IMF (2009) work the dates of financial crises were taken to be those identified by Reinhart and Rogoff (2008), who used a ‘narrative’ approach to find them. Here we need a measure that can be generated by any model augmented with financial effects. Because the financial stress measure aims to quantify the extra costs that firms encounter if they are required to borrow, one guide would be the size of the external finance premium. Thus, ideally, a crisis would be defined as occurring when the finance premium gets above a certain level but, as this is unlikely to be easy to determine, we simply investigate relationships as the level of the premium rises.

The features noted above require that one locate turning points in the level of economic activity in order to compute the characteristics of the business cycle, so as to locate the dates when an expansion or a recession started. For this purpose we use the BBQ program, which is described in Harding and Pagan (2002) and is a quarterly version of the method for locating turning points set out in Bry and Boschan (1971).<sup>7</sup>

## **4. Two Models with Financial-Real Linkages**

As outlined in the previous section there are many items that might be influenced by credit and many models that might be constructed to elucidate the interaction between credit and business cycles. A large model capturing all the possibilities might be desirable, but in this paper we focus upon examples of two widely used ways of capturing a number of these linkages.

### **4.1 The Gilchrist *et al* (2009) (GOZ) Augmentation of the SW Model**

The first model we examine, due to Gilchrist *et al* (2009), adds the impact of financial conditions upon investment using the financial accelerator mechanism described in Bernanke *et al* (1999) to the SW model. There is no specific role for collateral. To the SW model they add four equations:

---

<sup>7</sup> A modified version of the program, written by James Engel, is used and is available at <http://www.ncer.edu.au/data/>.

$$E_t \tilde{r}_{t+1}^K = \frac{1 - \delta}{\bar{R}_K + (1 - \delta)} E_t \tilde{q}_{t+1} + \frac{\bar{R}_K}{\bar{R}_K + (1 - \delta)} E_t \widetilde{mpk}_{t+1} - \tilde{q}_t \quad (3)$$

$$\tilde{s}_t \equiv E_t \tilde{r}_{t+1}^K - (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \quad (4)$$

$$\tilde{s}_t = \chi(\tilde{q}_t + \tilde{k}_t - \tilde{n}_t) + \varepsilon_t^{fd} \quad (5)$$

$$\tilde{n}_t = \frac{\bar{K}}{\bar{N}} \tilde{r}_t^K - \left( \frac{\bar{K}}{\bar{N}} - 1 \right) (\tilde{s}_{t-1} + \tilde{r}_{t-1} - \tilde{\pi}_t) + \theta \tilde{n}_{t-1} + \varepsilon_t^{NW}, \quad (6)$$

where the over-bar indicates a steady state value,  $r_t^K$  is the rate of return on capital,  $q_t$  is the real price of capital,  $mpk_t$  is the marginal product of capital,  $s_t$  is the external finance premium,  $k_t$  is the capital stock, and  $n_t$  is entrepreneurs' net worth. Of the coefficients,  $\delta$  is the depreciation rate of capital and  $\theta$  is the survival rate of entrepreneurs. Equation (4) defines the external finance premium as the difference between the expected rate of return on capital (which is determined by Equation (3)) and the real interest rate. Equation (5) shows how the external finance premium varies with the degree of leverage, which is governed by the parameter  $\chi$ . The shock  $\varepsilon_t^{fd}$  captures fluctuations in the supply of credit unrelated to the leverage of the entrepreneurs. The evolution of the net worth of entrepreneurs is given in Equation (6), with the first term reflecting the leveraged return of capital and the second the cost of debt. A certain fraction of wealth disappears as entrepreneurs disappear ( $(1 - \theta)\tilde{n}_{t-1}$ ). The shock  $\varepsilon_t^{NW}$  represents exogenous fluctuations to net worth.

## 4.2 The Iacoviello (2005) Model

The key feature of the Iacoviello model is that the housing asset acts as collateral and, as the loan-to-value ratio is fixed, the amount that can be borrowed is based on the value of housing. This model has been used as the basis for further studies, such as Iacoviello and Neri (2010) and Gerali *et al* (2010).

There are three types of agents of interest in this model – a patient consumer who lends and two borrowers: an impatient consumer and an entrepreneur. There is a collateral asset (housing) that is in fixed supply. Its services are consumed and also used in production. Because housing is in fixed supply there is no residential investment in aggregate but there is fixed capital investment. Both the impatient consumer and the entrepreneur are credit constrained, with borrowing limited



by a loan-to-value ratio which differs between them. Iacoviello estimates these parameters to be 0.89 for entrepreneurs and 0.55 for households. The decisions made by households and firms are much the same as those in the GOZ model except that credit constraints can limit expenditures, and so changes in the value of collateral can potentially have effects on cycles.

## 5. Business Cycle Characteristics of the Models

### 5.1 The GOZ Model

To assess the business cycle properties of the GOZ model we simulate data from it and apply the BBQ cycle-dating procedure to the simulated data. Because the real variables in the GOZ model are log deviations from a constant growth path we need to add back a trend growth term to the simulated data to get a series on the level of GDP that can be used to determine the business cycle. We use the growth rate assumed by GOZ. Table 1 contains the cycle output along with what we would get when BBQ is applied to quarterly per capita US GDP data over the period 1973:Q1–2009:Q4.<sup>8</sup> The Smets-Wouters results are found by using the parameters estimated by GOZ; in other words, they show the business cycle properties of the model when the financial accelerator mechanism is excluded.<sup>9</sup>

The SW and GOZ models both show a good match to the business cycle characteristics, and overall the differences between them are not great. If one looks at the outcomes for investment, these remain broadly the same, although perhaps surprisingly, the amplitude of the average investment expansion is less in the GOZ model. Thus the presence of a financial accelerator does not seem to have had a great effect upon business cycle outcomes, although it should be noted that these are averages and credit may be important in *particular cycles*.

---

8 Alberto Ortiz kindly provided us with a Dynare program that simulated an updated version of the model they use, and some of their data for the period 1973:Q1–2009:Q4. The parameter values set in that code are the posterior mean and are different to those reported in their paper, in part due to a shorter estimation period being used, namely 1985:Q1–2009:Q4. As this is quite short for cycle dating we focus on the longer sample when comparing the models to the data. If instead the shorter sample was used then expansions would be longer and contractions less severe.

9 Using the parameters from Smets and Wouters (2007), the results are not that different.

<b>Table 1: Business Cycle Characteristics – Data, SW and GOZ Models</b>			
	Data	SW model	GOZ model
<b>Durations (qtrs)</b>			
Expansions	13.6	13.3	14.8
Contractions	4.8	4.5	4.2
<b>Amplitude (%)</b>			
Expansions	9.2	8.9	9.0
Contractions	-2.8	-1.9	-1.6
<b>Cumulative amplitude (%)</b>			
Expansions	132.4	99.2	107.9
Contractions	-8.1	-7.0	-5.2

Some experiments can be conducted here. Doubling the standard deviation of the credit supply shocks in the GOZ model has a very small effect upon the cycle. It is necessary to make much bigger changes in order to have an impact, well outside the range of values of the external finance premium that have been observed. Thus, quadrupling the standard deviation reduces expansion length to 14.6 quarters and increases the amplitude of recessions, although only to  $-1.7$  per cent. But it does this by producing premia of 1 000 basis points (and more). At that level the probability of a recession is around 0.43, but one might think that this is rather low for such an extreme external finance premium. Doubling the coefficient  $\chi$  in Equation (5), which governs the sensitivity of the external finance premium to the entrepreneur's net worth, has little impact on the business cycle characteristics.

## 5.2 The Iacoviello Model

Using the parameter values provided in Iacoviello (2005) we simulate data from his model. Because Iacoviello 'detrended' the per capita GDP data with a band-pass filter before estimating the parameters of his model, the simulated data does not correspond to the level of GDP, which is necessary for business cycle

dating. To recover the latter from the filtered data is a non-standard problem.<sup>10</sup> Consequently, we decided to work with the filtered data, that is, to investigate the *growth cycle* rather than the business cycle. Table 2 therefore contains the growth cycle in output from the IAC model and from the data (found with the BBQ program), over Iacoviello's estimation period of 1974:Q1–2003:Q4. Clearly the fit on durations is quite good, but the amplitudes of growth cycle expansions and recessions are considerably smaller in the model than in the data. The growth cycles are fairly symmetric as the asymmetry in the business cycle comes from the fact that there is positive long-run growth and that has been filtered out here.

---

**Table 2: Growth Cycle Characteristics – Data and Iacoviello Model**

---

	Data	Iacoviello model
<b>Durations (qtrs)</b>		
Expansions	5.7	5.5
Contractions	4.8	5.5
<b>Amplitude (%)</b>		
Expansions	5.4	4.5
Contractions	–5.2	–4.5
<b>Cumulative amplitude (%)</b>		
Expansions	23.2	16.1
Contractions	–18.1	–16.5

---

## 6. Correspondence of GOZ Model Business Outcomes with Stylised Facts

We now seek to examine some of the characteristics listed in Section 3. In order to compare the model's output with the stylised facts, it will be necessary to find

---

<sup>10</sup> Landon-Lane (2002) studies this problem for the Hodrick-Prescott filter. We attempted to reconstruct the data by inverting the band-pass filter via a Moore-Penrose generalised inverse. Although an experiment with actual data seemed to recover the levels correctly, when applied to simulated data the growth rate in the reconstructed output series had negative serial correlation, which is contrary to actual US GDP data. Such negative serial correlation induces long expansions, as it implies a rapid bounceback from any negative growth rate. Hence we were finding that the Iacoviello model was producing very long business cycle expansions, although this might have been an artifact of our inversion procedure.

the credit growth rates implied by the model. Appendix A shows that the growth in real debt,  $\Delta \ln D_t^*$ , equals

$$\begin{aligned}\Delta \ln D_t^* &= \Delta q_t^* + \Delta k_t^* + \gamma^* + 100 \Delta \ln(1 - l_t^{-1}) \\ l_t &= \exp((q_t^* + k_t^* - n_t^*)/100 + \ln(\frac{\bar{K}}{N})),\end{aligned}\tag{7}$$

where  $\gamma$  is the long-term rate of growth of per capita output,  $l_t$  is leverage and an asterisk indicates these are measured in percentage form.

## 6.1 Credit and Output Growth Following a Financial Crisis

To assess the first relation documented by the IMF using simulated data we computed GDP growth and real credit growth over the first eight quarters of expansions. Let these be  $\Delta z_j$  and  $\Delta c_j$  respectively, where  $j$  indexes an expansion. Forming  $\phi_j = \Delta z_j - \Delta c_j$ , the stylised facts would be that the average of  $\phi_j$  from expansions that come after a financial crisis would be positive. Given that we use the size of the external finance premium ( $\tilde{s}_t$ ) as an index of the extent of a financial crisis, we would expect some relation between  $\phi_j$  and  $\psi_j$ , where  $\psi_j$  is the external premium when the  $j$ 'th expansion began. We used three measures of  $\psi_j$ : the value of the premium at the origin of the expansion (at time  $t_j$ ), and two averages based on current and past values,  $\frac{1}{2} \sum_{k=0}^1 \tilde{s}_{t_j-k}$  and  $\frac{1}{3} \sum_{k=0}^2 \tilde{s}_{t_j-k}$ . As the conclusions were the same in all cases we present the relation with the first of the three measures. Then the regression of  $\phi_j$  upon  $\psi_j$  and a constant gives (with  $t$ -statistics in parentheses)

$$\phi_j = \begin{matrix} -2.3 & + & 3.4 \psi_j, \\ (-2.8) & & (8.7) \end{matrix}$$

suggesting that credit growth is increasingly weaker than output growth over the first two years of expansion when the external premium at the beginning of the expansion is larger.

It is worth noting that the average credit growth over the first eight quarters of all expansions is 0.6 per cent versus the 4.3 per cent in output. But this hides an enormous variation. There are many simulations in which the growth in credit over the first two years of an expansion exceeds that of output. This remains true for a significant number of expansions which are preceded by a high external finance

premium, which we would interpret as a financial crisis. The growth rates in credit are extremely volatile, with a standard deviation of quarterly growth of 3.68 versus only 0.66 for output, although this volatility is also in the actual data used by GOZ.<sup>11</sup>

## 6.2 Recessions and Financial Crises

We can investigate the dependence of recessions upon the external finance premium by taking advantage of the binary nature of the recession indicator  $R_t$  ( $R_t = 1$  if the economy is in recession but zero otherwise). One possibility is to compute the probability of a recession as the external finance premium rises;  $P(R_t = 1 | \tilde{s}_t)$ . Assuming this has a Probit form it will be  $\Phi(\alpha + \beta \tilde{s}_t)$ , where  $\Phi(\cdot)$  is the cumulative standard normal distribution function. Figure 1 plots this as a function of  $s_t$  from simulations of the GOZ model. Note that the unconditional probability of an NBER recession over 1953:Q2–2009:Q4 was approximately 0.16. While the rise in the external premium does increase the probability quite substantially, it never gets to a standard critical value often used in predicting recessions of 0.5.

An alternative indicator of the relationship between recessions and the external finance premium comes from recognising, following Harding (2008), that the recession states  $R_t$  generated by BBQ (and also true for NBER recession indicators) follow a recursive process of the form

$$R_t = 1 - (1 - R_{t-1})R_{t-2} - (1 - R_{t-1})(1 - R_{t-2})(1 - \wedge_{t-1}) - R_{t-1}R_{t-2}\vee_{t-1}, \quad (8)$$

where  $\wedge_t$  is a binary variable taking the value unity if a peak occurs at  $t$  and zero otherwise, while  $\vee_t$  indicates a trough. By definition  $\wedge_t = (1 - R_t)R_{t+1}$  and  $\vee_t = (1 - R_{t+1})R_t$ . In BBQ,

$$\begin{aligned} \wedge_t &= 1(\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}) \\ \vee_t &= 1(\{\Delta y_t < 0, \Delta_2 y_t < 0, \Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0\}), \end{aligned}$$

---

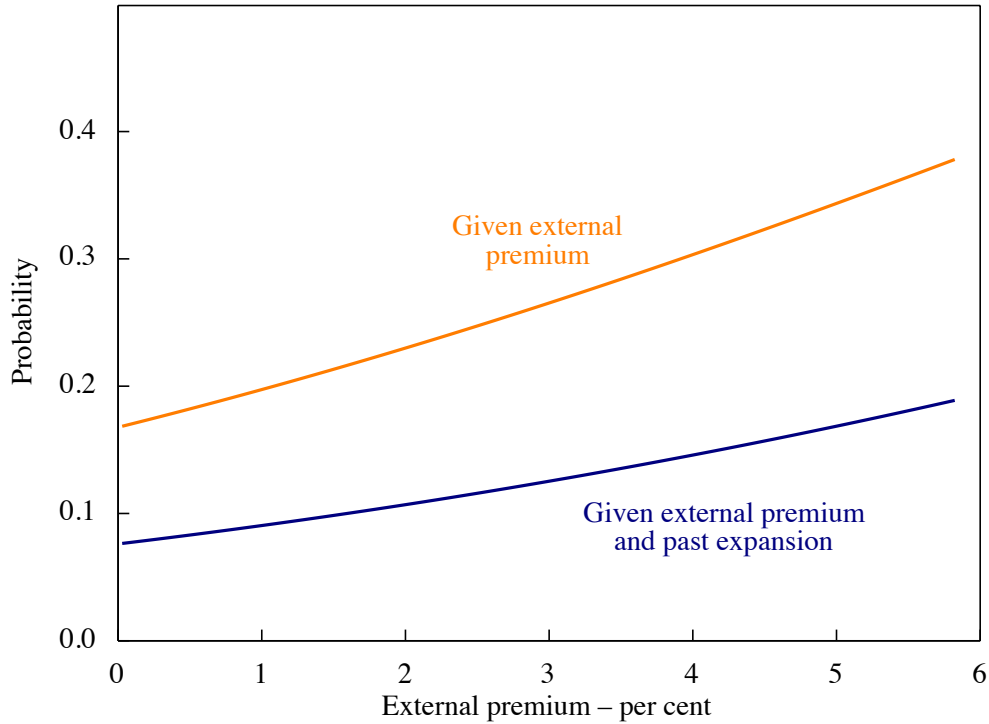
11 The extreme volatility comes from the factor  $100\Delta \ln(1 - l_t^{-1})$  in the growth of credit as set out in Equation (7). If one computes the standard deviation of  $100\Delta \ln(1 - l_t^{-1})$  from the data and the model we find these to be 5.53 and 4.89 respectively.

where  $\Delta_2 y_t = y_t - y_{t-2}$  will be six-monthly growth. Then one might condition upon the previous states as well as the external finance premium, to determine what the probability of going into a recession at time  $t$  is for a given finance premium and the knowledge that we were in expansion at  $t - 1$  and  $t - 2$ . This probability, using Equation (8), will be

$$\begin{aligned} \Pr(R_t = 1 | R_{t-1} = 0, R_{t-2} = 0, \tilde{s}_t) &= 1 - (1 - E(\wedge_{t-1} | \tilde{s}_t)) \\ &= E\{1(\Delta y_t < 0, \Delta_2 y_{t+1} < 0) | \tilde{s}_t\} \\ &= E\{1(\Delta y_t < 0)1(\Delta_2 y_{t+1} < 0) | \tilde{s}_t\}, \end{aligned}$$

where the conditioning on the past expansions has been suppressed. Assuming a Probit again, Figure 1 contains a plot of this against  $\tilde{s}_t$ . There is clearly a big difference between it and  $\Pr(R_t = 1 | \tilde{s}_t)$ . If it is known that one is in an expansion in the preceding two periods the rise in the probability of a recession, even for large values of the external finance premium, is quite small. Indeed, the result suggests that the external finance premium will not be very useful for predicting recessions, a point we come back to later.

**Figure 1: Probability of a Recession Given External Finance Premium**



### 6.3 The Duration of a Recession and the Size of the External Finance Premium

The next question we seek to examine is whether the duration of recessions depends upon the magnitude of the external finance premium. There are two ways one might do this. One is to relate the durations of recessions to the external premium. To do this, we simulated a long series of data and regressed the durations of recessions in this series against the external finance premium at the beginning of the recession. While this showed a positive relationship the connection was not strong – even large changes in the premium only caused the duration to increase by just a quarter. Another method is to compute  $\Pr(R_{t+m} = 1 | R_t = 1, \tilde{s}_t)$ , i.e. the probability that, in  $m$  periods time, the economy will still be in a recession given it was in recession at time  $t$ , conditional on the external finance premium. Table 3 shows what this probability is for three levels of  $\tilde{s}_t$  and for  $m = 2, 3, 4$ . This probability effectively is a measure of duration dependence.

External premium $\tilde{s}_t$ (basis points)	$m = 2$	$m = 3$	$m = 4$
25	0.51	0.36	0.27
300	0.53	0.37	0.28
485	0.53	0.38	0.29

It appears that the probability of a recession continuing only increases slightly with a larger initial external finance premium. Essentially, both computations address the often-quoted result that a recession associated with a financial crisis is around twice as long as one that does not have one. While we would think that a crisis would involve a high external interest rate premium, given that there would be little credit available, the GOZ model only delivers such a prediction in a very weak way. One would certainly associate a crisis with an elevated probability of recession, as seen in Figure 1, but its duration does not seem to depend much on the premium. One factor contributing to this might be the persistence in the growth in credit; the persistence in  $\{\Delta \ln(1 - (1/l_t))\}$ , which is used to form credit growth in Equation (7), is considerably smaller in the model than in the data.

## 6.4 Credit Growth and Recession Prediction

One might ask if there is any evidence that a recession can be predicted with the GOZ model. The probability is different to what was computed above as we are now looking at  $\Pr(R_{t+1}|\tilde{s}_t)$  and not  $\Pr(R_t|\tilde{s}_t)$ . We might also be interested in  $\Pr(R_{t+1}|R_t = 0, R_{t-1} = 0, \tilde{s}_t)$ . The latter equals

$$E\{1(\Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0)|R_t = 0, R_{t-1} = 0, \tilde{s}_t\},$$

pointing to the fact that predicting a recession involves successfully predicting *negative* quarterly and six-monthly growth over the two quarters following on from the prediction point. This is much stronger than the ability to predict growth rates of output *per se*. It may be that we predict positive ones quite well, and this can make the prediction record for output growth look rather good, even though recession prediction is a dismal failure.

There are some current difficulties in determining the predictions of the GOZ model since it was estimated using some data that were not available to us. In particular, the credit spread was one they constructed. To gain an idea of how useful the model might be we assume that the latter is well represented by the Baa spread.<sup>12</sup> The figures in GOZ suggest the Baa spread is related to their indicator, although they argue that their measure is a better predictor. With the Baa spread as  $\tilde{s}_t$  we evaluate  $\Pr(R_{t+1}|\tilde{s}_t)$ . Here the Baa spread used is that available at the beginning of the quarter for which a prediction is to be made. Although the spread is significant in a Probit model fitted to the recession indicator ( $t$  ratio of 3.5), it is clear from Figure 2 that it adds little to the predictive power. Even in 2008, it was not indicating a recession until the recession was well under way (the predicted probability in the first quarter of 2008 was just 0.23). This concurs with the finding of Harding and Pagan (2011) that many series recommended as useful for predicting recessions in fact have little predictive power, and suggests that the

---

<sup>12</sup> Because there are two interest rates in the model, and one of these is the policy rate, it might be better to use the spread over a three-month Treasury bill rate. But doing this does not change the results very much. The Baa spread is calculated using data from the Federal Reserve Board of Governors Statistical Release H.15.



external finance premium, a key part of the GOZ model, might be ineffective at predicting recessions.<sup>13</sup>

**Figure 2: Probability of a Recession Given Baa Spread**



Note: Shading denotes NBER recessions

To understand the limits of using models such as this for predicting recessions we observe that, at time  $t - 1$ , we would be predicting  $1(\Delta y_t < 0, \Delta_2 y_{t+1} < 0)$  using the information available at  $t - 1$ , that is, we aim to predict future growth outcomes. A check on whether a model would be able to predict such a quantity is to ask how important the unpredictable part of future shocks are to these growth outcomes. Shocks, such as technology, often have an autoregressive structure, and it is the innovation whose impact upon the business cycle we wish to determine. We therefore simulate the GOZ model turning off the contemporaneous innovations, that is, the model is run with the current innovations set to zero, although they are re-set to their actual values in later periods. To illustrate what is done, take an AR(1)  $z_t = \rho z_{t-1} + e_t$ , where  $e_t$  is white noise. Defining  $z_t^- = \rho z_{t-1}$ , we note that

<sup>13</sup> We also experimented with using the GOZ model itself to predict recessions, that is, we constructed an estimate of  $E(\Delta y_{t+1}|F_t)$  from the GOZ model, where  $F_t$  denotes the information set from the model excluding the external finance premium (i.e. we delete its influence). Then  $E(\Delta y_{t+1}|F_t)$  and the Baa spread were both used in the Probit model. Each variable was significant but there was no improvement in predictive power for recessions over that shown for the Baa spread.

$z_t$  and  $z_t^-$  differ only in that the current innovation is set to zero; in other words,  $z_t^-$  is the predicted value of  $z_t$ . Table 4 shows the business cycles from the GOZ model with current innovations present (equivalent to basing the computation on  $z_t$ ) and with them suppressed (equivalent to  $z_t^-$  and hence designated  $GOZ^-$ ). It is clear that the current innovations have an enormous effect upon the cycle characteristics. Abstracting from current innovations, expansions now become very long, and so there will be fewer recessions, leading to our conclusion that the GOZ model will predict fewer recessions if future shocks are not known.

	GOZ	GOZ <sup>-</sup>
<b>Durations (qtrs)</b>		
Expansions	14.8	33.9
Contractions	4.2	4.2
<b>Amplitude (%)</b>		
Expansions	9.0	15.9
Contractions	-1.6	-0.8
<b>Cumulative amplitude (%)</b>		
Expansions	107.9	461.8
Contractions	-5.2	-2.8

Note: GOZ<sup>-</sup> is the GOZ model with current innovations to shocks suppressed

#### 6.4.1 Relative performance of investment prediction

One of the observed cycle characteristics listed above was that there would be a stronger response by investment than output. We therefore studied the investment cycle present in the observed investment data used by GOZ, namely a chain-weighted index of per capita private fixed investment and durable goods. Here expansions were 12 quarters long on average and contractions were 6.5 quarters. So, while the investment cycle length is not far from that of GDP, the contractions are longer and the expansions are shorter. Figure 3 presents the probability of an investment recession as a function of the Baa spread,  $\Pr(R_{t+1} = 1|\tilde{s}_t)$ , and it is apparent that it rises very quickly with the spread. This is also true of the model, where the probability of an investment contraction is 0.47 when there is an (annualised) spread of 205 basis points, 0.58 when the spread is 305 points, and 0.78 when it is 515 basis points.

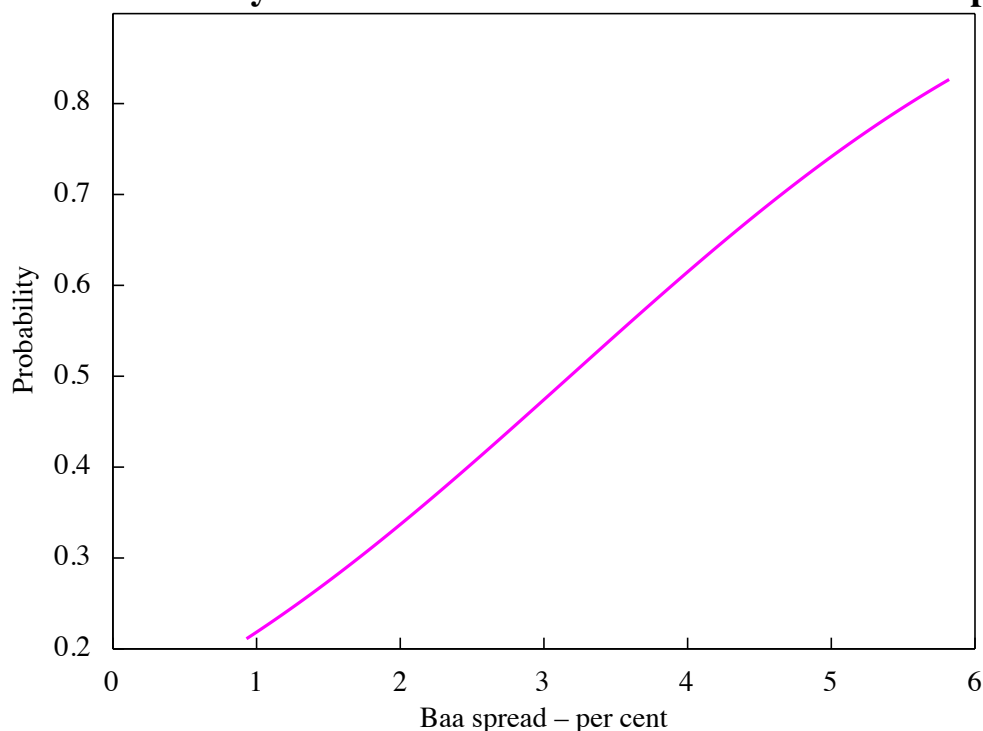
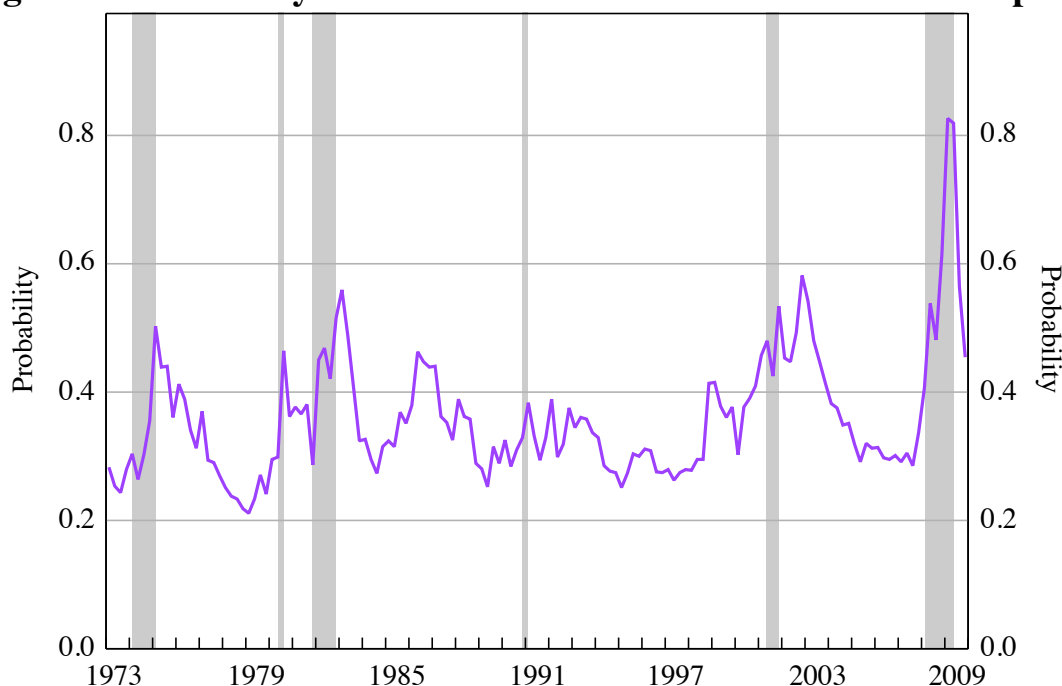
**Figure 3: Probability of an Investment Contraction Given Baa Spread**

Figure 4 shows the probability of an investment contraction given the Baa spread over the GOZ estimation period. A comparison with Figure 2 suggests that the probability of an investment recession tends to be higher than that of output recessions for a given spread, reflecting in part the fact that investment is only one component of output, and therefore a very large negative growth rate in investment is needed to cause a decline in output by itself. This is the case in the GOZ model; despite using a relatively wide definition of investment in estimation, the parameters of the model (in particular  $\alpha$ , which governs its weight in the production function) imply a steady-state investment-to-output ratio of around 6.5 per cent, which is less than fixed non-residential investment's share of output in the data. If  $\alpha$  is doubled then the model matches the business cycle characteristics of the data better and the effects of an increase in the external finance premium are greater. For example, an increase in the external finance premium from 25 to 300 basis points raises the probability that an economy in recession now will remain there two quarters in the future from 0.51 to 0.57, which is still a small increase but more than double the impact shown in Table 3. Similarly, the probability of a recession at large values of the external finance

premium is higher, but remains below 0.5.<sup>14</sup> As the role of investment in the GOZ model is more aligned with fixed non-residential investment, increasing  $\alpha$  much further is not realistic. In all, it appears that one needs to work with a broad set of investment expenditures, i.e. housing and consumer durables could be important to getting the quantitative financial effects right and these probably should be integrated into the structure of the model. In turn this implies that collateral effects might be important.

**Figure 4: Probability of an Investment Contraction Given Baa Spread**



Note: Shading denotes NBER recessions

## 7. Correspondence of Iacoviello Model Growth Cycle Outcomes with Stylised Facts

The ‘stylised facts’ used earlier pertain to the business cycle, and it is unclear what implications these would have for detrended output, namely the growth cycle, which is output in the Iacoviello model. Furthermore, there is no external finance premium in Iacoviello’s model, making it hard to define a crisis.

<sup>14</sup> At 5 per cent it is approximately 0.42.

Here we consider whether the collateral asset prices ( $\tilde{q}_t^h$ ) are a long way below their steady state levels, that is, the extent to which  $\tilde{q}_t^h$  is negative. Just as before there was a question of timing, in that one might wish to use lags of  $\tilde{q}_t^h$ , so we experimented with  $\tilde{q}_t^h$ ,  $\tilde{q}_{t-1}^h$  and  $\tilde{q}_{t-2}^h$ . The last seemed to give the best results. Thus we computed  $\Pr(R_t | \tilde{q}_{t-2}^h)$ . Somewhat surprisingly we found that, when the price of collateral was 10 per cent below steady state, the probability of a recession was 0.06, while if it was just 10 per cent above the probability was 0.94. So high values of the collateral price significantly raised the recession probability. This seems an extraordinarily sensitive reaction to the collateral price. To check the results we regressed simulated  $R_t$  against  $\tilde{q}_{t-2}^h$  and got the linear model  $0.50 + 0.06\tilde{q}_{t-2}^h$  which matches the Probit results well. A non-parametric fit produced the same outcome. It might be noted that the unconditional probability of a recession is 0.5, as the growth cycle from the model is virtually symmetric.

The feature noted in connection with the GOZ model about the importance of current shocks is again echoed here. Durations of growth cycle expansions and contractions are little changed, while the amplitudes are now 63 per cent of what they were when all shocks are included, that is, there is much less volatility.

Another feature of the Iacoviello model that is of interest is to examine what it tells us about the impact of variations in the loan-to-value ratios. One might think of this as an index of how easy it is to get credit. In the Iacoviello model the two loan-to-value ratios are set at 0.89 (entrepreneurs) and 0.55 (households), so we multiply these with a constant  $k$  in order to emulate a range of credit conditions. The values of  $k$  chosen are 0.1, 0.5, 0.9, 1.0 and 1.1, so that the fourth of these values uses the ratios in Iacoviello's (2005) work. Table 5 shows how the amplitudes of expansions and contractions for the growth cycle change for different values of  $k$  (the absolute amplitudes are virtually identical for a given  $k$  so we only present one, and the changes in the durations are relatively small). It could be regarded as surprising that easier credit leads to greater amplitudes because in the model it might have been expected to enable agents (such as impatient consumers) to better smooth their consumption, producing expenditures that are less volatile. Moreover, even very tough credit conditions ( $k = 0.1$ ) lead to an amplitude that is much the same as when  $k = 0.5$ . Thus, easier credit results in a much more volatile economy, and it is apparent that, as the loan-to-value ratio approaches unity, volatility goes up a great deal. In a sense this is a story about imbalances. Keeping strong credit standards may be key to ensuring stability.

---

**Table 5: Absolute Values of Amplitudes of Growth Cycles**  
Phases in Iacoviello model for degrees of credit availability

---

$k$	Amplitude
0.1	2.4
0.5	2.4
0.9	3.3
1.0	4.5
1.1	9.5

---

## 8. Conclusion

Models that incorporate financial features pertaining to credit and debt are increasingly appearing in the macroeconomic literature. To date much of the assessment of the success of this augmentation of traditional models has involved recording the magnitude of impulse responses to various financial shocks and whether the shocks can give a better account of the variation in output over time. Our paper provides a complementary approach, namely whether the augmented models provide a better explanation of the business or growth cycles and whether they can replicate some stylised facts about the relationship between recessions and credit. To demonstrate this approach we took two models representative of common ways of introducing financial factors into macroeconomic models – those of Gilchrist *et al* (2009) and Iacoviello (2005). While financial factors can play a role in particular cycles, generally it seems that the average cycle characteristics of these models are not affected much by their introduction. The Gilchrist *et al* model managed to replicate some of the stylised facts but failed to do so for others, for example, that credit crises produce long duration recessions. This points to the need to either add extra features or perhaps combine existing ones. Finally, successful prediction of recessions ultimately involves an ability to predict the signs of future output growth rates. To gauge whether the models examined here can predict recessions, we ask how important to current output growth rates are the component of current shocks that are unpredictable using past information. We find they are very important. Consequently, the models imply that future growth rates in output are largely dependent on future shocks. Since these are unpredictable using current information this severely limits the predictive ability of the two models.

## Appendix A: Derivation of Debt Growth Equation

Let leverage be  $l_t = \frac{Q_t K_t}{N_t}$ . Then

$$l_t = \frac{Q_t K_t}{Q_t K_t - D_t}$$

$$\implies D_t = Q_t K_t (1 - l_t^{-1}).$$

It immediately follows that

$$\begin{aligned} \Delta \ln D_t &= \Delta \ln(Q_t K_t) + \Delta \ln(1 - l_t^{-1}) \\ &= \Delta \tilde{q}_t + \Delta \tilde{k}_t + \gamma + \Delta \ln(1 - l_t^{-1}), \end{aligned}$$

where  $\tilde{q}_t = \ln(Q_t/\bar{Q})$  and  $\tilde{k}_t = \ln(K_t/\bar{K}\gamma^f)$ , since the steady state growth rate of capital will be the same as output ( $\gamma$ ). Designating the ratio  $\frac{K_t}{N_t}$  as  $R_{KN,t}$ , and using the fact that  $\bar{Q} = 1$ , an expression for  $l_t$  is available from

$$\begin{aligned} \ln l_t &= \ln Q_t + \ln R_{KN,t}. \\ \ln l_t - \ln \bar{l} &= \ln Q_t - \ln \bar{Q} + \ln R_{KN,t} - \ln \bar{R}_{KN}. \\ \ln l_t &= \tilde{q}_t + \tilde{k}_t - \tilde{n}_t + \ln \bar{R}_{KN}. \\ \therefore l_t &= \exp(\tilde{q}_t + \tilde{k}_t - \tilde{n}_t + \ln(\frac{\bar{K}}{\bar{N}})). \end{aligned}$$

Now Gilchrist *et al* (2009) measure variables in percentages so, designating these by a '\*', we get

$$l_t = \exp((q_t^* + k_t^* - n_t^*)/100 + \ln(\frac{\bar{K}}{\bar{N}})).$$

Hence we have

$$\begin{aligned} \Delta \ln D_t^* &= \Delta q_t^* + \Delta k_t^* + \gamma^* + 100 \Delta \ln(1 - l_t^{-1}) \\ l_t &= \exp((q_t^* + k_t^* - n_t^*)/100 + \ln(\frac{\bar{K}}{\bar{N}})). \end{aligned}$$

## References

**Aoki K, J Proudman and G Vlieghe (2004)**, ‘House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach’, *Journal of Financial Intermediation*, 13(4), pp 414–435.

**Aron J, JV Duca, J Muellbauer, K Murata and A Murphy (2010)**, ‘Credit, Housing Collateral and Consumption: Evidence from the UK, Japan and the US’, University of Oxford Department of Economics Discussion Paper Series No 487.

**Beneš J, A Binning, M Fukač, K Lees and T Matheson (2009)**, *K.I.T.T.: Kiwi Inflation Targeting Technology*, Reserve Bank of New Zealand, Wellington.

**Bernanke BS, M Gertler and S Gilchrist (1999)**, ‘The Financial Accelerator in a Quantitative Business Cycle Framework’, in JB Taylor and M Woodford (eds), *Handbook of Macroeconomics, Volume 1C*, Elsevier Science, Amsterdam, pp 1341–1393.

**Bry G and C Boschan (1971)**, *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*, Technical Paper No 20, National Bureau of Economic Research, New York.

**Christensen I and A Dib (2008)**, ‘The Financial Accelerator in an Estimated New Keynesian Model’, *Review of Economic Dynamics*, 11(1), pp 155–178.

**Davis MA and J Heathcote (2005)**, ‘Housing and the Business Cycle’, *International Economic Review*, 46(3), pp 751–784.

**Fukač M and A Pagan (2010)**, ‘Structural Macroeconometric Modeling in a Policy Environment’, in A Ullah and DEA Giles (eds), *Handbook of Empirical Economics and Finance*, Statistics: A Series of Textbooks and Monographs, Chapman & Hall/CRC, Boca Raton, pp 215–245.

**Gerali A, S Neri, L Sessa and FM Signoretti (2010)**, ‘Credit and Banking in a DSGE Model of the Euro Area’, *Journal of Money, Credit and Banking*, 42(Supplement s1), pp 107–141.

**Gertler M and N Kiyotaki (2010)**, ‘Financial Intermediation and Credit Policy in Business Cycle Analysis’, in BM Friedman and M Woodford (eds), *Handbook of Monetary Economics, Volume 3A*, Elsevier B.V., Amsterdam, pp 547–599.



**Gilchrist S, A Ortiz and E Zakrajšek (2009)**, ‘Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model’, Paper presented at Reserve Bank of Australia Research Workshop ‘Monetary Policy in Open Economies’, Sydney, 14–15 December.

**Greenlaw D, J Hatzius, A Kashyap and H Shin (2008)**, ‘Leveraged Losses: Lessons from the Mortgage Meltdown’, in *US Monetary Policy Forum Report No 2*, Rosenberg Institute, Brandeis International Business School and Initiative on Global Markets, University of Chicago Graduate School of Business, pp 7–59.

**Harding D (2008)**, ‘Detecting and Forecasting Business Cycle Turning Points’, Paper presented at the 5th Eurostat Colloquium on Modern Tools for Business Cycle Analysis, Jointly organised by Eurostat and European University Institute, Luxembourg, 29 September–1 October.

**Harding D and A Pagan (2002)**, ‘Dissecting the Cycle: A Methodological Investigation’, *Journal of Monetary Economics*, 49(2), pp 365–381.

**Harding D and A Pagan (2011)**, ‘Econometric Analysis and Prediction of Recurrent Events’, National Centre for Econometric Research Working Paper Series No 75.

**Iacoviello M (2005)**, ‘House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle’, *The American Economic Review*, 95(3), pp 739–764.

**Iacoviello M and S Neri (2010)**, ‘Housing Market Spillovers: Evidence from an Estimated DSGE Model’, *American Economic Journal: Macroeconomics*, 2(2), pp 125–164.

**IMF (International Monetary Fund) (2009)**, *World Economic Outlook – Crisis and Recovery*, World Economic and Financial Surveys, IMF, Washington DC.

**Landon-Lane J (2002)**, ‘Inverting the Hodrick-Prescott Filter’, *Computational Economics*, 20(3), pp 117–138.

**Leamer EE (2007)**, ‘Housing is the Business Cycle’, in *Housing, Housing Finance, and Monetary Policy*, A Symposium sponsored by the Federal Reserve Bank of Kansas City, Kansas City, pp 149–233.

**Liu Z, P Wang and T Zha (2009)**, ‘Do Credit Constraints Amplify Macroeconomic Fluctuations?’, Paper presented at Reserve Bank of Australia Research Workshop ‘Monetary Policy in Open Economies’, Sydney, 14–15 December.

**Muellbauer J (2010)**, ‘Household Decisions, Credit Markets and the Macroeconomy: Implications for the Design of Central Bank Models’, Bank for International Settlements Working Paper No 306.

**Reinhart CM and KS Rogoff (2008)**, ‘This Time is Different: A Panoramic View of Eight Centuries of Financial Crises’, NBER Working Paper No 13882.

**Smets F and R Wouters (2007)**, ‘Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach’, *The American Economic Review*, 97(3), pp 586–606.