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Abstract

The rational expectations equilibrium of a small open economy can be subject to indeterminacy if foreign monetary policy does not satisfy the Taylor principle. We study the implications of foreign-induced indeterminacy for the conduct of monetary policy in a small open economy. In the canonical sticky-price small open economy model, we find that indeterminacy arising in the large economy can increase the volatility of the small economy. Our main finding, however, is that 'smallness' is a property of the unique rational expectations equilibrium of the large economy, and not a general property of the small open economy model. If the *large* economy fails to anchor expectations, shocks to the small economy can affect the large one. This form of indeterminacy gives rise to a 'butterfly effect'. Additional assumptions are required to preserve the 'smallness' of the small economy.

JEL Classification Numbers: E30, E32, E52, E58, F41

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1. Introduction

> The flapping of a single butterfly's wing today produces a tiny change in the state of the atmosphere. Over a period of time, what the atmosphere actually does diverges from what it would have done. So, in a month's time, a tornado that would have devastated the Indonesian coast doesn't happen. Or maybe one that wasn't going to happen, does. (Stewart 1990, p 141)

A general agreement in the modern literature on monetary economics is that monetary policy should obey the Taylor principle: that the nominal short-term interest rate should rise eventually more than one-for-one with the rate of inflation. There is evidence that the success of monetary policy over the past two decades, compared to the problems in the 1970s, can be explained by reference to the Taylor principle.[1] In most sticky-price models, this principle ensures that beliefs themselves do not turn into independent sources of fluctuations.

In these models, a central bank that fails to satisfy the Taylor principle is unable to ensure a unique rational expectations equilibrium (REE) for the economy.[2] Such monetary policies lead to indeterminacy of the equilibrium: an economy for which many different outcomes are possible given the same fundamental situation. This problem of non-uniqueness has attracted considerable attention in the literature.[3]

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1 See, for example, Clarida, Galí and Gertler (2000), Mankiw (2002), Bernanke (2004) and Lubik and Schorfheide (2004).

2 See Woodford (2003) for a detailed description of the relationship between the Taylor principle and uniqueness of the REE. See also Benhabib, Schmitt-Grohe and Uribe (2001) for an example of a model in which the Taylor principle is not necessary for determinacy of the equilibrium.

3 We cannot possibly do justice to the literature. Instead, we point the interested reader to Sargent and Wallace (1973), Taylor (1977), Barro (1981), Pesaran (1987), Bernanke and Woodford (1997), Farmer (1999), and the references therein. Although these studies differ along various dimensions, they refer to indeterminacy of the REE. It is important to keep in mind as McCallum (1983) argues, however, that the non-uniqueness problem is a more general feature of dynamic models that involve expectations, and not a particular one attributable to the rational expectations hypothesis.

With a new Keynesian closed economy model, in which violations of the Taylor principle lead to multiple equilibria, Lubik and Schorfheide (2004) show that passive monetary policy better accounts for the dynamics of inflation and output in the United States prior to 1979. To the extent that indeterminacy – in a closed economy – is the result of an improper policy, determinacy could easily be restored by changing policy settings appropriately. As emphasised by Bullard and Singh (2006), however, good monetary policy can be insufficient to ensure determinacy of the REE in an open economy. Thus, an open economy may be exposed to non-fundamental fluctuations that ‘originate abroad’.

In general, indeterminacy of the REE can manifest itself in two non-exclusive ways. Non-fundamental disturbances may become additional sources of economic fluctuations and fundamental shocks may propagate differently. One of our goals is to study the implications of foreign-induced indeterminacy for the conduct of monetary policy in a small open economy. In particular, with a sticky-price small open economy model we address the following questions. How does the small economy respond to non-fundamental disturbances? Can monetary policy insulate, to some extent, the small economy from non-fundamental disturbances? To the best of our knowledge, no study has examined the implications of foreign indeterminacy for monetary policy in a small open economy. There is literature studying specific conditions for determinacy and indeterminacy in open economy models.[4] Our focus here is different. We study the dynamic behaviour of the economy and optimal policy responses under ‘inherited’ indeterminacy.

Surprisingly, however, our main finding is this: if the large economy fails to achieve a unique equilibrium, shocks to the small economy affect the large one. In other words, ‘loose’ expectations abroad create a channel through which shocks that originate in the small economy influence the large economy. We call this channel the ‘butterfly effect’. In this way, the theory gives a structural and elegant interpretation of sunspot shocks for the large economy.

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[4] Benigno and Benigno (2006) and Benigno, Benigno and Ghironi (2007) study how the indeterminacy regions of the parameter space vary with regard to different types of monetary policy rules in a dynamic general equilibrium model with two similar countries. De Fiore and Liu (2000) and Zanna (2003) study how determinacy of the equilibrium depends on the degree of openness of a small economy, among other things.

Another of our goals is to examine methodological aspects of solving small open economy models under rational expectations. As we discuss at length below, the 'butterfly effect' can be viewed as a result of an implicit assumption: expectations (in the small and large economy) are formed rationally with access to *full* information. Only if the equilibrium of the large economy is unique, is the small economy truly 'small'. Therefore, 'smallness' is a property of the unique REE of the large economy. It is not our goal here, however, to assess the empirical relevance of this mechanism.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 discusses indeterminacy. Section 4 presents our main findings and Section 5 concludes.

2. The Model

The model is a version of Galí and Monacelli's (2005) fully micro-founded, stochastic, dynamic, general equilibrium, sticky-price small open economy model. Some broad features of the model are: all output is tradable; prices are sticky as in Calvo (1983); there is full exchange rate pass-through; and there are complete securities markets.

We add foreign and domestic aggregate demand and supply shocks and keep the large economy in its structural form.[5] Instead of working through the details of the derivation, which are in Galí and Monacelli, we present the key log-linear aggregate relations.

2.1 The Large Economy

Variables with a *star superscript* correspond to the large economy, which can be described with a standard set of new Keynesian closed economy equations.[6]

Firms operate under monopolistic competition in the goods market and Calvo-price stickiness. Factor markets are competitive and goods are produced

[5] The terms *foreign* and *large* are used interchangeably.

[6] See Goodfriend and King (1997), Clarida, Galí and Gertler (1999), Woodford (2003) and Ireland (2004) for discussions of the new Keynesian closed economy model.

with a constant returns-to-scale technology. One can show that the Phillips curve in the large economy takes the form

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa x_t^* + v_{\pi,t}^* \tag{1}$$

where: π_t^* stands for the foreign inflation rate; x_t^* is the foreign output gap; $v_{\pi,t}^*$ is a foreign cost-push shock; the parameter κ is strictly positive and captures the degree of price rigidities; the household's discount factor, β, lies between zero and one; and E_t denotes expectations conditional on information at t.

The aggregate demand schedule (IS-curve) implies that the current level of the foreign output gap, x_t^*, depends on its expected future level, the *ex-ante* short-term real interest rate, foreign total factor productivity, a_t^*, and a foreign aggregate demand disturbance, $v_{x,t}^*$, as follows:

$$x_t^* = E_t x_{t+1}^* - \frac{1}{\sigma}\left(r_t^* - E_t \pi_{t+1}^*\right) - \phi_1(1-\rho_a^*)a_t^* + \frac{1-\rho_x^*}{\sigma} v_{x,t}^* \tag{2}$$

where: r_t^* is the foreign nominal short-term interest rate; σ is strictly positive and governs intertemporal substitution; ρ_a^* is the persistence of a_t^*; ρ_x^* is the persistence of $v_{x,t}^*$; and ϕ_1, defined for notational convenience, is $\frac{1+\varphi}{\sigma+\varphi}$, with $\varphi > 0$ governing the elasticity of labour supply.

Foreign monetary policy follows a Taylor rule of the form

$$r_t^* = \rho_r^* r_{t-1}^* + \alpha_\pi^* \pi_t^* + \alpha_x^* x_t^* + \varepsilon_{r,t}^* \tag{3}$$

where $\varepsilon_{r,t}^*$ is an independent and identically distributed (*iid*) foreign monetary policy shock, with zero mean and standard deviation $\sigma_{\varepsilon_r^*}$. Given the way in which the policy rule is written, α_π^* and α_x^* capture the short-run reaction of r_t^* to the deviation of foreign inflation from target (assumed to be zero) and the foreign output gap. So, values of $\alpha_\pi^*/(1-\rho_r^*)$ below unity correspond to violations of the Taylor principle and give rise to indeterminacy of the equilibrium.

The potential level of foreign output, $\overline{y}_t^*$, is the level that would prevail in the absence of nominal rigidities. For the large economy, it can be shown that the actual level of output, y_t^*, and the output gap, x_t^*, obey

$$x_t^* \equiv y_t^* - \overline{y}_t^* = y_t^* - \phi_1 a_t^*. \tag{4}$$

Foreign exogenous processes evolve according to

$$a_t^* = \rho_a^* a_{t-1}^* + \varepsilon_{a,t}^* \tag{5}$$
$$v_{\pi,t}^* = \rho_\pi^* v_{\pi,t-1}^* + \varepsilon_{\pi,t}^* \tag{6}$$
$$v_{x,t}^* = \rho_x^* v_{x,t-1}^* + \varepsilon_{x,t}^* \tag{7}$$

where: the shocks $\varepsilon_{a,t}^*$, $\varepsilon_{\pi,t}^*$ and $\varepsilon_{x,t}^*$ are *iid* with zero mean and standard deviations $\sigma_{\varepsilon_a^*}$, $\sigma_{\varepsilon_\pi^*}$ and $\sigma_{\varepsilon_x^*}$, respectively; the auto-regressive parameters, ρ_a^*, ρ_π^* and ρ_x^* are less than unity in absolute value.

2.2 The Small Open Economy

In the small open economy, the IS-curve implies that the output gap, x_t, is a function of its expected future value, the nominal interest rate, the expected rate of *domestically produced goods* inflation, the expected growth rate of foreign output, foreign and domestic aggregate demand shocks, and domestic total factor productivity. Following Galí and Monacelli (2005), one can show that the small open economy's IS-curve takes the form

$$\begin{aligned} x_t = {} & E_t x_{t+1} - \frac{1}{\sigma_\alpha}\left(r_t - E_t \pi_{h,t+1}\right) + \phi_3 E_t \Delta y_{t+1}^* + \\ & \frac{(1-\rho_x)(1-\phi_2)}{\sigma} v_{x,t} + \frac{1-\rho_x^*}{\sigma}\phi_3 v_{x,t}^* - \phi_4 (1-\rho_a) a_t \end{aligned} \tag{8}$$

where ρ_x and ρ_a are the persistence parameters of domestic demand and domestic productivity shocks, respectively. The parameters σ_α, ϕ_2, ϕ_3 and ϕ_4 are functions of deep parameters. In particular,

$$\begin{aligned} \sigma_\alpha &\equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} \\ \omega &\equiv \sigma\tau + (1-\alpha)(\sigma\iota - 1) \\ \phi_2 &\equiv \frac{\sigma_\alpha - \sigma}{\sigma_\alpha + \varphi} \\ \phi_3 &\equiv \alpha(\omega - 1) + \phi_2 \\ \phi_4 &\equiv \frac{1+\varphi}{\sigma_\alpha + \varphi} \end{aligned}$$

where: $\alpha \in [0,1]$ captures the degree of openness; τ is the intratemporal elasticity of substitution between foreign and domestically produced goods; and ι is the elasticity of substitution across varieties of foreign goods.[7]

The dynamics of *domestically produced goods* inflation, $\pi_{h,t}$, are governed by an analogous Phillips curve equation

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa_\alpha x_t + v_{\pi,t} \tag{9}$$

where: $\kappa_\alpha \equiv \lambda\left(\sigma_\alpha + \varphi\right)$; $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$; θ governs the degree of price stickiness; and $v_{\pi,t}$ is a cost-push shock.

Monetary policy in the small economy is assumed to follow a Taylor rule that sets the nominal interest rate, r_t, in response to its own lagged value, the deviation of *consumer price* inflation, π_t, from its target (assumed to be zero), and the output gap, x_t :

$$r_t = \rho_r r_{t-1} + \alpha_\pi \pi_t + \alpha_x x_t + \varepsilon_{r,t} \tag{10}$$

where $\varepsilon_{r,t}$ is an *iid* monetary policy shock with zero mean and standard deviation σ_{ε_r}.

The terms of trade, s_t, are defined (from the perspective of the large economy) as the price of foreign goods, $p_{f,t}$, in terms of the price of home goods, $p_{h,t}$. That is, $s_t = p_{f,t} - p_{h,t}$. Around a symmetric steady state the consumer price index is a weighted average of the form $p_t = (1-\alpha)p_{h,t} + \alpha p_{f,t}$. It is straightforward to show that $p_t = p_{h,t} + \alpha s_t$. From this equation it follows that *consumer price* inflation and *domestically produced goods* inflation are linked by the expression

$$\pi_t = \pi_{h,t} + \alpha \Delta s_t. \tag{11}$$

The nominal exchange rate, e_t, is defined as the price of foreign currency in terms of the domestic currency. The real exchange rate, q_t, in turn, is defined as $q_t \equiv e_t + p_t^* - p_t$. It then follows that changes in the nominal exchange rate, Δe_t, can be decomposed into changes in the real exchange rate and *consumer price* inflation differentials,

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^*. \tag{12}$$

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7 We refer the reader to Galí and Monacelli (2005) for the non-linear expressions that contain these structural parameters.

Positive values of Δe_t indicate a nominal depreciation of the domestic currency as the price of the foreign currency increases. Because the law of one price is assumed to hold $p_{f,t} = e_t + p_t^*$, which implies that the terms of trade can also be written as $s_t = e_t + p_t^* - p_{h,t}$. Combining these expressions, it is easy to show that the real exchange rate is proportional to the terms of trade. Thus,

$$\Delta q_t = (1-\alpha)\,\Delta s_t. \tag{13}$$

Complete international securities markets, together with the market clearing conditions, lead to the following relationship between the terms of trade, s_t, and output differentials and demand shock differentials:[8]

$$s_t = \sigma_\alpha(y_t - y_t^*) - \frac{\sigma_\alpha}{\sigma}\left(v_{x,t} - v_{x,t}^*\right). \tag{14}$$

The presence of the aggregate demand shock differential in Equation (14), $\left(v_{x,t} - v_{x,t}^*\right)$, alters the small economy's flexible price level of output, relative to Galí and Monacelli (2005).[9] The relationship between the actual level of output, y_t, and the output gap, x_t, satisfies the following equation:

$$x_t \equiv y_t - \bar{y}_t = y_t - \phi_2 y_t^* - \frac{\phi_2}{\sigma}(v_{x,t} - v_{x,t}^*) - \phi_4 a_t. \tag{15}$$

Finally, the exogenous domestic processes evolve according to

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_{a,t} && (16)\\ v_{\pi,t} &= \rho_\pi v_{\pi,t-1} + \varepsilon_{\pi,t} && (17)\\ v_{x,t} &= \rho_x v_{x,t-1} + \varepsilon_{x,t} && (18) \end{aligned}$$

where: the shocks, $\varepsilon_{a,t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$ are *iid* with zero mean and standard deviations σ_{ε_a}, σ_{ε_π} and σ_{ε_x}, respectively; the auto-regressive parameters, ρ_a, ρ_π and ρ_x, are less than unity in absolute value.

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[8] Demand shocks, $v_{x,t}$, enter the household's lifetime expected utility as follows: $E_0 \sum_{t=0}^{\infty} \beta^t e^{v_{x,t}} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$. Thus, one can show that aggregate demand disturbances enter the international risk-sharing condition as in Equation (14).

[9] One could show that the level of potential output in the small economy is given by $\phi_2 y_t^* + \frac{\phi_2}{\sigma}(v_{x,t} - v_{x,t}^*) + \phi_4 a_t$. If aggregate demand shocks were absent from our model, the expression for the output gap collapses back to that of Galí and Monacelli's.

2.3 Calibration

The benchmark calibration of the model yields a unique REE and resembles that of Galí and Monacelli (2005).[10] Our calibration is loosely based on data from the US and Australia and falls within the range of chosen values in the literature. The values assigned to the structural parameters are summarised in Table 1.

Table 1: Benchmark Calibration

Price stickiness	$\theta = 0.75$
Discount factor	$\beta = 0.99$
Intertemporal elasticity of substitution	$\sigma = 1.50$
Share of foreign goods in CPI basket	$\alpha = 0.40$
Elasticity of substitution between foreign varieties	$\tau = 1.1$
Elasticity of substitution between domestic and foreign goods	$\iota = 1.2$
Elasticity of labour supply	$\varphi = 2.0$
Interest rate smoothing	$\rho_r = 0.90$
Output gap response	$\alpha_x = 0.001$
Inflation response	$\alpha_\pi = 0.125$
Interest rate smoothing (large)	$\rho_r^* = 0.90$
Output gap response (large)	$\alpha_x^* = 0.001$
Inflation response (large)	$\alpha_\pi^* = 0.125$

The shape – but not the size – of the impulse responses are invariant to the standard deviations of the fundamental disturbances. The set of optimal policy results, however, are sensitive to these values.

The exogenous processes described by Equations (5), (6), and (7) and their domestic counterparts are known in the literature to be highly persistent.[11] We chose ρ_a^* to 0.95, $\rho_{v_x}^*$ to 0.96, $\rho_{v_\pi}^*$ to 0.98, and ρ_a, ρ_{v_π}, and ρ_{v_x} to 0.95.

Given the parameter values in Table 1, we set the standard deviations of the shocks in two steps. First, we calibrate the standard deviations of the large economy's shocks as follows: σ_{a^*} is set to 0.007 as suggested by Cooley and Prescott (1995).

10 Galí and Monacelli set $\sigma = \tau = \iota = 1$. For this special case, the small economy's real marginal cost is completely insulated from movements in foreign output. We chose to select a more general calibration, although our main findings hold in this special case as well.

11 See, for example, Ireland (2004).

Then, $\sigma_{v_{\pi}^{*}}$, $\sigma_{v_{x}^{*}}$ and $\sigma_{\varepsilon_{r}^{*}}$ are chosen to minimise the sum of squares deviations of the theoretical standard deviations of the interest rate, inflation, and the output gap from empirical counterparts.[12] The interest rate in the data is taken to be the quarterly average of the Federal funds rate, foreign inflation is measured as the quarterly growth rate of the US consumer price index, and the foreign output gap is measured as log deviations of US real quarterly GDP per capita from a linear trend over the sample period 1980:Q1–2006:Q4. This strategy yields the values summarised in Table 2.

Table 2: Benchmark Calibration – Foreign Shocks

$\sigma_{v_{x}^{*}} = 0.01982$
$\sigma_{a^{*}} = 0.00700$
$\sigma_{r^{*}} = 0.00002$
$\sigma_{v_{\pi}^{*}} = 0.00071$

Second, we take the large economy's parameter values as given and calibrate the standard deviation of the small economy's shocks in a similar way. The value of σ_a is also set to 0.007, and $\sigma_{v_{\pi}}$, $\sigma_{v_{x}}$ and $\sigma_{\varepsilon_{r}}$ are set to minimise the sum of squares deviations of the theoretical standard deviations of the small economy's interest rate, consumer price inflation, and the output gap from their empirical counterparts. For the small economy we use Australian data and take these to be the quarterly average of the nominal cash rate, the quarterly growth rate of the consumer price index, and log deviations of real quarterly GDP per capita from a linear trend; once again, all series are taken over the same sample period as before. This procedure yields the values summarised in Table 3.

Table 3: Benchmark Calibration – Domestic Shocks

$\sigma_{v_{x}} = 0.03713$
$\sigma_{a} = 0.00700$
$\sigma_{r} = 0.00066$
$\sigma_{v_{\pi}} = 0.00002$

12 The criterion that we seek to minimise is of the form: $\left(\sigma_r - \sigma_r^e\right)^2 + \left(\sigma_\pi - \sigma_\pi^e\right)^2 + \left(\sigma_x - \sigma_x^e\right)^2$, where σ_i stands for the model-generated standard deviation of variable i, and σ_i^e for its empirical counterpart.

3. Stable Rational Expectations Equilibria

3.1 Existence, Uniqueness and Multiplicity

For any variable, say π_t, the expectational error, η_t^{π}, is defined as $\pi_t - E_{t-1}\pi_t$, where E_{t-1} is the expectations operator conditional on information at $t-1$, and η_t^{π} satisfies $E_{t-1}\eta_t^{\pi} = 0$ for all periods t. As in Sims (2001), we collect the expectational errors in a $k \times 1$ vector η_t and write the model, given by Equations (1) to (18), in matrix form as follows:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi\varepsilon_t + \Pi\eta_t \tag{19}$$

where ε_t is a $l \times 1$ vector of fundamental serially uncorrelated random disturbances and the $n \times 1$ vector y_t contains the remaining variables, including conditional expectations.

Lubik and Schorfheide (2003) have extended Sims's (2001) solution for linear rational expectations models by characterising the full set of stable solutions. For completeness we provide a discussion below.

The QZ decomposition (generalised Schur decomposition) yields unitary matrices (complex matrices with orthonormal columns) Q and Z, and upper-triangular matrices Λ and Ω such that $\Gamma_0 = Q'\Lambda Z'$ and $\Gamma_1 = Q'\Omega Z'$. An important by-product of this decomposition is that it gives the generalised eigenvalues of Γ_0 and Γ_1 as the ratios of the diagonal elements of the matrices Λ and Ω. [13]

Next, we define a new set of variables, $w_t = Z'y_t$, and partition the resulting system into the m variables whose generalised eigenvalues are greater than 1, $w_{2,t}$, and those whose generalised eigenvalues are less than 1, $w_{1,t}$. Then, in Equation (19), we substitute the matrices Γ_0 and Γ_1 for their QZ decompositions and pre-multiply by Q to obtain

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} (\Psi\varepsilon_t + \Pi\eta_t)\,.$$

[13] A standard eigenvalue problem is of the form $Ax = \lambda x$. A generalised eigenvalue problem takes the form $Ax = \mu Bx$. The values of μ that satisfy this last equation are called the generalised eigenvalues of A and B.

In the expression above, the bottom set of equations, which can be rewritten as

$$\Omega_{22}^{-1}\Lambda_{22}w_{2,t} = w_{2,t-1} + \Omega_{22}^{-1}\left(Q_2\Psi\varepsilon_t + Q_2\Pi\eta_t\right) \tag{20}$$

governs the behaviour of the $m \times 1$ vector of purely explosive variables, $w_{2,t}$. For the system to be stable, the expectational errors, η_t, have to offset the impact that fundamental shocks, ε_t, have on the purely explosive variables, $w_{2,t}$. In other words, a stable solution requires that in all periods t,

$$Q_2\Pi\eta_t = -Q_2\Psi\varepsilon_t. \tag{21}$$

Every possible $l \times 1$ vector of fundamental shocks, ε_t, gives rise to a new $m \times 1$ vector $-Q_2\Psi\varepsilon_t$; and for every possible $-Q_2\Psi\varepsilon_t$, Equation (21) asserts that there exists a combination of the columns of $Q_2\Pi$ capable of producing exactly $-Q_2\Psi\varepsilon_t$. As Sims (2001) shows, a stable solution to Equation (19) exists if, and only if, each of the columns of $Q_2\Psi$ can be obtained as linear combinations of the columns of $Q_2\Pi$.[14]

Note that $Q_2\Pi$ has dimensions $m \times k$. So there are m equations and k expectational errors to be determined. If the number of explosive variables, m, equals the number of expectational errors, k, and each of the m equations is independent of one another, then $Q_2\Pi$ is a full rank matrix and the expectational errors are unique linear combinations of the fundamental shocks. That is,

$$\eta_t = -\left(Q_2\Pi\right)^{-1}Q_2\Psi\varepsilon_t. \tag{22}$$

In this case, Equation (19) has a unique solution and the dynamics of y_t are exclusively driven by fundamental shocks.

It is possible, however, that Equation (21) does not determine η_t in a unique manner. For instance, if k exceeds m, Equation (21) can be satisfied for infinitely many values of η_t. In this way, the system admits expectational errors that are unrelated to the fundamental disturbances.

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[14] Sims's condition is actually more general because it allows any pattern of serial correlation in ε_t. The condition reduces to Equation (21) under our assumption of serially uncorrelated disturbances. Unless otherwise stated, the existence condition given by Equation (21) always holds in our analysis. Stability also requires the initial condition $w_{2,0} = 0$.

Even in the complete absence of fundamental shocks, expectational errors can become a source of fluctuations. To see this, suppose that $k > m$ and $\varepsilon_t = 0$ for all t. Then Equation (21) reduces to the homogeneous system $Q_2 \Pi \eta_t = 0$. Since there are more unknowns than equations, it follows that $Q_2 \Pi \eta_t = 0$ has a non-trivial solution $(\eta_t \neq 0)$, and, in fact, infinitely many of them. Continue to assume that $\varepsilon_t = 0$ and $\eta_t \neq 0$ for all t, then Equation (19) becomes $\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Pi \eta_t$, which explicitly shows that the dynamics of y_t are, in this case, exclusively a function of non-fundamental shocks.

Formally, the variation in η_t that may arise under indeterminacy, and that is unrelated to the variation in ε_t, is the result of sunspot shocks, which we denote ξ_t.[15]

3.2 Expectations and Size

The small open economy model given by Equations (1) to (18) has a particular structure meant to capture the size differences of the two economies. The large economy, described by Equations (1) to (7), can be solved in isolation without reference to any other equation in the system. Thus, the large economy can be written as a self-contained system as follows:

$$\Gamma_0^* y_t^* = \Gamma_1^* y_{t-1}^* + \Psi^* \varepsilon_t^* + \Pi^* \eta_t^* . \tag{23}$$

Proceeding as before, the stability condition of the large economy is

$$Q_2^* \Pi^* \eta_t^* = -Q_2^* \Psi^* \varepsilon_t^* . \tag{24}$$

Note that if the solution to Equation (24) is unique, the k^* foreign expectational errors, η_t^*, are, exclusively, linear combinations of the l^* foreign fundamental shocks, ε_t^*.

[15] See Lubik and Schorfheide (2003) for an expression of the expectational errors as linear combinations of fundamental shocks, ε_t, and non-fundamental shocks, ξ_t. The only restriction on the distribution of sunspots shocks is that they follow a martingale difference sequence; that is, $E_{t-1}\xi_t = 0$.

It is possible to partition y_t into the n^* foreign variables, y_t^*, and $\bar{n}$ remaining ones, $\bar{y}_t$; ε_t and η_t can be partitioned in a similar manner, so that Equation (19) can be written as follows:

$$\begin{pmatrix} \Gamma_0^* & 0 \\ \Gamma_0^{21} & \Gamma_0^{22} \end{pmatrix} \begin{pmatrix} y_t^* \\ \bar{y}_t \end{pmatrix} = \begin{pmatrix} \Gamma_1^* & 0 \\ \Gamma_1^{21} & \Gamma_1^{22} \end{pmatrix} \begin{pmatrix} y_{t-1}^* \\ \bar{y}_{t-1} \end{pmatrix} + \begin{pmatrix} \Psi^* & 0 \\ \Psi^{21} & \Psi^{22} \end{pmatrix} \begin{pmatrix} \varepsilon_t^* \\ \bar{\varepsilon}_t \end{pmatrix} + \begin{pmatrix} \Pi^* & 0 \\ \Pi^{21} & \Pi^{22} \end{pmatrix} \begin{pmatrix} \eta_t^* \\ \bar{\eta}_t \end{pmatrix}. \tag{25}$$

The dimensions of the sub-matrices, Γ_0^*, Γ_0^{21},...,Π^{22}, are conformable to the partition. The stability condition in its partitioned form satisfies

$$\begin{pmatrix} Q_2^*\Pi^* & 0 \\ Q_2\Pi^{21} & Q_2\Pi^{22} \end{pmatrix} \begin{pmatrix} \eta_t^* \\ \bar{\eta}_t \end{pmatrix} = - \begin{pmatrix} Q_2^*\Psi^* & 0 \\ Q_2\Psi^{21} & Q_2\Psi^{22} \end{pmatrix} \begin{pmatrix} \varepsilon_t^* \\ \bar{\varepsilon}_t \end{pmatrix} \tag{26}$$

and the sub-matrices conform also to the partition obtained from the QZ decomposition: $m = m^* + \bar{m}$.

If the equilibrium of the large economy is unique, it follows that

$$\eta_t^* = - \left(Q_2^*\Pi^*\right)^{-1} Q_2^*\Psi^*\varepsilon_t^* \tag{27}$$

which, in turn, implies that we can solve the second set of equations in Equation (26) as

$$Q_2\Pi^{22}\bar{\eta}_t = \left(Q_2\Pi^{21} \left(Q_2^*\Pi^*\right)^{-1} Q_2^*\Psi^* - Q_2\Psi^{21}\right) \varepsilon_t^* - Q_2\Psi^{22}\bar{\varepsilon}_t. \tag{28}$$

Equations (27) and (28) highlight an important aspect of the structure of the model: uniqueness in the large economy is sufficient to ensure that its dynamics are driven only by its fundamental shocks, regardless of whether or not the small economy's expectational errors, $\bar{\eta}_t$, are uniquely determined by its fundamentals. As one would expect, indeterminacy which originates in the small economy does not affect the equilibrium dynamics of the large economy. The converse, as one would also expect, is that indeterminacy caused by the large economy affects the determination of $\bar{\eta}_t$ and the dynamics of the small economy.

Surprisingly, perhaps, indeterminacy arising in the large economy opens up a channel through which shocks to the small economy, $\bar{\varepsilon}_t$, influence the determination of η_t^*, and therefore the dynamics of the large economy. Put differently, foreign indeterminacy allows the small economy's shocks, $\bar{\varepsilon}_t$, to act exactly like non-fundamental shocks for the large economy. The small economy's shocks, $\bar{\varepsilon}_t$, can be described in this way because they do not influence the dynamics of the large economy under uniqueness. This is the 'butterfly effect': a situation in which the failure to pin down the equilibrium of the large economy allows developments in the small economy to affect developments in the large one.

To see this mechanism, consider Equation (26), and assume that $m^* < k^*$ and $\bar{m} = \bar{k}$. Clearly, indeterminacy in the large economy translates into indeterminacy for the whole system because $m < k$. In this case, $Q_2\Pi$ is not an invertible matrix because of the rank deficiency stemming from the large economy's equations. Nevertheless, the full set of solutions can be calculated with the generalised inverse of $Q_2\Pi$, which we denote by $(Q_2\Pi)^+$:[16]

$$\begin{pmatrix} \eta_t^* \\ \bar{\eta}_t \end{pmatrix} = -(Q_2\Pi)^+ Q_2\Psi \begin{pmatrix} \varepsilon_t^* \\ \bar{\varepsilon}_t \end{pmatrix}.$$

It is important to observe that the decrease in m^* caused by foreign indeterminacy (relative to the $m^* = k^*$ case), removes a set of zero-restrictions from Equation (26) – restrictions that were necessary to isolate the large economy from the small one.

Formally, the vector of expectational errors, η_t, belongs to $\mathbb{R}^k$. For the sub-vector $\eta_t^* \in \mathbb{R}^{k^*}$ to equate to the same value as the one that would have been obtained had we solved for the foreign system (Equation (23)) in isolation, none of the zero-restrictions that show up in the full rank version of Equation (26) can be removed. The 'butterfly effect' appears because, in solving for η_t in $\mathbb{R}^k$,

[16] The solution shown above is the minimum distance solution which satisfies Equation (21). The general solution to the inhomogeneous system $Q_2\Pi\eta_t = -Q_2\Psi\varepsilon_t$ is the sum of a particular solution of the inhomogeneous system and the general solution of the corresponding homogeneous system $Q_2\Pi\eta_t = 0$. Under the assumption that $Q_2\Pi$ is a full row rank matrix, $\left(Q_2\Pi\,(Q_2\Pi)'\right)^{-1}$ exists. Then the solution η_t can always be written as the sum of the generalised inverse solution of the inhomogeneous system and a solution of the homogeneous system: $\eta_t = -(Q_2\Pi)^+ Q_2\Psi\varepsilon_t + \left(I - (Q_2\Pi)'\left(Q_2\Pi\,(Q_2\Pi)'\right)^{-1} Q_2\Pi\right) z_t$, where the vector z_t is arbitrary (apart from its dimensionality).

foreign indeterminacy effectively takes away some of the zero-restrictions that were necessary to obtain the same solution for η_t^* that would have been obtained had we solved for the large economy in isolation, in $\mathbb{R}^{k^*}$.

If the large economy is solved in isolation under indeterminacy, the 'butterfly effect', of course, can never occur. To justify this approach requires the additional assumption that agents in the large economy form their expectations solely on the basis of information from that economy alone. In this case, the set of multiple solutions of the large economy can be computed with the generalised inverse of $Q_2^*\Pi^*$. The foreign expectational errors are then given by $\eta_t^* = -\left(Q_2^*\Pi^*\right)^+ Q_2^*\Psi^*\varepsilon_t^*$, in which case $\bar{\varepsilon}_t$, by construction, can never influence η_t^*.[17] Solving the large economy first and then using its solution as an exogenous process for the small economy, or solving them simultaneously, is equivalent only if the equilibrium of the large economy is unique.

Thus, uniqueness of the large economy's equilibrium constrains the formation of expectations to a subset of the full information set. 'Smallness' is then a property that emerges from the unique determination of the large economy's equilibrium, but not a general property of the system.

4. Results

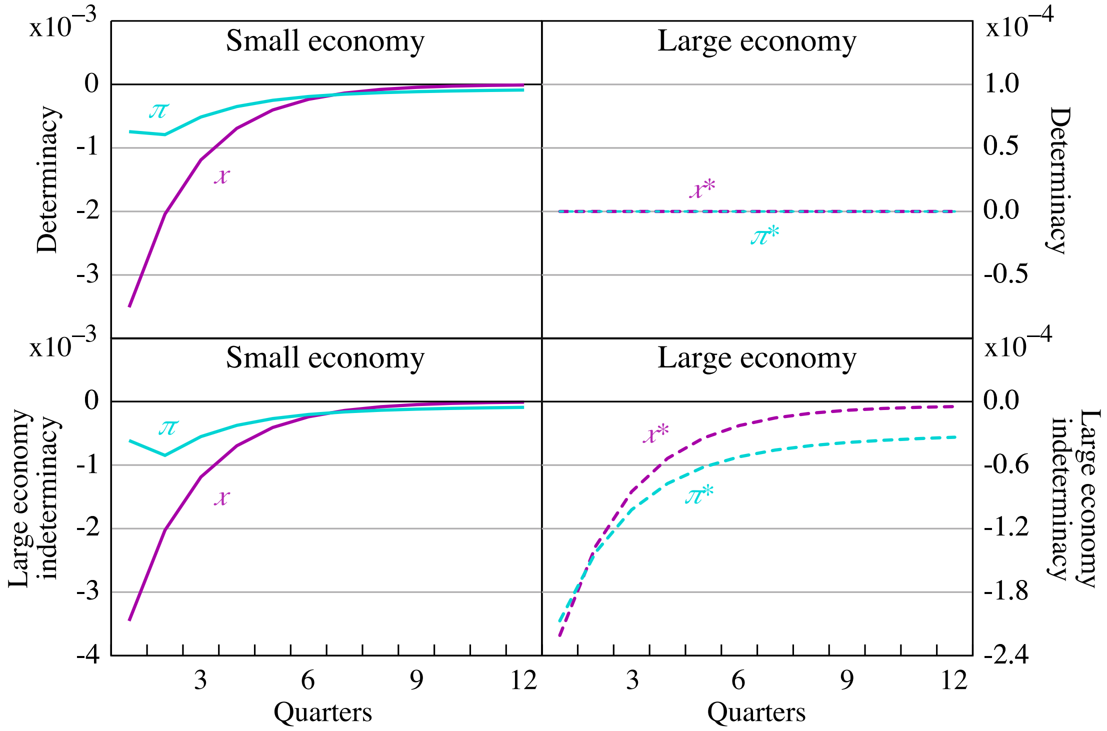
This section discusses two set of results: the dynamics of the model under determinacy and indeterminacy; and the implications for domestic monetary policy of foreign-induced indeterminacy.

4.1 Dynamics

Figure 1 illustrates the impulse response functions of inflation and the output gap for the small and large economies after a positive productivity shock, ε_t^a, to the small economy. The top panels of the figure show the responses under the benchmark calibration ($\alpha_\pi = 0.125$ and $\alpha_\pi^* = 0.125$), which yields a unique REE.

[17] The solution shown here is the minimum distance solution which satisfies Equation (24). The general solution to the inhomogeneous system given by Equation (24) is the sum of a particular solution of the inhomogeneous system and the general solution of the corresponding homogeneous system $Q_2^*\Pi^*\eta_t^* = 0$.

Figure 1: Impulse Responses to a Domestic Productivity Shock



The bottom panels show the same responses under foreign indeterminacy (with $\alpha_\pi^* = 0.075$).[18]

Under determinacy, the solution to Equation (19) is unique, and the effects of the shock to the small economy are similar to those found elsewhere in the literature: a positive productivity shock decreases inflation, raises output, and raises potential output even more. And as expected, the large economy's variables do not react to this domestic shock.[19]

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18 Of the infinitely many solutions, we select the one given by Sims's (2001) code, which coincides with Lubik and Schorfheide's (2003) solution where, M_1 (in their notation), is set to zero. It is important to note, however, that the 'butterfly effect', as we discussed above, is a general feature of any of the infinitely many solutions; and not only of the infinitely many solutions to this particular model, but of the infinitely many solutions of any model that satisfies the partition given in Equation (25). Obviously, as discussed above, the 'butterfly effect' is not a feature of the infinitely many solutions that would arise under domestically induced indeterminacy.

19 If the solution to Equation (19) is unique, then so is the solution to Equation (23).

The bottom panels of Figure 1 illustrate the 'butterfly effect'. Indeterminacy generated in the large economy allows foreign variables to respond to the same shock, ε_t^a. The responses of foreign variables are not large in magnitude; these are an order of magnitude smaller than those of domestic variables, but they are clearly not zero. Because this response would not have occurred if α_π^* had remained at its original determinacy consistent value, one can interpret ε_t^a as a sunspot shock for the large economy. In this way, the theory yields a plausible fundamental interpretation for sunspot shocks. This finding, that shocks from the small open economy operate like sunspot shocks for the large economy is, to the best of our knowledge, new in the literature.

Figure 2 examines the responses to two types of sunspot shocks. The top panels show the impulse responses of inflation and the output gap to a sunspot shock if indeterminacy is domestically induced ($\alpha_\pi = 0.075$). The bottom panels show impulse responses for the same variables to a sunspot shock if indeterminacy is induced abroad ($\alpha_\pi^* = 0.075$).

Figure 2: Impulse Responses to Sunspot Shocks

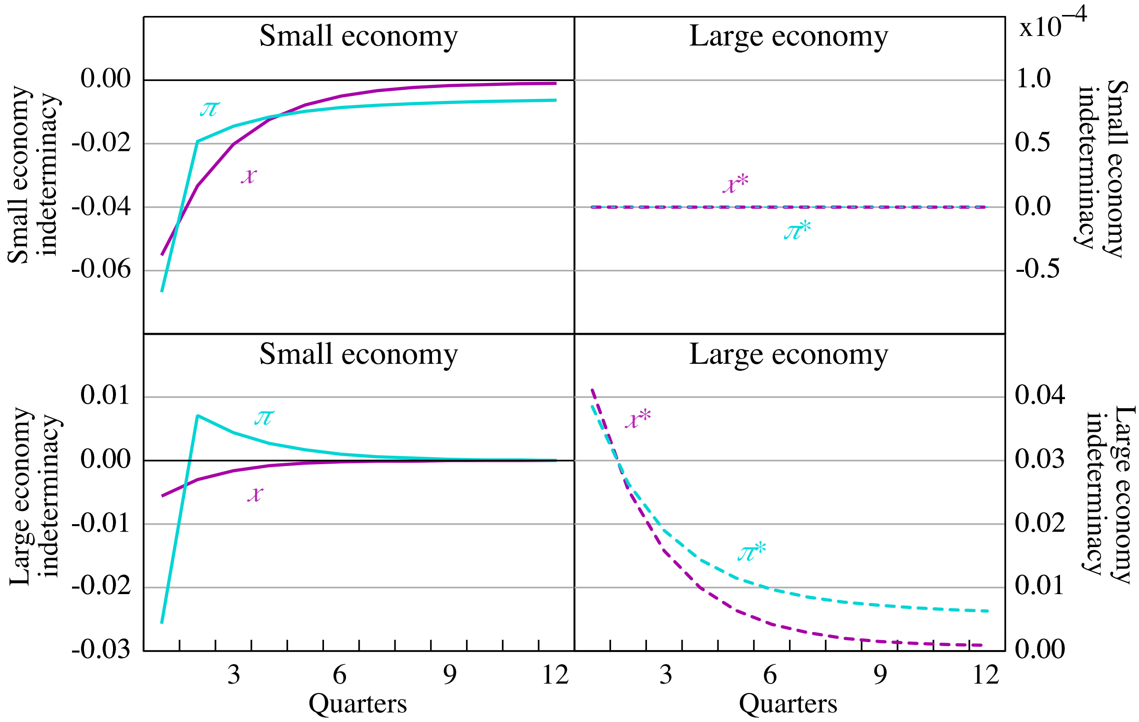
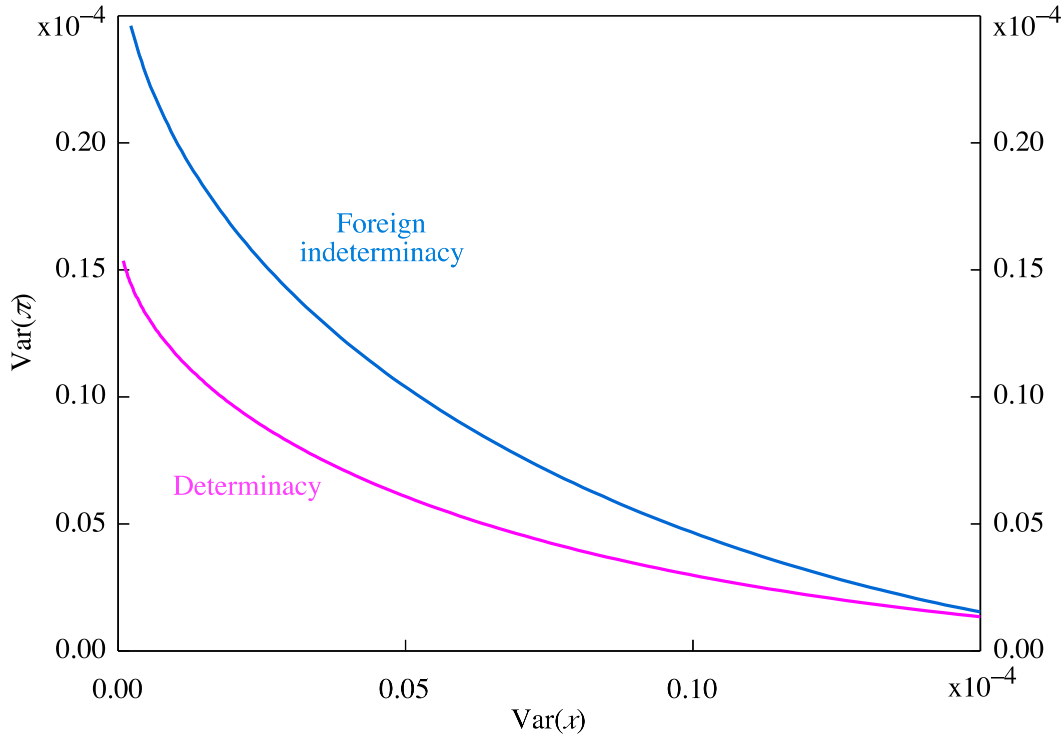


Figure 2 highlights important points that we have already discussed. It is not indeterminacy in itself that gives rise to a 'butterfly effect', but rather indeterminacy in the large economy. The top right-hand panel of Figure 2 shows that when monetary policy in the small economy is the source of indeterminacy, the large economy remains insulated from the effects of this shock. Neither fundamental nor sunspot shocks originating from the small economy affect the large economy as long as monetary policy in the large economy is consistent with determinacy. Only if the large economy pursues indeterminate policies do both economies become susceptible to sunspot fluctuations.

4.2 Taylor Curves

This section examines the extent to which foreign indeterminacy translates into additional volatility of the small open economy. It also examines the policy-maker's ability to mitigate the impact of (foreign) sunspot shocks on the domestic economy.

Figure 3: Taylor Curves



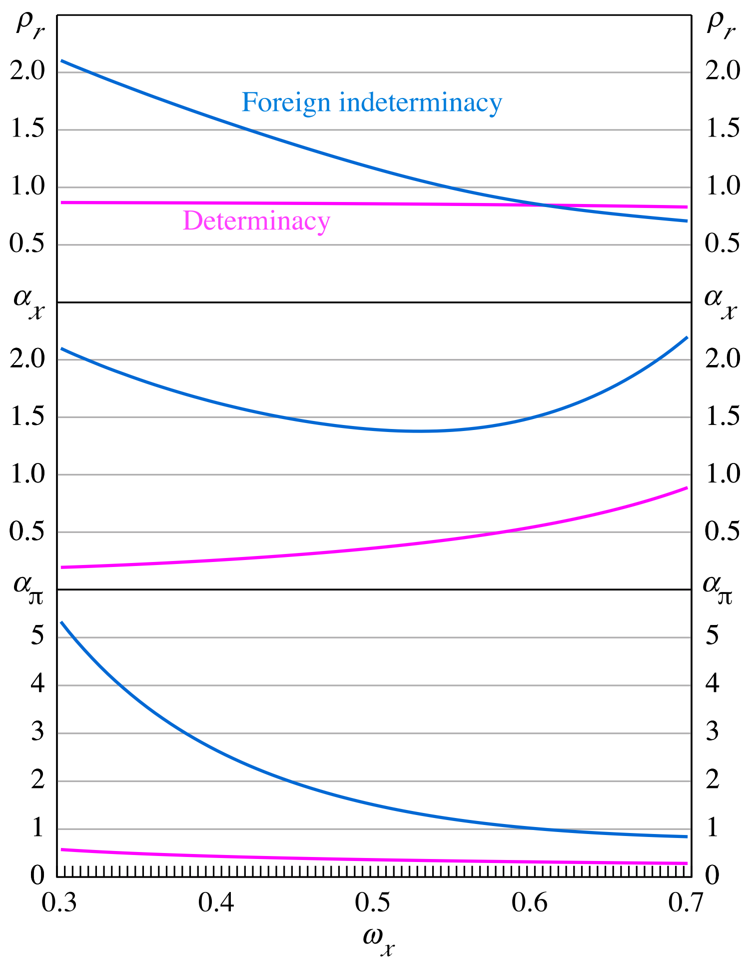
The objective of the policy-maker in the small economy is to set the parameters of the policy rule in Equation (10) so as to minimise a loss that is a weighted average of the variances of the output gap and consumer price inflation given by

$$\min_{\rho_r,\alpha_\pi,\alpha_x} \omega_x\sigma_x^2 + (1-\omega_x)\,\sigma_\pi^2$$

where ω_x denotes the weight attached to the stabilisation of the output gap. Figure 3 shows two Taylor curves: one for which all parameters, except those of the policy rule, of course, satisfy our benchmark calibration (with foreign determinacy); and another one for which foreign monetary policy violates Taylor's principle, $\alpha_\pi^* = 0.075$. For the latter, we add a sunspot shock with a standard deviation of 0.017, which is close to that of the foreign aggregate demand shock, and we use the same indeterminacy solution as before.

Foreign indeterminacy, and the sunspot shock associated with it, shifts the domestic policy-maker's efficient frontier and makes previously efficient points infeasible. In response to this higher volatility, we find that monetary policy's best response is to become more aggressive. This is illustrated in Figure 4, which shows the optimal parameter values of the policy rule associated with each of the Taylor curves of Figure 3.

Figure 4: Optimal Taylor Rule Coefficients



5. Conclusion

This paper studies some implications of indeterminacy of the rational expectations equilibrium for a small open economy. With a version of the canonical sticky-price small open economy model, we find that indeterminacy in the large economy can increase the volatility of the small economy as it exposes the small economy to endogenous volatility. We show that a plausible monetary policy response in this situation is likely to involve a more aggressive response to deviations of inflation from the target and of output from potential. We also show that fundamental shocks for the small economy can act like non-fundamental shocks for the large economy.

But, perhaps more importantly, we find that ‘smallness’ is not a general property of the model but, instead, a property of the large economy’s unique determination. This finding, we think, is methodologically important, since even if the ‘butterfly effect’ is thought to be merely an inconvenient theoretical result, it still requires careful consideration of the assumptions underpinning small open economy models. In particular, smallness can be guaranteed in one of two ways. First, by limiting the analysis to unique solutions which, in the case of the model presented here, can be achieved by satisfying the Taylor principle. Or, second, by restricting the information available to agents in the large economy to the set of information in that economy alone.

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