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Abstract

Expectations of the future play a key role in the transmission of monetary policy. Over recent years, a lot of theoretical and applied macroeconomic research has been based on the assumption of rational expectations. However, estimated models based on this assumption typically fail to capture the dynamics of the economy unless mechanical sources of persistence, such as habit formation in consumption and/or indexation to past prices, are imposed. This paper develops and estimates a small open economy model for Australia assuming two different types of expectations: rational expectations and learning. Learning – where expectations are formed by extrapolating from the historical data – can be an alternative means to generate the persistence observed in the data.

The paper has four key findings. First, learning does not reduce the importance of conventional mechanical forms of persistence. Second, despite this, the model with learning is able to generate real exchange rate dynamics that are consistent with empirical models but which are absent in standard theoretical models. Third, there is some tentative evidence that learning is preferred over rational expectations in terms of fitting the data. Fourth, since the adoption of inflation targeting, agents appear to be using a longer history of data to form their expectations, consistent with greater stability of inflation.

JEL Classification Numbers: E32, E52 , E63, F41

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1. Introduction

Forward-looking expectations are fundamental to the monetary policy transmission mechanism in modern macroeconomic models. The hypothesis of rational expectations is the standard paradigm for the formation of expectations in these models.[1] Rational expectations assumes that agents – firms, households and policy-makers – have complete knowledge of the economy including its precise structure (the model and its parameters). In reality, however, economic and policy decisions are made under incomplete knowledge about the economy. This paper studies situations where economic agents have less precise knowledge than is presumed by rational expectations. It is assumed that economic agents engage in learning as they try to improve their understanding of the economy and make forecasts upon which they base their decisions.

Rational expectations models, in which agents are completely forward-looking, can sometimes exhibit unrealistic dynamic properties; in particular, households and firms will adjust their behaviour immediately in response to future anticipated events. In reality the economy does not 'jump' about in this fashion. So in order to match the empirical features of the data, models require mechanical sources of persistence, such as habit formation in consumption and indexation to past prices. These features partly reflect the fact that it is often costly for households and firms to change their behaviour rapidly, but it can also be argued that such modelling techniques lack a firm theoretical grounding. A plausible alternative is that agents are not as well informed as rational expectations assumes. Instead, they have to form expectations on the basis of limited information. One possibility is that agents behave as if they use an econometric learning algorithm to form

1 For example, see Edge, Kiley and Laforte (2007) for the United States, Adolfson *et al* (2007) for Sweden, Christoffel, Coenen and Warne (2008) for the euro area and Jääskelä and Nimark (2008) for Australia.

their expectations.[2] Intuitively, when agents are uncertain about their economic environment, exogenous shocks lead to revisions of beliefs over time, which may draw out the effects of a shock.

Recently, a number of papers have attempted to quantify whether learning is important empirically using closed economy models. Milani (2007) finds that learning generates persistence that can be a substitute for the inertia generated by indexation and habit formation in an equivalent rational expectations model. Using a larger new Keynesian model with a similar learning mechanism, Slobodyan and Wouters (2009) find that learning fits the data equally well or better than rational expectations. However, they argue that learning complements the canonical model but does not provide a substitute source of persistence. They suggest that Milani's (2007) result is a product of using a model that has a much poorer fit under rational expectations. Murray (2008) takes this a step further to conclude that learning actually makes the model worse in terms of forecastability. He estimates a new Keynesian model that falls somewhere between Milani (2007) and Slobodyan and Wouters (2009) in terms of its size, and finds that learning accounts for some, but not all, persistence. However, unlike Slobodyan and Wouters (2009), he finds that impulse responses are different for the two expectation assumptions.

We extend this line of research by considering the effect of learning in an open economy model. A variable of particular interest is the real exchange rate. Evidence from vector autoregression (VAR) models suggests that the response of the real exchange rate to an unexpected change in monetary policy is delayed with a peak effect after about one year (Eichenbaum and Evans 1995; Faust and Rogers 2003). This stands in contrast to standard structural general equilibrium models for which the peak effect typically occurs within the quarter, followed by relatively rapid reversion to the mean, consistent with the theory of exchange rate overshooting (Dornbusch 1976). One commonly applied method of generating more persistence to match the observed exchange rate behaviour

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2 For a textbook treatment see Evans and Honkapohja (2001) and for a recent survey of articles see Evans and Honkapohja (2007) and Sargent, Williams and Zha (2006).

is to assume that financial markets are imperfectly integrated. This implies that the exchange rate is subject to a stochastic 'risk-premium' shock – which is added to the uncovered interest rate parity (UIP) condition (see, for instance, Benigno 2009). This potentially very persistent shock weakens the link between the exchange rate and its fundamental determinants.

An alternative method of matching the behaviour of the real exchange rate, while satisfying the UIP condition, may be provided by learning. The idea here is that the process of learning can slow down the adjustment of the real exchange rate to economic shocks.

To examine the implications of learning, we estimate a small open economy model for Australia. We examine the effect of the expectations assumption by comparing a model with constant-gain learning[3] to one with the standard rational expectations assumption. While we find that learning can replace some of the structural inertia in the model, it strengthens the role of habit formation somewhat. The impulse response functions in the learning model exhibit more persistence than those of the rational expectations model. At least part of this is due to learning rather than a shift in some of the estimated parameters. There is also some evidence that the learning model is preferred by the data. Further, we show that there has been a downward shift in the constant-gain learning parameter after the introduction of inflation targeting – that is, agents use a longer run of data when forming expectations – consistent with greater macroeconomic stability, particularly of inflation and interest rates.

The remainder of the paper is structured as follows. Section 2 outlines the key features of the model. Section 3 describes the solution of the model under rational expectations and learning as well as the estimation technique. Section 4 presents the estimation results for the different versions of the model and examines the impulse response functions of key variables to a monetary policy shock and a productivity shock. Section 5 extends the sample period and allows for a break in the speed of learning. Section 6 concludes.

3 Constant-gain learning does not weigh all earlier observations equally but discounts past data. This makes sense if the economy is subject to structural change over time.

2. A Small Open Economy DSGE Model

This section sketches the building blocks of the small open economy dynamic stochastic general equilibrium (DSGE) model that we estimate. The model closely follows Justiniano and Preston (2010) and Nimark (2007). The log-linearised equations are given in Appendix A.[4]

2.1 Households

The economy consists of a continuum of identical households who maximize their lifetime utility over consumption and leisure. Each household i has the following preference for lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{exp(v_t^d)(C_t(i) - \eta H_t)^{1-\gamma}}{(1-\gamma)} - \frac{N_t(i)^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where: β is the household's rate of time preference; C_t is the time t consumption bundle consisting of both domestically produced and imported consumption goods; $H_t = C_{t-1}$ captures external habit formation; N_t is hours of labour supplied; γ is the inverse elasticity of intertemporal substitution; φ is the inverse elasticity of labour supply; η is the degree of habit formation; and v_t^d is an exogenous consumption preference shock that follows the AR(1) process

$$\begin{aligned} v_t^d &= \rho_d v_{t-1}^d + \varepsilon_t^d \\ \varepsilon_t^d &\sim N(0, \sigma_d^2) \end{aligned} \tag{2}$$

Households are constrained by the following nominal budget constraint:

$$B_{t+1} + S_t B_{t+1}^* + P_t C_t = W_t N_t + \Pi_t + r_{t-1} B_t + r_{t-1}^* S_t B_t^* \Phi(a_t^*, v_t^s) \tag{3}$$

where expenditures appear on the left-hand side and sources of income on the right-hand side. Households work in a perfectly competitive labour market and earn a nominal wage (W_t) for every hour (N_t) of labour supplied. As owners of monopolistic firms, they also receive profits (Π_t). Households allocate their income each period between domestic bonds (B_{t+1}), foreign bonds (B_{t+1}^*) and

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[4] A step-by-step derivation is available from the authors on request.

consumption goods C_t (P_t denotes the CPI price index);[5] S_t is the nominal exchange rate defined such that an increase in S_t implies a depreciation of the domestic currency. For domestic bonds and foreign bonds the nominal yields are r_t and r_t^*, respectively. There is a premium $\Lambda(a_t^*, v_t^s) = e^{-\phi_a a_t^* + v_t^s}$ associated with having a net foreign position, which is a function of the real net holding of foreign bonds $\left(a_t^*\right)$ and a risk premium shock $\left(v_t^s\right)$. This is necessary to ensure a well-defined steady state (Benigno 2009).[6] The evolution of net foreign assets $\left(a_t^* = \frac{S_t B_{t+1}^*}{P_t^d}\right)$ at the aggregate level satisfies the following equation (where we have assumed a zero net supply of domestic bonds):

$$a_t^* = \frac{S_t P_t^x}{P_t^d} X_t - \frac{S_t P_t^*}{P_t^d} C_t^m + r_t^* \frac{S_t}{\pi_t^d S_{t-1}} a_{t-1}^* \Lambda(a_t^*, v_t^s) \tag{4}$$

The risk premium shock follows the AR(1) process

$$\begin{aligned} v_t^s &= \rho_s v_{t-1}^s + \varepsilon_t^s \\ \varepsilon_t^s &\sim N(0, \sigma_s^2) \end{aligned} \tag{5}$$

Arbitrage implies that the expected marginal utility from domestic bonds must match that of foreign bonds, which leads to the uncovered interest rate parity condition, adjusted for the foreign bond premium:

$$\mathbb{E}_t \left((\Lambda(a_t^*, v_t^s) \frac{S_{t+1}}{S_t} \right) = \frac{r_t}{r_t^*} \tag{6}$$

2.2 Firms

There are two types of firms: producers and importers. Producers manufacture a single differentiated good in a monopolistically competitive market. This good can be sold to the producers' domestic market or to importers in the foreign economy. Domestic exporters sell the final domestic good at price $P^x = P^d / S$ so there is

5 The consumption bundle C_t is a standard constant elasticity of substitution (CES) aggregated index of domestically produced and imported bundles C_t^d and C_t^m; P_t^d and P_t^m refer to the price indices of these bundles.

6 If the domestic economy is a net borrower, households are charged a premium on the foreign interest rate. On the flip side, net lender economies receive a premium on the foreign interest rate.

complete exchange rate pass-through in the export market. Output of production firms is given by:

$$Y_t = exp(v_t^a)N_t \tag{7}$$

where: Y_t is the quantity of the domestic good produced; N_t is hours of labour; and v_t^a is a productivity shock that follows the AR(1) process

$$\begin{aligned} v_t^a &= \rho_a v_{t-1}^a + \varepsilon_t^a \\ \varepsilon_t^a &\sim N(0, \sigma_a^2) \end{aligned} \tag{8}$$

Domestic importers purchase the foreign variety of the good and resell it in the domestic market. The import market is competitive and thus importers are unable to influence the price that they pay. However, they are assumed to have pricing power when selling to the domestic market.

We introduce price rigidities suggested by Galí and Gertler (1999) to capture the observed inertia in domestic and imported consumption good inflation. Each period, only a fraction of firms are able to change their price (a fraction θ^d of firms producing domestically and a fraction θ^m of importing firms do not change prices in a given period). A fraction ω of the domestic producers and importers that do change prices use a rule-of-thumb that links their price to lagged inflation (in their own sector). The rest $(1 - \omega)$ set their prices optimally. The log-linearised new Keynesian Phillips curves for domestic goods and imports are

$$\pi_t^j = \lambda^j mc_t^j + \mu_f^j \mathbb{E}\pi_{t+1}^j + \mu_b^j \pi_{t-1}^j + v_t^j \tag{9}$$

$$j \in \{d, m\}, \qquad \lambda^j = \frac{(1-\omega)(1-\theta^j)(1-\theta^j\beta)}{\theta^j + \omega(1-\theta^j(1-\beta))}$$

$$\mu_f^j = \frac{\beta\theta^j}{\theta^j + \omega(1-\theta^j(1-\beta))}, \mu_b^j = \frac{\omega}{\theta^j + \omega(1-\theta^j(1-\beta))}$$

where: j refers to domestic (d) or imported (m) good sectors; mc_t^j is the marginal cost of production; and π_t^j is inflation. There is a cost-push shock (v_t^m) in the import Phillips curve (but not in the domestic Phillips curve), which follows an AR(1) process of

$$\begin{aligned} v_t^m &= \rho_m v_{t-1}^m + \varepsilon_t^m \\ \varepsilon_t^m &\sim N(0, \sigma_m^2) \end{aligned} \tag{10}$$

2.3 Monetary Policy

The nominal interest rate (r_t) is set according to a Taylor rule based on lagged interest rates and time t–1 information on inflation and output:

$$r_t = \phi_r r_{t-1} + (1-\phi_r)[\phi_y y_{t-1} + \phi_\pi \pi_{t-1}] + v_t^r \tag{11}$$

where: all variables are expressed as logarithmic deviations from the steady state; y_t is the domestic output gap; π_t is CPI inflation; and v_t^r is an exogenous monetary policy shock.

McCallum (1999) argues that rules with this feature fit the data better because the informational assumptions are more realistic. Bullard and Mitra (2002) show that under learning this particular form of the Taylor rule has superior stability properties for a wide variety of parameter combinations.

2.4 The World Economy

The foreign economy (the rest of the world) is a large economy. We represent the foreign economy as an unrestricted VAR(1) of output, inflation and the nominal interest rate:

$$\begin{pmatrix} y_t^* \\ \pi_t^* \\ r_t^* \end{pmatrix} = M \begin{pmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ r_{t-1}^* \end{pmatrix} + \varepsilon_t^* \tag{12}$$

where ε_t^* is the world shock vector and y_t^*, π_t^*, r_t^* are respectively foreign output (linearly detrended), inflation and the nominal interest rate expressed as percentage deviations from their sample means. The foreign economy is treated as exogenous to the domestic economy and thus the coefficient matrix M is estimated separately from the rest of the model.

3. Estimating the Small Open Economy Model

We estimate the model using likelihood-based Bayesian methods. This approach has been employed to estimate rational expectations DSGE models in the recent literature.[7]

7 An and Schorfheide (2007) and Smets and Wouters (2003), for example.

3.1 The Law of Motion

Under both rational expectations and learning, the log-linear equations (given in Appendix A) can be summarised by the following system:

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + \Gamma_2 \widehat{E}_t x_{t+1} + \Gamma_3 v_t \tag{13}$$

$$v_t = \Gamma_4 v_{t-1} + \varepsilon_t \tag{14}$$

where: x_t is a vector of the model variables, a subset (x^f) of which are forward-looking variables (see Equation (A22)); v_t is a vector of exogenous variables (see Equation (A23)); ε_t is a vector of stochastic shocks; and $\widehat{E}$ is a possibly non-rational expectations operator. Matrices Γ_j are provided in Appendix A.

If the expectations operator, $\widehat{E}$, was rational we could proceed from here in the usual way and find a solution to the model using standard techniques such as those provided in Sims (2001) and Söderlind (1999). However, when $\widehat{E}$ is not rational we need to specify how agents form forecasts of future macroeconomic conditions. That is, we need an expression for $\widehat{E}x^f_{t+1}$ (where x^f_t is the vector of forward-looking variables in the model: $x^f_t = \left[c_t, \pi^m_t, \pi^d_t, \Delta s_t\right]'$).

For our learning formulation, we assume that agents know the structure of the economy (as embodied in Equations (A1)–(A16)) but are unsure of its parameters and how shocks propagate (that is, they do not know a, b or c below). We assume that agents update their beliefs about forward-looking variables according to:[8]

$$\widehat{E}_t x^f_{t+1} = a + b x^i_{t-1} + c\Gamma_4 v_t \tag{15}$$

where: x^i is a subset of model variables that are used to estimate x^f; and matrices a, b and c denote agents' time-varying estimates of the model's reduced form. Note that the learning rule in Equation (15) assumes that agents observe current values

8 In the learning literature, Equation (15) is known as the Perceived Law of Motion (PLM) of the agents.

of exogenous variables (that is, they know v_t). This is a common simplifying assumption made in learning models. However, we assume that private agents begin period t with estimates a, b and c based on data through t–1. That is, they know x^i_{t-1}. At t+1 agents add the new data point (x^i_t) to their information set to update their estimates of a, b and c using recursive least squares, for instance.

We consider two slightly different learning rules. One, which we describe as Minimum State Variable learning (MSV), is more restrictive and is used as a robustness check (see Section 4.3 and Appendix C for a discussion of these results). Under this approach, agents include in x^i_{t-1} only those variables which do not 'jump' (that is, those variables that correspond to non-zero columns in Γ_1). This learning rule is closely associated with the rational expectations version of the model.[9] So, under this approach, a lagged value of the nominal exchange rate is not used as an explanatory variable in x^i, even though a one-step-ahead forecast of it is formed.

The other learning rule relaxes this information assumption slightly and is used as our benchmark. We label this learning rule as 'VAR learning' since agents incorporate lagged values of all variables that are forecasted in x^f, including the lagged value of the exchange rate.

There are many choices of algorithms to estimate the parameter matrix $\Phi_t = [a_t, b_t, c_t]'$. Constant-gain learning is the standard algorithm used to model learning in empirical macroeconomics. This places more weight on recent information, which helps the model handle structural change.[10] The constant-gain learning rule is

$$\begin{aligned} \Phi_t &= \Phi_{t-1} + \bar{g} R_t^{-1} Z_{t-1} [(x^f_t)' - Z'_{t-1} \Phi_{t-1}] \\ R_t &= R_{t-1} + \bar{g} [Z_{t-1} Z'_{t-1} - R_{t-1}] \end{aligned} \tag{16}$$

where: $\bar{g}$ is the gain parameter; $Z_t = [1, x^i_{t-1}, v_t]'$ denotes data of the current period; Φ_t denotes the coefficient estimates; and R_t is the matrix of second moments of

9 This is often referred as the MSV solution, however, we also include a constant term a in agents' perceived law of motion, which implies that agents do not know the steady-state values of the economy.

10 Evans and Honkapohja (2001) show that under certain conditions the constant-gain learning equilibrium will converge to a distribution around the rational expectations equilibrium.

Z_t. Each period, agents adjust their estimates from the last period by a fraction $\bar{g}$ of the weighted difference between the realised x_t^f and the forecast made last period $Z_{t-1}'\Phi_{t-1}$. We follow Milani's (2007) approach and jointly estimate the gain parameter with the structural parameters.

Taking Equation (15) one step ahead gives the general form of the forecasting equation. This can be substituted into Equation (13) to get the implied 'Actual Law of Motion' (ALM) for the learning model:

$$\xi_t = A_t + F_t \xi_{t-1} + G_t \varepsilon_t \tag{17}$$

where $\xi_t = [v_t, x_t]$.

The ALM for both the learning model and the rational expectations model are mapped to the data using the measurement equation

$$Y_t^{data} = H\xi_t + e_t \tag{18}$$

where: Y_t^{data} contains the observable time series $y_t^*, \pi_t^*, r_t^*, y_t, \pi_t, r_t, c_t$ and Δq_t (the data are described in detail in Section 3.3); the matrix H maps the variables in the state vector (ξ_t) to the observed data; and e_t is measurement error specified as white noise. We allow for measurement error in all observable variables.

3.2 Forming the Posterior

The log posterior distribution of the parameters to be estimated (Θ) is given by

$$\Upsilon = \mathscr{L}(\Theta) + \mathscr{L}(Y^{data}|\Theta) \tag{19}$$

where $\mathscr{L}(\Theta)$ is the logarithm of the prior probability of the parameters Θ and $\mathscr{L}(Y^{data}|\Theta)$ is the log likelihood of observing Y^{data} given the parameters Θ. Details of the prior probabilities are provided in Section 3.3.

The likelihood is given by

$$\mathscr{L}(Y^{data}|\Theta) = -0.5\sum_{t=0}^{T}\left[n\ln 2\pi + \ln|\Omega_t| + u_t'\Omega_t^{-1}u_t\right] \tag{20}$$

where: n is the number of observable variables; Ω_t is the covariance matrix of the theoretical one-step-ahead forecast errors implied by a given parameterisation of the model; and u_t is a vector of the actual one-step-ahead forecast errors from predicting the variables in the sample Y^{data} using the model parameterised by Θ. The likelihood is computed using the Kalman filter.

The mode of the posterior distribution is found by numerically optimising Equation (19) using Bill Goffe's simulated annealing algorithm. The posterior distribution is found using a Random-Walk Metropolis-Hastings algorithm using 2 million draws (with a 25 per cent burn-in period).

3.3 Data Description and Priors

The model is estimated using data from 1993:Q1 to 2007:Q1. For the domestic variables we use: (linearly detrended) real non-farm GDP, the (demeaned) cash rate, (demeaned) CPI inflation, the (demeaned) real exchange rate appreciation and (linearly detrended) real household consumption. Linearly detrended trade-weighted G7 real GDP is used as data for foreign real GDP. Demeaned data for the foreign nominal interest rate and inflation are simple averages of US, euro area and Japanese interest rates and inflation series respectively. Details of the data used can be found in Appendix B.

The priors, which are chosen to conform with the constraints implied by theory, are described in the first three columns of Table 1. We fix the household discount rate β equal to 0.99 and the share of imports α equal to 0.18. Fairly loose gamma priors are imposed on the elasticity of labour supply (γ) and the intertemporal elasticity of substitution (φ). The price stickiness parameters (θ^d, θ^m) are assigned beta priors that are based on the mean quarterly duration found in Australian data (see Jääskelä and Nimark 2008) with a mean of 0.6 and a standard deviation of 0.05. The priors for the structural shocks (σ) are inverse gamma distributions with a mean of 0.001 and a standard deviation of 0.02, reflecting the fact that there is little prior information on the shocks. The sensitivity of our results to alternative priors is analysed in Section 4.3.

4. Results

4.1 Parameter Estimates

We first estimate the model described in Equations (A1)–(A16) for the 1993–2007 sample under the assumption of rational expectations. The estimates of the rational expectations parameters are used as starting values when estimating the learning model, which is based on the same prior information as used for the rational expectations model.[11] We then compare the results from the learning model to those from the rational expectations model.

4.1.1 Rational expectations

Table 1 presents the estimation results for the rational expectations model. A number of these are worth noting. The degree of habit formation in consumption (η) is much stronger than the 0.33 reported in Justiniano and Preston (2010). The rule-of-thumb parameter (ω) implies that both domestic and imported inflation are highly persistent processes. The estimated coefficients for the interest rate rule are broadly similar to those reported in other studies using Australian data (Jääskelä and Nimark 2008; Nimark 2007; and Kulish and Rees 2008). The persistence of the AR(1) processes are much lower in our model compared to Justiniano and Preston (2010) but higher or the same as in Nimark (2007). The most persistent estimated shock process is the risk premium shock (ρ^{s}). The upper bound of the 90 per cent confidence interval for this parameter exceeds 0.95; however the posterior distribution is rather wide, indicating substantial parameter uncertainty.

4.1.2 Constant-gain least squares learning

The estimation results for the model with learning are presented in Table 2. In short, our results show that learning is not a substitute for the structural sources of persistence and habit formation in consumption. A comparison of the estimates of the parameters for mechanical persistence (η – habits; ω – the share of the rule-of-thumb producers) shows a noticeable *increase* in both compared with the rational expectations model estimates.

[11] The initial value of Φ_t (Φ_0), in Equation (16) is computed using the structural presentation given in Equation (13) at the estimated rational expectations parameter values. The initial value of R_t (R_0) is the variance from the data series generated by the Kalman filter.

Table 1: Prior and Posterior Distributions for Rational Expectations Model

Parameters	**Prior**			**Posterior**			
	Distribution	Mean	Std dev	Mean	Std dev	5%	95%
			Households and firms				
α		0.18		0.18			
β		0.99		0.99			
γ	Gamma	1.20	0.20	1.221	0.198	0.913	1.566
η	Uniform	[0,1)		0.870	0.050	0.774	0.939
φ	Gamma	2.00	0.40	1.604	0.325	1.116	2.176
ω	Beta	0.20	0.05	0.267	0.063	0.167	0.373
δ	Gamma	1.50	0.10	1.510	0.098	1.351	1.674
δ_x	Gamma	1.50	0.10	1.497	0.099	1.339	1.662
θ_d	Beta	0.60	0.05	0.832	0.025	0.788	0.870
θ_m	Beta	0.60	0.05	0.609	0.048	0.528	0.687
ϕ_a	Gamma	0.10	0.05	0.117	0.065	0.043	0.242
			Taylor rule				
ρ_r	Beta	0.75	0.01	0.741	0.010	0.725	0.758
ϕ_π	Gamma	1.50	0.10	1.556	0.010	1.395	1.725
ϕ_y	Gamma	0.20	0.10	0.199	0.077	0.089	0.340
			Persistence of shocks				
ρ_a	Beta	0.60	0.20	0.585	0.061	0.481	0.682
ρ_s	Beta	0.60	0.20	0.625	0.308	0.172	0.978
ρ_d	Beta	0.60	0.20	0.678	0.084	0.531	0.806
ρ_m	Beta	0.60	0.20	0.111	0.050	0.039	0.200
			Std dev of shocks ($\times 10^{-2}$)				
σ_a	Inv gamma	0.1	2	0.097	0.040	0.054	0.175
σ_s	Inv gamma	0.1	2	0.109	0.063	0.052	0.227
σ_d	Inv gamma	0.1	2	0.290	0.220	0.101	0.707
σ_m	Inv gamma	0.1	2	0.127	0.083	0.060	0.265
σ_r	Inv gamma	0.1	2	0.034	0.005	0.027	0.042

Notes: The posterior statistics are based on 2 million draws using the Markov Chains Monte Carlo (MCMC) method with a 25 per cent burn-in period. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. Measurement errors (e_t in Equation (19)) are estimated assuming no prior information and are not shown here. The marginal likelihood is –1 554.

Table 2: Prior and Posterior Distributions for Learning Model

Parameters	Prior			Posterior				
	Distribution	Mean	Std dev	Mean (RE)	Mean	Std dev	5%	95%
				Households and firms				
α		0.18		0.18	0.18			
β		0.99		0.99	0.99			
γ	Gamma	1.20	0.20	1.221	1.284	0.074	1.152	1.399
η	Uniform	[0,1)		0.870	0.973	0.006	0.962	0.982
φ	Gamma	2.00	0.40	1.604	1.045	0.126	0.814	1.257
ω	Beta	0.20	0.05	0.267	0.370	0.047	0.294	0.456
δ	Gamma	1.50	0.10	1.510	1.498	0.059	1.396	1.601
δ_x	Gamma	1.50	0.10	1.497	1.504	0.066	1.396	1.628
θ_d	Beta	0.60	0.05	0.832	0.733	0.029	0.685	0.780
θ_m	Beta	0.60	0.05	0.609	0.653	0.032	0.595	0.703
ϕ_a	Gamma	0.10	0.05	0.117	0.242	0.051	0.181	0.344
$\bar{g}$	Uniform	[0,1)			0.0002	0.0001	0.0001	0.0003
				Taylor rule				
ρ_r	Beta	0.75	0.01	0.741	0.751	0.009	0.736	0.765
ϕ_π	Gamma	1.50	0.10	1.556	1.493	0.060	1.377	1.581
ϕ_y	Gamma	0.20	0.10	0.199	0.153	0.062	0.069	0.279
				Persistence of shocks				
ρ_a	Beta	0.60	0.20	0.585	0.878	0.043	0.801	0.944
ρ_s	Beta	0.60	0.20	0.625	0.360	0.140	0.213	0.714
ρ_d	Beta	0.60	0.20	0.678	0.789	0.113	0.552	0.894
ρ_m	Beta	0.60	0.20	0.111	0.510	0.085	0.366	0.676
				Std dev of shocks ($\times 10^{-2}$)				
σ_a	Inv gamma	0.1	2	0.097	0.076	0.021	0.049	0.116
σ_s	Inv gamma	0.1	2	0.109	0.102	0.043	0.057	0.180
σ_d	Inv gamma	0.1	2	0.290	0.107	0.040	0.061	0.181
σ_m	Inv gamma	0.1	2	0.127	0.071	0.017	0.048	0.102
σ_r	Inv gamma	0.1	2	0.034	0.033	0.005	0.027	0.042

Notes: The posterior statistics are based on 2 million draws using the Markov Chains Monte Carlo (MCMC) method with a 25 per cent burn-in period. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. Measurement errors (e_t in Equation (19)) are estimated assuming no prior information and are not shown here. The marginal likelihood is –1 840.

Indeed, this can be seen in Table 3, which compares the implied reduced-form Phillips curve coefficients (see Equation (9)) based on the (mean) rational expectations and learning parameter estimates. The forward-looking components are more prominent than the backward-looking components, though this is more so under rational expectations than under learning.

Table 3: Reduced-form Phillips Curve Coefficients

Parameter	Rational expectations	Learning
	Domestic goods	
λ^d : slope of the Phillips curve	0.020	0.042
μ_f^d : inflation – forward	0.751	0.660
μ_b^d : inflation – backward	0.243	0.336
	Imported goods	
λ^m	0.130	0.076
μ_f^m	0.690	0.633
μ_b^m	0.306	0.363

These findings are in contrast with Milani (2007), who finds that the parameters for structural sources of persistence uniformly decrease when learning is introduced to the model. However, our results are consistent with estimates based on larger DSGE models such as in Murray (2008) and Slobodyan and Wouters (2009). They find that learning either increases the parameter estimates on mechanical persistence or does not affect them.

We find that learning changes the degree of persistence of the shocks, in some cases quite substantially. However, the direction of these changes is not uniform. The persistence of the productivity (ρ_a), preference (ρ_d) and the cost-push (ρ_m) shocks increase while the persistence of the risk premium shock (ρ_s) decreases. The increase in the parameter related to net foreign assets (ϕ_a) suggests that learning cannot be used to replace the high amount of persistence required in the UIP condition to model the real exchange rate effectively. Milani (2007) and Murray (2008) both find that the estimates of the persistence of shocks are quite different under learning; some are larger and some are smaller. Slobodyan and Wouters (2009) find that most estimated persistence parameters remain unchanged or fall.

We find that the standard deviation of the shocks is smaller or the same under learning. However, Milani (2007) and Murray (2008) find that they are broadly similar or increase while Slobodyan and Wouters (2009) find the majority are similar but those that change do not do so in a consistent way.

A comparison of the 90 per cent confidence intervals shows that uncertainty regarding most parameter estimates is lower under learning than under rational expectations. Only, the estimates of the persistence of the cost-push shock (ρ_m) and consumption preference shock (ρ_d) are less precise.

4.2 The Transmission and Propagation of Shocks

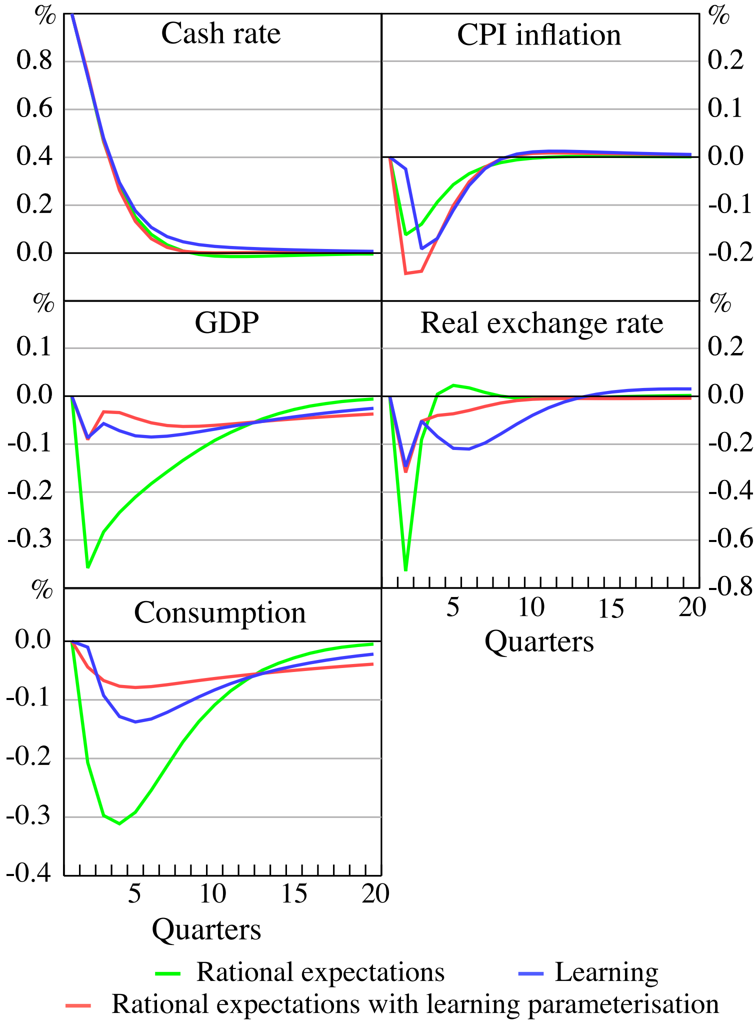
To gauge the role of learning in transmitting different types of shocks we examine the impulse responses of macroeconomic variables in the model.

4.2.1 Monetary policy shock

Impulse responses for key variables to an unexpected 100 basis point tightening of monetary policy are shown in Figure 1. In both the rational expectations model and the learning model the shock leads to a fall in output, inflation and consumption and an appreciation of the real exchange rate. The shape of the responses in the learning model are similar to the rational expectations model but generally show more persistence and inertia. One particularly interesting response is that of the real exchange rate. While the peak effect occurs in the first quarter in both cases, under learning the effect is much more prolonged. This is consistent with empirical evidence provided in data-driven SVAR models (see Liu (2008) for Australia, and Eichenbaum and Evans (1995) and Faust and Rogers (2003) for the United States). Although our parameter estimates indicate that learning does not preclude the need to add a risk premium to the 'pure' UIP condition, the impulse responses suggest that it enhances the dynamics of the model in propagating policy shocks. All of the impulse response functions show that the peak occurs later and that it takes longer for the shock to completely propagate through the system under learning than under rational expectations. The magnitudes of the real exchange rate, consumption and GDP responses are more subdued in the learning model relative to the rational expectations model. This is confirmed by the parameter estimates, which suggest that consumption decisions are less responsive to the expected real interest rate under learning.[12] However, the CPI responds by

[12] From Equation (A13) the relevant coefficient is $\frac{1-\eta}{(1+\eta)\gamma}$, the mean of which is 0.012 under learning compared with 0.057 under rational expectations.

Figure 1: Impulse Responses to Monetary Policy Shock



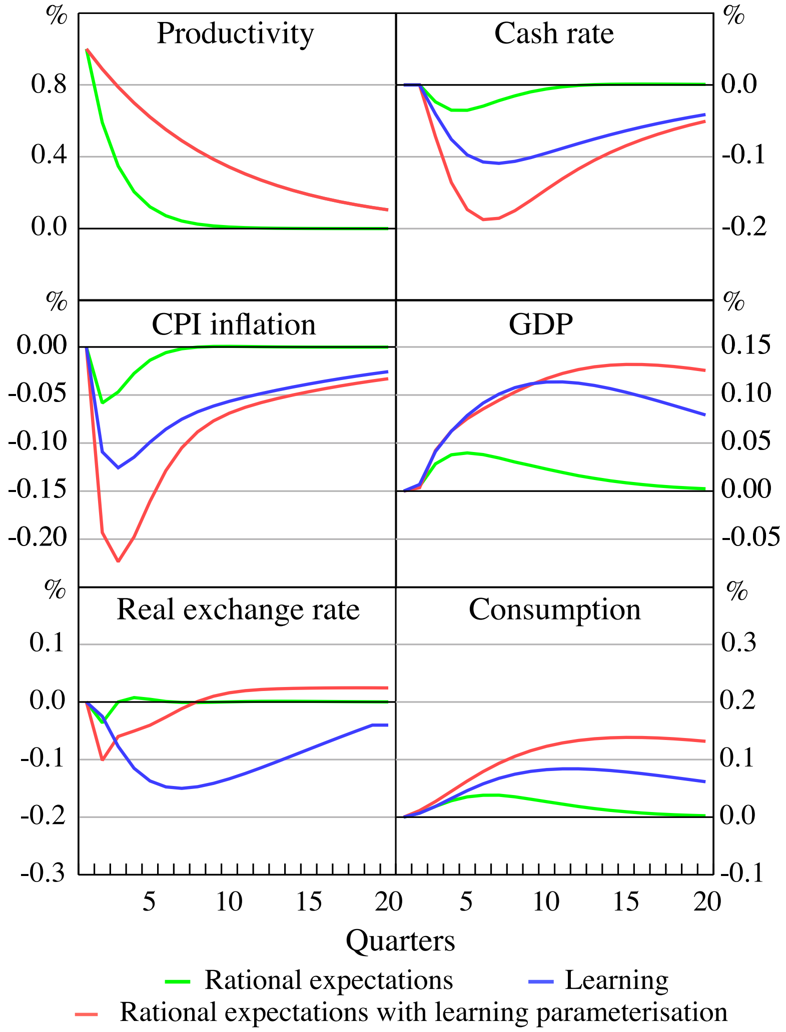
more, which is partly due to increased structural inflation persistence (a larger rule-of-thumb parameter, ω) that makes inflation more sensitive to movements in the marginal cost (the sensitivity of CPI inflation is a weighted average of λ^d and λ^m; see Equation (A9)).

It may be that the change in the dynamic response is due to the fact that the parameters of mechanical sources of persistence are higher in the learning model, rather than because of learning *per se*. To demonstrate that the change in dynamics is, at least in part, due to the learning process, we plot the impulse response functions for an economy where expectations are rational but use the estimated structural parameters implied by the learning model (given in Table 2). For the real exchange rate and GDP it is clear that the changed shape of the impulse response can be attributed to learning. In the absence of learning, this parameterisation (shown in red) generally results in much smaller responses but, more importantly, with a similar shape as under the estimated rational expectations model (shown in green). Hence the learning mechanism is leading to more persistence in the economy. In the learning model, all the forward-looking variables (consumption, inflation and the exchange rate) are more persistent. As a result, the cash rate response also returns more slowly to baseline. It is worth noting that the majority of the reduction in the size of the response between the rational expectations and the learning models can be attributed to the change in the parameters. Learning actually amplifies the fluctuations: the magnitude of the responses in the rational expectations model with learning parameterisation is more subdued than under learning. From this exercise we conclude that learning, by itself, changes the propagation of the monetary policy shock, extending the response of the exchange rate, output and consumption, in addition to delaying the response of inflation and consumption.

4.2.2 Productivity shock

Figure 2 shows impulse responses of key variables to a one standard deviation productivity shock. It is necessary to emphasise at the outset that the shock is much *more* persistent in the learning model, which creates an additional consideration when comparing impulse response functions. The difference in the decay of the shock is shown in the top-left panel in Figure 2. As expected, the increase in productivity increases GDP and consumption. Inflation falls, and the cash rate is lowered; the real exchange rate appreciates. Once again, learning results in a more protracted and persistent response in the real exchange rate, suggesting that learning may slow down the adjustment to the productivity shock. We repeat the exercise conducted in the previous section and use the learning parameterisation in the rational expectations model; these impulse response functions will, among other things, include the effect of the increase in the persistence of the productivity shock. As before, the real exchange rate response is more persistent and pronounced under learning. Due to the larger appreciation of the exchange rate, and more modest decline in the cash rate, the responses of output, consumption and inflation are more subdued and short-lived under learning.

Figure 2: Impulse Responses to Productivity Shock



4.3 Robustness: Looser Priors

To generate plausible dynamics in the rational expectations model several parameters ($\omega, \theta_d, \theta_m$ and ρ_r) require tight priors. As a robustness check, we re-estimate both the rational expectations and learning models with a set of looser priors on the rule-of-thumb (ω), Calvo (θ_d, θ_m) and interest rate smoothing (ρ_r) parameters. For the first three, the standard deviation is doubled from 0.05 to 0.1, and for the fourth it is increased from 0.01 to 0.05. The results for the re-estimated rational expectations model are detailed in Table C1 in Appendix C. There are several noticeable changes in the parameter estimates. With these looser priors, the estimate of a proportion of the rule-of-thumb firms (ω) has risen from 0.27 to 0.89 and the uncertainty in the estimate of this parameter has tripled. The persistence of the interest rate (ρ_r) falls from 0.74 to 0.66.[13] The paths of the impulse response functions also move and the confidence intervals are wider.

Table 4 summarises the reduced-form Phillips curve coefficients in the rational expectations and learning models under these looser priors. In contrast to the earlier baseline results (Table 3), in the learning model the Phillips curves are *less* backward-looking.

Table 4: Reduced-form Phillips Curve Coefficients – Looser Priors

Parameter	Rational expectations	Learning
	Domestic goods	
λ^d : slope of the Phillips curve	0.007	0.011
μ_f^d : inflation – forward	0.434	0.805
μ_b^d : inflation – backward	0.566	0.189
	Imported goods	
λ^m	0.002	1.056
μ_f^m	0.473	0.530
μ_b^m	0.526	0.465

The difference in the propagation of shocks between the 'tight' and 'loose' prior specifications for the rational expectations and learning models is also interesting. Table 5 shows the difference between the cumulative median impulse response for the two prior sets. This is reported as the sum of the absolute value of the difference in the response at each quarter for the first 20 quarters. For the monetary policy shock the difference between the prior specifications is lower under learning than under rational expectations for three of the five impulse responses. However, for the productivity shock the difference between the prior specifications is always larger under learning than it is under rational expectations. Table 5 also shows the sum of the absolute differences in the width of the 90 per cent confidence intervals for each period. Again, the change in the width of the confidence intervals between the prior choices is mostly smaller for the monetary policy shock under learning, while for the productivity shock it is always larger under learning than it is under rational expectations.

[13] The learning model is also estimated under the looser prior specifications (using the same starting values as before). The results are detailed in Table C2. ω, a proportion of the rule-of-thumb firms is lower, but the habit formation parameter (η) stays high, and is higher than in rational expectations version of the model.

Table 5: Comparison of the Impulse Response Functions for the Different Prior Choices

Variable	Median[(a)]		Confidence interval width[(b)]	
	Rational expectations	Learning	Rational expectations	Learning
	Monetary policy shock			
Cash rate	0.29	0.40	1.41	1.80
CPI inflation	0.48	0.37	0.76	0.12
Output	0.76	0.40	1.23	0.80
Consumption	0.65	0.23	1.38	0.88
Real exchange rate	1.30	1.50	2.40	1.83
	Productivity shock			
Cash rate	0.22	1.05	0.48	0.85
CPI inflation	0.25	0.91	0.51	0.90
Output	0.44	1.35	1.02	1.18
Consumption	0.32	0.86	0.95	0.43
Real exchange rate	0.68	3.73	1.65	2.09

Notes: (a) The sum of the absolute value of the differences between the tight prior and looser prior impulse response functions at each period. Only the first 20 quarters are considered. Calculated as $d = \sum_{t=1}^{20} |IRF_t^{i,lp} - IRF_t^{i,tp}|$, where $IRF_t^{i,lp}$ denotes the response of variable i at time t under 'loose' prior, and $IRF_t^{i,tp}$ denotes the same metric under the 'tight' prior.
(b) The sum of the absolute value of the difference between the width of the confidence interval at each period. Calculated as $\sum_{t=1}^{20} |(|(IRF_t^{i,lp,ub} - IRF_t^{i,lp,lb})| - |(IRF_t^{i,tp,ub} - IRF_t^{i,tp,lb})|)|$, where ub (lb) denotes the upper (lower) bound.

Overall, then, the evidence is mixed as to whether the priors do more or less work under learning than they do under rational expectations. However, comparing the fit of the rational expectations and learning models (carried out using Geweke's (1999) Modified Harmonic Mean measure of the marginal likelihood) suggests that data marginally prefer the learning model in the case of the looser set of priors.

4.4 Robustness: MSV Learning

In this section we summarise the estimation results assuming MSV learning. That is, we endow the agents with the reduced-form specification that they would use under the rational expectations equilibrium when forming expectations about future variables.

Tables C3 and C4 provide the parameter estimates under MSV learning assuming the two different prior specifications as before. (These should be compared with Tables 1 and C1, respectively.) A quick glance at these parameter estimates shows that the learning model relies at least as much as the rational expectations model on the mechanical sources of persistence. Habit formation is stronger and most of the shock processes are more persistent under learning. But whether or not inflation is more persistent under learning is unclear; in the 'loose' prior specification (Table C4) the reduced-form coefficients of the Phillips curves for both domestic and imported goods imply more forward-looking behaviour under learning, but with the 'tight' prior (Table C3) this is less clear-cut. Figures C1 and C2 plot impulse responses to monetary and productivity shocks. Even though the MSV learning rule does not contain a lagged exchange rate term, the real exchange rate does not peak immediately after the productivity shock. With regard to the monetary policy shock, it appears that the tighter prior slows down the adjustment of the exchange rate; under the looser prior with MSV learning the exchange rate behaves more in line with that of the rational expectations model (Figure 1).

To sum up the results of Section 4, it seems that mechanical sources of persistence are needed to match the inertial behaviour in data, however there is some evidence that the data prefer the learning model in the case of looser priors.

5. Longer Sample: Break in the Gain Parameter

It can be argued that the gain parameter varies over time and is lower in periods of macroeconomic stability. This makes sense because a higher gain parameter means that the past is discounted faster; if there is less stability, the recent past is more informative than the more distant past. It is therefore plausible that more credible monetary regimes, with lower and less volatile inflation, should be associated with a lower gain parameter. Accordingly, we look for a break in the gain parameter in 1993, when the Reserve Bank of Australia adopted the current inflation-targeting framework. To do this, we re-estimate the learning model from 1984 (the floating of the Australian dollar) onwards. The learning process is initialised using the rational expectations parameterisation estimated over the longer sample, and all the parameters of the learning model are estimated. $\bar{g}_{LS}$, the gain parameter for this longer sample (without allowing for the regime shift), is 0.0013, much higher than the gain parameter $\bar{g}_{IT}$ for the shorter post-inflation-targeting sample, which is 0.0002 (Tables 2 and 6). While this is consistent with a shift to a more credible regime, the long sample incorporates the regime shift, which by itself could push up the gain parameter.

To deal with this we examine the evidence of the effect of the regime shift on learning by allowing the gain parameter to break at the time of the adoption of the inflation target. That is:

$$\bar{g} = \begin{cases} \bar{g}_1, & t < 1993:Q2 \\ \bar{g}_2, & t \geq 1993:Q2 \end{cases} \tag{21}$$

The likelihood of this 'break' model is given as the sum of the likelihood of the whole sample and the two sub-samples

$$\mathscr{L} = \mathscr{L}(Y^{data}_{1983Q2:2007Q1}|\Theta;\bar{g}_0) + \mathscr{L}(Y^{data}_{1983Q2:1992Q4}|\Theta;\bar{g}_1) + \mathscr{L}(Y^{data}_{1993Q1:2007Q1}|\Theta;\bar{g}_2) \tag{22}$$

where Θ denotes a vector of all the other model parameters, which are assumed to be sample-invariant. (This is why the whole sample component appears in the likelihood function.)[14] When the break is taken into consideration, the gains $\bar{g}_1$ and $\bar{g}_2$ both fall below that for the long-sample model without the break ($\bar{g}_{LS}$). This supports the proposition that the estimated gain parameter may be 'too high' in estimates over long samples with regime shifts that are ignored. The gain in the pre-inflation-targeting period, $\bar{g}_1$, is 0.0004 and the gain after the regime shift, $\bar{g}_2$, is lower at 0.0003. This is consistent with the notion that the inflation-targeting regime is a more stable period in many respects than that which came before.

[14] Other parameters may also have shifted in response to the move to inflation targeting, but it is beyond the scope of this paper to test for such breaks.

Table 6: Gain Parameters

Parameters	**Prior**	**Posterior**			
	Distribution	Mode	Std dev	5%	95%
		Baseline: Inflation-targeting period, no break			
$\bar{g}_{IT}$	Uniform [0,1)	0.0002	0.0001	0.0001	0.0003
		Long sample, no break			
$\bar{g}_{LS}$	Uniform [0,1)	0.0013	0.0011	0.0001	0.0031
		Break model			
$\bar{g}_1$	Uniform [0,1)	0.0004	0.0001	0.0002	0.0006
$\bar{g}_2$	Uniform [0,1)	0.0003	0.0003	0.0001	0.0010

Notes: The posterior statistics are based on 2 million draws using the Markov Chains Monte Carlo (MCMC) method with a 25 per cent burn-in period. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. Measurement errors (e_t in Equation (19)) are estimated assuming no prior information and are not shown here. All parameters described in Table 2 were estimated jointly with the gain parameters using the prior information described in Table 2.

6. Conclusion

Rational expectations models assume that economic agents are perfectly informed about the structure of the economy. In this paper we relax this assumption and estimate the effect of learning on the propagation mechanism in a small open economy model. When private agents learn about the economy it is reasonable to assume that they form expectations of macroeconomic variables using statistical forecasting models, which are continuously re-estimated as new data become available. Our results show that learning does enhance the empirical fit of a small open economy model for Australia. Milani (2007) claims that learning is a replacement for the standard ad hoc sources of structural inertia such as price indexation and habit formation in consumption in a stylised closed economy model. However, we find that learning complements rather than replaces these structural features. This is consistent with Slobodyan and Wouters (2009) and Murray (2008) who analyse the effects of learning in relatively large closed economy settings.

Unlike these two papers, however, we show that learning results in impulse response functions that are consistent with those seen in more data-driven models. This is particularly noticeable for the response of the real exchange rate. However, a very persistent risk premium shock must still be added to the uncovered interest rate parity condition in order to fit the real exchange rate.

We also find that since the adoption of inflation targeting, agents appear to be using a longer history of data to form their expectations, consistent with more stable inflation and interest rates.

Although there are still many aspects of learning that require further study, our results suggest that the incorporation of learning into standard structural models warrants further investigation.

Appendix A: Log Linear Equations

Real exchange rate

$$\Delta q_t = \Delta s_t + \pi_t^* - \pi_t \tag{A1}$$

Export demand

$$c_t^x = y_t^* - \delta^x \tau_t^* \tag{A2}$$

Domestic consumption demand

$$c_t^d = c_t + \alpha \delta \tau_t^{md} \tag{A3}$$

Import demand

$$c_t^m = c_t - \delta(1-\alpha)\tau_t^{md} \tag{A4}$$

Real marginal cost of imported goods

$$mc_t^m = -\tau_t^* - \tau_t^{md} \tag{A5}$$

Real marginal cost of domestic goods

$$mc_t^d = \varphi y_t - (1+\varphi)v_t^a - v_t^d + \gamma(1-\eta)^{-1}(c_t - \eta c_{t-1}) + \alpha \tau_t^{md} \tag{A6}$$

The relative price of goods produced domestically sold to the world

$$\tau_t^* = \tau_{t-1}^* + \pi_t^d - \pi_t^* - \Delta s_t \tag{A7}$$

Domestic production

$$y_t = (1-\alpha)c_t^d + \alpha c_t^x \tag{A8}$$

CPI Inflation

$$\pi_t = (1-\alpha)\pi_t^d + \alpha \pi_t^m \tag{A9}$$

The relative price of imported goods for the domestic consumer

$$\tau_t^{md} = \tau_{t-1}^{md} + \pi_t^m - \pi_t^d \tag{A10}$$

Flow budget constraint

$$a_t^* = \beta^{-1} a_{t-1}^* + c^x c_t^x - c^m (c_t^m + \tau_t^*) \tag{A11}$$

Monetary policy

$$r_t = \phi_y y_{t-1} + \phi_\pi \pi_{t-1} + \phi_i r_{t-1} + v_t^r \tag{A12}$$

Consumption

$$c_t = \frac{1}{1+\eta}(\mathbb{E}_t c_{t+1} + \eta c_{t-1}) - \frac{1-\eta}{(1+\eta)\gamma}(r_t - \mathbb{E}_t \pi_{t+1}) + \frac{1-\eta}{(1+\eta)\gamma}(v_t^d - \mathbb{E}_t v_{t+1}^d) \tag{A13}$$

Inflation of domestically produced goods

$$\pi_t^d = \mu_f^d \mathbb{E}_t \pi_{t+1}^d + \mu_b^d \pi_{t-1}^d + \lambda^d mc_t^d \tag{A14}$$

Inflation of imported produced goods

$$\pi_t^m = \mu_f^m \mathbb{E}_t \pi_{t+1}^m + \mu_b^m \pi_{t-1}^m + \lambda^m mc_t^m + v_t^m \tag{A15}$$

Uncovered interest rate parity

$$r_t - r_t^* = \mathbb{E}_t \Delta s_{t+1} - \phi_a a_t^* + v_t^s \tag{A16}$$

Shock processes

$$v_t^a = \rho_a v_{t-1}^a + \varepsilon_t^a, \varepsilon_t^a \sim \left(0, \sigma_a^2\right) \tag{A17}$$

$$v_t^s = \rho_s v_{t-1}^s + \varepsilon_t^s, \varepsilon_t^s \sim \left(0, \sigma_s^2\right) \tag{A18}$$

$$v_t^d = \rho_d v_{t-1}^d + \varepsilon_t^d, \varepsilon_t^d \sim \left(0, \sigma_d^2\right) \tag{A19}$$

$$v_t^m = \rho_m v_{t-1}^m + \varepsilon_t^m, \varepsilon_t^m \sim \left(0, \sigma_m^2\right) \tag{A20}$$

$$v_t^r = \varepsilon_t^r, \varepsilon_t^r \sim \left(0, \sigma_r^2\right) \tag{A21}$$

The model can be expressed in the following form

$$
\begin{aligned}
\Gamma_0 x_t &= \Gamma_1 x_{t-1} + \Gamma_2 \widehat{E}_t x_{t+1} + \Gamma_3 v_t \\
v_t &= \Gamma_4 v_{t-1} + \varepsilon_t
\end{aligned}
$$

where

$$
\begin{aligned}
x_t &= [\Delta q_t, c_t^x, c_t^d, c_t^m, mc_t^m, mc_t^d, \tau_t^*, y_t, \pi_t, \tau_t^{md}, a_t^*, r_t, c_t, \pi_t^d, \pi_t^m, \Delta s_t] && \text{(A22)} \\
v_t &= [y_t^*, \pi_t^*, r_t^*, v_t^a, v_t^s, v_t^d, v_t^m, v_t^r] && \text{(A23)}
\end{aligned}
$$

$$
\Gamma_0 = \begin{bmatrix}
1 & 0_{1\times7} & -1 & 0_{1\times6} & 1 & & & & \\
0 & 1 & 0_{1\times5} & \delta^x & 0_{1\times8} & & & & \\
0_{1\times2} & 1 & 0_{1\times6} & -\alpha\delta & 0_{1\times2} & -1 & 0_{1\times3} & & \\
0_{1\times3} & 1 & 0_{1\times5} & \delta(1-\alpha) & 0_{1\times2} & -1 & 0_{1\times3} & & \\
0_{1\times4} & 1 & 1 & 0_{1\times2} & 1 & 0_{1\times6} & & & \\
0_{1\times5} & 1 & 0 & -\varphi & 0 & -\alpha & 0_{1\times2} & -\gamma(1-\eta)^{-1} & 0_{1\times3} \\
0_{1\times6} & 1 & 0_{1\times6} & -1 & 0 & 1 & & & \\
0 & -\alpha & -(1-\alpha) & 0_{1\times4} & 1 & 0_{1\times8} & & & \\
0_{1\times8} & 1 & 0_{1\times4} & -(1-\alpha) & -\alpha & 0 & & & \\
0_{1\times9} & 1 & 0_{1\times3} & 1 & -1 & 0 & & & \\
0 & -c^x & 0 & c^m & 0_{1\times2} & c^m & 0_{1\times3} & 1 & 0_{1\times5} \\
0_{1\times11} & 1 & 0_{1\times4} & & & & & & \\
0_{1\times11} & -\frac{1-\eta}{(1+\eta)\gamma} & 1 & 0_{1\times3} & & & & & \\
0_{1\times5} & -\lambda^d & 0_{1\times7} & 1 & 0_{1\times2} & & & & \\
0_{1\times4} & -\lambda^m & 0_{1\times9} & 1 & 0 & & & & \\
0_{1\times10} & \phi_a & 1 & 0_{1\times14} & & & & &
\end{bmatrix};
$$

$$\Gamma_1 = \begin{bmatrix} 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 12} & -\gamma\eta(1-\eta)^{-1} & 0_{1\times 3} \\ 0_{1\times 6} & 1 & 0_{1\times 9} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 9} & 1 & 0_{1\times 6} \\ 0_{1\times 10} & \beta^{-1} & 0_{1\times 5} \\ 0_{1\times 7} & \phi_y & \phi_\pi & 0_{1\times 2} & \phi_r & 0_{1\times 4} \\ 0_{1\times 12} & -\eta(1-\eta)^{-1} & 0_{1\times 3} \\ 0_{1\times 13} & \mu_b^d & 0_{1\times 2} \\ 0_{1\times 14} & \mu_b^m & 0 \\ 0_{1\times 16} \end{bmatrix};$$

$$\Gamma_2 = \begin{bmatrix} 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 16} \\ 0_{1\times 8} & \frac{1-\eta}{(1+\eta)\gamma} & 0_{1\times 3} & (1+\eta)^{-1} & 0_{1\times 3} \\ 0_{1\times 13} & \mu_f^d & 0_{1\times 2} \\ 0_{1\times 14} & \mu_f^m & 0 \\ 0_{1\times 15} & 1 \end{bmatrix};$$

$$\Gamma_3 = \begin{bmatrix} 0 & 1 & 0_{1\times6} & & \\ 1 & 0_{1\times7} & & & \\ 0_{1\times8} & & & & \\ 0_{1\times8} & & & & \\ 0_{1\times8} & & & & \\ 0_{1\times3} & -(1+\varphi) & 0 & -1 & 0_{1\times2} \\ 0 & -1 & 0_{1\times6} & & \\ 0_{1\times8} & & & & \\ 0_{1\times8} & & & & \\ 0_{1\times8} & & & & \\ 0_{1\times8} & & & & \\ 0_{1\times7} & 1 & & & \\ 0_{1\times5} & (1-\rho_d)\frac{1-\eta}{(1+\eta)\gamma} & 0_{1\times2} & & \\ 0_{1\times8} & & & & \\ 0_{1\times6} & 1 & 0 & & \\ 0_{1\times4} & 1 & 0_{1\times3} & & \end{bmatrix}; \Gamma_4 = \begin{bmatrix} M_{3\times3} & & \\ 0_{1\times3} & \rho_a & 0_{1\times4} \\ 0_{1\times4} & \rho_s & 0_{1\times3} \\ 0_{1\times5} & \rho_d & 0_{1\times2} \\ 0_{1\times6} & \rho_m & 0 \\ 0_{1\times8} & & \end{bmatrix}$$

Appendix B: Data Description and Sources

Inflation ($\pi_t^{cpi,trim}$): trimmed mean consumer price index excluding taxes and interest (RBA).

Consumption (c_t): real seasonally adjusted household final consumption expenditure (ABS Cat No 5206.0).

Real exchange rate (Δq_t): the change in real trade-weighted exchange rate (RBA).

Nominal interest rate (r_t): overnight cash rate, averaged over the quarter (RBA).

Output (y_t): real seasonally adjusted non-farm GDP (ABS Cat No 5206.0).

World Output (y_t^*): G7 trade-weighted real GDP (RBA).

World inflation (π_t^*): G7 trade-weighted headline CPI inflation (RBA).

World interest rate (r_t^*): average of euro area, US and Japanese short-term nominal interest rates (RBA).

Appendix C: Robustness Checks

Table C1: Looser Prior and Posterior Distributions for Rational Expectations Model

Parameters	Prior			Posterior			
	Distribution	Mean	Std dev	Mean	Std dev	5%	95%
			Households and firms				
β		0.99		0.99			
γ	Gamma	1.20	0.20	1.075	0.212	0.971	1.667
η	Uniform	[0,1)		0.830	0.057	0.730	0.911
φ	Gamma	2.00	0.40	1.411	0.366	1.249	2.441
ω	Beta	0.20	0.10	0.893	0.175	0.290	0.934
δ	Gamma	1.50	0.10	1.593	0.102	1.399	1.737
δ_x	Gamma	1.50	0.10	1.311	0.101	1.306	1.637
θ_d	Beta	0.60	0.10	0.692	0.074	0.651	0.901
θ_m	Beta	0.60	0.10	0.811	0.081	0.647	0.913
ϕ_a	Gamma	0.10	0.05	0.166	0.057	0.064	0.243
			Taylor rule				
ρ_r	Beta	0.75	0.05	0.659	0.047	0.619	0.773
ϕ_π	Gamma	1.50	0.10	1.544	0.1035	1.390	1.727
ϕ_y	Gamma	0.20	0.10	0.178	0.082	0.081	0.349
			Persistence of shocks				
ρ_a	Beta	0.60	0.20	0.721	0.079	0.519	0.780
ρ_s	Beta	0.60	0.20	0.558	0.218	0.120	0.951
ρ_d	Beta	0.60	0.20	0.621	0.092	0.520	0.820
ρ_m	Beta	0.60	0.20	0.169	0.038	0.086	0.207
			Std dev of shocks ($\times 10^{-2}$)				
σ_a	Inv gamma	0.1	2	0.071	0.161	0.067	0.466
σ_s	Inv gamma	0.1	2	0.129	0.073	0.059	0.271
σ_d	Inv gamma	0.1	2	0.124	0.139	0.081	0.464
σ_m	Inv gamma	0.1	2	0.059	0.058	0.060	0.233
σ_r	Inv gamma	0.1	2	0.036	0.005	0.027	0.042

Notes: The posterior statistics are based on 2 million draws using the Markov Chains Monte Carlo (MCMC) method with a 25 per cent burn-in period. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. Measurement errors (e_t in Equation (19)) are estimated assuming no prior information and are not shown here. The marginal likelihood is –1 569.

Table C2: Looser Prior and Posterior Distributions for Learning Model

Parameters	**Prior**			**Posterior**			
	Distribution	Mean	Std dev	Mean	Std dev	5%	95%
				Households and firms			
α		0.18		0.18			
β		0.99		0.99			
γ	Gamma	1.20	0.20	1.344	0.188	1.048	1.664
η	Uniform	[0,1)		0.972	0.012	0.950	0.988
φ	Gamma	2.00	0.40	1.210	0.275	0.792	1.694
ω	Beta	0.20	0.10	0.205	0.053	0.113	0.285
δ	Gamma	1.50	0.10	1.638	0.102	1.475	1.813
δ_x	Gamma	1.50	0.10	1.434	0.097	1.280	1.598
θ_d	Beta	0.60	0.10	0.882	0.032	0.825	0.929
θ_m	Beta	0.60	0.10	0.236	0.057	0.153	0.338
ϕ_a	Gamma	0.10	0.05	0.258	0.064	0.162	0.374
$\bar{g}$	Uniform	[0,1)		0.0002	0.0001	0.0001	0.0003
				Taylor rule			
ρ_r	Beta	0.75	0.05	0.759	0.045	0.684	0.829
ϕ_π	Gamma	1.50	0.10	1.494	0.096	1.338	1.660
ϕ_y	Gamma	0.20	0.10	0.189	0.090	0.065	0.350
				Persistence of shocks			
ρ_a	Beta	0.60	0.20	0.643	0.128	0.430	0.848
ρ_s	Beta	0.60	0.20	0.231	0.125	0.081	0.487
ρ_d	Beta	0.60	0.20	0.648	0.112	0.450	0.821
ρ_m	Beta	0.60	0.20	0.953	0.022	0.912	0.983
				Std dev of shocks ($\times 10^{-2}$)			
σ_a	Inv gamma	0.1	2	0.091	0.030	0.053	0.152
σ_s	Inv gamma	0.1	2	0.071	0.022	0.045	0.114
σ_d	Inv gamma	0.1	2	0.252	0.107	0.113	0.450
σ_m	Inv gamma	0.1	2	0.178	0.113	0.065	0.429
σ_r	Inv gamma	0.1	2	0.033	0.005	0.027	0.041

Notes: The posterior statistics are based on 2 million draws using the Markov Chains Monte Carlo (MCMC) method with a 25 per cent burn-in period. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. Measurement errors (e_t in Equation (19)) are estimated assuming no prior information and are not shown here. The marginal likelihood is –1 567.

Table C3: Tighter Prior and Posterior Distributions for MSV Learning Model

Parameters	Prior			Posterior			
	Distribution	Mean	Std dev	Mean	Std dev	5%	95%
			Households and firms				
α		0.18		0.18			
β		0.99		0.99			
γ	Gamma	1.20	0.20	1.327	0.156	1.067	1.605
η	Uniform	[0,1)		0.977	0.006	0.966	0.985
φ	Gamma	2.00	0.40	0.938	0.163	0.658	1.223
ω	Beta	0.20	0.05	0.255	0.040	0.193	0.328
δ	Gamma	1.50	0.10	1.491	0.083	1.352	1.634
δ_x	Gamma	1.50	0.10	1.504	0.084	1.369	1.655
θ_d	Beta	0.60	0.05	0.733	0.024	0.670	0.771
θ_m	Beta	0.60	0.05	0.605	0.038	0.532	0.661
ϕ_a	Gamma	0.10	0.05	0.224	0.052	0.145	0.322
$\bar{g}$	Uniform	[0,1)		0.0003	0.0001	0.0002	0.0005
			Taylor rule				
ρ_r	Beta	0.75	0.01	0.750	0.010	0.734	0.766
ϕ_π	Gamma	1.50	0.10	1.502	0.083	1.366	1.648
ϕ_y	Gamma	0.20	0.10	0.165	0.071	0.070	0.307
			Persistence of shocks				
ρ_a	Beta	0.60	0.20	0.740	0.131	0.452	0.888
ρ_s	Beta	0.60	0.20	0.389	0.136	0.192	0.659
ρ_d	Beta	0.60	0.20	0.432	0.074	0.337	0.576
ρ_m	Beta	0.60	0.20	0.764	0.081	0.661	0.935
			Std dev of shocks ($\times 10^{-2}$)				
σ_a	Inv gamma	0.1	2	0.077	0.035	0.045	0.067
σ_s	Inv gamma	0.1	2	0.098	0.034	0.058	0.091
σ_d	Inv gamma	0.1	2	3.394	1.632	1.238	3.156
σ_m	Inv gamma	0.1	2	0.077	0.020	0.051	0.073
σ_r	Inv gamma	0.1	2	0.032	0.004	0.026	0.032

Notes: The posterior statistics are based on 2 million draws using the Markov Chains Monte Carlo (MCMC) method with a 25 per cent burn-in period. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. The marginal likelihood is –1 830.

Table C4: Looser Prior and Posterior Distributions for MSV Learning Model

Parameters	Prior			Posterior			
	Distribution	Mean	Std dev	Mean	Std dev	5%	95%
			Households and firms				
α		0.18		0.18			
β		0.99		0.99			
γ	Gamma	1.20	0.20	1.295	0.201	0.978	1.643
η	Uniform	[0,1)		0.894	0.082	0.723	0.977
φ	Gamma	2.00	0.40	1.588	0.337	1.081	2.176
ω	Beta	0.20	0.10	0.669	0.091	0.506	0.805
δ	Gamma	1.50	0.10	1.588	0.103	1.422	1.763
δ_x	Gamma	1.50	0.10	1.463	0.097	1.306	1.627
θ_d	Beta	0.60	0.10	0.765	0.064	0.655	0.864
θ_m	Beta	0.60	0.10	0.696	0.073	0.570	0.809
ϕ_a	Gamma	0.10	0.05	0.293	0.082	0.166	0.437
$\bar{g}$	Uniform	[0,1)		0.0001	0.0001	0.0001	0.0002
			Taylor rule				
ρ_r	Beta	0.75	0.05	0.754	0.049	0.670	0.830
ϕ_π	Gamma	1.50	0.10	1.502	0.100	1.341	1.672
ϕ_y	Gamma	0.20	0.10	0.188	0.094	0.065	0.367
			Persistence of shocks				
ρ_a	Beta	0.60	0.20	0.697	0.127	0.460	0.872
ρ_s	Beta	0.60	0.20	0.788	0.150	0.503	0.968
ρ_d	Beta	0.60	0.20	0.700	0.119	0.479	0.871
ρ_m	Beta	0.60	0.20	0.256	0.121	0.076	0.465
			Std dev of shocks ($\times 10^{-2}$)				
σ_a	Inv gamma	0.1	2	0.143	0.081	0.067	0.121
σ_s	Inv gamma	0.1	2	0.081	0.029	0.048	0.074
σ_d	Inv gamma	0.1	2	0.441	0.265	0.141	0.380
σ_m	Inv gamma	0.1	2	0.044	0.008	0.033	0.043
σ_r	Inv gamma	0.1	2	0.036	0.005	0.027	0.033

Notes: The posterior statistics are based on 2 million draws using the Markov Chains Monte Carlo (MCMC) method with a 25 per cent burn-in period. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. The marginal likelihood is –1 813.

Figure C1: Impulse Responses to Monetary Shock

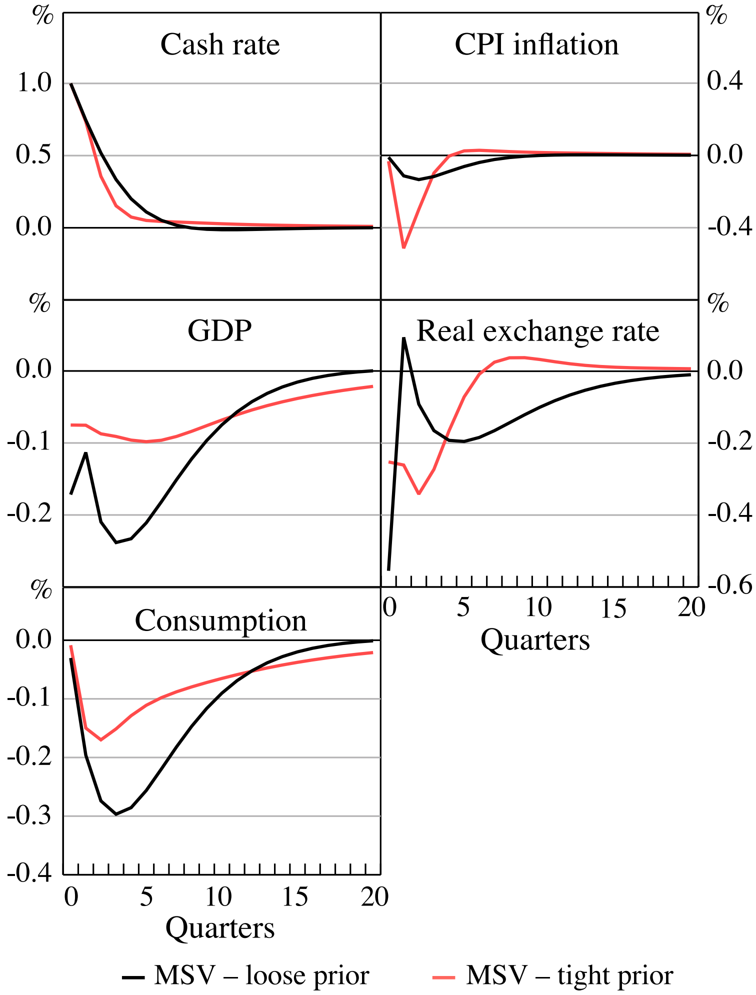
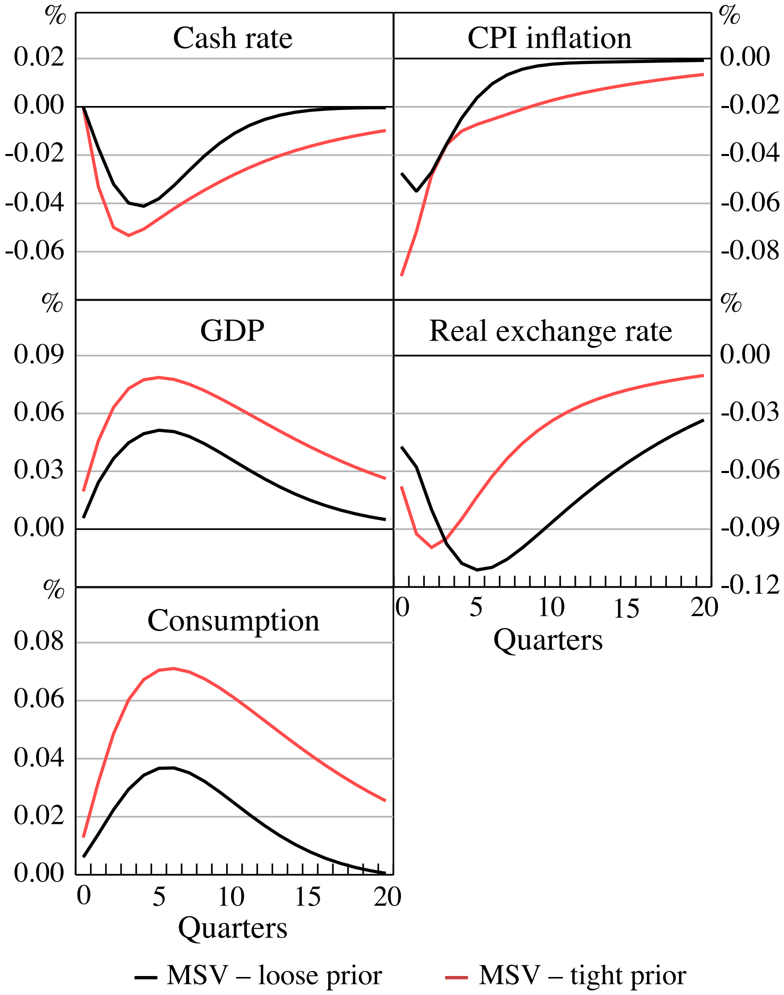


Figure C2: Impulse Responses to Productivity Shock



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