A Model of Intermediation, Money, Interest, and Prices*

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Abstract

A model integrates a modern implementation of monetary policy into an incomplete-markets monetary economy. Monetary policy (MP) sets corridor rates and conducts open-market operations and fiscal transfers. These tools grant independent control over credit spreads and inflation. Through the influence on spreads, MP affects the evolution credit, output, and the wealth distribution. Classic experiments illustrate how different instruments have effects through different channels and provide some policy insights: (a) MP can move real loan and deposit rates (both in the long and short-run), (b) opening credit spreads can be desirable, (c) negative reserves rates can increase the lending rates, (d) fiscal transfers can be recessionary if anticipated.

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1 Introduction

> Now, toward the end of my career as at the beginning, I see myself as a monetarist. My contributions to monetary theory have been to incorporate the quantity theory into modern modeling. For the empirically well established predictions —long-run links— this job has been accomplished. On the harder questions of monetary economics — the real effects of monetary instability, the roles of inside and outside money, this work contributes examples but little in empirically successful models. It is understandable that in the leading operational macroeconomic models today— the RBC and the New Keynesian models—money as a measurable magnitude plays no role at all, but I hope we can do better than this in the future.
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> —Robert E. Lucas, 2013
>
> —Final paragraph in the introduction to Collected Papers in Monetary Economics

In modern economies, monetary policy (MP) operates through the provision of reserves and a corridor system of policy rates.[1] A prevalent view has is that these tools influence bank credit and, thus, impact real activity (Bernanke and Blinder, 1992). Although there is empirical support for this view (Kashyap and Stein, 2000; Drechsler et al., 2017), its theoretical foundations are still being laid out. This paper presents a incomplete-markets model where credit (inside money) is intermediated by banks that hold reserves for liquidity reasons (outside money). MP is implemented through a corridor system and open-market operations (OMO). The paper articulates how these tools lead to predictions about credit, money, borrowing and lending rates, and prices, in an incomplete-markets economy.

The main insight of this paper is that a corridor system and OMO, are a way to independently implement a credit spread and inflation targets, and opening a spread is desirable. The control over credit spreads is a notion of the credit channel. The control over inflation, relates to other, better traveled, transmission mechanisms.[2] Although the view that central banks affect bank credit volumes and spreads is ubiquitous, there is a shortage of dynamic general-equilibrium models that formalize this view. Filling this gap is important. For one thing, during booms, policy circles debate whether *MP can sow the seeds of crises,* whereas during crises, whether MP is akin to *pushing on a string*. How should MP preclude a credit crunch but unleash credit if happens, is at the core of policy debates. This paper suggests that a countercyclical credit spread can mitigate credit crunch, at the cost of a less efficient ex-ante insurance.

The building block is a canonical heterogeneous-agent continuous-time environment. This is an endowment economy where households face idiosyncratic risk, as in a Hugget economy

[1] A corridor system is a rate on discount-window loans and interest on reserves. The discount rate is the rate at which a central bank lends reserves to banks that are below their reserve requirement. The rate on reserves is the rate at which banks are remunerated by holding reserve balances at the central bank.

[2] A narrative description of different transmission channels of MP is found in Bernanke and Gertler (1995)'s "Inside the Black box". Kashyap and Stein (2000) presented evidence on the credit channel by exploiting differences in the cross-section of liquidity ratios across banks. Bindseil (2014) describes the modern implementation of MP through banks across countries.

(Huggett, 1993). To speak about productive efficiency, we let households choose between a safe endowment process, or from riskier but more profitable process. It is never efficient to chose the safe endowment because endowment risk is idiosyncratic. However, when households approach borrowing limits, they switch to the inefficient choice.[3] This mechanism maps GDP to the fraction of borrowing constrained agents. Although simplistic, this mechanism captures how financial stress leads to an output cost.

Credit is nominal and intermediated by a fringe of competitive banks. In addition to deposits and loans, banks hold reserves for precautionary reasons. Ultimately, banks are pass-through entities and all of the action comes from MP which has the power to control bank credit. This power stems from an institutional feature: whereas loans are always held by the issuer bank, deposits circulate. Banks use reserves to settle deposit transfers. The potential shortage of reserves by some banks, opens the door for an interbank market that operates with frictions (á la Ashcraft and Duffie, 2007; Afonso and Lagos, 2015). As a result, some banks face the risk of being forced to borrow reserves at a discount-window rate set by policy. The spread in corridor rates and the overall quantity of reserves set by MP translate intro a cost on providing intermediation. Any induced cost is ultimately passed on to households. The power to influence credit spreads is a notion of the bank credit-channel.

Similar notions of the credit channel already appear in recent work by Bianchi and Bigio (2017a) and in Piazzesi and Schneider (2018). The novelty is to place the credit channel in the context of an incomplete-markets economy. The richness of incomplete-markets economies delivers a broad set of implications and clarifies the connection with other transmission mechanisms. In this incomplete-market economy, real effects of MP emerge from the household's precautionary motive. Because MP indirectly affects the distribution of wealth, it influences the mass of agents that approach borrowing constraints, and consequently, output.

The paper shows how this economies can be solved in real terms because the deposit, money and loans markets collapse into a market for real credit. The real credit market is influenced by a real spread, which is effectively controlled by MP. As a result, MP influences the real interest rate in the short run. The model delivers an analytic expression for the real credit spread as a function policy corridor spread and OMO. There is also an expression for nominal interests on deposits and loans, which have the interest on reserves as a base rate. Since real spreads influence the real deposit rate, the interest on reserves grants independent control over inflation.

The model is also explicit about a reserve satiation regime, and a zero-lower bound on deposit rates. In a satiation regime, nominal borrowing and lending rates equal the interest on reserves

[3] Although this is a positive paper, MP can be motivated by the desire to provide insurance and to reduce the productive inefficiency. Policy may want to trade-off these goals over time. A Central Bank may want to induce a real borrowing/lending spreads that produces less efficient risk-sharing against the ability to have room to lower credit spreads when credit-market conditions worsen. With other credit market imperfections, this is even more important.

and MP is neutral.[4] With additional frictions, a control over nominal rates can produce effects through the interest-rate, inflation-cost, and debt-deflation channels, all of which can be thought of operating independently.[5]

When the CB opens a credit spread, it lowers the effective rate faced by savers, but increases the rate of borrowers. Since borrowers are more interest senstive, this effect lowers the equilibrium real interest rate. A positive spread also generates fiscal revenues, which in an incomplete-markets economy generates non-Ricardian effects. A positive spread introduces a trade-off. Spreads improve equality but, as we know from incomplete-market economies, this translates into a reduction in ex-ante insurance. In this economy, a positive spread can improve economic efficiency, because it reduces the number of agents at their borrowing limit, which increases output. We also show that a countercyclical spread can be desirable, especially when we activated a demand externality.

Once we conceive that MP operations can induce a real spread and affect inflation independently we begin to challenge many preconceived views about MP. We challenge (a) the idea that MP is long-run neutral[6], and (b) the idea that monetary aggregates are not independent and interest rates are not independent policy instruments, two working restrictions in classic empirical work. The model can rationalize several empirical regularities: the presence of a liquidity effect, and a higher loan-to-deposit rate elasticity to policy changes. A third (normative) implication is that a credit spread target can be desirable: although credit spreads limit risk sharing, it can mitigate the impact of a credit crunch. Simply put, the model prescribes a trade-off between a the depth of a crisis and the amount of risk-insurance. A final (also normative) implication is a warning against the use of unconventional policies: negative interests on reserves can increase credit spreads and amplify the effects of a credit crunch.[7] Fiscal transfers, on the other hand, can have reverse effect than intended if they are anticipated.

The organization is as follows. A connection with the literature is presented in Section 2. Section 3 lays out the core model. Section 4 describes the determination of credit, interest and prices in the model and how these affect real output. That section also derives implementation conditions for MP. Section 5 presents our study on MP regimes. Section 6 incorporates real wage rigidity. This extension motivates as to think of how MP can activate real spreads to limit the extent of a crisis. Section 7 concludes.

[4]Different from Woodford (1998), the control over nominal rates is achieved without open-market operations, but by setting the interest on reserves. A related result is independently discussed in Hall and Reis (2017).

[5] In each case, the model would need an additional ingredient: nominal rigidities, cash transactions, and long-term debt, respectively.

[6]This feature is also true in other incomplete-market economies with money, but the reason is not the spread, but the effect of real money balances on credit markets.

[7] When MP lowers corridor rates, to the point where deposit rates are zero, currency becomes a perfect substitute of deposits, for households. This feature induces a zero-lower bound on deposit rates, and this constraint alters the sign of the effects of reductions in corridor rates.

2 Connection with the Literature

The title of this paper emphasizes a departure from the two most common approaches in monetary economics. One approach emphasizes the connection between *money and prices*, and the other between *interest and prices*. In the first approach, money plays a transactions role (Lucas and Stokey, 1987; Lagos and Wright, 2005) and there is a tight connection between prices and the quantity of (outside) money. By contrast, the real rate is fixed, and any real effects follows because inflation is a transactions tax. The other common approach, the new-Keynesian approach, is all about the connection between *interest and prices*. Under that framework, MP controls real rates directly because prices are rigid. However, monetary aggregates play no role, and MP does not affect credit; not directly at least. The model in this paper establishes a meaningful connetction between *intermediation, money, interest and prices* and delivers the different policy insights we already highlighted. Although the insights are different, the paper also shows that the credit-channel can be studied independently or together with the inflation-tax or interest-rate channels that appear in other approaches.

After 2008, there's been an increased interest on how MP interacts with credit markets. That gap is being filled and heterogeneous agent models were a natural starting point.[8] In fact, the first generation of heterogeneous agent studies, the Lucas (1980) and Bewley (1983) papers, were interested in MP, and not in heterogeneity per se. The goal was to study the stationary price of outside money. However, neither model was interested on how MP affects credit.[9] Credit, of course, has a tradition in heterogeneous agent models (Huggett, 1993; Aiyagari, 1994). The model here differs from those classic frameworks because here all assets are nominal, credit is intermediated by banks, and MP affects credit through its influence on banks.

One recent generation of papers introduced nominal rigidities into heterogenous agent models. To replicate the credit crunch of 2008, Guerrieri and Lorenzoni (2012), studies the tightening of borrowing limits in a Bewley economy with nominal rigidities.[10] These models are appealing because, as an artifact of heterogeneity, MP responses depend on the distribution of wealth.

[8] Models that feature credit must provide a motive for credit. One way is to endow agents with different technologies as in Bernanke and Gertler (1989) and the other is make them subject to idiosyncratic risk. To establish a connection between MP and credit markets, models must have features by which MP impacts credits. A first such model is Bernanke et al. (1999) which incorporated nominal rigidities into the two-sector economy of Bernanke and Gertler (1989). In Bernanke et al. (1999) MP was capable of moving real rates because of nominal rigidities. In that model, and models that follow it, Christiano et al. (2009), credit imperfections amplify the effects of the interest-rate channel—through the financial accelerator. However, the effect on credit spreads is not an independent instrument, as it is here.

[9] In both models, there was a constant supply of outside money. Lucas (1980) studied a stable price equilibrium. Bewley (1983) focused on the case where money earned an interest rate financed with lump-sum taxes, so interest rate had redistributive consequences as it was funded with lump-sum transfers. Ljungqvist and Sargent (2012, Chapter 18) describes shows how policies in Bewley (1983) models are akin to changes in borrowing limits in economies with pure credit.

[10] Following up on that work, McKay et al. (2015) compare the effects of forward-guidance policies in representative agent new-Keynesian models and incomplete markets economies.

Auclert (2016) for example studies how the number of borrowing constrained agents influences income sensitivity. Kaplan et al. (2016) introduce illiquid assets which disconnect interest rate elasticities from the distribution of wealth.[11] MP operates through the interest-rate channel in all of these model. A distinction with the current paper is that here the credit-channel affects credit directly.

Another set of works introduces currency transactions in models that feature a credit market. A lesson is that when inside money is an imperfect substitute for currency, the inflation-tax can spill over credit markets (see for example Berentsen et al., 2007; Williamson, 2012; Gu et al., 2015). Rocheteau et al. (2016) work in a money-search environment with non-degenerate currency holdings, and study how MP affects activity by changing the relative value of outside money. The model here abstracts from the inflation-tax channels, but it could be adapted to feature transactions as in Rocheteau et al. (2016). Gomes et al. (2016) features a different channel. That model present a model where MP affects credit markets through a debt-deflation effect. That paper postulates a Fisher equation and studies how inflation can affect credit markets when debt has a long-term maturity.

Thc credit-channel in this paper is not new. The model here incorporates the implementation of MP in Bianchi and Bigio (2017a) into a Hugget economy. Bianchi and Bigio (2017a) is one of the first models to articulate a notion of the credit channel and how MP functions through corridor rates. That paper has a rich description of bank decisions, whereas the non-financial side is static. The banking sector in this paper is a simplified version that operates as a direct pass-through. Instead, here the non-financial sector has the dynamic features of incomplete-market economies. That distinction is meaningful. In Bianchi and Bigio (2017a) the dynamic effects of MP follow the evolution of bank net worth and the dynamic decisions of banks; here, the dynamics follow from the evolution of household wealth.[12] The nature of incomplete-markets here leaves room for a normative dynamic use of the credit channel that is not present in that paper. Piazzesi and Schneider (2018) feature a similar implementation of MP, but the focus of the latter paper is the effects on asset prices.

Our model also shares common elements with Brunnermeier and Sannikov (2012). In Brunnermeier and Sannikov (2012), agents face undiversified investment risk. A natural demand for currency emerges without intermediaries. The presence of intermediaries allows some diversification because intermediaries can exchange equity of inside money depending on intermediary net worth. However, reductions in intermediary net worth can reduce the supply of money and thereby increase exposure to idiosyncratic risk. With a decline in the supply of inside money,

[11] Greenwald (2016) and Wong (2016) how interest rate sensitivities to mortgage refinancing.

[12] For example, in that model, one-time policy shocks have dynamic effects because they affect bank equity. Here, dynamic effects track the evolution of wealth. The price level is also determined differently. Whereas in that paper, the size of banks relative to the rest of the economy determines the demand for real reserves, and thus the price level—through a quantity-theory equation. Here, the price level is determined by the evolution of real household wealth.

idiosyncratic risk increases and output falls as this leads to misallocation across sectors and less investment. MP in that paper achieves two things: first, it stabilizes asset prices and redistributes wealth towards intermediaries. In that paper, MP is implemented in two ways, either via helicopter drops or through interest payments on outside money holdings. However, that models doesn't feature a channel where spreads are affected directly.[13]

3 Environment

Time t is continuous and runs to infinity, $t \in [0, \infty)$. The economy features three sets of agents: the public, banks, and a central bank. There is a single produced good. The unit of account are units of outside money and the price of goods is P_t.

The CB is both a monetary and a fiscal authority: the CB determines policy rates, open-market operations and makes/collects (lump-sum) transfers to/from households. The sources of uncertainty are (i) idiosyncratic production shocks and (ii) a financial shock. Policy responses can be expected or unexpected.

Notation. Individual state variables are denoted with lower case letters. Aggregate nominal state variables in capital letters. Aggregate real variables are written in calligraphic font. The rest of the section presents the environment without digressions, and leaves discussions towards the end.

The Public. A measure-one continuum of households that face a consumption-savings decision.[14] Preferences are described by:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} U\left(c_t\right) dt\right]$$

where $U\left(c_t\right) \equiv \left(c_t^{1-\gamma} - 1\right) / \left(1 - \gamma\right)$ is the instantaneous utility.

Each household operates a production technology. They are heterogeneous because their income is stochastic Although all assets are nominal, the individual state variable is, s_t, the stock of real financial claims. There's a distribution $f\left(s, t\right)$ of real financial wealth. Positive wealth is held in deposits, a_t^h, or currency, m_t^h. Negative wealth is represents loans, l_t^h. The nominal rate on deposits is i_t^a and the nominal rate on loans is i^l. The balance sheet of the household is presented in Appendix A.

Households operate production with an intensity $u \in \{L, H\}$. An intensity is chosen every

[13]Other related work includes Silva (2016), that focuses on open-market operations and the effects of expected inflation. In Buera and Nicolinni (2016), the identity between borrowers and lenders is determined by a threshold interest rate. Furthermore, there is an explicit role for outside money as a transactions instruments and MP has real effects by affecting the stock of risk-free bonds which, in turn, affects the threshold identity of borrowers and lenders.

[14]In the extension of section 3, we also imbed a labor supply decision.

instant. We refer to $u = L$ as the low intensity and $u = H$ as the high intensity. The choice of u determines the production rate, $y(u)$. Production is higher under the high utilization,$y(H) > y(L) > 0$. However, if $u = H$, the household faces idiosyncratic risk.[15] This idiosyncratic risk is $\sigma(u) dZ_t$ where dZ_t is the white noise associated with a Brownian motion. Each household faces it's own idiosyncratic risk Z_t, but it controls the level of risk via u. We assume that $\sigma(H) > \sigma(L) = 0$.[16]

Households can borrow with some limitations. In particular, credit is limited by two numbers: (i) a constant *debt limit* $\bar{s} \leq 0$ and (ii) a potentially time-varying *borrowing limit* $\tilde{s}_t \geq \bar{s}$. The debt limit produces the constraint $s_t \geq \bar{s}$. The borrowing limit is introduced to allow for a credit crunch. Namely, in $s \in [\bar{s}, \tilde{s}_t]$, household can roll over their debts, including accrued interests, but not borrow more principal. Formally, this constraint is $ds_t \geq r_t s_t dt$. If we combine the borrowing constraint with the household's budget constraint, we obtain:

$$c_t dt \geq dw_t \text{ in } s \in [\bar{s}, \tilde{s}_t].$$

Unless $u_t = L$, the household faces the random shock w_t. Hence, the constraint forces $u_t = L$ and, thus, the borrowing limit is equivalent to:

$$u_t = L \text{ and } ds_t \geq r_t s_t dt \text{ in } s \in [\bar{s}, \tilde{s}_t].$$

A credit crunch is a decline in $\tilde{s}_t$.

Real household income is the sum of individual production and transfers. Real transfers are T_t. Thus, households earn $h(u,t) = y(u) + T_t$. The stochastic component of real household income is $\sigma(u) dZ_t$. Hence, the stochastic process from real income is $dw_t = h(u,t)\, dt + \sigma(u) dZ_t$. The law of motion of real wealth is:

$$ds_t = \left(r_t(s_t) \left(s_t - \frac{m_t^h}{P_t} \right) - c_t \right) dt + dw_t \text{ where } r_t = \begin{cases} r_t^a \text{ if } s_t > 0 \\ r_t^l \text{ if } s_t \leq 0 \end{cases}.$$

The real rates $\{r_t^a, r_t^l\}$ are defined as $r_t^a \equiv i^a - \dot{P}_t / P_t$ and $r_t^l \equiv i^l - \tau^l - \dot{P}_t / P_t$, where $\dot{P}_t / P_t$ is the inflation rate. The corresponding Hamilton-Jacobi-Bellman (HJB) equation of the household problem is:

[15]This shock can be interpreted as risky output or as demand risk. Demand risk can be introduced easily by assuming that products are heterogeneous and aggregated via an Armington aggregator.

[16]It is worth saying that this idiosyncratic risk is born by the household and cannot be diversified due to incomplete markets. This induces a Pareto inefficiency when household's chose $u = L$. This follows because Brownian innovations have mean zero. Hence, if agents could diversify this risk, they would want to, and this would create an extra benefit of $y(H) - y(L)$.

Problem 1 *[Household's Problem] The household's value and policy functions are the solutions to:*

$$\rho V(s,t) = \max_{\{c,u,m\}} \; U(c) + V_s'(r_t(s)\left(s - \frac{m}{P_t}\right) - \frac{\dot{P}_t}{P_t}m - c + h(u,t)) + \frac{1}{2}V_s''\sigma^2(u) + V_t$$

subject to: $u = L$ *and* $c \in [0, h(u,t)]$ *in* $s \in [\bar{s}, \tilde{s}_t]$.

The household's optimal policy is easy to characterize. The choice between risky and safe endowments is separable from the consumption and portfolio choices. The only portfolio choice is to decide how much currency to hold when they have positive wealth. This choice depends only on the nominal deposit rate: they hold only deposits when the nominal deposit rate is positive, they are indifferent between currency and deposits only if the nominal deposit rate is zero, and they strictly prefer currency if deposits are negative. The latter case never occurs in equilibrium. Consumption is given by a simple first-order condition: $U'(c) = V_s'$. Finally, the risky endowment is chosen whenever:

$$\frac{Y(H) - Y(L)}{\frac{1}{2}\sigma^2(H)} \geq \frac{V_s''}{V_s'} = \gamma \frac{c_t'(s,t)}{c(s,t)}.$$

The interpretation is that as long as the precautionary motive is not strong, household only select the safe technology when they are forced to. For the rest of the paper, we assume and verify that this condition holds.

Let $c(s,t), u(s,t)$ and $m^h(s,t)$ be the solutions to the household's problem. The drift of the household's real wealth is

$$\mu(s,t) \equiv r_t(s)\left(s - m^h(s,t)/P_t\right) - c(s,t) + h(u,t).$$

The volatility of household wealth is $\sigma_s^2(s,t) \equiv \sigma^2(u(s,t))$. The the path of the real wealth distribution, $f(s,t)$, is the solution to the following Kolmogorov-Forward equation:

$$\frac{\partial}{\partial t}f(s,t) = -\frac{\partial}{\partial s}[\mu(s,t)f(s,t)] + \frac{1}{2}\frac{\partial^2}{\partial s^2}\left[\sigma_s^2(s,t)f(s,t)\right]. \quad (1)$$

Banks. Banks are intermediaries between households with positive and negative wealth. There is free entry and perfect competition among banks.[17] At t, banks issue nominal deposits a_t^b, nominal loans l_t^b, and maintain reserves balances m_t^b. An individual bank's balance sheet is described in Appendix A. Their aggregate holdings of deposits, loans and reserves, are denoted by A_t^b, L_t^b, and M_t^b, respectively.

The CB sets a reserve requirement coefficient $\varrho \in [0,1]$. Banks must hold reserves equal

[17] Thus, banks operate without equity. Adding a bank equity would require and additional state variable. Restrictions such as capital requirements or limited participation would produce bank profits.

to ϱ fraction of deposits. If not, a bank is in violation of its reserve requirements. However, the balance of reserves is not entirely under the control of a bank; similar to Bianchi and Bigio (2017a); Piazzesi and Schneider (2018), banks are subject to random payments shocks.

A payment shock occurs within a small time interval Δ, to be taken to zero. Between t and $t+\Delta$ a bank receives or loses deposits from/to other banks. Net deposit flows are settled with reserves. Payment shocks take values $\omega \in \{-\delta, \delta\}$ with equal probability. If $\omega = \delta$, a bank receives δa_t deposits and is credited δa_t reserves from other banks. If $\omega = -\delta$, the bank transfers δa_t deposits and δa_t debited to other banks. Thus, the reserve balance balance at $t+\Delta$, for the bank that receives ω :

$$b_{t+\Delta} = m_t^b + \omega a_t - \varrho \left(a_t + \omega a_t\right).$$

Banks with reserve deficits ($b_{t+\Delta} < 0$) can borrow from banks in surplus ($b_{t+\Delta} > 0$). The inter-bank loans is a dynamic search market. Because of search frictions, banks cannot borrow their entire deficits from other banks, and thus resort to the CB's discount window. We study the problem of the bank as the size of time intervals vanishes $\Delta \to 0$. This limit yields convenient expressions.[18]

The average benefit (cost) of an excess (deficit) reserve balances, b, is:

$$\chi(b) = \begin{cases} \chi^- b & \text{if } b \leq 0 \\ \chi^+ b & \text{if } b > 0 \end{cases}. \tag{2}$$

The coefficients, $\{\chi^-, \chi^+\}$, are endogenous objects whose expressions are presented in Appendix B. For now, we note that χ summarizes the costs of borrowing and lending that follow from the search frictions, the overall distribution of reserves, and policy parameters. Bank profits between t and $t+\Delta$ are:

$$\pi_t^b = \Delta\left(i_t^l l_t^b + i_t^m - i_t^a a_t^b + \mathbb{E}\left[\chi_t\left(b_{t+\Delta}\right)|\theta_t\right]\right).$$

Since profits are proportional to Δ, policy functions are independent of Δ—and thus constant as $\Delta \to 0$. Naturally, CB policies affect bank decisions via the influence on $\dot{\chi}_t$. The problem of an individual bank is:

Problem 2 *[Bank's Problem] A bank solves:*

$$\pi_t^b \quad = \quad \max_{\{a,m,l\} \geq \mathbb{R}_+^3} i_t^l l + i_t^m m - i_t^a a + \mathbb{E}\left[\chi_t\left(b\left(a,m\right)\right)|\theta_t\right]$$

[18]Note that the $b_{t+\Delta}$, is a random that we are treating as a stochastic process. If we were to track b_t as a function of time, this stochastic process would not be well defined. This is beause this process would jump discretely, in every instant. However, treating $b_{t+\Delta}$ as the single realization of a random variable is a well defined object.

subject to $l + m = a$ and

$$b(a,m) = \begin{cases} m - \varrho a + (1-\varrho)\,\delta a \text{ with probability } 1/2 \\ m - \varrho a - (1-\varrho)\,\delta a \text{ with probability } 1/2 \end{cases}.$$

Central Bank. The CB has a net asset position (in nominal terms) given by,

$$E_t = L_t^f - M_t.$$

The net-asset position are, L_t^f, the loans held by the CB, minus the monetary base, $M_t \geq 0$. In real terms, the CB's net-asset position is $\mathcal{E}_t = E_t/P_t$. An open-market operation (OMO) is a simultaneous increase in M_t and L_t^f. The CB can hold $L_t^f < 0$.[19] The CB can make transfers, denoted by $P_t T_t$. The corresponding accounting entries are presented in Appendix A.

In addition to these operations, the CB sets a lending rate i_t^{dw} for discount window loans and a rate on reserves i_t^m. The CB faces a solvency restriction, $i_t^{dw} - i_t^m \geq 0$ and $i^{dw} \geq 0$.[20] The pair $\{i_t^m, i_t^{dw}\}$ are called the corridor rates. The distance $\iota_t = i_t^{dw} - i_t^m$ is the corridor spread.

The income flow of the CB is:

$$\pi_t^f = i_t^l L_t^f - i_t^m (M_t - M0_t) + \iota_t \left(1 - \psi_t^-\right) B_t^-. \tag{3}$$

The first two terms are the interest-rate income and expenses. The CB earns (pays) $i_t^l L_t^f$ on its holdings (issuances) of loans. The CB also pays an interest on reserves i_t^m on the money supply held by reserves—the currency stock, $M0_t$, does not earn interests. The third term, $\iota_t \left(1 - \psi_t^-\right) B_t^-$, is the income earned at the discount window loans—$\left(1 - \psi_t^-\right) B_t^-$ the aggregate amount of discount loans, that we describe below.

The net-asset position evolves according to

$$dE_t = \underbrace{\pi_t^f dt - P_t T_t}_{\text{CB profits - transfers}} = \underbrace{dM_t - dL_t^f}_{\text{unbacked transfers}}.$$

Markets. The CB supplies outside money, M_t. Outside money is held as reserves by banks,

[19] There is no distinction between private and public loans. In fact, whenever $L_t^f < 0$, an increase in L_t^f is interpreted as conventional open-market operation. Instead, when $L_t^f > 0$, an increase L_t^f is an unconventional open-market operation.

[20] The spread $i_t^{dw} - i_t^m \geq 0$ because a negative corridor spread would enable banks to borrow from the discount-window and lend back the CB making arbitrage profits. If $i^{dw} < 0$,banks could borrow reserves and lend reserves as currency to households swapping the currency for deposits at zero rates. This operation would produce another arbitrage for the bank.

or currency by the public. The aggregate stock of currency is:

$$M0_t \equiv \int_0^\infty m_t^h(s) f(s,t)\, ds.$$

Equilibrium in the outside-money market is:

$$M0_t + M_t^b = M_t. \tag{4}$$

The credit market has two sides, a deposit and a loans market. In the deposit market, households hold deposits supplied by banks. In the loans market, households obtain loans supplied by banks. The distinction between the loans and deposits is that they clear with different interest rates. The deposit market clears when:

$$A_t^b = \int_0^\infty \underbrace{a_t^h(s)}_{P_t s - m_t^h(s)} \cdot f(s,t)\, ds. \tag{5}$$

The left is the supply of deposits and the right is the deposit demand. The loans market clears when:

$$L_t^b + L_t^f = \int_{-\infty}^0 l_t^h(s) f(s,t)\, ds. \tag{6}$$

The monetary aggregates are: M_t is the monetary base, $M0_t$ is the currency outstanding and the higher aggregate is $M1_t \equiv A_t^b + M0_t$.

The interbank market is the market where banks exchange reserve positions. By the end of each t, each bank maintains a reserve balance b_t. A fraction of those balances, the amount f_t, are lent (or borrowed) at the interbank market. If the bank is in deficit, $b_t - f_t$ is borrowed from the CB's discount window at a cost i_t^{dw}. The corresponding amounts traded in the interbank market and borrowed from the discount window depend on the search probabilities $\{\psi_t^+, \psi_t^-\}$. In particular,

$$f = \begin{cases} -\psi^- b & \text{if } b \le 0 \\ \psi^+ b & \text{if } b > 0 \end{cases} \quad \text{and} \quad b - f = \begin{cases} -(1-\psi^-) b & \text{if } b \le 0 \\ 0 & \text{if } b > 0 \end{cases}.$$

We define the aggregate deficit and surplus of reserves by:

$$B_t^- = -\int b_t \mathbb{I}_{[b>0]} G_t(b) \qquad \text{and} \qquad B_t^+ = \int b_t \mathbb{I}_{[b>0]} G_t(b).$$

Clearing in the interbank market requires that:

$$\psi_t^- B_t^- = \psi_t^+ B_t^+. \tag{7}$$

The interbank market is over-the-counter (OTC) market (as in Ashcraft and Duffie, 2007; Afonso and Lagos, 2012) and, thus, there are many interest rates. The average interbank rate is endogenous and equal to $\bar{\imath}_t^f$. Given trading probabilities, the policy rates and the average rate $\bar{\imath}_t^f$, the average rate earned on positive (negative) positions determine (2):

$$\chi_t^- = \psi_t^- \bar{\imath}_t^f + (1-\psi_t^-)\iota_t, \text{ and } \chi_t^+ = \psi_t^+ \bar{\imath}_t^f.$$

We adopt the formulation in Bianchi and Bigio (2017b) which presents an explicit solution to $\left\{\psi_t^+, \psi_t^-, \bar{\imath}_t^f\right\}$. The microfoundation in Bianchi and Bigio (2017b) follows Afonso and Lagos (2012), under special assumptions that deliver analytic expressions for $\left\{\psi_t^+, \psi_t^-, \bar{\imath}_t^f\right\}$.[21]

Let $\theta_t = B_t^- / B_t^+$ denote the market tightness. The probabilities and rates $\left\{\psi_t^+, \psi_t^-, \bar{\imath}_t^f\right\}$ depend θ_t and an efficiency parameter, λ. This parameter captures the clearing speed of the OTC market. Given an interbank-market tightness, θ, we obtain, $\bar{\theta}$, the post-trade tightness. These ratios are related via:

$$\bar{\theta}(\theta) \equiv \begin{cases} 1 + (\theta - 1)\exp(\lambda) & \text{if } \theta > 1 \\ 1 & \text{if } \theta = 1 \\ \left(1 + \left(\theta^{-1} - 1\right)\exp(\lambda)\right)^{-1} & \text{if } \theta < 1 \end{cases}. \tag{8}$$

With this function, we obtain the average cost function in (2) through:

$$\chi^+(\theta, \iota) = \iota \left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{1/2} \left(\frac{\theta^{1/2}\bar{\theta}(\theta)^{1/2} - \theta}{\bar{\theta}(\theta) - 1}\right) \text{ and } \chi^-(\theta, \iota) = \iota \left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{1/2} \left(\frac{\theta^{1/2}\bar{\theta}(\theta)^{1/2} - 1}{\bar{\theta}(\theta) - 1}\right), \tag{9}$$

[21] The idea in the Afonso and Lagos (2012) model is that banks in surplus and deficit trade in sequential trading rounds. During each round, a number of matches between deficit and surplus banks are formed. Upon a match, banks bargain over the rate on interbank loan. The outside option depends on the matching probabilities of the following rounds and the outside options of subsequent rounds. Matching probabilities evolve depending on the evolution of matches. The number of matches depend on the volume of deficit and surplus balances that haven't matched at previous rounds.

because the time-varying coefficients of (2) are $\chi_t^+ = \chi^+(\theta_t, \iota_t)$, and $\chi_t^- = \chi^-(\theta_t, \iota_t)$. The formulas for $\{\psi_t^+, \psi_t^-, \bar{\iota}^f\}$ are presented in Appendix B. This object is critical for control over credit spreads. Figure 12 in Appendix D presents a depiction of the formula (9).[22]

Finally, the goods market clears when:

$$\int_{-\infty}^{\infty} y(u(s,t)) f(s,t)\, ds \equiv Y_t = C_t \equiv \int_{-\infty}^{\infty} c(s,t) f(s,t)\, ds. \tag{10}$$

Equilibrium. A price path-system is the vector functions $\{P(t), i^l(t), i^a(t)\} : [0,\infty) \to \mathbb{R}_+^3$. A policy path is the set of functions $\left\{L_t^f, M_t, E_t, i_t^{dw}, i_t^m, T_t, \tau_t\right\} : [0,\infty) \to \mathbb{R}_+^7$. Next, we define an equilibrium path.

Definition 1 *[Perfect Foresight Equilibrium.] Given initial condition for the distribution of household wealth $f_0(s)$, for E_0 and P_0, and a policy path, an equilibrium is (a) a price system, (b) a path of real wealth distribution $f(s,t)$, (c) aggregate bank holdings $\{L_t^b, M_t^b, A_t^b\}_{t\geq 0}$, and (d) household's policy rule and value function, $\{c(s,t), u(s,t), m^h(s,t), V(s,t)\}_{t\geq 0}$:*

1. *the solution to the bank's problem is $\{A_t^b, M_t^b, L_t^b\}_{t\geq 0}$,*
2. *the household's policy rule and value functions solve the household's problem,*
3. *the government's policy path satisfies the governments budget constraint (3)*
4. *the law of motion for $f(s,t)$ is consistent with (1)*
5. *all the asset markets and the goods market clear.*

We characterize some features of the equilibrium dynamics of the model in the next section. A steady state occurs when $\frac{\partial}{\partial t} f(s,t) = 0$ and $\{r_t^a, r_t^l\}$ are constant. An asymptotically stable path is an equilibrium path where $\{r_t^a, f(s,t), r_t^l\}$ asymptotically approaches a steady-state.

3.1 Discussion of Environment Features

Financial Architecture. The financial architecture of the model capture some institutional features of financial markets. In practice, banks issue deposits in two transactions. A first transaction is an effective swap of liabilities with the public. For example, when banks lend, banks effectively issue deposits to borrowers (a bank liability) in exchange loans (a liability of the public). This swap captures the process of inside money creation. Then deposits circulate from borrowers to savers as borrowers purchase consumption from savers. This circulation gives rise to the positions in the interbank market. The second transaction is that households can exchange

[22] Observe that $\chi^-(\theta,\iota)$-$\chi^+(\theta,\iota)$=$\iota\left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{1/2}\left(\frac{\theta-1}{\bar{\theta}(\theta)-1}\right)$¿0 because the sign of $\theta-1$ and $\bar{\theta}(\theta)-1$ is the sames.

deposits for currency.—in the model, currency is automatically transferred into CB reserves. This transactions allows the model to be explicit about a deposit zero-lower bound (DZLB).[23]

Financial Constraints. The distinction between borrowing and debt limits has a technical motivation. The technical reason is that formulation allows to study an unexpected credit crunch. Suppose we want to study a credit crunch with only a debt limit. If there is an unexpected change change in debt limits, because income flows continuously, there would be a positive mass of households violating their debt limit in the instant of the credit crunch. This inconvenience does not apply when the borrowing limit $\tilde{s}_t$ moves unexpectedly. In the latter case, households now face a problem insuring risk, but are not forced to reduce their debt stock immediately.

There is also an economic motivation to make a distinction between borrowing and debt limits. When a bank extends the principal of loan, it increases the bank's liabilities. This is different than rolling over a loan. In the case of a rollover, banks earn interest income which increases their equity in an accounting sense. During financial crises, banks may wish to allow debt roll-overs, but may not want to extend more principal precisely because more principal consumes regulatory capital. In addition, if a bank forces a sudden loan repayment, it can trigger a loan default. Defaults are costly for banks, because defaults lead to underwritings that also subtract regulatory capital. The formulation of financial constraints in the model, is motivated by these observations, although the model is not explicit about bank capital.[24]

Time-Zero Price as a Parameter. Our definition of equilibrium is non-standard treats time-zero price as given. The reason to fix the time-zero price is because otherwise, like in any model with an nominal asset, the model has multiplicity. The reason is that a time-zero price determines the real distribution of wealth. Our approach is to think of P_0 as stemming from a steady-stat price level reached in the past. This we think of time-zero as a steady state with a well determined price level. In fact, the equilibrium definition can be adapted to begin a steady state.

[23] It is not the usual argument of rulling out the arbitrage where individuals can borrow at the bond rate and deposit in currency. Instead, here, by convention deposits are a claim on currency so they are exchanged at par. If the deposit rate is positive, it will not be in their interest to hold currency. If banks offer a negative deposit rate, households would convert all deposits into currency. When deposit rates are zero, banks are indifferent between exchanging deposits for reserves on the margin. Below, describe the economy is affected by this constrain and describe how if the CB charges negative rates on reserves, this induces an increase in spreads.

[24] This phenomenon is called evergreening. We do not model this explicitly, but we are guided by this economic interpretation. Our constraint is consistent with the interpretation. Caballero et al. (2008) discuss evergreening feature in a model of zombie lending.

4 Implementation

From Nominal to Real Variables. Next, we derive some results that enable us to obtain a set of implementation conditions for . Define the *liquidity ratio* as

$$\Lambda_t \equiv M_t^b / A_t.$$

The market tightness of the interbank market can be written as a function of the liquidity ratio:

$$\theta_t = \theta(\Lambda_t) \equiv \frac{\sum_{z\in\{-1,1\}} -\frac{1}{2}\min\{\varrho - \Lambda_t - \delta z, 0\}}{\sum_{z\in\{-1,1\}} \frac{1}{2}\max\{\Lambda_t - \varrho + \delta z, 0\}}.$$

We obtain the following Lemma:

Lemma 1 *[χ function] The coefficients of the liquidity cost function, χ_t, are a function of the policy corridor, ι_t, and the liquidity ratio, Λ_t. Two monetary aggregates $\{M_t^b, A_t\}$ feature the same liquidity ratio produce the same liquidity cost function.*

Explicit formulas for $\{\chi_t^+, \chi_t^-\}$ as functions of Λ_t are presented in Appendix F.1. The illustrated are plotted in Figure 13 in Appendix D which depicts the market tightness and $\{\chi_t^+, \chi_t^-\}$ as functions of Λ_t. An immediate consequence of Lemma 1 is an analytic expression for the nominal loan and deposit rates, as functions of $\{i_t^m, \Lambda_t, \iota_t\}$:

Proposition 1 *[Nominal Rates and Real Spread] Given $\{i_t^m, \Lambda_t, \iota_t\}$ the equilibrium rates $\{i_t^l, i_t^a\}$ are:*

$$i_t^l = i^l(i_t^m, \Lambda_t, \iota_t) \equiv i_t^m + \underbrace{\frac{1}{2}\left[\chi^+(\Lambda_t, \iota_t) + \chi^-(\Lambda_t, \iota_t)\right]}_{\text{liquidity value of reserves}} \tag{11}$$

$$i_t^a = i^a(i_t^m, \Lambda_t, \iota_t) \equiv i_t^m + \underbrace{\frac{1}{2}(1-\varrho)\left[(1+\delta)\chi^+(\Lambda_t, \iota_t) + (1-\delta)\chi^-(\Lambda_t, \iota_t)\right]}_{\text{liquidity value of reserves-liquidity cost of deposits}}. \tag{12}$$

The equilibrium real credit spread, $r_t^l - r_t^a$, is:

$$\Delta r_t = \Delta r(\Lambda_t, \iota_t) \equiv \varrho\frac{\chi^+(\Lambda_t, \iota_t) + \chi^-(\Lambda_t, \iota_t)}{2} + (1-\varrho)\delta\frac{\chi^-(\Lambda_t, \iota_t) - \chi^+(\Lambda_t, \iota_t)}{2}. \tag{13}$$

The equilibrium rates (11) and (12) guarantee that banks earn zero profits.

Proposition 1 establishes that both the nominal borrowing and lending rates equal the nominal interest on reserves plus a liquidity premium. In either formula, the nominal interest on reserves acts as a base rate. The liquidity premia over the rate on reserves constant depend, through χ, on the liquidity ratio, Λ, and corridor spread, ι. It is easy to verify that because

$\chi^- \geq \chi^+$, there is a positive spread $i^l_t \geq i^a_t$. Consider the liquidity premium that determines the loans rate: loans have to earn a premium over the interest on reserves because, on the margin, an additional holding of reserves either earns χ^+ if the bank is in surplus or saves the bank χ^- if the bank is in deficit—each scenario occurs with equal probability which explains the 1/2.[25] That premium is a liquidity value of reserves. The liquidity premium of deposits emerges because, an additional unit of deposits produces a marginal balance of reserves of $\varrho + (1-\varrho)\delta$ if the withdrawal shocks takes deposits out of the bank or $\varrho - (1-\varrho)\delta$ if it brings funds. In either scenario, the marginal effects are χ^- and χ^+ respectively. The premium earned by deposits are the difference between this liquidity cost of deposits and the liquidity value of reserves.

The premia between loans and deposits translate into real a real spread, (13), which is also as function of Λ and ι. This result is automatic because any spread between two nominal rates equals the spread between real rates.

Notice that if the CB is able to affect real spread, and banks earn zero profits as established by Proposition 1, the profits from the spread must be earned by the CB. The next proposition exploits this observation and shows how to express the law of motion of the net-asset position $\mathcal{E}_t$ without reference to nominal variables:

Proposition 2 *[Real Budget Constraint] Consider equilibrium in all asset markets, then $\mathcal{E}_t$ satisfies:*

$$d\mathcal{E}_t = \left(\underbrace{(r^a_t + \Delta r_t)}_{\text{return on CB balance sheet}} \mathcal{E}_t + \underbrace{\Delta r_t \int_0^\infty s f\,(s,t)\, ds}_{\text{discount window profits}} - T_t \right) dt, \tag{14}$$

with $\mathcal{E}_0$ given.

Proposition 2 is a law of motion in real terms for the CB's position. The first term is the portfolio returns earned by CB earns (losses) which equal the real lending rate r^l_t times the net asset position. The CB earns some operational profits from the discount window.

An additional restriction is a long-run long-run solvency constraint for the CB. In particular, there's limit $lim_{t\to\infty}\mathcal{E}_t \geq \underline{\mathcal{E}}$, for some minimum $\underline{\mathcal{E}}$ that guarantees that the CB can raise enough revenues and satisfy $d\underline{\mathcal{E}} = 0$—essentially, the model features a Laffer curve for CB revenues. It must be the case that at $\underline{\mathcal{E}}$ discount-window revenues cover any balance sheet costs. This condition is equivalent to assuming that the the CB's liabilities are not worth zero in equilibrium. Although we don't solve for $\underline{\mathcal{E}}$, in all of the equilibria we study, the policy path converges to a stable government net-asset position and $lim_{t\to\infty} d\mathcal{E}_t = 0$.

Another restriction is that $\mathcal{E}_t \leq \bar{s}$, which is equivalent to saying that the CB cannot save more

[25] More precisely, we have to consider the probability that the bank is in surplus or deficit. However, since all banks are identical, $\chi^+ = \chi^+$ when the interbank market is has only one side.

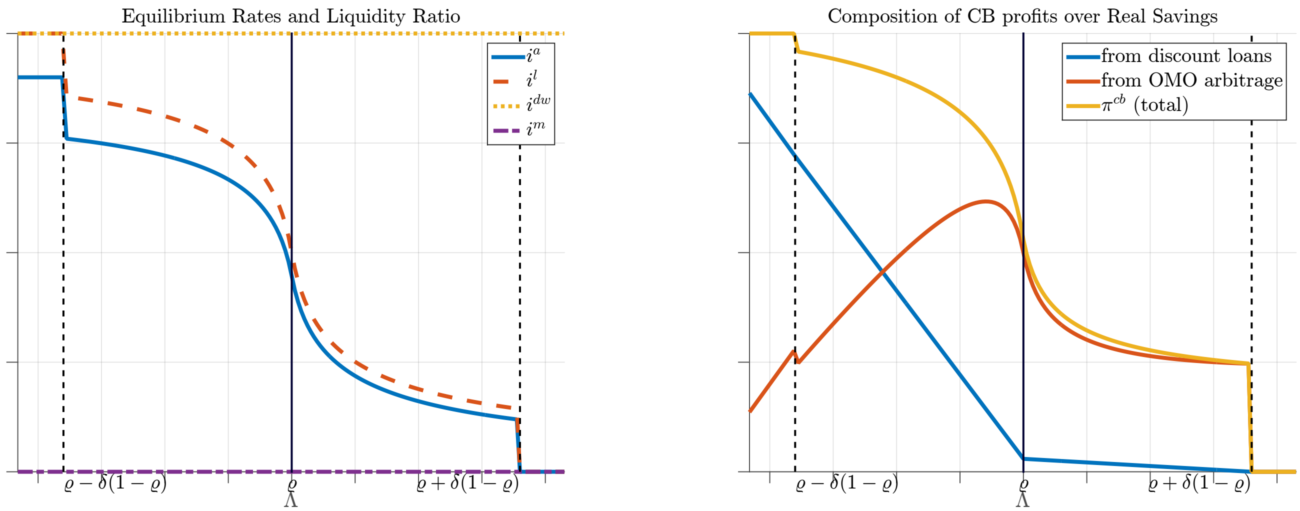


Figure 1: Market Rates and CB profits as Functions of Induced Liquidity

than the highest possible supply of loans, which is attained when all agents hit their borrowing limits.

Figure 1 displays the formulas in Propositions 1 and the components of CB profits as functions of Λ_t. The figure displays two panels, the left panel plots $\{i_t^l, i_t^a\}$ as functions of (11) and (12) for fixed policy rates$\{\iota, i^m\}$. Both rates lie in between i^m and i^{dw}.[26] Both rates feature a spread for $\Lambda \leq \varrho + (1-\delta)\,\varrho$—we discuss what occurs at this limit next. The credit spread decreases with the liquidity ratio. The right panel shows the components of the CB's profits—normalized by the stock of deposits.[27]

The next proposition shows that all market-clearing conditions can be summarized via a single market-clearing condition in terms of real wealth. This proposition is critical to obtain a computation algorithm for the model.

Proposition 3 *[Real Wealth Clearing] Let nominal rates be given by (11) and (12), and let the liquidity ratio be given by Λ_t. Then, market-clearing in real terms*

$$-\int_{-\infty}^{0} sf(s,t)\,ds = \int_{0}^{\infty} sf(s,t)\,ds + \mathcal{E}_t \;\; \textit{for } t \in [0,\infty), \tag{15}$$

[26]Credit risk or illiquidity is enough to produce rates above those bands.

[27] The first source are revenues obtained from discount window loans. As CB provides more liquidity, its discount-window profit per unit deposit declines. The second source of revenues is the arbitrage from open-market operations. This source produces the typical Laffer curve that emerges in monetary models. For a fixed amount of deposits, the higher the liquidity ratio, the CB exploits an arbitrage between the loans and the rate on reserves. The larger the open-market operations, the greater the arbitrage. However, as the CB provides more liquidity, the spread $i_t^l - i_t^m$ drops, which explains decreasing profit part of the plot. This is different from the typical Laffer curve that follows from seigniorage. With a decreasing demand for real balances in inflation, monetary models feature a Laffer curve because more inflation provides more marginal revenues. However, an opposite force emerges from reducing the value of real balances.

implies market clearing in all asset markets. Furthermore, if (15) and the Kolmogorov-Forward equation (1) hold, then, the goods market clearing condition (10) also holds.

Given a spread, market clearing in real financial claims is consistent with one real deposit rate r_t^a.

Implementation. From equation (13), we learn that the real spread Δr_t is a function of the liquidity ratio and the corridor spread, $\{\Lambda_t, \iota_t\}$. The policy corridor $\{\iota_t\}$ is directly chosen by the CB. A natural question is then, to what extent can the CB control the real spread? Next, we establish that the CB can control the liquidity ratio via a choice of OMO's. Yet, this control reaches a limit when liquidity induces a DZLB.

In this economy, households can convert deposits into currency. Banks cannot.[28] As a result, although the CB can set $i_t^m < 0$, the deposit rate is always positive $i_t^a \geq 0$. We can solve for another liquidity ratio, a liquidity ratio such that, using (12), produces a $i_t^a = 0$. When rates on reserves are positive, $i_t^m \geq 0$, the deposit rate is always positive, regardless Λ_t. Hence, when rates are positive, there is no liquidity ratio that produces a deposit rate of zero. For $i_t^m < 0$, there does exist a liquidity ratio consistent with zero interests on deposits.[29] Formally, we define Λ_t^{zlb} as the threshold liquidity that triggers the DZLB, which equals:

$$\Lambda^{zlb}\left(i_t^m, \iota_t\right) \equiv \begin{cases} \min\left\{\Lambda | 0 = i_t^m + \frac{1}{2}\left(1-\varrho\right)\left[\left(1-\delta\right)\chi^{+}\left(\Lambda, \iota_t\right) + \left(1+\delta\right)\chi^{-}\left(\Lambda, \iota_t\right)\right]\right\} & i_t^m \leq 0 \\ \infty & i_t^m > 0 \end{cases}.$$

To illustrate how the DZLB limits the control over the liquidity ratio, we define the *monetary-base liquidity ratio*, Λ_t^{MB}, as

$$\Lambda_t^{MB} \equiv \frac{M_t}{A_t}.$$

Different from the banks' liquidity ratio Λ_t, the monetary-base liquidity ratio Λ_t^{MB} is defined in terms of the total monetary base, the sum of reserves and currency, and not only reserves, in its numerator. We can express the monetary-base liquidity ratio in terms of real objects:

$$\Lambda_t^{MB} = \frac{\left(E_t + L_t^f\right)/P_t}{A_t/P_t} = \frac{\mathcal{E}_t + \mathcal{L}_t^f}{\int_0^\infty s f\left(s,t\right) ds} \equiv \Lambda^{MB}\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right).$$

With these threshold points, we can establish the following implementation result:

[28] We assume that banks can't hold currency wither by regulation, taxation or physical costs.

[29] By construction, the currency held by households satisfies:

$$M0_t = \left(\Lambda_t^{MB} - \Lambda_t^{zlb}\right) P_t \int_0^\infty s f\left(s,t\right) ds > 0.$$

Proposition 4 *[Implementation Conditions] Consider an equilibrium path for the real deposit rate, the real spread, the distribution of real wealth and inflation,* $\left\{r_t^a, \Delta r_t, f_t, \dot{P}_t/P_t\right\}_{t\geq 0}$. *This path is a sufficient statistic for the equilibrium allocation. To implement the desired equilibrium path, the CB chooses* $\left\{i_t^m, \iota_t, \mathcal{L}_t^f, T_t\right\}$ *subject to the following restrictions:*

1. *The liquidity ratio is given by:*

$$\Lambda_t = \Lambda\left(i_t^m, \iota_t, \mathcal{E}_t, f_t, \mathcal{L}_t^f\right) \equiv \min\left\{\Lambda^{zlb}\left(i_t^m, \iota_t\right), \Lambda^{MB}\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right)\right\}.$$

2. *The real spread* Δr_t *is given by (13) and* $\{\Lambda_t, \iota_t\}$.

3. *The real rate* r_t^a *solves (15), given* $f(s,t)$*, the real spread* Δr_t*, the transfers* T_t *and the inflation rate* $\dot{P}_t/P_t$.

4. *The distribution of real wealth,* f_t *, evolves according to (1) with* f_0 *given.*

5. *The real net-asset position,* $\mathcal{E}_t$*, satisfies (14) with* $\mathcal{E}_0$ *given.*

6. *Finally, the inflation rate is given by:*

$$\dot{P}_t/P_t \;=\; i_t^m + \frac{1}{2}\left[\chi^+\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right) + \chi^-\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right)\right] - \left(\Delta r_t + r_t^a\right). \quad (16)$$

Proposition 4 describes the set of allocations that can be induced by the CB. The allocations are affected by the CB because it can indirectly control the real spread either through changes in corridor rates or by affecting the liquidity ratio via OMO. The liquidity ratio cannot be increased beyond Λ_t^{zlb} because beyond that point, any OMO translates into an increase in currency without effects on real spreads. At that point the CB loses spreads as a policy instrument. The real spread, given a distribution of wealth or a net-asset position, pins the real deposit rate of the economy. The real deposit rate is consistent with market clearing, and the evolution of the net-asst position. Since the real rates are pinned down by market clearing, the interest on reserves determines the inflation rate. Through market clearing, the size of $\mathcal{E}_t$ also influences the real interest rate of the economy.

In addition to the DZLB which limits the control over the liquidity ratio, there are two other conditions where the interbank market is inoperative. One region occurs when the CB produces a reserve-satiation regime—when every bank has enough reserves to meet their reserve requirements. This regime occurs when $\Lambda_t \geq \bar{\Lambda} \equiv \varrho + (1+\delta) > 0$. The other region is the scarcity regime that occurs when all banks are in deficit. This regions is given by $\Lambda_t \leq \underline{\Lambda} \equiv \varrho + (1-\delta)$.

It is immediate to obtain a relationship between the satiation liquidity threshold and the

DZLB liquidity threshold, depending on whether interest on reserves are positive. In particular:

$$sign\left(i_t^m\right) = sign\Big(\Lambda^{zlb}\left(i_t^m, \iota_t\right) - \bar{\Lambda}\Big).$$

In words, when the rate on reserves is positive, the satiation point is reached before the *monetary-base liquidity ratio* reaches DZLB point. If the rate is negative, the DZLB is reached before. Intuitively, when the interest rate on reserves is $i_t^m = 0$, the DZLB coincides with the satiation point, because banks are indifferent between holding reserves in exchange for deposits at zero rates, and households are indifferent between holding currency or deposits. Since banks are satiated, they have no additional uses for reserves. When the rate on reserves is positive, the deposit rate will be positive because banks would attract any currency holdings to convert currency into reserves. When rates on reserves are negative, there is a liquidity ratio such that prior to reaching satiation, banks would charge zero-deposit rates.

This observation is useful to understand the following Proposition about the effects of policy.

Proposition 5 *[Properties of Equilibrium Rates and Spreads] The equilibrium rates and spreads are characterized by the following relations (liquidity premia):*

1. *If $\Lambda_t \in \left(\underline{\Lambda}, \min\left\{\bar{\Lambda}, \Lambda^{zlb}\left(i_t^m, \iota_t\right)\right\}\right)$:*

$$i^{dw} > i^l > i^a > i^m \text{ and } 0 < \Delta r < \iota.$$

2. *If $\Lambda_t \leq \underline{\Lambda}$, then $i^{dw} = i^l = i^a$ and $\Delta r = 0$.*
3. *If $\Lambda_t \geq \bar{\Lambda}$, then $i^l = i^a = i^m$ and $\Delta r = 0$.*
4. *If $\Lambda_t = \Lambda^{zlb}\left(i_t^m, \iota_t\right)$ then $i^l > i^a = 0$ and $\Delta r > 0$.*

The equilibrium rates and spreads feature the following policy effects:

1. *If $\Lambda_t \in \left(\underline{\Lambda}, \min\left\{\bar{\Lambda}, \Lambda^{zlb}\left(i_t^m, \iota_t\right)\right\}\right)$:*

$$\left\{\frac{\partial i^l}{\partial \mathcal{L}_t^f}, \frac{\partial i^a}{\partial \mathcal{L}_t^f}, \frac{\partial \Delta r}{\mathcal{L}_t^f}\right\} < 0, \left\{\frac{\partial i^l}{\partial i_t^m}, \frac{\partial i^a}{\partial i_t^m}, \frac{\partial \Delta r}{\partial i_t^m}\right\} = \{1, 1, 0\}, \left\{\frac{\partial i^l}{\partial \iota_t}, \frac{\partial i^a}{\partial \iota_t}, \frac{\partial \Delta r}{\partial \iota_t}\right\} = \{1, 1, 1\}.$$

2. *If $\Lambda_t < \underline{\Lambda}$, then*

$$\left\{\frac{\partial i^l}{\partial \mathcal{L}_t^f}, \frac{\partial i^a}{\partial \mathcal{L}_t^f}, \frac{\partial \Delta r}{\mathcal{L}_t^f}\right\} = 0, \left\{\frac{\partial i^l}{\partial i_t^m}, \frac{\partial i^a}{\partial i_t^m}, \frac{\partial \Delta r}{\partial i_t^m}\right\} = \{0, 0, 0\}, \left\{\frac{\partial i^l}{\partial \iota_t}, \frac{\partial i^a}{\partial \iota_t}, \frac{\partial \Delta r}{\partial \iota_t}\right\} = \{1, 1, 1\}.$$

3. *If* $\Lambda_t > \bar{\Lambda}$, *then*

$$\left\{\frac{\partial i^l}{\partial \mathcal{L}_t^f}, \frac{\partial i^a}{\partial \mathcal{L}_t^f}, \frac{\partial \Delta r}{\mathcal{L}_t^f}\right\} = 0, \left\{\frac{\partial i^l}{\partial i_t^m}, \frac{\partial i^a}{\partial i_t^m}, \frac{\partial \Delta r}{\partial i_t^m}\right\} = \{0,0,0\}, \left\{\frac{\partial i^l}{\partial \iota_t}, \frac{\partial i^a}{\partial \iota_t}, \frac{\partial \Delta r}{\partial \iota_t}\right\} = \{1,1,1\}.$$

4. *If* $\Lambda_t = \Lambda^{zlb}(i_t^m, \iota_t)$ *then*

$$\left\{\frac{\partial i^l}{\partial \mathcal{L}_t^f}, \frac{\partial i^a}{\partial \mathcal{L}_t^f}, \frac{\partial \Delta r}{\mathcal{L}_t^f}\right\} = 0, \frac{\partial i^l}{\partial i_t^m} > 0, \frac{\partial i^a}{\partial i_t^m} = 0, \frac{\partial \Delta r}{\partial i_t^m} > 0, \frac{\partial i^l}{\partial \iota_t} > 0, \frac{\partial i^a}{\partial \iota_t} = 0, \frac{\partial \Delta r}{\partial \iota_t} = 0.$$

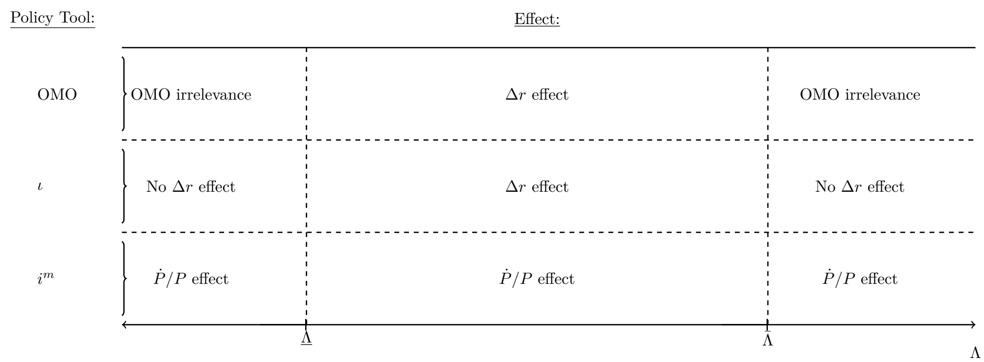
Proposition 5 establishes the direction of effects of the three tools that the CB can alter to affect the real spread: the rate on reserves, the corridor spread and the size of its balance sheet. The qualitative properties are sketched in Figure 4. The qualitative properties depend on the sign of i_t^m: when this rate is positive the liquidity ratio is bounded by its satiation limit whereas when it is negative, it is bounded by the DZLB limit.

In either case, when liquidity is scarce enough to dry up the interbank market, OMO or changes in the discount window do not carry out real effects. When liquidity is greater than $\underline{\Lambda}$,but below the satiation or DZLB limits, the CB can implement a real spread by selecting the OMO that induce real assets, $\mathcal{L}_t^f$, consistent with a liquidity ratio that produces a targeted spread. Alternatively, for fixed Λ_t, the CB can select ι_t that delivers a desired spread. In addition, the CB can select a rate on reserves to achieve a desired level of inflation.

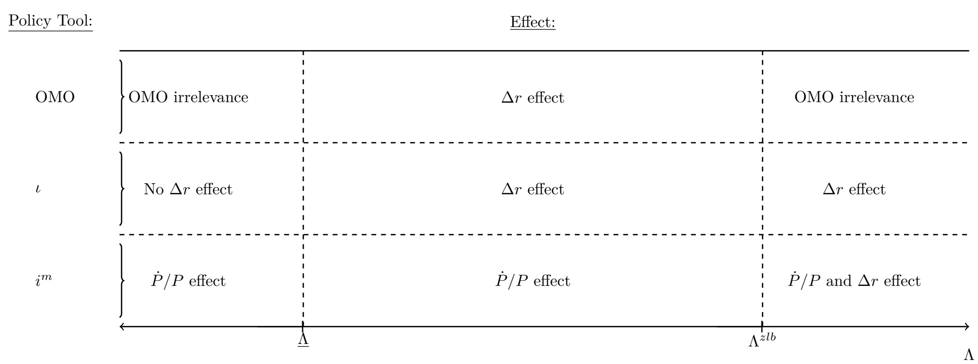
When $i_t^m > 0$ it is possible to have$\Lambda_t > \Lambda$. In that region, OMO satisfy a classic Wallace irrelevance. Furthermore, increases in the corridor spread have no effect. Yet the rate on reserves controls inflation.

An interesting property emerges when $i_t^m < 0$, because that choice opens the possibility of a DZLB. The DZLB is triggered when the liquidity ratio $\Lambda_t = \Lambda_t^{zlb}$. In that region, OMO are irrelevant because for any increase in CB liabilities keeps the liquidity ratio constant as households absorb the increase by holding currency, as we explained above. The interesting property is that the real spread remains open at the DZLB. The reason is that negative rates on reserves act like a tax on deposit holdings: since the deposit rate is fixed at zero, banks require a higher lending rate to compensate them. Hence, changes in the rate on reserves produce a joint effect on the real spread and inflation. Increased in the corridor spread, also alter the spread because the DZLB regime some banks still access the discount window.

This result is different from the DZLB that emerges in models with cash-in advance constraints. In those models, a DZLB emerges if the CB floods the public with savings instruments so that the asset clears at negative rates. This opens the door to an arbitrage.[30] The main take-away from this section is that, unless constrained the DZLB or the satiation/scarticity limites, whenever there's a positive corridor spread, CB's have the power to affect real spreads and inflation and a long-run interest rate, as independent targets. We discuss the policy implications of this result next.



(a) Positive rate on reserves ($i^m > 0$)



(b) Negative rate on reserves ($i^m < 0$)

Figure 2: Summary of Policy Effects

Appendix C presents an algorithm that builds on this proposition that we use to solve the model. Appendix C presents how the solution algorithm has to be adapted to admit currency holdings at the DZLB.

Alternative Implementations. It is worth discussing a few alternative implementations of monetary policy in the model. Bindseil (2014) presents an overview of implementation practices

[30] In Rognlie (2016), negative rates are possible because there are costs of holding physical currency. He also models currency holdings. If banks could transform reserves into cash at zero cost, then $i_t^m \geq 0$. However, if banks have a physical holding cost as in Rognlie (2016), the increase in lending rates would follow.

around the world. In the model, one alternative to control the real spread directly through OMO is to target an interbank-market rate $\bar{i}^{f}$. In the model, the interbank rate is, by definition: $\bar{i}^{f} = i_{t}^{m} + \chi^{+}(\Lambda, \iota) / \psi^{+}(\theta(\Lambda))$. For a given ι, we can obtain a value of Λ consistent with a target $\bar{i}^{f}$. Thus, there is also map from a policy target $\bar{i}^{f}$ to a real spread Δr_{t}. So long interest on reserves, i^{m}, and, $\bar{i}^{f}$, the CB can achieve target inflation and a real spread independently. In practice, central banks are explicit about a nominal interbank target. The interpretation of our model is that this affects the real spread too.

According to Bindseil (2014), CB's adopt either a floor systems or corridor systems. These are both particular cases of the model. In a corridor system, the corridor spread, ι_{t}, is typically kept fixed. In a floor system $i^{m} = 0$, buth the CB changes $i^{dw} = \iota$. In the model, a corridor system or a floor system allow to target inflation and the real spread independently, as long as long as Λ_{t}—or equivalently $\bar{i}^{f}$—the interbank market are kept as separate instruments.

If for example, a country runs a corridor system, but fixes the distance between the interbank rate, the CB loses an instrument, and can either target inflation or a real spread. Prior to 2008, the US adopted a system a floor system while keeping a constant gap between the interbank market and the discount rate. This is like fixing $\iota - \bar{i}^{f}$ to a constant. Many countries that adopt corridor systemts, target a liquidity ratio such that the interbank market fall in between both policy rates, $\bar{i}^{f} = i^{m} + \frac{1}{2}\iota$. In both cases, the CB loses the ability to target spreads and inflation independently. A change in the inflation target alters real spreads. These observation are important. For one thing, they suggest that adopting either regime, by targeting inflation were also moving real spreads, perhaps inadvertently.

Does it matter how we implement a given credit spread? In the model, the spread can be obtained by moving the corridor spread ι, or via OMO. We could be tempted to argue that these instruments have different fiscal consequences, so it matters because they alter the ability to redistribute. In fact, this intuition is wrong, and we can answer the question negatively.

Corollary 1 *[No Fiscal Consequence of an implementation choice] Consider two policies $\{\iota_{t}, \Lambda_{t}\}$ that implement the same real spread target, Δr_{t}. Both are consistent with the same discount window profits, and hence produce the same fiscal revenue.*

Price-Level Determination and Consistent Transfers. The model inherits classical properties money financed deficits in Bewley economies (Bewley (1983); Ljungqvist and Sargent (2012, , Chapter 18.11)). First, because the economy is nominal, there is a continuum of equilibrium indexped by time-zero prices: for given nominal claims, the time-zero price indexes a real distribution of wealth. Second, a version of the quantity theory holds here. Fix the path of the real net-asset position, $\mathcal{E}_{t}$, and real transfers, then we can scale every nominal variable by a scalar and obtain the same equilibrium. Third, changes in the growth rate of unbacked monetary issuances produce an increase in inflation. For example, assume a policy from t onwards, the CB increases T. Then inflation will increase at a constant rate as long as the CB increases i_{t}^{m} at the same amount. If the CB increases transfers but keeps the real rate constant, it effectively changes the real value of transfers and thus also the real rate. Similarly, an increase in i_t^m is consistent with a higher inflation rate only nominal transfers increase to maintain real transfers constant.

5 From Instruments to Transmission Channels

Calibration. Although there is much to incorporate into the model before a serious quantitative assesment, we calibrate the model to illustrate the dynamic features of the model, and get a quantitative sense. The examples we present are based on the following calibration which is summarized in table 5. The calibration has the US economy prior to the Great Recession in mind.

The risk aversion coefficient of risk-aversion and intertemporal-elasticity, γ, is set to 2, a standard number in macroeconomic models. The time discount, ρ, is set to 4%, which yields a real deposit rate of approximately 1.5% at steady state. The high intensity endowment flow is set to 1, which is a normalization. The low intensity endowment is set to 0.7 which produces an output drop of about 10% in a credit crunch where the borrowing limit is set to zero. This is a benchmark number in line with the output drops recorded in the worst recessions. The volatility under the high intensity endowment is set to $\sigma(H) = 1$, to obatain that private savings are about 4 times output. This number is in line with macro models where savings to output (capital) is that amount. We think of the net asset position in terms of the consolidated government. For that reason, we set th net asset position $\mathcal{E}_{ss}$ is set to equal 20% of private assets, and this number yields a level of public debt to GDP of 80%, in line with the value for the US prior to the Great Recession. The debt limit is $\bar{s}$ is 10 times the income generated by the low endowment. This produces a debt-to-income ratio of 10, for the worse household, in line with numbers used by the literature. The amount of real assets held by the CB, $\mathcal{L}_t^f$, is set to zero. This is because prior to the Great Recession, the asset size of the Fed's balance sheet was very small. This choice is a normalization, because as we showed earlier, we can implement a spread with OMO or a policy corridor spread. The interbank-market efficiency, λ, is set to 2.1 following Bianchi and Bigio (2016). The rate on reserves is set to r_{ss}^a so that there is no inflation at steady state. The discount window rate is to produce a spread of $\Delta r_{ss}^a = 2\%$. The rate on reserves is set to 2% which generates approximately no inflation at steady state. Finally, the discount window rates

Steady-state Moments. As a sanity check, we report some moments produced by the model that are not targeted. The moments are reported in table 5. The model produces a share of agents at their borrowing limit of 10% whereas 40% of households are indebted. This numbers produce an output efficiency loss of 3.5%, which follows from the high endowment normalizatoin and per capita GDP of 0.965. The CB's operation profits are 4.2%. In the US, the Fed's transfers to the Federal Government are similar to Corporate Tax revenues, which are about 1.8% of GDP.

Parameter	**Description**	**Reference/Target**
$\gamma = 2$	Risk aversion	Literature Standard
ρ=0.06	Time discount	2% real risk-free rate
$y(H) = 1$	High intensity endowment drift	Normalization
$y(L) = 0.8$	Loan maturity	
$\sigma(H) = 0$	High intensity endowment drift	Savings to GDP ratio $A_{ss}/Y_{ss} = 4$
$\mathcal{E}_{ss}= -0.2 \cdot A_{ss}$	Net asset position	50% public to private debt
$\bar{s} = 10/y(L)$	Debt limit	Maximal loan to income ratio of 12
$\tilde{s}_{ss} = \bar{s}$	Leverage constraint	No shock at steady state
$\mathcal{L}_t^f = 0$	Organic emission	Normalization
λ=2.1	Interbank-market match Efficiency	Follows Bianchi and Bigio (2016)
$i^m = r^a_{ss}$	Reserve rate	0% inflation target
$i^w = 5\%$	Discount rate	real spread of 2%

Table 1: Parameter Values

Moment	**Value**
Fraction of households at debt limit	11.1%
Fraction of households in debt	39.6%
Output efficiency loss	3.5
CB operational revenue	4.2%
CB interest-rate expense	−1.3%
Wealth quantiles/per-capita GDP $\{Q_{10}, Q_{25}, Q_{50}, Q_{75}, Q_{90}\}$	$\{-10.3, -30.8, 1.45, 5.21, 8.58\}$

Table 2: Untargetted Moments

Since the model does not have operational costs for banks or the CB, this figure which is twice as high as in the data, is a reasonable approximation. The interest expense on the CB's poisition is 1.3% of GDP. Finally, we report levels of wealth over GDP measured as wealth over per-capita income, at different quantiles. The model misses the fat-tail features of the wealth distribution that we see in the data.

Doctrines. A monetary doctrine is a set of MP rules. In this section, we study three doctrines. First, we describe a doctrine where MP eliminates credit spreads. Then, we study the effects of unbacked fiscal transfers. The third doctrine targets a credit spread. Table 3 presents a summary of the instruments employed under each doctrine and the channels under which they operate.

	Instrument				Channel		
Doctrine	i_t^m	T_t	ι_t	L_t^f	Fisherian	Non-Ricardian	Credit
Nominal Rate Target (sec 5.1)	✓				✓		
Fiscal Transfers (sec 5.2)		✓				✓	
Credit Target (sec 5.3)		✓	✓	✓		✓	✓

Table 3: Instruments and Transmission Channels

5.1 Nominal Rate Target the Fisherian Channels

We begin with a doctrine where MP neutralizes the effect on real spreads and its fiscal consequences, but where the CB controls a single prevalent nominal rate in the economy, by controlling i^m. Thus, we label this doctrine, the nominal rate target. This doctrine is a useful benchmark because it induces neutrality, so we can study the pure effects of a credit crunch. We have the following:

Corollary 2 *Consider a policy path such that $E_t = T_t = 0$, all t. Let the CB either:*

(a) *satiate banks with reserves, $\Lambda_t > \bar{\Lambda} \equiv \varrho + (1+\delta)$, or*

(b) *eliminate the corridor spread, $\iota_t = 0$,*

at all t, then an equilibrium features:

1. ***(No Spread)*** $\Delta r_t = 0$

2. ***(Neutrality)*** *The evolution of $\{r^a_t, f(s,t)\}$ is unaffected by policy.*

3. ***(Fisherian Transmission)*** *Inflation is controlled by i^m_t through the Fisher equation (16).*

Corollary 2 presents a special case of Proposition 4. The Corollary describes two conditions under which the CB can control inflation without effects on credit markets: when the CB satiates banks with reserves or eliminates the corridor spread. The outcome of these policies a zero credit spread, and policy neutrality. However, the CB can still control inflation. This Corollary is a microfoundation for a nominal control without reference to open-market operations.[31] In essence, under a nominal rate target, the CB effectively controls the unit of account. With a nominal-rate target, the CB can implement an inflation target or price-level target if wants.

The control of the nominal rate relates to three transmission mechanisms that have been studied by the literature. The transmission mechanism in the New-Keynesian model is the real interest-rate channel: With nominal rigidities, the Fisher equation relates changes in nominal rates to changes in real rates. The equilibrium real rate is consistent with the household's Euler equations. In the model of this paper, adding nominal rigidities would produce an interest-rate channel like othre heterogeneous-agent new-Keynesian model (Guerrieri and Lorenzoni, 2012; Kaplan et al., 2016). Proposition 2 is a banking microfoundation for the control of nominal-interest rates. The model with the aggregate demand externality is solved in the same way.

In the CIA and money-search frameworks, real rates are fixed by discount factors. Inflation follows directly from a choice of nominal rates, again according to the Fisher equation. In those

[31] In the Proposition, we set $E_t = 0$ for the sake of exposition, but the result holds with little loss in generality. Under condition (b), if the CB eliminates the corridor system, the CB can control inflation even if it issues zero reserve balances. This is sometimes called the Wicksellian doctrine. The belief that MP should be conducted eliminating all interbank market frictions, Woodford (2001).

models, inflation acts like tax on transactions because transactions are carried out in currency. The model can be adapted to introduce sporadic cash-transactions as in Rocheteau et al. (2016), and produce an inflation tax-channel.

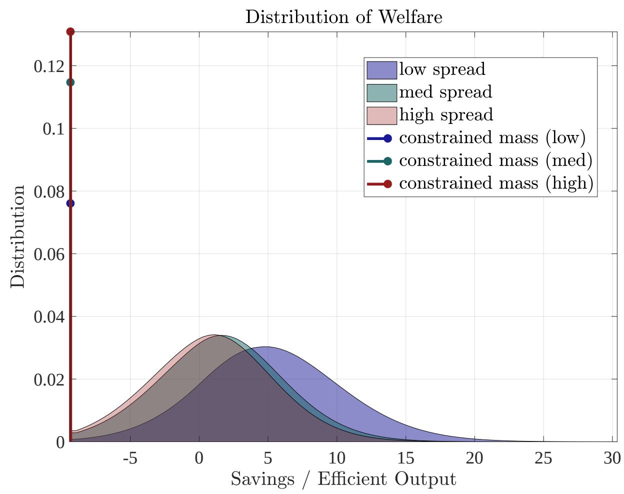
A final channel is the debt-defaltion channel. In the model, debt is paid instantaneously. With long-term debt, unexpected changes in policy rates i_t^{m} would not be neutral, as in (see for example Gomes et al., 2016). The reason is that a surprise in inflation alters the real distribution of debt. Our model can be adapted along those lines to allow for a debt-deflation channel.

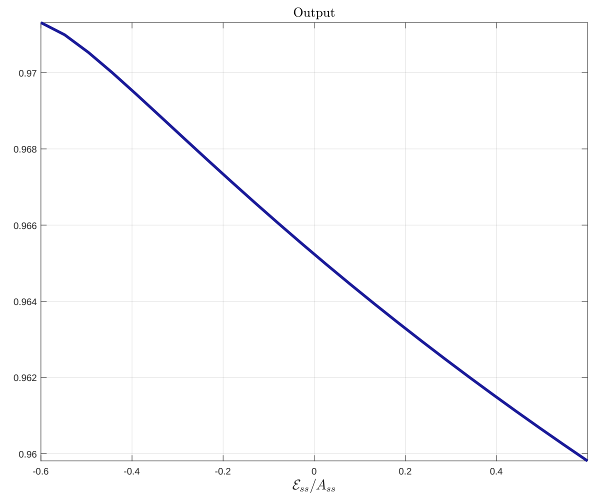
5.2 Fiscal Transfers and the Non-Ricardian Channel

This section is concerned with the effects of unbacked fiscal transfers. There are two reasons to study fiscal transfers. First, although the ultimate goal is to study the credit channel, the credit channel necessarily produces fiscal revenues. Fiscal revenues must be distributed at some point and thus, the credit channel necessarily has a fiscal component. The second reason is that, when the DZLB is reached, fiscal transfers are the only available tool the CB's arsenal. In fact, a program of "helicopter drops" was advocated during recent years. Fiscal transfers are the traditional tool studies in monetary models, so this section establishes a connection with classic monetary-fiscal analysis.

For now, we maintain the assumption that $\iota = 0$, allow for a non-zero net-asset position. We compare first, versions of the same economy for different levels of the net-asset position. Figure 3 reports the level of output and the real wealth distribution for different levels of the net asset position. The model shares the feature that a reduction in the net-asset position is akin to a relaxation of the borrowing limit, a result discovered in Bewley (1983, , Chapter 18.11) and discussed in (Ljungqvist and Sargent (2012, Chapter 18.11)). We can observe in Panel (a) that the lower the net-asset position, the distribution spreads out. A negative asset position is consistent with a higher rates. Panel (b) shows that output increases as the net-asset position falls because there is a lower mass of households at the debt limit. The fact that welfare improves as the CB increases its negative asset position is akin to an increase in debt limis, or conducting a policy that gets closer to the Friedman rule. However, we observe that to generate that effect, the CB needs lump-sum transfers, an instrument that may not be part of its toolkit in practic.

Next, we study an experiment with an a one-time increase in fiscal transfers. The program is anounced at time zero, but it is actually carried out, the following year. The economy is initiated at a steady-state net-assets $E_{ss} = -0.2 \times A_{ss}$. This initial condition produces corresponding transfers equivalent to $T_{ss} = r^{a}_{ss} \times E_{ss}$. When the policy is anounced, transfers are raised five-time when the program is initiated. At the initial impact, the policy is reverted with a reversal rate. At the end of the program, which lasts a year, transfers fall to a path that allow net assets to drop back to their steady state level. The policy variables in this program are shown in Figure 4.



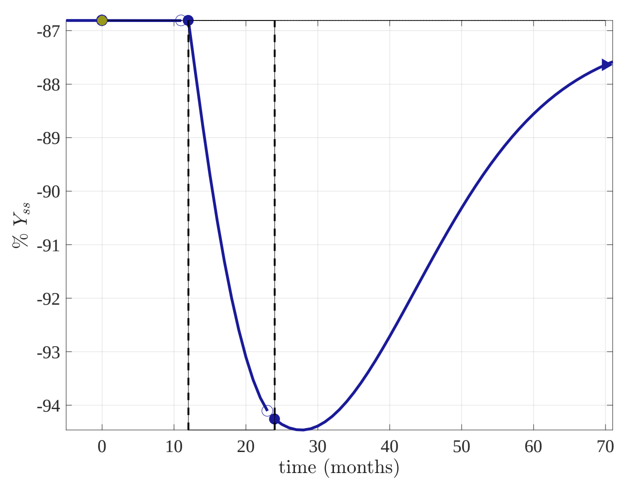


(a) Wealth Distribution for three different levels of $\mathcal{E}_t / A_{ss}$

(b) Output as function of $\mathcal{E}_t / A_{ss}$

Figure 3: Fiscal Transfer Policy.

(a) Net Government Position $\mathcal{E}_t$



(b) Fiscal Transfers T_t

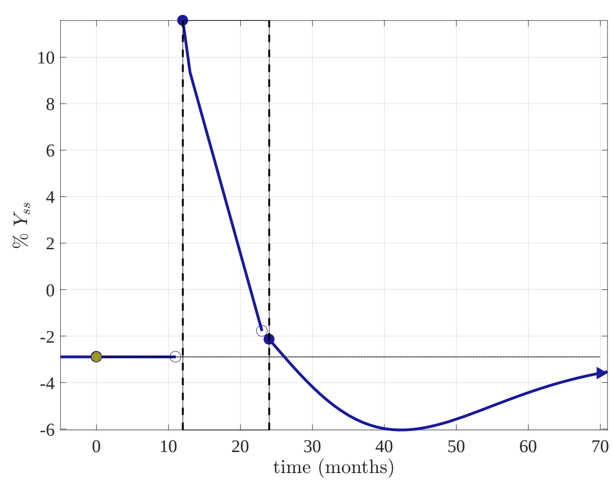


Figure 4: Fiscal Transfer Policy.

Figure 5 plots the responses of macroeconomic variables after the program.[32] Once the policy is anounced, agents expect a fiscal transfer that will materialize the following year. In the expectation of the shock, we observe an increase in the common real interest (Panel c), a growth in credit (Panel b) and a decline in GDP (Panel d). The sign of the effects switch entirely when the policy is implemented.

The reason for the effect prior to the implementation is that fiscal transfers reduce the precautionary behavior by borrowers. At a steady state, being at and abandoning the borrowing limit regions is expensive in marginal utility terms. For that reason, without transfers, agents have a strong precautionary motive to accumulate savings. When borrowers expect thetransfer, they are aware that if in the unlucky state of reaching a borrowing limit, fiscal transfers will push them away from that region. This aid declines their precautionary savings and produces a relative increase in the demand for debt relative to the supply of funds by savers. This pattern results in an increase in real rates. Although the policy induces more credit, it is recessionary in the ex-ante phase: the reason is that because the reduction in precautionary behavior leads more agents to effectively hit their borrowing limits (see the wealth distribution in Panel (a)). In that state, constrained borrowers switch technology and output falls.

Once the policy is implemented, we observe a decline in real rates, a decrease in private borrowing, and an expansion of production. This results from the push away from the borrowing constrained region, produced by the fiscal transfer. Upon the program, real rates suddenly increase despite a reduction in the volume of loans. The reason is the expansion of output because the precautionary motive is eased and agents switch to more efficient production. The transfer increases the volume of deposits: borrowers use the transfer to clear their debts. Savers exchange their transfers for deposits. As a result, banks increase their reserve balances. By period 24, the the policy gradually reverses transfers and the net asset position to steady state. Output and credit variables mirror the behavior of the expansion phase. The policy is clearly non-Ricardian because the market incompleteness.

[32]As explained in section xxx, the operation is carried out with the FA increasing it's debt, the CB increasing reserves and holding public debt.

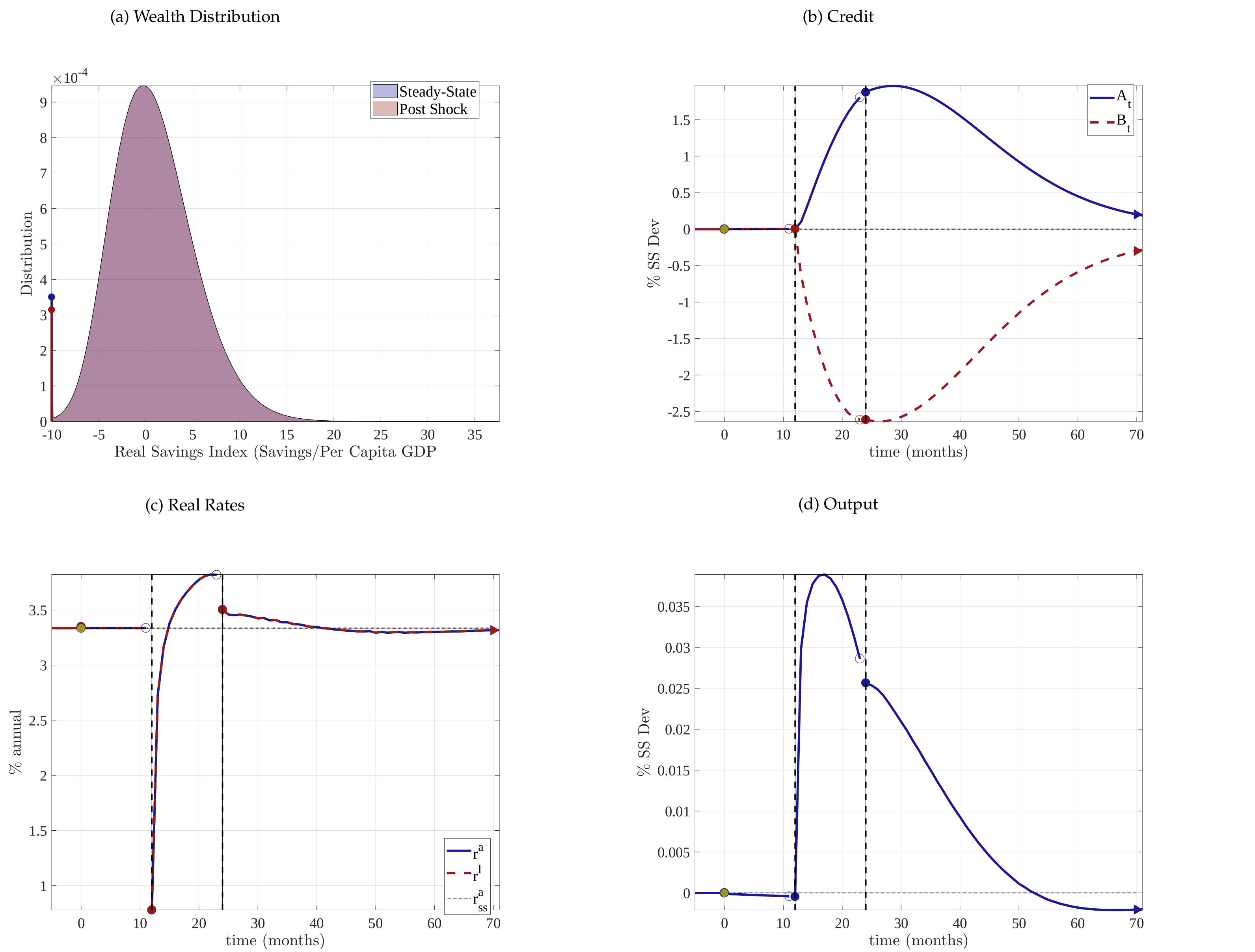


Figure 5: Macroeconomic Aggregates after a Fiscal Transfer.

5.3 Credit Spread Target and the Credit Channel

We now present the effects of MP when its tools activate the credit channel. We first consider a policy where the CB targets a reduction in the real credit spread. To illustrate an ex-ante phase, the policy is announced at time 0, but becomes effective after 1 year, and the policy lasts for another year. As explained earlier, the policy has both the effect of a reduction in credit spreads and alters the profits of the CB. Because the CB earns profits, the FA distributes them as lump-sum taxes. In the previous examples, we showed how these effects where small. The policy targets a positive spread in steady state of 250bps. By period fifteen, open-market operations are carried out and these reduce the credit spread to 100bps. As before, we proceed with a description of ex-ante and ex-post effects.

The ex-ante macroeconomic policy effects are shown in Figure 6. The effects of an expected reduction in credit spreads stimulate borrowers. The reason is that, as with expected transfers, the precautionary motive of borrowers is eased. This produces an increase in the demand for loans relative to the supply of deposits. However, since savers are more elastic to real rates, the real rate increases in response to the expected decline in spreads. We can see that credit expands faster than savings, and this has to do with a reduction in the CG net asset position. Because agents expect an the easing of credit spreads, they reach their borrowing limits faster, something that produces a decline output.

This policy is achieved thanks to a large scale open-market operation (panel (a) in Figure 7). Notice that since the policy assumes fixed nominal interest-rate on reserves, the policy is deflationary given the increase in real rates—the increase in the liquidity ratio reduces both nominal rates. This leads to a deflationary episode. Note that the credit channel operates throughout the supply-side as it leads to a direct effect over bank's balances. An ex-ante deflation is observed in panel (b) of Figure 7.

Immediately after the policy is implemented, the reduction in the real spread pushes a mass of borrowers away from the constrained region and starts a phase of output recovery. The expansion in credit continues during the period of low spreads, but slowly begins to revert. This follows because the period of low spreads is also known to end. This aspect is also shown in the declining path of real rates.

The easing of credit spreads allows a large mass of agents to leave the borrowing-constrained region. This produces an expansion that eventually overcomes the prior reduction in the level of output. As the policy is reverted, real rates are normalized and the distribution of real wealth stabilizes. The economy converges to a one where the price level is lower than before, although reserves in real terms are back to steady state.

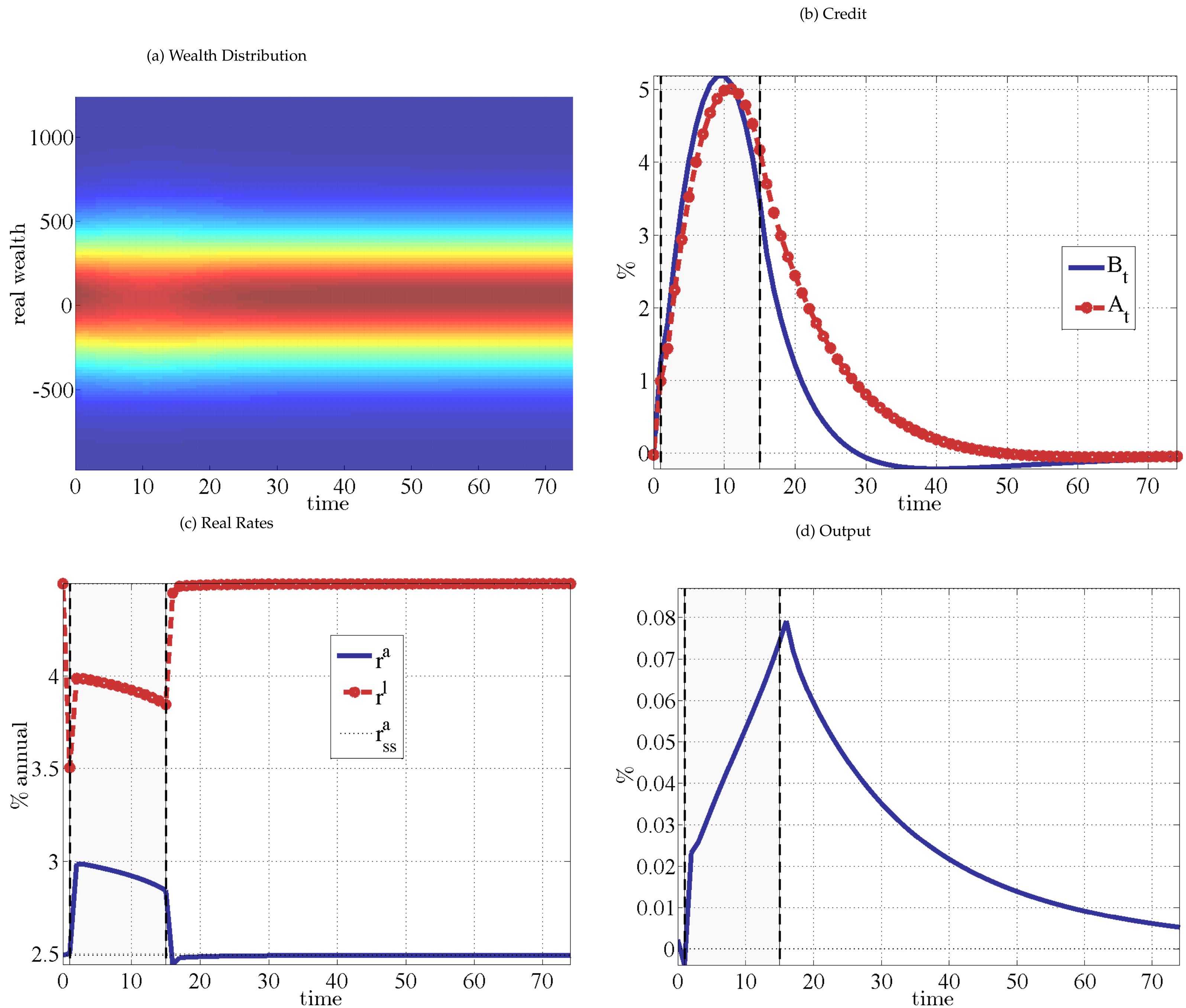


Figure 6: Macroeconomic Aggregates after a Credit Crunch (Credit Target).

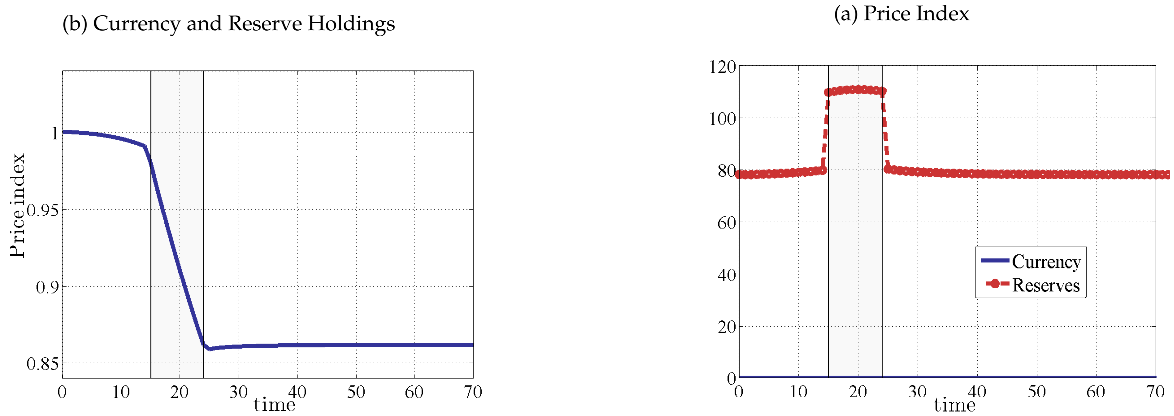


Figure 7: Components of (Outside) Money Supply and Inflation.

Liquidity Effect and Rate Sensitivity.

5.4 DZLB and Satiation Limits

Open-Market Operation that Lead to Satiation.

We now study the effects of a credit crunch in conjunction with an aggressive open-market operation. The policy is aggressive enough to lead the economy to a zero-lower bound on deposit rates. The macroeconomic patterns are described in Figure 8. By time 15, we observe a a credit crunch. Prior to the crunch, the patter follows a similar path to the one that occurs in absence of a policy reaction: output expands as the volume of credit drops, something that shows in a partial decline in real rates. When the shock is realized, monetary policy reacts with an aggressive program of open-market operations. This expansion is observed in Figure xxx. The result is a reduction in credit spreads during the crunch. The presence of the zero-lower bound is reflected in the increase in cash holdings by households. The spread is still active because the policy is conducted with a reduction in the interest in reserves. An even stronger expansion leads only to an increase in currency.

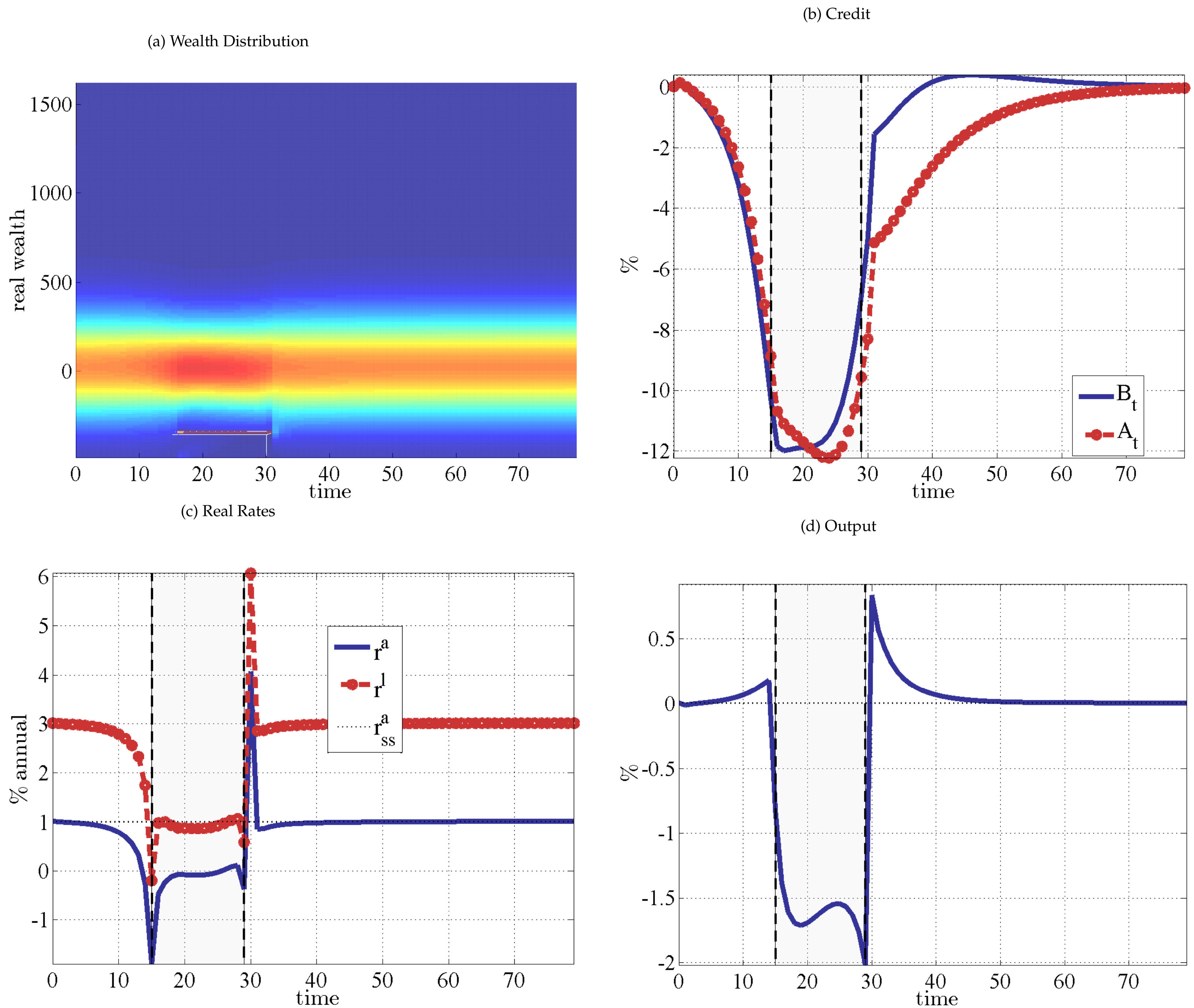


Figure 8: Macroeconomic Aggregates after a Credit Crunch (Wicksellian Doctrine).

Deposit Zero-Lower Bound.

Negative Rate on Reserves. In this model, negative rate on reserves, are detrimental. A negative rate on reserves will be the prescription in a model with nominal rigidities as means to containing deflation. In this model, it is a policy mistake because it leads to an aggravation of credit spreads. Cite in proposition and Brunnermeier Coby. To see this, Figure 9, performs the same experiment as before, but in conjunction with the large scale operation, the policy is conducted together with reduction in the rate on reserves. As we can observe, the policy leads to an aggravation of credit spreads, as anticipated by Proposition xxx.

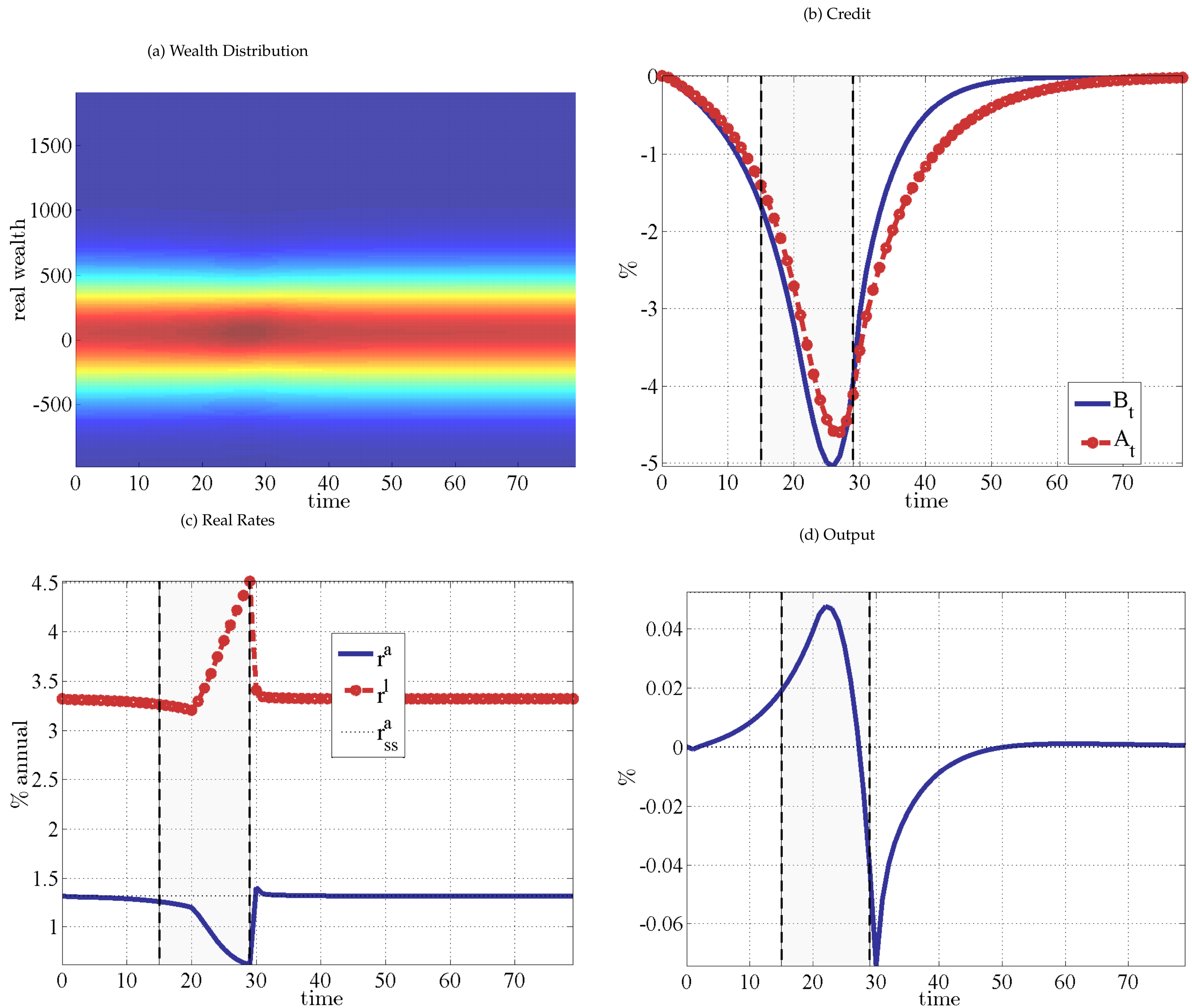


Figure 9: Macroeconomic Aggregates after a Credit Crunch (Wicksellian Doctrine).

Discussion: Fiscal Transfers at the DZLB? So far we described how at the zero-lower bound on deposits, open-market operations only increase the volume of currency in the economy. By contrast, a further reduction in real rates, deepens the effects. MP is left only with fiscal transfers as a tool. This tool can be seen as a helicopter drop, a direct injection of liquidity to households. As we noted, if the policy is anticipated, the policy can have recessionary effects.

5.5 Credit Channel: Normative Analysis

Effects of Credit Crunch. We now study a tightening of the borrowing limit. Under an inflation target doctrine, there are no real effects from policy. This also will help us understand the results under other doctrines. We introduce a temporal in $\tilde{s}_t$ that occurs at $t = 15$, but is anticipated at time 0 (credit crunch). The wealth distribution is initiated at steady state. We present the effects of anticipated shocks because as the dynamics after the shock date are similar to the dynamics that follow if the shock is unanticipated. Thus, anticipated shocks allow us to illustrate the main forces in anticipation of the shock and after the arrival of an unanticipated shock.

Figure 10 shows the dynamics of the real-side of the economy during the experiment. The effects can be divided into ex-ante and ex-post effects. When the credit crunch is expected, borrowers want to avoid a wealth position that falls above the borrowing limit. If they do, borrowers will have to adopt the safe, but unproductive technology. In preparation to that event, borrowers increase their desire to save away from the constraint. Naturally, if borrowers increase their savings, savers must reduce their savings along the transition. This leads to the compression in the distribution of wealth that appears in Panel (a). Panel (b) shows how both real deposits and loans fall along the transition. In equilibrium, real interest rates must fall, to discourage savers from savings. The threat of falling in the borrowing-constrained region induces borrowers to save, despite the low interest rate regime. Rates fall gradually reaching a trough when the shock arrives. Panel (c) shows the path of real rates. In the ex-ante phase, output actually expands (Panel (d)). The reason is the desire to avoid the borrowing constrained region during the crunch, leads to a lower mass of households at the constrained region, prior to the crunch. This allows those households to produce more efficiently soon after they abandon the region.

Upon the credit crunch, a mass of households is found in the borrowing constrained region, $s_t \in [\bar{s}, \tilde{s}_t]$. This can be seen through the mass concentration at the borrowing constrained region. This forces those households to switch to the safe technology. The consequence an immediate output collapse. Output continuously falls during crunch, as more and more households are dragged into the borrowing constrained region. The expectation of a recovery produces an increasing path of real interest rates—consumption smoothing—but as the crunch vanishes, interest rates jump back to accommodate the increased demand for credit. The evolution of the credit volume is interesting: at an initial phase, credit continues to decrease during the crunch. However, as the recovery is expect, credit begins to expand as borrowers wish to to smooth consumption—borrowers are more sensitive to interest rates than savers.

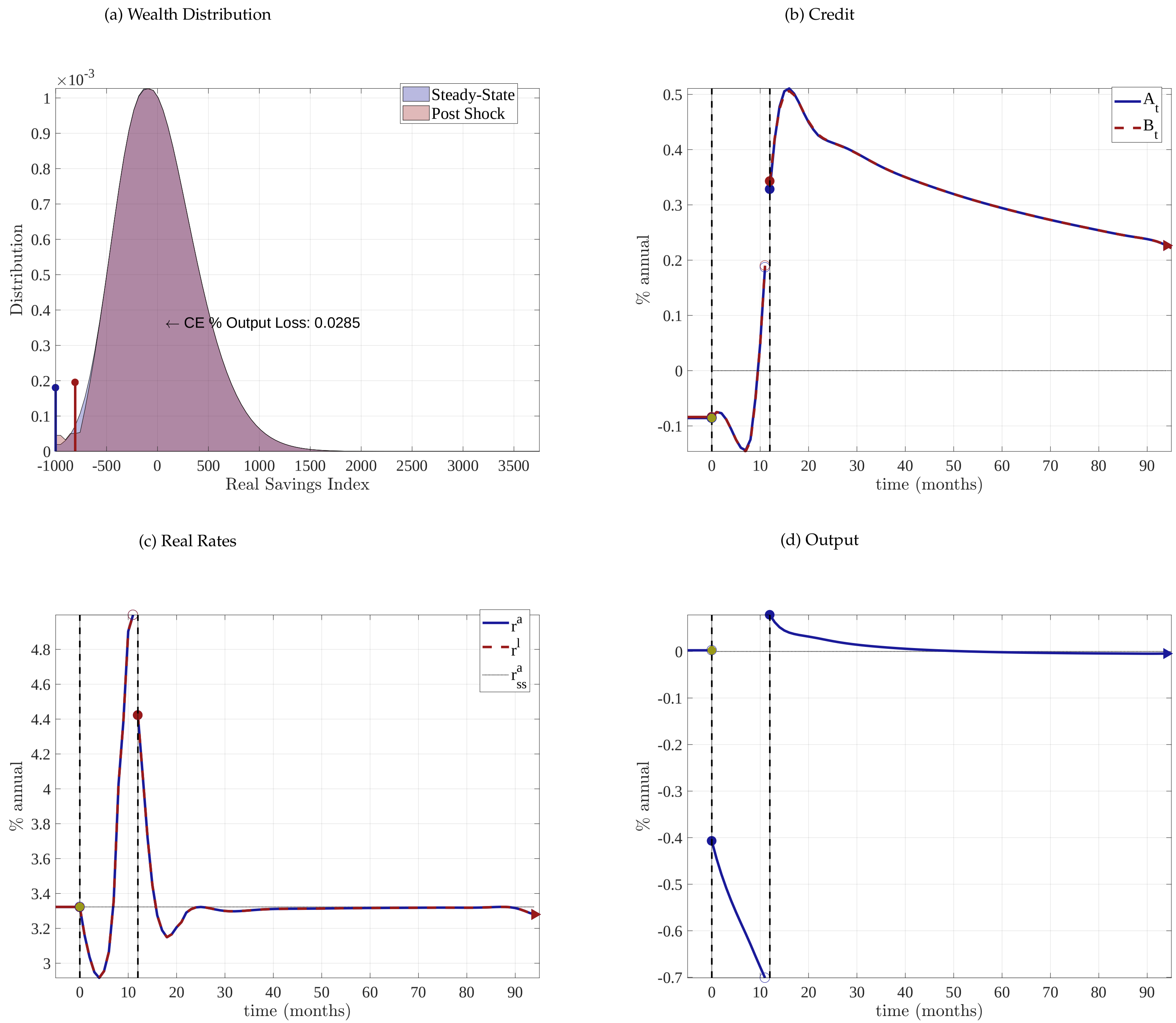


Figure 10: Macroeconomic Aggregates after a Credit Crunch (Nominal Rate Target).

Note: The figure is built using....

6 Aggregate Demand Externality

In this section, we modify our model to allow for a labor-demand externality. This externality will amplify the effects of the credit-crunch and will make a stronger point to lower spreads. We show that in this case, monetary is more powerful than in normal instances.

For that purpose, we now endow households with a continuum mass $\bar{n}$ of labor endowments. Like in Hansen (1985) and Rogerson (1988), labor endowments are indivisible —so a unit of labor is active or inactive. This supply is perfectly inelastic. Also, the labor endowment of one household cannot be employed in the firm owned by that household. The only role for labor is to introduce an aggregate demand externality that is one of two motivations for an active monetary policy.

In this model, the only reason to activate the safe technology is to avoid hitting a borrowing constraint. Since if every entrepreneur chooses the safe technology is equivalent to choosing the total number of workers in the economy.

Each technology requires a specific amount of workers $n(u)$. Naturally, $n(H) > n(L)$ and we also normalize $n(H) = \bar{n}$, so if all entrepreneurs operate with the high intensity technology, all workers are employed.

The Labor Market. The labor market suffers an imperfection because there is a labor hold-up problem as in Caballero and Hammour (1998). Once an entrepreneur hires a worker output becomes specific to the worker. In particular, the workers at a firm can threat the household to divert the fraction $(1-\eta_l)$ output of the firm. Thus, after being hired, workers are in a position to bargain over the total output produced per unit of time.[33] As a result, ex-post output must split into η_l destines to the entrepreneur and $1-\eta_l$ to the worker. More importantly, η_l captures the extent of a demand externality. If $\eta_l = 1$, the firm's choice of utilization has only an incidence on its own income, but $\eta_l < 1$, the firm's choice has an incidence on other household's income. Since real wages given u are fixed, workers and firms cannot contract on a technology. Instead, the firm unilaterally chooses a technology and then splits output accordingly.

Since each household has a continuum of workers, labor income risk is perfectly diversified. Considering this diversification workers of each household receive a common labor income flow of:

$$w_t^l = (1-\eta_l)\int_0^\infty y(u(s,t))\, f_t(s)\, ds.$$

[33]This construction can be approximated by a limit. Suppose that technologies are fixed over specific time intervals $\Delta t, 2\Delta t$, ... For every interval, assume that once the technology is chosen and workers are hired, contracts are negotiated on the spot and according to a bargaining problem. In particular, workers may threaten the entrepreneur not to work in which case they receive no output. Presumably, this hold-up problem leads to an output split according to some Nash-bargaining problem —also a la Rubinstein. In that case, output is divided in η and $(1-\eta)$ shares to entrepreneurs and workers correspondingly.

Because the technology choice affects the amount of workers hired, the model can feature unemployment. where $f_t(s)$ is the distribution of wealth and $u(s,t)$ technology choice of the employing household with wealth s at time period t. The only change in the household's problem is that now its real income flow is the sum of firm profits, labor income, and net transfers. Thus, per unit of time they earn:

$$h(u_t, t) = \eta_l y(u_t) + w_t^l + T_t$$

where $\eta_l y_t(u)$ is the household's entrepreneurial wealth, w_t^l the wage described earlier and T_t.

Labor market-clearing must be consistent with a level of unemployment:

$$\Upsilon(t) = \int_0^\infty \left[1 - (\mathbb{I}[u(s,t) = H] - 1)\frac{n(L)}{\bar{n}}\right] f_t(s)\, ds.$$

6.1 Credit Channel: Normative Analysis and the AD Externality

n this section we present the effects of a credit crunch once we activate the demand externality. Figure 11 presents the macroeconomic effects of a credit crunch under three scenarios: the first scenario is the response according to Figure xxx where there's no spread policy. The second scenario activates the externality. The final scenario is the case when the aggregate demand externality is present and the CB activates a credit spread an maintains it open. We can observe two things: First, that the presence of the aggregate demand externality amplifies the extent of the crisis. Second, that by producing an active spread ex-ante, the CB is able to contain the impact of the crisis. The reason is that the CB suppresses credit ex-ante. This means that the economy will feature less borrowing, but when the crunch is realized, less agents hit their borrowing constraints. The policy is so strong, that the impact is mitigated that the path is even smoother than without the externality. This observations lead us to conclude that in the model with a demand externality, MP should trade-off ex-ante inefficiencies against the depth of an ex-post crisis.

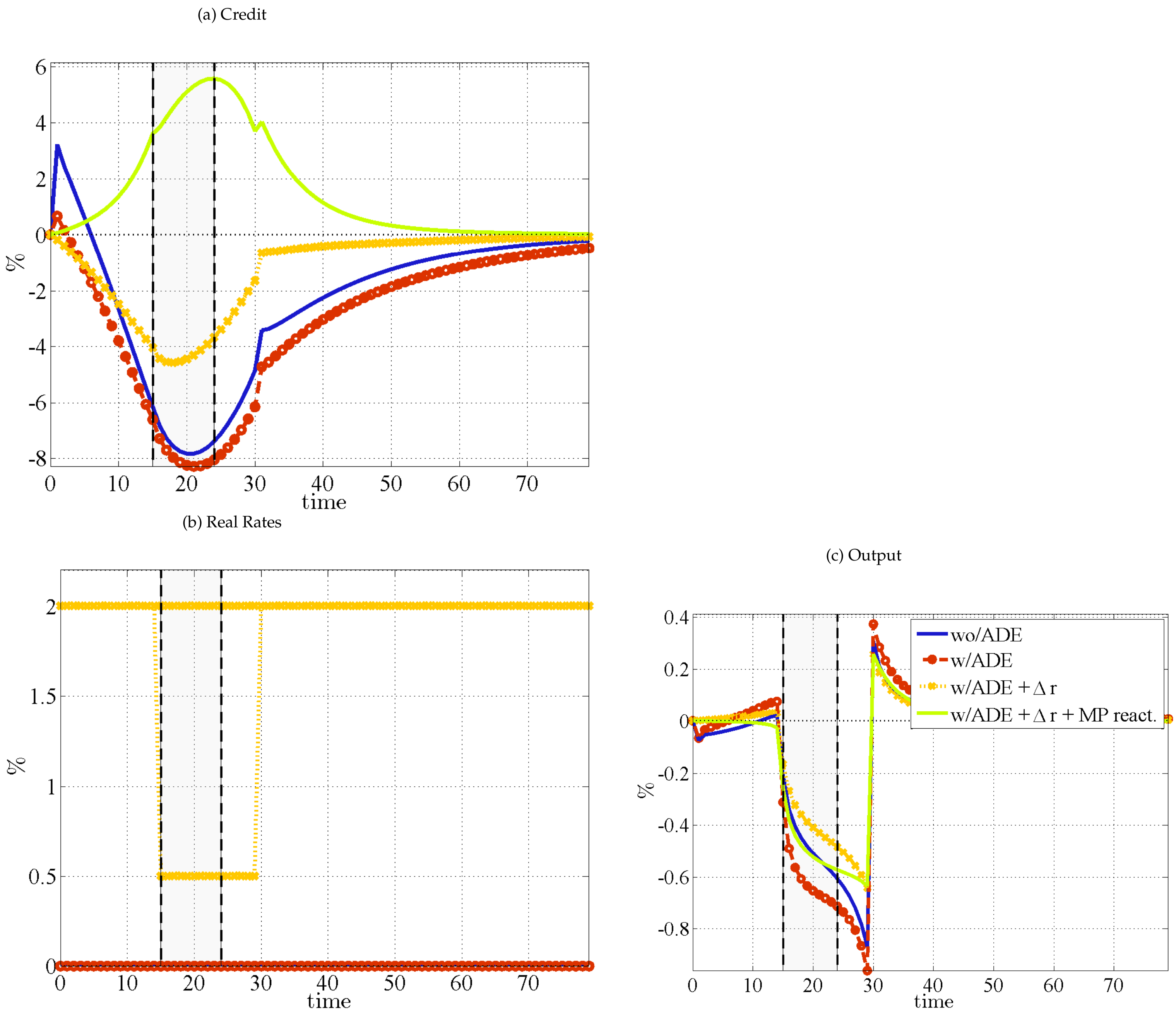


Figure 11: Macroeconomic Aggregates after a Credit Crunch - Comparison.

7 Conclusion

We began this paper with a quote from Robert Lucas. We can conclude by returning to that quote. Lucas's quote is inspiring because it summarizes his contributions to monetary economics, but also shows dissatisfaction with the lack of credit in monetary models. For him, his agenda was successful in rationalizing the long-run connection between money growth and inflation, a correlation that was disputed in the past. Introspectively, Lucas shows genuine scientific dissatisfaction with the workhorse models he help build, emphasizing the lack of a connection of monetary models with credit markets.

Our paper actually builds on one of Lucas earlier models, Lucas (1980). Ours is one of the several recent attempts to integrate credit into monetary theory. Here, outside money (reserves) are an input for inside money creation (deposits and loans). We tried this attempt trying to stay close to modern implementations of monetary policy. We also tried to articulate how different policy tools are tide to different transmission mechanisms stressed by the literature. We drew lessons for policy that we hope can be qualified empirically in the future, they present serious warnings on how monetary policy should be conducted.

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Appendix

A Accounting in the Model

A.1 Balance Sheets

Household Balance Sheet. The household's balance sheet in in nominal terms is:

Assets	Liabilities
m_t^h	l_t^h
a_t^h	$P_t s_t$

.

Bank Balance Sheet. The balance sheet of an individual bank is:

Assets	Liabilities
m_t^b	a_t^b
l_t^b	

.

CB Balance Sheet. The balance sheet of the central bank is:

Assets	Liabilities
L_t^f	M_t
	E_t

Accounting of OMO. To make this interpretation more clear, consider F_t is an outstanding amount of nominal bonds issued by the FA. Let $F_t^{cb} < F_t$ be the stock of government bonds held at the Central Bank. In that case, the balance sheet of the CG is

Assets	Liabilities
F_t^{cb}	$M_t + F_t$
	E_t

=

Assets	Liabilities
$F_t^{cb} - F_t$	M_t
	E_t

.

Thus, $L_t^f = F_t^{cb} - F_t < 0$ is the stock of government bonds held by banks and E_t is the stock of government liabilities net of CB purchases. A conventional open-market operation is simply an increase in F_t^{cb} funded with an increase in M_t. From the government's income flow, we can see that this operation would yield profits to the CB if there's a spread $i_t^l > i_t^m$.

A.2 Flow of Funds Identities

Lemma 2 *If the deposit, loans and money markets clear, then:*

$$P_t \int_0^{\infty} sf(s,t)ds = P_t \int_{-\infty}^{0} sf(s,t)ds - E_t. \tag{17}$$

Proof. The deposits and loans markets clearing condition requires:

$$A_t^b = \int_0^{\infty} a_t^h(s) f(s,t) ds \tag{18}$$

$$L_t^b + L_t^f = \int_{-\infty}^{0} l_t^h(s) f(s,t) ds, \tag{19}$$

and clearing in the money market requires:

$$M_t^b + M_t^0 = M_t \tag{20}$$

We also have that the budget constraint (balance sheet) of banks satisfies the following identity:

$$A_t^b = L_t^b + M_t^b. \tag{21}$$

Real household assets are held as nominal deposits or currency, hence:

$$P_t \int_0^{\infty} s f(s,t) ds = \int_0^{\infty} a_t^h(s) f(s,t) ds + M0_t. \tag{22}$$

and, similarly for liabilities:

$$P_t \int_{-\infty}^{0} s f(s,t) ds = \int_{-\infty}^{0} l_t^h(s) f(s,t) ds. \tag{23}$$

Once we combine (18), (19) and (21), we obtain a single condition:

$$\int_0^{\infty} a_t^h(s) f(s,t) ds = \int_{-\infty}^{0} l_t^h(s) f(s,t) ds - L_t^f + M_t^b. \tag{24}$$

This condition can be expressed in terms of real household wealth, with the use of (22) and (23):

$$P_t \int_0^{\infty} s f(s,t) ds = P_t \int_{-\infty}^{0} s f(s,t) ds - L_t^f + M_t^b + M0_t.$$

If we use the money market clearing-condition, (4), and employ the definition of net-asset position of the CB, we obtain (17). **QED.**

B Formulas for Interbank Market Trades

The parameter λ captures a the matching efficiency of the interbank market.[34] The corresponding trading probabilities for surpluses and deficit positions along a trading session are:

$$\psi^{+}(\theta) \equiv \begin{cases} 1-e^{-\lambda} & \text{if } \theta \geq 1 \\ \theta\left(1-e^{-\lambda}\right) & \text{if } \theta < 1 \end{cases}, \qquad \psi^{-}(\theta) \equiv \begin{cases} \frac{1-e^{-\lambda}}{\theta} & \text{if } \theta > 1 \\ 1-e^{-\lambda} & \text{if } \theta \leq 1 \end{cases}.$$

The resulting average interbank market rate is determined by the average of Nash bargains over the positions and is given by:

$$\bar{i}^{f}\left(\theta, i^{m}, \iota\right) \equiv \begin{cases} i^{m}+\iota-\left(\left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta}-1\right)\left(\frac{\theta}{\theta-1}\right)\left(\frac{\iota}{e^{\lambda}-1}\right) & \text{if } \theta > 1 \\ i^{m}+\iota(1-\eta) & \text{if } \theta = 1 \\ i^{m}+\iota-\left(1-\left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta}\right)\left(1+\frac{\theta/\bar{\theta}(\theta)}{1-\theta}\right)\left(\frac{\iota}{e^{\lambda}-1}\right) & \text{if } \theta < 1 \end{cases}$$

where η is a parameter associated with the bargaining power of banks with reserve deficits. It can be verified that

$$\psi_t^{-} B_t^{-} = \psi_t^{+} B_t^{+}, \tag{25}$$

which is a market clearing condition for the interbank market. Thus, the path for $\left\{\psi_t^{+}, \psi_t^{-}, \bar{\imath}^{f}\right\}$ is given by $\psi_t^{+} \equiv \psi^{+}(\theta_t)$, $\psi_t^{-} \equiv \psi^{-}(\theta_t)$ and $\bar{\imath}_t^{f} \equiv \bar{\imath}^{f}(\theta_t)$. In the paper, we set $\eta = 1/2$.

B.1 The functions $\{\chi^{+}, \chi^{-}\}$

Here we present an example of the formula 9. The left panel presents a mapping from θ to $\bar{\theta}$. The right panel plots $\{\chi_t^{+}, \chi_t^{-}\}$ for a given $\{\eta, \lambda, \iota\}$.[35] The formulas for $\{\chi^{+}, \chi^{-}\}$ show how the average costs of intermediation depend on the policy corridor ι on the policy spread ι and the amount of outside money M_t. The reason why the CB can affect outcomes is because, in turn, these intermediation costs affect bank decisions.

Figure 13 has two panels. The left panel plots the numerator and denominator in the definition of $\theta(\Lambda_t)$. As Λ_t increases, on aggregate, banks have less reserve deficits and a greater reserve surplus. There are bounds at the left and right of the figure, at the points where all banks are in deficit and at the point where all banks are satiated with reserves. The right panel shows the map from Λ to $log(\theta_t)$. Because θ_t is only a function of the liquidity ratio, (9) we obtain $\chi_t^{+} = \chi^{+}(\theta(\Lambda_t), \iota_t)$ and $\chi_t^{-} = \chi^{-}(\theta(\Lambda_t), \iota_t)$.

[34] This can be shown very easily using a differential form.

[35] As the interbank market is tighter average rates for short and long positions increase and approach the width of the corridor window. Instead, as the tightness drops, both rates get closer to zero. We use these formulas later to map the stance of monetary policy to a market tightness, and through the interbank market spread, we obtain formulas for real spreads.

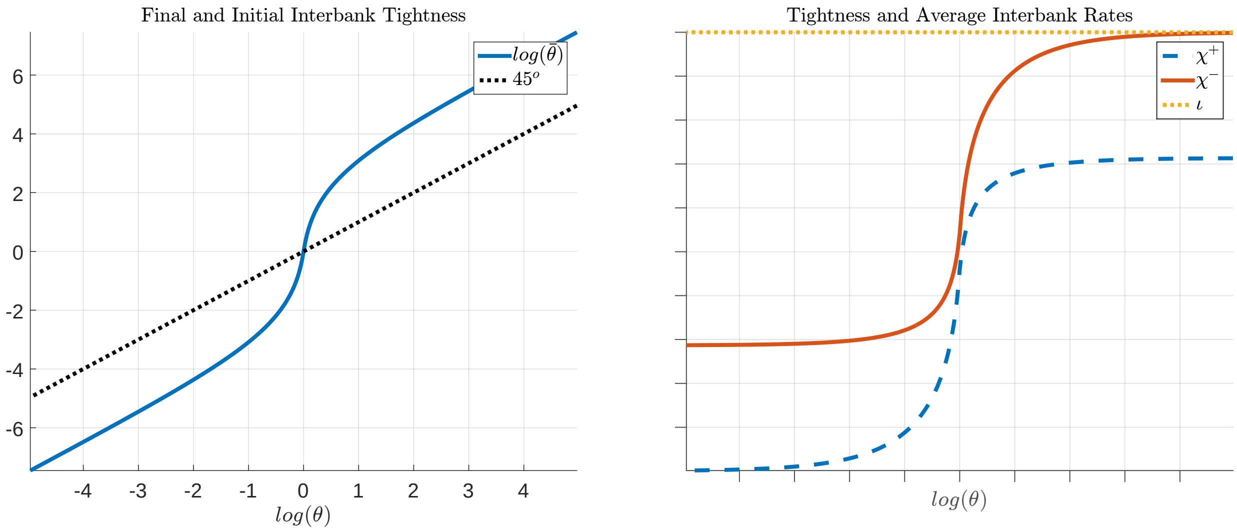

Figure 12: Interbank-Market Conditions and Interbank-Market tightness.

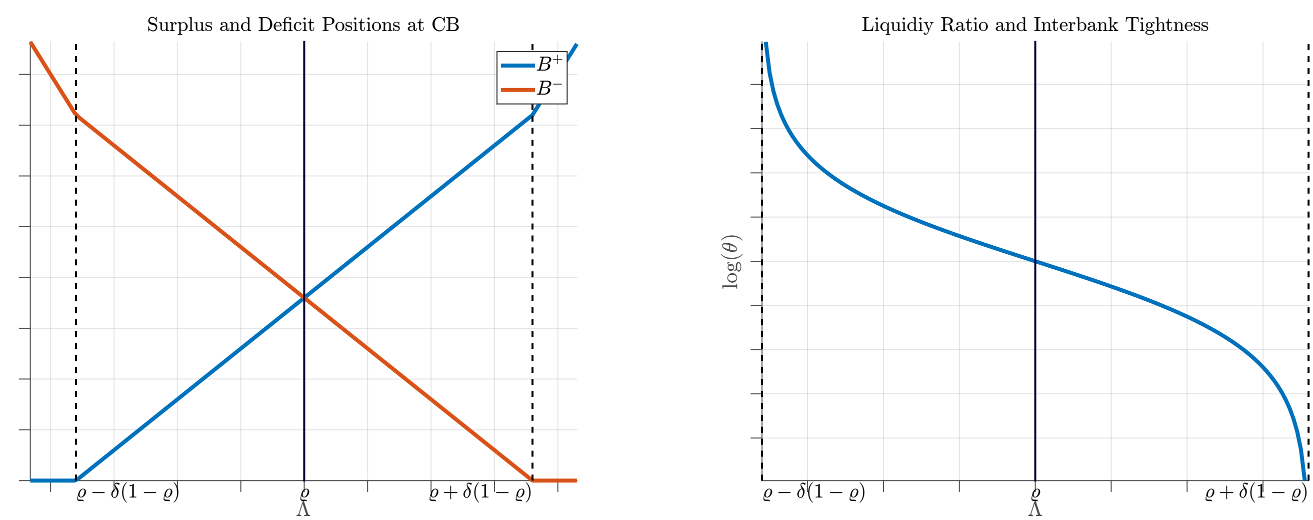

Figure 13: Interbank-Market Conditions and Liquidity Ratio.

C Solution Algorithm

Propositions 1, 3 and 2 are the objects we need to solve the model. They allow us to solve the model entirely by solving for the equilibrium path of a single price. For example, we can solve the model by solving the path for a real deposit rate r_t^a. The spread Δr_t follows immediately from Proposition 1 if we know the path for ι_t and Λ_t set by the CB. The real spread gives us r_t^l. To solve the household's problem, we need the path for $\left\{r_t^a, r_t^l, T_t\right\}$. The path for T_t is must be consistent with (14) and this yields a path for real government liabilities, $\mathcal{E}_t$. Then, $\mathcal{E}_t$ together with the evolution of $f(s,t)$ obtained from the household's problem, yield two sides of one equation enters 15. The rate equilibrium rate r_t^a must be the one that solves 15 implicitly.

Before we study the effects of monetary policy under different policy doctrines, we want to explain the implementability constraints faced by the CB. Then, we briefly discuss the behavior of the model at the deposit zero-lower bound on deposits rates and when the CB satiates the economy with reserves.

Solution at ZLB. [TBA]

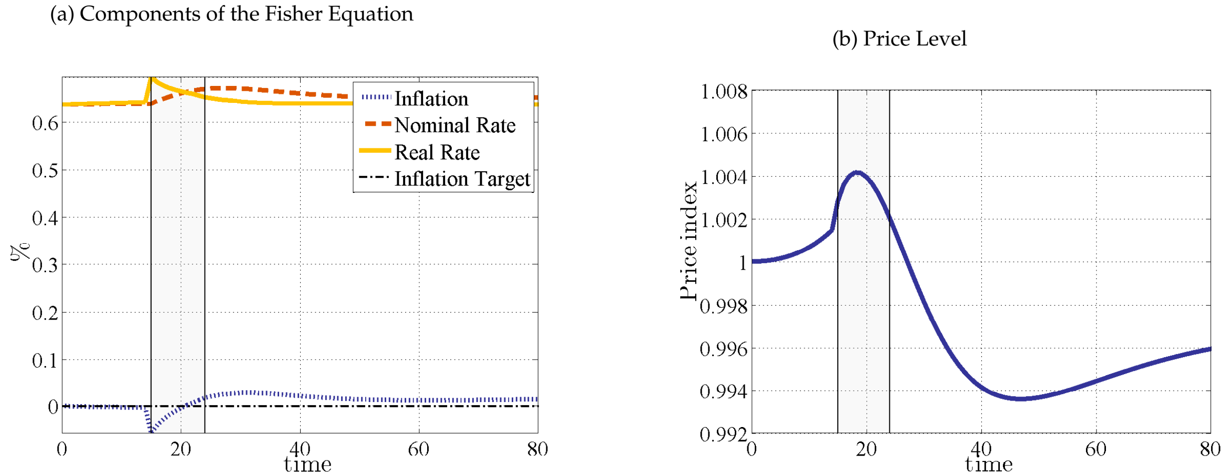

Figure 14: Components of the Fisher Equation and Price Level (Credit Crunch under a Nominal Rate Target).

D Additional Plots

Price Level and Components of $M0_t$ after a Fiscal Transfer.

Price Level and Components of $M0_t$ after a Satiation Policy.

Price Level and Components of $M0_t$ at a DZLB.

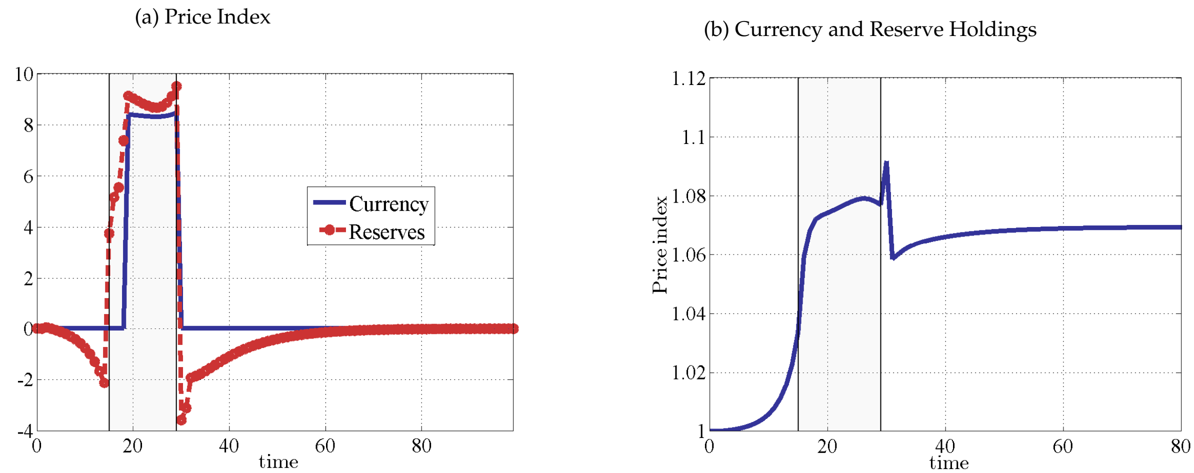

Figure 15: Transitions at the Satiation Limit.

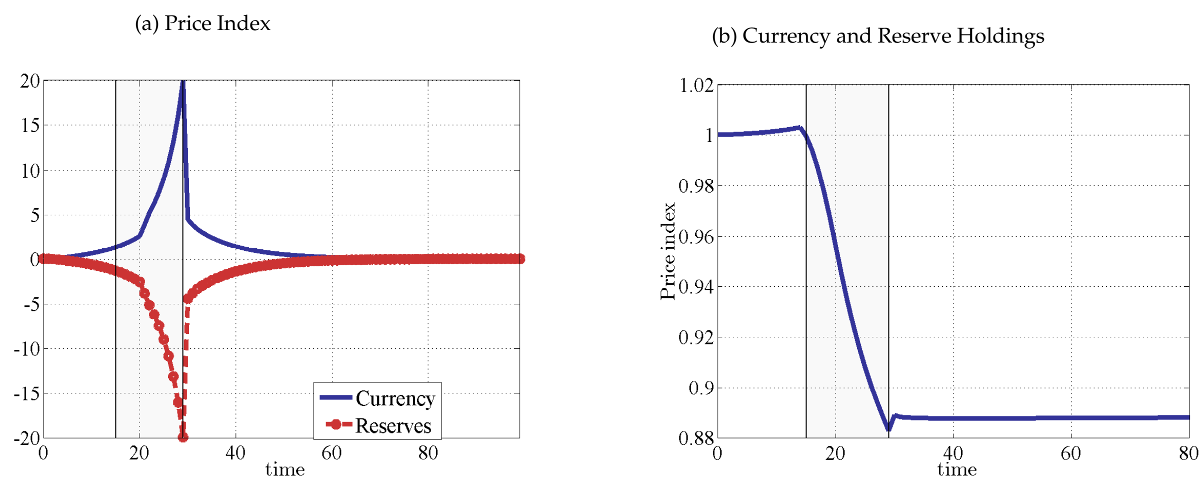

Figure 16: Transitions with lower rates.

E Additional Plots

F Proofs

F.1 Proof of Lemma 1

The proposition establishes thatχ_t is a function of the corridor spread $\imath_t$ and the liquidity ratio Λ_t only. Since the balance of reserve for an individual bank is $b_t = m_t^b + \omega a_t - \varrho(a_t + \omega a_t) = m_t^b - \varrho a_t + (1-\varrho)\omega a_t$ where $\omega \in \{-\delta, \delta\}$, and these occur with equal probability, then the aggregate balances of reserves for surplus and deficit banks are:

$$
\begin{aligned}
B_t^+ &= \sum_{z\in\{-1,1\}} \frac{1}{2} \max\{M_t^b - \varrho A_t + (1-\varrho)\delta z A_t, 0\} \text{ and} \\
B_t^- &= -\sum_{z\in\{-1,1\}} \frac{1}{2} \min\{M_t^b - \varrho A_t + (1-\varrho)\delta z A_t, 0\}.
\end{aligned} \tag{26}
$$

Consider the interbank market tightness, $\theta_t = B_t^-/B_t^+$. Dividing the numerator and denominators by A_t allows us to write:

$$
\theta_t = \frac{B_t^-/A_t}{B_t^+/A_t} = \frac{-\sum_{z\in\{-1,1\}} \min\{\Lambda_t - \varrho + (1-\varrho)\delta z, 0\}}{\sum_{z\in\{-1,1\}} \max\{\Lambda_t - \varrho + (1-\varrho)\delta z, 0\}} \equiv \theta(\Lambda_t).
$$

where we used (26) and the definition of Λ_t to obtain the last equality, before the definition of the implicit function $\theta(\Lambda_t)$. Define$\bar{\bar{\theta}}(\Lambda_t) \equiv \bar{\theta}(\theta(\Lambda_t))$ as the implicit end-of-day tightness—as a function of the liquidity ratio. Hence we can replace, $\theta(\Lambda_t)$ and $\bar{\bar{\theta}}(\Lambda_t)$, into the definitions of $\chi_t^+(\theta_t)$, $\chi_t^-(\theta_t)$, and χ_t. We obtain:

$$
\begin{aligned}
\chi^+(\Lambda_t, \imath_t) &\equiv \imath_t \left(\frac{\bar{\bar{\theta}}(\Lambda_t)}{\theta(\Lambda_t)}\right)^\eta \left(\frac{\theta(\Lambda_t)^\eta \bar{\bar{\theta}}(\Lambda_t)^{1-\eta} - \theta(\Lambda_t)}{\bar{\theta}(\Lambda_t) - 1}\right) \\
\chi^-(\Lambda_t, \imath_t) &\equiv \imath_t \left(\frac{\bar{\bar{\theta}}(\Lambda_t)}{\theta(\Lambda_t)}\right)^\eta \left(\frac{\theta(\Lambda_t)^\eta \bar{\bar{\theta}}(\Lambda_t)^{1-\eta} - 1}{\bar{\bar{\theta}}(\Lambda_t) - 1}\right).
\end{aligned}
$$

Thus, Lemma 1 follows. **QED.**

F.2 Proof of Proposition 1

Take $\{\Lambda_t, i_t^m, \imath_t\}$ as given. By Lemma 1, $\{\chi^+, \chi^-\}$ are also given. Consider an individual bank's problem:

$$
\begin{aligned}
&\max_{a,l\geq 0} i_t^l l + i_t^m(a-l) - i_t^a a + \mathbb{E}\left[\chi_t[b(a, a-l)]\right] \\
&\max_{a,l\geq 0} i_t^l l + i_t^m(a-l) - i_t^a a + \frac{1}{2}\Big[\chi^+(\Lambda_t, \imath_t)(a - l - \varrho a + (1-\varrho)\delta a) + \chi^-(\Lambda_t, \imath_t)(a - l - \varrho a - (1-\varrho)\delta a)\Big] \\
&\max_{a,l\geq 0} \Big[i_t^l - i_t^m - \frac{1}{2}\left(\chi^+(\Lambda_t, \imath_t) - \chi^-(\Lambda_t, \imath_t)\right)\Big] l - \Big[i_t^a - i_t^m - \frac{1}{2}(1-\varrho)\left((1+\delta)\chi^+(\Lambda_t, \imath_t) - (1-\delta)\chi^-(\Lambda_t, \imath_t)\right)\Big] a
\end{aligned}
$$

The problem is linear. Thus, a necessary condition for a positive and finite supply of loans and deposits are conditions (11) and (12). Since in equilibrium the demand of deposits and loans is finite, the result follows. Substitute (11) and (12), and the bank earns zero expected profits from any choice of $\{a, l\}$. Now, observe that by, definition of real rates, $r_t^l = i_t^l - \dot{P}_t/P_t$ and $r_t^a = i_t^a - \dot{P}_t/P_t$. Hence, $\Delta r_t = i_t^l - i_t^a$. Thus, the expression for the real spread follows immediately from subtracting the right-hand side of (12) from the right-hand side of (11). This concludes the proof of Proposition 1. **QED.**

F.3 Proof of Proposition 2

The profits of the CB are given by:

$$\pi_t^f = i_t^l L_t^f - i_t^m \left(M_t - M0_t\right) + \iota_t \left(1 - \psi_t^-\right) B_t^-.$$

Note that the earnings from discount-window loans equal the average payment in the interbank market, and thus:

$$\iota_t \left(1 - \psi_t^-\right) B_t^- = -\mathbb{E}\left[\chi_t \left(b \left(A_t, A_t - L_t\right)\right)\right]. \tag{27}$$

By Proposition xxx, banks earn zero profits in expectation. Thus,

$$-\mathbb{E}\left[\chi_t \left(b \left(A_t, A_t - L_t\right)\right)\right] = i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t. \tag{28}$$

Thus, substituting (27) and (28) into the expression for π_t^f above yields:

$$\begin{aligned}\pi_t^f &= i_t^l L_t^f - i_t^m \left(M_t - M0_t\right) + i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t^b. \\ &= i_t^l L_t^h - i_t^a A_t^h,\end{aligned}$$

where we used the clearing condition in the money market, $M_t^b + M_t^0 = M_t$, the deposit market, $A_t^b = A_t^h$, and the loans market, $L_t^b = L_t^h + L_t^f$. Now, observe that:

$$\pi_t^f = -i_t^l P_t \int_{-\infty}^{0} s f(s,t) ds - i_t^a \left(P_t \int_0^{\infty} s f(s,t) ds - M0_t.\right),$$

but we know from the household's problem that $i_t^a M0_t$=0. Hence, profits are given by:

$$\pi_t^f = -i_t^l P_t \int_{-\infty}^{0} s f(s,t) ds - i_t^a P_t \int_0^{\infty} s f(s,t) ds.$$

Divide (17) by the price level to obtain:

$$-\int_{-\infty}^{0} s f(s,t) ds = \int_0^{\infty} s f(s,t) ds + \mathcal{E}_t.$$

and thus:

$$\pi_t^f = \left(i_t^l - i_t^a\right) P_t \int_0^{\infty} s f(s,t) ds + i_t^l E_t = \Delta r_t P_t \int_0^{\infty} s f(s,t) ds + i_t^l E_t.$$

Dividing both sides by the price level leads to:

$$\frac{\pi_t^f}{P_t} = \Delta r_t \int_0^{\infty} s f(s,t) ds + i_t^l \mathcal{E}_t = \Delta r_t \int_0^{\infty} s f(s,t) ds + \left(r_t^a + \Delta r_t + \frac{\dot{P}_t}{P_t}\right) \mathcal{E}_t. \tag{29}$$

Then, note that:

$$d\mathcal{E}_t = \frac{dE_t}{P_t} - \frac{\dot{P}_t}{P_t}\mathcal{E}_t = \frac{\pi_t^f}{P_t} - T_t - \frac{\dot{P}_t}{P_t}\mathcal{E}_t.$$

But, a substitution of (29) yields:

$$d\mathcal{E}_t = \left((r_t^a + \Delta r_t)\mathcal{E}_t + \Delta r_t \int_0^\infty sf(s,t)ds - T_t \right) dt.$$

This proofs Proposition 2. **QED.**

F.4 Proof of Proposition 3

The accounting identities in section xxx, show that if all markets clear, the real market clears. Then,

Divide (17) by the price level to obtain:

$$-\int_{-\infty}^{0} sf(s,t)ds = \int_0^\infty sf(s,t)ds + \mathcal{E}_t, \text{ for } t \in [0,\infty).$$

The proposition establishes that if this condition holds, all asset markets clear. To proceed with the proof, argue that if the condition holds, but one of the markets doesn't clear, we reach contradiction.

To see that, observe that if condition (xxx) holds, then taking time derivatives we obtain:

$$0 = \frac{\partial}{\partial t}\left[\int_{-\infty}^{\infty} sf(s,t)ds\right] + \frac{\partial}{\partial t}\left[\mathcal{E}_t\right],$$

Then we have:

$$0 = \int_{-\infty}^{\infty} s\frac{\partial}{\partial t}\left[f(s,t)\right]ds + \frac{\partial}{\partial t}\left[\mathcal{E}_t\right],$$

but recall that if the KFE equation holds, then:

$$0 = \int_{-\infty}^{\infty} s\left(-\frac{\partial}{\partial s}\left[\mu(s,t) f(s,t)\right]\right) + \frac{1}{2}\left(\frac{\partial^2}{\partial s^2}\left[\sigma_s^2(s,t) f(s,t)\right]\right)ds + \frac{\partial}{\partial t}\left[\mathcal{E}_t\right].$$

Now, observe that, if we employ the integration by parts formula:

$$-\int_{-\infty}^{\infty} s\frac{\partial}{\partial s}\left[\mu(s,t) f(s,t)\right]ds = -s\mu(s,t) f(s,t)|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \mu(s,t) f(s,t)\, ds.$$

We know that

$$-s\mu(s,t) f(s,t)|_{-\infty}^{\infty} = 0$$

and that

$$\int_{-\infty}^{\infty} \mu(s,t) f(s,t)\, ds = \int_{-\infty}^{\infty} \left[r_t(s)\left(s - m^h(s,t)/P_t\right) - c(s,t) + h(u(s,t),t)\right] f(s,t)\, ds.$$

Then, note that:

$$\int_{-\infty}^{\infty} r_t(s)\, sf(s,t)\, ds = r_t^l \int_{-\infty}^{\infty} r_t(s)\, sf(s,t)\, ds - \int_0^\infty \Delta r_t(s)\, sf(s,t)\, ds.$$

And that, by definition:

$$\frac{M0_t}{P_t} = \int_{-\infty}^{\infty} \frac{m^h\left(s,t\right)}{P_t} f\left(s,t\right) ds.$$

and

$$\int_{-\infty}^{\infty} \left(-c\left(s,t\right) + h\left(u\left(s,t\right),t\right)\right) f\left(s,t\right) ds = Y_t - C_t.$$

The term:

$$\frac{1}{2}\int_{-\infty}^{\infty} \left(\frac{\partial^2}{\partial s^2}\left[\sigma_s^2\left(s,t\right) f\left(s,t\right)\right]\right) ds = \frac{1}{2}\frac{\partial}{\partial s}\left[\sigma_s^2\left(s,t\right) f\left(s,t\right)\right]\Big|_{-\infty}^{\infty} = 0.$$

Thus, we are left with:

$$r_t^l \int_{-\infty}^{\infty} r_t\left(s\right) s f\left(s,t\right) ds - \int_0^{\infty} \Delta r_t\left(s\right) s f\left(s,t\right) ds - \frac{M0_t}{P_t} + Y_t - C_t + \frac{\partial}{\partial t}\left[\mathcal{E}_t\right] = 0.$$

But then, given the law of motion for real equity [TBA]. **QED.**

F.5 Proof of Proposition 4

Along an equilibrium path for $\{r_t^a, \mathcal{E}_t, f_t, \Delta r_t, \tau_t^l, T_t\}$ the set of implementable nominal interbank rates and inflation rates is the set of $\{\dot{P}_t/P_t, \bar{i}_t^f\}$ where

$$\frac{\dot{P}_t}{P_t} = i_t^m + \frac{1}{2}\left[\chi^+(\Lambda_t, \imath_t) + \chi^-(\Lambda_t, \imath_t)\right] - \Delta r_t - r_t^a \tag{30}$$

$$\bar{i}_t^f = \chi^+(\Lambda_t, \imath_t)/\psi^+(\theta(\Lambda_t)) \tag{31}$$

for any $\{i_t^m, \imath_t, \mathcal{L}_t^f\}$ such that

$$\begin{aligned}
\Delta r_t &= \Delta i_t - \tau_t^l \\
\Delta i_t &= \varrho \frac{\chi^+(\Lambda_t, \imath_t) + \chi^-(\Lambda_t, \imath_t)}{2} + \delta(1-\varrho)\frac{\chi^-(\Lambda_t, \imath_t) - \chi^+(\Lambda_t, \imath_t)}{2} \\
\mathcal{L}_t^f &\leq \int_{-\infty}^{0} s f(s,t) ds, \quad (\imath_t, i_t^m) \in \mathbb{R}_+^2.
\end{aligned}$$

Equations (30) and (31) steams form definitions for nominal, real and interbank rate. The implementation constraint $\mathcal{L}_t^f \leq \int_{-\infty}^{0} s f(s,t) ds$ simply tells that there must be enough private liabilities to set $\mathcal{L}_t^f$. **QED.**

F.6 Proof of Proposition 2

The interbank market is satiated with reserves if $\Lambda_t \geq \bar{\Lambda} = \varrho + (1-\varrho)\delta$. Then the interbank market tightness is $\theta(\Lambda_t) = 0$ for any $\Lambda_t \geq \bar{\Lambda} = \varrho + (1-\varrho)\delta$. First, we must take the following limit

$$\lim_{\theta \to 0} \frac{\bar{\theta}(\theta)}{\theta} = \lim_{\theta \to 0} \frac{1}{\theta[1 + (\theta^{-1} - 1)\exp(\lambda)]} = \lim_{\theta \to 0} \frac{1}{\theta + (1-\theta)\exp(\lambda)} = \exp(-\lambda)$$

Then, given (η, λ), for any $\Lambda_t \geq \bar{\Lambda}$:

$$\chi^+(\Lambda_t, \imath_t) = \lim_{\theta \to 0} \imath_t \theta \left(\frac{\bar{\theta}(\theta)}{\theta} \right)^\eta \left(\frac{[\bar{\theta}(\theta)/\theta]^{1-\eta} - 1}{\bar{\theta}(\theta) - 1} \right) = 0$$

$$\chi^-(\Lambda_t, \imath_t) = \lim_{\theta \to 0} \imath_t \left(\frac{\bar{\theta}(\theta)}{\theta} \right)^\eta \left(\frac{\theta[\bar{\theta}(\theta)/\theta]^{1-\eta} - 1}{\bar{\theta}(\theta) - 1} \right) = \imath_t \exp(-\eta\lambda)$$

Although $\chi_t^- > 0$, there are not banks with reserves deficit, thus

$$\mathbb{E}\left\{ \chi_t[b(a, a-l)] | \theta_t \right\} = \chi^+(\Lambda_t, \imath_t)\,(a - l - \varrho a) = 0$$

Hence, the bank's problem becomes

$$\pi_t^b = \max_{a,l} (i_t^l - i_t^m) l_t - (i_t^a - i_t^m) a_t$$

and by FOCs we obtain that $i_t^m = i_t^a = i_t^l = \bar{i}_t^f$. **QED.**

F.7 Proof of Proposition ??