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Research Discussion Paper

Terms of Trade Shocks and Incomplete Information

Daniel Rees

RDP 2013-09

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Abstract

The terms of trade are subject to both permanent and transitory shocks. Particularly for commodity-producing small open economies, it is sometimes argued that the inability of agents to determine which of these shocks are permanent and which are transitory leads to more macroeconomic volatility than would be the case if agents had perfect information about the persistence of these shocks. I set up a small open economy model in which agents have imperfect information about the persistence of terms of trade shocks and estimate the parameters of the model using Australian data. The results point to the existence of large informational frictions. In fact, agents' beliefs about the future path of the terms of trade following transitory and permanent shocks are almost identical. However, the results also suggest that incomplete information causes agents to respond more cautiously to terms of trade shocks. Consequently, consumption, output and the trade balance are less volatile under incomplete information than they are under full information.

JEL Classification Numbers: C32, E32, F41, Q33
Keywords: terms of trade, imperfect information, small open economy, real business cycle

Table of Contents

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Daniel Rees

1. Introduction

> Commodity prices are typically much more volatile than those of other goods or services.
>
> ...
>
> [M]any of the price movements last just long enough to convince investors and governments that 'this time it is different'. And there is always a chance that some day it *will* be different. In the intervening period, long-range investments may have been set in train, new facilities built, and workers relocated ... If prices stay high (or low) for a sufficiently long time, these reallocations of capital and labour could well be warranted and yield valuable returns ... The trouble is that businesses, households, and policy-makers often get caught out ... The inherent difficulty associated with predicting how long a boom (or bust) might last, and how high (or low) prices might go, makes the process extremely risky. Critics worry that a commodity-based economy will constantly find itself in motion, never quite settling down.
>
> – John Murray, Deputy Governor of the Bank of Canada, 6 May 2010
> (Murray 2010)

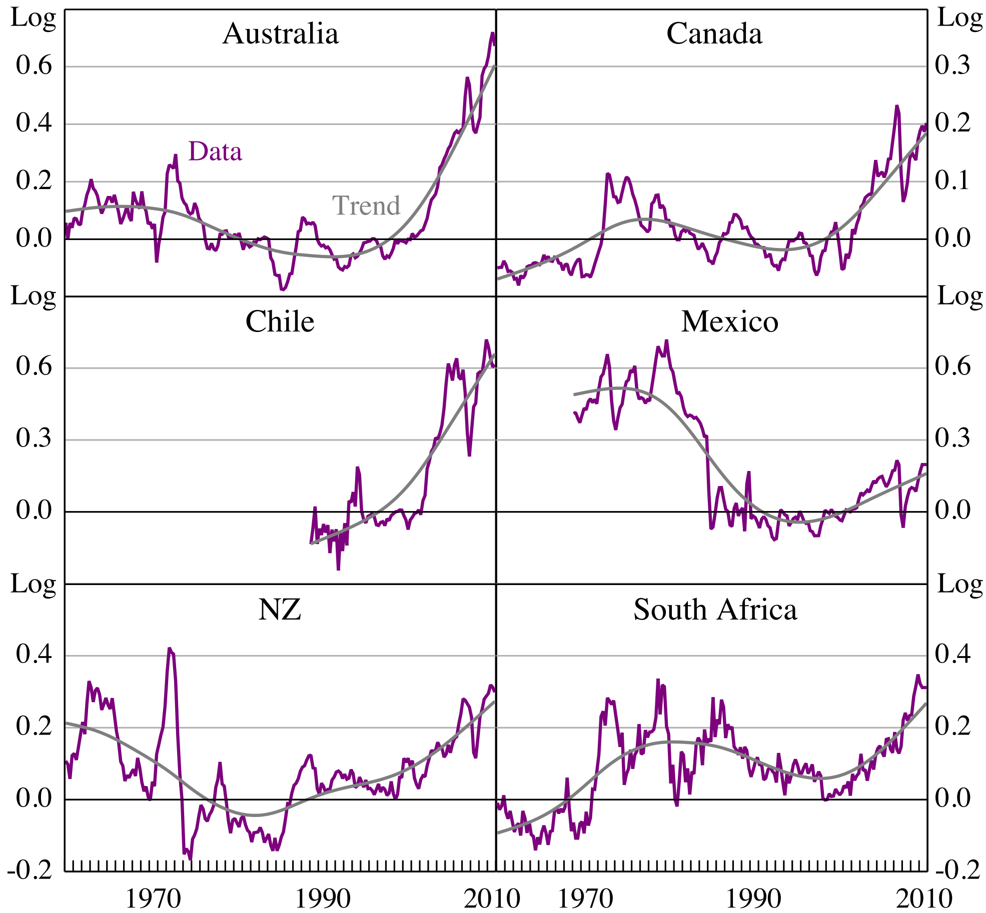
The terms of trade of many commodity-producing small open economies can be characterised by a succession of slow-moving long-run trends, augmented by high-frequency transitory fluctuations. As an illustration, Figure 1 shows the terms of trade – defined as the ratio of export prices to import prices – for six small open economies between 1961 and 2010. For each country, the data line represents the level of the terms of trade in logs, while the trend line shows an HP-filtered trend.[1] Although the exact patterns differ across countries, each has experienced periods in which the trend terms of trade persistently decreased as well as periods in which it persistently increased. Changes in the trend terms of trade are often large. For

1 Although the data are quarterly, the trend was calculated using a smoothing parameter of 64 000 rather than the usual 1 600. This reflects the fact that commodity price cycles – which drive the terms of trade in these economies – are typically longer than business cycles.

example, the trend terms of trade decreased by around 50 per cent in Mexico during the 1980s and increased by over 50 per cent in Australia during the 2000s. Deviations from trend are also substantial. During the early 1970s, New Zealand's terms of trade was at times 30 per cent above its trend level.

Figure 1: Terms of Trade

Selected economies, 2003:Q1 = 0



Note: The trend line is calculated using an HP filter with a smoothing parameter of 64 000

Sources: See Appendix A

The presence of both persistent and transitory movements in the terms of trade matters because the optimal response to a terms of trade shock depends upon the persistence of the shock. A positive terms of trade shock is similar to a positive productivity shock in that it allows an economy to sustainably increase consumption without a corresponding increase in factor inputs. A simple permanent income model would suggest that consumption-smoothing households will respond to a temporary increase in the terms of trade by saving some of the windfall and increasing consumption by the annuity value of the shock. In contrast, a permanent increase in the terms of trade will induce a larger immediate consumption response and a smaller increase in saving.[2]

But in order for households and firms to respond in this textbook manner, they must first be able to identify which shocks are permanent and which are transitory. There is some reason to believe that they can do so. Unlike many other drivers of macroeconomic fluctuations – such as productivity or consumption preference shocks – the terms of trade are observable. Moreover, for many countries, changes in the terms of trade reflect broad global economic developments. For example, the increases in the terms of trade during the 2000s for the economies shown in Figure 1 were largely due to rising commodity prices, driven by strong economic growth in countries such as China and India (Kearns and Lowe 2011; Plumb, Kent and Bishop 2013; Kilian and Hicks forthcoming). To the extent that agents recognise the underlying causes of changes in the terms of trade, it seems plausible to think that they are able to forecast the persistence of these changes accurately.

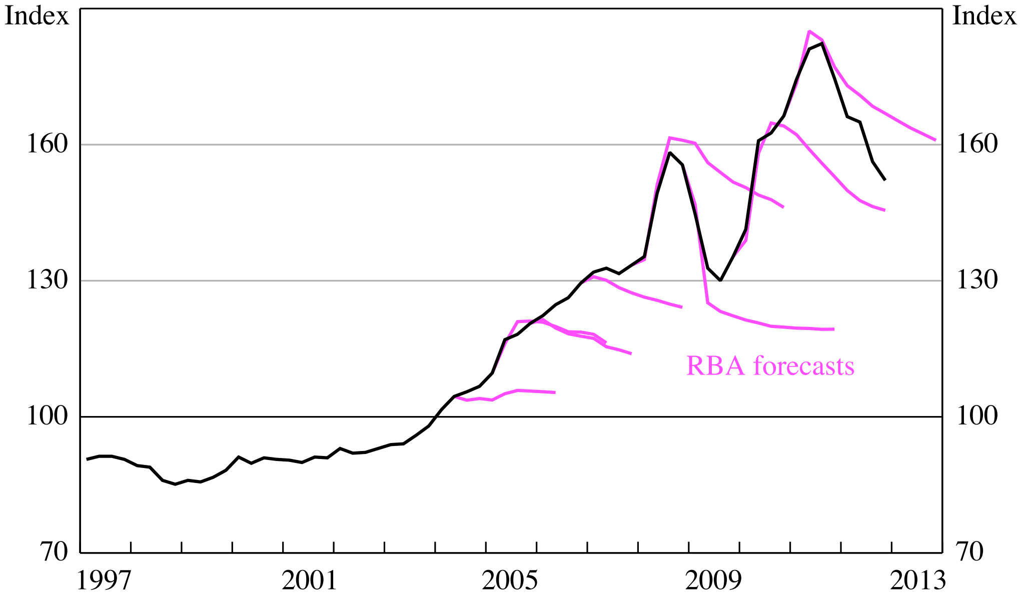
And yet there is also evidence which suggests that identifying the persistence of terms of trade shocks is difficult. Consider Figure 2. This shows the evolution of the terms of trade in Australia during the 2000s, as well as successive forecasts of the the terms of trade published by the Reserve Bank of Australia (RBA). It is striking how consistently the forecasts underestimated the persistence of increases in the terms of trade despite the fact that many of the underlying drivers of the terms of trade boom were at least partly observable.[3] Of course, a number of interpretations of Figure 2 are possible. It may be that the persistence of terms of trade shocks are predictable, but that forecasters made mistakes during the recent boom. For example, most forecasters appear to have underestimated the effect of strong growth in emerging market economies in Asia on demand for commodities. Also, forecasters may have overestimated the speed with which additional supply would come on stream. Alternatively, the prices of Australia's exports may have experienced a succession of positive, but temporary, shocks. Under this interpretation, forecasts like those in Figure 2 were optimal, but

2 The responses to temporary and permanent terms of trade shocks may differ from this simple permanent income example, depending, for example, on consumers' willingness to substitute intertemporally and between tradeable and non-tradeable goods. Nonetheless, the key point that the optimal responses to transitory and permanent shocks differ is generally true.

3 It is worth noting that the RBA forecasts were not unusual in underestimating the persistence of the increase in the terms of trade.

Australia's terms of trade merely received an unusual sequence of shocks. The results of this paper, however, suggest an alternative interpretation for the patterns of Figure 2, namely that the terms of trade does experience persistent shocks, but that it is difficult to identify these shocks in real time.

Figure 2: Forecasts of Australia's Terms of Trade
2003/04 average = 100



Source: Plumb *et al* (2013)

To reach this conclusion, I augment an otherwise standard small open economy model to include incomplete information about the persistence of terms of trade shocks. I then estimate the model using Bayesian methods on Australian data. The results suggest that agents face considerable difficulties in untangling the persistence of terms of trade shocks. In fact, agents' beliefs about the future path of the terms of trade are largely independent of the type of terms of trade shock that hits the economy. Consequently, it should come as no surprise that the response of the economy to terms of trade shocks differs substantially from that implied by models in which agents are perfectly informed about the nature of these shocks.

As well as documenting the existence of incomplete information about the persistence of terms of trade shocks, I also examine its implications for macroeconomic volatility. As the quotation at the beginning of this paper illustrates, it is often argued that an inability to forecast accurately the persistence of commodity price shocks exacerbates the macroeconomic volatility of small open economies. I demonstrate that, at least in the model used in this paper, this is not the case. This is because, while incomplete information about the persistence of terms of trade shocks increases the volatility of investment, it also encourages households to respond more cautiously to changes in the terms of trade. This makes consumption, the trade balance and output less volatile than they would be if agents had full information.

This paper is related to several strands of literature. Most directly, it complements work examining the effects of incomplete information about the composition of structural shocks, as in Angeletos and La'O (2010) and Blanchard, L'Huillier and Lorenzoni (forthcoming). An application of this methodology to international macroeconomics is found in Boz, Daude and Durdu (2011), who estimate open economy real business cycle models for Canada and Mexico that include uncertainty about the persistence of productivity shocks. This paper contributes to this literature in two ways. First, it provides empirical evidence of the existence of incomplete information about the persistence of an economically meaningful shock that has not previously been examined. Beyond this modest goal, the paper may also shed light on the pervasiveness of informational frictions about other shocks. Because terms of trade shocks are observable and can be rationalised in terms of broader economic developments, it seems plausible that households and firms have more information about these shocks than they do about other, unobserved shocks. Consequently, estimates of the extent of uncertainty regarding the persistence of terms of trade shocks may well represent a lower bound of the uncertainty regarding other shocks.

The paper also contributes to the literature examining the determinants of business cycles in small open economies. Aguiar and Gopinath (2007) demonstrate that a small open economy business cycle model can better match the moments of macroeconomic variables in developing economies if augmented with persistent shocks to the growth rate of productivity, which accumulate over time, to accompany standard transitory mean-reverting productivity shocks. Boz *et al* (2011) demonstrate that a similar result can be obtained with smaller productivity shocks if one assumes that agents have incomplete information about whether shocks are temporary or permanent. An open question in both of these papers is why some economies should experience more persistent, or less observable, shocks than others. This paper provides a potential answer to this question by highlighting the difficulty of identifying the persistence of commodity price shocks. If developing economies are more exposed to commodity price movements than advanced economies, then commodity price shocks could provide one explanation for why the nature of shocks to developing and advanced economies appears to differ.

This paper is also related to the literature describing the response of small open economies to terms of trade shocks. Key theoretical papers in this literature include Harberger (1950) and Laursen and Metzler (1950), who use a simple Keynesian approach, and Sachs (1981), Obstfeld (1982) and Svensson and Razin (1983), who examine the response to a terms of trade shock in an intertemporal optimisation setting. A number of papers have examined these relationships empirically. Otto (2003) constructs structural VAR models for a number of small open economies to examine the effect of transitory terms of trade shocks on the trade balance. He concludes that a positive terms of trade shock generally leads to an improvement in the trade balance, consistent with a basic consumption-smoothing model of the current account in a model with only transitory shocks. Kent and Cashin (2003) separate economies into those whose terms of trade shocks are typically permanent and those whose terms of trade shocks are typically transitory. They find that a positive terms of trade shock leads to a deterioration in the current account in the former economies and an improvement in the latter. They argue that their results are also consistent with standard intertemporal approaches to the current account in which agents smooth their consumption in response to transitory shocks and adjust consumption and investment more substantially in response to persistent shocks.

Other papers have examined the importance of terms of trade shocks as a source of macroeconomic fluctuations. The empirical results here are inconclusive. Based on structural VARs estimated for a number of developing countries, Broda (2004) concludes that terms of trade shocks typically explain less than 10 per cent of output volatility in developing countries. In contrast, using a simulated real business cycle model, Mendoza (1995) finds that terms of trade disturbances explain 56 per cent of output fluctuations in developing countries and 33 per cent of output fluctuations in advanced economies.

To some extent, the results of this paper reinforce those of the previous empirical literature. For example, I find that transitory positive terms of trade shocks lead to an increase in net exports while permanent positive terms of trade shocks trigger a decrease in net exports. However, as outlined in Blanchard *et al* (forthcoming), if agents have incomplete information about the persistence of shocks then it is not possible for an econometrician to identify permanent and transitory shocks in the data. Consequently, the finding that agents are largely unable to differentiate between permanent and transitory terms of trade shocks raises questions about the identification of these shocks in other papers.[4]

The paper proceeds as follows. Section 2 outlines the model and clarifies the information structure. Section 3 describes the estimation and summarises the key results. Sections 4 and 5 discuss the implications of the empirical results for the response of the economy to terms of trade shocks. Section 6 reports a series of robustness checks and Section 7 presents conclusions.

2. A Small Open Economy Model

The basic setup is a standard small open economy model with incomplete markets, similar to those in Mendoza (1995) and Aguiar and Gopinath (2007). I augment the model by assuming that agents are imperfectly informed about the contribution of permanent and transitory shocks to the observed terms of trade, requiring agents to solve a signal extraction problem.

In the model, households choose consumption, saving and labour supply to maximise lifetime utility. Households consume two goods – a good produced in their home economy, and an imported or foreign-produced good. The relative price of the two goods is the terms of trade, which is assumed to be exogenous to developments in the home economy. Households can invest in two assets – physical capital and a one-period non-contingent bond traded in international capital markets. The price of the bond is set exogenously, except for a small risk premium included to ensure that the economy's net foreign debt is stationary. There is one firm in the model, which features production with endogenous capital and labour. I augment the model with permanent and transitory productivity shocks and include capital adjustment costs. These features help the model to fit the data, but play little role in the analysis.

4 This issue may be less of a concern for Kent and Cashin (2003) as they do not identify individual transitory or permanent shocks. Their approach can be viewed as implicitly assuming that agents have no information about the persistence of individual terms of trade shocks and merely expect the persistence of the average shock. It turns out that this assumption about the information structure is not a bad approximation to the results of this paper.

2.1 The Environment

2.1.1 Firms

The economy features a single perfectly competitive firm that produces a tradeable good using a Cobb-Douglas production technology of the form:

$$Y_t = A_t K_t^{\alpha} \left(X_t N_t\right)^{1-\alpha} \tag{1}$$

where Y_t denotes output in period t, K_t denotes capital and N_t denotes hours worked. A_t and X_t are productivity shifters. The process, A_t, is stationary and follows a first-order autoregressive process in logs. In what follows, I use lower-case letters to represent log deviations from a variable's steady state, so that $a_t = \ln A_t - \ln A^*$ where A^* is the steady state value of A_t. The evolution of a_t then follows:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a;\ \varepsilon_t^a \sim N\left(0, \sigma_a^2\right) \tag{2}$$

The second productivity shock, X_t, is non-stationary. Let

$$M_t \equiv \frac{X_t}{X_{t-1}} \tag{3}$$

I assume that the logarithm of M_t follows a first-order autoregressive process of the form:

$$m_t = \left(1-\rho_m\right)\mu + \rho_m m_{t-1} + \varepsilon_t^m;\ \varepsilon_t^m \sim N\left(0, \sigma_m^2\right) \tag{4}$$

The parameter μ measures the deterministic growth rate of the productivity factor X_t. The parameters $\rho_a, \rho_m \in [0,1)$ govern the persistence of a_t and m_t. Somewhat loosely, I refer to a_t and m_t as transitory and permanent productivity shocks, respectively.

Profit maximisation by the firm ensures that factor prices reflect marginal value products:

$$W_t = (1-\alpha) P_t^H \frac{Y_t}{N_t} \tag{5}$$

$$R_t^k = \alpha P_t^H \frac{Y_t}{K_t} \tag{6}$$

where W_t is the nominal wage, P_t^H is the price of the home-produced good and R_t^k is the rate of return to capital.

2.1.2 Households

Households maximise expected lifetime utility given by:

$$\sum_{t=1}^{\infty} \beta^t \left(\ln C_t - A_L \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{7}$$

where β is the household's rate of time preference, C_t is consumption, N_t is hours worked, φ is the inverse of the labour supply elasticity and A_L is a constant used to calibrate average labour supply in the model to match that in the data.

The household's consumption bundle is a Cobb-Douglas aggregate of home- and foreign-produced goods,

$$C_t = \frac{\left(C_t^H\right)^{1-\eta} \left(C_t^F\right)^{\eta}}{(1-\eta)^{1-\eta}\,\eta^{\eta}} \tag{8}$$

where C_t^H are home-produced goods and C_t^F are foreign-produced goods. The parameter $\eta \in (0,1)$ governs the relative weights of home- and foreign-produced goods in the household's consumption bundle. Let P_t be the consumer price index corresponding to C_t. Then,

$$P_t = \left(P_t^H\right)^{1-\eta} \left(P_t^F\right)^{\eta} \tag{9}$$

where P_t^F is the price of the foreign-produced good. Household optimisation ensures that the demand for home- and foreign-produced goods is given by:

$$C_t^H = (1-\eta)\left(\frac{P_t}{P_t^H}\right) C_t \tag{10}$$

$$C_t^F = \eta \left(\frac{P_t}{P_t^F}\right) C_t \tag{11}$$

Households have access to two assets: domestic capital and a single-period, risk-free bond, denominated in the foreign good. The household's period-by-period budget constraint is:

$$Q_t B_{t+1} + P_t C_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - \mu\right)^2 K_t \leq W_t N_t + R_t^k K_t + B_t \tag{12}$$

where Q_t denotes the price of one-period risk-free bonds, B_{t+1} denotes the stock of bonds acquired in period t, I_t denotes gross investment and ϕ is a parameter that controls the cost of adjusting the size of the capital stock. The capital stock evolves according to the law of motion:

$$K_{t+1} = (1-\delta) K_t + I_t \tag{13}$$

where $\delta \in [0,1)$ denotes the depreciation rate of capital.

To ensure that the solution to the model is stationary, I assume that the country faces a debt-elastic interest rate premium as in Schmitt-Grohé and Uribe (2003). Specifically,

$$\frac{1}{Q_t} = 1 + r^* - \psi\left(e^{B_{t+1}/X_t\left(P_t^H\right)^{\frac{1}{1-\alpha}} - \bar{B}} - 1\right) \tag{14}$$

where r^* is the exogenous foreign rate of interest on a risk-free bond and $\bar{B}$ is the steady-state foreign asset level.

Household utility maximisation implies the following first order conditions:

$$\frac{1}{C_t} = \lambda_t P_t \tag{15}$$

$$N_t^{\varphi} = \lambda_t W_t \tag{16}$$

$$\begin{aligned} \beta \mathbb{E}_t \left\{\lambda_{t+1}\left[1-\delta+R_{t+1}^K\right]\right\} &= \lambda_t\left(1+\phi\left(\tfrac{K_{t+1}}{K_t}-\mu\right)\right) \\ +\beta \mathbb{E}_t &\left\{\lambda_{t+1}\left[\tfrac{\phi}{2}\left(\tfrac{K_{t+2}}{K_{t+1}}-\mu\right)^2 - \phi\tfrac{K_{t+2}}{K_{t+1}}\left(\tfrac{K_{t+2}}{K_{t+1}}-\mu\right)\right]\right\} \end{aligned} \tag{17}$$

$$Q_t \lambda_t = \beta \mathbb{E}_t\left\{\lambda_{t+1}\right\} \tag{18}$$

where λ_t is the Lagrange multiplier on the household's budget constraint.[5]

2.1.3 Relative prices

I take the price of the foreign good, P_t^F, as the numeraire and normalise it to 1. I define the terms of trade, S_t, as the relative price of home-produced goods in terms of foreign-produced goods. It follows from the definition of the consumer price index that:

$$S_t = P_t^H;\ P_t = S_t^{1-\eta}$$

The home economy is assumed to be small in the sense that it is a price-taker on world markets. Consequently, changes in its terms of trade are exogenous to domestic variables. The terms of trade are assumed to follow the process,

$$S_t = Z_t \Gamma_t \tag{19}$$

The first component, Z_t, represents a transitory shock to the terms of trade, which is assumed to follow a first-order autoregressive process in logs. That is,

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z;\ \varepsilon_t^z \sim N\left(0, \sigma_z^2\right) \tag{20}$$

The second component, Γ_t is a permanent terms of trade shock. Let,

$$G_t \equiv \frac{\Gamma_t}{\Gamma_{t-1}} \tag{21}$$

I assume that the logarithm of G_t follows a first-order autoregressive process of the form:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g;\ \varepsilon_t^g \sim N\left(0, \sigma_g^2\right) \tag{22}$$

The decomposition of the terms of trade outlined in Equations (19)–(22) is extremely flexible and encompasses many of the assumptions about the evolution of the terms of trade used in other papers. For example, if $\sigma_g^2 = 0$, the terms of

5 Note that in taking first order conditions with respect to the foreign debt level, I have assumed that agents take the interest rate on foreign assets as given – that is, they do not internalise the effect of their decisions on their borrowing costs. For a discussion of the implications of internalisation of the risk premium, see Lubik (2007).

trade is subject to purely transitory shocks, while if $\sigma_z^2 = 0$ and $\rho_g = 0$ then the terms of trade follows a random walk.[6]

2.1.4 Market clearing

Market clearing requires that the quantity of goods produced in the home economy equals the consumption of these goods at home and abroad. This is ensured by the current account condition:

$$Q_t B_{t+1} + P_t C_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - \mu\right)^2 K_t \leq P_t^H Y_t + B_t \tag{23}$$

2.1.5 Equilibrium

An equilibrium is a sequence of quantities $\{C_t, N_t, I_t, Y_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty}$, prices $\left\{W_t, R_t^K, Q_t, P_t, P_t^H, S_t\right\}_{t=0}^{\infty}$ and exogenous processes $\{A_t, X_t, Z_t, \Gamma_t\}_{t=0}^{\infty}$ such that (i) firms maximise profits, which implies Equations (5) and (6), (ii) households maximise utility, which implies Equations (15)–(18), and (iii) markets clear, given by Equation (23), subject to the technological and resource constraints in Equations (1), (13), (14) and (19) and the exogenous processes given in Equations (2), (4), (20) and (22).

2.2 Information Structure

I assume that agents have complete information about all aspects of the economy other than the components of the terms of trade, about which they are imperfectly informed. In particular, I assume that agents can observe the level of the terms of trade but cannot observe Z_t or Γ_t directly. Reflecting the fact that agents are likely to have some information about the persistence of these shocks, I assume that agents receive a noisy signal regarding the permanent terms of trade shock. I refer to this signal as h_t such that $h_t = g_t + \varepsilon_t^h$ where ε_t^h are independently and identically distributed with mean zero and variance σ_h^2 . The agents' information set as of time t includes the entire history of terms of trade shocks and signals; $\mathscr{I}_t \equiv \left\{S_t, h_t, S_{t-1}, h_{t-1}, \ldots\right\}$.

6 The terms of trade will also follow a random walk if $\rho_g = \rho_z = \rho$ and $\rho\sigma_g^2 = (1-\rho)^2\sigma_z^2$.

In the model, agents form expectations about the decomposition of the terms of trade using the Kalman filter. To implement this, I represent the agent's filtering problem in state space form using the decomposition in Boz *et al* (2011). First, I define the growth rate of the terms of trade as:

$$\begin{aligned}\Delta s_t &\equiv \ln s_t - \ln s_{t-1} \\ &= z_t - z_{t-1} + g_t\end{aligned}$$

The measurement equation includes a reformulation of this definition as well as the definition of the noise process:

$$\underbrace{\begin{bmatrix}\Delta s_t \\ h_t\end{bmatrix}}_{\mathscr{S}_t} = \underbrace{\begin{bmatrix}1 & -1 & 1 \\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix}z_t \\ z_{t-1} \\ g_t\end{bmatrix}}_{\mathscr{X}_t} + \underbrace{\begin{bmatrix}0 & 0 & 0 \\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{D}} \underbrace{\begin{bmatrix}\varepsilon_t^z \\ \varepsilon_t^g \\ \varepsilon_t^h\end{bmatrix}}_{U_t} \tag{24}$$

The transition equation summarises the evolution of the unobserved variables and is given by:

$$\underbrace{\begin{bmatrix}z_t \\ z_{t-1} \\ g_t\end{bmatrix}}_{\mathscr{X}_t} = \underbrace{\begin{bmatrix}\rho_z & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho_g\end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix}z_{t-1} \\ z_{t-2} \\ g_{t-1}\end{bmatrix}}_{\mathscr{X}_{t-1}} + \underbrace{\begin{bmatrix}1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0\end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix}\varepsilon_t^z \\ \varepsilon_t^g \\ \varepsilon_t^h\end{bmatrix}}_{U_t} \tag{25}$$

where $U_t \sim N(0, \Upsilon)$ and $\Upsilon \equiv \begin{bmatrix}\sigma_z^2 & 0 & 0 \\ 0 & \sigma_g^2 & 0 \\ 0 & 0 & \sigma_h^2\end{bmatrix}$.

The Kalman filter can then be used to express the consumers' beliefs about the components of the terms of trade in recursive form as:

$$\mathscr{X}_{t|t} = (\mathbb{I} - \mathbf{KC}) A \mathscr{X}_{t-1|t-1} + \mathbf{K}\mathscr{S}_t \tag{26}$$

where $\mathbb{I}$ is an identity matrix and K is the Kalman gain, calculated as:

$$\mathbf{K} = \mathbf{LC}' \left(\mathbf{CLC}' + \mathbf{DQD}'\right)^{-1} \tag{27}$$

and $\mathbf{L}$ is the steady-state error covariance matrix, calculated as the solution to:

$$\mathbf{L} = \mathbf{ALA}' - \mathbf{ALC}'\left(\mathbf{CLC}'\right)^{-1}\mathbf{CLA}' + \mathbf{BQB}' \tag{28}$$

Equations (26)–(28) fully characterise learning.

2.3 Model Solution

I solve the model by taking a log-linear approximation to the equilibrium conditions derived in the previous section.[7] The solution of the model follows Uhlig (1999) and Blanchard *et al* (forthcoming). Let $\mathscr{Y}_t$ denote the endogenous variables controlled by the agent. The economic model can be represented as the stochastic difference equation:

$$\mathbf{F}E_t\left\{\mathscr{Y}_{t+1}\right\} + \mathbf{G}\mathscr{Y}_t + \mathbf{H}\mathscr{Y}_{t-1} + \mathbf{M}\mathscr{S}_t + \mathbf{N}E_t\mathscr{S}_{t+1} = 0 \tag{29}$$

where $\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$, $\mathbf{M}$ and $\mathbf{N}$ are matrices of parameters and $\mathscr{S}_t$ is the vector of observable variables described in section 2.2. The unique stable solution of the model is:

$$\mathscr{Y}_t = \mathbf{P}\mathscr{Y}_{t-1} + \mathbf{Q}\mathscr{S}_t + \mathbf{R}\mathscr{X}_{t|t} \tag{30}$$

where $\mathscr{X}_{t|t}$ represents the agents' expectation of the unobserved states described in section 2.2. The matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$ can be found by solving the three matrix equations:

$$\begin{aligned} \mathbf{FP}^2 + \mathbf{GP} + \mathbf{H} &= 0 \\ (\mathbf{FP} + \mathbf{G})\mathbf{Q} + \mathbf{M} &= 0 \\ (\mathbf{FP} + \mathbf{G})\mathbf{R} + \mathbf{F}(\mathbf{QC} + \mathbf{R})\mathbf{A} &= 0 \end{aligned}$$

where the matrices $\mathbf{A}$ and $\mathbf{C}$ are as defined in Section 2.2.

3. Estimation

I estimate the model using Bayesian methods. This section outlines the estimation strategy, including the choice of priors, and explains how the variables of the theoretical model map into observable time series.

7 Appendix B outlines the model's steady state and the log-linearised equations.

3.1 Measurement

The initial stage of the estimation is to map the model's variables, which are generally unobservable, into observable variables that can be used to estimate the model's parameters. To do this, I first express the log-linear equilibrium conditions, derived in the previous section, in state space form as:

$$\tilde{\xi}_t = \mathbf{T}\tilde{\xi}_{t-1} + \nu_t \tag{31}$$

$$\tilde{Y}_t = \Xi + \mathbf{V}\tilde{\xi}_t + \zeta_t \tag{32}$$

$$\begin{bmatrix} \nu_t \\ \zeta_t \end{bmatrix} \sim N\left(0, \begin{bmatrix} \mathbf{W} & 0 \\ 0 & \mathbf{X} \end{bmatrix}\right) \tag{33}$$

where the state vector $\tilde{\xi}_t = \{\mathscr{Y}_t, \mathscr{S}_t, \mathscr{X}_{t|t}\}$ collects the model's theoretical variables and the vector $\tilde{Y}_t$ collects the observed variables used to estimate the model. Equation (31) governs the transition of the state variables, while Equation (32) maps the state into observable variables. The matrices $\mathbf{T}$, Ξ, $\mathbf{V}$ and $\mathbf{W}$ are functions of the parameters of the model.

The observable variables I use to estimate the model are the growth rates of real GDP, private consumption, private gross fixed capital formation and the terms of trade as well as the level of the trade balance-to-GDP ratio, $\frac{NX_t}{Y_t}$. That is:

$$\tilde{Y}_t = \begin{bmatrix} \Delta \ln Y_t & \Delta \ln C_t & \ldots \\ \Delta \ln I_t & \Delta \ln S_t & \frac{NX_t}{Y_t} \end{bmatrix} \tag{34}$$

All variables are expressed in per capita terms and are seasonally adjusted. I estimate the model using quarterly Australian data over the period 1973:Q1–2012:Q2.[8] The starting point reflects the first quarter for which per capita national accounts data are available for Australia. This is somewhat earlier than the starting date for most Australian DSGE models, which typically use data spanning the period after the adoption of a floating exchange rate in 1983 or inflation targeting in 1993. A later starting date is appropriate for models containing nominal interest rates or inflation, whose behaviour is likely to be affected by changes in the conduct of monetary policy. In contrast, the model in this paper contains no nominal variables. And, given the presence of long-lived

8 Appendix A outlines the data sources used in the estimation.

trends in the terms of trade, it seems preferable to use a longer time series to estimate the model.

Following Jääskelä and Nimark (2011), the covariance matrix, $\mathbf{X}$, of the vector of measurement errors, ζ_t, in Equation (32) are set to $E_t\left[\tilde{Y}_t\tilde{Y}_t'\right] \times 0.1$ so that 10 per cent of the variance of the data series is assumed to come from measurement errors.

3.2 Bayesian Estimation

I estimate the parameters of the model using Bayesian methods that combine prior information with information from the data. The estimation works in the following way. Denote the vector of parameters to be estimated as Θ. The log posterior distribution of the parameters to be estimated is given by:

$$\Upsilon = \mathscr{L}(\Theta) + \mathscr{L}\left(\tilde{Y}_t|\Theta\right) \tag{35}$$

where $\mathscr{L}(\Theta)$ is the log of the prior probability of observing a given vector of parameters and $\mathscr{L}\left(\tilde{Y}_t|\Theta\right)$ is the log likelihood of observing the dataset $\tilde{Y}_t$ for a given parameter vector. This likelihood is given by:

$$\mathscr{L}\left(\tilde{Y}_t|\Theta\right) = -0.5\sum_{t=0}^{T}\left[p\ln(2\pi) + \ln|\Omega| + u_t'\Omega^{-1}u_t\right] \tag{36}$$

where p is the dimension of $\tilde{Y}_t$, Ω is the covariance matrix of the theoretical one-step-ahead forecast errors implied by a given parameterisation of the model and u_t is the vector of actual one-step-ahead forecast errors.

The numerical procedure I use to estimate the posterior distribution follows the methodology outlined in An and Schorfheide (2007). In computing the posterior distribution, I set the number of Metropolis-Hastings draws equal to 500 000, and select these after discarding an initial 250 000 burn-in draws.

3.2.1 Priors

For the AR(1) parameters of the exogenous processes, I assign beta priors with a mean of 0.8 and standard deviation of 0.1. Using the beta distribution for these priors ensures that the estimated parameters lie between 0 and 1, consistent with economic theory. I assign inverse gamma priors with a mean of 5×10^{-3} and a standard deviation of 0.01 to the standard deviations of the exogenous processes. Finally, for the capital adjustment cost parameter, ϕ, I assign a truncated normal prior, with a mean of 7.5 and standard deviation of 2.5.[9]

The theoretical model, of course, contains a number of additional parameters. Many of these are likely to be poorly identified using only the observed data series included in the model but have been estimated many times previously. Rather than rely on imprecise estimates of these parameters, I calibrate them using values determined by previous research or economic theory. In a Bayesian framework, calibration can be thought of as a very tight prior. Table 1 outlines the calibrated parameters.

Table 1: Calibrated Parameters

Parameter	Value	Description
β	0.99	Discount factor
δ	0.02	Depreciation rate
μ	1.0045	Steady-state technology growth rate
$\tilde{B}$	1.10	Steady-state foreign debt level
α	0.33	Capital share of income
η	0.20	Imports share of consumption
φ	1.00	Inverse Frisch labour supply elasticity
ψ	0.001	Portfolio adjustment cost

The parameters for the discount factor, depreciation rate and capital share of income are standard for a model estimated on quarterly data. The parameter for μ broadly conforms to the average quarterly growth rate of GDP per capita over the sample period. The parameter for $\tilde{B}$ is set to ensure that the model matches the average net export-to-GDP ratio seen in the data, while that for η matches the import share of consumption. The parameter for ψ is set as a small value that ensures that the model is stationary while having only a minor impact on the dynamics of the model. Finally, the parameter for φ is taken from Jääskelä and Nimark (2011).

9 The truncation ensures that ϕ is greater than 0. In the estimation, the bulk of the posterior distribution of this parameter lies far away from the truncation point.

3.3 Posterior Distribution

Table 2 shows the main results of estimation. The transitory terms of trade shock is reasonably persistent, with a posterior mean of the AR(1) coefficient ρ_z equal to 0.84. The permanent shock is marginally less so, with ρ_g equal to 0.77. In terms of the magnitude of the shocks, the standard deviation of transitory terms of trade shocks, σ_z, is quite large at 1.25 per cent, while the standard deviation of the permanent terms of trade shock, σ_g, is much smaller at just 0.22 per cent. Nonetheless, a shock to ε_g ultimately has a much larger and more lasting impact on the terms of trade. A positive shock to ε_z causes a once-off increase in the terms of trade which then diminishes, although the high value of ρ_z implies that it takes some time for the terms of trade to return to its initial level following the shock. In particular, the half-life of this shock is around six quarters, and the terms of trade does not return to its trend level for several years. In contrast, a positive shock to ε_g increases the terms of trade on impact and then continues to increase the terms of trade further, albeit at a diminishing rate, over time. The accumulation continues over several quarters, and the terms of trade ultimately settles at a level around five times the level of the initial impulse five years after the initial shock.

Table 2: Prior and Posterior Distributions – Incomplete Information Model

		Prior		**Posterior**			
Parameter	**Distribution**	Mean	SD	Mode	Mean	5%	95%
Exogenous processes – AR(1) coefficients							
ρ_a	Beta	0.80	0.10	0.98	0.98	0.97	0.99
ρ_m	Beta	0.80	0.10	0.65	0.62	0.42	0.81
ρ_z	Beta	0.80	0.10	0.85	0.84	0.74	0.92
ρ_g	Beta	0.80	0.10	0.81	0.77	0.64	0.88
Exogenous processes – standard deviations ($\times 10^{-2}$)							
σ_a	Inv Gamma	0.50	1.00	0.69	0.69	0.60	0.78
σ_m	Inv Gamma	0.50	1.00	0.15	0.17	0.10	0.25
σ_z	Inv Gamma	0.50	1.00	1.24	1.25	1.06	1.44
σ_g	Inv Gamma	0.50	1.00	0.18	0.22	0.11	0.34
σ_h	Inv Gamma	0.50	1.00	1.19	1.31	0.64	2.11
Other parameters							
ϕ	Trunc Normal	7.50	2.50	9.67	9.76	7.87	11.75
Log marginal density		–1 774.2					

The standard deviation of the noise shocks, σ_h is also large, at 1.31 per cent. This suggests that agents receive a fairly weak signal about the persistence of terms of trade shocks.

Although the remaining parameter estimates are not the focus of this paper, it is comforting to note that the results seem plausible and are broadly consistent with other empirical estimates. In particular, the magnitude of the transitory productivity shocks are estimated to be larger than those of the permanent productivity shocks, which is consistent with the estimates for Canada in Aguiar and Gopinath (2007), although the persistence of these shocks are slightly larger than in that study. The results also imply large capital adjustment costs. This is a common finding in the open economy literature. In the absence of these adjustment costs, the ability to finance the accumulation of imported capital using foreign borrowing without requiring an accompanying decrease in consumption would lead the model to predict implausibly large investment volatility.

4. Response of the Economy to Terms of Trade Shocks

In this section I first show how incomplete information affects the response of the economy to terms of trade shocks and then discuss its implications for aggregate macroeconomic volatility.

4.1 Dynamic Responses to Terms of Trade Shocks

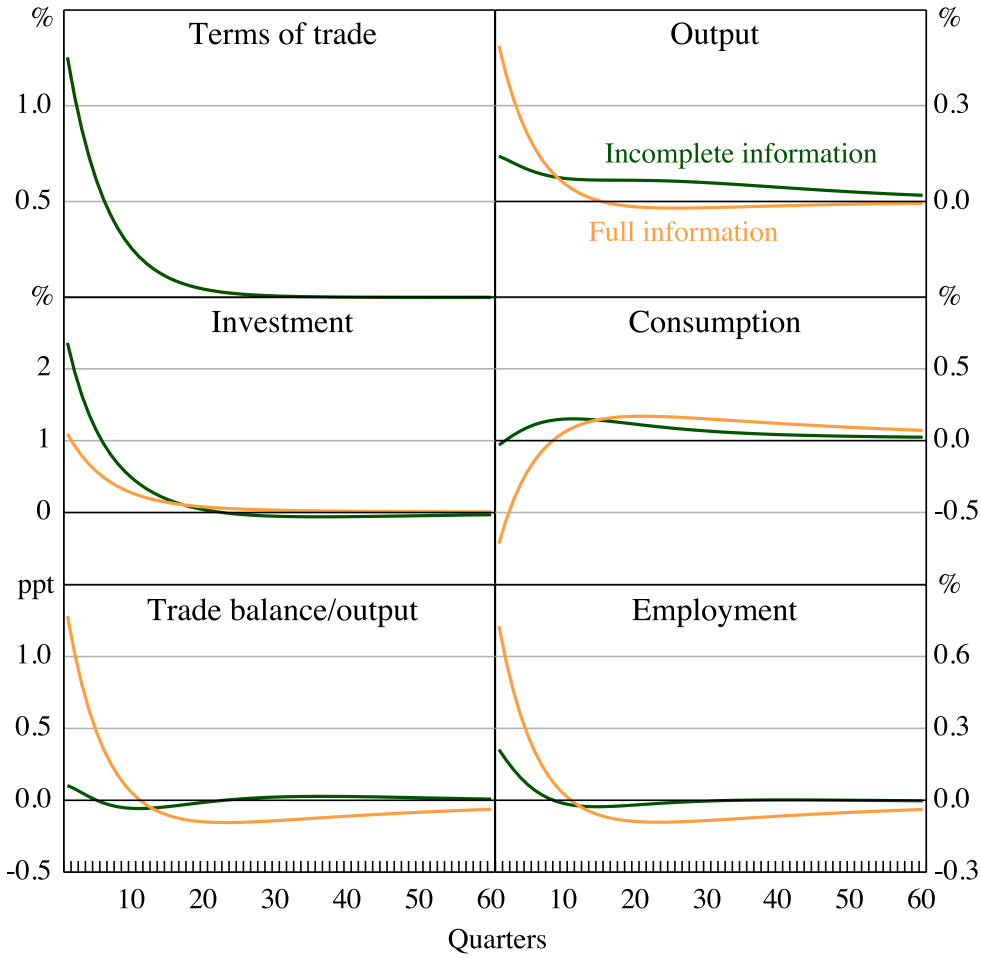
4.1.1 Transitory terms of trade shocks

Figure 3 shows the response of the economy to a positive one standard deviation transitory terms of trade shock. I focus first on the response of the economy when agents have incomplete information about the persistence of these shocks.

Initially, the shock increases the terms of trade by around 1.3 per cent. In subsequent quarters, the terms of trade decrease and after six years stabilise at their original level. The shock increases the price of the economy's output relative to the price of consumption and investment goods. This induces households to invest more and increases real wages, which leads to an expansion in labour supply and production. The employment boom is short-lived, however, and within two years employment falls below its trend level. This reflects the fact that the increase in the terms of trade makes households in the economy wealthier. As they receive disutility from working, households choose to convert some of their increased wealth into additional leisure. In contrast, the investment boom is more persistent and it takes five years for investment to return to trend. Output remains slightly above trend for a considerable period, reflecting the increase in the size of the economy's capital stock. The impact of the terms of trade boom on the trade balance is quite small, with an initial increase followed a few quarters later by a small decrease.

Figure 3: Impulse Response Function

Response to a one standard deviation transitory shock



Consumption also responds positively to the shock before gradually reverting to trend. This response reflects two factors. First, the higher terms of trade increases the wealth of domestic residents. All other things equal, this would lead them to increase their consumption. However, to the extent that agents expect that some of the increase in the terms of trade will eventually dissipate, the shock also creates the expectation of a decrease in the price of the consumption good in terms of import prices.[10] As the bond price in this model is exogenous and denominated in terms of importables, the expected decrease in the relative price of the consumption good increases the real interest rate faced by domestic households. This induces households to postpone consumption.[11] The substitution effect of a higher real interest rate is greatest immediately following the shock, while in later quarters the income effects of greater wealth dominate the consumption response.

It is instructive to compare the response of the economy under incomplete information to its response under full information.[12] In the full information case, the initial responses of employment and output are substantially larger than they are under incomplete information. That is, with full information, agents realise that the shock is transitory and bring forward production to take advantage of the temporarily high export prices. Meanwhile, the shock causes an initial contraction in consumption. This is larger than in the incomplete information case for two reasons. First, under full information agents are confident that the increase in the terms of trade will be short-lived. Consequently, the expected increase in their wealth is smaller than it would be if they anticipated that the terms of trade would be persistently higher. Second, fully informed agents are also confident that that the price of the home-produced consumption good relative to the imported good's price will decrease in the future. Consequently, the real interest rate is also higher under full information than it is under incomplete information. Both the smaller positive income effect and larger substitution effect will tend to depress consumption in the full information case relative to the incomplete information case.

In contrast, the increase in investment is smaller under full information. This largely reflects the impact of the capital adjustment costs, which dampen the

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10 In log-linear terms, $\tilde{p}_t = (1-\eta)\tilde{s}_t$, so that the CPI is proportional to the terms of trade in this model. The transitory terms of trade shock increases the CPI on impact, but agents expect deflation in subsequent periods as the terms of trade decline.

11 It should be noted that while this mechanism is general, the sign and magnitude of the consumption response are sensitive to the parameterisation of the utility function. For example, in a similar model, Mendoza (1995) assumes an intertemporal elasticity of substitution of 2.6, which causes consumption to increase following a transitory terms of trade shock.

12 To calculate the response under the full information case, I use the parameter estimates from Table 2 but set the standard deviation of the terms of trade noise shocks, σ_h, equal to zero. In Section 6 I re-estimate the model parameters under the assumption of full information.

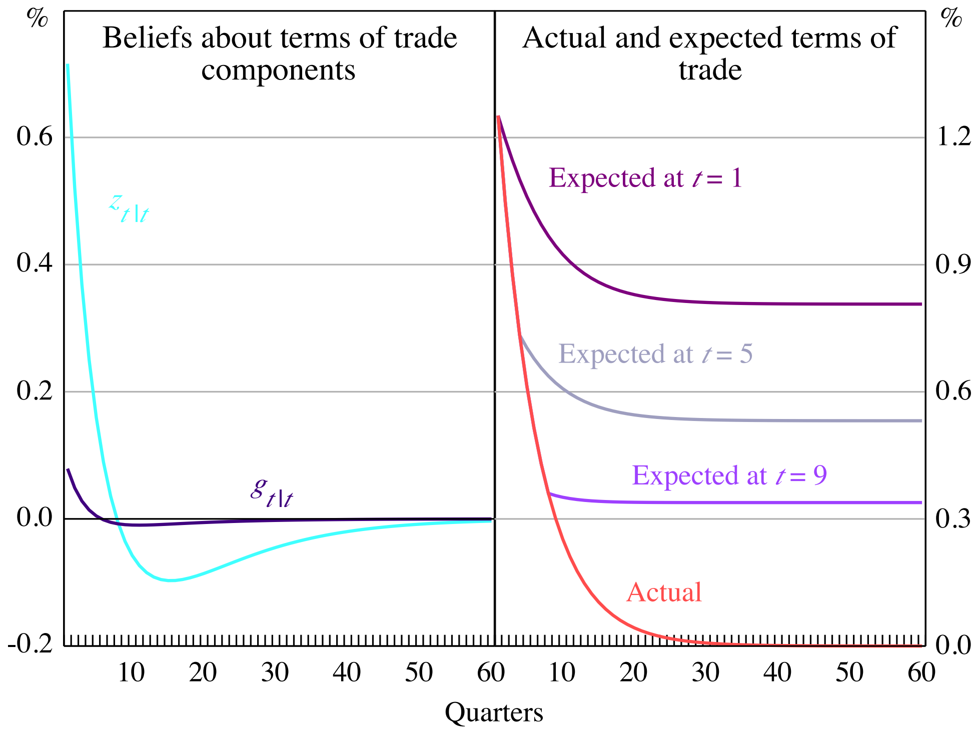
response of investment to transitory shocks. Agents do not wish to pay large costs to expand the capital stock during a terms of trade boom and then to pay these costs again when the capital stock shrinks as commodity prices fall. The combination of a larger increase in output, smaller increase in investment and decrease in consumption implies a greater initial increase in the trade balance in the full information case compared to the incomplete information case. After two years, households start to draw down on the foreign assets that they accumulate through the increased trade balance, and use the proceeds to fund additional consumption.

The response of the economy under full information reflects a standard consumption smoothing response to a temporary increase in income. Agents produce more when the relative price of output is high and save part of the windfall to fund higher consumption when it is cheaper in the future. To understand the response of agents under incomplete information, it is necessary to examine their beliefs. These are illustrated in Figure 4. The left panel shows how agents' beliefs about the two components of the terms of trade shock, z_t and g_t, evolve following a transitory shock. Agents have some success in identifying the shock. They attribute over half of the 1.3 per cent increase in the terms of trade to the transitory shock and only a small proportion to the permanent shock.[13] Agents are less successful in inferring the evolution of z_t and g_t in future periods. But they still correctly attribute most of the evolution in the terms of trade to transitory shocks.

Given that agents correctly identify transitory shocks as the main cause of the observed changes in the terms of trade, why do their reactions differ so much between the full information and incomplete information cases? The key to understanding this is to recall that the permanent shock increases the terms of trade in future periods as well as on impact. Hence, even a small initial increase in agents' beliefs about g_t can translate into a large increase in the expected long-run level of the terms of trade. To illustrate this, the right panel of Figure 4 shows the actual path of the terms of trade as well as agents' expectations about the evolution of the terms of trade calculated in the period when the shock hits, as well as after five and nine quarters. Although agents initially attribute only a small portion of the shock to the permanent component, agents initially believe

13 Letting the symbol ˆ refer to agents' beliefs about the components of the terms of trade, the sum of $\hat{z}_t$ and $\hat{g}_t$ does not equal the change in the terms of trade because agents also adjust their beliefs regarding $\hat{z}_{t-1}$ following the shock. Specifically, on impact, $\hat{z}_{t-1}$ is equal to around –0.8, so that $\hat{z}_t - \hat{z}_{t-1} + \hat{g}_t = \Delta s_t$.

Figure 4: Beliefs Following Terms of Trade Shock
Response to a one standard deviation transitory shock



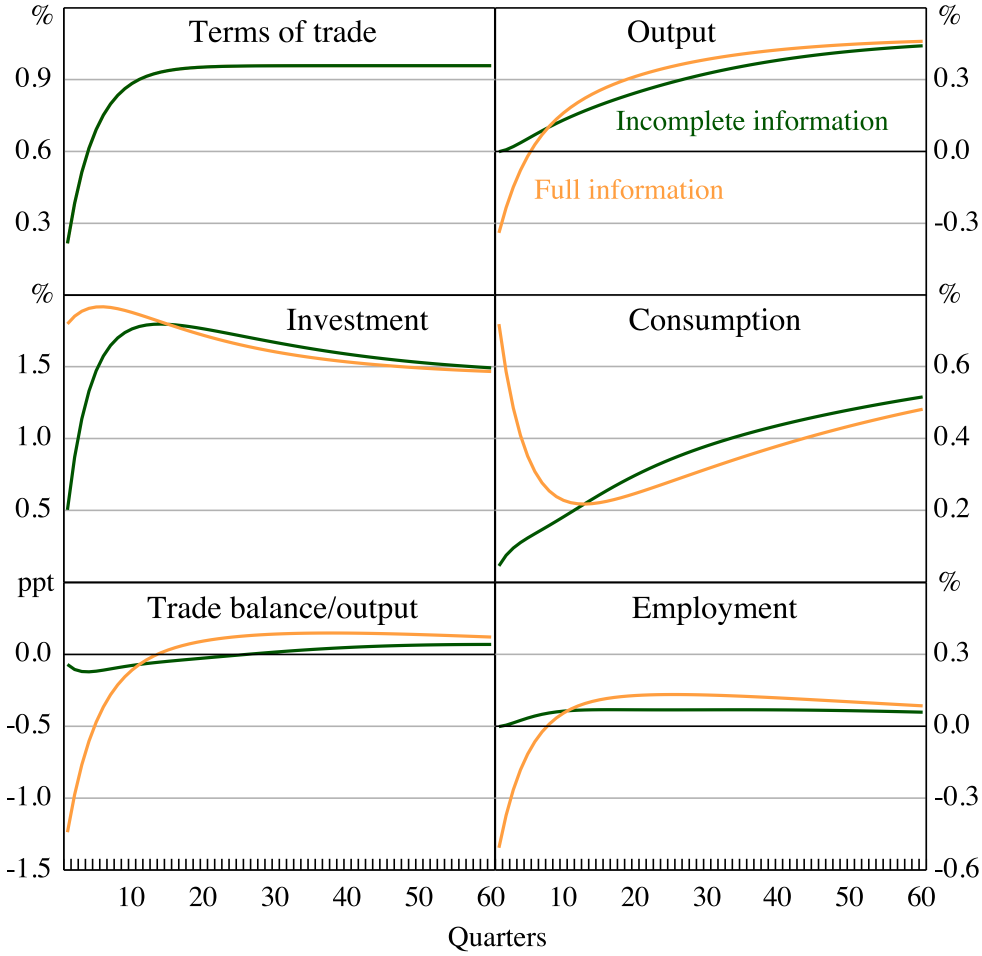
that this small permanent shock is ultimately expected to leave the terms of trade 0.8 per cent above its initial level. In subsequent quarters, as the terms of trade starts to fall, agents revise down their expectations, but continue to believe that some of the increase in the terms of trade will be permanent. This explains why, in the incomplete information case, agents in the economy feel less urgency to work and save more in the near term to take advantage of the high terms of trade than they do in the full information case.

4.1.2 Permanent terms of trade shocks

Turning to the permanent shock, Figure 5 shows the economy's response to a one standard deviation shock to ε_g. The shock increases the terms of trade by 0.2 per cent on impact, and accumulates over time so that the terms of trade ultimately settles at around 1 per cent above its initial level. Focusing first on the incomplete information case, output and investment both increase following the shock. The expansion in output is initially small and accumulates over time. In contrast, the initial investment response is large, and then diminishes. While output and investment increase permanently following the shock, employment eventually returns to trend. It takes a long time to do so, however, and 15 years after the shock employment remains above trend. Consumption initially responds little to the shock, but then increases over time. Since the investment boom is larger than the increase in revenue from the higher terms of trade, the economy's trade balance decreases for some time following the shock, although it ultimately increases once the investment boom passes.

Figure 5: Impulse Response Function

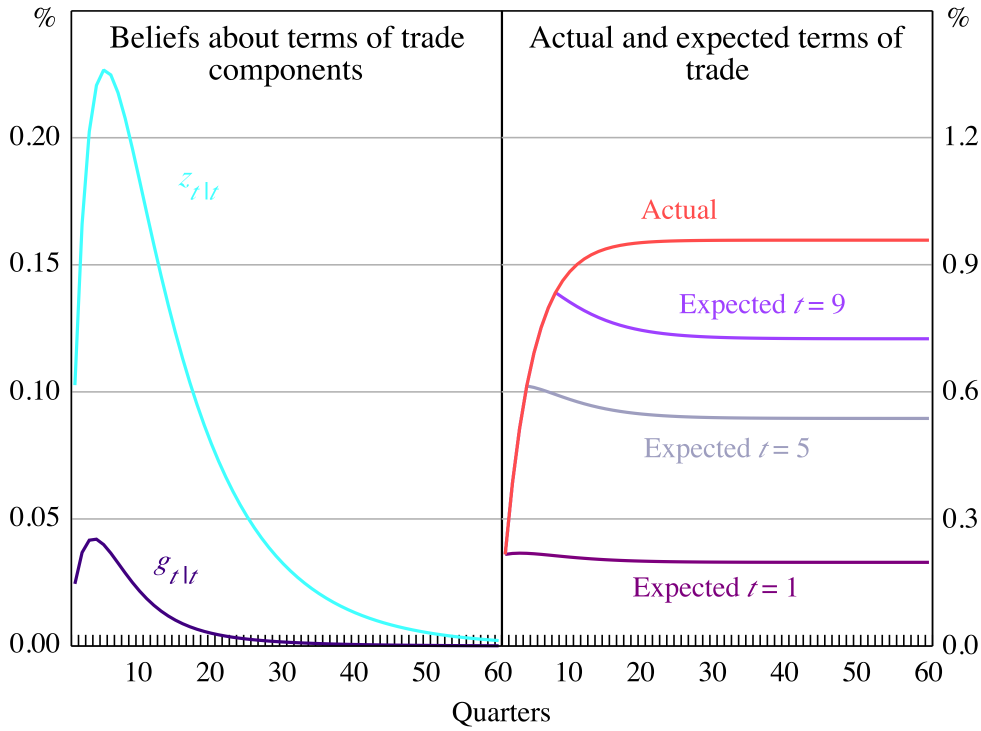
Response to a one standard deviation persistent shock



Once again, it is informative to examine the economy's response to the shock under full information. In this case, employment and output both decrease following the shock and only return to their initial level after two years. Following that, however, the response of output in the full and incomplete information cases are broadly similar. In contrast, the investment boom is larger than under incomplete information. This reflects the fact that, when agents are confident that the terms of trade will increase in the future, they are more willing to make long-term investments today. Agents also increase consumption by more under full information. A stronger response of consumption and investment, and weaker response of output, translates into a larger initial decline in the trade balance. This is offset by a stronger increase in the trade balance in future years.

Figure 6 shows agents' beliefs about the composition of the permanent shock under incomplete information. Agents make substantial errors in interpreting this shock. Initially, they attribute most of the increase in the terms of trade to changes in its transitory component. And even as the terms of trade increase further in future periods, agents continue to attribute only a small proportion of these changes to innovations to the persistent component of the terms of trade. Indeed, immediately following a permanent shock agents' expectations about the future evolution of the terms of trade (shown in the right-hand panel of Figure 6) aren't substantially different from their beliefs following a transitory shock. Agents expect that most, but not all, of the initial increase in the terms of trade will be permanent. As the terms of trade rise further in the future, agents revise up their estimate of the long-run level of the terms of trade. But they still fail to forecast subsequent increases. Or, put another way, the high estimated standard deviation of the noise shock means that agents struggle to distinguish between persistent and transitory terms of trade shocks in real time.

Figure 6: Beliefs Following Terms of Trade Shock
Response to a one standard deviation persistent shock

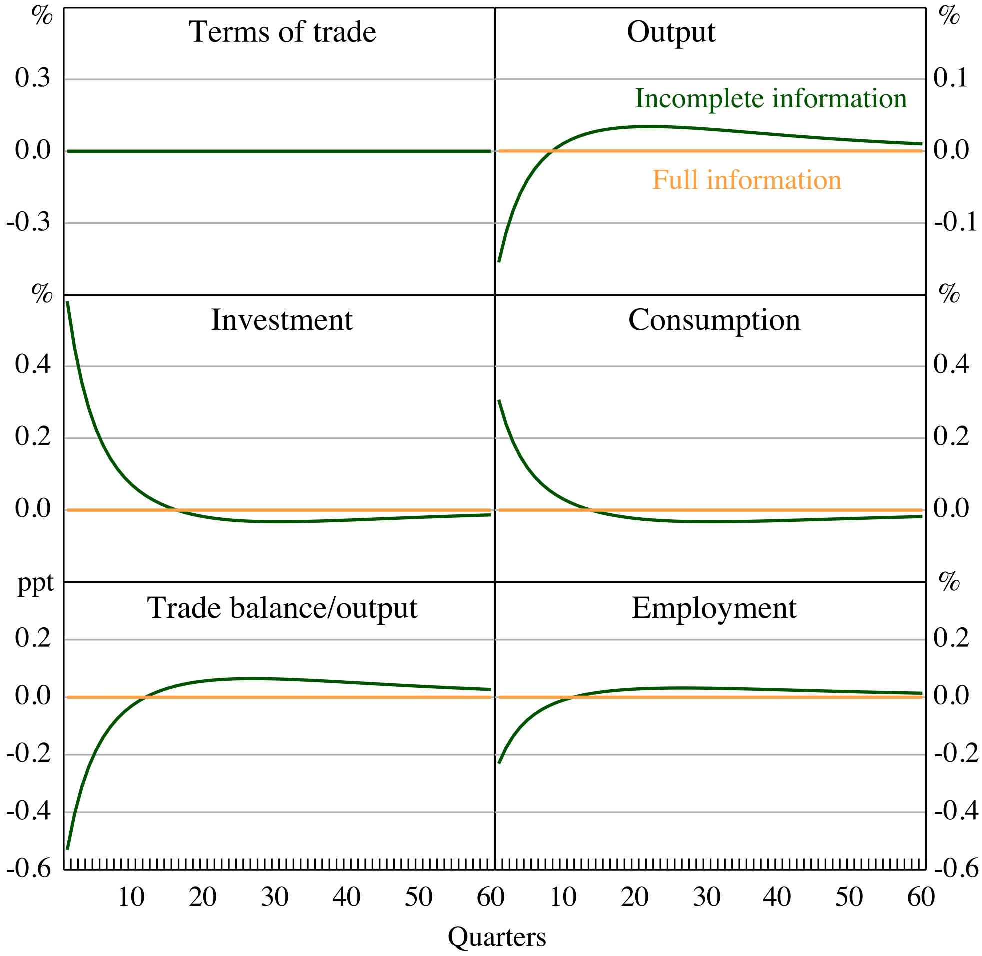


4.1.3 Noise shocks

As a further exercise, Figure 7 shows the response to a noise shock. Although it is difficult to assign a structural interpretation to this shock, it can be thought of as a signal that the terms of trade will increase permanently that ultimately proves to be unfounded. Agents respond to a noise shock because they cannot be sure that it is not genuine. In particular, agents consume more in anticipation of higher future income and invest more in anticipation of higher prices for their output in the future. In the near term, they also work and produce less, expecting that future increases in export prices will make such a decrease in production sustainable. Agents fund their additional consumption and investment by borrowing from abroad. This translates into a decrease in the trade balance. When they realise that the signal was misleading and that the terms of trade will not increase, agents are forced to draw back on consumption, and to work and produce more to repay their accumulated foreign borrowing.

Although one must be cautious about interpreting this shock, it is interesting that the behaviour of the economy is similar to that of an economy running a ‘bad’ current account deficit, of the type described by Blanchard and Milesi-Ferretti (2009), in which private saving decreases in anticipation of an income boom that does not occur.[14]

Figure 7: Impulse Response Function
Response to a one standard deviation noise shock



4.2 Implications for Volatility

The previous section demonstrated how incomplete information alters the macroeconomic effects of permanent and transitory terms of trade shocks. In light of these results, one might wonder whether incomplete information makes the economy more or less sensitive to terms of trade shocks. Specifically, if agents have incomplete information about the persistence of terms of trade shocks, is the

14 Examining the behaviour of household and firm expectations in the lead up to balance of payments crises to see whether this mechanism is empirically relevant would be a useful avenue for further research.

variance of macroeconomic variables higher or lower and do terms of trade shocks contribute more or less to macroeconomic volatility?

One can imagine why either outcome might be possible. Under incomplete information, agents' forecasts of future terms of trade movements are likely to be less accurate than if they have full information. As agents in the model make consumption and investment decisions in a forward-looking manner, these mistakes may force them to adjust their consumption patterns and expand or contract their investment projects. Moreover, under full information, agents do not respond to terms of trade noise shocks. On the other hand, the impulse responses in Figures 3 and 5 suggest that the response of output, consumption, employment and the trade balance to both transitory and persistent terms of trade shocks is more muted under incomplete information than it is under full information, although the investment response appears more volatile.

It turns out that, at the estimated parameter values, the model suggests that incomplete information reduces macroeconomic volatility and diminishes the importance of terms of trade shocks as a source of macroeconomic fluctuations. We can see this in Table 3, which compares the standard deviations of the growth rates of output, consumption, investment and the trade balance in the data and in the model. Under incomplete information, the model suggests a degree of macroeconomic volatility broadly comparable to that seen in the data. Under full information, the standard deviation of output growth increases by around a quarter, while the volatility of consumption growth and the trade balance double. In contrast, the volatility of investment increases under incomplete information.

Table 3: Moments – Incomplete Information Model

	Standard deviation			**Per cent of variance explained by:**				
Variable	Data	Incomplete information	Full information	Incomplete information			Full information	
				ε_z	ε_g	ε_h	ε_z	ε_g
$\Delta \ln Y_t$	0.94	0.82	1.02	3.1	0.7	4.2	25.3	14.7
$\Delta \ln C_t$	0.75	0.70	1.24	1.2	1.5	21.5	39.0	36.5
$\Delta \ln I_t$	2.90	3.15	2.73	61.3	5.4	3.8	17.3	43.5
$\Delta \ln NX/Y_t$	1.28	1.44	3.29	3.1	19.4	39.0	45.5	47.2

The intuition for this result comes from the fact that, under full information, transitory terms of trade shocks cause large changes in the timing of production and consumption, but comparatively modest changes in investment. If agents expect that an increase in the terms of trade will be temporary, they will work and produce more to maximise income while prices are high. In contrast, permanent shifts in the terms of trade induce smaller intertemporal changes in production and consumption.[15] However, when agents are unable to observe the persistence of shocks, they react more cautiously to temporary terms of trade shocks. As the measured variance of these transitory shocks is high, this caution reduces macroeconomic volatility. The increase in the volatility of investment in the incomplete information case is largely due to the existence of capital adjustment costs in the model. Under full information, agents are unwilling to pay these costs in response to temporary shocks and will choose a smooth path for investment in response to permanent shocks. In contrast, imperfectly informed agents will invest more during transitory terms of trade booms and, after initially under-investing during the early stages of persistent booms, will increase investment rapidly when commodity prices remain high.

Table 3 also shows the proportion of the variance of each of the variables explained by terms of trade shocks.[16] Under incomplete information, terms of trade shocks explain a relatively modest proportion of the variance of output and consumption, but a large proportion of the variance of investment and the trade balance. Terms of trade noise shocks also account for a large portion of the variance of consumption and the trade balance. In the absence of noise shocks, the terms of trade becomes a much more important driver of the variances of output growth and consumption and a smaller driver of investment. The contribution of terms of trade shocks to the variance of the trade balance also increases. There is also a change in the relative contribution of transitory and permanent terms of trade shocks, with the relative contribution of permanent shocks to investment increasing and to the trade balance decreasing.

These results help to reconcile the conflicting results of Broda (2004) and Mendoza (1995) regarding the importance of terms of trade disturbances as a source of macroeconomic fluctuations. The baseline results correspond to Broda's finding that terms of trade shocks explain a relatively modest proportion of output

15 Permanent shifts may, however, cause changes in the sectoral composition of output, for example between the tradeables and non-tradeables sectors or among tradeable industries. Examining these changes is not the focus of this paper and is left for future research.

16 The remainder of the variance not explained by terms of trade shocks is explained by productivity.

volatility. In contrast, the results under the assumption of full information are much closer to those in Mendoza, who also assumes that agents have full information about the persistence of terms of trade shocks.

5. What Shocks Have Occurred?

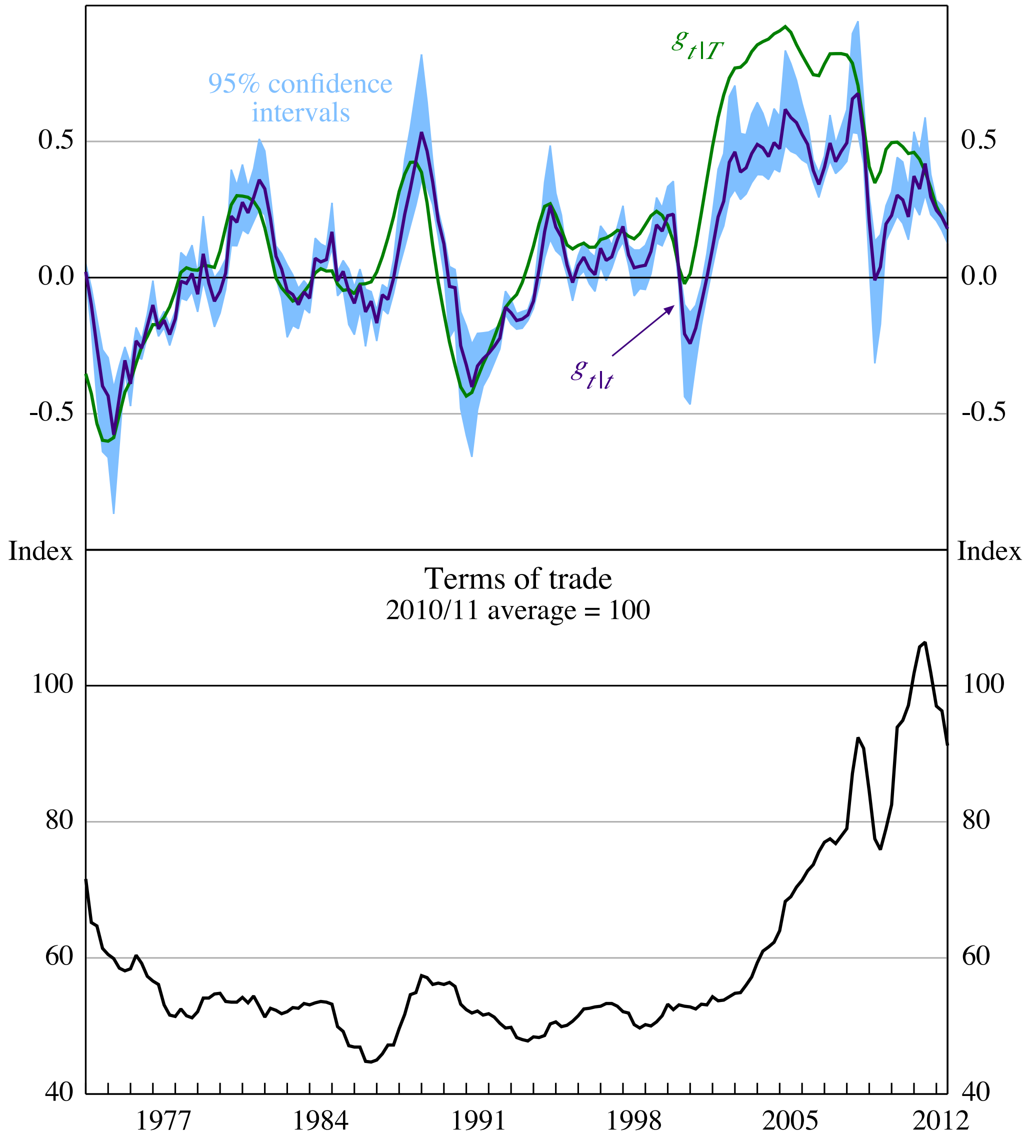
An advantage of estimating a structural model is that it sheds light on whether observed movements in the terms of trade have reflected changes in its transitory or persistent components. In this section, I discuss these estimates.

Figure 8 plots a series of estimates for the permanent component of the Australian terms of trade. The first shows the median estimate of $g_{t|T}$, derived by applying the Kalman smoother to the posterior distribution of the parameters. This is the model's estimate of the permanent component of terms of trade shocks, calculated using all information in the sample. In contrast, the second corresponds to the median estimate $g_{t|t}$. This is the model's estimate of agents' real time beliefs about the permanent component of terms of trade shocks. The shaded area represents the 95 per cent confidence interval for this value.

For the first part of the sample, the two lines move together fairly closely, although the real-time estimates appear to lag the full sample estimates slightly. That is, the model suggests that the beliefs that agents held about the permanent component of the terms of trade, formed given the information available to them at the time, were fairly close to what the model now suggests, given the full sample of data. The slight lag reflects the informational frictions in the model, which mean that agents generally only perceive changes in the long-run trend of the terms of trade after these changes have occurred. Nevertheless, agents' mistakes appear for the most part to have been fairly short-lived.

The pattern in the 2000s is different. Throughout this decade, the model suggests that agents' estimates of the permanent component of the terms of trade were systematically lower than what the model now suggests was the case. This is consistent with the terms of trade forecasts presented in Figure 2, and suggests that throughout this episode agents attributed more of the run-up in the terms of trade to its temporary component than was actually the case.

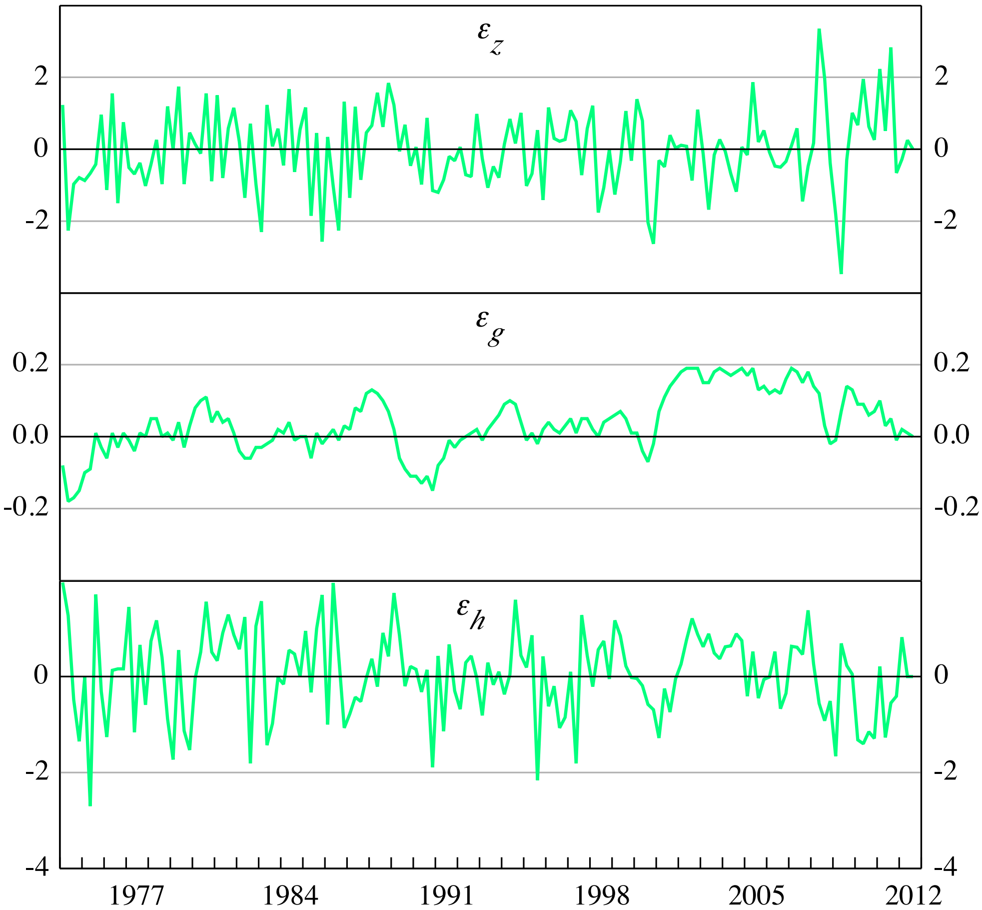
Figure 8: Persistent Component of Terms of Trade



Sources: ABS; author's calculations

One can also use the Kalman smoother to derive estimates of the time series of structural shocks to the terms of trade. These are shown in Figure 9. Of particular interest is the sequence of large permanent shocks to the terms of trade in the 2000s. The model also estimates that there were large negative shocks to the permanent component of the terms of trade in the mid 1970s and late 1980s and, to a lesser extent, around the year 2000. It is also interesting to note that the model suggests that there were large transitory shocks to the terms of trade in 2007/08 – that is, at least part of the large increases in the terms of trade at this time were transitory. Moreover, the model attributes much of the recovery in the terms of trade after 2009 to a series of positive transitory shocks.

Figure 9: Terms of Trade Shocks



6. Robustness Checks

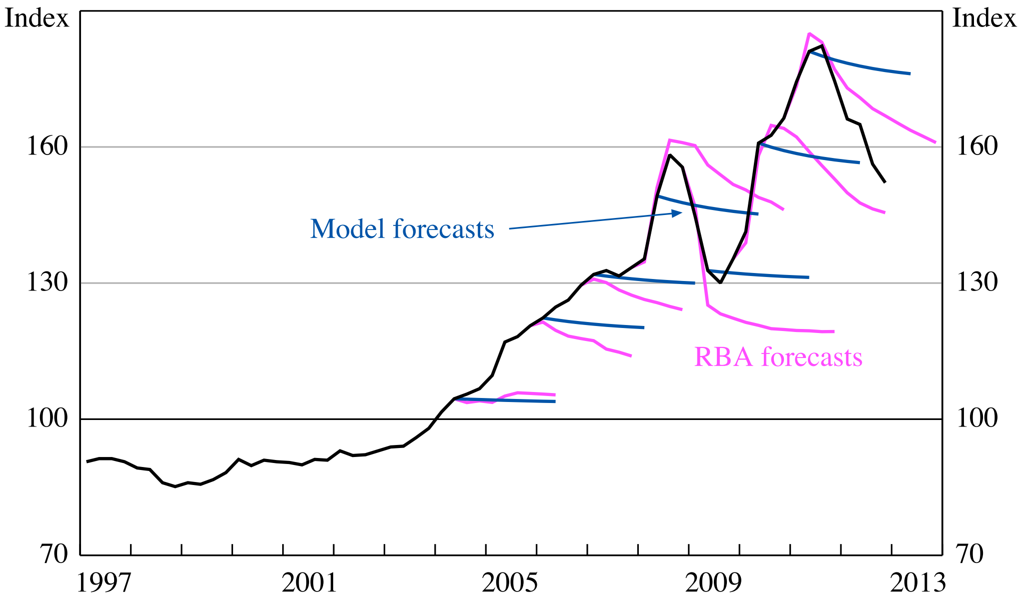
In this section, I demonstrate the usefulness of the model's forecasts for the evolution of the terms of trade by comparing them to real-world forecasts. I also compare the results presented in Section 4 to those from a model estimated under the assumption that agents have full information.

6.1 How Credible are the Model's Terms of Trade Forecasts?

The forecasting process for the terms of trade in the model is extremely simple. One might be concerned that the large noise shocks that the model implies are the result of excluding other sources of information that agents might use to forecast the terms of trade. To examine this, for each draw from the posterior distribution, I use the Kalman filter to back out the forecasts that agents in the model would have made for the evolution of the terms of trade, given their beliefs. Figure 10 shows the median forecasts at various times over the past decade. It appears that the model's forecasts are reasonably close to those produced by the RBA (as shown in Figure 2), which are themselves close to those produced by other government and private sector forecasts. This suggests that the model's forecasting process is a reasonable approximation to that used by real-world agents and that the results of the paper are not driven by artificially constraining the information sets of agents in the model.

Figure 10: Terms of Trade

2003/04 average = 100



Sources: ABS; RBA; author's calculations

6.2 Comparison to Full Information Model

As a further exercise, I compare the results of the incomplete information model presented in this paper to results from a model estimated assuming that agents have full information about the persistence of terms of trade shocks. For this exercise, I re-estimate the model in Section 2, using the same priors, but restrict the standard deviation of terms of trade noise shocks, σ_h, to equal zero. The estimation results are presented in Appendix C.

The parameter estimates relating to the terms of trade processes differ from the incomplete information model in several respects. In particular, the estimated persistence of the transitory terms of trade shock is considerably higher in the full information model, while the persistence of the permanent terms of trade shock is lower. The estimated magnitude of the permanent terms of trade shock is also somewhat smaller in the full information model. In contrast, the parameter estimates relating to the productivity shocks and capital adjustment costs are almost identical between the two models. The log marginal density for this model, computed using the Geweke (1999) modified harmonic mean estimator, is lower than in the incomplete information model, suggesting that the latter has better relative model fit.

As a further test of the relative merits of the incomplete and full information models, I examine their ability to replicate the dynamic interactions between the terms of trade and domestic macroeconomic variables that we see in the data. To do this, I first estimate a vector autoregression (VAR) using HP-filtered Australian data over the period 1973:Q1–2012:Q2 of the form:

$$AY_t = v + B(L)Y_t + u_t \tag{37}$$

where $Y_t' = (tot, y, c, i, nx/y)$ is a vector of stationary endogenous variables, v is a vector of constants, $u_t' = \left(u^{tot}, u^{y}, u^{c}, u^{i}, u^{nx/y}\right)$ is an error vector, A is a matrix, $B(L)$ is a matrix polynomial in the lag operator and $var(u_t) = \Omega$. I restrict the matrices A and $B(L)$ so that the domestic variables do not affect the terms of trade, either contemporaneously or with a lag. This is consistent with the assumption throughout this paper that the terms of trade is exogenous with respect to domestic economic developments. Having estimated the VAR, I calculate the impulse response of the domestic variables to a one standard deviation shock to the terms of trade. As discussed in Section 1, under the null hypothesis of incomplete information about the persistence of terms of trade shocks, it is not possible to give a structural interpretation to this shock. However, the exercise still provides a useful summary of the dynamic reduced form relationships between empirical variables.

I then simulate 156 observations of synthetic data (equivalent to the sample size of the empirical data) for the terms of trade, output, consumption, investment and the trade balance-to-GDP ratio using the partial and full information models, in each case taking a random draw from the posterior parameter distribution to simulate the model. For each model, I estimate a VAR as in Equation (37) using the synthetic data and calculate impulse responses to a terms of trade shock. I then repeat this process 10 000 times for each model.

Figure 11 shows the responses to a one standard deviation terms of trade shock produced by the VAR, as well as the median and one standard deviation confidence bands of the theoretical responses generated by the incomplete information model. The theoretical response of GDP to the shock in the model is almost identical to the response seen in the data. The responses of investment and net exports in the model also display a similar pattern to those seen in the data, although the magnitude of the model's investment response is too large while the net exports response is too weak. The consumption response is weaker in the model than in the data, although after the second period the data responses lie within the one standard deviation confidence bands of the model responses.

Figure 11: Impulse Response Function

Response to a one standard deviation terms of trade shock

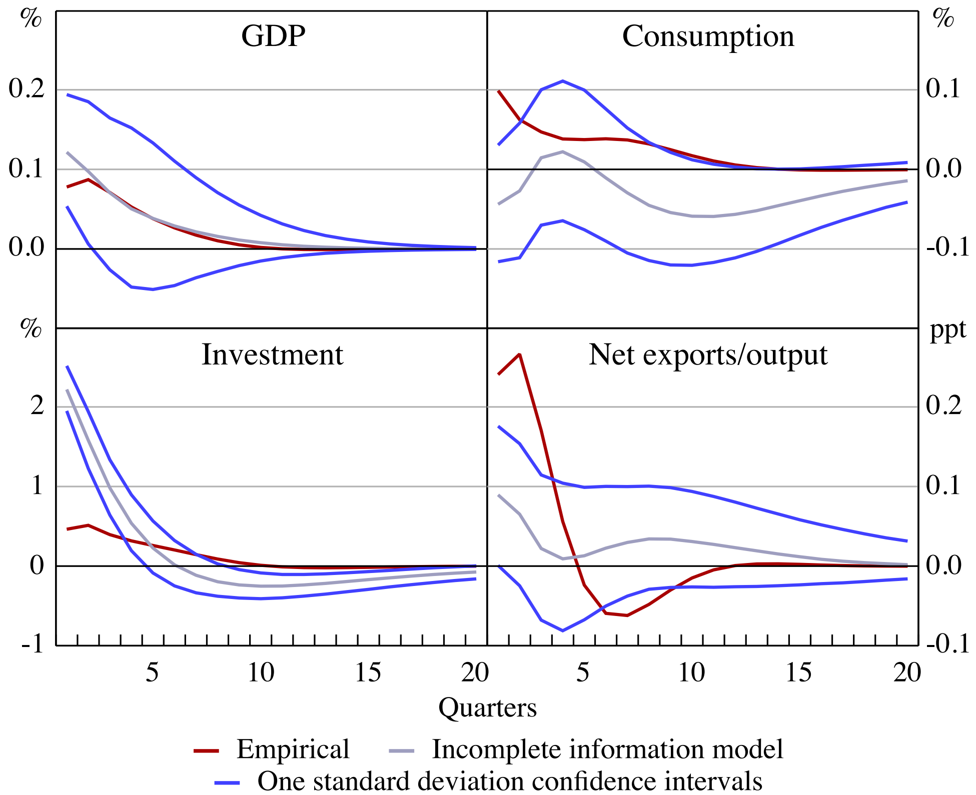
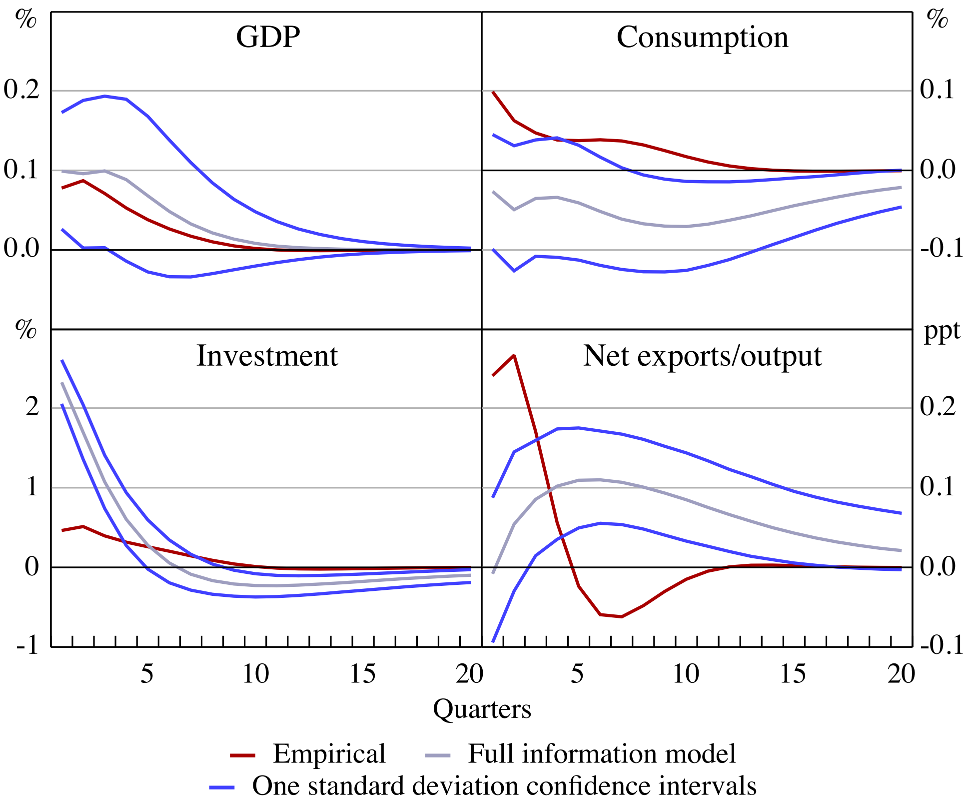


Figure 12 shows the responses for the full information model. Once again, the GDP response produced by the model is extremely close to that in the data, while the model's investment response is similar in pattern but too large in magnitude compared to the data. However, the full information model implies a significantly weaker consumption response than is observed in the data. And the pattern of the model's net exports response also differs considerably from that seen in the data, responding too little immediately following the shock and too much later on. In sum, these results suggest that the incomplete information model comes closer to reproducing the dynamic relationships between the terms of trade and other macroeconomic variables that we see in the data than does the full information model.

Figure 12: Impulse Response Function

Response to a one standard deviation terms of trade shock



7. Conclusion

This paper has examined the extent to which agents are uncertain about the persistence of terms of trade shocks and described the effect of these shocks when agents have incomplete information. The results suggest that agents find it difficult to identify whether terms of trade disturbances are permanent or transitory in real time. In fact, the empirical results suggest that agent's expectations about the evolution of the terms of trade are largely invariant to the type of shock that hits the economy. A corollary of this result is that we should not expect households or firms to respond to terms of trade shocks in a first-best manner. Instead, at least in the model presented in this paper, in response to a temporary positive shock agents will consume more and produce less than in a full information environment. And, in response to a permanent shock, agents will consume less and produce more than they would if they had full information. But despite the fact that agents make mistakes in identifying the source of terms of trade shocks, incomplete information about these shocks does not increase macroeconomic volatility.

A number of extensions to this work deserve consideration. First, it may be worthwhile to replicate the estimation for other small open economies, including those featured in Figure 1. In particular, it would be interesting to learn whether the extent of incomplete information, and the effect of terms of trade noise shocks, differs between an advanced economy like Australia and a developing small open economy like Brazil or Chile. It may also be interesting to extend the model to include a non-traded sector and to incorporate nominal rigidities, including sticky wages and prices into the model. The former extension might reveal whether incomplete information about relative prices in the tradeable sector could have spillover effects to the rest of the economy. In particular, it may be interesting to see whether incomplete information about terms of trade movements could cause 'Dutch disease' type effects on the non-commodity sectors of the economy. Finally, the inclusion of nominal rigidities could allow one to examine how uncertainty about the persistence of terms of trade shocks affects monetary policy.

Appendix A: Data Sources and Definitions

The dataset spans the quarters 1973:Q1 to 2012:Q2. The start date is chosen because quarterly estimates of the Australian population, published by the Australian Bureau of Statistics (ABS), are not available before that date.

Consumption

Final consumption expenditure, expressed in chain volume terms and seasonally adjusted divided by population ('Australian National Accounts: National Income, Expenditure and Product', ABS Cat No 5206.0)

Gross domestic product

Real gross domestic product, expressed in chain volume terms and seasonally adjusted divided by population (ABS Cat No 5206.0)

Investment

Gross fixed capital formation, expressed in chain volume terms and seasonally adjusted divided by population (ABS Cat No 5206.0)

Population

Total resident population ('Australian Demographic Statistics', ABS Cat No 3101.0)

Net exports

Ratio of nominal net exports to nominal gross domestic product (ABS Cat No 5206.0)

Terms of trade

Australia: terms of trade index, seasonally adjusted (ABS Cat No 5206.0)

For the other economies in Figure 1, with the exception of Canada, terms of trade data was sourced from national statistical agencies. I retrieved data for Canada from the OECD. For Brazil, New Zealand and South Africa, published terms of trade indices were used. For Canada, I constructed a terms of trade index by dividing the exports of goods and services deflator by the imports of goods and services deflator. For Mexico, the terms of trade index was constructed by dividing the export price index by the import price index. The raw data for Canada, New Zealand and South Africa were quarterly. For Brazil and Mexico, I constructed a quarterly series using quarterly averages of monthly data. Sources for the individual countries are:

Brazil: Institute for Applied Economic Research (www.ipeadata.gov.br)

Canada: OECD (www.oecd.org)

Mexico: Instituto Nacional de Estadística y Geografia (www.inegi.org.mx)

New Zealand: Statistics New Zealand (www.stats.govt.nz)

South Africa: South African Reserve Bank (www.resbank.co.za).

Appendix B: Steady State and Log-linearised Equations

B.1 Steady State

Before deriving the non-stochastic steady state of the model, I normalise the following variables to be in a form that is stationary: $\tilde{Y}_t = Y_t / \left(X_{t-1} S_{t-1}^{\frac{\alpha}{1-\alpha}} \right)$, $\tilde{C}_t = C_t / \left(X_{t-1} S_{t-1}^{\frac{\alpha+\eta(1-\alpha)}{1-\alpha}} \right)$, $\tilde{K}_t = K_t / \left(X_{t-1} S_{t-1}^{\frac{1}{1-\alpha}} \right)$, $\tilde{B}_t = B_t / \left(X_{t-1} S_{t-1}^{\frac{1}{1-\alpha}} \right)$ and $\tilde{I}_t = I_t / \left(X_{t-1} S_{t-1}^{\frac{1}{1-\alpha}} \right)$, where a ˜ denotes a stationary variable. Using these normalisations, the non-stochastic steady state of the model is given by:

$$
\begin{aligned}
\tilde{Y} &= \tilde{K}^{\alpha} \left(\mu N^{1-\alpha} \right) && \text{(B1)} \\
(\mu + \delta - 1) \tilde{K} &= \tilde{I} && \text{(B2)} \\
\frac{1}{Q} &= 1 + r^{*} && \text{(B3)} \\
\tilde{C} A_L N^{1+\varphi} &= (1-\alpha) \tilde{Y} && \text{(B4)} \\
\alpha \frac{\tilde{Y}}{\tilde{K}} &= \frac{\mu}{\beta} + \delta - 1 && \text{(B5)} \\
Q &= \frac{\beta}{\mu} && \text{(B6)} \\
(Q\mu - 1) \tilde{B} + \tilde{C} + \tilde{I} &= \tilde{Y} && \text{(B7)}
\end{aligned}
$$

where I have replaced the wage rate and the rate of return on capital in the solution to the consumers' problem with the marginal products of labour and capital from the firm's profit maximisation conditions, given by Equations (5) and (6).

B.2 Log-linearised Equilibrium Conditions

To solve the model, I log-linearise the model around its non-stochastic steady state derived in the previous section. The log-linearised equilibrium conditions are:

Production:

$$\tilde{y}_t = \alpha \left(\tilde{k}_t + m_t \right) + (1-\alpha) (a_t + n_t) \tag{B8}$$

Intratemporal optimisation:

$$(1+\varphi)\,n_t+\tilde{c}_t=\tilde{y}_t+\eta\Delta s_t \tag{B9}$$

Bond market Euler equation:

$$\begin{aligned}\tilde{c}_t-q_t &= E_t\left\{\tilde{c}_{t+1}\right\}+\omega\Delta s_t \\ &\quad +(1-\eta)\,E_t\left\{\Delta s_{t+1}\right\}+m_t\end{aligned} \tag{B10}$$

Capital Euler equation:

$$\begin{aligned}\tilde{c}_t+\phi\mu\tilde{k}_t &= E_t\left\{\tilde{c}_{t+1}\right\}+(1+\phi\mu)\,m_t-\left(1-\eta-\frac{1+\phi\mu}{1-\alpha}\right)\Delta s_t \\ &\quad -\frac{\beta\alpha\hat{Y}}{\mu\hat{K}}E_t\left\{\tilde{y}_{t+1}\right\}+\left(1-\eta-\frac{\beta\phi\mu}{1-\alpha}-\frac{\beta\alpha\bar{Y}}{\mu\bar{K}}\right)E_t\left\{\Delta s_{t+1}\right\} \\ &\quad +\left(\phi\mu\,(1+\beta)+\frac{\beta\alpha\bar{Y}}{\mu\bar{K}}\right)\tilde{k}_{t+1}-\beta\phi\mu E_t\left\{\tilde{k}_{t+2}\right\} \\ &\quad -\beta\phi\mu E_t\left\{m_{t+1}\right\}\end{aligned} \tag{B11}$$

Capital accumulation:

$$\mu\tilde{k}_{t+1}+\mu m_t+\frac{\mu\Delta s_t}{1-\alpha}=(1-\delta)\,\tilde{k}_t+\frac{\tilde{I}}{\tilde{K}}\tilde{i}_t \tag{B12}$$

Risk-free rate of return:

$$q_t=\psi\tilde{b}_{t+1} \tag{B13}$$

Current account:

$$\begin{aligned}&\tilde{C}\,(\tilde{c}_t+(1-\eta)\,\Delta s_t)+\tilde{I}\tilde{i}_t \\ &+Q\tilde{B}\mu\left(q_t+\tilde{b}_{t+1}+m_t+\tfrac{\Delta s_t}{1-\alpha}\right)=\tilde{Y}\,(\tilde{y}_t+\Delta s_t)+\tilde{B}\tilde{b}_t\end{aligned} \tag{B14}$$

where lower case letters denote a variable's log-deviation from its steady state, that is $d_t=\ln D_t-\ln D^*$.

Appendix C: Estimation Results – Full Information Model

Table C1: Prior and Posterior Distributions – Full Information Model

		Prior		Posterior			
Parameter	**Distribution**	Mean	SD	Mode	Mean	5%	95%
Exogenous processes – AR(1) coefficients							
ρ_a	Beta	0.80	0.10	0.98	0.98	0.97	0.99
ρ_m	Beta	0.80	0.10	0.59	0.59	0.38	0.79
ρ_z	Beta	0.80	0.10	0.98	0.98	0.98	0.99
ρ_g	Beta	0.80	0.10	0.74	0.70	0.57	0.81
Exogenous processes – standard deviations ($\times 10^{-2}$)							
σ_a	Inv Gamma	0.50	1.00	0.70	0.70	0.61	0.79
σ_m	Inv Gamma	0.50	1.00	0.15	0.16	0.10	0.24
σ_z	Inv Gamma	0.50	1.00	1.20	1.20	1.06	1.40
σ_g	Inv Gamma	0.50	1.00	0.13	0.15	0.09	0.21
Other parameters							
ϕ	Trunc Normal	7.50	2.50	10.50	10.47	8.41	12.62
Log marginal density		–1 782.3					

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