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**Ageing, Retirement  
and Savings: A General  
Equilibrium Analysis**

*Mariano Kulish,  
Kathryn Smith and  
Christopher Kent*

RDP 2006-06

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# **AGEING, RETIREMENT AND SAVINGS: A GENERAL EQUILIBRIUM ANALYSIS**

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## **Abstract**

This paper studies the macroeconomic consequences of ageing in an overlapping-generations model with endogenous retirement. We study the behaviour of the economy when population ageing is driven by movements in fertility, changes in longevity, and a combination of both. To gauge the economic implications of these demographic changes we calibrate the model to match key features of the Australian economy. With either a fall in fertility or a rise in longevity, population ageing increases capital intensity in the long run. When fertility and longevity operate together, the increase in capital intensity is more than additive, and the share of life spent in retirement stays roughly constant. The dynamic response of the economy is sensitive to the relative strength of the two factors that drive ageing.

JEL Classification Numbers: D91, J13, J26

Keywords: baby boom, endogenous retirement, longevity, OLG (overlapping generations)

# Table of Contents

1.	Introduction	1
2.	Related Literature	4
3.	The Model	7
3.1	Households	8
3.2	Firms	11
3.3	Aggregation and Equilibrium	11
4.	Calibration	12
5.	Results	16
5.1	Permanent Fall in Fertility	16
5.2	Baby Boom and Bust	20
5.3	Increase in Longevity	21
5.4	Combined Change in Fertility and Longevity	23
5.5	Sensitivity Analysis	24
6.	Conclusion	28
	Appendix A: Technical Appendix	29
A.1	The Household Problem	29
A.2	The Solution Method	31
	Appendix B: Data Sources for Calibration	34
	References	35

# AGEING, RETIREMENT AND SAVINGS: A GENERAL EQUILIBRIUM ANALYSIS

Mariano Kulish, Kathryn Smith and Christopher Kent

## 1. Introduction

It is well known that there will be significant changes in the demographic structure of the world over coming decades. It is also well known that these demographic changes will have consequences for many aspects of the economy. Perhaps less well appreciated is that the economic consequences of ageing depend on the strength of the two factors that drive the ageing process: falling fertility and rising longevity. There are numerous studies addressing the economic effects of ageing.<sup>1</sup> Bryant (2004) discusses some differences between declining fertility and increasing longevity in terms of their macroeconomic effects. However, no studies (to the best of our knowledge) have analysed the combined implications of changes in fertility and longevity in a general equilibrium model with endogenous retirement decisions.

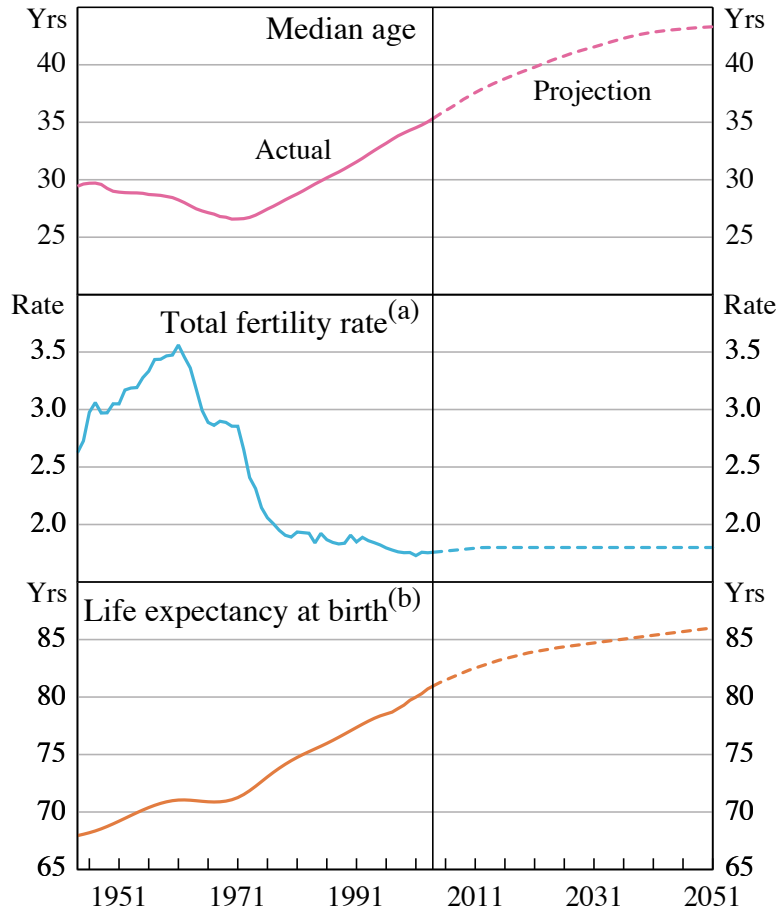
While it is difficult to get a precise measure of the relative contribution of fertility and longevity to population ageing, we can get a rough idea by examining projections derived from demographic models. Figure 1 shows a projected rise in the median age for Australia's population of about 13 years from the start of the baby boom<sup>2</sup> to 2051.<sup>3</sup> The elevated level of the total fertility rate over the two decades or so of the baby boom actually helped *delay* population ageing in Australia, while one of the genuine causes of ageing is the lower levels of the fertility rate that prevailed from 1965 onwards. If fertility had not declined in this way, the Productivity Commission's model would have projected a rise in the median age of around 4 years – suggesting that a sizeable portion of the estimated

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<sup>1</sup> See, for example, Kotlikoff (1989), Yoo (1997), Miles (1999), Brooks (2002), Bloom, Canning and Graham (2002), Bloom, Canning and Moore (2004) and Poterba (2004).

<sup>2</sup> This is the period 1946–1965 when the level of the total Australian fertility rate is generally considered to have been very high relative to its own past (Productivity Commission 2005).

<sup>3</sup> These projections were derived from data and models provided by the Productivity Commission.

**Figure 1: Median Age, Fertility and Longevity – Australia**

Notes: (a) Babies per woman  
 (b) Average of men and women  
 Source: Productivity Commission

rise of 13 years over 1946–2051 can be attributed to falling fertility. Somewhat less of the rise in the median age can be attributed to the rise in longevity. If longevity had been constant throughout the period, the demographic model would have projected a rise in the median age of about seven years.<sup>4</sup> In short, both the drop in fertility and the rise in longevity will contribute significantly to ageing, but in Australia's case, the effect of the former is projected to be somewhat larger than the latter.

<sup>4</sup> Notice that the sum of the implied changes in the median age due to longevity and fertility do not sum to the projected change in the median age of 13 years. The difference arises because the two demographic factors interact. Also, any estimate that isolates the contribution to ageing from individual factors is sensitive to the assumptions about the demographic variables over the forecasting period.

These two demographic factors have potentially quite different effects on the economy, largely because of their different implications for labour supply and demand. In particular, a decline in fertility, other things equal, reduces the size of the working-age population relative to the stock of capital, putting upward pressure on wages. In contrast, rising longevity – to the extent that it is also accompanied by improvements in health (at all ages) – encourages individuals to delay retirement, increasing the size of the working-age population and putting downward pressure on wages.

Of course, these immediate effects are not the full story. In the case of a drop in fertility, we need to account for adjustments in wages and rates of return to capital when determining the ultimate impact on labour supply and capital accumulation. And in the case of a rise in longevity, the initial downward impact that a longer lifespan has on the wage rate could even be reversed in the long run. The key feature of longevity is that, in addition to providing extra labour (with healthier workers able to work for longer), it can also add substantially to labour demand in the long run. This is because longevity is likely to raise the absolute number of years a person spends in retirement. To finance this longer retirement, individuals need to accumulate more wealth during their working years. These extra savings add to the capital stock, raising the demand for labour. So it is the potential for a relatively larger capital stock that means increased longevity could lead to higher real wages in the long run.

Whether the changes in labour supply or demand contribute most to the change in wages as a result of increased longevity will depend on a range of factors, including the extent of changes in fertility and longevity, the relationships between longevity and health, and preferences over consumption and leisure. This paper incorporates these factors in a general equilibrium model. Our goal is to investigate responses to ageing in an economy populated by rational, fully informed individuals, in which there is no taxation and no government intervention. In this way, our results can be seen as suggestive of ‘first-best’ responses to ageing.

To study how combined changes in longevity and fertility affect the economy, we augment an otherwise standard overlapping generations (OLG) model by incorporating endogenous retirement. We study the response of the economy to changes in fertility, longevity, and the two combined. We calibrate the model’s

parameters to match key features of the Australian economy and find that ageing, driven by either falling fertility or increased longevity, ultimately raises the capital-to-labour ratio, increases wages, decreases rates of returns, and increases income and consumption per capita. However, the transition to this new steady state is more complex than the long run suggests, with wages falling initially (and rates of return initially rising) in the cases where a rise in longevity contributes to population ageing.

The rest of the paper is structured as follows. Section 2 reviews related literature. Section 3 describes the OLG model. Section 4 discusses the calibration of the model. Section 5 analyses the results and Section 6 concludes.

## **2. Related Literature**

Our work relates to two strands of the literature. One of these focuses on the impact of changes in fertility (baby ‘booms’ and/or ‘busts’) on key aspects of the economy.<sup>5</sup> The other strand concentrates on the economic effects of increased longevity. We discuss some central features of both of these strands in turn.

The standard workhorse for economic questions related to demographic changes is the OLG model, which distinguishes between individuals according to their stage of life. At the heart of the OLG model is the life-cycle hypothesis, which states that individuals prefer to smooth consumption over their lifetime. This implies that saving rates will typically be low early in life when income is low, rise as individuals move through their peak earning years, then decline and become negative in retirement as people draw down on accumulated assets.

Poterba (2004) offers a simple starting point for understanding the effect that changes in fertility rates have on the economy.<sup>6</sup> In this model, when the baby boom generation retires and goes to sell their assets to the smaller subsequent generation, the price of the asset falls. Hence the baby boom generation experiences a lower

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<sup>5</sup> Economic variables also affect demographic variables. See Zhang and Zhang (2005) and the references therein for models in which demographic variables are to some extent endogenously determined.

<sup>6</sup> Poterba’s model assumes (among other things) two generations, constant saving rates, and an asset in fixed supply.



return on their asset holdings in their retirement years than do previous and subsequent generations.

While Poterba's model highlights an important link between asset prices or rates of return and the age structure of the population, it ignores several relevant complications. For example, we might think that optimising behaviour, a variable supply of capital, bequests, portfolio choices over risky assets, borrowing constraints, international capital flows, endogenous retirement, and pension schemes (to name a few) might be important in determining the relationship between fertility rates and the economy. A number of studies have incorporated some of these factors into stylised models to explore the effects of changes in fertility on asset markets.

Abel (2003) relaxes simple assumptions about capital supply and bequests. He shows that, in an OLG model where the supply of capital is variable, a baby boom reduces the rate of return relative to what it would have been in a steady state with a constant birth rate. He shows that those born into a baby boom cohort experience less attractive returns on their capital than those born at other times. When individuals are allowed to leave bequests, the basic results still hold, although these are sensitive to the specification of the bequest motive. Bohn (2006) presents a dynastic OLG model where bequests are endogenous. He shows that ageing can reduce bequests, which works to stabilise the capital-to-labour ratio and hence the rate of return.

Yoo (1997) also experiments with a variable capital supply. He calibrates an OLG model with inelastic labour supply and exogenous retirement in which consumers live for 55 periods and work for 45. He finds that a rise in the fertility rate, followed by a decline, initially raises and then lowers asset prices. The effects are sensitive to whether or not capital is in fixed supply. The price of the asset rises 35 per cent when capital is in fixed supply and 15 per cent when the supply is variable.

Brooks (2002) augments a four-period OLG economy with a portfolio decision over risky and riskless assets. The four generations alive at any one time are children, young workers, old workers and retirees. Agents supply labour inelastically and retire in their last period. The model is calibrated so that older individuals prefer to hold less of the risky asset. A simulated baby boom affects the equilibrium level of both risky and riskless asset returns, but the returns on the risky asset change by half as much as the riskless return. Overall, baby

boomers earn returns on retirement savings about 100 basis points below current returns, but in terms of lifetime utility they are slightly better off than other cohorts. This reflects the fact that, by short-selling the riskless asset, baby-boom workers are able to supply capital as well as labour. This offsets the movement that would otherwise occur in relative factor prices if the strategy of short-selling the riskless asset were not available. Constantinides, Donaldson and Mehra (2002) show that imposing borrowing constraints on the young magnifies the effect that fertility changes have on capital markets by preventing these kinds of short-selling strategies.

Geanakoplos, Magill and Quinzii (2002) also incorporate a portfolio decision over a risky and riskless asset. They study a calibrated three-period OLG endowment economy to investigate the relationship between fertility changes and the equity market. The main finding is that actual equity market movements in the United States are two to three times larger than their demographic model can explain.

Börsch-Supan, Ludwig and Winter (2003) use a multi-country OLG model to study the effects of ageing on international capital flows. Their long-term demographic projections for several world regions suggest that capital flows from fast-ageing countries to the rest of the world are likely to be substantial. While factors of production could move to mitigate some of the effects of ageing, closed economy analyses remain valid precisely because ageing is a global phenomenon.<sup>7</sup>

A second strand of the ageing literature has studied the economic impacts of rising life expectancy. Kotlikoff (1989) uses a general equilibrium model with exogenous retirement to investigate the effect of rising life expectancy on key macroeconomic variables such as output per capita and capital intensity. He finds that proportional increases in the age of retirement and the age of death raise capital intensity and output per capita.

Recently, Bloom *et al* (2004) have studied the effects of increases in longevity on optimal retirement and saving decisions in a partial equilibrium model. Retirement is motivated by an increasing disutility from work throughout life, which is interpreted as capturing individuals' age-specific health status. They show that increases in longevity reduce saving rates and result in a less-than-proportional

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<sup>7</sup> See also Börsch-Supan (2005), which examines how different speeds of ageing in different regions affect trade and factor movements.

increase in the retirement age. These results are driven by the wealth effect from compound interest: a higher lifespan means that individuals' savings earn the same rate of interest for longer. This increases lifetime income and raises consumption of both market goods and leisure. However, these results might not necessarily hold in a general equilibrium setting where the return to capital is endogenous.

Kotlikoff, Smetters and Walliser (2001) investigate different scenarios for life expectancy in an elaborate version of the Auerbach and Kotlikoff (1987) model which includes intragenerational heterogeneity. The authors study the potential of ageing-related capital deepening to lessen ageing-related fiscal pressure in the US, and investigate the fiscal implications of a number of demographic changes including alternative life expectancy scenarios. They find that the need to save for longer retirement stimulates capital accumulation. However, this additional capital increases labour demand and leaves the capital-to-labour ratio unchanged in the long run.

Finally, it is worth noting that calibrated general equilibrium studies can be a valuable tool for assessing the effects of ageing on the macroeconomy. This is because of the difficulties suffered by empirical studies in this area. Regardless of whether macro- or microeconomic data are used, the empirical findings appear to be sensitive to both the definition of demographic variables and the specification of the econometric model. For example, Bergantino (1998) finds that the age structure of the population has a significant effect on post-war equity price fluctuations in the US, while Poterba (2001) finds very limited support for this kind of relationship.<sup>8</sup>

### **3. The Model**

At any point in time, the economy is populated by a finite number of generations and a representative profit-maximising firm. Every period the oldest generation dies and one new generation enters the economy. Households live for  $T$  periods, choose to work for  $T' \leq T$ , consume, and supply capital and labour to competitive factor markets. Agents have perfect foresight and there is no government or foreign sector.

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<sup>8</sup> For an overview of the empirical literature, see Miles (1999), Poterba (2004) and the references therein.

### 3.1 Households

Individuals within a generation are equal in every respect. Agents start and end life with no wealth, as there are no bequests and no uncertainty about the time of death. Formally, an agent born at time  $t$  maximises lifetime utility

$$\max \sum_{s=1}^T \beta^{s-1} \left( \frac{1}{1-\rho} c_{s,t+s-1}^{1-\rho} + v(s, T) \frac{1}{1-\rho} l_{s,t+s-1}^{1-\rho} \right) \quad (1)$$

by choosing sequences of consumption and leisure,  $\{c_{s,t+s-1}, l_{s,t+s-1}\}_{s=1}^T$  subject to a period budget constraint of the form

$$a_{s+1,t+s} = (1 - l_{s,t+s-1})e_s w_{t+s-1} + R_{t+s-1} a_{s,t+s-1} - c_{s,t+s-1} \quad (2)$$

and period inequality constraints on leisure of the form

$$l_{s,t+s-1} \leq 1 \quad (3)$$

as well as initial and terminal conditions on individual wealth.<sup>9</sup> In the equations above:  $c_{s,t}$  is time  $t$  consumption of an agent  $s$  years old;  $a_{s,t}$  is the beginning of period  $t$  stock of wealth of an agent  $s$  years old;  $R_t = 1 + r_t - \delta$  is the rate of return to capital between  $t$  and  $t + 1$ , where  $\delta \in (0, 1)$  is the depreciation rate of the capital stock between  $t$  and  $t + 1$ ;  $e_s$  is an age-specific constant that captures differences in human capital or productivity across cohorts;  $\beta \in (0, 1)$  is the household's subjective discount factor; and the parameter  $\rho > 0$  governs the degree of inter- and intra-temporal substitution.

As in Bloom *et al* (2004), the disutility of working depends on an individual's health status, which in turn is negatively related to age and is captured in the function  $v(s, T)$ . Consistent with the weight of evidence on this issue (Sickles and Taubman 1986; Fogel 1994, 1997; Costa 1998; Mestdagh and Lambrecht 2003; Cai and Kalb 2004), we assume that increases in life expectancy are associated with improved health status. In particular we assume that the function is given by

$$v(s, T) = \left( b_1 \frac{s}{T} \right) \int_{-\infty}^s \frac{1}{\sqrt{2\pi b_3 T}} \exp \left( -\frac{1}{2} \left( \frac{x - b_2 T}{b_3 T} \right)^2 \right) dx \quad (4)$$

---

<sup>9</sup> The agent is born with no wealth ( $a_{1,t} = 0$ ), and dies with no wealth ( $a_{T+1,t+T} = 0$ ).

where the parameters  $b_1$ ,  $b_2$ , and  $b_3$  are strictly positive. The function  $v$  is the cumulative distribution function of a normal random variable with mean  $b_2T$  and standard deviation  $b_3T$ , scaled by  $b_1\frac{s}{T}$ . We chose this specification for a number of reasons. First,  $v$  is an increasing function of age,  $s$ , and a decreasing function of life expectancy,  $T$ . Therefore as the individual ages, the function magnifies the disutility from work that arises because of deteriorating health. Second,  $v$  is homogeneous of degree zero in  $s$  and  $T$ .<sup>10</sup> This has the important implication that the disutility from work does not depend on absolute age, but rather on an agent's age relative to their lifespan. In other words, the disutility from work of an agent 40 years old with a lifespan of 60 years is equivalent to that of an individual 60 years old with a lifespan of 90 years.<sup>11</sup> Finally, we can choose the mean and standard deviation in  $v$  ( $b_2T$  and  $b_3T$ ) so that agents' labour supply decisions match observed age-specific participation rates.

For an individual that works for the first  $T'$  periods, the Kuhn-Tucker first-order conditions yield a solution of the form

$$c_{s+1,t+s} = (\beta R_{t+s})^{1/\rho} c_{s,t+s-1} \quad s = 1, 2, \dots, T-1 \quad (5)$$

$$l_{s,t+s-1} = \begin{cases} \left( \frac{v(s,T)}{e_s w_{t+s-1}} \right)^{1/\rho} c_{s,t+s-1} & s = 1, 2, \dots, T' \\ 1 & s = T'+1, \dots, T \end{cases} \quad (6)$$

---

<sup>10</sup> One way to ensure that a function is homogeneous of degree zero is if it is the product of two functions that both have this property. In the case of  $v$ , it is easy to see that the term  $b_1\frac{s}{T}$  is homogeneous of degree zero in  $s$  and  $T$ . The other term, the cumulative distribution function of a normal random variable, also has this property. This can be seen by considering that every cumulative normal distribution evaluates to  $\frac{1}{2}$  at its mean, and contains the same amount of probability within any given number of standard deviations as any other cumulative normal distribution. Changing the values of  $s$  and  $T$  by the same proportion would result in the same number of standard deviations as at their original values, and thus the same value of the cumulative density function, which implies homogeneity of degree zero as required.

<sup>11</sup> Bloom *et al* (2004) use the function  $v(s,T) = a \exp(s/T)$ , and assume that the agent chooses an indicator variable that takes the value of one when working and zero when retired. In this way, the agent either works full-time or retires. The problem of directly attaching a function of this form and not using an indicator variable is that labour elasticities would vary unrealistically with age.

where an expression for initial consumption ( $c_{1,t}$ ) can be found in Appendix A. Equation (5) is the Euler equation for consumption. It shows that when the interest rate is equal to the inverse of the discount factor, the consumer desires a flat lifetime consumption path. An even higher rate of interest would give rise to an upward-sloping consumption profile. At a utility maximum, the consumer is unable to gain from feasible shifts of consumption between periods. A one-unit reduction in present consumption lowers lifetime utility by  $c_{s,t}^{-\rho}$ , the marginal utility of present consumption. This saved consumption unit can be converted into  $R_t$  units of consumption in the following period, raising lifetime utility by  $\beta R_t c_{s+1,t+1}^{-\rho}$ . Equation (5) states that at an optimum the agent equates these quantities.

It is worth emphasising that, unlike many OLG models,  $T'$  is not exogenous but must be determined by the individual as part of the solution. The smooth nature of  $v$  allows us to focus on cases where the agent works initially and retires later on because paths for leisure that would imply expected reversals of retirement would not be optimal. However, some agents might reverse their retirement decisions in the presence of unanticipated changes in parameters such as  $T$  and  $e_s$ . For example, in the presence of an unanticipated rise in life expectancy, a retired agent might rejoin the workforce to avoid a drastic decline in consumption over their (now longer) life. Indeed, situations such as this occur in the simulations below.

Equation (6) shows that, other things equal, a reduction in the disutility from work,  $v(s, T)$ , induces the individual to demand less leisure and favour a later retirement. In general equilibrium, there are second-round effects because the change in  $v(s, T)$  affects aggregate labour supply and puts downward pressure on wages. Depending on the relative strength of the income and substitution effects, this might offset some of the move towards later retirement, as the incentives to work are now lower. Equation (6) also shows that, even if there are no changes in individual preferences caused by changes in  $T$ , factors that alter the path of wages and interest rates – such as a baby boom or technological change – could also change retirement decisions.

### 3.2 Firms

There is a single competitive production sector using capital and labour as inputs into a Cobb-Douglas production function with constant returns to scale

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \quad (7)$$

where  $Y_t$ ,  $K_t$ , and  $L_t$  are the aggregate levels of output, capital and efficient labour at time  $t$ , and  $\alpha \in (0, 1)$ . The variable  $A$  captures total factor productivity and is to some degree a scaling constant. However, for a given profile of human capital,  $e_s$ , increases in  $A$  have real effects that go beyond mere changes in the unit of account. As in Auerbach and Kotlikoff (1987), the model lacks a well-defined steady state when  $A$  grows at a constant rate over time.<sup>12</sup>

As a result of profit maximisation we obtain the standard factor-demand curves which can be written in intensive form as

$$r_t = \alpha Ak_t^{\alpha-1} \quad (8)$$

$$w_t = (1 - \alpha)Ak_t^\alpha \quad (9)$$

where  $k_t = K_t/L_t$  is capital per efficient worker.

### 3.3 Aggregation and Equilibrium

If  $N_{s,t}$  denotes the number of individuals  $s$  years old that are alive in period  $t$ , then the total population,  $N_t$ , is simply  $\sum_{s=1}^T N_{s,t}$ . The effective work force at time  $t$  is

$$L_t = \sum_{s=1}^T (1 - l_{s,t}) e_s N_{s,t} \quad (10)$$

---

<sup>12</sup> If  $A$  grows over time, wages will grow over time and the consumption-leisure ratio will trend towards ever-increasing or ever-decreasing labour force participation. Auerbach and Kotlikoff argue that, in the long run, an ever-increasing  $A$  would lead to an absurd result. While the technical issue about an ever-increasing  $A$  cannot be ignored in our model, we would like to emphasise that the model implies that technical improvements reduce the age of retirement. This is important because, even though life expectancy has been growing in most countries, the effective age of retirement has fallen or stayed constant in a number of them.

We assume that cohorts grow at a constant rate  $n$  governed by the law of motion  $N_{s,t} = (1+n)N_{s,t-1}$ .<sup>13</sup> As this growth rate  $n$  determines the relative size of the different cohorts, we interpret it as a fertility parameter. In steady state, where  $T$  and  $n$  are constant, the rates of cohort and population growth will be identical, and the total population would evolve according to  $N_t = (1+n)N_{t-1}$ .

In equilibrium, the supply of capital (the aggregate wealth of agents in period  $t$ ) must be equal to the capital stock that firms demand at  $t$ .

$$K_t = \sum_{s=1}^T N_{s,t} a_{s,t} \quad (11)$$

The dynamics of the economy are governed by the evolution of the aggregate stock of capital. Combining the capital accumulation constraint with the resource constraint of the economy, we obtain the equilibrium law of motion of the economy

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t. \quad (12)$$

## 4. Calibration

We choose values for the parameters so that the model's initial steady state would resemble key features of the current Australian economy. Parameter values (Table 1) were selected and constructed as follows:

### *Life expectancy ( $T$ )*

The simple average of life expectancy at birth for males and females in 2003 is 80 years. As agents in our model begin life and work at the same time, and as it is reasonable to think that agents enter the workforce at around 20 years of age, we subtract 20 years from this lifespan to obtain an initial life expectancy of 60 in our model.

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<sup>13</sup> Alternatively we could write  $N_{s,t} = (1+n)N_{s+1,t}$  by noticing that  $N_{s,t-1} = N_{s+1,t}$ .



**Table 1: Calibration of the OLG Model – Initial Steady State**

Variable <sup>(a)</sup>	Description	Value
$T$ (years)	Life expectancy	60
$n$	Cohort growth rate	0.012
$e_s$	Human capital profile	See Figure 2
$b_1$	Parameter of $v(s, T)$	3.0
$b_2$	Parameter of $v(s, T)$	0.7
$b_3$	Parameter of $v(s, T)$	0.03
$\beta$	Discount factor	0.97
$\rho$	Utility function parameter	3.5
$A$	Total factor productivity	0.4
$1 - \alpha$	Labour's share of income	0.55
$\delta$	Depreciation rate	0.052

Note: (a) See Appendix B for detailed data sources and methods.

### *Cohort growth rate ( $n$ )*

Our benchmark cohort growth rate (1.2 per cent per year) was calibrated so that the median age in the model is that of the 2004 Australian population, conditional on life expectancy. This strategy also yields a steady-state growth rate for the model population which is in line with the current growth rate of the Australian population.

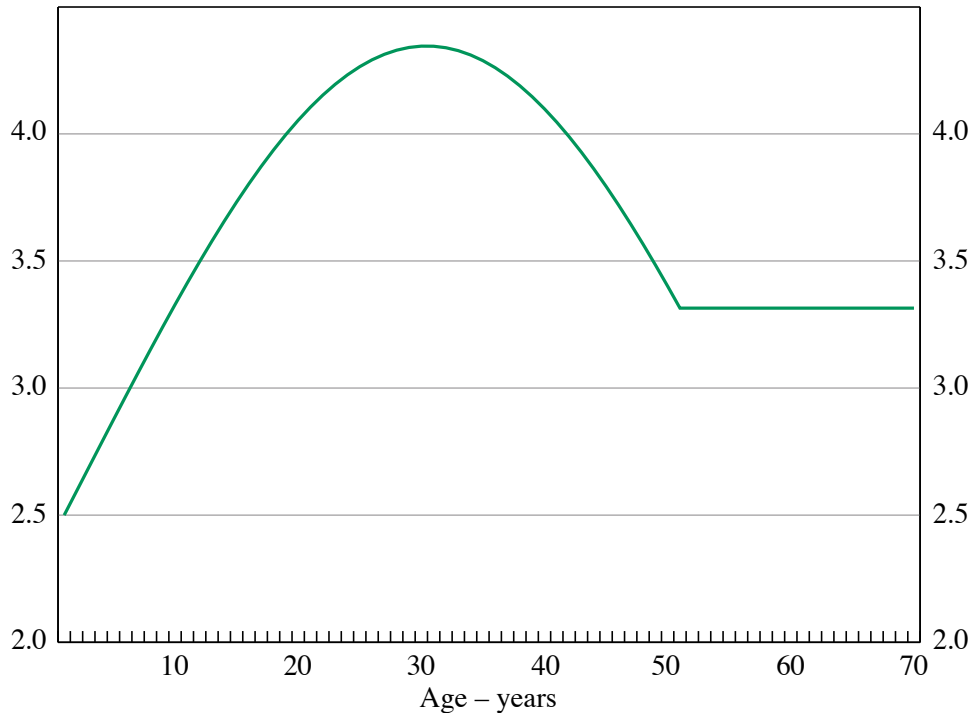
### *Human capital profile ( $e_s$ )*

We approximate the level of human capital at each point in an agent's life using male and female age-wage equations estimated by the Australian Bureau of Statistics (ABS). These equations use cross-sectional data with the hourly wage as the dependent variable, and experience and education as independent variables. We assume no post-school qualifications (this alters only the level and not the growth of human capital over time) and construct hourly wages by age for a representative agent by weighting the estimated male and female wages by their share of hours worked.<sup>14</sup>

<sup>14</sup> We experimented with adjusting females' work experience for child-rearing as discussed in Reilly, Milne and Zhao (2005). As the differences between the adjusted and unadjusted series are minor, we use the unadjusted values in our benchmark steady state.

In the data used to estimate the ABS age-wage equations, the maximum work experience is 50 years, while agents can work for up to 60 in our benchmark steady state. Hence we needed to make an assumption about the out-of-sample path of human capital. We chose to hold human capital constant at its last observed value (Figure 2).

**Figure 2: Human Capital Profile,  $e$**



Note: In all figures, unless otherwise specified, the unit of measurement is in terms of the numeraire good (that is, output).

*Time-varying weight on leisure:  $v(s, T)$*

We choose  $b_1$ ,  $b_2$  and  $b_3$  in Equation (4) so that the leisure choices of an agent who lives their entire life under the conditions of the initial steady state broadly resemble the pattern of age-specific participation rates currently prevalent in Australia. At present, labour force participation begins to decline quite sharply when Australians are aged around 50 (30 in our model), and the *average* age of complete retirement is around 59 years (which means that some fraction of people aged 60 and above would still be in the workforce). We therefore choose  $b_1$ ,  $b_2$  and  $b_3$  (3.0, 0.7, 0.03) so that agents in the benchmark steady state begin to withdraw from the labour force at around 30 but are *fully* retired after 41 years, having worked for two-thirds of their lives.

*Other utility function parameters: ( $\beta$ ,  $\rho$ )*

We choose a value of  $\beta$  (0.97) which is within the relatively narrow range of values used in the literature. In contrast, there is not much agreement with respect to the inter- and intra-temporal elasticities of substitution. For example, Auerbach and Kotlikoff (1987) discuss a range of studies where the inter-temporal elasticity varies from less than 1 to more than 14.<sup>15</sup> We choose a value for  $\rho$  (3.5) that generates a reasonable value for the capital-to-output ratio in the benchmark steady state. Our values of both  $\beta$  and  $\rho$  are comparable with those used by Auerbach and Kotlikoff. Our value of  $\rho$  implies an inter- and intra-temporal elasticity of substitution of 0.28. Auerbach and Kotlikoff set these values to 0.25 and 0.4 respectively.

*Total factor productivity ( $A$ )*

As discussed above,  $A$  is a scaling coefficient on output in OLG models of this kind. We chose a value of  $A$  (0.4) that generates broadly reasonable values for endogenous variables such as the capital-to-output ratio and the age of retirement in the benchmark steady state.

*Labour share of income ( $1 - \alpha$ )*

Our value of 0.55 is the average of compensation of employees as a share of total factor income over 1995–2005.<sup>16</sup>

*Real rate of capital depreciation ( $\delta$ )*

We choose 0.052, the average depreciation rate of the aggregate capital stock over 1995–2005. We average over this recent period because the depreciation rate has trended upwards since the early 1990s.

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<sup>15</sup> Most of these studies refer to models that do not have leisure in the utility function.

<sup>16</sup> This measure excludes labour's share of gross mixed income and net taxes on labour, which are properly included in labour's share of income. As these are positive in Australia, our measure probably slightly overstates the value of  $\alpha$ .

## 5. Results

We now discuss the results from four scenarios, each of which deals with a different set of unanticipated demographic changes (Table 2). The first two scenarios involve changes in fertility. Of these, the first is a permanent fall in fertility. The second is a 20-year increase in fertility followed by a permanent fall to a value lower than the initial level – this is the baby ‘boom and bust’ characteristic of many developed countries after the second World War. The third scenario is an increase in longevity, which we model as an increase in  $T$ . It is more realistic to expect a rise in longevity to play out over the course of many years and be at least partially anticipated. However, even recently, official projections of longevity have been revised considerably over a short period of time.<sup>17</sup> The final scenario combines the second and third to examine the economic implications of an increase in longevity coupled with a baby boom and bust.

**Table 2: Summary of Scenarios**

Scenario	Values of demographic parameters				
	Fertility parameter (per cent)			Longevity (years)	
	$n_1$	$n_2$	$n_3$	$T_1$	$T_2$
5.1	1.2	0.0	0.0	60	60
5.2	1.2	2.4	0.0	60	60
5.3	1.2	1.2	1.2	60	70
5.4	1.2	2.4	0.0	60	70

### 5.1 Permanent Fall in Fertility

In this scenario the fall in fertility is permanent, so the model shifts to a new steady state.<sup>18</sup> In steady state, aggregate variables grow at the rate of population growth,  $n$ , so a reduction in fertility decelerates the growth rate of these variables.

<sup>17</sup> For example, between 2001 and 2004, the Government Actuary’s Department in the United Kingdom raised their projections for life expectancy for those reaching age 65 in 2050 by 4 years for women and 4½ years for men (Hills 2006).

<sup>18</sup> The model takes around 150 years to reach the new steady state in this scenario. Although population growth stabilises after around 70 years, the model takes longer than this to reach the new steady state. This is because, even after population growth has stabilised, there are agents still alive who have chosen their consumption and leisure sequences conditional on the non-steady-state price sequences of the transition period.

However, in the transition, different variables decelerate at different rates – and some even accelerate for a short while.

The fall in fertility *acts like* a reduction in labour supply. Workers become relatively more scarce than they were initially, so the capital-to-labour ratio rises relative to the initial steady state (Figure 3). Therefore, wages rise from their initial level, and interest rates fall. These changes in factor prices induce shifts in the savings and retirement behaviour of agents. For the latter, we find that the substitution effect dominates the income effect from higher wages. As a result, agents retire slightly later in the new steady state, with the share of life spent working (defined as  $\sum_{s=1}^T (1 - l_{s,t})/T$ ) rising from 62.8 per cent to 63.0 per cent.

**Figure 3: Transitional Dynamics – Fall in Fertility**

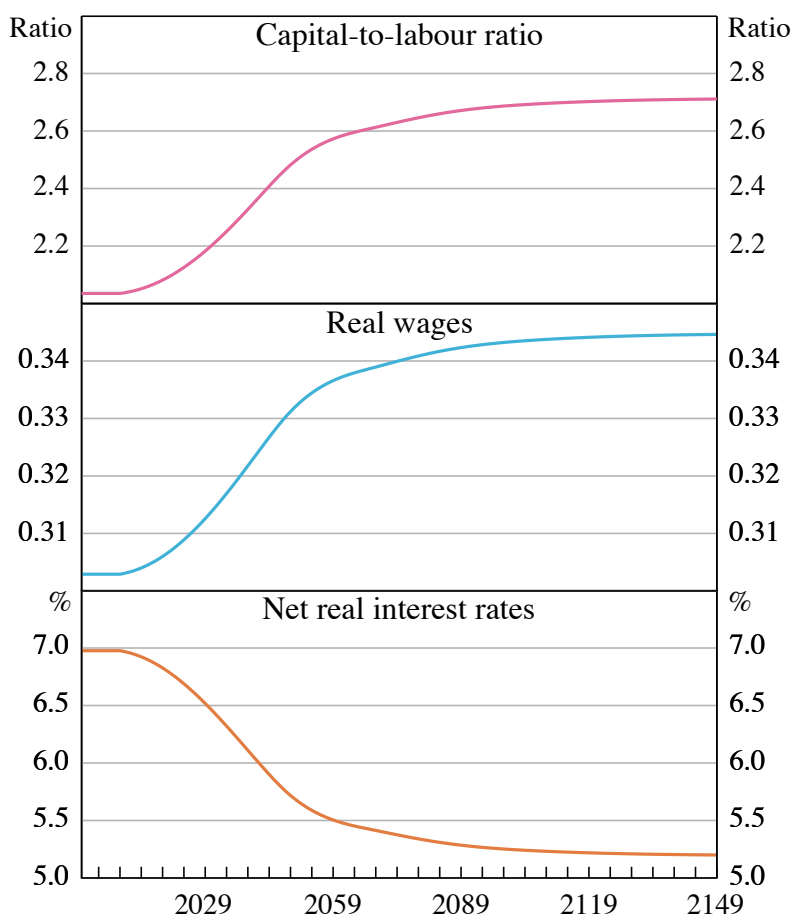
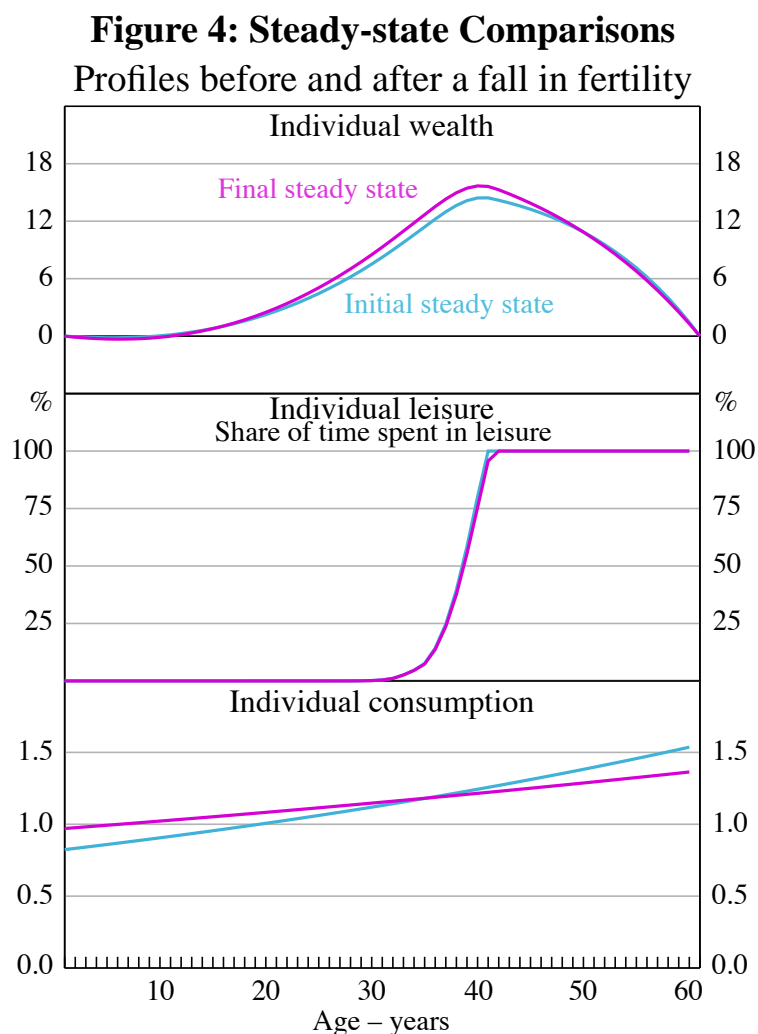


Figure 4 shows the lifetime profile of wealth, leisure and consumption in the two steady states. As we would expect, the consumption profile becomes flatter in response to lower interest rates. Higher wage rates induce agents to supply more labour and help them to accumulate more wealth over their lifetime. This in turn

allows them to finance more consumption over their lifetime (notwithstanding the lower return to capital). In fact, agents in the final steady state are better off, in terms of lifetime utility, than agents in the initial steady state.

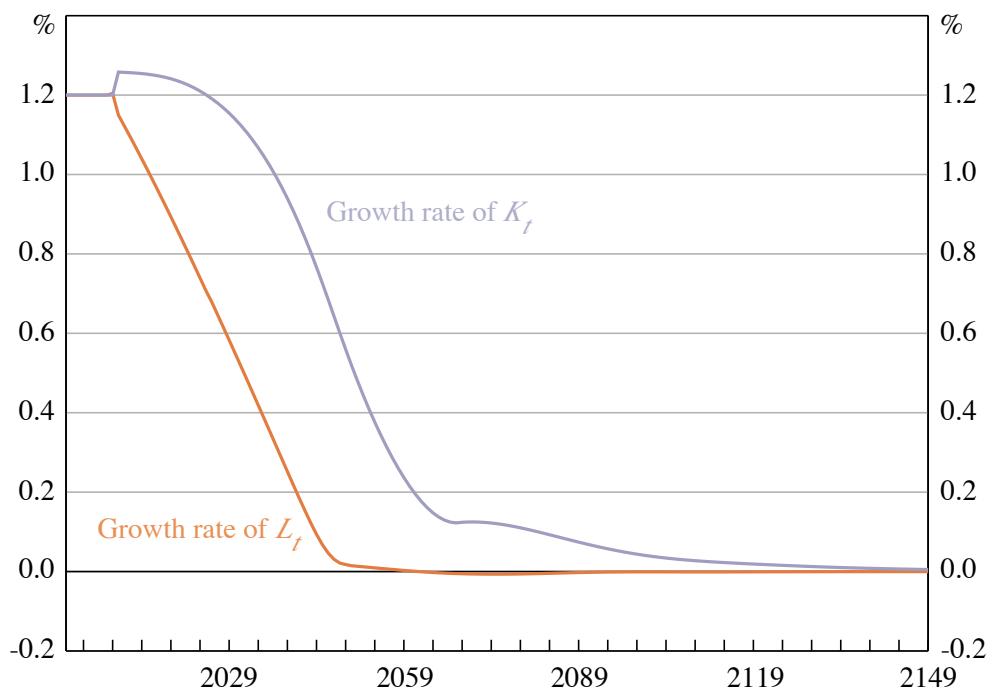


Note: The last period shown is 61 years when the person has died and has no wealth left.

To help understand the transition between the initial and final steady states, we can examine the growth rates of the capital stock and the labour supply (Figure 5). With the capital-to-labour ratio rising during the transition, we know that the growth rate of the capital stock must be above the growth rate of the efficient workforce during this period. In fact, the growth rate of capital rises above its initial level for about 15 years. That is, the growth rate of aggregate savings actually rises for a time. Different cohorts make very different contributions to this aggregate result. While younger cohorts increase consumption and reduce their

saving rates during this period, this is more than offset by an opposing response of middle-aged and older cohorts.

**Figure 5: Fall in Fertility**



These responses from different cohorts occur because the changes in factor prices affect these two groups differently. Those in their middle and old age do not benefit as much as younger generations from the increase in wages, because much (or all) of their working life has already passed. Also, lower interest rates decrease current and future income, especially for those already retired, who depend entirely on income from capital. Middle-aged and older cohorts react by reducing consumption and increasing savings. Younger cohorts are harmed less by lower returns to capital since they have accumulated little or no wealth. For them, the increase in wages together with a high time endowment allows them to initially increase consumption and reduce saving.<sup>19</sup>

Although this model incorporates non-standard features, the result that a decline in population growth (with unchanged longevity) leads to a higher capital-to-labour ratio is similar to the result from a standard two-period Diamond OLG model.

<sup>19</sup> Nevertheless, young people at the time of the change are worse off than subsequent generations. This is because factor prices adjust gradually. Hence those who are very young when the shock hits do not benefit from higher wages as much as future cohorts, and by the time they have accumulated a substantial quantity of savings, interest rates have fallen considerably.

## 5.2 Baby Boom and Bust

Here there is an increase in fertility that lasts for 20 years, followed by a permanent fall in fertility.<sup>20</sup> During the boom, agents act as if the higher fertility rate were to last forever. In other words, both of the changes in fertility are unanticipated.<sup>21</sup>

Our model behaves symmetrically in the sense that the effects of the initial boom are opposite to those discussed above. The boom acts as an increase in the labour supply. This change in the capital-to-labour ratio puts downward pressure on wages and upward pressure on interest rates. Following the argument above, but working in reverse, the growth rate of aggregate savings falls initially.

After 20 years of transition towards the new steady state implied by the higher fertility rate, there is an unexpected fall in fertility. Except for the position of the economy at the time of the change, the dynamics are exactly those of our first scenario. The following baby bust eventually reverses the effects of the temporary boom, leading ultimately to the same steady state as before.

Figure 6 illustrates the transition for all scenarios. Comparing the paths for capital intensity, wages, and the interest rate for this scenario with those for the previous one, we can see that the baby boom delays the onset of the new steady state caused by the permanent fall in fertility. Furthermore, the baby boom causes interest rates and wages to initially move in the opposite direction.

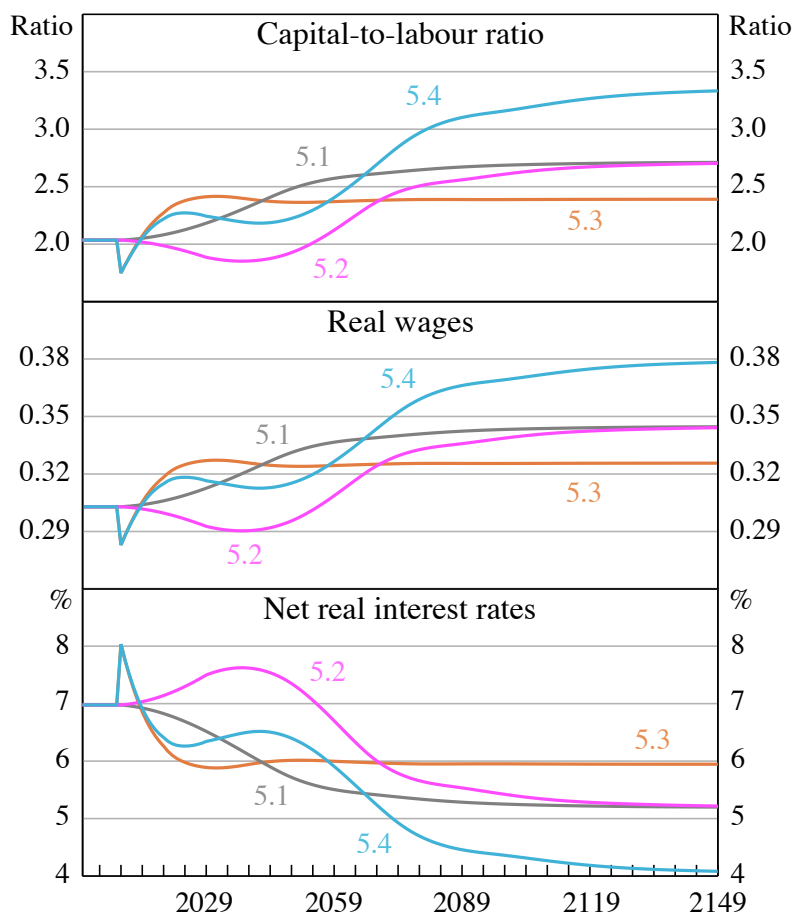
The economy eventually converges to the same final steady state as in Scenario 5.1, in which agents are better off than they were initially. However, the transitional dynamics in the two scenarios have quite different implications for the welfare of different cohorts.

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<sup>20</sup> We choose a boom of 20 years to match the duration of the baby boom in Australia.

<sup>21</sup> This simulation involves an additional complexity since it requires us to calculate a virtual future path of the economy which would not occur, but which is necessary to establish behaviour during the boom years.



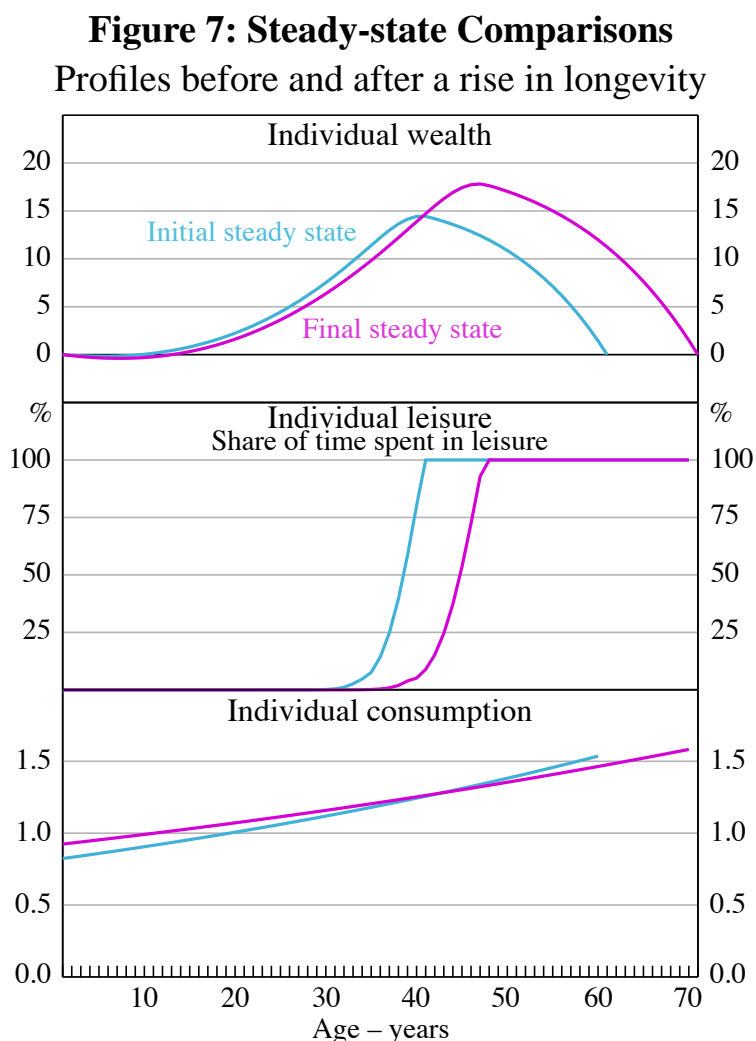
**Figure 6: Transitional Dynamics – Scenarios 5.1, 5.2, 5.3 and 5.4**

### 5.3 Increase in Longevity

Here there is a permanent, unexpected increase in agents' lifespan,  $T$ . This is accompanied by an improvement in health, an assumption we relax later on. As agents are healthier and live longer, they retire later in life, increasing the aggregate labour supply. In response, wages jump down and interest rates jump up (Figure 6). However, these initial effects on factor prices are gradually unwound, so that eventually, wages rise and interest rates fall, relative to the initial steady state. This is because, in the longer run, increased longevity also raises the aggregate demand for labour. The mechanism at work is simple. A longer lifespan raises the absolute number of years agents spend in retirement. Agents must save more for this longer retirement, and these additional savings raise the aggregate capital stock. This in turn raises the marginal product of labour, which pushes up wages, and reduces the return on capital.

Interestingly, the share of life spent working is slightly lower in the new steady state. Agents are retired for 23 rather than 20 years, but work for 62.6 per cent rather than 62.8 per cent of their lives.

Figure 7 illustrates the differences in wealth accumulation, leisure, and consumption in the two steady states. Overall, agents accumulate more wealth, retire later, and have a flatter consumption profile.

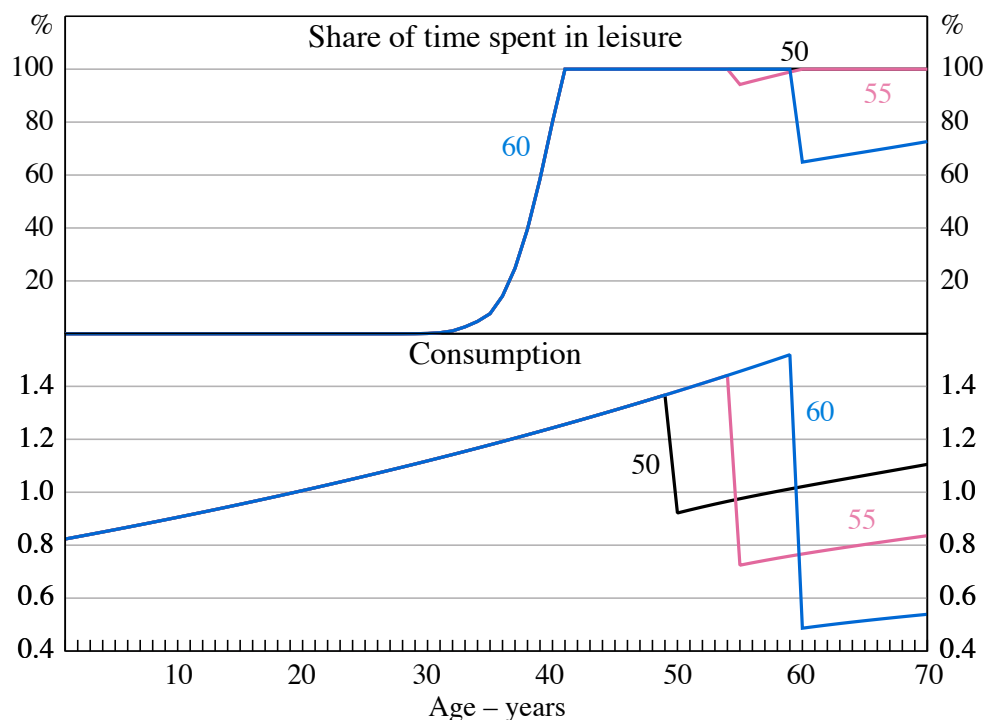


Note: The last period shown is 71 years when (under the higher longevity scenario) the person has died and has no wealth left.

We assume that all agents alive at the time of the change experience the same absolute increase in longevity. Both the fact that the change in longevity occurs suddenly rather than gradually, and the fact that it affects all agents equally, are not entirely realistic. Even so, the key results regarding the direction of changes in variables of interest should hold in a more realistic setting.

In our model, the labour supply response of older agents of different ages to the unexpected increase in longevity is particularly interesting. This is because there is a trade-off between reducing consumption and supplying labour which is wealth- and age-dependant. For example, those people who are 55 or above are already well into their retirement, having almost completely dissaved, and must return to work in order to finance consumption in these extra last years of their lives (Figure 8). As these people are at the end of their lives, their time-varying weight on leisure is very high. Agents aged about 50 at the time of the change also have a relatively high weight on leisure, but they have a larger stock of wealth than older retirees, so they choose to only cut consumption rather than return to work.

**Figure 8: Leisure and Consumption Profiles with Increased Longevity**  
For agents aged 50, 55 and 60 at time of change



#### 5.4 Combined Change in Fertility and Longevity

This scenario combines Scenarios 5.2 and 5.3 to investigate the economic effects of changes in fertility *and* an increase in longevity, which roughly characterises the demographic changes of the past half-century or so. We know that these two scenarios by themselves have the same sorts of effects: both an increase in longevity and a permanent baby bust will eventually raise wages, lower interest rates, and raise income and consumption per capita. More interesting is the fact

that the effect of changes in fertility and longevity on variables such as the capital-to-labour ratio are not additive: the combined effect is greater than the sum of the two individual effects (Figure 6). In particular, the capital-to-labour ratio increases by 0.67 in Scenario 5.2, 0.35 in Scenario 5.3, and 1.32 in Scenario 5.4. However, the combined effect of a change in fertility and longevity on the steady-state retirement age is less than the sum of the two individual effects.

During the early part of the transition to the new steady state, there is a period of lower wages and higher interest rates driven by relatively abundant labour. In this scenario the increased labour supply has two sources: the temporary rise in fertility associated with the baby boom and the higher labour supply associated with increased longevity. Eventually, the lower fertility rates and the need to finance consumption over longer retirements raises the capital-to-labour ratio, increases wages and reduces rental rates on capital.

## 5.5 Sensitivity Analysis

Tables 3 and 4 illustrate the sensitivity of the steady-state capital-to-labour ratio and retirement behaviour to the demographic parameters in our model. We include some additional steady states not computed above. Comparing the capital intensities in Table 3 confirms that the results from the scenarios presented above hold more broadly. That is, both lower fertility rates and longer lives increase the capital intensity of the economy, and the effects of combined changes in longevity and fertility are superadditive. This result is, to the best of our knowledge, new in the literature.

$n$ (per cent)	$T$ (years)				
	60	65	70	75	80
2.4	1.55	1.65	1.75	1.85	1.94
1.2	2.04	2.21	2.39	2.60	2.77
0.0	2.71	3.03	3.36	3.73	4.12
-1.2	3.67	4.21	4.81	5.48	6.20

Notice that in Table 3 changes in  $T$  do not leave the capital-to-labour ratio unchanged. One could choose quarters instead of years and recalibrate the rest of the parameters (by altering their units of account accordingly) so as to leave the capital-to-labour ratio unchanged. However, for a given choice of the unit of account for time, a change in  $T$  will not require a change in the the unit of account of the other parameters. For example, the rate of depreciation, and the magnitude of total factor productivity will be unchanged. For this reason, changes in  $T$  lead to a change in the capital-to-labour ratio.<sup>22</sup>

**Table 4: Share of Life Spent Working in Various Steady States**

$n$ (per cent)	Per cent				
	$T$ (years)				
	60	65	70	75	80
2.4	62.5	62.3	62.2	62.0	62.0
1.2	62.8	62.7	62.6	62.6	62.5
0.0	63.0	63.0	63.0	62.9	62.9
-1.2	63.2	63.2	63.2	63.2	63.2
-2.4	63.3	63.3	63.3	63.3	63.4

Table 4 shows the sensitivity of the time spent working to changes in the demographic parameters. For a given  $T$ , lower fertility decreases the share of life spent in retirement. This largely reflects the impact that higher wages associated with lower values of  $n$  have on labour supply decisions. Also notice that, for a given value of  $n$ , the share of life spent working can stay constant, rise, or fall with increases in longevity. This is because two opposing forces are at work. A longer lifespan provides more years over which to accumulate wealth from compounding interest income, but in general equilibrium a longer retirement increases the supply

<sup>22</sup> In Blanchard's (1985) model, where there is uncertainty about death, the effective discount rate becomes a function of the horizon (life expectancy) of agents. Although there is no uncertainty of this kind in our model, increasing  $\beta$  with  $T$  does not alter the result above – namely, that the capital-to-labour ratio rises with  $T$ . This result is a general one. It is easy to see why it holds in a very simple two-period OLG model, with non-productive, non-depreciating capital, a discount factor of one, a production function that is linear in labour, and assumptions about health such that people work for the first half of their lives. In this case, doubling the lifespan (to four periods) will double the (equilibrium) capital-to-labour ratio as agents now have to fund two consecutive periods in retirement. However, a four-period model could replicate the capital-to-labour ratio of the two-period model if agents were able to work in the first period, retire in the second, return to work in the third and retire again in the fourth.

of capital and lowers the interest rates over which to compound. Interestingly, unlike Bloom *et al*'s (2004) partial equilibrium analysis in which interest rates and wages remain constant, the share of life spent working in our model could move in either direction.

We also examine the sensitivity of our results to different parameter values and assumptions about health and human capital. Under the assumption that health does not improve when life expectancy rises, the capital-to-labour ratio converges to an even higher level than before.<sup>23</sup> An increase in lifespan of 10 years increases the steady-state capital-to-labour ratio from 2.04 to 2.39 when health improves and from 2.04 to 3.50 when there is no health improvement (Figure 9). With constant health, retirement behaviour is virtually unchanged relative to the initial steady state. That is, labour supply does not increase as much as in Scenario 5.3. Agents have an extra 10 years of consumption to finance but the amount of time spent in the workforce remains almost unchanged. As a result, they need to accumulate more wealth during their working lives.

With unchanged health, there is still an initial increase in the labour supply when longevity increases. The increased labour supply is mainly driven by older retirees' need to finance additional years of consumption despite their high disutility from working.

In steady state the profile of human capital operates (jointly with  $A$ ) as a scaling parameter. However, in the face of (unexpected) lifespan changes, the profile of human capital is an important determinant of labour supply decisions. We investigate an increase in longevity and health under the assumption that, instead of rising over a person's lifetime, human capital remains constant (at the initial value of 2.5 in the benchmark calibration). We find that this lower level of human capital induces a stronger labour supply response to a change in  $T$ .

Our results are robust to a wide range of values for the other parameters.<sup>24</sup> For example, the qualitative results of Scenario 5.4 are robust to variation in the household's discount factor  $\beta$ ; a rise in  $\beta$  from 0.97 to 0.99 increases the final

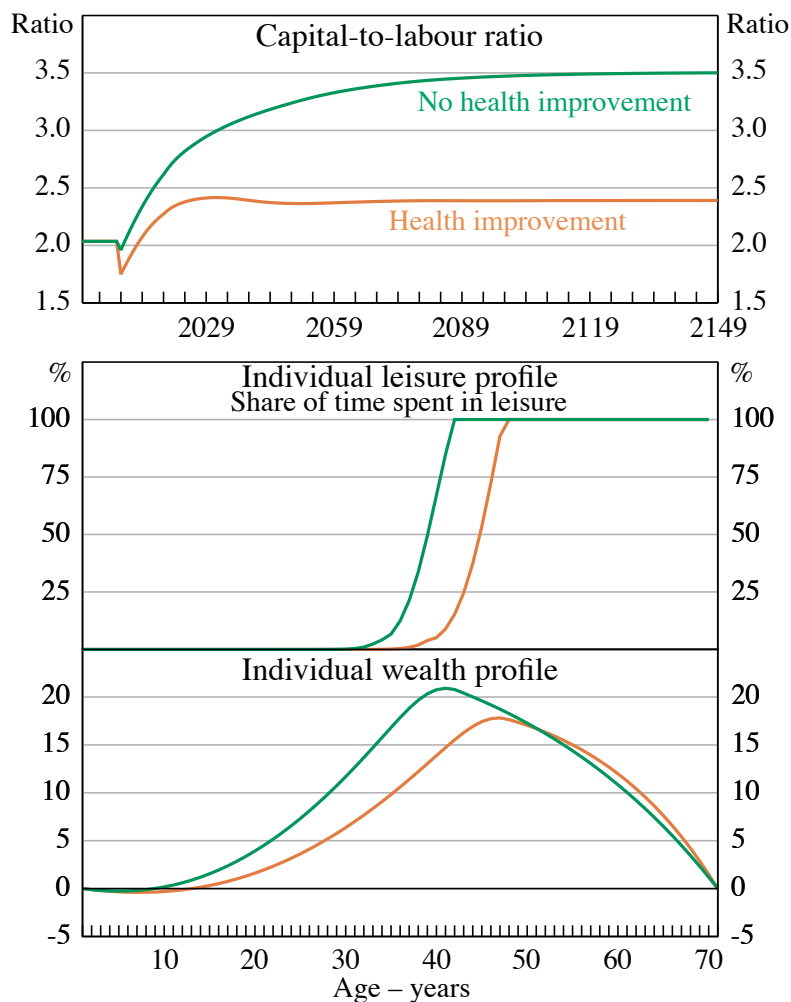
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<sup>23</sup> We model constant health by relaxing the assumption that  $v(s, T)$  is homogenous of degree zero in  $s$  and  $T$ . We limit the dependence of  $v(\cdot)$  on  $T$  by keeping the 'mean' and 'standard deviation' at their previous values (that is,  $b_2 T_1$  and  $b_3 T_1$  respectively).

<sup>24</sup> These additional results are available upon request.

steady-state capital-to-labour ratio from 3.33 to 5.09 and preserves the shape of the transition paths. The results are also robust to a lower value of  $\rho$  (1.5 compared to 3.5).<sup>25</sup>

**Figure 9: A Rise in Longevity (10 Years) – With and Without Better Health**



Note: The last period is 71 years when the person has died and has no wealth left.

<sup>25</sup> Much lower values of  $\rho$  were problematic for the convergence of the solution algorithm. In our model a lower  $\rho$  increases both the intra- and inter-temporal elasticities of substitution. Hence, the sensitivity of the inner and outer loops of the solution algorithm (illustrated in Appendix A) increase jointly when  $\rho$  falls.

## 6. Conclusion

In this paper we study the macroeconomic consequences of ageing. We emphasise the distinction between the drivers of ageing that is often ignored in the literature. Both longevity and fertility influence the economy independently, but they also operate together by magnifying the effects of ageing in a number of respects. Moreover, during the transition to a new steady state, these two factors have very different implications for the behaviour of wages and real interest rates.

Healthy lifespan extensions increase the absolute number of years in retirement, but the fraction of life spent in the workforce could rise or fall. A longer lifespan provides more years over which to accumulate capital from compounding interest income. But, in general equilibrium, a longer time spent in retirement increases the supply of capital and lowers the interest rates over which to compound.

We find that a permanent fall in the fertility rate increases capital intensity, raising wages and lowering interest rates, and delays retirement. When life expectancy increases, the economy also converges to an equilibrium with a higher capital intensity, but the transition to the steady state looks quite different. If health improves hand-in-hand with life expectancy (which appears plausible), the economy would initially undergo a period of relatively low wages and high interest rates. This effect would be reversed in the long run, as capital accumulates when workers build a larger pool of savings to fund more years in retirement. When fertility falls and lifespans increase at the same time, the capital-to-labour ratio converges to a level which is higher than the sum of the two acting alone, and the transition to the new steady state involves periods of relatively abundant labour and low wages.



## Appendix A: Technical Appendix

### A.1 The Household Problem

The household problem is to maximise Equation (1) subject to the period budget constraint, Equation (2), inequality constraints on leisure given by Equation (3), initial and terminal conditions on individual wealth ( $a_{1,t} = a_{T+1,t+T} = 0$ ), and non-negativity constraints on consumption and leisure.

Let  $\lambda$  and  $\mu_{s,t+s-1}$  be the Lagrange multipliers associated with the lifetime budget constraint and the period  $t + s - 1$  inequality constraint on leisure, respectively. With *a priori* knowledge about the functional form of  $v(s, T)$  we can infer that  $c_{s,t+s-1} > 0$  and  $l_{s,t+s-1} > 0$ . Lifetime resources would be exhausted along the optimal path, so the lifetime budget constraint would be active. With this in mind, the Kuhn-Tucker first-order conditions of the problem can be written as

$$\beta^{s-1} c_{s,t+s-1}^{-\rho} - \frac{\lambda}{R_t^{t+s-1}} = 0 \quad (\text{A1})$$

$$v(s, T) \beta^{s-1} l_{s,t+s-1}^{-\rho} - \frac{\lambda e_s w_{t+s-1}}{R_t^{t+s-1}} - \mu_{s,t+s-1} = 0 \quad (\text{A2})$$

$$\sum_{s=1}^T \frac{(1 - l_{s,t+s-1}) e_s w_{t+s-1}}{R_t^{t+s-1}} - \sum_{s=1}^T \frac{c_{s,t+s-1}}{R_t^{t+s-1}} = 0 \quad (\text{A3})$$

$$1 - l_{s,t+s-1} \geq 0 \quad (\text{A4})$$

$$0 = \mu_{s,t+s-1} (1 - l_{s,t+s-1}) \quad (\text{A5})$$

$$\mu_{s,t+s-1} \geq 0 \quad c_{s,t+s-1} \geq 0 \quad l_{s,t+s-1} \geq 0 \quad (\text{A6})$$

where  $R_t^{t+s-1} \equiv \prod_{i=t+1}^{t+s-1} R_i$ .

There are three main cases to consider with respect to the solution. One in which the constraint on leisure, Equation (A4), never binds; one in which Equation (A4) is always active; and one in which Equation (A4) is initially not active, but becomes active later on. The first case lies at the interior of the opportunity set and poses no difficulty. It is easy to show that the second case is not optimal if initial and terminal wealth are zero. In this case, the agent can never consume as no income is ever generated. So we can rule out the first and second case and concentrate on the third in which the agent works a given number of periods and retires from then on.

Assume that Equation (A4) is not active for  $s = 1, \dots, T'$  and is active for  $s = T' + 1, \dots, T$ . In this case, the plan for consumption and leisure implicitly incorporates the time of retirement. One can view the agent as choosing the cut-off period,  $T'$ , after which the constraint ceases to be inactive. In this case we can rewrite the lifetime constraint as follows

$$\sum_{s=1}^{T'} \frac{(1 - l_{s,t+s-1}) e_s w_{t+s-1}}{R_t^{t+s-1}} = \sum_{s=1}^T \frac{c_{s,t+s-1}}{R_t^{t+s-1}}$$

Define the variables  $Q'_t$  and  $H'_t$  as

$$Q'_t \equiv \sum_{s=1}^{T'} \frac{e_s w_{t+s-1}}{R_t^{t+s-1}}$$

$$H'_t \equiv \sum_{s=1}^{T'} v(s, T)^{1/\rho} (e_s w_{t+s-1})^{\frac{\rho-1}{\rho}} (\beta^{s-1})^{\frac{1}{\rho}} (R_t^{t+s-1})^{\frac{1-\rho}{\rho}}$$

$$+ \sum_{s=1}^T (\beta^{s-1})^{\frac{1}{\rho}} (R_t^{t+s-1})^{\frac{1-\rho}{\rho}}$$

The first-order conditions of the problem can be combined with the lifetime budget constraint and the above definitions to arrive at an expression for the household's first-period consumption of the form

$$c_{1,t} = \frac{Q'_t}{H'_t}$$

with an expression for  $c_{1,t}$  the optimal path for consumption and leisure satisfies Equations (5) and (6)

$$c_{s,t+s-1} = \left( \beta^{s-1} R_t^{t+s-1} \right)^{1/\rho} c_{1,t} \quad s = 1, 2, \dots, T$$

$$l_{s,t+s-1} = \begin{cases} \left( \frac{v(s, T)}{e_s w_{t+s-1}} \right)^{1/\rho} c_{s,t+s-1} & s = 1, 2, \dots, T' \\ 1 & s = T' + 1, \dots, T \end{cases}$$

Note that this is not a closed form analytical solution for  $c_{s,t}$  and  $l_{s,t}$  because  $T'$  is not a fixed parameter of the household's problem but, rather, a choice variable.

Formally,  $T'$  never enters the problem or forms part of the solution. Rather, it is a convenient indicator of when the Lagrange multipliers associated with the leisure constraints are zero, and allows us to express the solution without reference to the Lagrange multipliers.

Although analytical expressions are not available in the case of OLG models of large dimensions, the problem can be solved numerically. The next section discusses the solution method.

## A.2 The Solution Method

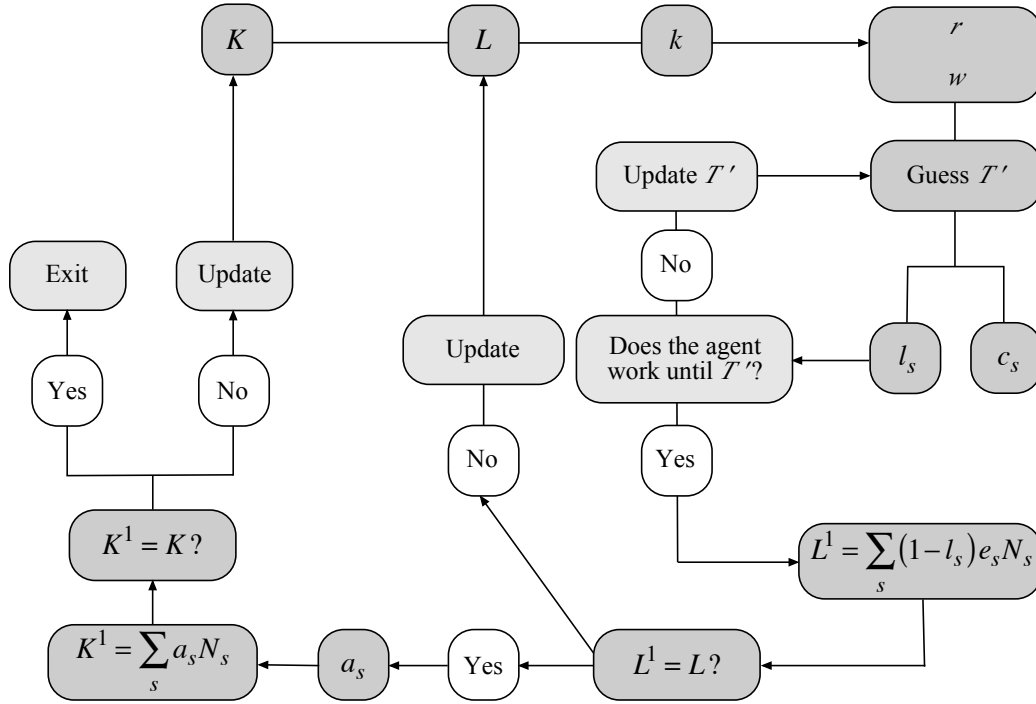
Solving for the steady state of the model involves solving a system of non-linear equations and inequality constraints. We solve for the equilibrium of the economy in the initial steady state using Gauss-Seidel iterations.<sup>26</sup> With respect to  $T'$  we use an initially constrained approach, as described in Intriligator (1971), in that our initial guess of the leisure profile is inside the agent's opportunity set. Our initial guess is that the agent does not retire (i.e.,  $T' = T$ , which implies  $l_{s,t+s-1} < 1$  for all  $s$ ).

The algorithm for solving the steady state is illustrated in Figure A1. We start with an initial guess of the aggregate capital stock and labour supply. With these values in hand we calculate factor prices. We then use our initial guess of  $T'$  to calculate leisure and consumption sequences. The leisure sequence gives us a new guess for  $T'$ . If this new guess is not equal to our initial one, we update our guess of  $T'$  and recalculate leisure and consumption sequences. Otherwise, we update labour supply and get a new value for the capital-to-labour ratio and factor prices. Once labour supply converges, we calculate individual wealth and aggregate it to get a new guess of the aggregate capital stock. A fixed point of this algorithm yields the steady-state capital-to-labour ratio.

To solve for the equilibrium transition path we use a strategy similar to Auerbach and Kotlikoff (1987). Finding the transition path is conceptually like finding the steady state. Some additional complications are worth mentioning. As the economy undergoes a transition in which conditions change over time, it is necessary to solve explicitly for each year. And because agents are forward-looking, it is necessary to solve for equilibrium in all transition periods

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<sup>26</sup> Matlab programs for the steady state and transition paths are available upon request.

**Figure A1: Steady-state Algorithm**

simultaneously. In our simulations we generally give the economy around 250 years to adjust to the final steady state. After 250 years, we constrain the economy to attain its final steady state. The idea is to allow the economy to settle down by itself well before 250 years.<sup>27</sup>

Agents that are alive at the time of the change need to be treated differently. At the time of the change they are ‘reborn’ with an ‘initial’ wealth equal to whatever they had accumulated up to that point, and a ‘shorter’ lifespan equal to  $T$  minus what they had already lived. One appealing property of any steady state in the model is that it can be interpreted both in its cross-sectional dimension or in its time dimension. For example, the steady-state leisure profile can be seen as the time  $t$  leisure that each agent of age  $s$  takes, or as the leisure profile that an individual born at  $t$  can expect to have if conditions do not change throughout their life. A complication of the transition path is that, since conditions are changing over time, individuals born at different times might choose different retirement ages.

<sup>27</sup> In particular, with experiments that involve increases in  $T$  it is necessary to make sure that the economy has sufficient time to adjust. This is because, the larger the value of  $T$ , the longer the economy takes to arrive at the new steady state. Numerical simulations suggest that the time the economy needs to converge grows proportionately more than  $T$ .

It is necessary to keep track of every single generation's age of retirement, which implies that a single guess for the age of retirement ( $T' + 1$ ) is not sufficient. In other words, it could well happen that at some time  $t$  all agents alive might choose different retirement ages.

In cases where we are modelling a baby boom followed from a baby bust there is an initial set of parameters, an intermediate one, and a final one. It is therefore necessary to calculate a transition path which would never occur, but which influences expectations of future prices. People behave as if the intermediate steady state would last forever, and are surprised later on with a new set of changes. In this case, it is necessary to calculate a virtual transition path, and use the conditions on that path as the initial conditions when the second change occurs.

## Appendix B: Data Sources for Calibration

*Life expectancy ( $T$ ):* average calculated from ABS Cat No 3302.0, Table 7.3.

*Cohort growth rate ( $n$ ):* we calculate a value of  $n$  consistent with the 2004 population in Model 4 from the Productivity Commission's (2005) report into population ageing. Here, the median age of the population aged 20 or over is 44 years and life expectancy (under the medium scenario) is 81 years. Substituting this life expectancy into our OLG model (after subtracting 20), one of a range of values of  $n$  that yields this median age is 1.2 per cent.

*Human capital profile ( $e_s$ ):* we use the 1999 coefficient estimates presented in Tables 6.2 and 6.3 for Equation (12) from Reilly *et al* (2005). We construct hourly wages by age for a representative agent by weighting male and female wages by their share of hours worked in 1999 from ABS Cat No 6291.0.55.003 data cube E06, 'Employed Persons by Sex, Industry, State, Status in Employment'. This wage profile is normalised so that wages in the first year of life are 2.5.

*Time-varying weight on leisure ( $b_1$ ,  $b_2$  and  $b_3$ ):* average ages of retirement for males and females in 2002 are from 'Comparison of Methods for Measuring the Age of Withdrawal from the Labour Force', ABS Research Paper 1351.0.55.009. The weights on male and female retirement ages are the averages of each gender's share of the labour force for the four quarters of 2002 from ABS Cat No 6202.0. To determine the age when labour-force participation begins to decline more rapidly we construct a series of age-specific participation rates for a representative agent using data from ABS Cat No 6291.0.55.001 data cube LM8, 'Labour Force Status by Sex, State, Age, Marital Status'. The representative agent is a weighted average of male and female labour-force participation where the weights are each gender's share of the total labour force over the same period (the March quarter 2002), also from ABS Cat No 6202.0. Visual inspection of this series shows that labour-force participation begins to decline more quickly at around age 50.

*Labour share of income ( $1 - \alpha$ ):* average over 1995–2005 calculated from ABS Cat No 5206.0, Table 41.

*Real rate of capital depreciation ( $\delta$ ):* average over 1995–2005 calculated from depreciation rates inferred from ABS Cat No 5204.0, Table 69.

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