THE EQUATIONS OF THE RBA82 MODEL OF THE AUSTRALIAN ECONOMY

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ABSTRACT

This paper presents the RBA82 model of the Australian economy. The derivation of the model's equations is described in some detail, and the fit of the model examined. A comprehensive bibliography is included which lists all papers published to date on the RBII project.

I. INTRODUCTION

This paper sets out the estimating equations of the most recent version of the RBII model of the Australian economy; this version is called RBA82. It is estimated on data from 1959 to 1980, and does not attempt to model structural changes that have taken place in the Australian economy since 1982. It is, however, used as the starting point for the study of the recent changes in financial markets by Fahrer and Rankin (1984). The name RBA82 is used for the estimated model to differentiate it from the simulation model used in this later work.

Section II describes the individual equations and their derivations. Since the first such description,[1] many aspects of the model have altered: the monetary transmission mechanism, to incorporate several short-term interest rates and a more complex specification for capital flows; the equations for factor demands; the treatment of expectations, by the introduction of expected sales; the modelling of taxes; and the treatment of farm product. Section III evaluates the fit of the model.

There are two appendices. The first lists the model equations and parameter estimates; the second details the techniques applied in order to estimate the model with (discrete) quarterly data.

II. EQUATIONS OF THE RBII MODEL

The theoretical framework of RBII identifies three sectors: the private (household and corporate) sector, the

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1. Jonson, Moses and Wymer (1976) presented and described the structure of the first version of the model, called RBA76. This version was examined at a conference held in 1977, the proceedings of which are reported in Norton, ed. (1977). A second version of the model was presented and applied to simulation analysis in Jonson and Trevor (1979), and Jonson, McKibbin and Trevor (1981, 1982).

consolidated banking system and the government sector; and the foreign sector.

It is assumed that the various sectors make the following decisions (a distinction between firms and households is retained for exposition):

i. firms - expected sales; expected private sales; total supply net of farm product and the split of this supply between domestic production and imports; the rate of change of the factors of production (capital and labour); and prices (both for domestic production and exports);

ii. households - consumption; investment in dwellings; supply of labour; non-bank take-up of government securities;

iii. consolidated banking system and government - bank advances; the rate of interest on government securities; and the $A/$US exchange rate. The latter two are modelled as policy reaction functions;

iv. foreign - exports of goods and services; and net capital inflow;

v. jointly determined - interest rates on 90 day bank bills, bank advances, and money; average weekly earnings; the price of government current expenditure; and personal income taxes.

The behavioural equations will be discussed in this order.

II.1 BEHAVIOURAL EQUATIONS

II.1.i Supply, desired stocks, factors of production and price decisions by firms

RBII now distinguishes between farm (y_f) and non-farm (y_{nf}) product with the former exogenous. Firms are assumed to supply goods and services to meet expected current non-farm sales (proxied as expected sales less farm product).

Data on expected sales are, however, not available on a basis consistent with the model's definitions. The model assumes that firms' expectations of total sales are generated by an adaptive mechanism:

$$D\log s^e = \bar{\alpha}_{22} (\log \hat{s}^e / s^e)$$

$$\hat{s}^e = s_0^e \, s$$

where $\bar{\alpha}_{22} = 0.5$. Given the expectations generating process, the value of α_{22}, data for sales, and that at some point sales and expected sales are assumed equal, then data for expected sales can be generated. This specification cannot be substituted out, so the above equation is listed in the model specification (equation (26) in Appendix 1) as an identity.

Firms plan to meet expected non-farm sales with domestic production or imports.[2] It is assumed that there is some steady state ratio (i_0) of imports to expected non-farm sales but that over shorter time horizons desired imports can diverge from this ratio due to shifts in relative prices, exchange rate expectations and shifts in taste. That is:

$$\hat{i} = [i_0 \{ EP_i (1.0+t_3) / P\}^{\beta_{18}} e^{\beta_{19} Q_i}] (s^e - y_f) \; .$$

Desired domestic supply (non-farm domestic production) can therefore be defined as:

$$\hat{y}_{nf} = (s^e - y_f) - \hat{i}$$

2. It should be recalled that, since the distinction between firms and households is used only for exposition purposes, this represents the decision of the aggregative private sector.

It is assumed that firms attempt to maintain relatively smooth production plans. Consequently, actual non-farm domestic production and imports are assumed to adjust to their desired levels by a first order partial adjustment process.

In practice, actual sales will differ from expected sales. Also lagged adjustment implies that production plans are not always achieved. As part of the process of maintaining smooth supply decisions firms will allow part of any short-run unexpected shifts in sales to be met by changes in the stock of inventories. Inventories therefore act as a buffer in the firms' supply decision.[3] It is further assumed that firms have a higher propensity to import than to change domestic production plans in response to a change in inventories.

This is modelled by adding inventory disequilibrium, defined as the ratio between desired and actual inventories, (that is, $\log(\hat{v}/v)$), to the output and imports equations but with independent parameters.

Desired inventories are homogenous of degree one in private expected sales:

$$\hat{v} = v_o s_p^e$$

where[4]

$$s_p = s - g_1 .$$

3. It will be seen below that helping to smooth the production process will also smooth the demand for factors of production.

4. Private sales is defined as sales less government current expenditure. Data for private expected sales is formed by the same procedure as that for expected sales (see equation (27) in Appendix 1). Desired inventories are a function of private expected sales rather than expected sales as government current expenditure is composed almost entirely of services valued by the wages paid to employees.

The imports and output equations are thus written as:

$$D\log y_{nf} = \alpha_6 \log (\hat{y}_{nf}/y_{nf}) + \beta_{20} \log(\hat{v}/v)$$

and

$$D\log i = \alpha_5 \log (\hat{i}/i) + \beta_{17} \log(\hat{v}/v).$$

Once firms have determined their desired level of domestic production they can determine their demands for factors of production (capital (K) and employment (L)). It is assumed that, at any given instant, the stock of capital and employment are fixed and so firms determine their desired rates of change of capital ($\hat{k}$) and labour ($\hat{\ell}$). That is, firms form their desired values of:[5]

$$k = D\log K = \beta_7 \ k_1 + (1.0-\beta_7)k_2$$

and

$$\ell = D\log L \ .$$

It is hypothesised that there will be a particular $\hat{k}_1$ and $\hat{\ell}$ for a given discrepancy between the real marginal product and the real marginal cost to the firm of each factor of production. The desired rate of growth of capital is also assumed to be a function of uncertainty. In the model this factor is measured by the excess of the inflaton rate (DlogP) over its steady state growth rate ($\lambda_2-\lambda_1$). That is:

$$\hat{k}_1 = \beta_8 \ (mpk-r_k) + \beta_9 \ (D\log P - (\lambda_2-\lambda_1))$$

$$\ell = \beta_{33}(mp\ell-\bar{\beta}_{34}w_r).$$

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5. The stock of mining capital (K_{minv}) and its rate of growth (k_2) are set exogenously in RBII.

where mpk and mpℓ are the marginal products of capital and labour respectively, and r_k and w_r are the respective real marginal costs.[6]

In RBII the production technology assumed for the calculation of marginal products is Cobb-Douglas with constant returns to scale and neutral technical progress.[7] That is:

$$y_{nf} = y_0 e^{\lambda_3 t} L^{(1.0-\bar{\beta}_{10})} K^{\bar{\beta}_{10}} .$$

From an empirical point of view, it is assumed that a more appropriate measure of the marginal product of capital is obtained by evaluating the marginal product using desired private output. The marginal products derived from this production function are thus:

$$mpk = \bar{\beta}_{10}(\hat{y}_{nf}-g_1)/K$$

and

$$mp\ell = (1.0-\bar{\beta}_{10})\ \hat{y}_{nf}/L .$$

The marginal costs are also equal to average costs, defined as:

$$r_k = (r_b/4.0) - DlogP$$

$$w_r = W(1.0+t_4)/P$$

where r_b and W are the nominal interest rate and nominal wage rate respectively, P is the price level, and t_4 is the average rate of payroll tax.

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6. β_{34} is a scaling factor to account for differences in units between mpℓ and w_r.

7. The parameters of the production function are fixed prior to the estimation of the rest of the model, on the basis of historical averages.

Dynamics take the form of partial adjustment. For employment the speed of adjustment also responds to a measure of capacity - in this case measured by the ratio between actual output and normal (or potential) output.[8] The capital and employment equations are given by equations (2) and (11) in Appendix 1. Equations (26) and (27) are identities that define k and ℓ in terms of the levels of capital (K) and employment (L). Thus it can be seen that the four equations (2), (26), (11) and (27) are equivalent to a pair of second-order equations in the levels of capital and employment.

The price of domestic production is specified as a geometric weighted sum of the prices for non-farm production and farm production. The latter is proxied by the price of exports of goods and services. It is assumed that firms determine the before-tax desired price of non-farm production as a mark-up over normal unit labour costs where the latter takes into account the costs to the employer of payroll tax. That is:

$$\hat{P}_{bt} = P_{bt_0}\{1.0/(1.0 - \beta_{10})\}WL(1.0+t_4)/\hat{y}_{nf} \ .$$

Actual before tax prices adjust to this desired level, but this adjustment is modified by two other factors.

Buffering, which was discussed in relation to the firms supply decision earlier, will typically have the effect of smoothing both quantities and prices. Consequently, disequilibrium in firms' buffer assets will have an impact on

8. Normal output is defined as the level that could be produced with existing capital and employment given the assumed production function .

firms' pricing decisions. Price behaviour should therefore be influenced by the discrepancy between desired and actual inventories. Firms may also use their liquidity position as a buffer asset. Thus it might be expected that the discrepancy between firms' desired and actual money balances will also impinge on decisions concerning production, factor demands and pricing.

Firm's disequilibrium in real money balances is proxied by aggregate private sector disequilibrium in real money balances. Attempts to find an empirically significant role for this variable in the factor demand equations and supply decisions have not been successful. However, such an effect is found in the price equation.

The change in before-tax prices is therefore modelled as:

$$\text{Dlog } P_{bt} = \alpha_7 \log(\hat{P}_{bt}/P_{bt}) + \beta_{22} \log(P\hat{m}/M) + \beta_{23} \log(\hat{v}/v).$$

Desired real money balances are defined as a function of real income and a vector of interest rates:

$$\hat{m} = m_0 \; y \; e^{\beta_4 r_m + \beta_5 r_b + \beta_6 r_{b\ell}}$$

The desired price of exports of goods and services is assumed to be a weighted sum of wool prices and a general index of world prices. Wool prices represent a major commodity price over which domestic suppliers have some impact. Dynamics are added by a partial adjustment process and the rate of adjustment is also assumed to be affected by inventory disequilibrium. The equation for the price of exports of goods and services is equation (8) in Appendix 1.

II.1.ii Labour supply, household expenditure and asset demands

It is assumed that households have some steady-state desired participation rate N_o. In the short run, however, desired participation is assumed to vary from the steady state rate depending on the extent to which real wages net of direct taxes vary from trend real wages.[9] That is:[10]

$$\hat{N} = N_o\{W(1.0-t_1)/(P_d w_o e^{\lambda_4 t})\}^{\beta_{37}} Z .$$

The partial adjustment of actual labour supply to this desired value is supplemented by a discouraged worker effect, proxied by the ratio of employment to labour supply. That is:

$$D\log N = \bar{\alpha}_{12} \log(\hat{N}/N) + \beta_{36} \log(L/\phi_o N) .$$

The equation for households' nominal consumption expenditure is:

$$D\log(P_d d) = \alpha_1 \log(\hat{Pd}/P_d d) + \beta_1 \log(Pm/\hat{M}).$$

Desired real consumption is a function of real disposable income (deflated by P_d rather than P) and a real rate of interest. Actual consumption adjusts to this desired level, modified by the effect of monetary disequilibrium, which represents the role of money as a buffer asset for the household sector.

Increases in the household sector's real stock of dwellings are assumed to be determined by a lagged adjustment to a desired housing stock (which is a function of real

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9. Nominal wages are deflated by the expenditure deflator.

10. As can be seen from Appendix 1, the estimate of the long run relative price elasticity of labour supply is negative, indicating a backward sloping supply of labour curve.

disposable income and the cost of borrowing), modified by excess supply in the labour market which proxies excess capacity in the economy. That is:

$$D\log K_h = \alpha_3 \log(\hat{K}_h/K_h) + \beta_{11} \log(L/\phi_o N)$$

$$\hat{K}_h = K_{h_o}\, y_d\, e^{\beta_{12} r_a}$$

The desired nominal demand for government securities by the non-bank private sector is assumed to be homogeneous of degree one in the price level. This allows the nominal demand to be written as the product of the price level and the real demand for bonds. The latter is specified as a function of real income and relative prices. Therefore:

$$\hat{b} = b_o y\, e^{\beta_{40} r_b + \beta_{41} r_m + \beta_{42}(r_b^e - r_b)}$$

The term $(r_b^e - r\ )$ measures the expected change in the interest rate on bonds. r_b^e is assumed to be the actual future bond rate in estimation (as in the study by Jonson (1972)), although in simulation it is endogenised as the "rational" expectation of the bond rate obtained from the bond rate reaction function.[11]

The partial adjustment of bonds to their desired level is also affected by monetary disequilibrium (the private sector's buffer asset) and a dummy variable to capture the introduction of Australian Savings Bonds. That is:

$$D\log B = \alpha_{13} \log(P\hat{b}/B) + \beta_{38} \log(P\hat{m}/M) + \beta_{39}\, QSB$$

Net Australian private capital owned by overseas residents is assumed to respond to a discrepancy between

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11. See the discussion of the r_b reaction function below.

desired and actual stocks. The desired level of net foreign-owned capital is also assumed to be homogeneous of degree one in prices and income and of degree zero in interest rates:

$$\hat{f} = f_o \, y \, e^{\beta_{46} [r_{be} - r_{eu}]} \, E/_{\hat{E}}{}^{\beta_{47}} \, e^{\beta_{48} QF}$$

$$\text{where} \quad \hat{E} = E_o \, P/_{P_w}$$

represents the expected rate of depreciation of the exchange rate.

This adjustment is also affected by the state of the trade account which is proxied by the ratio of nominal exports to imports and the rate of change of the interest differential, and by monetary conditions as measured by the proportional excess of M3 over its trend value. That is:

$$DlogF = \log(P\hat{f}/F) + \beta_{43} \log M/_{Moe^{\lambda_2 t}}$$

$$+ \beta_{44} \log(\gamma_o P_x x/EP_i i) + \beta_{45} D(r_{b\ell} - r_{eu})$$

II.1.iii <u>Consolidated Banking System and Government Sector</u>

The rate of growth in bank advances is assumed to be supply determined but the rate of change is influenced by demand factors, proxied by nominal income relative to trend and the rate of change of the value of inventories.

Banks are assumed to take as given the level of deposits, proxied by the volume of money. Banks are required to hold reserves equal to a proportion of their deposits, that is, hM. It is assumed that banks allocate the remainder of their portfolio, (1.0-h)M, between advances and holdings of government securities on the basis of relative returns. The bank advances equation is thus:

$$DlogA = \bar{\alpha}_{15} \log(\hat{A}/A) + \beta_{49} \log(Py/P_o y_o \; e^{\lambda_2 t})$$

$$+ \beta_{50} Dlog(Pv) + \beta_{51} QA$$

$$\hat{A} = A_o \; (1.0-h)Me^{\beta_{52}(r_b - r_a)}$$

Two policy instruments are modelled with policy reaction functions. The two variables are the rate of interest on long (ten year) government securities and the \$A/\$US exchange rate. The underlying economic determinants of each instrument are captured in the own partial adjustment term. In the short run, behaviour is influenced by other objectives. The reaction functions are given as equations (18) and (19) of the model specification.

The desired bond rate is assumed to be a weighted sum of private sector interest rates (proxied by the bill rate) and the long-run equilibrium bond rate which is in turn assumed to be the sum of a constant real rate and the annualised growth rate of money:

$$\hat{r}_b = r_{b_o} + \beta_{67} \; 4.0 \; DlogM + (1.0-\beta_{67})r_{bl}$$

The other objectives represented in the bond rate reaction function are employment conditions, monetary targets (since 1975), and the behaviour of foreign reserves.[12]

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12. The expected bond rate is, as explained above, endogenised for simulation by applying a "rational" expectations mechanism to the core of the bond rate reaction function, on the assumption that $\hat{r}_b$ is expected to remain unchanged. Thus, in the discretised form of the model used for simulation,

$$E[J_{+1} \; rb] = \frac{\alpha_{21}}{1 + \alpha_{21}} J\hat{r}_b + \frac{1 - 1/2\;\alpha_{21}}{1 + 1/2\;\alpha_{21}} Jrb$$

which is used to replace r_b^e in the equation for non-bank holdings of government securities. (J denotes a two-period moving average.)

The desired exchange rate is assumed to be given by a general purchasing power parity. The other influence in the reaction function is a short-run interest disparity term.

Exchange rate expectations in $\hat{f}$, $\hat{r}_{bl}$, and $\hat{r}_a$ are assumed to be derived from the purchasing power parity relationship.[13] The reaction functions also include synthetic variables to allow for discrete changes in the instruments which cannot otherwise be captured by the continuous-time specification and in the exchange rate equation an additional dummy variable for the period to 1971 when the exchange rate was fixed.

II.1.iv Foreign Sector

The desired demand for Australian exports of goods and services depends on world activity (proxied by world trade) and relative prices. That is:

$$\hat{x}^d = x_o^d \; x_w (P_x/EP_w)^{\beta_{16}} .$$

The elasticity with respect to world trade is unitary so the above equation implies that Australia's share of world trade is solely a function of relative prices.

The partial adjustment of actual exports to this demand for exports is supplemented by two supply factors. The first is seasonal conditions in the rural economy, proxied by the ratio of farm product to total product. The second is the level of excess demand in the domestic economy, proxied by inventory disequilibrium. The export equation is:

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13. A detailed treatment of the nature and role of exchange rate expectations is provided in Jonson, McKibbin and Trevor (1982).

$$\mathrm{Dlog}x = \log(\hat{x}^d/x) + \beta_{13}\log(\hat{v}/v)$$

$$+ \beta_{14}\,QDS + \beta_{15}\log(\mu_0 y_f/y)\ .$$

II.1.v <u>Jointly Determined</u>

Wage demands are assumed to be homogenous of degree one in prices so that nominal average weekly earnings adjust so as to equate the real marginal product of labour with the real wage rate. This adjustment is influenced by three other factors: first, the state of the labour market (measured by the ratio of employment to desired labour supply); second, the assumed role of the Arbitration Commission in influencing expectations as proxied by the ratio of real award wages to trend and the rate of change of this variable; and third, inflationary expectations proxied by the level of disequilibrium money balances. That is:

$$\mathrm{Dlog}W = \alpha_{10}\log(\hat{W}/W) + \beta_{29}\log(L/\phi_0\hat{N})$$

$$+ \beta_{30}\log(w_a/w_{a_0}e^{\lambda_5 t})$$

$$+ \beta_{31}(\mathrm{Dlog}w_a - \lambda_5) + \beta_{32}\log(Pm/\hat{M})$$

The price of government current expenditure adjusts over time to its desired level which is assumed to be a weighted sum of wages and the price of output. This specification allows for the high proportion of public sector salaries in government current spending. The specification is given as equation (9) of Appendix 1.

The bank bill rate, advances rate and money rate are short-term market-determined rates of interest. Their desired values are assumed to be weighted sums of an equivalent foreign rate adjusted for exchange rate expectations and another domestic rate. Partial adjustment processes provide dynamics. In the advances and money rate equations, terms in Dlog (M/A) are included to represent the free liquidity of the banking system.[14] These equations are (17), (18) and (19) of the model specification given in Appendix 1.

There are obvious problems of modelling personal income taxes in a continuous-time framework. The approach followed in RBII is to model the rate of change of tax collections as a weighted sum of the current change and exponentially weighted lagged levels of the equilibrium levels of taxes as some collections are at source but others adjust slowly.[15] That is:

$$D\log T_{11} = a_{16}\log(\hat{T}_{11}/T_{11}) + \beta_{49}(D\log\hat{T}_{11} - \lambda_2) .$$

Desired taxes are defined as the product of a tax rate and an income base, that is:

$$\hat{T}_{11} = T_{11_0} t_1 \; WL .$$

In a progressive income tax system, the tax rate t_1 is not wholly exogenous. In RBII, t_1 is defined as:

$$t_1 = t_{11} W^{\bar{\beta}_{54}}$$

where t_{11} represents the average level of tax rates and $(1.0+\beta_{54})$ represents the progressivity coefficient.

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14. Here, M represents the level of deposits with banks.

15. See Brady (1978b).

II.2 IDENTITIES

RBII emphasises the role of buffer stocks in the dynamic adjustment of the economy. The levels of the buffer stocks (money and inventories) are determined residually from the balance sheet identities in the model, and thus represent the net outcomes of a number of individual decisons by all sectors. Disequilibria in buffer stocks are then specified to modify the decisions of the private sector; this feedback ensures the long-run equality of actual and desired holdings of the buffer stocks. This is the most convenient method of modelling the buffer stock effects, since it avoids the need to specify the complex short-run demand functions for money and inventories which the approach implies.[16]

II.2.1 The Determination of Money

The supply of money is obtained in RBII as the residual item in the economy's financial balance sheet. There are three sectoral flow constraints, so only two need to be imposed in the model. These are discussed in turn.

(i) Foreign Sector.

The flow constraint appears as the foreign reserves identity setting the rate of change of reserves equal to the rate of exports (in nominal terms) less the rate of imports (in nominal terms) plus the rate of change of the net stock of private Australian capital owned by foreigners plus the rate of change of the net stock of Australian government capital owned by

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16. The expressions for $\hat{m}$ and $\hat{v}$ are thus long-run in nature, while the short-run demands are identically equal to actual stocks. Jonson (1976) gives a more complete exposition of the buffering concept.

foreigners. Using the notation of the model this can be written as:

$$DR = P_x x - EP_i i + DF + DF_g$$

which is equation (20) in Appendix 1.

(ii) Consolidated Banking System and the Government Sector.

The government sector flow constraint can be written as:

$$G\text{-}T = DB_{RBA} + DB_B + DB_P + 0$$

where G is government expenditure

T is government taxes

B_{RBA} is bonds held by the RBA

B_B is bonds held by the banks

B_p is bonds held by the private, non-bank sector

0 is other financing.

This equation can be rewritten as:

$$DB_{RBA} = G\text{-}T - DB_B - DB_p + 0.$$

The consolidated balance sheet of the private banks and the monetary authorities can be written to show that the rate of change in the volume of money (liabilities of these sectors) is equal to:

$$DM3 = DR - DF_g + DB_{RBA} + DB_B + DA + DMISC$$

where MISC is other miscellaneous assets and liabilities of the consolidated banking sector. The two sectors' flow constraints can be consolidated into identities determining the changes in domestic credit (C) and money:

$$DC = G\text{-}T - DB + DA + DMISC$$

and

$$DM3 = DR - DF_g + DC$$

which are equations (21) and (22) of the model.

The buffering role of money is modelled by the inclusion of monetary disequilibrium in the equations for household expenditure, product prices, earnings and non-bank holdings of government securities.

II.2.2 The Determination of Inventories

RBII models the supply of output and the major components of demand. In this framework, the national income identity is used to determine the actual rate of change of inventories as the residual which equates supply and demand in the short run. The national income identity thus appears as equation (23) in Appendix 1, as the equation for inventories.

The buffering role of inventories is captured by the effects of inventory disequilibrium on exports, imports, output and product and export prices.

III. THE FIT OF THE MODEL

The parameters of RBA82 are estimated from a log-linear discrete-time approximation to the original model,[17] using quarterly data from 1959(3) to 1980(4). The full twenty-nine equation system is estimated simultaneously using the FIML package RESIMUL developed by Wymer (1968).

A measure of the goodness of fit of the model as a complete system is given by the Carter-Nagar system R^2, which is 0.6501; the chi-square test rejects the hypothesis that the model is not consistent with the data.[18]

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17. The techniques used are detailed in Appendix 2.

18. The system R^2 has the same interpretation as traditional single-equation R^2 measures, and the chi-square test rejects the null hypothesis that the true system R^2 is zero. See Carter and Nagar (1977).

Table 1 shows the fit of the equations of the model as estimated, in dynamic simulation over two sample periods.[19] It is clear that the model performs well throughout, and is noticeably better for price variables (except export prices) over the shorter period of 1974-1980.

TABLE 1: MEASURES OF FIT, LINEARISED RBA82 MODEL

	RMSE* 1959(4)-80(4)	RMSE* 74(1)-80(4)
log d	2.05	2.62
k	.30	.29
log K_h	1.06	1.33
log x	4.85	4.22
log i	7.00	7.22
log y	2.65	2.42
log P	4.56	2.59
log P_x	6.60	8.61
log P_g	4.82	3.05
log w	3.83	1.71
l	.43	.43
log N	.80	.46
log B	6.95	6.32
log F	10.61	7.06
log A	5.36	3.55
log T_{11}	9.99	6.66
r_{bl}	.73	1.01
r_a	.63	.92
r_m	.78	.96
r_b	.79	.96
log E	5.27	5.88
log R	17.72	15.43
log C	3.70	2.49
log M	2.76	3.28
lov v	5.01	4.03
log S^e	1.93	2.16
log Sp^e	2.20	2.45
log K	3.83	3.07
log L	1.47	1.56

* x10^2. For variables in logarithms, the RMSE is approximately equal to the Root Mean Squared Percentage Error of the original variables (d, K_h, x, i, ...).

19. The full period is 1959(4)-1980(4); the first quarter is used to initialise the simulation.

An additional perspective on the fit of RBA82 is given by Table 2, which compares the RMSE of key equations of RBA76 and RBA84.[20]

TABLE 2: COMPARISON OF ROOT MEAN SQUARE ERRORS, RBA76 AND RBA82 (LINEARISED MODELS)

	RMSE* 1963(2) - 1974(4) RBA76	RBA82
log d	2.62	0.90
k	0.27	0.30
log x	8.01	4.27
log i	6.66	6.19
log y	3.14	1.88
log P	3.28	4.68
log W	3.98	4.65
log L	0.81	0.80
log R	22.21	20.63
log M	7.77	1.73
log v	12.62	4.12

* $\times 10^2$

RBA82 outperforms RBA76 in almost all cases. The exceptions are wages and prices; Table 1 suggests that the fit of RBA82 for these variables is better in the late 1970s. It is quite possible that RBA76 would perform poorly, relative to RBA82, if simulated beyond the end of 1974.

Similarly, RBA82 would be of limited usefulness in simulations into the 1980s since the structure of the economy (and of the financial system in particular) has changed considerably. The model would need to be altered to allow for floating exchange and bond rates, and policy reaction functions

20. The simulation period 1963(2)-1974(4) was that reported for RBA76 in Jonson, Moses and Wymer (1976), from where the first column is taken.

for exchange intervention and bond tender sizes, as well as shifts in private sector behaviour brought about by these changes in policy structure. Re-estimation, using the systems estimation technique favoured to date in the RBII project, is not feasible due to lack of data, and alternative methods using simulation models (not directly estimated in the traditional way) must be explored. It is envisaged that the major application of the RBA82 structure as outlined in this paper will be as the starting point from which such models are developed.

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APPENDIX 1: RBII SPECIFICATION[1]

1. Household Expenditure

$$D\log(P_d d) = \alpha_1 \log(P\hat{d}/P_d d) + \beta_1 \log(P\hat{m}/M)$$

$$\hat{d} = d_o y_d e^{\beta_2[(r_a/4.0) - D\log P]}$$

$$y_d = y - T_1/P + c$$

$$P_d = P_{d_o}[EP_i(1+t_3)]^{\beta_3} P^{(1.0-\beta_3)}$$

$$\hat{m} = m_o\, y e^{\beta_4 r_m + \beta_5 r_b + \beta_6 r_{bl}}$$

2. Rate of Growth of Business Fixed Capital Stock

$$Dk = \alpha_2 (\hat{k} - k)$$

$$\hat{k} = \beta_7 \hat{k}_1 + (1.0-\beta_7)\hat{k}_2$$

$$\hat{k}_1 = \beta_8 (mpk - r_k) + \beta_9 (D\log P - (\lambda_2 - \lambda_1))$$

$$\hat{k}_2 = D\log K_{minv} - (\lambda_2 - \lambda_1)$$

$$mpk = \bar{\beta}_{10} (\hat{y}_{nf} - g_1)/K$$

$$r_k = (r_b/4.0) - D\log P$$

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1. A subscript of zero (o) indicates a constant. A bar (-) above a parameter indicates that it is set prior to the estimation of the other parameters of the system. D is the differential operation d/dt, e is the exponential operator, and log is the logarithmic (to base e) operator. A variable with a hat (^) above it indicates the desired value of the variable.

3. Stock of Dwellings

$$D\log K_h = \alpha_3 \log(\hat{K}_h/K_h) + \beta_{11} \log(L/\phi_o N)$$

$$\hat{K}_h = K_{h_o} y_d e^{\beta_{12} r_a}$$

4. Exports of Goods and Services

$$D\log x = \alpha_4 \log(\hat{x}^d/x) + \beta_{13} \log(\hat{v}/v) + \beta_{14} QDS$$

$$+ \beta_{15} \log(\mu_o y_f/y)$$

$$\hat{x}^d = x_o^d \, x_w(P_x/EP_w)^{\beta_{16}}$$

$$\hat{v} = v_o s_p^e$$

5. Imports of Goods and Services

$$D\log i = \alpha_5 \log(\hat{i}/i) + \beta_{17} \log(\hat{v}/v)$$

$$\hat{i} = [i_o[EP_i(1.0+t_3)/P]^{\beta_{18}} e^{\beta_{19} Q_i}](s^e - y_f)$$

6. Domestic Production

$$y = y_{nf} + y_f$$

$$D\log y_{nf} = \alpha_6 \log(\hat{y}_{nf}/y_{nf}) + \beta_{20} \log(\hat{v}/v)$$

$$\hat{y}_{nf} = [1.0-\hat{i}](s^e - y_f)$$

7. Price of Domestic Production

$$D\log P = \beta_{21} D\log P_{bt} + \beta_{21} D\log(1.0+t_6) + (1.0-\beta_{21}) D\log P_x$$

$$D\log P_{bt} = \alpha_7 \log(\hat{P}_{bt}/P_{bt}) + \beta_{22} \log(P\hat{m}/M)$$

$$+ \beta_{23} \log(v\hat{/}v)$$

$$\hat{P}_{bt} = P_{bt_o}[1.0/(1.0-\bar{\beta}_{10})]WL(1.0+t_4)/\hat{y}_{nf}$$

8. Price of Exports of Goods and Services

$$D\log P_x = \alpha_8 \log(\hat{P}_x/P_x) + \beta_{24} \log(\hat{v}/v)$$

$$\hat{P}_x = P_{x_0}(EP_{w1})^{\beta_{25}}[EP_i(1.0+t_3)]^{(1.0-\beta_{25})}$$

9. Price of Government Current Expenditure

$$D\log P_g = \alpha_9 \log(\hat{P}_g/P_g)$$

$$\hat{P}_g = P_{g_0} W^{\beta_{26}} P^{(1-\beta_{26})}$$

10. Average Weekly Earnings

$$D\log W = \alpha_{10} \log(\hat{W}/W) + \beta_{29} \log(L/\phi_0\hat{N})$$

$$+ \beta_{32} \log(P\hat{m}/m) + \beta_{30} \log(w_a/w_{a_0}e^{\lambda_5 t})$$

$$+ \beta_{31} (D\log w_a - \lambda_5)$$

$$\hat{W} = W_0(1.\bar{0}-\beta_{10}) P\hat{y}_{nf}/L$$

11. Rate of Growth of Employment

$$D\ell = \alpha_{11} (\hat{\ell}-\ell) + \beta_{33}\log(y/\bar{y})$$

$$\hat{\ell} = \beta_{34} (mp\ell - \bar{\beta}_{35}w_r)$$

$$mp\ell = (1.0-\bar{\beta}_{10}) \hat{y}_{nf}/L$$

$$w_r = W(1.0+t_4)/P$$

$$\bar{y} = y_0 e^{\lambda_3 t} L^{(1.0-\bar{\beta}_{10})} K^{\bar{\beta}_{10}}$$

12. Labour Supply

$$D\log N = \bar{\alpha}_{12} \log(\hat{N}/N) + \beta_{36} \log(L/\phi_o N)$$

$$\hat{N} = N_o \{W(1.0-t_1)/P_d w_o e^{\lambda_4 t}\}^{\beta_{37}} Z$$

13. Non-Bank Holdings of Government Securities

$$D\log B = \alpha_{13} \log(P\hat{b}/B) + \beta_{38} \log(P\hat{m}/M) + \beta_{39} QSB$$

$$\hat{b} = b_o y e^{\beta_{40} + \beta_{41} r_m + \beta_{42} (r_b^e - r_b)}$$

14. Net Australian Private Capital Owned by Overseas Residents

$$D\log F = \alpha_{14} (\log P\hat{f}/F) + \beta_{43} \log(M/Moe^{\lambda_2 t})$$

$$+ \beta_{44} \log(\gamma_o P_x x/EP_i i) + \beta_{45} (Dr_{bl} - Dr_{eu})$$

$$\hat{f} = f_o \; ye^{\beta_{46}(r_{bl} - r_{eu}) + \bar{\beta}_{47}\xi + \beta_{48} QF}$$

$$\xi = \log(\hat{E}/E)$$

15. Bank Advances

$$D\log A = \alpha_{15} \log(\hat{A}/A) + \beta_{49} \log(Py/P_o y_o e^{\lambda_2 t})$$

$$+ \beta_{50} D\log(Pv) + \beta_{51} QA$$

$$\hat{A} = A_o (1.0-h) M e^{\beta_{52}(r_b - r_a)}$$

16. Personal Income Taxes

$$D\log T_{11} = \alpha_{16} \log(\hat{T}_{11}/T_{11}) + \beta_{53} (D\log \hat{T}_{11} - \lambda_2)$$

$$\hat{T}_{11} = T_{11_o} t_1 WL$$

$$t_1 = t_{11} W^{\beta_{54}}$$

17. Bank Bill Rate

$$Dr_{bl} = \alpha_{17} (\hat{r}_{bl} - r_{bl}) + \beta_{55} DQr_{bl}$$

$$\hat{r}_{bl} = r_{bl_o} + \bar{\beta}_{56} r_{eu} + (1.0 - \bar{\beta}_{56}) r_m + \bar{\beta}_{57} \xi + \beta_{58} QE$$

18. Bank Advances Rate

$$Dr_a = \alpha_{18}(\hat{r}_a - r_a) + \beta_{59} DlogM/A$$

$$\hat{r}_a = r_{ao} + \beta_{60} r_b + (1-\beta_{60}) r_w + \beta_{61} \xi$$

19. Money Rate

$$Dr_m = \alpha_{19}(\hat{r}_m - r_m) + \beta_{27} Dlog M/A + \beta_{28} Qr_m$$

$$\hat{r}_m = r_{mo} + r_b$$

20. Bond Rate

$$Dr_b = \alpha_{20} (\hat{r}_b - r_b) + \bar{\beta}_{62} \log(L/\phi_o N) + \beta_{63} \log(M/M^t) + \beta_{64} \log(R/\theta_o M) + \beta_{65} (DlogR - \lambda_2) + \beta_{66} QS$$

$$\hat{r}_b = r_{b_o} + \beta_{67} 4.0 DlogM + (1.0 - \beta_{67}) r_{bl}$$

21. ($A/$US) Exchange Rate

$$DlogE = \alpha_{21} \log(\hat{E}/E) + \beta_{68} (r_{bl} - r_{eu}) + \beta_{69} QUS + \beta_{70} QER + \beta_{71} Q60$$

$$\hat{E} = E_o (P/P_w)$$

22. Foreign Reserves

$$DR = P_x x - EP_i i + DF + DF_g$$

23. Domestic Credit

$$DC = P_g g_1 + Pg_2 + Pc + I - T_1 - T_2 - DB + DA + DMISC$$

$$T_1 = T_{11} + T_{12}$$

$$T_{12} = t_5 CTB$$

$$T_2 = T_{21} + T_{22}$$

$$T_{21} = t_2 Pd$$

$$T_{22} = T_{22_0} t_4 WL$$

24. Volume of Money

$$DM = DR + DC - DF_g$$

25. Inventories

$$Dv = y + i - s$$

$$= y + i - d - DK - DK_h - x - g_1 - g_2 - g_3 - sd$$

26. Expected Sales

$$D\log s^e = \bar{\alpha}_{22} \log(\hat{s}^e/s^e)$$

$$\hat{s}^e = s_o^e \, s$$

$$s = d + x + DK + DK_h + g_1 + g_2 + g_3 + sd$$

27. Private Expected Sales

$$D\log s_p^e = \bar{\alpha}_{23} \log (\hat{s}_p^e / s_p^e)$$

$$\hat{s}_p^e = s_{p_o}^e \, s_p$$

$$s_p = s - g_1$$

28. Business Fixed Capital Stock

$$D\log K = k$$

29. Employment

$$D\log L = \ell$$

VARIABLES USED IN RBII

A	bank advances to private sector
B	government bonds held by private non-bank groups
c*	real cash benefits to persons
C	domestic credit
CTB*	effective company tax base
d	real household consumption expenditure
E	exchange rate ($A/$US)
F	net Australian private capital owned by overseas residents
F_g*	net Australian government capital owned by overseas residents
F	net Australian private capital owned by overseas residents
g_1*	real government current expenditure
g_2*	real government capital expenditure
g_3*	real public authorities capital expenditure
h*	required liquidity ratio of the banking sector
i	real imports of goods and services
I*	interest payments on government debt
k	proportionate change in the real stock of business fixed capital
k_1	proportionate change in the real stock of non-mining business fixed capital
k_2	proportionate change in the stock of mining business fixed capital
K	real stock of business fixed capital
K_h	real stock of dwellings
K_{minv}*	stock of mining capital
ℓ	proportionate change in employment
L	employment
m	real stock of money (M/P)
mpk	marginal product of capital
mpℓ	marginal product of labour
M	stock of money (M3)
M^{t}*	target stock of money (M3)
MISC*	miscellaneous items in the money supply identity
N	labour supply
P	price of domestic output
P_{bt}	price of domestic output net of indirect taxes
P_d	consumption deflator
P_g	price of government consumption expenditure
P_i*	Australian import prices ($US)
P_w*	world prices ($US)
P_{wl}*	price of wool ($US)
P_x	price of exports
QA*	synthetic variable for growth of bank advances, 1973
QDS*	synthetic variable for U.S. dock strike, 1969
QE*	synthetic variable for expectations about the exchange rate, 1972-6
QER*	synthetic variable for timing of exchange rate changes, 1972, 1973, 1974, 1976
QF*	synthetic variable for capital inflow during "resources boom", 1980
Q_i*	synthetic variable for growth of imports, 1974-1980
Ql*	dummy variable for shake-out effect in labour market, 1974-6
Q60*	synthetic variable for period of fixed exchange rate, 1959-1971
Qr_{bl}*	dummy variable for increases in commercial bill rate, 19
Qr_m*	synthetic variable for rise in r_m, 1973
QS*	synthetic variable for increases in official interest rates, 1961, 1973
QSB*	dummy variable for the introduction of Australian Saving Bonds 1976(1)-(2)
QUS*	dummy variable for devaluation of $US, 1973
r_a	interest rate on bank advances
r_b	interest rate on 10 year government bonds
r_b^e	expected next-period interest rate on 10 year government bonds
r_{bl}	interest rate on 90 day commercial bills
r_{eu}*	interest rate on 90 day Eurodollar bills
r_k	real marginal cost of capital
r_m	interest rate on trading bank fixed deposits
r_w*	interest rate on 10 year US government bonds
R	gold and foreign exchange reserves
s	sales
s^e	expected sales
s_β^e	private expected sales
sd*	real statistical discrepancy
t*	time trend starting in 1959(3)
t_{11}*	index of income tax rate schedule
t_2*	average rate of tax on consumption
t_3*	average rate of tariffs
t_4*	average rate of payroll tax
t_5*	statutory company tax rate
t_6*	average rate of tax on expenditure
T_1	receipts of direct taxes
T_{11}	receipts of personal income tax
T_{12}	receipts of company tax
T_2	receipts of indirect taxes
T_{21}	receipts of sales tax
T_{22}	receipts of payroll tax
v	real stock of inventories of goods
w_a*	index of real award wages

real marginal cost of labour

index of average earnings

real exports of goods and services

real demand for exports of goods and services

real supply of exports of goods and services

real world exports of goods and services

real domestic output (net of depreciation)

real normal domestic output (net or depreciation)

real disposable income

real farm output (net of depreciation)

real non-farm output (net of depreciation)

population of working age

expected rate of depreciation

TABLE A1.1 PARAMETER ESTIMATES[2]

Parameter	Estimate	Asymptotic Standard Error
α_1	0.112	0.027
α_2	0.688	0.141
α_3	0.024	0.002
α_4	0.834	0.158
α_5	0.207	0.060
α_6	0.900	0.145
α_7	0.413	0.093
α_8	0.538	0.098
α_9	1.435	0.257
α_{10}	0.232	0.048
α_{11}	2.169	0.351
α_{12}	0.130	-
α_{13}	0.101	0.025
α_{14}	0.056	0.014
α_{15}	0.206	0.020
α_{16}	0.415	0.105
α_{17}	0.225	0.057
α_{18}	0.219	0.025
α_{19}	0.447	0.051
α_{20}	0.272	0.024
α_{21}	0.048	0.016
α_{22}	0.500	-
α_{23}	0.500	-
β_1	-0.029	0.013
β_2	-5.775	1.554
β_3	0.200	-
β_4	14.642	6.457
β_5	-18.807	6.981
β_6	-6.470	2.595
β_7	0.934	0.024
β_8	0.489	0.072
β_9	-0.391	0.063
β_{10}	0.400	-
β_{11}	0.009	0.007
β_{12}	-1.019	0.254
β_{13}	-0.117	0.183
β_{14}	0.061	0.057
β_{15}	0.214	0.061
β_{16}	-0.333	0.067
β_{17}	0.645	0.147
β_{18}	-1.065	0.173
β_{19}	0.015	0.003
β_{20}	0.188	0.061
β_{21}	0.928	0.009
β_{22}	-0.079	0.031
β_{23}	0.180	0.046
β_{24}	0.019	0.154
β_{25}	0.231	0.048

Parameter	Estimate	Asymptotic Standard Error
β_{26}	0.718	0.014
β_{27}	-0.130	0.024
β_{28}	0.005	0.001
β_{29}	0.200	0.026
β_{30}	0.048	0.025
β_{31}	0.381	0.057
β_{32}	-0.044	0.017
β_{33}	0.258	0.057
β_{34}	0.081	0.014
β_{35}	1.56611	-
β_{36}	0.139	0.028
β_{37}	-0.327	0.079
β_{38}	-0.065	0.035
β_{39}	-0.051	0.009
β_{40}	7.348	8.489
β_{41}	-19.606	6.755
β_{42}	-83.331	17.666
β_{43}	-0.503	0.059
β_{44}	-0.113	0.024
β_{45}	0.250	0.155
β_{46}	6.765	2.501
β_{47}	-0.500	-
β_{48}	0.163	0.016
β_{49}	0.057	0.009
β_{50}	0.172	0.042
β_{51}	0.029	0.003
β_{52}	-7.605	0.890
β_{53}	0.558	0.191
β_{54}	0.5436	-
β_{55}	0.043	0.003
β_{56}	0.500	-
β_{57}	0.050	-
β_{58}	0.022	0.015
β_{59}	-0.027	0.015
β_{60}	0.530	0.083
β_{61}	0.050	0.005
β_{62}	0.200	-
β_{63}	0.127	0.025
β_{64}	-0.012	0.001
β_{65}	-0.019	0.003
β_{66}	0.002	0.001
β_{67}	0.293	0.022
β_{68}	-0.082	0.056
β_{69}	0.098	0.012
β_{70}	0.084	0.006
β_{71}	0.018	0.006

2. Estimation period is 1959(3) - 1980(4). Parameters without a standard error are imposed prior to estimation.

IMPOSED STEADY STATE GROWTH RATES

Variable[3]	Notation	Growth Ra
d, K, K_h, x, i, y, v, g^*_1, g^*_2, g^*_3, c^*, s^e, s^e_p, sd^*, y^*_f, x^*_w	λ_1	(.012)
P, P_x, P^*_i, P^*_w, P^*_{w1}	$\lambda_2 - \lambda_1$	(.009)
P_g, W	$\lambda_2 - \lambda_1 + \lambda_4$	(.0165
N, L, Z^*	$\lambda_1 - \lambda_4$	(.0045
Trend rate of technical progress	λ_3	(.0045
Trend rate of growth of labour productivity	$\lambda_4 = \lambda_3/(1 - \beta_{10})$	(.0075
w_a	λ_5	(.0040
B, F, A, T_{11}, R, C, M, CTB^*, F^*_g, $MISC^*$, M^{t*}, I^*, K^*_{minv}	λ_2	(.021)
E, r_b, r_{b1}, r^*_w, r^*_{eu}, r_m, r_a, k, ℓ, h^*, t^*_{11}, t^*_2, t^*_3, t^*_4, t^*_5, t^*_6	-	(0.0)

3. An asterisk (*) next to a variable indicates that it is exogenous.

APPENDIX 2: NOTES ON THE ESTIMATION OF THE RBII MODEL

RBII is non-linear and specified in continuous time with considerable use of within and across equation restrictions.[1] Economic data are usually observed at discrete intervals of time and so the continuous-time model cannot be estimated directly. Further, non-linearities increase the computational costs substantially which can be reduced by linearising the model for estimation. This appendix discusses the estimation of RBII with Wymer's RESIMUL programme.[2] This programme calculates full information maximum likelihood (FIML) estimates of the parameters of the linear, approximate discrete-time analogue to the continuous-time system.

In its general form, the RBII model can be written (after reducing its second order equations to pairs of first-order equations by the addition of suitably defined variables, and solving out zero-order equations) as a system of first order, non-linear differential equations, that is:

$$F[D\underset{\sim}{y}(t),\ \underset{\sim}{y}(t),\ \underset{\sim}{z}(t),\ \underset{\sim}{\theta}] = \underset{\sim}{\omega}(t) \qquad (1)$$

where

D is the differential equation $\frac{d(\ )}{dt}$;

$\underset{\sim}{y}(t)$ is a vector of endogenous variables;

$\underset{\sim}{z}(t)$ is a vector of exogenous variables;

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1. In the language of Challen and Hagger (1979), RBII belongs to the Phillips-Bergstrom-Wymer class of models.

2. See Wymer (1968).

$\underset{\sim}{\theta}$ is a vector of constant parameters; and

$\underset{\sim}{\omega}(t)$ is a vector of disturbance terms.[3]

A recursive system of linear first order differential equations is derived from (1), by linearising the model around some point, such as the steady state path, to give:

$$D\underset{\sim}{y}(t) = f\ [\underset{\sim}{y}(t),\ \underset{\sim}{z}(t),\ \underset{\sim}{s}(t),\ \underset{\sim}{\theta}] + \underset{\sim}{\omega}(t) \tag{2}$$

where

$\underset{\sim}{s}(t)$ is a vector of linearisation errors defined to ensure equality of F[ ] and f[ ].

The system of equations (2) can be written in matrix notation as:

$$DY(t) = A(\Theta)Y(t) + B(\Theta)Z(t) + C(\Theta)S(t) + T(t) \tag{3}$$

where A, B and Care matrices whose elements are functions of the constant parameter set Θ. These functions of parameters (which may be non-linear) will incorporate any restrictions implicit in the non-linear models as well as restrictions resulting from the linearization of the model.

As the model is specified in continuous time the endogenous and exogenous variables are defined at a point in time t. Consequently, if the model contains flow variables in addition to stock variables, the former are equivalent to 'a rate of change per unit of time at time t'. Such flow variables are not observable instantaneously. In general, economic data are only observable at discrete points in time or for flow variables as integrals over some observation

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3. Some of the elements of $\underset{\sim}{\omega}(t)$ may be assumed to be zero corresponding to identities in the system.

interval. Consequently a discrete-time analogue of the continuous-time model has to be derived.

An approximate discrete-time analogue to the continuous system is derived by integrating (3) over the observation interval (t-1, t) using the following approximations:

$$\int_0^1 DY(t-s)ds = \Delta Y_t \quad ; \quad \text{and}$$

$$\int_0^1 Y(t-s)ds = JY_t$$

where

$\Delta = 1-L;$

$J = \frac{1}{2}(1+L);$ and

L is the lag operator ($LX_t = X_{t-1}$).

This yields the set of first-order difference equations:

$$\Delta Y_t = A(\theta)JY_t + B(\theta)JZ_t + C(\theta)JS_t + T_t \,. \tag{4}$$

In this form Y and Z are still defined at a point in time. Consequently if the model includes both flow and stock variables it will be necessary to integrate (4) over the observation interval to give measurable flow variables. This gives:

$$\Delta \overset{\circ}{Y}_t = A(\theta)J\overset{\circ}{Y}_t + B(\theta)J\overset{\circ}{Z}_t + C(\theta)J\overset{\circ}{S}_t + \Omega_t \tag{5}$$

where

$$\overset{\circ}{X}_t = \int_{t-1}^{t} X(s)ds = \frac{1}{2}(1+L)X_t = JX_t \,.$$

For flow variables $\overset{\circ}{X}_t$ will be the observed data, for stocks $\overset{\circ}{X}_t$ will be the first order moving average (FORMA) of end of period stocks.

This second integral leads to the error term Ω_t being serially correlated. Wymer (1968) shows that the disturbance term is approximately:

$$\Omega_t = \Psi_t + 0.268\ \Psi_{t-1} \tag{6}$$

where Ψ_t is a serially uncorrelated random disturbance. As this moving average process is independent of the parameters of the model, the system may be transformed by the inverse of this process to give a model with serially uncorrelated residuals. Expanding the inverse of (6) as a Taylor series gives the approximation:

$$\Psi_t = \Omega_t - 0.268\ \Omega_{t-1} + (0.268)^2\ \Omega_{t-1} - (0.268)^3\ \Omega_{t-3}\ . \tag{7}$$

Accordingly, the discrete-time data can be 'pre-whitened' by filtering all the variables by a continuous systems moving average transformation (COSMA).

Thus the model estimated by FIML is:

$$\Delta\tilde{Y}_t = A(\Theta)\ J\tilde{Y}^0_t + B(\Theta) J\tilde{Z}^0_t + C(\Theta) J\tilde{S}^0_t + \Psi_t \tag{8}$$

where ~ above a variable indicates that it has been COSMA'd and Ψ_t is white noise.

It is equation (8) that is estimated directly in RESIMUL.