Deriving implied time-variation in DSGE coefficients from a kernel-estimated TVP-VAR *

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Abstract

This paper uses kernel methods (presented in Giraitis et al. (2011)) to estimate a 7 variable TVP-VAR on the US dataset constructed by Smets and Wouters (2007a) [SW]. Echoing the work of Christiano et al. (2005) we compute time variation in the impulse response to monetary policy shocks (here identified using sign restrictions) and fit the SW model to these time varying impulse response functions, deriving this way time-variation in the DSGE parameters. We find that many parameters change substantially, particularly those defining nominal rigidities, but also investment adjustment costs. Monetary policy parameters change, but not as much as is evident in other studies of the Great Moderation and its causes, contrary to the 'indeterminacy' theory of the Great Inflation.

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1 Introduction

This paper presents estimates of time-variation in the coefficients of the widely cited DSGE model in Smets and Wouters (2007a) (referred to hereafter as SW). The estimation strategy has three parts. It starts by estimating a time-varying parameter, reduced-form VAR [TVP-VAR] in the same 7 observed variables for the US as the DSGE model, using kernel methods that we proposed in previous work (Giraitis et al. (2011)) that, unlike other methods, can handle without difficulty a large VAR like this. Next, we use sign-restrictions to identify monetary policy (and other) shocks (in a fashion derived from the work of Uhlig (2005), Canova and Nicolo (2002) and others) for each observation in our sample period, corresponding to each of the 'instantaneous VARs' estimated by the TVP VAR. Finally, a minimum-distance estimator is used (see, for example, Theodoridis (2011)) to fit the DSGE model to these impulse response functions, once again, *for each observation in the sample period*. We therefore map changes in reduced form macro-dynamics (as manifest in the TVP-VAR) into implied changes in the structural DSGE parameters. Our approach is a deliberate echo of the work of Christiano et al. (2005) [hereafter CEE]. They estimated a DSGE model - the precursor to SW - by choosing the parameter constellation that minimised the distance between the impulse responses to a monetary policy shock in the DSGE model and a fixed-coefficient VAR. We do the same but starting out with a time-varying coefficient VAR that produces correspondingly time-varying DSGE parameter estimates. The connection with CEE's pioneering work is not perfect: we use sign restrictions to identify the monetary policy shock, while Christiano et al. (2005) used a Cholesky identification scheme appropriate for the timing implied by the limited participation mechanism built into their DSGE model (and not appropriate for SW); we fit the SW model, Christiano et al. (2005) of course fit their own, essentially the SW model less six of the shocks, but with an additional working capital channel. But these differences are details.

We find very substantial fluctuations in the DSGE parameters that best fit the evolving impulse response functions. It is useful at this point to give a flavour of some of our results: The probability that prices and wages are *not* reset in a given period varies between 0.6 and 0.85 for prices, and between 0.6 and 0.9 for wages. There are also significant changes in the coefficient that wage and price setters load onto past inflation when indexing between price changes: from 0.45 to 0.7 in the case of wages, and from 0.25-0.7 in the case of prices. We compute that there is an increase in implied values for h, which encodes habits in consumption, from around 0.75 to 0.85: a less dramatic change than for the nominal rigidities parameters, but, given the predominance of this parameter in determining overall persistence in the reduced form for inflation and other variables, still an important shift. The investment adjustment costs parameter rises precipitously in the boom years before 2000 and falls just as quickly thereafter. Interestingly a parameter that is probably most securely micro-founded, the discount rate, is estimated to be pretty constant. We also detect variations in the parameters defining the behaviour of monetary policy. The standard deviation of monetary policy shocks varies between 0.18 and 0.1; the persistence in these shocks varies between 0.2 and 0.45. The coefficients in the policy rule are, in absolute terms, more stable than some of the others coefficients, but they still each vary by as much as 0.2 from their maximum to their minimum values. There is no indication that there was ever a period where monetary policy was insufficiently responsive to inflation such that there was model indeterminacy, in contrast to findings of some other work on the Great Moderation (see, for example, Lubik and Schorfheide (2004) and Clarida et al. (2000)). Relatedly, we find that there are times when monetary policy pre-Volcker appears as or even more counter inflationary than that post-Volcker, a result that might seem surprising from the perspective of studies of monetary policy change that use split-sample analysis, eg Clarida et al. (2000).

2 Connections to existing work

What we do and find in our paper relates to several strands of thought in the literature in empirical macroeconomics. We make these connections below, distinguishing between the methodological literature on characterising and detecting structural change, and the empirical literature on uncovering and explaining structural change in VAR and DSGE model parameters.

2.1 Methodological literature on characterising structural change

The first line of work we want to emphasise is methodological. This paper is the latest in a series of papers we have written seeking to comment on the standard method for estimating stochastic time varying parameter VAR models in macroeconometrics, by offering an alternative, kernel-based method. These papers include Kapetanios and Yates (2011), which reworked the analysis of evolving inflation persistence in Cogley and Sargent (2005) using kernel methods; Giraitis et al. (2011) which derives the theoretical results on consistency and asymptotic normality of the kernel estimator for an AR model where the coefficients follow a bounded random walk and Giratis et al. (2012) which extends these results to the case of a VAR with stochastic volatility.

The standard method for estimating time varying coefficient VARs was presented by Cogley and Sargent (2005), Cogley et al. (2010) and others that followed them in a number of papers, including, for example: Benati and Surico (2008), Gali and Gambetti (2009a), Benati and Mumtaz (2007) and Mumtaz and Surico (2009) and many more. This method estimates the sequence of VAR parameters and volatilities by formulating the VAR as a state-space model (with the law of motion for the VAR parameters taking the place of the state transition equation) using the Gibbs sampler to characterise the joint posterior density. Most macro practitioners (certainly those cited above and more) are using the Carter and Kohn (1994) algorithm or analagous, which draws an entire sequence of parameters in the transition equation of the state-space model, and wish to enforce the restriction that at all points in the sample the hypothetical VAR corresponding to the values of the state at any point is instantaneously stationary (on the grounds that instances that breach this condition are not economically meaningful). As is well known (and discussed, for example, in Koop and Potter (2011)) this method can quickly become very slow, or entirely intractable in macro applications with persistent data, due to a failure to obtain enough, or even any draws, with applications where the VAR has a dimension of 5 or more. Koop and Potter (2011) present an alternative set of ('single move') algorithms that draw states (VAR parameters) one period at a time, easing this problem substantially, but at the computational cost of the chain mixing more slowly.

Our alternative kernel estimator is not subject to this problem. It delivers point estimates of the VAR parameter sequence (and confidence intervals) directly, not indirectly via characterising a posterior. The stationarity problem is not eliminated, of course. The frequentist user of the kernel method might find that having estimated the VAR parameter sequence that there are some values for which point estimates violate the stationarity condition. In which case, one would either proceed or stop, discarding the VAR as not meaningful. The Bayesian user of kernel methods would inevitably find that, without informed priors, some probablity mass were attached to the event that the VAR is non-stationary, in which case suitably informed priors could rule this out. (Note that we don't explain how a Bayesian would use our estimates, and leave this particular application of non-parametric Bayesian econometrics to future work).

These practical benefits - stressed in our earlier work - come into play in the application presented in this paper because we have such a large dimension VAR (7 variables). We assert that this would be problematic for sampling methods to handle estimating a TVP VAR of this dimension, but TVP-VARs of this size and even much greater can be executed easily with our method.

Aside from the considerable practical benefits, we stressed in previous work that for the frequentist user, our kernel estimator has good theoretical properties such as consistency and asymptotic normality in the presence of persistent but stochastic time varying coefficients. Analogous results are not available for likelihood estmates using the Gibbs-sampling/state-space approach in the presence of the economically motivated restrictions bounding the VAR coefficients inside the unit circle.

Of course, the debate about how best to characterise structural change is broader than simply a choice between kernel versus Gibbs sampler based methods for estimating stochastic time varying coefficient models. This debate should be seen in the context of the larger literature spanning other methods for describing structural change. This embraces at least three literatures, including i) the literature on smooth, deterministic change, exemplified by Priestley (1965), Dahlhaus (1996) and Robinson (1991); ii) the literature on estimating VARs with parameters that follow a Markov process [see, for example, Sims and Zha (2006)]; and iii) the literature on identifying infrequent and abrupt, structural change, (see, for example Chow (1960), Brown et al. (1974) and Ploberger and Kramer (1992)).

2.2 Substantive literature on detecting changes in DSGE parameters

The second line of work which is relevant for this paper concerns the substantive empirical findings in our paper. The broad framework for this work is that it asks whether DSGE parameters are time varying or not, or, to put it more pejoratively, and borrowing the title of Fernandez-Villaverde and Rubio-Ramirez (2008), asks "how structural are structural parameters?" The stakes are high, here, because the hope of DSGE modellers was that the model construction had bridged the gap between the early RBC models of Kydland and Prescott (1982) and others by incorporating various nominal and real frictions, yet without violating the insistence on microfoundations of that school of thought.

Time-variation in the parameters defining these descendants of the early purist models provides prima-facie evidence that the microfoundations are suspect.

2.2.1 Mapping from changes in the reduced from VAR to DSGE parameters

There are three variants of this kind of work. The first, which includes our paper, first estimates reduced form time-variation and then maps that into, or seeks to interpret this as caused by, time variation in DSGE parameters. The closest paper in this vein to ours in execution is Hofmann et al. (2010). They estimate a 4 variable TVP-VAR using the industry standard Gibbs-Sampling algorithm and identify technology and demand shocks using sign restrictions. They then take three snapshots of the implied estimated impulse responses (at the beginning, middle and end of their sample) and to these they fit a New Keynesian model with sticky prices, sticky wages (indexation in both) and habits in consumption. The model could be described as a Smets-Wouters model without capital formation. Their three point estimates show changes in DSGE parameters that are larger than those we uncover (see Table 1 of their paper). For example: median estimates of the price indexation parameter are 0.15 for 1960, 0.8 for 1974 and 0.17 for 2000. And for wages the analogous figures are 0.3, 0.91 and 0.17.

The main point of departure from this paper for us is the use of the kernel estimator to generate the sequence of reduced form VAR coefficients. As a consequence this allows us to estimate a larger, 7-variable VAR on an (updated) Smets and Wouters (2007a) dataset. The hope is that by using more data we can improve identification. Our paper also differs on a number of details. First, we allow *all* the parameters of the SW model to change over time, whereas Hofmann et al. (2010) fix some of their parameters at calibrated values. In particular, they fix the discount rate, the elasticity of labour supply, and the mark-ups in product and labour markets. Our results provide more support for fixing the discount rate than the elasticity of labour supply, which does show considerable movement across the sample. Second, we chose to fit the DSGE model to an identified monetary policy shock, as opposed to the technology shock used by Hofmann et al. (2010).[1] We make this choice on two grounds. First, we expect that the response of real variables to a monetary shock should be the acid test of the extent of nominal rigidities, a key focus for those scrutinising the validity of the CEE/SW model. If the classical dichotomy held, and prices were flexible, real variables would be invariant to a monetary policy shock. Secondly, we want to preserve a parallel with the original paper by CEE, who fitted a DSGE model to the impulse response to a monetary policy shock recovered from a fixed coeffcient VAR.

Several other papers adopt this same general approach of making connections between time-varying reduced form VAR dynamics and changes in structural DSGE parameters. Cogley and Sargent (2005) build a 3 variable time-varying coefficient VAR to characterise shifts in macroeconomic dynamics. Later, in their joint work with Primiceri, these author seek to find structural explanations via a small DSGE model.[2] In a similar vein, Sargent and Surico (2011) interpret shifts in the money growth - inflation correlation as accountable for by changes in the monetary policy rule in a smaller scale sticky price RBC model. In Cogley et al. (2012), changes in the correlation between nominal interest rates and inflation, and inflation persistence are associated with shifts in the degree of indexation of firms' prices, shifts which come about because of changes in the monetary regime. Gali and Gambetti (2009b) estimate a TVP-VAR involving labour productivity and hours work, and uncover changes in the impulse response to identified technology shocks. The fixed-coefficient literature to which these two papers address themselves was an argument about key parameters of the DSGE model that should be taken as the DGP. If hours worked did fall, as Gali's striking 1999 paper found[3], following a technology shock, then this suggested either that technology shocks were not major contributors to the business cycle (indicating a small value for the parameter governing the variance of these shocks) or, for example, that prices were sticky (whereupon the conventional result in the flex price RBC

[1] In order gain extra precision in the identification of the monetary policy shock, we also identify technology, labour supply and aggregate demand shocks. For an exposition of how precision in identifying one shock is improved by identifying other shocks, see Paustian (2007)

[2] Cogley et al. (2010).

[3] See: Gali (1999).

model that hours rise after a technology shock is overturned). The TVP-VAR results are therefore to be interpreted as alluding to potential changes in, eg, the volatility of technology shocks and the degree of stickiness in prices.

2.2.2 Estimating time-varying DSGE parameters directly

The second variety of work on time-variation in DSGE parameters estimates changes in DSGE parameters directly rather than indirectly via a mapping from a reduced form VAR. Within this category of papers, there are two tactics. One is to embed time-variation into the DSGE model itself. Another is simply to estimate the DSGE model on different samples.

Fernandez-Villaverde and Rubio-Ramirez (2008) build a DSGE model that includes stochastic processes for policy rule and price/wage stickiness parameters, processes over which agents in the model form rational expectations. They find abundant evidence of time variation. In one sense, our exercise is more limited and less constructive: we don't offer a DSGE model with full-blown time variation in place of the fixed coefficient variety, a model that would be a potential replacement for the fixed-coefficient DSGE model. Instead we undertake the narrower task of shedding light on whether the benchmark fixed coefficient variety is deficient or not, specifically asking whether the coefficients in the fixed coefficient DSGE model are really fixed. In another sense, our paper offers something new relative to Fernandez-Villaverde and Rubio-Ramirez (2008). Freed from the computational burden of computing expectations over the time-variation in the DSGE parameters, we can look for time-variation in *all* of the model's (19) parameters at the same time. By contrast, Fernandez-Villaverde and Rubio-Ramirez (2008) allow only *one* parameter at a time to move.[4] Note too that they use full information methods; we resort to partial information methods, partly to preserve the parallel with CEE's fixed coefficient work; and because of the benefits claimed for this approach (insurance against model misspecification), though we of course note the costs (for example aggravating identification problems).

Cogley and Sbordone (2008) is another example of a DSGE model that embodies explicit time-variation. They estimate a VAR with a time-varying trend inflation rate, imposing on the VAR cross equation restrictions implied by a version of the New Keynesian model linearised around a time-varying trend. This time-variation allows the model to explain the persistence in inflation well despite having no "backward-lookingness" in the form of indexation. This paper therefore makes an intimate connection between a TVP-VAR and a DSGE model related to the one considered here, re-interpreting the previous result that the data need indexation in the Philips Curve as reflecting the fact that the model omits time varying trend inflation.

Justiniano and Primiceri (2008) estimate a DSGE model with time-varying volatilities of the structural shocks in the model, and interpret the Great Moderation through the lens of this model. Born and Pfeifer (2011) likewise estimate time-variation in volatilites, with a particular focus on the changing volatility of monetary and fiscal policies.

The second tactic for estimating time-variation in DSGE coefficients is, as described above, to estimate DSGE models on different samples. Smets and Wouters (2007b) estimate their model over two sub-samples of US data and concludde that (variances of shocks aside), structural DSGE parameters are

[4]This was confirmed to us in email correspondence with Fernandez-Villaverde.

stable. Benati (2008) estimates a small New Keynesian model on various subsamples corresponding to different monetary regimes. He finds that the indexation parameter, corresponding to inflation persistence in the Philips Curve, varies substantially between monetary regimes, and therefore adduces that the reduced form property of inflation persistence derives, ultimately, not from indexation, but from the behaviour of monetary policy. Canova and Sala (2009) estimates the simplest New Keynesian model on rolling samples using full information Bayesian methods. He finds evidence that policy and private sector parameters change, and also instability in the variance of the shocks. Canova and Ferroni (2011) conduct a similar exercise using the Smets and Wouters (2007a) model, augmented to allow for real balances to affect consumption and for money growth to enter the policy rule. Giacomini and Rossi (2009) report rolling regression estimates of the Smets and Wouters (2003) model in the course of developing a KLIC based method of conducting rolling comparisons of the performance of competing models. Castelnuovo (2012) estimates on US data a rolling-sample version of the model of Andres et al. (2006) (a New Keynesian model without capital, but with habits, indexation, and costs of adjusting porfolios to bring in a role for money).

Also worth mentioning is the literature on detecting instabilities in DSGE model parameters. An example is Inoue and Rossi (2011). They develop and test an algorithm for recursively identifying sets of stable and unstable parameters in a DSGE model. They test for joint stability of all parameters, and, if this rejects, eliminate the parameter with the lowest individual p-value (corresponding to a test that this individual parameter is stable), and re-test for joint stability, proceeding like this until that test does not reject. This identifies a set of stable parameters. They apply this to a New Keynesian model, and find widespread evidence of parameter instability, including changes in parameters defining nominal rigidities, habits and monetary policy parameters.

The subsample and rolling-regression literature here is connected with our paper not just in terms of its substantive focus (DSGE parameter change) but also methodologically. Appropriately specified kernel functions can produce as special cases either rolling regressions or subsample estimates. Kernel estimators therefore nest these alternatives. For example, a kernel function that equal-weights all observations within a window, and zero-weights those outside it is a rolling-regression. Which, then, would be most appropriate? A richer kernel, or those that produce rolling regressions? The answer to this depends on the kind of structural change one is trying to model, which, unfortunately, is not known ex ante! Loosely speaking, one might say that provided the structural change is sufficiently gradual, richer (eg normal) kernels will be optimal, rather than rolling-regression kernels. Note that flat, rolling-regression kernels are not optimal in the case that we consider, where change in the DSGE parameters is derived from asssuming at the outset persistent, stochastic evolution of the reduced-form VAR parameters.

2.3 Diagnosing the causes of the Great Moderation

One of the focal points of the literature on structural change in macroeconomic dynamics has beent to to try to diagnose the causes of the Great Moderation - crudely, the phenomena including the rise and fall of the mean, variance and persistence of inflation and the fall in the volatility of output. Many of the papers mentioned above seek to quantify the contribution of shock volatilities versus parameter change, and to identify and those factors directly attributable to policy and those not. Two prominent, early papers were McConnell and Perez-Quiros (2000) and Stock and Watson (2002).

A survey of some of the subsequent literature is Velde (2004)

Our paper makes a limited contribution to this literature in so far as it quantifies changes in monetary policy. As already noted above, we find that there is not such a dramatic difference between pre and post-Volcker monetary policy rules as sometimes reported in other papers; and we also find no evidence that monetary policy was such as to have led to indeterminacy at any time in the sample. We cannot say much more than this -eg to speculate about the relative contributions of different kinds of shocks - because of the way we estimated the DSGE model. Recall that we are fitting it to the impulse response to a monetary policy shock, and not taking a stand on, and therefore not producing estimates of changes in the rest of the stochastic structure of the model. So our paper speaks to the contribution of 'good luck' to the Great Moderation (ie factors unrelated to policy) only indirectly, by noting the relative constancy of the monetary regime in the US.

3 Econometric framework for estimating the TVP-VAR and DSGE parameters

In this Section we set out the components of the analytical toolkit used to derive the time-varying DSGE coefficient estimates. There are three elements of this. First, the estimation of the reduced form TVP-VAR; second, the identification of the structural shocks from the reduced form shocks, and the computation of the associated impulse responses; and third, the estimation of the time-varying DSGE parameters. There is nothing new in this toolkit presented in this paper; the justification for the kernel estimator in the case of potentially stochastic time varying coefficient models is set out in Giraitis et al. (2011); using sign restrictions to identify monetary policy and other shocks derives form work by many (see our citations in the introduction); estimating the DSGE model using impulse resposne function matching is the basis of Rotemberg and Woodford (1998) and Christiano et al. (2005), and an account of the theoretical basis for this can be found in Theodoridis (2011). The next sections serve only to clarify notation and make the paper self contained for readers not familiar with all the components.

3.1 Time-Varying Estimation of Reduced Form VAR Models

We start by considering the multivariate dynanic autoregressive model given by

$$\boldsymbol{y}_t = \boldsymbol{\Psi}_{t-1}\boldsymbol{y}_{t-1} + \boldsymbol{u}_t, \quad t = 1, 2, \cdots, n, \tag{3.1}$$

where $\boldsymbol{y}_t$ is an $m-$dimensional vector, $\boldsymbol{u}_t = (u_{1t}, ..., u_{mt})'$ and $\boldsymbol{\Psi}_{t-1} = [\psi_{t-1,ij}]$. It is important for the theoretical properties of our proposed estimator to bound the eigenvalues of $\boldsymbol{\Psi}_{t-1}$ to lie in the interval $(-1, 1)$. There are a variety of ways to implement such a bounding. One such way, that mirrors the bounding of Giraitis et al. (2011) for univariate processes, can be implemented by defining

$$\tilde{\boldsymbol{\Psi}}_{t-1} = [\tilde{\psi}_{t-1,ij}], \qquad \tilde{\psi}_{t,ij} = \tilde{\psi}_{t-1,ij} + v_{\psi t,ij}, \quad t = 1, \cdots, n; \;\; i, j = 1, \cdots, m$$

where $v_{\psi t,ij}$ is a zero mean i.i.d. sequence with finite variance. Then,

$$\psi_{t-1,ij} = \psi \frac{\tilde{\psi}_{t-1,ij}}{\max_{0\leq i\leq t-1}\sum_{j=1}^{m}|\tilde{\psi}_{t-1,ij}|}, \quad t = 1, 2, \cdots, n,$$

where $0 < \psi < 1$. This ensures that the maximum eigenvalue of $\boldsymbol{\Psi}_{t-1}$ is bounded above by one in absolute value.

To estimate the coefficients $\Psi_1, \cdots, \Psi_n$, we suggest as an estimator a weighted sample autocorrelation at lag 1 given by

$$\hat{\boldsymbol{\Psi}}_t := \left(\sum_{k=1}^{n} K(\frac{t-k}{H_\Psi})y_k y_{k-1}'\right)^{-1}\left(\sum_{k=1}^{n} K(\frac{t-k}{H_\Psi})y_{k-1} y_{k-1}'\right),$$

where $K(x) \geq 0$, $x \in \mathbb{R}$ is a continuous bounded function. This estimator is simply a generalisation of a rolling window estimator given by

$$\hat{\boldsymbol{\Psi}}_t := \left(\sum_{k=t-H_\Psi}^{t+H_\Psi} y_k y_{k-1}'\right)^{-1}\left(\sum_{k=t-H_\Psi}^{t+H_\Psi} y_{k-1} y_{k-1}'\right),$$

which is a local sample correlation of y_t's at lag 1, based on $2H_\Psi + 1$ observations $y_{t-H_\Psi}, \cdots, y_{t+H_\Psi}$. For the bandwidth we assume that

$$H_\Psi \to \infty \text{ and } H_\Psi = o_p(n). \tag{3.2}$$

K is a kernel function such that $K(x) \geq 0$, $x \in \mathbb{R}$ is continuous and bounded, such for some $c > 1$,

$$K(x) \leq \exp(-cx^2), \quad x \to \infty. \tag{3.3}$$

Giraitis et al. (2011) show that under certain regularity conditions, and in the case of a univariate model, this kernel estimator delivers consistent estimates: ie it can be shown that

$$||\hat{\boldsymbol{\Psi}}_t - \boldsymbol{\Psi}_t|| = O_p\left(\left(\frac{H_\Psi}{n}\right)^{1/2}\right) + O_p\left(\left(\frac{1}{H_\Psi}\right)^{1/2}\right). \tag{3.4}$$

Giratis et al. (2012) suggest that analogous results hold in the multivariate context considered here.

Next, we consider the case of a time varying variance for $\boldsymbol{u}_t$ and ways to estimate it. We set

$$\boldsymbol{u}_t = \boldsymbol{H}_t \boldsymbol{\epsilon}_t \tag{3.5}$$

where $\boldsymbol{H}_t = [h_{t,ij}]$, $E(\boldsymbol{\epsilon}_t) = 0$, $E(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t') = \boldsymbol{\Sigma}_\epsilon$, and

$$\tilde{\boldsymbol{H}}_t = [\tilde{h}_{t,ij}], \qquad \tilde{h}_{t,ij} = \tilde{h}_{t-1,ij} + v_{ht,ij}, \quad t = 1, \cdots, n; \;\; i, j = 1, \cdots, m$$

where $v_{ht,ij}$ is a zero mean i.i.d. sequence with finite variance. Then,

$$h_{t,ij} = \frac{\tilde{h}_{t,ij}}{\sqrt{t}}, \quad t = 1, 2, \cdots, n,$$

We suggest the fowish to obtain an estimator for $\mathbf{\Sigma}_t = \boldsymbol{H}_t \mathbf{\Sigma}_\epsilon \boldsymbol{H}_t'$. We suggest the following estimator. Define

$$\hat{\boldsymbol{u}}_t = \boldsymbol{y}_t - \hat{\mathbf{\Psi}}_{t-1} \boldsymbol{y}_{t-i}$$

Then,

$$\hat{\mathbf{\Sigma}}_t = \sum_{k=1}^{n} K_h(\frac{t-k}{H_h}) \hat{\boldsymbol{u}}_t \hat{\boldsymbol{u}}_t'$$

where

$$H_h \to \infty \text{ and } H_h = o_p\left(H_\Psi\right). \tag{3.6}$$

Once again, work in progress that will form a new paper suggests that under certain regularity conditions $\hat{\mathbf{\Sigma}}_t$ is a consistent estimator of $\mathbf{\Sigma}_t$, such that:

$$||\hat{\mathbf{\Sigma}}_t - \mathbf{\Sigma}_t|| = O\left(\left(\frac{H_\Psi}{n}\right)^{1/2}\right) + O\left(\left(\frac{H_h}{H_\Psi}\right)^{1/2}\right) + O\left(\left(\frac{1}{H_\Psi}\right)^{1/2}\right). \tag{3.7}$$

Given the above, the estimated reduced form impulse response function for the time-varying VAR model at time t and horizon h, is given by

$$\boldsymbol{R}\left(k; t, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Sigma}}\right) = \hat{\mathbf{\Psi}}_t^k \hat{\mathbf{\Sigma}}_t^{1/2}.$$

3.2 Recovering structural shocks from the reduced form shocks using sign restrictions

Given an estimated reduced form impulse response function we wish to provide a structural interpretation to the VAR shocks and thereby obtain an estimated structural impulse response function. The aim is to factorise the covariance matrix of the m-dimensional reduced form error, at time t, denoted by $\mathbf{\Sigma}_t$, as

$$\mathbf{\Sigma}_t = \boldsymbol{B}_t \boldsymbol{B}_t'$$

where $\boldsymbol{B}_t$ is usually either given by a Cholesky factorisation or, in our case, an eigenvector-eigenvalue decomposition of the form $\boldsymbol{P}_t \boldsymbol{D}_t \boldsymbol{P}_t'$ where $\boldsymbol{P}_t$ is a matrix whose columns contain the eigenvectors of $\mathbf{\Sigma}_t$ and $\boldsymbol{D}_t$ is a diagonal matrix containing the eigenvalues of $\mathbf{\Sigma}_t$. Then, $\boldsymbol{B}_t = \boldsymbol{P}_t \boldsymbol{D}_t^{1/2}$. Of course, there exist multiple such factorisations since for any nonsingular orthogonal matrix $\boldsymbol{Q}_t$, we have

$$\mathbf{\Sigma}_t = \boldsymbol{B}_t \boldsymbol{Q}_t \boldsymbol{Q}_t' \boldsymbol{B}_t'$$

As is well known, $n(n-1)/2$ restrictions are sufficient to fully specify a unique $\boldsymbol{Q}_t$. A number of schemes deriving from insights from theoretical models have been proposed to specify these $n(n-1)/2$ restrictions. But given the controversy that each has prompted, an alternative approach has become popular, which rather than seeking to identify a unique $\boldsymbol{Q}_t$, instead aims to identify a set of $\boldsymbol{Q}_t$'s that instead satisfy particular sign restrictions for the impulse responses, computed as:

$$\boldsymbol{R}\left(k; t, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Sigma}}\right) = \hat{\mathbf{\Psi}}_t^k \hat{\boldsymbol{B}}_t \boldsymbol{Q}_t. \tag{3.8}$$

The benefit of using sign restrictions is that they can be selected to be 'robust' in that they apply to many different varieties of business cycle model (see for example, Peersman and Straub (2009)), and that therefore one increases the chance of uncovering, within the set, a shock of the kind that the research intends to find, rather than something that would only be a shock of the kind required in the event that one model among many that are possible happens to obtain in reality. The cost, of course, of giving up uniquness in identification, is, of course, that one no longer has uniqueness (!) and istead many possible impulse responses, leaving the researcher with an imprecise conclusion about the impact and the importance of the shock in question.

The practical problem then is to undertake a search of the possible $\boldsymbol{Q}_t$'s. We do this by searching through Givens rotations, where these are of the following form:

$$\boldsymbol{I}_{pq}^{n}(\theta) = \begin{pmatrix} 1 & \dots & 0 & \dots & 0 & \dots & 0 \\ \dots & \ddots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & cos(\theta) & \dots & -sin(\theta) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & sin(\theta) & \dots & cos(\theta) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \dots & 0 & \dots & 1 \end{pmatrix}, 0 \leq \theta \leq \pi/2$$

where $p < q$ denote the positions on the diagonal taken by $cos(\theta)$. These matrices respect the desired property that $\boldsymbol{I}_{pq}^{n}(\theta)\boldsymbol{I}_{pq}^{n\prime}(\theta) = \boldsymbol{I}$. Then, $\boldsymbol{Q}_t$ is parameterised as

$$\boldsymbol{Q}_t(\theta) = \prod_{i=1}^{n-1} \prod_{j=i+1}^{n} \boldsymbol{I}_{ij}^{n}(\theta_{ij}) \tag{3.9}$$

where $0 \leq \theta_{ij} \leq \pi/2$ and θ is the $n(n-1)/2$ vector containing all the scalar θ_{ij}. It is obvious that

$$\boldsymbol{Q}_t(\theta)\boldsymbol{Q}_t(\theta)' = \boldsymbol{I}.$$

In fact, we found it beneficial to search through not single Givens rotations, but multiple products of such rotations. This increases the number of satisficing sign restrictions, something that can sometimes be beneficial when imposing many restrictions, or restrictions at multiple horizons. Thus, we extend the parameterisation in (3.9) to the following

$$\boldsymbol{Q}_t(\theta^{(1)}, ..., \theta^{(m)}) = \prod_{s=1}^{m} \boldsymbol{Q}_t(\theta^{(s)}) = \prod_{s=1}^{m} \prod_{i=1}^{n-1} \prod_{j=i+1}^{n} \boldsymbol{I}_{ij}^{n}(\theta_{ij}^{(s)}) \tag{3.10}$$

The result of this sheme, to recap, is a set of rotations($\mathbf{Q}'s$) and a corresponding set of impulse response function. These are mapped into a set of DSGE parameter estimates by, for each $\mathbf{Q}$, computing the resulting impulse response function $\mathbf{R}$ and then finding the DSGE parameter vector θ that minimises the distance between the DSGE impulse response and the VAR impulse response. To emphasis, we do this estimation for each rotation, ie we do not first construct a summary statistic of the IRF's (eg a median). Precisely what we mean, and the theoretical basis for this minimum distance procedure, we set out briefly in the next section.

3.3 Minimum Distance Estimation of the DSGE parameters

In this section, we describe formally the minimum distance estimation [MDE] procedure we use to map from the time varying structural impulse response functions to the time varying DSGE parameter estimates, (which will be familiar to many from the work of Rotemberg and Woodford (1998), Christiano et al. (2005) and many others), and set out the theoretical basis for so doing, established in other work, an account of which can be found in Theodoridis (2011).

The solution of any linearised DSGE model, like the one described in Section 4.2, can be written in the following state-space format

$$\boldsymbol{y}_t = \boldsymbol{\Xi}(\theta)\,\boldsymbol{x}_t \tag{3.11}$$

$$\boldsymbol{x}_t = \boldsymbol{\Phi}(\theta)\boldsymbol{x}_{t-1} + \boldsymbol{\Lambda}(\theta)\boldsymbol{\omega}_t \tag{3.12}$$

where equation (3.12) describes the evolution of the state vector $\left(\boldsymbol{x}_t \in \mathbb{R}^{dx}\right)$[5], equation (3.11) relates the vector of the observable variables $\left(\boldsymbol{y}_t \in \mathbb{R}^{dy}\right)$ with the states of the economy, and $\boldsymbol{\omega}_t \in \mathbb{R}^{d\omega}$ denotes the vector of the structural errors, which are normally distributed with zero mean and $\boldsymbol{I}_{d\omega}$ covariance matrix[6], ie $\boldsymbol{\omega}_t \sim N\left(\boldsymbol{0}_{d\omega}, \boldsymbol{I}_{d\omega}\right)$[7]. Finally, the elements of the matrices $\boldsymbol{\Xi}(\theta)$, $\boldsymbol{\Phi}(\theta)$ and $\boldsymbol{\Lambda}(\theta)$ are non-linear functions of the structural parameter vector $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and $\boldsymbol{\Theta}$ is a compact subset of $\mathbb{R}^{d\theta}$.

Equations (3.11) and (3.12) can be used to analyse the effects of the shocks disturbing the economy, in other words, to study agents' optimal responses to small structural perturbations. This type of analysis, which lies at the core of the DSGE modelling, reveals the dynamic properties of the model and improves researchers' intuition about the cyclical behaviour of the model. This information is summarised by the following $kdy^2 \times 1$ vector valued DSGE impulse response function

$$\boldsymbol{R}(k,t,\boldsymbol{\theta}) \equiv \left(vec\left(\frac{\partial \boldsymbol{y}_{t+1}}{\partial \boldsymbol{\omega}_t}\right)', ..., vec\left(\frac{\partial \boldsymbol{y}_{t+k}}{\partial \boldsymbol{\omega}_t}\right)'\right)' = \left(\boldsymbol{\Lambda}(\boldsymbol{\theta})' \otimes \boldsymbol{\Xi}(\boldsymbol{\theta}) \otimes \boldsymbol{I}_k\right)\boldsymbol{b}(k;\boldsymbol{\theta}) \tag{3.13}$$

where $\boldsymbol{b}(k;\boldsymbol{\theta}) \equiv \left(vec\left[\boldsymbol{\Phi}(\boldsymbol{\theta})\right]', ..., vec\left[\boldsymbol{\Phi}(\boldsymbol{\theta})^k\right]'\right)'$.

We can now use the time-series model (3.1) to obtain the VAR counterpart of $\boldsymbol{R}(k,t,\boldsymbol{\theta})$ which is given by (3.8). Given (3.4), (3.7) and the continuity of $\boldsymbol{R}(k;t,\boldsymbol{\Psi},\boldsymbol{\Sigma})$ with respect to $\boldsymbol{\Psi},\boldsymbol{\Sigma}$, we have that

$$\left\|\boldsymbol{R}(k;t,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t) - \boldsymbol{R}\left(k;t,\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Sigma}}\right)\right\| = o_p(1). \tag{3.14}$$

This property allows us to use $\boldsymbol{R}\left(k;t,\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Sigma}}\right)$ to derive an estimate of the structural parameter vector θ_t (to emphasise, for each t in our sample period) as follows

$$\hat{\boldsymbol{\theta}}_t \equiv \arg\min_{\boldsymbol{\theta}} \left[\boldsymbol{R}(k,t,\boldsymbol{\theta}) - \boldsymbol{R}\left(k;t,\hat{\boldsymbol{\Psi}}_t,\hat{\boldsymbol{\Sigma}}_t\right)\right]' \mathcal{W} \left[\boldsymbol{R}(k,t,\boldsymbol{\theta}) - \boldsymbol{R}\left(k;t,\hat{\boldsymbol{\Psi}}_t,\hat{\boldsymbol{\Sigma}}_t\right)\right] \tag{3.15}$$

where $\mathcal{W}$ is a positive definite matrix. We assume that the above problem has a unique solution, and we also define $\boldsymbol{\theta}_t$ to be the unique solution of

$$\arg\min_{\boldsymbol{\theta}} \left[\boldsymbol{R}(k,t,\boldsymbol{\theta}) - \boldsymbol{R}(k;t,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t)\right]' \mathcal{W} \left[\boldsymbol{R}(k,t,\boldsymbol{\theta}) - \boldsymbol{R}(k;t,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t)\right] \tag{3.16}$$

[5] $\mathbb{R}$ denotes the real line and da the dimension of the vector a.

[6] I_{da} stands for the $da \times da$ identity matrix.

[7] $\xrightarrow{D}$ implies convergence in distribution, while $\xrightarrow{P}$ convergence in probability.

Thus: for consistency, for each period t, there must be only one DSGE parameter vector, that minimises the distance between the DSGE and the estimated VAR impulse responses; and only one DSGE parameter vector that minimises the distance between the DSGE and *true* VAR impulse responses.

The following quantity (the square root of the distance function) -

$$Q\left(k;\boldsymbol{\theta}_t,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t\right)\equiv\mathcal{W}^{1/2}\left[\boldsymbol{R}\left(k,t,\boldsymbol{\theta}_t\right)-\boldsymbol{R}\left(k;t,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t\right)\right] \tag{3.17}$$

- is used to illustrate that our structural estimate is a consistent estimate. We assume that:

1. $Q\left(k;\cdot,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t\right)$ is continuous in a neighborhood of $\boldsymbol{\theta}_t\in\boldsymbol{\Theta}$
2. The rank of $\nabla_{\boldsymbol{\theta}}Q\left(k;\boldsymbol{\theta}_t,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t\right)$ equals the dimension of $\boldsymbol{\theta}_t$ and it is constant in a neighborhood of $\boldsymbol{\theta}_t$
3. The dimension of $Q\left(k;\boldsymbol{\theta}_t,\boldsymbol{\Psi}_t,\boldsymbol{\Sigma}_t\right)$ is greater than or equal to the dimension of $\boldsymbol{\theta}_t$
4. There exists $\alpha>0$ and $M=O_p\left(1\right)$ such that for all $\tilde{\boldsymbol{\theta}}_t,\boldsymbol{\theta}_t'\in\boldsymbol{\Theta}$ and all $\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Sigma}},$

$$\left|\left\|Q\left(k;\tilde{\boldsymbol{\theta}}_t,\hat{\boldsymbol{\Psi}}_t,\hat{\boldsymbol{\Sigma}}_t\right)\right\|^2-\left\|Q\left(k;\boldsymbol{\theta}_t',\hat{\boldsymbol{\Psi}}_t,\hat{\boldsymbol{\Sigma}}_t\right)\right\|^2\right|\leq M\left\|\tilde{\boldsymbol{\theta}}_t-\boldsymbol{\theta}_t'\right\|^{\alpha} \tag{3.18}$$

Then given (3.14) and by theorem 2.1 of Newey and McFadden (1986),

$$\left\|\hat{\boldsymbol{\theta}}_t-\boldsymbol{\theta}_t\right\|\xrightarrow{p}0. \tag{3.19}$$

Our MDE procedure and the assumptions underpinning its consistency are thus identical to the standard MDE procedures used in fixed coefficient VAR and DSGE analyses, with the exception that it is carried out, and the assumptions invoked, for each of the 'instantaneous VARs' defined by the sequence of parameters in the TVP-VARs which the kernel estimator (described earlier) produces. This procedure mirrors what Hofmann et al. (2010) did, with the only difference being that, they were using a smaller dimension VAR, using more familiar state-space/Gibbs sampling methods to estimate the TVP-VAR, a smaller dimension DSGE model, calibrating some of the parameters, and picking out just a few of the instantaneous VARs articulated by their TVP-VAR.

Note that the standard costs and benefits of using MDE or related procedures also apply here in our time-varying parameter implementation. We at no point invoke the truth or falsehood of the DSGE model in establishing consistency. That result simply states that whatever (true or false) DSGE model best matches the true set of instantaneous VAR impulse responses will anchor our estimated DSGE parameters, given ample data.

This concludes the theoretical discussion of our estimation method. Of course many choices have to be made to operationalise the above approach. These include the choice of the variables in the VAR model, the sign restrictions and the DSGE model used. These will be discussed in detail in the next section.

4 Empirical implementation

As noted earlier, we use the 7 variable quarterly dataset for the US compiled by SW, comprising: quarterly growth in GDP, CPI inflation, hours worked, quarterly growth in investment, quarterly growth in consumption, quarterly growth in real wages and the Fed Funds rate. The dataset in the 2007 AER depository is updated to 2010Q2.

4.1 Identification and computation of the impulse response functions

We fit the DSGE model to the impulse response function to a monetary policy shock. We consider this to be the key moment for estimating parameters defining nominal frictions, which we suspect - not least because of the previous work by Fernandez-Villaverde and Rubio-Ramirez (2008) and Hofmann et al. (2010) - are the least microfounded of all of the DSGE parameters. Plus, this preserves a parallel with the CEE paper in 2005 which fits a fixed coefficient variety of a similar DSGE model to the IRF to a monetary policy shock. In contrast to CEE, we will identify this shock using sign restrictions, following (in the sense of being inspired by, rather than implementing the exact restrictions imposed) Faust (1998), Uhlig (2005), Canova and Nicolo (2002) and others.

We identify four shocks in total, including, in addition to the monetary policy shock: a technology shock, a labour supply shock, and a demand shock. We identify these additional shocks to increase the precision with which the monetary policy shock is identified, noting the results in Paustian (2007).

The relevant sign restrictions are encoded in the table below, where the column headings refer, for those not familiar, to (consumption, investment, GDP, hours worked, inflation, real wages and the nominal interest rate respectively):

	Δc	Δi	Δy	h	π	$\Delta(w/p)$	i
mon pol	-	-	-		-		+
technology		+	+		-		
labour supply			+	+	-	-	
demand	+		+		+	+	+

Unpacking the table in words: a monetary policy shock is taken to be a shock that, if interest rates rise, induces inflation, output, investment and consumption to fall, with the responses on hours worked and the real wage left free. Technology shocks are identified to be such that an *improvement* in technology causes inflation to fall, output and investment to rise, with the response of interest rates, hours worked and consumption left free. (Leaving the hours worked responses free is compelling here given the prior controversy about that response, and the ease with which this sign can be reversed by appropriate parameterisation of a DSGE model). An outward shock to labour supply is identified as one that leads to hours worked increasing at the same time as the real wage falls, and output increases, with other responses left free. This scheme bears some resemblance to the one used by Chang and Schorfheide (2003): they took an outward shift in labour supply to be something that reduced labour productivity: this is consistent with us assuming that real wages fall. A demand shock is one that induces positive comovement in interest rates, inflation, output and consumption, with the responses of hours and investment left free.

In estimating and identifying the impulse response to the monetary policy (and other) shocks there are two sources of uncertainty. The first is the sampling variability experienced in estimating the reduced form VAR parameters, and the variance-covariance matrix of the VAR residuals. The second is the uncertainty associated with the fact that many rotations of the variance covariance of the reduced form residuals will satisfy the sign restrictions above. We report only uncertainty generated from this second, rotational source. This is for expositional clarity, but we might note in addition that with increasingly long samples the sampling uncertainty would of course disappear, while the rotational uncertainty would not, so, in a sense, the latter is the dominant source of variability.

4.2 Brief review of the Smets-Wouters (2007) model

To make the paper self-contained, this subsection briefly discusses some of the key linearized equilibrium conditions of Smets and Wouters (2007a) model. Readers who are interested in how these arrive from solving consumer and firms' decision problems are recommended to consult the references mentioned above directly. All the variables are expressed as log deviations from their steady-state values, $\mathbb{E}_t$ denotes expectation formed at time t, '$-$' denotes the steady state values and all the shocks (η_t^i) are assumed to be normally distributed with zero mean and unit standard deviation.

The demand side of the economy consists of consumption (c_t), investment (i_t), capital utilisation (z_t) and government spending $\left(\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g\right)$ which is assumed to be exogenous. The market clearing condition is given by

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \tag{4.1}$$

where y_t denotes the total output and Table (1) [displayed at the end of the paper] provides a full description of the model's parameters. The consumption Euler equation is given by

$$\begin{aligned} c_t = {} & \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \left(1 - \frac{\lambda/\gamma}{1+\lambda/\gamma}\right) \mathbb{E}_t c_{t+1} + \frac{(\sigma_C - 1)\left(\bar{W}^h \bar{L}/\bar{C}\right)}{\sigma_C (1+\lambda/\gamma)} (l_t - \mathbb{E}_t l_{t+1}) \\ & - \frac{1-\lambda/\gamma}{\sigma_C (1+\lambda/\gamma)} (r_t - \mathbb{E}_t \pi_{t+1}) \end{aligned} \tag{4.2}$$

where l_t is the hours worked, r_t is the nominal interest rate and π_t is the rate of inflation. If the degree of habits is zero ($\lambda = 0$), equation (4.2) reduces to the standard forward looking consumption Euler equation. The linearised investment equation is given by

$$i_t = \frac{1}{1+\beta\gamma^{1-\sigma_C}} i_{t-1} + \left(1 - \frac{1}{1+\beta\gamma^{1-\sigma_C}}\right) \mathbb{E}_t i_{t+1} + \frac{1}{(1+\beta\gamma^{1-\sigma_C})\gamma^2 \varphi} q_t \tag{4.3}$$

where i_t denotes the investment and q_t is the real value of existing capital stock (Tobin's Q). The sensitivity of investment to real value of the existing capital stock depends on the parameter φ (see, ?). The corresponding arbitrage equation for the value of capital is given by

$$q_t = \beta\gamma^{-\sigma_C}(1-\delta)\mathbb{E}_t q_{t+1} + \left(1 - \beta\gamma^{-\sigma_C}(1-\delta)\right)\mathbb{E}_t r_{t+1}^k - (r_t - \mathbb{E}_t \pi_{t+1}) \tag{4.4}$$

where $r_t^k = -(k_t - l_t) + w_t$ denotes the real rental rate of capital which is negatively related to the capital-labour ratio and positively to the real wage.

On the supply side of the economy, the aggregate production function is define as

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha) l_t\right) \tag{4.5}$$

where k_t^s represents capital services which is a linear function of lagged installed capital (k_{t-1}) and the degree of capital utilisation, $k_t^s = k_{t-1} + z_t$. Capital utilization, on the other hand, is proportional to the real rental rate of capital, $z_t = \frac{1-\psi}{\psi} r_t^k$. The accumulation process of installed capital is simply described as

$$k_t = \frac{1-\delta}{\gamma} k_{t-1} + \frac{\gamma - 1 + \delta}{\gamma} i_t \tag{4.6}$$

Monopolistic competition within the production sector and Calvo-pricing constraints gives the following New-Keynesian Phillips curve for inflation

$$\begin{aligned} \pi_t = & \frac{i_p}{1+\beta\gamma^{1-\sigma_C} i_p} \pi_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1+\beta\gamma^{1-\sigma_C} i_p} \mathbb{E}_t \pi_{t+1} \\ & - \frac{1}{\left(1+\beta\gamma^{1-\sigma_C} i_p\right)} \frac{\left(1-\beta\gamma^{1-\sigma_C}\xi_p\right)\left(1-\xi_p\right)}{\left(\xi_p\left(\left(\phi_p - 1\right)\varepsilon_p + 1\right)\right)} \mu_t^p + \varepsilon_t^p \end{aligned} \tag{4.7}$$

where $\mu_t^p = \alpha\left(k_t^s - l_t\right) - w_t$ is the marginal cost of production and $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \sigma_p \eta_t^p - \mu_p \sigma_p \eta_{t-1}^p$ is the price mark-up price shock which is assumed to be an ARMA(1,1) process. Monopolistic competition in the labour market also gives rise to a similar wage New-Keynesian Phillips curve

$$\begin{aligned} w_t = & \frac{1}{1+\beta\gamma^{1-\sigma_C}} w_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1+\beta\gamma^{1-\sigma_C}} \left(\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}\right) - \frac{1+\beta\gamma^{1-\sigma_C} i_w}{1+\beta\gamma^{1-\sigma_C}} \pi_t \\ & + \frac{i_w}{1+\beta\gamma^{1-\sigma_C}} \pi_{t-1} - \frac{1}{1+\beta\gamma^{1-\sigma_C}} \frac{\left(1-\beta\gamma^{1-\sigma_C}\xi_w\right)\left(1-\xi_w\right)}{\left(\xi_w\left(\left(\phi_w - 1\right)\varepsilon_w + 1\right)\right)} \mu_t^w + \varepsilon_t^w \end{aligned} \tag{4.8}$$

where $\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1-\lambda}\left(c_t - \lambda c_{t-1}\right)\right)$ is the households' marginal benefit of supplying an extra unit of labour service and the wage mark-up shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \sigma_w \eta_t^w - \mu_w \sigma_w \eta_{t-1}^w$ is also assumed to be an ARMA(1,1) process.

Finally, the monetary policy maker is assumed to set the nominal interest rate according to the following Taylor-type rule

$$r_t = \rho r_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right] + r_{\Delta y}\left[\left(y_t - y_t^p\right) + \left(y_{t-1} - y_{t-1}^p\right)\right] + \varepsilon_t^r \tag{4.9}$$

where y_t^p is the flexible price level of output and $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \sigma_r \eta_t^r$ is the monetary policy shock.[8]

[8] The flexible price level of output is defined as the level of output that would prevail under flexible prices and wages in the absence of the two mark-up shocks.

5 Empirical Results

5.1 Time-varying impulse response functions to a monetary policy shock

Our estimated impulse responses - holding the variance of the reduced form shocks constant - are reported in Figures 1 and 2 below. Figure 1 and 2 plots 3d charts of the IRFs through time. For each period, and for each horizon, we are reporting the median across impulse responses that satisfy the sign restrictions. (For the moment we defer the task of addressing the well known comment by Fry and Pagan (2007) on this practice for future work).

The broad picture - in part by construction, since some of the signs are imposed - is as follows: a contractionary policy shock - that therefore causes the short rate to rise - reduces inflation, investment, consumption and output. The sign on real wages and hours worked were left free, and note that these both fall. Hours and inflation show signs of a hump-shaped response (the peak response not being on impact, but later). All charts show pronounced time-variation in the IRFs. For example, there is a clear increase in the magnitude of the effect of the policy shock on some real variables in the early 2000s, relative to periods just prior to that.

Figure 3 depicts the IRFs in a slightly different way. We calculate the cumulated sum of these responses and, for each IRF, and for each time period, we plot four lines, the first, fourth, eighth and twelfth period responses. If the responses are defined as R_i, then the cumulated responses are defined as:

$$
\begin{aligned}
CR_1 &= R_1 \\
CR_4 &= \sum_{i=1}^{4} R_i \\
CR_8 &= \sum_{i=1}^{8} R_i \\
CR_{12} &= \sum_{i=1}^{12} R_i
\end{aligned}
$$

Loosely speaking these lines could be viewed as the integral of the response at each horizon. If $CR_8 = CR_{12}$ then the variable has returned back to its steady-state in period $Q = 8$ and it stays there. If $CR_8 < 0$ ($CR_8 > 0$) and $CR_{12} > CR_8$ ($CR_{12} < CR_8$) then the series has returned to its steady state and moves to the opposite direction.

Figure 3 brings out more starkly some of the more detailed time-variation occurring in the IRFs to the monetary policy shock. As the chart shows, the cumulative impact at all horizons and for most responses shows quite a bit of variation. To take a few examples: if we look at the charts for consumption, output, hours ["labour"], we can see that for some periods the turquoise line is below the red line (implying that the monetary policy shock is still propagating and being amplified out to 12 quarters) yet at other times the reverse is true (implying that the shock has started to dissipate by 12 quarters). Another example: for most charts, the cumulative impulse at all horizons varies a lot. The impact of the shock on the growth in investment out to 12 quarters varies between -0.2 and -1.2; the impact on output at 12 quarters varies between -0.2 and 0. The impact on real wages out to 4 quarters varies from -0.1 to 0.2.

A necessary condition for us to get time variation in the implied DSGE estimates is that these impulse responses to a monetary shock vary: this condition is clearly satisfied in our case, so we proceed therefore to the step of mapping from the IRFs to the DSGE parameter estimates.

5.2 Time Varying DSGE parameters

Our benchmark estimation results are presented in Figures 4 and 5. The charts plot the median and 16%-84% confidence sets that result from fitting the entire set of rotations of the reduced form shocks that satisfy the sign restrictions. Marked as a blue diamond are the SW estimates produced from their full information Bayesian Maximum Likelihood procedure, which we report as a comparison. These SW estimates very often are different from the average of our "sub-sample" estimates. This is to be expected: our estimates differ not only because they are sub-sample, but because SW used full information/Bayesian estimates, with informative priors, and ours are partial information, classical estimates.

We report on the parameter estimates in several blocks; nominal rigidities; real side of the model; monetary and fiscal policy.

5.2.1 Nominal rigidities.

We estimate very pronounced changes in the parameters defining nominal wage and price rigidity. ξ_p, the "Calvo parameter"for prices, which encodes the probability of not re-setting prices, is estimated to be about 0.72 in 1955, falls steadily to a low point of 0.6 in 1985 (a period which, roughly speaking, captures the 'Great Inflation'), and then rises sharply to 0.83, by 2005 (a period which brackets the 'Great Moderation'), before falling back sharply to 0.7 by 2010.[9] There is strong circumstantial evidence, as also pointed out by Fernandez-Villaverde and Rubio-Ramirez (2008) and Hofmann et al. (2010), that this parameter is a reduced form for some underlying state-dependent model of prices in which the frequency of price changes is inversely related to inflation itself. The equivalent parameter for wages, ξ_w follows a very similar path indeed, as we would expect if this speculation about the underlying state-dependent pricing model is correct, since wage inflation has followed a similar path to price inflation, but note that despite the pronounced time-variation in both parameters we find that the probability of not re-setting is always lower for prices than wages.[10]

The indexation parameter is perhaps the most controversial aspect of the DSGE model: micro evidence on prices strongly suggests that there is no indexation; yet indexation in prices and wages greatly improves the fit of the DSGE model to macro time series. i_p records the coefficient in the one argument linear rule that firms use to multiply with last period's inflation to index prices. We estimate that this begins in 1955 at 0.4, rising to almost 0.7 in 1985 or so, before falling to a low point of around 0.25 in 2005, and then rising sharply again to 0.5 by 2010. This parameter is very closely correlated with the estimated path for the Calvo price parameter ξ_p, for reasons which are obviously not interpretable through the lens of the time-invariant DSGE model of nominal rigidities. This parameter varies a lot with monetary regimes, as Benati (2008) showed, but our estimates reveal that there is also

[9] In terms of the price average duration the last two numbers imply that this varies from, approximately, 2 to 8 quarters, respectively (red diamonds in Figure 4).

[10] Mapping the minimum and the maximum value of ξ_w to wages average duration we realise that wages were reset every 2 and 10 quarters, respectively (red diamonds in Figure 4).

clearly a lot of instability *within* institutionally-defined regimes. For example, it is not the case that indexation-induced persistence is greater pre than post-Volcker. Estimated indexation varies a lot in both sub-regimes (Note too that we are separately estimating the contribution of monetary policy to inflation persistence here, and will report on the policy block of parameters below). The equivalent parameter for wages, i_w follows a very similar path, but fluctuating in a slightly narrower range (0.4-0.7, as opposed to 0.25-0.7 for prices).

These parameter fluctuations echo those found in Fernandez-Villaverde and Rubio-Ramirez (2008) and Hofmann et al. (2010). Relative to the latter, which is the closest paper to ours in execution, we find slightly smaller fluctuations in the parameters defining nominal rigidity. There are still quite a few differences between their method and ours to account for the mildly contrasting results: we use kernel methods to estimate the reduced form VAR, they use the random coefficients model; we use a 7 variable VAR and they use 4 variables; we allow all parameters to vary over time, they fix many at calibrated values; our identification scheme differs from their in some details; and we fit only to a monetary policy shock.

Our results emphasise that more research may be needed to refine the nominal rigidities in the canonical DSGE model, echoing many previous papers. It is well known that the details of optimal monetary policy depend a lot on the nature of nominal rigidities. Examples being: the stickier are wages relative to prices, the more weight the authorities should place on nominal wage stabilisation relative to price stabilisation (Erceg et al. (2000)); the presence of indexation implies the authorities should stabilise a quasi-difference of inflation involving the indexation parameter itself (Woodford (2003)). Finding such a large amount of variation in the nominal rigidity parameters is disquieting since they are important for optimal policy.

5.2.2 Real economy parameters.

There are several points worth noting here. First, on h, the parameter that encodes habits in consumption. This parameter is estimated at about 0.77 in 1955 and fluctuates between this value and about 0.83 until the early 2000s, when it jumps from a trough of 0.75 to 0.88. These changes are smaller than for the indexation parameters that are engineered to generate persistence on the nominal side, but in terms of half-lives of consumption these are still pretty large changes. Consumption ends up more backward-looking and less sensitive to real interest rate changes than it begins the sample. The inverse intertemporal elasticity of substitution (σ_c) fluctuates quite a bit, showing a particularly large fall from a peak in 1970 of about 1.25 to a trough of about 0.8 in 1980. The inverse Frisch elasticity of labour supply (σ_L) shows a marked fall between 1985 when it peaks around 5.25, to the early 2000s when it troughs at around 2. The parameter governing the costs of adjusting investment (Φ) is pretty flat for most of the sample, but then shows a large rise from a trough of around 2 in 1990 to a peak of about 5 in 2005, before falling back sharply to 2 again. The greater this parameter, the more detached is investment from the traditional cost of finance manifest in Tobin's Q. This suggests that the DSGE model had a hard time to explain the boom investment during the 1990s, and the subsequent 'post Y2K' bust in the 2000s. Interestingly, the discount rate, β, is found to be relatively constant. We draw comfort from this. There is a wide body of evidence, macro/finance and micro/experimental, that the discount rate is close to but less than 1. So this is probably the most micro-founded parameter of all in the DSGE model. Stepping back: where the DSGE model is most micro-founded, we find the parameter estimates to be flat; where it is least well-microfounded, we find the parameter to be quite variable. We take our flat β to strengthen the case for interpreting our results as indicating something useful about the DSGE model's failings. If everything were moving, even parameters that were relatively solidly evidenced outside the model, we could be more sceptical that this estimated variation was just noise, or indicative of poor identification. Note that the flat β confirms ex post that the data would in some sense support the fixing of β in Hofmann et al. (2010).

5.2.3 Policy parameters.

Monetary policy is assumed to have been characterised by an interest rate rule such that the interest rate responds to its own lag, a term in the inflation rate, the output gap and the change in the output gap (sometimes known as the 'speed limit'). The responsiveness of interest rates to inflation, r_π, is the least precisely estimated of the monetary policy parameters, and moves around the least relative to the confidence sets that we compute, staying within a range of 1.7 to 1.9. On the face of it our estimates fail to confirm previous work that argues that the Great Inflation was caused by a breach of the Taylor principle. We effectively discard all DSGE parameter constellations that imply indeterminacy by penalising these heavily in the minimum distance procedure. But the profile of the estimates of the monetary policy and other parameters suggests that this constraint is never binding: if it were we would expect the minimisation to drive one of the parameters somewhere in the model to a boundary. We discard separately (even amongst otherwise stationary solutions) candidate parameter sets in which r_π is less than 1, but this coefficient is never driven to that boundary in estimation.[11]

The coefficient on lagged interest rates, ρ, fluctuates between 0.75 and 0.95, which we take to be quite large movements, especially relative to the small confidence sets we estimated. Interestingly, the coefficient on lagged interest rates rises at the point when the indexation parameters for prices and wages rise. It has been remarked in previous work Dittmar et al. (2005) that interest rate inertia can generate reduced form inflation persistence and therefore *substitute* for persistence imparted by indexation in price and wage setting. But our estimates tell a story of policy induced and nominal rigidity-induced persistence going hand in hand, acting as *complements*. The coefficients on the level and change in the output gap also vary somewhat, between 0.04 and 0.14 for the level term and 0.15 and 0.3 for the speed limit term. The most pronounced movement in the monetary policy parameters occurs in the early 200s, a time when the Fed was keeping rates as low as possible to avoid the threat of deflation. Our estimates pick this up as being associated with a rise in interest rate inertia and the responsiveness to the speed limit, and a fall in the response to the level of the output gap.

Finally, the volatility of the estimated monetary policy shock process falls steadily through the sample period, from about 0.18 in 1955 to about 0.12 by 2010. We are used to reading that this volatility is markedly higher pre than post Volcker, but our graphs tell a slightly different story. True enough, this volatility is, on average, higher pre than post-Volcker. But its fall starts in 1970, and is actually complete by 1985 when the volatility troughs. Thereafter, for the bulk of the Volcker regime, it rises, until Greenspan arrives.

Taken together, the variation we detect in policy parameters is considerably less than that uncovered

[11] We appreciate that a value >1 is not going to be necessary or sufficient to guarantee stability, but it is nevertheless interesting that our estimation generates values always >1, when others, using estimates of the interest rate reaction function only, or structural models, have found differently.

in much of the other work whose focus was understanding the causes of the Great Inflation and its subsequent moderation. At face value (and noting that this is conditional on taking literally the notion that the world was characterised by a sequence of private sector blocks of the DSGE model that also differed each period) this suggests less of a role for monetary policy in explaining changes in macro dynamics than other accounts which take different approaches.

5.3 Sensitivity: time varying Σ

It has been common for researchers estimating TVP-VARS using the Gibbs Sampling algorithm and the random coefficients model to allow for time varying volatilities. Especially following the comments by Sims on the first Cogley-Sargent work in this vein [Sims (2001)] where he worried that the apparent changes in the reduced form VAR dynamics had been amplified by imposing that the VAR had time-invariant volatilities. In this section, we comment on whether the results described above survive us allowing the volatilities to be time varying. On the whole, the results survive. We still find substantial instability in the DSGE parameter estimates, and the paths of the estimates largely follow those derived under the assumption of fixed volatilities. These findings are evident from Figures 6 and 7, which plot the median for the time-invariant volatilities against the confidence sets for those estimated assuming time-varying volatilities.

6 Conclusions

In this paper, we have discussed an approach to the estimation of time-varying DSGE models that begins by estimating a time-varying parameter VAR on the 7 variable data set SW used to estimate their medium scale DSGE model. Such an estimation is currently impractical using the widely known Gibbs-Sampling algorithms to estimate stochastic time-varying coefficient models, given the need to impose that at each point in time the implied VAR satisfies stationarity conditions. In order to proceed, we have deployed a kernel estimator, explained in prior work, (Kapetanios and Yates (2011), Giraitis et al. (2011)) that is known to deliver consistent estimates of the VAR parameters, and has no problem handling large dimension systems.

Armed with this estimated TVP-VAR, we have identified monetary policy shocks using sign restrictions, and computed the impulse response to those shocks implied by each of the instantaneous VARs that the TVP-VAR produces, for each observation period in our data set. We have then fit the SW model to these impulse response functions using a minimum distance procedure. In doing this, we are, roughly speaking, replicating what CEE did for their model, (similar in design to SW), for impulse responses to a monetary policy shock that were recovered from a fixed-coefficient VAR. We therefore map the time-varying evolution in reduced form dynamics into time-variation in the structural parameters of the DSGE model.

We find that many of the parameters of the model are subject to substantial time-variation. Those that move most are those that define the nominal rigidities in the model: the Calvo and indexation parameters for prices and wages. These have long been the focus of criticism from outside the DSGE community (see, for example, Chari et al. (2009)), and recognised by those practising within it to be work in progress. But other parameters appear to move a lot too. For example, the parameter that defines the cost of adjusting investment. Smaller, but still economically meaningful movements are detected in the parameter that encodes habits in consumption, and those that define the behaviour of the monetary policy authorities.

It is hardly surprising that the changing reduced form VAR parameters translate into changes in the DSGE parameters. Small samples are noisy. And we know that the DSGE model itself suffers from identification problems. We should surely expect these estimates to move a round quite a bit over time. So is there really anything to be read into these estimates? We believe they deserve attention for a few reasons. First, the movements are smooth, relatively low frequency and, in the case of the parameters defining nominal rigidities, correlate in a systematic way with inflation, which, as others have noted, hints at an underlying state-dependent model for pricing. These movements look less like those one would expect if the underlying cause was small-sample noise. Second, the parameters that move most are those that are accepted to be the least well micro-founded, and whose justification relies most on being necessary to fit the (fixed-coefficient) macro time series dynamics: those defining nominal rigidities and investment adjustment costs. Those that move least - for example the discount rate, are those for which there is a wide body of prior evidence that this is an invariant behavioural parameter.

A final feature of our results that may be of interest is the relatively small movements in the parameters that define monetary policy compared to those estimated by papers in the Great Moderation literature. For example, taken literally, we find no support for the argument that there have been periods where interest rates do not satisfy the "Taylor Principle", and that the Great Inflation was therefore the result of a period where policy had led to indeterminacy. We would not want to push this interpretation too far though, since it is stretching it somewhat to characterise central bank policy as implementing an entirely different but parsimonious rule for rates each period in the sample.

These results confirm finding in two other similar papers, particularly Fernandez-Villaverde and Rubio-Ramirez (2008) and Hofmann et al. (2010). Those papers, as well as recording changes in monetary policy behaviour, put the spotlight on the nominal rigidities in the DSGE model by recording that estimates of those parameters moved over time, and in ways correlated with inflation. We have found this too. Although, as we have emphasised already, our work also suggests that other parameters on the real side seem to vary substantially too.

Appendices

Charts

Figure 1: IRFs Constant Σ_v: A

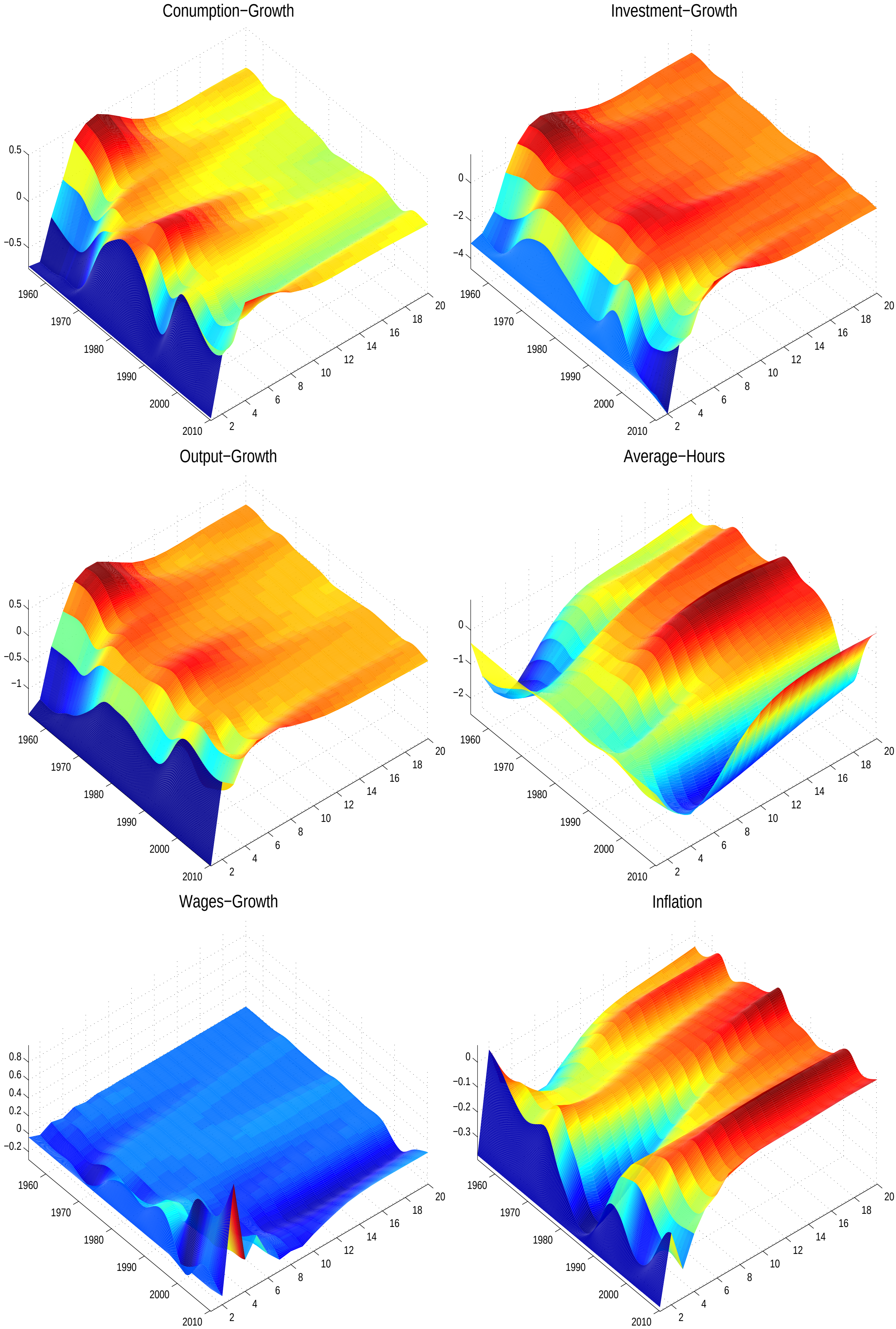


Figure 2: IRFs Constant Σ_v: B

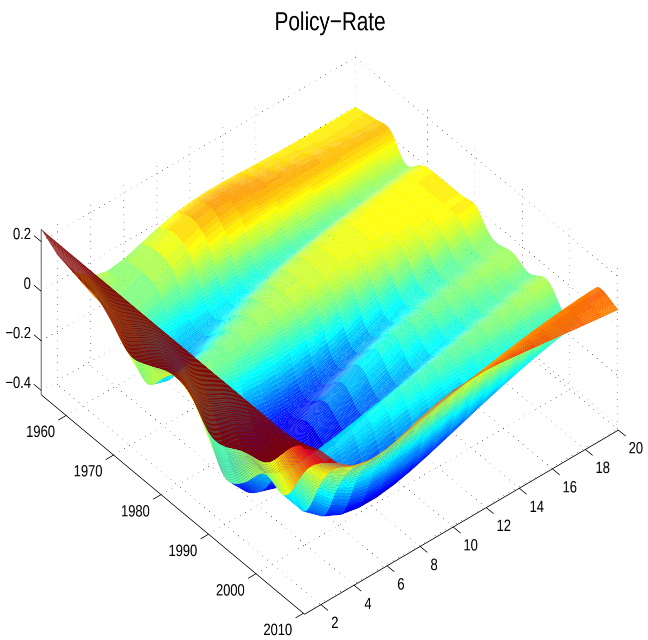


Figure 3: IRFs Summary Constant Σ_v

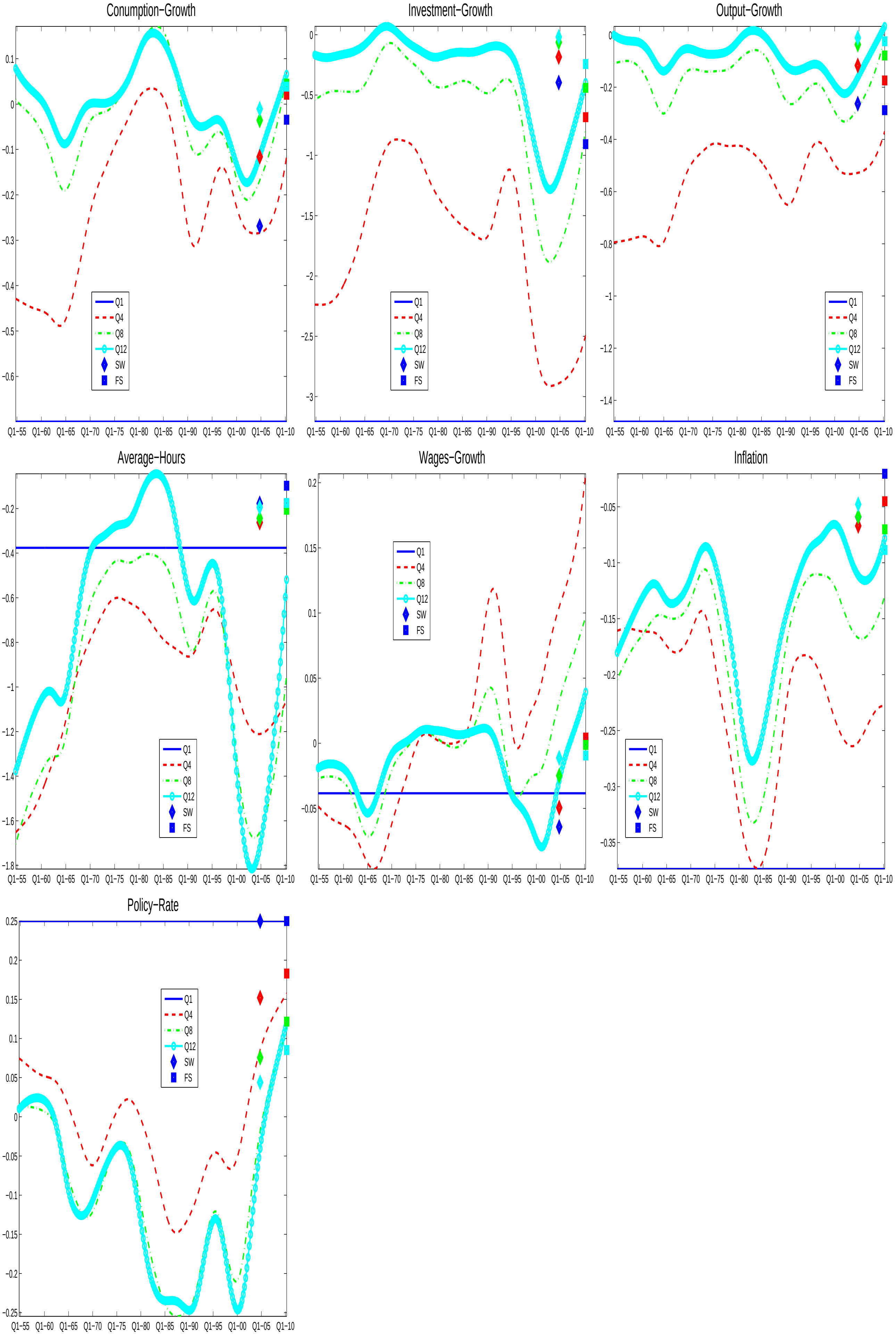


Figure 4: Structural Parameters Constant Σ_v: A

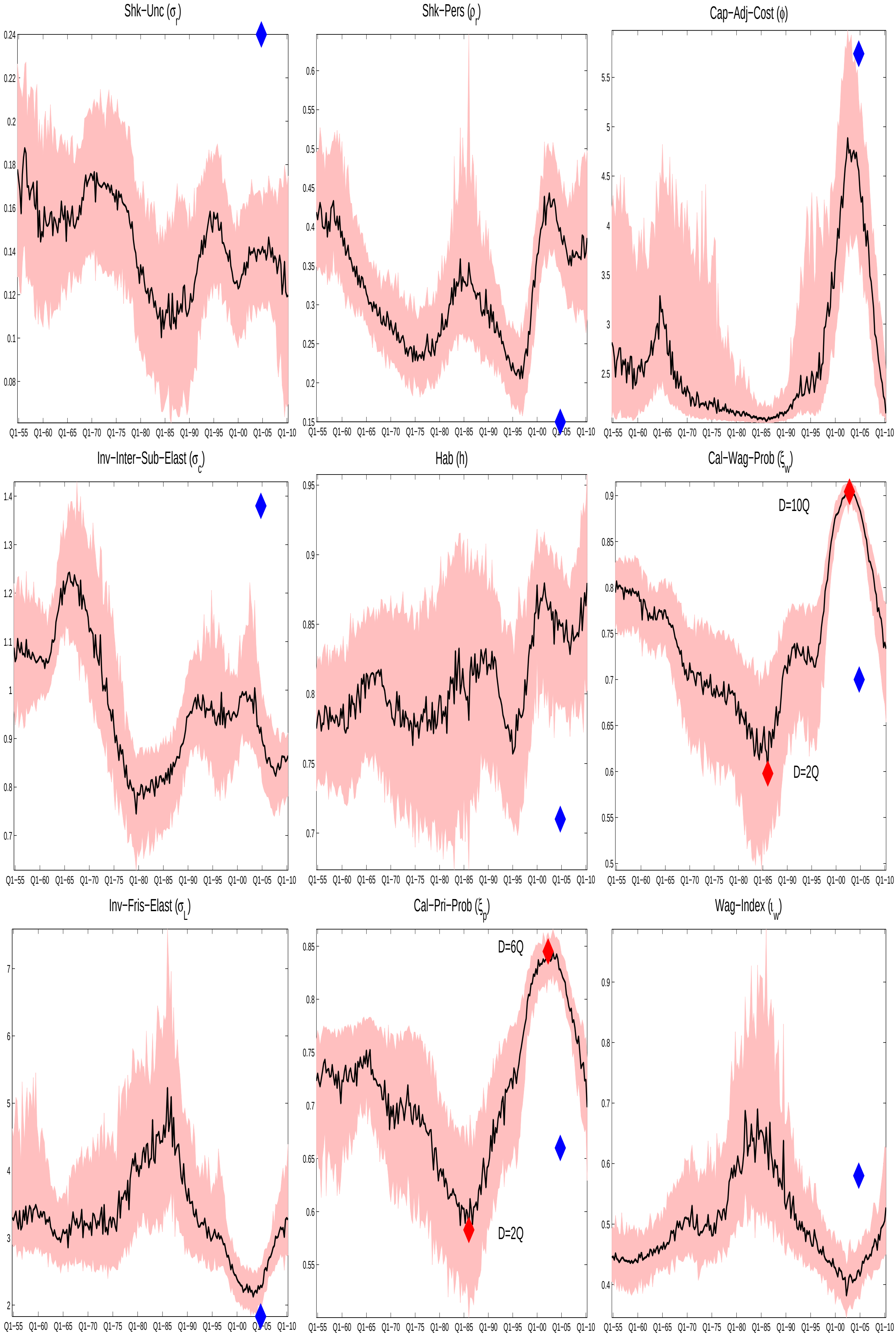


Figure 5: Structural Parameters Constant Σ_v: B

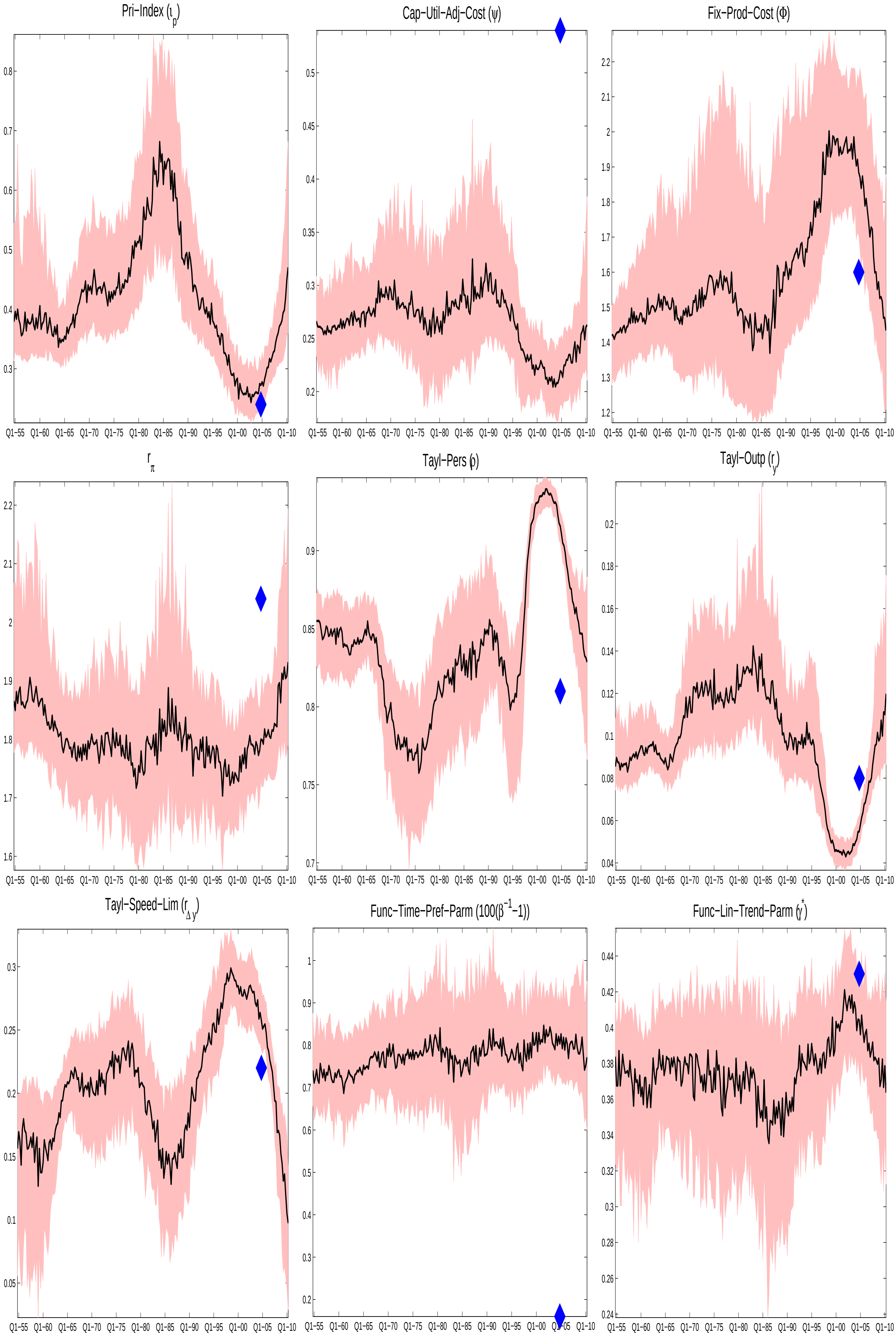


Figure 6: Structural Parameters Time-Varying Σ_v: A

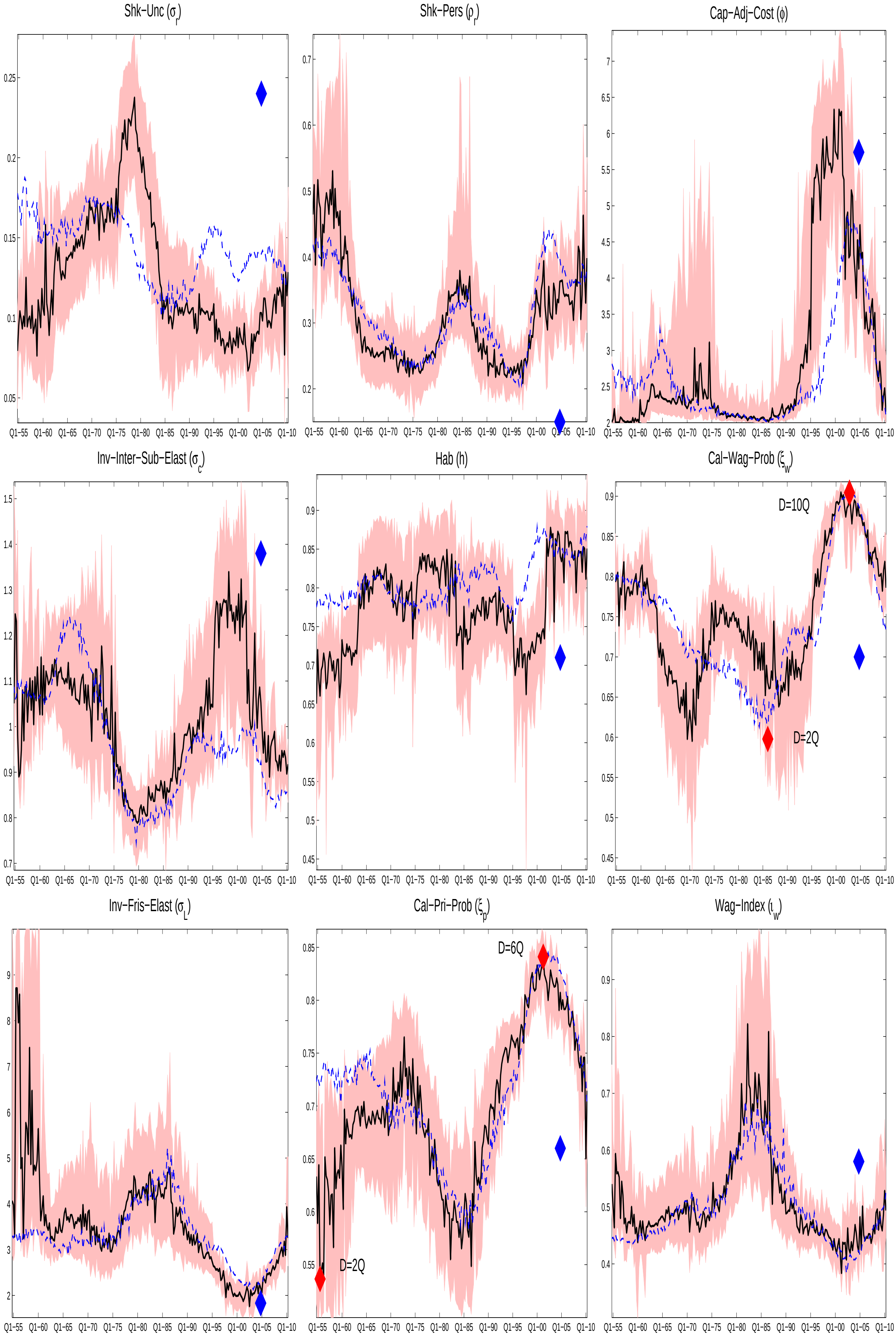


Figure 7: Structural Parameters Time-Varying Σ_v: B

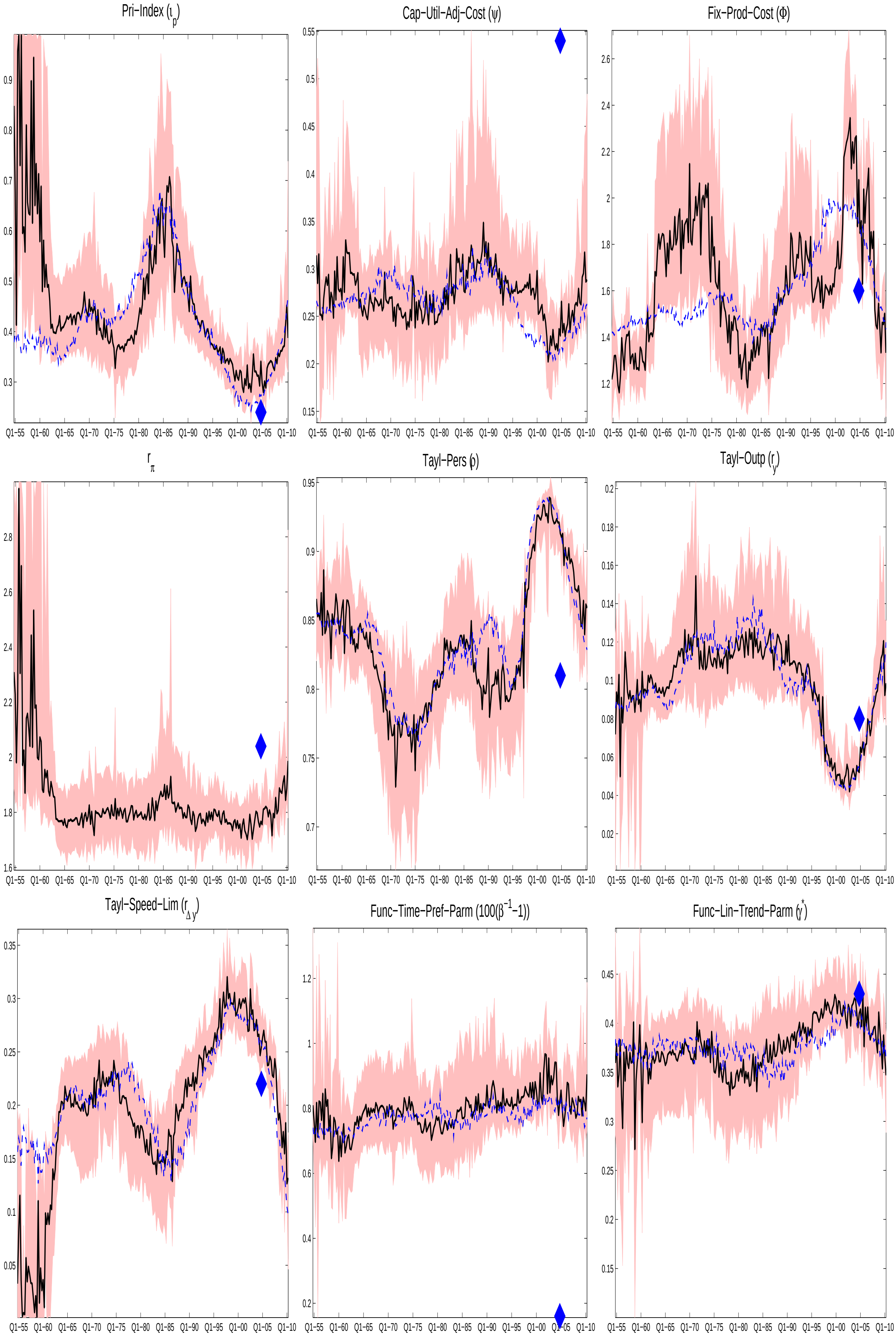


Figure 8: IRFs Time-Varying Σ_v: A

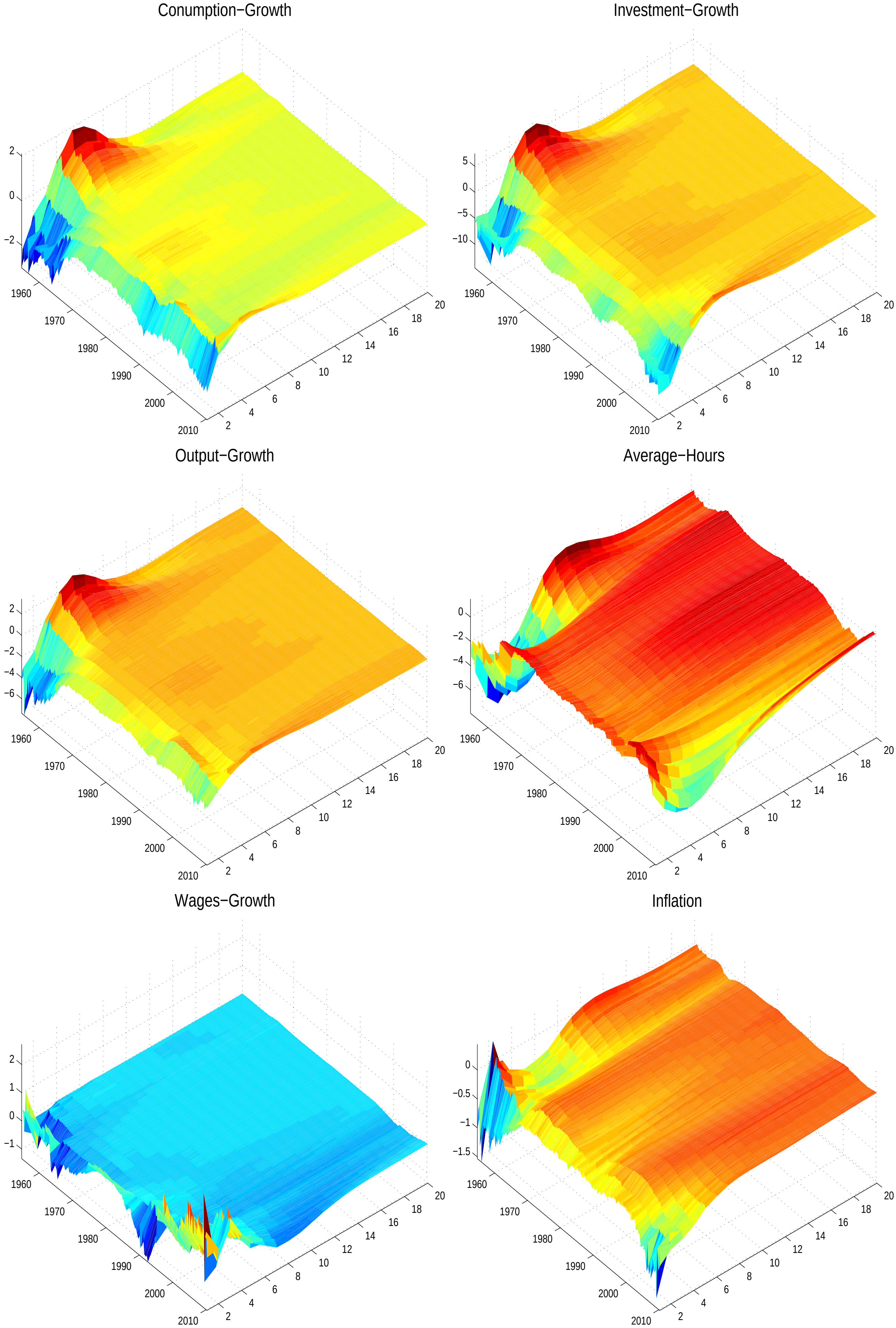


Figure 9: IRFs Time-Varying Σ_v: B

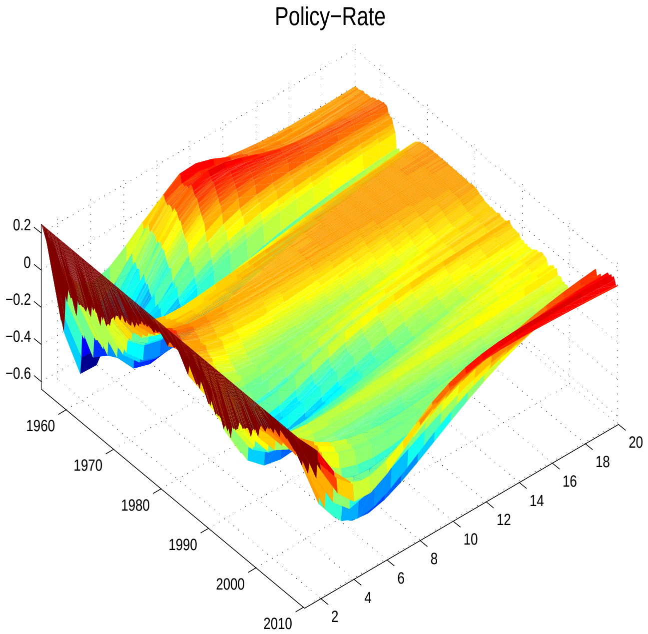
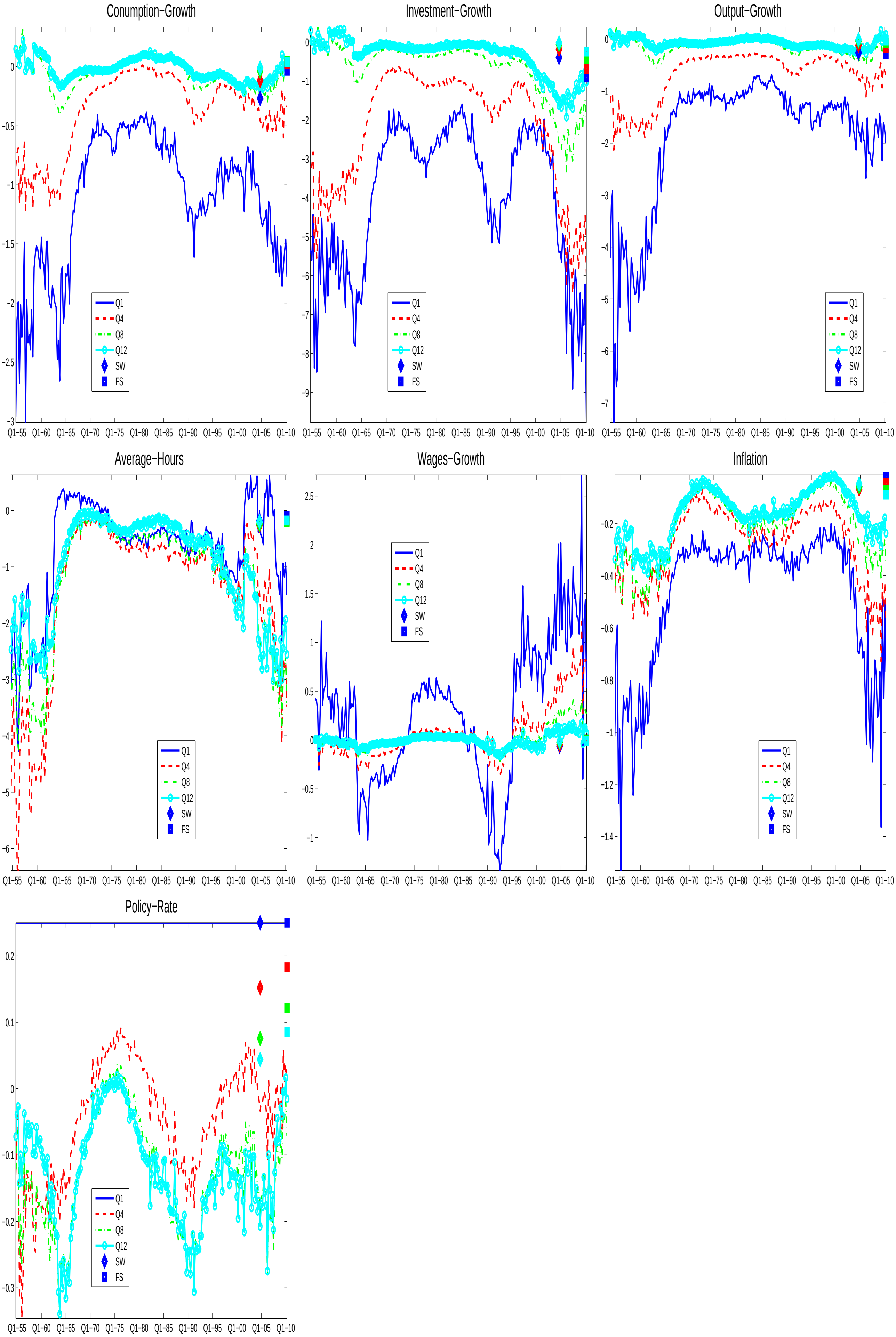


Figure 10: IRFs Summary Time-Varying Σ_v



Tables

Table 1: Parameter descriptions and estimated values from **?**

Symbols	Description	M_0
γ	Steady State Growth Rate	1.00
π	Steady State Inflation	1.00
Φ	Fixed Cost	1.50
S"	Steady State Capital Adjustment Cost Elasticity	5.74
α	Capital Production Share	0.19
σ	Intertemporal Substitution	1.38
h	Habit Persistence	0.71
ξ_w	Wages Calvo Parameter	0.70
σ_l	Labour Supply Elasticity	1.83
ξ_p	Prices Calvo Parameter	0.66
i_w	Wage Indexation	0.58
i_p	Price Indexation	0.24
z	Capital Utilisation Adjustment Cost	0.27
ϕ_π	Taylor Inflation Parameter	2.04
ϕ_r	Taylor Inertia Parameter	0.81
ϕ_y	Taylor Output Gap Parameter	0.08
ϕ_{dy}	Taylor Output Gap Change Parameter	0.22
ρ_g	Government Spending Shock Persistence	0.97
ρ_{ms}	Policy Shock Persistence	0.15
ρ_p	Price Mark-up Shock Persistence	0.89
ρ_w	Wage Mark-up Shock Persistence	0.96
ma_p	Price Mark-up MA Term	0.69
ma_w	Wage Mark-up MA Term	0.84
σ_g	Government Spending Shock Uncertainty	0.53
σ_{ms}	Policy Shock Uncertainty	0.24
σ_p	Price Mark-up Shock Uncertainty	0.14
σ_w	Wage Mark-up Shock Uncertainty	0.24

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