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Reconciling Microeconomic and Macroeconomic Estimates of Price Stickiness

Adam Cagliarini, Tim Robinson and Allen Tran

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Authors: cagliarinia and robinsont at domain rba.gov.au

Economic Publications: ecpubs@rba.gov.au

Abstract

This paper attempts to reconcile the high estimates of price stickiness from macroeconomic estimates of a New-Keynesian Phillips Curve (NKPC) with the lower values obtained from surveys of firms' pricing behaviour. This microeconomic evidence also suggests that the frequency with which firms adjust their prices varies across sectors. The paper shows that in the presence of this heterogeneity, estimates of aggregate price stickiness from microeconomic and macroeconomic data should differ. Heterogeneity in firms' pricing decisions, as well as a more realistic production structure, is introduced into an otherwise standard New-Keynesian model. Using a model calibrated with microeconomic pricing survey data for Australia, the paper shows that estimates of the NKPC considerably overstate the true degree of price stickiness and may falsely suggest that some prices are indexed to past inflation. These problems arise because of a type of misspecification and a lack of suitable instruments.

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RECONCILING MICROECONOMIC AND MACROECONOMIC ESTIMATES OF PRICE STICKINESS

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1. Introduction

The degree of price stickiness has a major influence on the behaviour of inflation and is an important determinant of the effects of monetary policy; all else equal, the stickier are prices, the larger is the response of economic activity to a monetary policy shock.

A common way of modelling price stickiness is to assume that the opportunity for firms to reset their prices in any particular period is a random event. The probability that they are unable to do so is known as the Calvo probability (Calvo 1983), which provides a natural measure of the degree of price stickiness. Aggregate measures of the Calvo probability have been estimated using both macroeconomic and microeconomic data. The two types of data tend to deliver very different estimates of the degree of aggregate price stickiness. The aim of this paper is to understand how and why these differences occur.

One approach to making inferences about an economy-wide Calvo probability is to estimate a New-Keynesian Phillips Curve (NKPC) using aggregate data on inflation and either output or real marginal costs. When prices are stickier, larger changes in output are required to change the rate of inflation. In other words, the Calvo probability is inversely related to the slope of the Phillips Curve. We will refer to an estimate of the Calvo probability obtained from the NKPC as θ_{NKPC}^{macro}; typically this is estimated using quarterly data to be at least 0.75, which implies that the average duration between price changes is at least four quarters.[1]

A second approach is to use data from surveys of firms or microeconomic-level price data (for example, see Blinder *et al* 1998). By measuring the average time that a price remains unchanged within a sector and taking a weighted average of these durations across sectors, it is possible to calculate an economy-wide average duration of prices. An aggregate measure of the Calvo probability can

1 For a range of values, see Dennis (2006) and Schorfheide (2008).

then be inferred from the average duration of prices, which we refer to as θ^{micro}. Microeconomic studies using the data which underlie the United States' CPI suggest that prices remain fixed on average for between one to three quarters (Bils and Klenow 2004; Klenow and Kryvtsov 2008; Nakamura and Steinsson 2008). This duration implies that the quarterly θ^{micro} is around 0.5.

The microeconomic data also reveal that considerable heterogeneity exists in the frequency with which prices are reset across sectors – see Klenow and Kryvtsov (2008). We show that in the presence of this heterogeneity, aggregate estimates of the Calvo probability from microeconomic and macroeconomic studies should not be expected to be equal. Further, we derive an aggregate measure of the Calvo probability that should be obtained from the aggregate data and used in macroeconomic modelling when heterogeneity exists, which we label θ^{macro}_{theory}.

A stark finding of our paper is that θ^{macro}_{theory} is *lower* than θ^{micro}, whereas (as already noted) θ^{macro}_{NKPC} is typically much higher than θ^{micro}. We argue that θ^{macro}_{NKPC} is a poor estimator of θ^{macro}_{theory} due to econometric problems stemming from the heterogeneity in price stickiness evident in the microeconomic data.

Imbs, Jondeau and Pelgrin (2007) also suggest that the divergence between the macroeconomic and microeconomic estimates of the frequency of price resetting could reflect heterogeneity across sectors, but focus on estimating an aggregate Calvo probability using sectoral panel data. Our alternative approach is to introduce various types of heterogeneity into a structural model.

The introduction of heterogeneity raises the issue of how firms interact with each other and consumers. One approach, which we adopt, is to allow firms to use the output of all other firms as intermediate inputs, which is known as roundabout production (Basu 1995). We show that in economies with both roundabout production and heterogeneity, the conventional measure of real marginal costs, namely labour's share of income, is no longer suitable and ignoring this contributes to the upwards bias of θ^{macro}_{NKPC}.

Because the standard NKPC is often unable to capture the persistence evident in inflation, a lag of inflation is often included. This is motivated by the possibility that some firms index their prices to past inflation instead of setting their prices optimally when they have the opportunity to reset prices (see, for example, Galí and Gertler 1999). These indexation assumptions are used despite there being no empirical microeconomic evidence of such behaviour. The resulting Phillips curve is referred to as the hybrid NKPC; estimates of these suggest that 80 per cent of firms index their prices to past inflation (see Schorfheide 2008). We show that a more realistic model with roundabout technology can generate the persistence in inflation evident in the data without resorting to ad hoc assumptions about the behaviour of prices. Further, if heterogeneity is also present, estimates of the hybrid NKPC will falsely suggest that the indexation of some prices exists when in reality there is none.

In the next section, the relationship between estimates of the Calvo probability obtained using macroeconomic and microeconomic data is clarified. Section 3 briefly describes the model used to generate data for the econometric analysis presented in Section 4. The macroeconomic implications of those estimates are discussed in Section 5 and Section 6 concludes.

2. Comparisons of Economy-wide Calvo Probabilities

Macroeconomic estimates of price stickiness typically rely on the assumption that all firms have the opportunity to reset their prices with the same probability. We relax this assumption and show that, at least in theory, the macroeconomic estimate of aggregate price stickiness should be less than the microeconomic-based estimate. In practice, the opposite is found – NKPC-based estimates tend to imply much more price stickiness than microeconomic-based estimates.

Under the Calvo pricing mechanism, a firm faces a probability θ_j (the Calvo probability) that it will not be given the opportunity to re-optimise the price it charges for its output in a given period. For such a firm, the output price would have an average duration given by

$$D(\theta_j) \equiv \frac{1}{1-\theta_j}, \tag{1}$$

which is a strictly increasing and convex function in the Calvo probability, θ_j. Since $D(\theta)$ is non-linear, we show below that inferring an economy-wide Calvo probability from an economy-wide average duration will yield an estimate which differs from that which is relevant for determining the slope of the aggregate

NKPC. The discrepancy between these two Calvo probabilities turns out to be economically significant, as they imply substantially different degrees of aggregate price stickiness.

2.1 Microeconomic-based Estimates

Microeconomic-based studies typically calculate the average length of time that prices are fixed for a particular product j, and compute a weighted average of these average durations to produce an economy-wide estimate for the average duration of prices, denoted by $\mathbb{E}(D(\theta_j))$ (see, for example, Klenow and Kryvtsov 2008; Nakamura and Steinsson 2008). Some economists then infer a Calvo probability, θ^{micro}, from this statistic. If the Calvo probability were homogenous across firms, θ^{micro} would reflect the economy-wide average Calvo probability, denoted by $\mathbb{E}(\theta_j)$. However, given the non-linearity of Equation (1) described above, Jensen's inequality tells us that when firms face different Calvo probabilities, θ^{micro} will be greater than the average Calvo probability $\mathbb{E}(\theta_j)$, that is

$$\begin{aligned} D(\theta^{micro}) \equiv \mathbb{E}(D(\theta_j)) &> D(\mathbb{E}(\theta_j)) \\ \Rightarrow \mathbb{E}(\theta_j) &< \theta^{micro}. \end{aligned} \tag{2}$$

$\mathbb{E}(\theta_j)$ and θ^{micro} are equal only in the absence of heterogeneity.

2.2 Macroeconomic-based Estimates

Using a similar argument, we can compare macroeconomic-based estimates of the Calvo probability inferred from an aggregate NKPC to the average Calvo probability from microeconomic data.

Assume that there are a finite number of sectors in the economy with firms in any given sector facing the same Calvo probability, but with Calvo probabilities varying across sectors. Each sector j, with Calvo probability θ_j, would therefore have its own NKPC given by

$$\pi_{j,t} = \frac{(1-\theta_j)(1-\beta\theta_j)}{\theta_j} mc_{j,t} + \beta \mathbb{E}_t \pi_{j,t+1}, \tag{3}$$

where: $\pi_{j,t}$ is inflation in sector j; β is the subjective discount factor of households; and $mc_{j,t}$ is the real marginal cost faced by firms in sector j. A NKPC for aggregate inflation, π_t, is obtained by weighting each sectoral NKPC according its weight in the price index (w_j)

$$\pi_t = \sum_{j=1}^{N} w_j \left(\frac{(1-\theta_j)(1-\beta\theta_j)}{\theta_j} mc_{j,t} + \beta \mathbb{E}_t \pi_{j,t+1} \right). \tag{4}$$

Let the coefficient on marginal costs for a particular sector's NKPC be denoted as

$$\lambda(\theta_j, \beta) = \frac{(1-\beta\theta_j)(1-\theta_j)}{\theta_j}.$$

We can decompose the coefficient on marginal costs as follows,

$$\lambda(\theta_j, \beta) = \bar{\lambda} + e_{\lambda,j},$$

where $\bar{\lambda}$ is a weighted average of the coefficients on marginal costs across the sectors; and $e_{\lambda,j}$ is the deviation from this average for a particular sector. We can then write the aggregate NKPC as

$$\pi_t = \bar{\lambda} mc_t + \beta \mathbb{E}_t \pi_{t+1} + \sum_j w_j e_{\lambda,j} mc_{j,t}, \tag{5}$$

where: $mc_t \equiv \sum_j w_j mc_{j,t}$; $\bar{\lambda} \equiv \sum_j w_j \lambda_j$; and the last term can be thought of as an 'error' term. Assuming that we can estimate $\bar{\lambda}$ without bias, and taking β as given, we can infer the macroeconomic estimate of the Calvo probability, θ^{macro}_{theory}, which solves $\lambda(\theta^{macro}_{theory}, \beta) = \bar{\lambda}$. Since $\lambda(\theta, \beta)$ is decreasing and convex in θ, we obtain

$$\begin{aligned} \lambda(\theta^{macro}_{theory}, \beta) \equiv \mathbb{E}(\lambda(\theta_j, \beta)) &\geq \lambda(\mathbb{E}(\theta_j), \beta) \\ \Rightarrow \theta^{macro}_{theory} &\leq \mathbb{E}(\theta_j). \end{aligned} \tag{6}$$

2.3 Comparing Microeconomic and Macroeconomic Estimates

From Equations (2) and (6) we have

$$\theta^{macro}_{theory} < \mathbb{E}(\theta_j) < \theta^{micro}, \tag{7}$$

with equality holding if all firms face the same Calvo probability. Therefore, macroeconomic estimates of the Calvo probability should be lower than corresponding microeconomic estimates. However, the evidence we cited in the introduction goes in the opposite direction; in practice, macroeconomic estimates of the Calvo probability (θ^{macro}_{NKPC}) are typically much *larger* than microeconomic estimates (θ^{micro}).

Part of the tension between the microeconomic and macroeconomic estimates of the Calvo probability reflect how θ^{micro} itself is constructed. A better way to utilise the microeconomic data in order to produce an estimate of the Calvo probability is to separately derive Calvo probabilities for each sector from the duration of that sector and then weight these appropriately. This is $\mathbb{E}(\theta_j)$.

However, $\mathbb{E}(\theta_j)$ is not the relevant estimate for the purpose of determining the slope of an aggregate NKPC. This is θ^{macro}_{theory}, which can be derived from the weighted average of the coefficients on marginal costs in the sectoral NKPCs.

Given the results from a pricing survey undertaken by the Reserve Bank of Australia presented later in Table 1 (Section 3.2), the Calvo probability consistent with this approach is $\hat{\theta}^{macro}_{theory} = 0.30$, implying that prices are fixed for just over 4 months on average, whereas the commonly used Calvo probability from the average duration statistic is $\hat{\theta}^{micro} = 0.59$, suggesting prices are fixed for just over 7 months on average, and $\mathbb{E}(\theta_j) = 0.48$. These are all much smaller than the estimate of $\hat{\theta}^{macro}_{NKPC} = 0.94$ from the NKPC for Australian data in Kuttner and Robinson (forthcoming).

As we will show later, estimates of price stickiness from aggregate NKPCs (θ^{macro}_{NKPC}) are high primarily for two reasons. First, in the presence of roundabout production and heterogeneity in factor shares, the aggregate NKPC is misspecified when the aggregate labour share is used as the measure of aggregate marginal costs. Second, heterogeneity implies that commonly used instruments may be weak and invalid, and this is exacerbated if roundabout production also exists.

Note that these problems are not specific to GMM and apply to all estimates of *aggregate* NKPCs which fail to account for roundabout production and heterogeneity.

3. The Data-generating Process

In this section, we sketch the model we use as the data-generating process for our evaluation of the effect of heterogeneity and roundabout production on the GMM estimates of the NKPC. We also discuss parameter selection, compare moments of the model to moments in the data and briefly explore the properties of the model with and without heterogeneity and roundabout production.

3.1 The Model

The model is similar to those used by Carvalho (2006) and Nakamura and Steinsson (2009), which both incorporate heterogeneity in the average duration of prices across firms. However, our approach differs in a number of respects. For example, we deviate from Nakamura and Steinsson (2009) by assuming that pricing is time-dependent.[2] Our model also incorporates a wider number of shocks than both Carvalho (2006) and Nakamura and Steinsson (2009), many of which are typically found in DSGE models.

The basic foundations of the monetary model described below are standard in the literature (for example, Ireland 2004). We add roundabout production as well as heterogeneity in Calvo price setting and technology. The model is described briefly below (a full description is provided in Appendix A).

The model contains the following agents:

- households, who are modelled in a standard way;
- final-goods firms, that produce final goods from intermediate goods;
- intermediate-goods firms, that produce intermediate goods for other intermediate-goods firms and final-goods firms; and

2 They do, however, consider a limiting case with large adjustment costs as an approximation to the Calvo model, which they refer to as the CalvoPlus model.

- a monetary authority.

Final-goods producers are perfectly competitive and use constant elasticity of substitution technology to produce final goods from intermediate goods, which they obtain from intermediate-goods firms in a particular sector. There is a continuum of monopolistically competitive intermediate-goods producers, indexed on the interval $(0,1]$. We segment this interval into a finite number of partitions, each representing a particular sector of the economy.

A characteristic of modern economies is a complex interdependence between firms within and across sectors of the economy.[3] In our model, goods produced by intermediate-goods firms in one sector can be used by other firms in that sector, by other intermediate-goods firms operating in other sectors and by the final-goods producers who only source their inputs from that sector. Intermediate-goods producers also use labour supplied by households. The monetary authority sets the interest rate as a function of inflation and growth in consumption (value added).

Each sector of the economy is characterised by its own production technology and Calvo probability (θ_j). Production of intermediate goods is characterised by Cobb-Douglas technology, with the factor shares for labour and for intermediate goods varying across sectors. This means that in the steady state, labour cost shares, marginal costs and prices will be sector-dependent.

The model incorporates four sources of uncertainty (in addition to that implied by the Calvo pricing mechanism):

- a consumption preference (demand) shock;
- an aggregate non-stationary technology process for intermediate-goods producers;
- sector-specific technology shocks for intermediate-goods producers; and

[3] Input-output tables clearly show that a large share of the output of firms is used as an input by other firms from the same sector. As an example, firms in the business services sector in Australia source about 70 per cent of their intermediate inputs from other firms in that same sector. In the typical tiered approach to modelling production, the output of firms in a sector is used only by firms in other sectors.

- a monetary policy shock.

3.2 Parameter Selection

The parameters of the model are mostly calibrated, reflecting our goal of creating a model to simulate realistic data, rather than to precisely estimate the true data-generating process.

A key part of the model is the heterogeneity in the Calvo probability across sectors. We calibrate the Calvo probability for each sector by using data collected by the Reserve Bank of Australia through a survey of firms. Briefly, the survey recorded 438 responses to a questionnaire on price setting from June 2000 to April 2006.[4] The survey contains the median number of price adjustments over 12 months and we compute the implied quarterly Calvo probabilities (θ_j) from the frequency of price changes reported by firms.

The survey asks the question 'how many times has the firm actually changed prices in the last 12 months?', which does not distinguish between price changes caused by sales or product substitutions. We assume that there is no indexation of prices consistent with microeconomic price behaviour in databases such as Dominick's Finer Food Database and the CPI Research Database (Midrigan 2007; Klenow and Kryvtsov 2008).[5] Table 1 presents the median number of price adjustments over 12 months and the associated Calvo probability for each sector.

The size of sector j is denoted by γ_j. In conjunction with the labour shares of income, these size parameters govern sectors' steady-state shares of labour and intermediate inputs. We choose these size parameters to try and match these shares to the data. Because there are 9 size parameters and essentially 28 moments, we cannot match all of these moments. Instead, we minimise the weighted average of the difference between the implied steady shares and those in the data. The moments reflecting each sector's share of labour, intermediate input shares and

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4 As the survey tries to match population characteristics, the number of responses from firms in the utilities and agriculture sectors is low.

5 'Backward indexation of prices, an assumption which, as far as I know, is simply factually wrong, has been introduced to explain the dynamics of inflation' (Blanchard 2009, p 25). Also see Chari, Kehoe and McGrattan (2009).

Table 1: Calvo Probability by Sector[a]

Sector	Average duration (quarters)	Calvo probability (θ_j)
Agriculture	4	0.75
Construction	$1\frac{1}{3}$	0.25
Manufacturing	2	0.50
Mining	4	0.75
Utilities	4	0.75
Wholesale and retail trade	1	<0.25 (0.1)[b]
Transport and storage	4	0.75
Business services	4	0.75
Household services	4	0.75
Tourism	4	0.75
Memo items:		
$\mathbb{E}(\theta_j)$		0.48
$\hat{\theta}^{micro}$		0.59
$\hat{\theta}^{macro}_{theory}$		0.30

Notes: (a) The Calvo probability (θ_j) is the probability that an intermediate-goods firm in sector j cannot reoptimise its price in a given quarter.
(b) Note that the wholesale and retail trade sector results imply that prices change more frequently than quarterly. Since the survey asks firms how often they changed *all* prices and in our model firms only produce one good, it is possible that prices for single goods are fixed for longer than a quarter. Consequently, we set the Calvo probability to 0.1 which implies prices are minimally sticky. If instead we assume that the wholesale and retail trade is a flexible-price sector, the results are quantitatively similar.

Sources: RBA; authors' calculations

gross revenue are taken from the Australian Bureau of Statistics (ABS) over the period 1995–2003. The economy-wide steady-state share of value added in gross output, $\frac{C}{Y}$, is 0.41 in the data whereas our choice of parameters implies 0.48. The results are shown in Table 2.[6]

In order to calibrate each sector's technology parameters, we use the annual experimental estimates of multifactor productivity (MFP) by sector as published by the ABS in 2007. Because the sectoral technology is assumed to follow a stationary quarterly AR(1) process in our model, we quadratically interpolate the data to obtain a quarterly series. We then detrended the logarithm of the quarterly series and estimated the AR(1) process for each sector from the filtered series. The

6 Note that the labour shares of income for the various sectors are lower than the aggregate measure from the national accounts as they are parameters in a gross output, rather than value-added, production function.

Table 2: Average Shares of the Total Economy for Each Sector
1995–2003[a,b]

	Gross revenue[c]		Hours worked		Intermediate inputs	
Sector	Data	Calibrated	Data	Calibrated	Data	Calibrated
Agriculture	0.05	0.06	0.09	0.05	0.04	0.06
Construction	0.14	0.15	0.12	0.10	0.17	0.17
Manufacturing	0.28	0.29	0.22	0.25	0.33	0.30
Mining	0.06	0.05	0.02	0.03	0.04	0.05
Utilities	0.03	0.03	0.01	0.02	0.03	0.03
Wholesale and retail trade	0.18	0.19	0.28	0.26	0.16	0.17
Transport and storage	0.08	0.08	0.08	0.07	0.08	0.09
Business services	0.12	0.07	0.09	0.12	0.08	0.06
Household services	0.05	0.05	0.07	0.06	0.05	0.05
Tourism	0.02	0.02	0.03	0.03	0.02	0.02

Notes: (a) Numbers presented are each sector's share of the total over the period 1995–2003 and their corresponding calibrated values. Values do not sum to 1 in the table because of rounding errors.
(b) Due to the lack of multifactor productivity estimates for some sectors, we exclude the following five sectors from our calculations: property and business services, government administration and defence, education, health and community services, and personal and other services.
(c) Excludes payments to capital.

Sources: ABS – *Australian National Accounts: Input-Output Tables* (Cat No 5209.0.55.001); *Labour Force, Australia, Detailed, Quarterly* (Cat No 6291.0.55.003); *Experimental Estimates of Industry Multifactor Productivity* (Cat No 5260.0.55.002); authors' calculations

calibrated values for the steady-state shares and estimated parameters describing sector technology are displayed in Table 3. We also use the market sector MFP series to calibrate the standard deviation of aggregate technology shock.

We set the Frisch elasticity of labour supply to one-half, consistent with the parameter values used in Carvalho (2006). Based on the industrial organisation and international trade literature, we set the elasticity of substitution to 4 (a one-third mark-up) as in Nakamura and Steinsson (2009). See Table 4 for a list of calibrated and estimated parameters not available directly from the data.

The log-linearised rational expectations model is solved using the method outlined in Sims (2002). We estimate the remaining parameters of the model, namely the monetary policy rule parameters, the persistence of the preference shocks and the standard deviations of some aggregate shocks, by maximum likelihood using the Kalman filter (see Hamilton 1994). The Kalman filter is required because we treat most of the variables in the model as unobservable; for example, many

Table 3: Technology Parameters for Each Sector

Sector	Persistence (ρ_j)	Technology shocks std deviation (σ_{zj})	Labour share of income (α_j)
Agriculture	0.83	3.91	0.29
Construction	0.88	1.46	0.24
Manufacturing	0.86	0.60	0.29
Mining	0.80	1.71	0.24
Utilities	0.93	0.53	0.24
Wholesale and retail trade	0.88	0.50	0.39
Transport and storage	0.87	0.61	0.29
Business services	0.91	0.61	0.44
Household services	0.79	0.69	0.35
Tourism	0.91	0.80	0.39

Sources: ABS – *Information Paper: Experimental Estimates of Industry Multifactor Productivity* (Cat No 5260.0.55.001); updated in Cat No 5260.0.55.002; authors' calculations

of the real variables in the log-linearised model have been detrended by the unobserved non-stationary aggregate technology series. The rational expectations solution to the model dictates how the model variables evolve over time and the relationships between them. The Kalman filter is used to estimate the unobserved series in a manner consistent with both the reduced form of the model and the observed variables, which include growth in GDP per capita, the overnight cash rate and headline CPI inflation (excluding taxes and volatile items) over the period 1993:Q1 to 2007:Q4.

3.3 Properties of the Model

3.3.1 Hazard functions

The hazard function for price changes describes the probability of a price change at time t given that the price has not changed between period 0 and period $t-1$. Comparing the characteristics of the hazard function from a model to that estimated from microeconomic data is a simple way of assessing the validity of the model's description of price-setting behaviour.

Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008), using the microeconomic data underlying the US CPI, find that the aggregate hazard

Table 4: Calibrated and Estimated Behavioural Parameters

Parameter	Description	Value	Std error
	Calibrated		
β	Household discount factor	0.99	–
ε	Elasticity of substitution	4.00	–
η	Frisch Elasticity of labour supply	0.50	–
σ_z	Std deviation of aggregate technology shock	0.44	–
	Estimated		
ρ_i	Persistence of the nominal interest rate	0.71	0.04
ϕ_π	Long-run response of policy rate to inflation	1.16	0.13
ϕ_g	Long-run response of policy rate to growth	0.21	0.11
ρ_a	Persistence of preference shocks	0.89	0.07
σ_i	Std deviation of interest rate shock	0.12	0.02
σ_a	Std deviation of preference shock	0.40	0.11

Source: authors' calculations

function is downward-sloping. That is, prices are less likely to change the longer they remain unchanged. Klenow and Kryvtsov (2008) argue that the result is likely to reflect heterogeneity, and once this is taken into account the estimated aggregate hazard function is flat.

The Calvo model predicts that the probability of a firm changing its price is independent of its duration, and therefore hazard functions, both at the product and aggregate level, are flat. Alternatively, by allowing for heterogeneity in the Calvo probability across sectors our model is capable of replicating the results of Klenow and Kryvtsov (2008), namely flat product-level hazard functions and a downward slope for the aggregate hazard function, which we demonstrate analytically in Appendix C. Our model therefore can generate an important characteristic of the microeconomic pricing data that the standard Calvo model cannot.

3.3.2 Empirical results

We compare the properties of four variations of our model in order to disentangle the effects of heterogeneity and roundabout production:

- (Baseline) A version of our model with labour as the only input to intermediate-goods production (that is, no roundabout production) and sectors that are homogeneous in every other respect.

- (Heterogeneous) This version adds heterogeneity in the Calvo probability and sectoral technology shocks but there is no roundabout production technology in the model.[7]

- (Roundabout) Roundabout technology is added to the baseline model so that intermediate-goods producers use other intermediate goods in production. However, this variation treats all sectors as homogenous so that the Calvo probabilities are uniform across sectors and there are no sector-specific technology shocks.[8]

- (Full) The full model includes roundabout technology, heterogeneity in Calvo probabilities and production technology as well as sector-specific technology shocks.

Our results can be summarised by the moments of the various models and the impulse response functions.

Table 5 presents actual and simulated moments for some of the key variables, namely growth in GDP per capita (g_t), inflation (π_t) and the nominal interest rate (i_t). The full model broadly matches the univariate moments of the data shown (compare the first column – the actual data – and the last). The magnitudes of the contemporaneous cross-correlations are not as well matched, but are correctly signed.

An effect of roundabout production is to increase the persistence of inflation to the magnitude that is evident in the data – the autocorrelation of inflation is 0.42 in the roundabout model, compared to 0.20 in the baseline model (Table 5). This occurs because the output of each intermediate-goods firm, whose price is sticky,

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7 Consequently, labour is the only factor of production, and its share of income, α_j, is 1 for all j. The absence of intermediate inputs also implies that $\frac{C}{Y} = 1$.

8 We set the Calvo probability to 0.3 in the roundabout and baseline models, which is the θ^{macro}_{theory} reported in Table 1.

Table 5: Moments of Observed and Simulated Series

Parameter	Actual	Baseline	Heterogeneous	Roundabout	Full
$Var(g_t)$	0.33	0.22	0.32	0.25	0.33
$Var(\pi_t)$	0.05	0.13	0.16	0.08	0.06
$Var(i_t)$	0.05	0.02	0.04	0.02	0.03
$Corr(g_t, \pi_t)$	–0.01	0.03	–0.20	0.16	0.06
$Corr(g_t, i_t)$	–0.12	0.07	–0.04	0.03	–0.02
$Corr(i_t, \pi_t)$	0.27	0.33	0.43	0.24	0.32
$Corr(g_t, g_{t-1})$	–0.04	–0.01	0.00	–0.04	–0.03
$Corr(i_t, i_{t-1})$	0.93	0.89	0.89	0.87	0.87
$Corr(\pi_t, \pi_{t-1})$	0.42	0.20	0.23	0.42	0.48

Notes: The moments are asymptotic except for those for actual data. The sample for the actual data is 1993:Q1 to 2007:Q4.

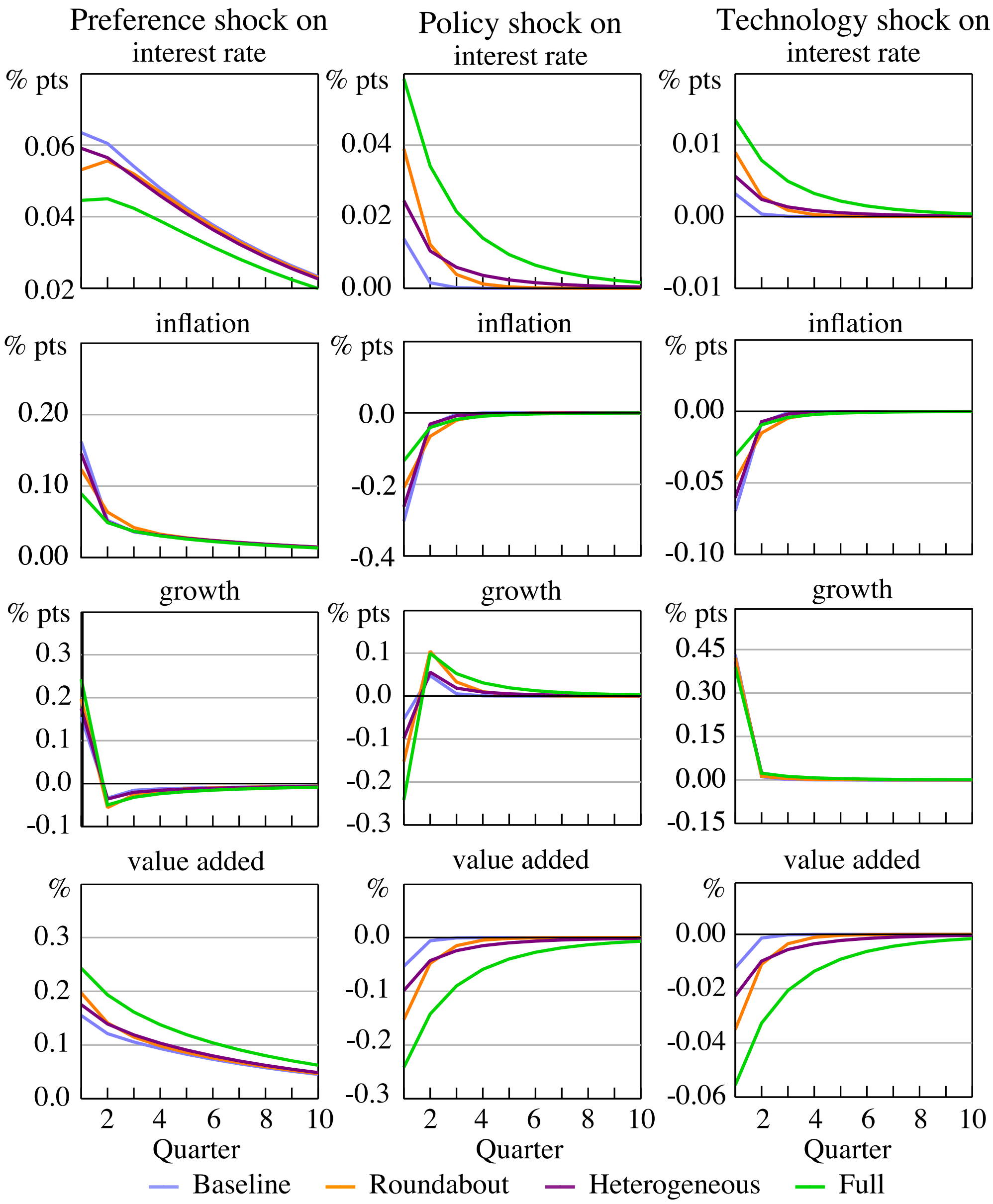
Sources: ABS; authors' calculations

is an input to the production of other goods. Consequently, the marginal costs of intermediate-goods firms, and hence inflation, are more persistent.

It is also noticeable that volatility of growth in value added is larger in the heterogeneous model. This may partially reflect that in addition to introducing variation in the Calvo parameter, this model also incorporates a larger set of shocks than the baseline model, such as sector-specific technology processes. The volatility of growth in value added in the full model is in line with that observed in the data.

We can also compare the four models by examining their impulse response functions. Figure 1 illustrates the paths of key variables relative to their steady-state values. To facilitate comparison, the standard deviations of the shocks are those estimated for the full model. Note that the initial response of the interest rate across the models to the same-sized monetary policy shock can be different because the response of inflation, and other variables, in the model will differ. Also note that a positive aggregate technology shock increases growth in value added and reduces inflation. Because this is a permanent shock, the steady-state level of value added increases, however, since prices are sticky initially, there is not a commensurate increase in actual value added, which falls below its steady-state level.

Figure 1: Comparison of Impulse Response Functions



It is clear that heterogeneity and roundabout production make a difference to the response of value added (which is equivalent to consumption in these models). This is because both heterogeneity and roundabout production lead to relatively more muted responses of inflation to the shocks; as prices take longer to respond, the bulk of the short-run effects of shocks manifests in quantities rather than prices. When heterogeneity and roundabout production are combined the two features appear to increase the response of value added to aggregate shocks even further.

Heterogeneity leads to muted price responses because the firms with stickier prices have a disproportionately large effect on those firms with more flexible prices (Carvalho 2006).[9] This is consistent with profit maximisation since firms generally try not to change their prices too much relative to their competitors. The firms with stickier prices change their prices by less in response to shocks. Even though firms in the more flexible-price sectors can alter their prices more frequently, they have an incentive to keep their prices close to their slow moving competitors. The result is that aggregate prices in a model with heterogeneity in price setting appear to respond less to shocks compared to a model where all firms adjust their prices with the same average frequency. This is apparent in Figure 1 with inflation responding by less in the heterogenous model compared with the baseline model, and by less in the full model compared with the roundabout model.

The inertia in prices implied by heterogeneity and roundabout production implies that unexpected changes in monetary policy will have a larger affect on output (value added). Indeed, we find that when only policy shocks are present, the variance of value added increases by almost thirty-three times (from 0.003 to 0.09 percentage points) when heterogeneity and roundabout production are included. This is consistent with Nakamura and Steinsson (2009) who find that the presence of these two features increases the potency of monetary policy. In contrast, using a single-sector menu cost model, Golosov and Lucas Jr (2007) found that '... in none of the simulations we conducted did monetary shocks induce large or persistent real responses'.

4. Estimating the Aggregate NKPC via GMM

In this section, we examine the implications of estimating aggregate NKPCs when the true data-generating process contains either heterogeneity, roundabout production or both. These Phillips curves are estimated using GMM, following the approach of Galí and Gertler (1999).

[9] Heterogeneity in the Calvo probability was also studied in a structural model by Carvalho (2006). He found that in order to replicate the empirical impulse responses from a model incorporating heterogeneity, the degree of price stickiness would have to be increased by approximately threefold in a model that ignores this source of heterogeneity. Our paper shows that failing to account for heterogeneity will result in common econometric methods of estimating the NKPC – such as the Generalised Method of Moments (GMM) used by Galí and Gertler (1999) – sizeably overstating the degree of price stickiness.

4.1 Quantifying the Inconsistency in GMM Estimates of the NKPC

In order to quantify the effect of heterogeneity and roundabout production on estimates of the aggregate NKPC, we use a Monte Carlo approach. We generate 1 000 datasets, each with 1 000 periods, from each of the four models of Section 3, and then use GMM to estimate a single NKPC with these data. We use 1 000 periods to focus on the asymptotic properties of the estimator; results for 120-period data are qualitatively similar.

GMM is widely used to estimate the NKPC because it is able to deal with the presence of expected inflation. Inflation can be decomposed into expected inflation, $\mathbb{E}_t\pi_{t+1}$, which is not observed, and the forecast error, v_{t+1}: $\pi_{t+1} = \mathbb{E}_t\pi_{t+1} + v_{t+1}$. The assumption of rational expectations is that people use all available information at time t when forming their expectations. This implies that the forecast errors should be unbiased ($\mathbb{E}_t v_{t+1} = 0$) and uncorrelated with all time t information, so that $\mathbb{E}_t(v_{t+1}z_t) = 0$, where z_t is an instrument (information which was available at time t). As this should hold across time, the unconditional expectation, $\mathbb{E}(v_{t+1}z_t) = 0$, is the orthogonality (moment) condition used by GMM.

Following Galí and Gertler (1999), we also instrument for marginal costs, which they motivate as an attempt to deal with the measurement error which is likely to exist in their variables representing real marginal costs. However, it is also likely that real marginal costs are endogenous, particularly in models that incorporate roundabout production. This effectively restricts our instruments, which are described below, to be dated $t-1$ or earlier.

We estimate the NKPC in its hybrid form where a fraction of firms who are able to change their prices in any given period do so mechanically by indexing to past inflation.[10] This allows us to determine whether estimates imply a role for indexation, even when none exists. Explicitly, this hybrid NKPC is of the form:

$$\pi_t = \frac{\beta\theta}{\phi}\mathbb{E}_t(\pi_{t+1}) + \frac{(1-\beta\theta)(1-\theta)(1-\omega)}{\phi}mc_t + \frac{\omega}{\phi}\pi_{t-1}, \tag{8}$$

[10] This form of the hybrid NKPC is due to Galí and Gertler (1999), and is more flexible than the form in Christiano, Eichenbaum and Evans (2005) as the model is free to reject the presence of indexation.

where: ω is the share of firms that index their prices; $\phi = \theta + \omega[1 - \theta(1 - \beta)]$; and mc_t is real marginal cost.

The exercise performed here is different to that presented in Imbs *et al* (2007). They simulate data from separate two-equation models for each of the sectors and estimates the reduced form aggregate NKPC via Maximum Likelihood.[11] Instead, we simulate data from a macroeconomic model with microeconomic foundations with interdependent sectors, which allows us to analyse the importance of the different sources of heterogeneity for the parameter estimates.

Galí and Gertler (1999) use the labour share of income as a measure for real marginal costs. They also use a large instrument set, including lags of inflation, real marginal costs, nominal wages growth, interest rates and detrended output. In contrast, we use a smaller instrument set for our baseline and roundabout models, as the New-Keynesian model without heterogeneity (and calibrated as described above) generates very high correlations amongst some of the potential instruments. (An alternative approach would be to incorporate more shocks into the model.) In the heterogeneous and full models we are able to use a larger instrument set, similar to Galí and Gertler (1999), as the introduction of heterogeneity considerably reduces the degree of correlation between the potential instruments. In particular, heterogeneity in factor shares and technology shocks across the sectors reduces the absolute correlation between the aggregate labour share and inflation.

The estimates based on our simulated data are presented in Table 6 and graphically using kernel density estimates in the figures that follow. The restricted instrument set includes 4 lags of inflation, growth in value added and the cash rate. The full instrument set also includes 2 lags of the labour share and wage inflation. We use a two-step GMM estimator, with a 12-lag Newey West heteroskedasticity and autocorrelation-consistent estimate of the covariance matrix, akin to Galí and Gertler (1999). We constrain the parameters β, θ^{macro} and ω to be between 0 and 1, so as to be economically meaningful. As GMM is sensitive to the parameterisation used in the moment conditions, we report estimates using two different parameterisations (as described in the notes to Table 6).

[11] The two-equation model consists of the NKPC and an AR(2) process for marginal costs.

Table 6: Monte Carlo Results from GMM Estimates of the Aggregate NKPC from Various Models

Parameter	True	Models: Baseline	Roundabout	Heterogeneous	Full
		Parameterisation 1			
β	0.99	1.00 (0.15)	1.00 (0.07)	1.00 (0.11)	1.00 (0.01)
θ^{macro}	0.30	0.38 (0.18)	0.42 (0.21)	0.81 (0.07)	0.85 (0.04)
ω	0.00	0.02 (0.04)	0.08 (0.10)	0.09 (0.05)	0.26 (0.05)
		with $\beta = 0.99$ imposed			
θ^{macro}	0.30	0.47 (0.28)	0.42 (0.19)	0.81 (0.09)	0.83 (0.04)
ω	0.00	0.05 (0.04)	0.08 (0.09)	0.09 (0.05)	0.24 (0.06)
		Parameterisation 2			
β	0.99	0.73 (0.14)	0.82 (0.10)	1.00 (0.06)	1.00 (0.01)
θ^{macro}	0.30	0.26 (0.03)	0.27 (0.02)	0.73 (0.04)	0.79 (0.03)
ω	0.00	0.00 (0.00)	0.00 (0.01)	0.03 (0.03)	0.19 (0.05)
		with $\beta = 0.99$ imposed			
θ^{macro}	0.30	0.24 (0.05)	0.29 (0.03)	0.72 (0.04)	0.77 (0.03)
ω	0.00	0.00 (0.01)	0.00 (0.01)	0.03 (0.03)	0.17 (0.05)

Notes: The median estimate is presented for each model and the estimates in parentheses are the standard deviations of the distribution of the parameters. The results in the lower panel of each half of the table are estimated with the restriction that $\beta = 0.99$. The moment conditions used to estimate the parameters are specified using two parameterisations:

(1)

$$\mathbb{E}\left\{\left(\pi_t - \frac{\omega}{\phi}\pi_{t-1} - \frac{(1-\beta\theta)(1-\theta)(1-\omega)}{\phi}mc_t - \frac{\beta\theta}{\phi}\pi_{t+1}\right)\mathbf{z_t}\right\} = 0,$$

where $\phi = \theta + \omega[1-\theta(1-\beta)]$ and $\mathbf{z_t}$ is the vector of instruments.

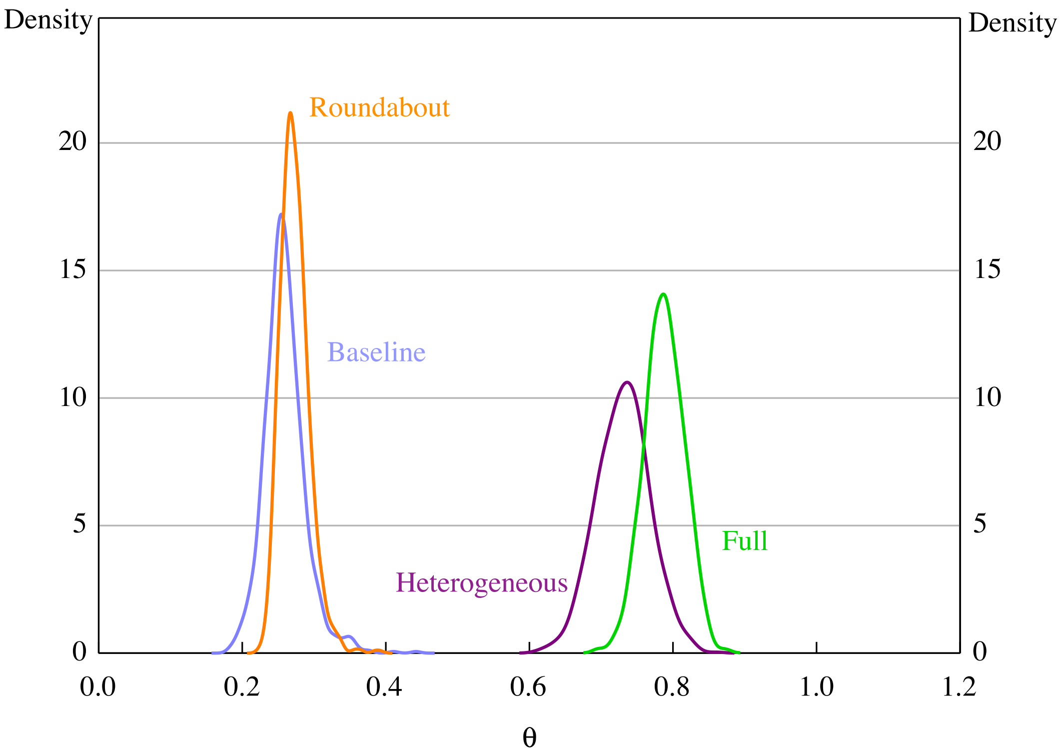
(2)

$$\mathbb{E}\{(\phi\pi_t - \omega\pi_{t-1} - (1-\beta\theta)(1-\theta)(1-\omega)mc_t - \beta\theta\pi_{t+1})\mathbf{z_t}\} = 0.$$

The estimates of the discount factor, β, are imprecise, except for the full model, where they are approximately one for both parameterisations. The results for the other parameters, however, are generally little changed if we constrain β to be its true value (0.99) (Table 6).

A common criticism of NKPCs is the need to include indexation in an ad hoc manner in order to capture the persistence of inflation evident in the data. Estimates of the NKPC based on data from the full model using either parameterisation falsely suggest that around 20 per cent of firms index their prices even though this is not a feature of the data-generating process (Table 6). These incorrect estimates of the level of indexation seems to come about through the

Figure 2: Distribution of the Estimated Calvo Parameter (θ)



Note: Uses the second parameterisation of the moment condition, shown in Table 6

interaction of heterogeneity and roundabout production as they are far larger in the full model than in the other models.

The difference between the theoretically true macroeconomic value for the Calvo probability and the estimates of the macroeconomic Calvo probability from the heterogeneous (full and heterogeneous) models is striking (Figure 2). Based on the theory presented in Section 2, the estimates of the macroeconomic Calvo probability should be about 0.30. The estimates from the heterogeneous models using either parameterisation are not even remotely close, with the median estimates for the full model being 0.85 and 0.79 respectively. Failing to account for the effect of heterogeneity on the estimates, as is commonly done, may lead one to falsely overstate the true extent of nominal rigidities. The fact that the estimated macroeconomic Calvo probability in both heterogeneous models is larger than the 'true' NKPC based Calvo probability, whereas theoretically we would expect it to be less, suggests a large econometric bias. Possible sources of this bias include the misweighting of marginal costs and using unsuitable instruments.

4.2 Implications of Misweighting Marginal Costs

A common measure of real marginal costs used in the literature is the aggregate labour share of income (or revenue) (see, for example, Galí and Gertler 1999; Sbordone 2002). In the baseline model this is consistent with theory. However, this is not true when both roundabout production and heterogeneity are present. Instead, one should use a gross revenue-weighted average of sectoral real marginal costs, reflecting the weighting used in the construction of the aggregate price index. Using the (misweighted) aggregate labour share of income results in inconsistent estimates of the Calvo probability. The extent of this inconsistency can be gauged by comparing these 'misweighted' estimates to those obtained by repeating the Monte Carlo exercise with the correctly weighted series (Figure 3). It is apparent that the misweighting, while important, is not a major factor in the overestimation of the Calvo probability; the median estimate using the second parameterisation falls from 0.79 to around 0.65, still well above 0.3 (Table 7). Interestingly, the misweighting does appear to be an important source of the erroneous finding of indexation of prices in the full model; if the correct marginal costs series is used with the second parameterisation the indexation parameter more than halves in size and is no longer significantly different from zero. [12]

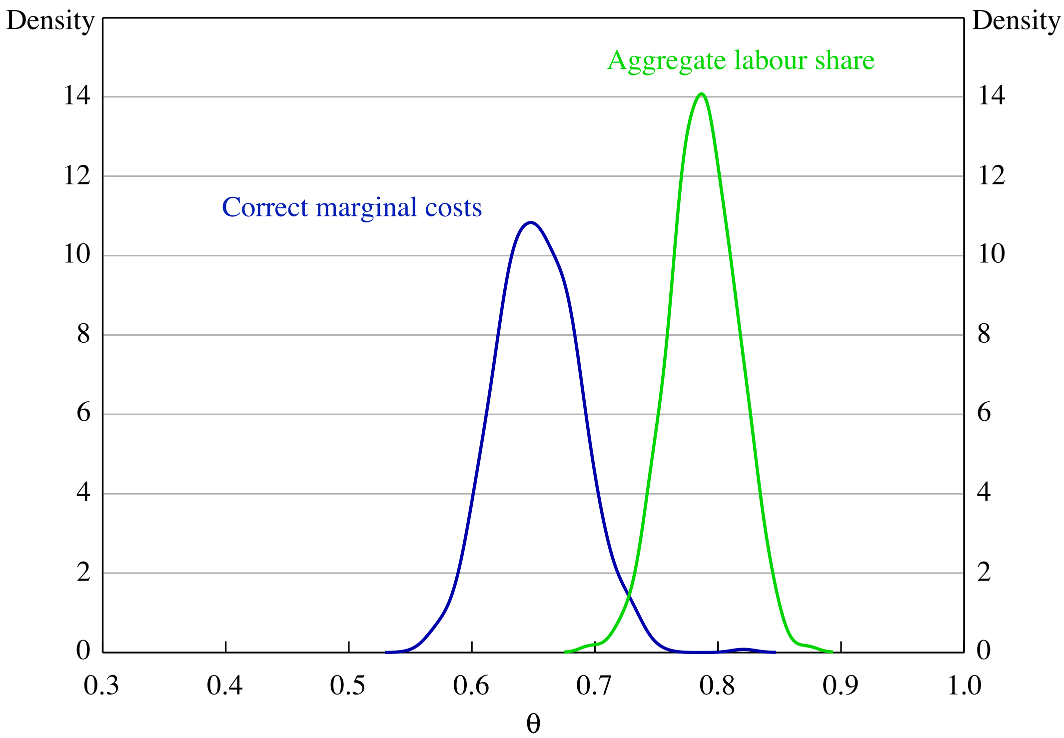
Table 7: Monte Carlo Results from GMM Estimates of the Aggregate NKPC from the Full Model

		Measure of marginal costs used	
Parameter	True	Labour share	Correctly weighted
β	0.99	1.00 (0.01)	0.96 (0.06)
θ^{macro}	0.30	0.79 (0.03)	0.65 (0.03)
ω	0.00	0.19 (0.05)	0.07 (0.05)
		Imposing $\beta = 0.99$	
θ^{macro}	0.30	0.77 (0.03)	0.66 (0.04)
ω	0.00	0.17 (0.05)	0.08 (0.05)

Notes: The median estimate is presented for each model and the estimates in parentheses are the standard deviations of the distribution of the parameters. The results in the lower panel of the table are estimated with the restriction that $\beta = 0.99$. Uses the second parameterisation of the moment condition, shown in Table 6.

[12] Alternatively, with the first parameterisation the magnitude of the indexation parameter is reduced by around one-third, but remains significantly different from zero.

Figure 3: Distribution of the Calvo Probability with Correctly Measured Marginal Costs: Full Model



Note: Uses the second parameterisation of the moment condition, shown in Table 6

4.3 Lack of Suitable Instruments

Another possible source of the inconsistent parameter estimates is that the NKPC may only be weakly identified because the instruments used for future inflation and marginal costs are weak. Mavroeidis (2005) argues that this is the case for the instruments which are commonly used in the estimation of the NKPC. The results in Table 6 and Figure 2 suggest that for the baseline and roundabout models weak instruments are not a large problem. In the models with heterogeneity, it appears that weak instruments may be a problem; this problem is compounded by the fact that the instruments might not only be weak, but also invalid.

As described in Section 2 (Equation (5)), the aggregate NKPC can be expressed as a weighted sum of sectoral NKPCs (ignoring indexation):

$$\pi_t = \bar{\lambda} mc_t + \beta \mathbb{E}_t \pi_{t+1} + \sum_j w_j e_{\lambda,j} mc_{j,t}. \tag{9}$$

This demonstrates that the presence of heterogeneity introduces an 'error' term, $\sum_j w_j e_{\lambda,j} mc_{jt}$.[13] As discussed above, we can decompose inflation into expected inflation and the forecast error, $\pi_{t+1} = E_t(\pi_{t+1}) + v_{t+1}$. Combining these we can define a broader error term, u_t, which includes both the forecast error, v_{t+1}, and the error induced by heterogeneity, that is $u_t \equiv -\beta v_{t+1} + \sum\limits_j w_j e_{\lambda,j} mc_{jt}$. u_t is the error term which is relevant for estimation.[14]

Recall that, given rational expectations, our GMM estimation approach assumes that

$$\mathbb{E}(u_t|\mathbf{z_t}) = 0, \tag{10}$$

which implies that the errors are uncorrelated with the instruments, $\mathbf{z_t}$.

We also need the instruments to be correlated with the variables for which we are instrumenting. Therefore, we need instruments that are uncorrelated with the weighted sectoral marginal costs but are correlated with aggregate marginal costs. Among the variables available in our model, it is difficult to conceive of instruments which can satisfy this condition.[15] This problem is an example of the aggregation bias first noted by Pesaran and Smith (1995) and applied specifically to aggregate NKPCs by Imbs *et al* (2007).

The lack of valid instruments depends on the existence of heterogeneity in the Calvo probabilities. If the Calvo probabilities were the same across sectors, the coefficients on marginal costs would be constant ($\lambda_j = \bar{\lambda}$, and $e_{\lambda,j} = 0$ for all j). Therefore, the second component of u_t would disappear; it would only be comprised of the expectation error, as is normally assumed. Consistent with this, the estimates of the Calvo probability for the baseline and roundabout models are broadly around their true values (Table 6 and Figure 2); it is when heterogeneity is introduced that the estimates are upwardly biased. Also, note that if the $e_{\lambda,j}$ and mc_{jt} were independent then it would be possible to express the moment

[13] For the models in this paper, the weights are each sector's share of gross revenue.

[14] As we allow for indexation and estimate the hybrid NKPC, $u_t \equiv -\frac{\beta\theta}{\phi} v_{t+1} + \sum\limits_j w_j e_{\lambda,j} mc_{jt}$, where $\phi = \theta + \omega[1 - \theta(1-\beta)]$, as before.

[15] Dees *et al* (2009) use a VAR to generate global or foreign instruments that may, in principle, be correlated to aggregate marginal costs and exogenous with respect to sectoral marginal costs.

condition as $\mathbb{E}(u_t|\mathbf{z_t}) = \sum_j \mathbb{E}(e_{\lambda,j}|\mathbf{z_t})\mathbb{E}(w_j mc_{jt}|\mathbf{z_t}) = 0$, which would be satisfied if $\mathbb{E}(e_{\lambda,j}|\mathbf{z_t}) = 0$. However, the distributions of the slopes of the sectoral Phillips curves and marginal costs are not independent; both depend on the distribution of the Calvo probabilities.

In models with heterogeneity, developments in one sector can affect other sectors and the size of the effects will depend on the distribution of the Calvo probabilities. Without roundabout production, sectors affect each other through the effect of their pricing decisions on consumer demand for other goods and the effect of their demand for labour on wages, and therefore the marginal costs faced by firms in other sectors. This is the reason why the distribution of the Calvo probabilities can matter. As an example, suppose there is a relatively large sector with flexible prices. A positive productivity shock in that sector would enable its prices to fall while its demand for labour would rise. At the same time, demand for output from other sectors would fall (as consumers buy relatively more of the now cheaper goods) and so they will reduce their labour demand. With a relatively large flexible price sector, the net effect on wages can be positive, resulting in higher marginal costs for other sectors. If the flexible price sector was relatively small, the net effect might be lower wages and lower marginal costs for other sectors. With roundabout technology, there is an additional direct effect on other firms since the prices charged by one sector will affect marginal costs in other sectors.

Note that this aggregation bias applies to any other estimation methods used to estimate aggregate NKPCs that ignore heterogeneity. For instance, Lindé (2005) uses a Full Information Maximum Likelihood approach to account for expected inflation. However, since the aggregate NKPC implicitly includes an error term that is likely to be correlated with aggregate marginal costs, estimates from this approach will also tend to be biased. Unless the estimation procedure corrects for the correlation between the regressors and the unobserved error, which is difficult as quality sector-level data are rarely available, the estimates of the NKPC remain likely to be biased.

In summary, it appears that the inconsistency that plagues estimates of the coefficient on marginal costs in the aggregate NKPC when heterogeneity is present primarily reflects model misspecification. There are two aspects to this. First, incorrect weighting affects the standard measure of marginal costs (the labour share of income) and, second, the instruments may be weak or not valid.

4.4 Sensitivity Analysis

To assess the robustness of our results we vary a number of parameters of the model and repeat the exercises performed above for the full model. First, we eliminate sector specific shocks completely. In this case, sectoral marginal costs remain heterogeneous as factor shares differ across sectors. Second, we reduce the heterogeneity in the Calvo probability by moving each sector roughly 0.1 closer to the average while maintaining the overall mean. Third, we lower the overall level of price stickiness by reducing each sector's Calvo probability, where possible, by 0.1. Finally, we estimate the purely forward-looking NKPC instead of the hybrid NKPC. The results in Table 8 show that the overstatement of price stickiness in estimates of the aggregate NKPC is robust to these alternative choices of parameters.

Table 8: Robustness Tests for the Full Model

Parameter	True	No sector-specific shocks	Less Calvo heterogeneity	Less price stickiness	Forward-looking NKPC
β	0.99	0.93 (0.05)	1.00 (0.05)	1.00 (0.04)	1.00 (0.00)
θ	0.30	0.64 (0.04)	0.79 (0.03)	0.77 (0.03)	0.77 (0.02)
ω	0.00	0.05 (0.05)	0.31 (0.05)	0.19 (0.05)	–

Notes: The median estimate is presented for each model and the brackets denote the standard deviation of the estimates. Uses the second parameterisation of the moment condition.

5. Macroeconomic Implications

We have shown that ignoring heterogeneity in the frequency of price resetting when estimating the NKPC leads to the aggregate Calvo parameter being considerably overstated, and that this bias is greater if roundabout production is a characteristic of the economy. To examine the macroeconomic consequences more broadly, we can compare the impulse response functions (to the same sized shocks) from the full model to those of a modified baseline model (Figure 4). The modification is that the forward-looking Phillips curve is replaced with parameter values similar to those estimated using GMM, namely $\beta = 0.99$, $\theta = 0.77$, and $\omega = 0.17$ (Table 6). This model is akin to the ubiquitous three-equation New-Keynesian model.

Although the two models are quite different, the impulse response functions are surprisingly similar. The modified model appears to be able to mimic the effects of heterogeneity and roundabout production in the full model with the combination of a larger Calvo probability and indexation. The main difference between the Phillips curves of the two models lies in their interpretation. Taken literally, one would falsely interpret the GMM estimates as implying that prices change every seven quarters on average and that roughly 20 per cent of firms choose to index their prices to past inflation. Instead, the results should be interpreted just as a set of parameters which can replicate a more realistic and complex data-generating process.[16]

6. Conclusions

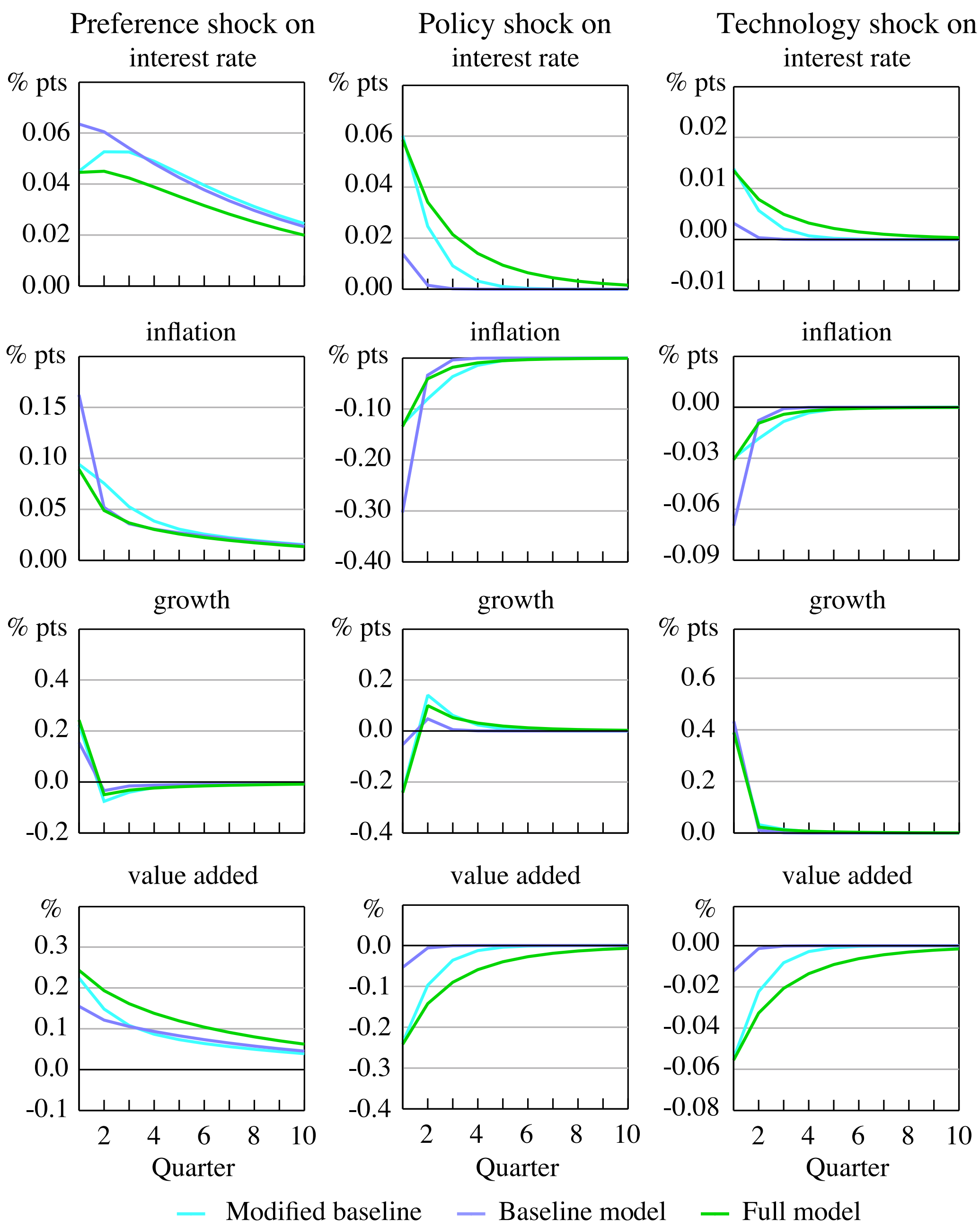
The New-Keynesian Phillips Curve has become a central part of models used for analysis of the macroeconomy and understanding the behaviour of inflation. However, a tension exists between the low frequency of price resetting that the standard estimates of the aggregate NKPC imply and the relatively high estimates from microeconomic data on firms' pricing behaviour. Furthermore, to improve the fit of the NKPC, indexation of prices to past inflation is often introduced, even though there is scant microeconomic evidence of such behaviour.

Using a Monte Carlo approach, we have argued that heterogeneity in the frequency of price resetting amongst firms, which is evident in the microeconomic data, is one source of the differences in the estimates of price stickiness. The presence of heterogeneity leads to econometric complications for the estimation of the NKPC. In particular, while it is necessary to instrument for both expected inflation and real marginal costs, the instruments that have commonly been used in the literature are unlikely to be suitable when heterogeneity exists. The instruments may be likely to be both weak, as previously demonstrated by Mavroeidis (2005), and invalid.

A second source of these differences is the more complex production structure of the economy than is often assumed in macroeconomic models. Roundabout production, whereby the output of each firm can either be consumed or used as an input to production by another firm, is a realistic way of describing the production

[16] Note that these parameters were obtained through conventional GMM estimation of the NKPC, rather than using Carvalho's (2006) approach of matching the impulse response functions of a model excluding heterogeneity to one including it.

Figure 4: Impulse Responses from GMM Estimates



process. We have argued that when both heterogeneity and roundabout production are present it is no longer appropriate to use the labour share of income as a measure of real marginal costs when estimating the NKPC; doing so is likely to result in estimates which falsely suggest that some firms index their prices to past inflation.

In summary, we have shown that estimates of the aggregate NKPC obtained in the standard way should not be given a structural interpretation. Any inference from such estimates about the frequency with which prices are reset is likely to be misleading. Acknowledging this resolves the tension between the microeconomic and macroeconomic estimates of price stickiness that exists in the literature.

Appendix A: The Model

The economy consists of a unit interval of identical households, N final-goods producers, a continuum of intermediate-goods producers and a monetary authority.

There is a continuum of intermediate-goods producers, indexed on the unit interval $(0,1]$. Each sector of the economy is represented by a sub-interval $(\psi_{j-1},\psi_j]$, where $0=\psi_0<\psi_1<\ldots<\psi_N=1$. The sub-intervals are not necessarily of the same length, so the measure of sector j is given by $\gamma_j=\psi_j-\psi_{j-1}$. A final-goods producer in sector j only uses intermediate goods produced by intermediate-goods firms belonging to the sub-interval $(\psi_{j-1},\psi_j]$.

A.1 Households

Given initial holdings of bonds, B_{-1}, and money, H_{-1}, the sequence of wages, prices for final consumption goods, prices for the aggregate consumption good and nominal interest rates $\{W_t,\{P_{j,t}\}_{j=1}^N,P_t^c,I_t\}_{t=0}^\infty$ and the sequence of monetary transfers and dividends $\{T_t,V_t\}_{t=0}^\infty$, the sequence of aggregate consumption, consumption of final consumption goods, money holdings, labour supply and bond holdings, $\{c_t,\{c_{j,t}^d\}_{j=1}^N,H_t,l_t^s,B_t\}_{t=0}^\infty$ solves the following intertemporal maximisation problem

$$\max_{\{c_t,\{c_{j,t}^d\}_{j=1}^N,H_t,l_t^s,B_t\}_{t=0}^\infty}\mathbb{E}_0\sum_{t=0}^\infty\beta^t\left(a_t\ln c_t+\ln\frac{H_t}{P_t^c}-\frac{1}{1+\frac{1}{\eta}}(l_t^s)^{1+\frac{1}{\eta}}\right)$$

subject to

$$T_t+V_t+H_{t-1}+B_{t-1}+W_tl_t^s\geq P_t^cc_t+\frac{B_t}{I_t}+H_t \tag{A1}$$

$$c_t=\left(\sum_{j=1}^N\gamma_j^{\frac{1}{\varepsilon}}\left(c_{j,t}^d\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A2}$$

$$P_t^cc_t=\sum_{j=1}^NP_{j,t}c_{j,t}^d. \tag{A3}$$

Using the Lagrange multiplier Λ_t on the budget constraint, we get the following first-order conditions

$$(l_t^s)^{\frac{1}{\eta}} = \frac{W_t}{P_t^c}\frac{a_t}{c_t} \tag{A4}$$

$$\frac{P_t^c c_t}{a_t H_t} = 1 - \frac{1}{I_t} \tag{A5}$$

$$\frac{a_t}{c_t} = \beta \mathbb{E}_t \left(\frac{a_{t+1}}{c_{t+1}} \frac{P_t^c}{P_{t+1}^c} I_t \right) \tag{A6}$$

$$T_t + V_t + H_{t-1} + B_{t-1} + W_t l_t^s = P^c c_t + \frac{B_t}{I_t} + H_t \tag{A7}$$

$$c_{j,t}^d = \gamma_j \left(\frac{P_{j,t}}{P_t^c} \right)^{-\varepsilon} c_t, \text{ for } j = 1, \ldots, N, \text{ and} \tag{A8}$$

$$\Lambda_t = \frac{\beta^t a_t}{P_t^c c_t}. \tag{A9}$$

We also have

$$P_t^c = \left(\sum_{j=1}^{N} \gamma_j P_{j,t}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

We assume the preference process evolves as follows

$$a_t = a_{t-1}^{\rho_a} e^{\varepsilon_{a,t}}.$$

A.2 Final-goods Firms

There are N sectors, indexed by j. Final-goods firms are perfectly competitive and make zero profits in equilibrium. They sell final goods to households that are produced using inputs from intermediate producers from their sector.

In period t, final-goods firms take as given their total production, $c_{j,t}^s$, and prices for their output and intermediate-goods prices, $P_{j,t}, \{P_t(k)\}_{k \in (\psi_{j-1}, \psi_j]}$, and solve the following cost-minimisation problem

$$\min_{\{c_t^d(k)\}_{k\in(\psi_{j-1},\psi_j]}} \int_{\psi_{j-1}}^{\psi_j} P_t(k)c_t^d(k)dk$$

subject to

$$c_{j,t}^s \leq \left(\left(\frac{1}{\gamma_j}\right)^{\frac{1}{\varepsilon}} \int_{\psi_{j-1}}^{\psi_j} c_t^d(k)^{\frac{\varepsilon-1}{\varepsilon}} dk\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

There is also a zero-profit condition that must also be satisfied

$$P_{j,t}c_{j,t}^s - \int_{\psi_{j-1}}^{\psi_j} P_t(k)c_t^d(k)dk = 0.$$

The first-order conditions become

$$c_t^d(k) = \frac{1}{\gamma_j}\left(\frac{P_t(k)}{P_{j,t}}\right)^{-\varepsilon} c_{j,t}^s, \qquad k \in (\psi_{j-1}, \psi_j],$$

where

$$P_{j,t} = \left(\frac{1}{\gamma_j}\int_{\psi_{j-1}}^{\psi_j} P_t(k)^{1-\varepsilon}dk\right)^{\frac{1}{1-\varepsilon}}.$$

A.3 Intermediate-goods Firms

Intermediate-goods firms are monopolistically competitive. Intermediate-goods producers in sector j can only change their prices in any given period with Calvo probability θ_j. Once prices have been determined, intermediate-goods firms produce to meet demand for their good from final-goods producers and other intermediate-goods producers. Demand for labour and other intermediate goods are determined by cost minimisation. Firm i in sector j takes wages, W_t, and the price for the aggregate intermediate good, P_t^m, as given to solve the following cost-minimisation problem

$$\min_{l_{i,t}^d, m_{i,t}^d} W_t l_{i,t}^d + P_t^m m_{i,t}^d$$

subject to:

$$y_t(i) \leq \left(z_{j,t} z_t l_{i,t}^d\right)^{\alpha_j} (m_{i,t}^d)^{(1-\alpha_j)},$$

where: $z_{j,t}$ is sector-specific productivity; z_t is the state of aggregate productivity; $l_{i,t}^d$ is labour demanded by firm i; and $m_{i,t}^d$ is firm i's demand for the aggregate intermediate good. The production constraint is binding and the Lagrange multiplier on the production constraint, $\Omega_{j,t}$, is the nominal marginal cost and is sector specific. First-order conditions are:

$$\begin{aligned}
l_{i,t}^d &= \Omega_{j,t} \alpha_j \frac{y_t(i)}{W_t} \\
m_{i,t}^d &= (1-\alpha_j)\Omega_{j,t} \frac{y_t(i)}{P_t^m} \\
\Omega_{j,t} &= MC_{j,t} = \frac{1}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}} \frac{1}{(z_{j,t} z_t)^{\alpha_j}} W_t^{\alpha_j} (P_t^m)^{1-\alpha_j}.
\end{aligned}$$

We define:

$$m_{i,t}^d = \left(\sum_{j=1}^{N} \gamma_j^{\frac{1}{\varepsilon}} (m_{i,t}^{j,d})^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $m_{k,t}^{j,d}$ is the demand for output from sector j by firm k, which can be expressed as follows:

$$m_{i,t}^{j,d} = \left(\left(\frac{1}{\gamma_j}\right)^{\frac{1}{\varepsilon}} \int_{\psi_{j-1}}^{\psi_j} m_{i,t}^d(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

So firms decide how to allocate their expenditure on intermediate goods across sectors, then decide within a sector from which firms it will source its intermediate inputs, given intermediate-goods prices, $\{P_t(i)\}_{i\in(0,1]}$, so that

$$P_t^m m_{i,t}^d = \sum_{j=1}^N P_{j,t} m_{i,t}^{j,d}$$
$$P_{j,t}^m m_{i,t}^{j,d} = \int_{\psi_{j-1}}^{\psi_j} P_t(k) m_{i,t}^d(k) dk.$$

From cost minimisaton, we find that:

$$m_{i,t}^{j,d} = \gamma_j \left(\frac{P_{j,t}^m}{P_t^m}\right)^{-\varepsilon} m_{i,t}^d$$
$$m_{i,t}^d(k) = \frac{1}{\gamma_j}\left(\frac{P_t(k)}{P_{j,t}^m}\right)^{-\varepsilon} m_{i,t}^{j,d} \quad i \in (\psi_{j-1}, \psi_j].$$

Using these results, we can derive the following expressions for the aggregate price index, the sectoral price indices and an expression for a firm's demand for intermediate inputs

$$P_{j,t}^m = \left(\frac{1}{\gamma_j}\int_{\psi_{j-1}}^{\psi_j} P_t(k)^{\frac{\varepsilon-1}{\varepsilon}} dk\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A10}$$

$$P_t^m = \left(\sum_{j=1}^N \gamma_j (P_{j,t}^m)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \tag{A11}$$

$$m_{i,t}^d = \left(\sum_{j'=1}^N \gamma_j^{\frac{1}{\varepsilon}} (m_{i,t}^{j',d})^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A12}$$

$$= \left(\int_0^1 m_{k,t}^d(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A13}$$

When firms are able to reset their prices, they solve the following problem:

$$P_t(k)^* \in \arg\max_{P_t(k)} \mathbb{E}_t \sum_{n=0}^{\infty} \frac{\Lambda_{t+n}}{\Lambda_t} \theta_j^n (P_t(k) - \Omega_{j,t+n}) y_{t+n}(k).$$

First-order conditions give us:

$$P_t(k)^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{n=0}^{\infty} \Lambda_{t+n} \theta_j^n \Omega_{j,t+n} (P_{t+n}^m)^{\varepsilon} y_{t+n}}{\mathbb{E}_t \sum_{n=0}^{\infty} \Lambda_{t+n} \theta_j^n (P_{t+n}^m)^{\varepsilon} y_{t+n}}.$$

Dividends distributed to households are just period t profits:

$$V_t(k) = P_t(k) y_t(k) - W_t l_{k,t}^d - \int_0^1 P_t(i) m_{k,t}^d(i) di,$$

and

$$V_t = \int_0^1 V_t(k) dk.$$

In equilibrium, total dividends to households will equal nominal value added less total nominal payments to labour ($P_t^c c_t - W_t l_t^s$).

We assume that the aggregate technology and sectoral technology processes evolve as follows

$$z_t = z_{t-1} e^{\mu_z + \varepsilon_{z,t}}, \text{ and } z_{j,t} = z_{j,t-1}^{\rho_{z,j}} e^{\varepsilon_{z,j,t}} \quad \text{for } j = 1, \ldots, N,$$

where: μ_z is the average growth rate of aggregate technology; $\varepsilon_{z,t}$ is the shock to aggregate technology, and $\varepsilon_{z,j,t}$ is the sector-specific technology shock.

A.4 Monetary Authority

The monetary authority follows the following policy rule:

$$I_t = I_{t-1}^{\rho_i} \left(\frac{1}{\beta} e^{-(1-\phi_g)\mu_z} \Pi_t^{\phi_\pi} g_t^{\phi_g} \right)^{1-\rho_i} e^{\varepsilon_{i,t}},$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ and $g_t = \frac{c_t}{c_{t-1}} \frac{z_{t-1}}{z_t}$.

Define the gross rate of growth of the money supply to be $\Xi_t = \frac{H_t}{H_{t-1}}$. Given its target for the nominal interest rate, nominal transfers T_t is given by $(\Xi_t - 1) H_{t-1}$ and Ξ_t is determined endogenously by the money demand equation and money market clearing:

$$\Xi_t = \frac{I_t}{I_t - 1} \frac{P_t c_t}{a_t H_{t-1}}.$$

A.5 Market-clearing Conditions

There are N markets for N final goods, a unit interval of markets for intermediate goods, a labour market, a bond market and a money market.

$$c_{j,t}^d = c_{j,t}^s \qquad \text{for } j = 1, \ldots, N \tag{A14}$$

$$y_t(k) = c_t^d(k) + \int_0^1 m_{i,t}^d(k)di \qquad \text{for } k \in (0,1] \tag{A15}$$

$$l_t^s = \int_0^1 l_{k,t}^d dk \tag{A16}$$

$$B_t = 0 \tag{A17}$$

$$H_t = H_{t-1} + T_t. \tag{A18}$$

A.5.1 Price indices

We define two price indices. The first, P_t should be familiar to the reader. The second, P_t^+, is introduced for convenience.

$$\begin{aligned} P_t \equiv P_t^c &= \left(\sum_{j=1}^N \gamma_j P_{j,t}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \\ &= \left(\sum_{j=1}^N \int_{\psi_{j-1}}^{\psi_j} P_t(k)^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}} \\ &= \left(\int_0^1 P_t(k)^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}} \\ &= P_t^m. \end{aligned}$$

$$
\begin{aligned}
P_{j,t}^{+} &= \left(\frac{1}{\gamma_j}\int_{\psi_{j-1}}^{\psi_j} P_t(k)^{-\varepsilon} dk\right)^{-\frac{1}{\varepsilon}} \\
P_t^{+} &\equiv \left(\sum_{j=1}^{N} \gamma_j (P_{j,t}^{+})^{-\varepsilon}\right)^{-\frac{1}{\varepsilon}} \\
&= \left(\int_0^1 P_t(k)^{-\varepsilon} dk\right)^{-\frac{1}{\varepsilon}}.
\end{aligned}
$$

A.5.2 Intermediate goods market-clearing

Using the final goods market-clearing condition:

$$
\begin{aligned}
y_t(k) &= c_t^d(k) + \int_0^1 m_{i,t}^d(k) dk \\
&= \frac{1}{\gamma_j}\left(\frac{P_t(k)}{P_{j,t}}\right)^{-\varepsilon} c_{j,t}^s + \int_0^1 \frac{1}{\gamma_j}\left(\frac{P_t(k)}{P_{j,t}}\right)^{-\varepsilon} m_{i,t}^{j,d} di. \\
&= \left(\frac{P_t(k)}{P_t}\right)^{-\varepsilon} c_t + \left(\frac{P_t(k)}{P_{j,t}}\right)^{-\varepsilon} \int_0^1 m_{i,t}^d di \\
y_{j,t} = \int_{\psi_{j-1}}^{\psi_j} y_t(k) dk &= \int_{\psi_{j-1}}^{\psi_j} \left(\frac{P_t(k)}{P_t}\right)^{-\varepsilon} c_t dk + \int_{\psi_{j-1}}^{\psi_j} \left(\frac{P_t(k)}{P_{j,t}}\right)^{-\varepsilon} \int_0^1 m_{i,t}^d didk \\
&= \gamma_j \left(\frac{P_{j,t}^{+}}{P_t}\right)^{-\varepsilon} c_t + \gamma_j \left(\frac{P_{j,t}^{+}}{P_t}\right)^{-\varepsilon} \int_0^1 m_{i,t}^d di \\
&= \gamma_j \left(\frac{P_{j,t}^{+}}{P_t}\right)^{-\varepsilon} y_t.
\end{aligned}
$$

where:

$$
\begin{aligned}
y_t &\equiv c_t + m_t^d, \qquad \textit{and} \\
m_t^d &= \int_0^1 m_{i,t}^d di.
\end{aligned}
$$

Since we can use the first-order conditions of the intermediate-goods firm k belonging to sector j to get:

$$y_t(k) = \left(z_t z_{j,t} \frac{\alpha_j \Omega_{j,t}}{W_t} \right)^{\frac{\alpha_j}{1-\alpha_j}} m^d_{k,t}.$$

We can also express sectoral output as follows:

$$y_{j,t} = \int_{\psi_{j-1}}^{\psi_j} y_t(k) dk = \left(\frac{z_t z_{j,t} \alpha_j \Omega_{j,t}}{W_t} \right)^{\frac{\alpha_j}{1-\alpha_j}} m^d_{j,t},$$

where:

$$m^d_{j,t} = \int_{\psi_{j-1}}^{\psi_j} m^d_{k,t} dk.$$

So:

$$m^d_t = \sum_{j=1}^{N} m^d_{j,t}.$$

A.5.3 Labour market-clearing

Labour demand by each firm is given by

$$l^d_{k,t} = \frac{\alpha_j \Omega_{j,t} y_t(k)}{W_t}.$$

Aggregating this across sector j:

$$\begin{aligned}
l^d_{j,t} &= \int_{\psi_{j-1}}^{\psi_j} l^d_{k,t} dk \\
&= \frac{\alpha_j \Omega_{j,t}}{W_t} \int_{\psi_{j-1}}^{\psi_j} y_t(k) dk. \\
&= \frac{\alpha_j \Omega_{j,t}}{W_t} y_{j,t} \\
l^d_t &= \sum_{j=1}^{N} l^d_{j,t}.
\end{aligned}$$

A.5.4 Sectoral price indices

In any given period, a firm in sector j has probability $(1-\theta_j)$ that it can change its price. This implies that the price index for sector j evolves as follows:

$$P_{j,t}^{1-\varepsilon} = \frac{1}{\gamma_j} \int_{\psi_{j-1}}^{\psi_j} P_t(k)^{1-\varepsilon} dk = \theta_j P_{j,t-1}^{1-\varepsilon} + (1-\theta_j)(P_t(k)^*)^{1-\varepsilon}.$$

Similarly, for the alternative price index:

$$(P_{j,t}^{+})^{-\varepsilon} = \theta_j (P_{j,t-1}^{+})^{-\varepsilon} + (1-\theta_j)(P_t(k)^*)^{-\varepsilon}.$$

A.6 Transformations and Normalisations

Due to the growth in aggregate technology, we detrend some variables to make them stationary so

$$\begin{array}{ccccc} \tilde{c}_t = \frac{c_t}{z_t} & \tilde{w}_t = \frac{W_t}{z_t P_t} & \tilde{y}_{j,t} = \frac{y_{j,t}}{z_t} & \tilde{y}_t = \frac{y_t}{z_t} & \tilde{h}_t = \frac{H_t}{z_t P_t} \\ \tilde{m}_{j,t}^d = \frac{m_{j,t}^d}{z_t} & \tilde{m}_t^d = \frac{m_t^d}{z_t} & \tilde{\Lambda}_t = \beta^{-t} z_t P_t \Lambda_t & r_{j,t} = \frac{P_{j,t}}{P_t} & r_{j,t}^{+} = \frac{P_{j,t}^{+}}{P_t} \\ & \Pi_t = \frac{P_t}{P_{t-1}} & \Pi_{j,t} = \frac{P_{j,t}}{P_{j,t-1}} & \omega_{j,t} = \frac{\Omega_{j,t}}{P_{j,t}}. & \end{array}$$

We also define:

$$r_{j,t}^{+} = \frac{P_{j,t}^{+}}{P_t} \quad \Pi_{j,t} = \frac{P_{j,t}}{P_{j,t-1}}$$

A.7 Summary of Non-linear Equations

This sub-section summarises the first-order conditions and market-clearing conditions required to solve the model at the sectoral level (rather than at the firm level). For completeness, and the interest of the reader, we present equations for sectors subject to the Calvo mechanism and those with flexible pricing, even though the latter are not used.

$$
\begin{aligned}
l_t^{\frac{1}{\eta}} &= \frac{\tilde{w}_t a_t}{\tilde{c}_t} \\
\frac{a_t}{\tilde{c}_t} &= \beta \mathbb{E}_t \left(\frac{a_{t+1}}{\tilde{c}_{t+1}} \frac{1}{\Pi_{t+1}} I_t e^{\mu_z + \varepsilon_{z,t+1}} \right) \\
\frac{\tilde{c}_t}{a_t \tilde{h}_t} &= 1 - \frac{1}{I_t} \\
\tilde{\Lambda}_t &= \frac{a_t}{\tilde{c}_t} \\
\tilde{h}_t &= \Xi_t \tilde{h}_{t-1} \frac{1}{\Pi_t} e^{-\mu_z - \varepsilon_{z,t}} \\
g_t &= \frac{\tilde{c}_t}{\tilde{c}_{t-1}} e^{-\mu_z - \varepsilon_{z,t}} \\
I_t &= I_{t-1}^{\rho_i} \left(\frac{1}{\beta} e^{-(1-\phi_g)\mu_z} \Pi_t^{\phi_\pi} g_t^{\phi_g} \right)^{1-\rho_i} e^{\varepsilon_{i,t}}.
\end{aligned}
$$

For $k \in (\psi_{j-1}, \psi_j]$ and $j = 1, \ldots, N$:

$$
\begin{aligned}
\omega_{j,t} &= \frac{1}{\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}} \frac{1}{(z_{j,t})^{\alpha_j}} \frac{\tilde{w}_t^{\alpha_j}}{r_{j,t}} \\
l_{j,t}^d &= \frac{\alpha_j \omega_{j,t}}{\tilde{w}_t} \tilde{y}_{j,t} \qquad j = 1, \ldots, N \\
\tilde{y}_{j,t} &= \gamma_j \left(r_{j,t}^+ \right)^{-\varepsilon} \tilde{y}_t \qquad j = 1, \ldots, N \\
\tilde{m}_{j,t}^d &= (1-\alpha_j) \omega_{j,t} r_{j,t} \tilde{y}_{j,t}.
\end{aligned}
$$

Sticky price sector indices follow

$$
\begin{aligned}
1 &= \theta_j \Pi_{j,t}^{\varepsilon-1} + (1-\theta_j) \left(r_{j,t}(k)^* \right)^{1-\varepsilon} \\
r_{j,t}(k)^* &= \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{n=0}^{\infty} (\beta \theta_j)^n \frac{\tilde{\Lambda}_{t+n}}{\Pi_{t,t+n}} \omega_{j,t+n} \Pi_{j,t,t+n}^{1+\varepsilon} \tilde{\bar{y}}_{j,t+n}}{\mathbb{E}_t \sum_{n=0}^{\infty} (\beta \theta_j)^n \frac{\tilde{\Lambda}_{t+n}}{\Pi_{t,t+n}} \Pi_{j,t,t+n}^{\varepsilon} \tilde{\bar{y}}_{j,t+n}} \\
r_{j,t} &= r_{j,t-1} \frac{\Pi_{j,t}}{\Pi_t} \\
(r_{j,t}^+)^{-\varepsilon} &= \theta_j \left(\hat{r}_{j,t-1}^+ \frac{1}{\Pi_t} \right)^{-\varepsilon} + (1-\theta_j) \left(r_{j,t}(k)^* r_{j,t} \right)^{-\varepsilon},
\end{aligned}
$$

where $\Pi_{j,t,t+n} = \frac{P_{j,t+n}}{P_{j,t}}$, $\Pi_{t,t+n} = \frac{P_{t+n}}{P_t}$, $\tilde{\bar{y}}_{j,t+n} = c_{j,t}^s + \int_0^1 m_{i,t}^{j,d} di$ and $r_{j,t}(k)^* = \frac{P_t(k)^*}{P_{j,t}}$.

Flexible price sector prices:

$$
\begin{aligned}
r_{j,t}(k)^* &= \frac{\varepsilon}{\varepsilon-1}\omega_{j,t} \\
\Rightarrow 1 &= \frac{\varepsilon}{\varepsilon-1}\omega_{j,t} \\
\Rightarrow r_{j,t} &= \frac{\varepsilon}{\varepsilon-1}\frac{1}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}}\left(\frac{\tilde{w}_t}{z_{j,t}}\right)^{\alpha_j} \\
r_{j,t} &= r_{j,t-1}\frac{\Pi_{j,t}}{\Pi_t}.
\end{aligned}
$$

The market-clearing and aggregation equations become:

$$
\begin{aligned}
l_t &= \sum_{j=1}^{N} l_{j,t}^d \\
\tilde{y}_t &= \tilde{c}_t + \tilde{m}_t^d \\
\tilde{m}_t^d &= \sum_{j=1}^{N} \tilde{m}_{j,t}^d \\
1 &= \sum_{j=1}^{N} \gamma_j r_{j,t}^{1-\varepsilon}.
\end{aligned}
$$

Stochastic processes are

$$
\begin{aligned}
z_{j,t} &= z_{j,t}^{\rho_j} e^{\varepsilon_{zj,t}}, \text{and,} \\
\log(a_t) &= \rho_a \log(a_{t-1}) + \varepsilon_{a,t}.
\end{aligned}
$$

A.8 Log-linearised Equations

Using the equations above, we denote the log-deviation from trend for variable x to be $\hat{x}$.

$$\frac{1}{\eta}\hat{l}_t = \hat{a}_t - \hat{c}_t + \hat{w}_t \tag{A19}$$

$$(1-\rho_a)\hat{a}_t - \hat{c}_t = \hat{i}_t - \mathbb{E}_t\left(\hat{c}_{t+1} + \hat{\pi}_{t+1}\right) \tag{A20}$$

$$\hat{c}_t - \hat{a}_t - \hat{h}_t = \left(\frac{1}{\bar{I}-1}\right)\hat{i}_t \tag{A21}$$

$$\hat{h}_t = \hat{\xi}_t + \hat{h}_{t-1} - \hat{\pi}_t - \varepsilon_{z,t} \tag{A22}$$

$$\hat{i}_t = \rho_i\hat{i}_{t-1} + (1-\rho_i)(\phi_\pi\hat{\pi}_t + \phi_g\hat{g}_t) + \varepsilon_{i,t} \tag{A23}$$

$$\hat{g}_t = \hat{c}_t - \hat{c}_{t-1} + \varepsilon_{z,t}. \tag{A24}$$

Note that $\bar{I} = \frac{1}{\beta}e^{-\mu_z}$.

Sectoral variables, market-clearing and aggregation equations

$$\hat{l}_{j,t} = \hat{\omega}_{j,t} - \hat{w}_t + \hat{y}_{j,t} \qquad j = 1,\ldots,N \tag{A25}$$

$$\hat{m}_{j,t} = \hat{\omega}_{j,t} + \hat{r}_{j,t} + \hat{y}_{j,t} \qquad j = 1,\ldots,N \tag{A26}$$

$$\hat{\omega}_{j,t} = -\alpha_j\hat{z}_{j,t} + \alpha_j\hat{w}_t - \hat{r}_{j,t} \qquad j = 1,\ldots,N \tag{A27}$$

$$\hat{y}_{j,t} = -\varepsilon\hat{r}^+_{j,t} + \hat{y}_t \qquad j = 1,\ldots,N \tag{A28}$$

$$\hat{y}_t = \overline{\left(\frac{C}{Y}\right)}\hat{c}_t + \overline{\left(\frac{M}{Y}\right)}\hat{m}_t \tag{A29}$$

$$\hat{l}_t = \sum_{j=1}^{N}\overline{\left(\frac{l_j}{l}\right)}\hat{l}_{j,t} \tag{A30}$$

$$\hat{m}_t = \sum_{j=1}^{N}\overline{\left(\frac{m_j}{m}\right)}\hat{m}_{j,t} \tag{A31}$$

$$0 = \sum_{j=1}^{N}\gamma_j\bar{r}_j^{1-\varepsilon}\hat{r}_{j,t} \tag{A32}$$

$$\hat{r}_{j,t} = \hat{r}_{j,t-1} + \hat{\pi}_{j,t} - \hat{\pi}_t \qquad j = 1,\ldots,N \tag{A33}$$

$$\hat{r}^+_{j,t} = \hat{r}_{j,t} + \theta_j(\hat{r}^+_{j,t-1} - \hat{r}_{j,t-1}) \qquad j = 1,\ldots,N, \tag{A34}$$

where $\overline{\left(\frac{C}{Y}\right)}$ and $\overline{\left(\frac{M}{Y}\right)}$ represent the steady-state shares of value added and intermediate input of gross output.

Flexible prices are set as a constant mark-up over nominal marginal costs while sticky price inflation evolves according to the New-Keynesian Phillips Curve.

Flexible price sectors:

$$\hat{r}_{j,t} = \alpha_j(\hat{w}_t - \hat{z}_{j,t}) \qquad j = 1,\ldots,N^f. \tag{A35}$$

Sticky price sectors:

$$\hat{\pi}_{j,t} = \frac{(1-\beta\theta_j)(1-\theta_j)}{\theta_j}\hat{\omega}_{j,t} + \beta\mathbb{E}_t\hat{\pi}_{j,t+1} \qquad j = N^f+1,\ldots,N. \tag{A36}$$

Driving variables:

$$\hat{z}_{j,t} = \rho_{z,j}\hat{z}_{j,t-1} + \varepsilon_{z,j,t} \qquad j = 1,\ldots,N \tag{A37}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{a,t}. \tag{A38}$$

In summary, we have $8N + 11$ endogenous variables and equations, which completes the model.

Appendix B: Heterogeneous Calvo Probabilities and the Hazard Function

Suppose we have an economy with N goods. The prices for each of the N goods evolve according to the Calvo mechanism. This means that the number of price changes in a given time interval follows a Poisson process. We assume that each item's price is changed at different rates. Good i's price is changed at Poisson rate λ_i so that

$$P(N(t+s)-N(s)=n)=e^{-\lambda_i t}\frac{(\lambda_i t)^n}{n!},$$

where $N(t+s)-N(s)$ represents the number of events on the interval $(s,s+t]$. The probability of at least one price change in each period (say $t=1$) is given by

$$1-P(N(s+1)-N(s)=0)=1-e^{-\lambda_i}.$$

The duration of prices follows an exponential distribution and so the hazard function for the duration of good i's price should be flat.

Denote the cumulative distribution function (CDF) for the time between price changes for each good by $F_i(t)$. Now if we collect the duration of prices for each of the N goods and pool these times, the CDF of this pooled population becomes:

$$F(t)=\sum_{i=1}^{N}\omega_i F_i(t),$$

where $\sum_i \omega_i = 1$ and $\omega_i > 0 \quad \forall i$. ω_i could be set to some weight according to the relative sample sizes for each good or the weight of the good in the CPI, but we remain agnostic as to where they come from.

The hazard function is defined as the probability of a change at t divided by the survival probability

$$h(t)=\frac{f(t)}{1-F(t)},$$

where $f(t)=\frac{dF}{dt}=\sum_i \omega_i f_i(t)$.

We know that $F_i(t) = 1 - e^{-\lambda_i t}$, so

$$\begin{aligned}
h(t) &= \frac{\sum_i \omega_i \lambda_i e^{-\lambda_i t}}{\sum_i \omega_i e^{-\lambda_i t}} \\
\frac{dh}{dt} &= \frac{\left(\sum_i \omega_i (-\lambda_i^2) e^{-\lambda_i t}\right)\left(\sum_i \omega_i e^{-\lambda_i t}\right) - \left(\sum_i \omega_i \lambda_i e^{-\lambda_i t}\right)\left(\sum_i \omega_i (-\lambda_i) e^{-\lambda_i t}\right)}{\left(\sum_i \omega_i e^{-\lambda_i t}\right)^2} \\
&= \frac{\left(\sum_i \omega_i \lambda_i e^{-\lambda_i t}\right)^2 - \left(\sum_i \omega_i (\lambda_i^2) e^{-\lambda_i t}\right)\left(\sum_i \omega_i e^{-\lambda_i t}\right)}{\left(\sum_i \omega_i e^{-\lambda_i t}\right)^2}.
\end{aligned}$$

If $\lambda_i = \lambda \ \forall i$, then $\frac{dh}{dt} = 0$. This means that if the prices of all goods in an economy are changed at the same rate, the hazard function should be flat.

If not, then the hazard function is downward-sloping.

To determine the slope of the hazard function, we only need to sign the numerator of its derivative (the denominator is always positive).

So the hazard function is downward-sloping if

$$\left(\sum_i \omega_i \lambda_i e^{-\lambda_i t}\right)^2 \leq \left(\sum_i \omega_i \lambda_i^2 e^{-\lambda_i t}\right)\left(\sum_i \omega_i e^{-\lambda_i t}\right).$$

Now, the Cauchy-Schwartz inequality states that

$$\sum |a_i b_i| \leq \left(\sum |a_i|^2\right)^{\frac{1}{2}} \left(\sum |b_i|^2\right)^{\frac{1}{2}}.$$

Since the terms we are dealing with are all positive, we can ignore the absolute value delimiters. If we set

$$a_i = \omega_i^{\frac{1}{2}} \lambda_i e^{-\frac{\lambda_i t}{2}}, \text{and}, \tag{B1}$$

$$b_i = \omega_i^{\frac{1}{2}} e^{-\frac{\lambda_i t}{2}}, \tag{B2}$$

we obtain:

$$\sum \omega_i \lambda_i e^{-\lambda_i t} \leq \left(\sum \left(\omega_i^{\frac{1}{2}} \lambda_i e^{-\frac{\lambda_i t}{2}} \right)^2 \right)^{\frac{1}{2}} \left(\sum \left(\omega_i^{\frac{1}{2}} e^{-\frac{\lambda_i t}{2}} \right)^2 \right)^{\frac{1}{2}},$$

or

$$\left(\sum \omega_i \lambda_i e^{-\lambda_i t} \right)^2 \leq \left(\sum \left(\omega_i \lambda_i^2 e^{-\lambda_i t} \right) \right) \left(\sum \left(\omega_i e^{-\lambda_i t} \right) \right).$$

In other words, the hazard function for a pooled sample should be downward-sloping.

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