TIME-CONSISTENT POLICY: A SURVEY OF THE ISSUES

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ABSTRACT

Expectations play an important role in the recent literature on the theory of macroeconomic policy. Rather than approaching the problem of macroeconomic policy formulation from the point of view of a policymaker mechanically controlling an economy, the recent literature has focussed on the strategic interaction of policymaking and private sector behaviour. It has also focussed on the role and interaction of political and economic institutions in understanding the theory of macroeconomic policy. A useful device for examining the many issues has been the application of game theory.

The purpose of this paper is to present the insights from this rapidly developing literature in a unified framework and in a less technical way than the original contributions.

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1. Introduction

The impact on the economy of a change in macroeconomic policy depends partly on the expectations surrounding the implementation of the policy. This point has been recognized for many years, but it has become more relevant in the economics literature with the widespread use of models containing forward-looking agents.

There are two main reasons why a policy announcement may not be credible[1]: the policy may be seen to be politically or economically infeasible; or the policy may be feasible but it may be believed that the policymaker has no incentive to carry through with the policy.

A policy of the second type is referred to in the literature as a "time-inconsistent" policy and is the focus of this paper. Specifically, a "time-inconsistent" policy is a policy which future governments (or the current government in future periods) will have an incentive to change in the future. If agents understand the incentives of the policymaker, then a time-inconsistent policy is not credible. A time-consistent policy is credible because the private sector understands that the policymaker has no incentive to change the policy once it is announced. In an important paper, Kydland and Prescott (1977) showed that the optimal policy (found by optimal control methods) in a model with forward looking agents would generally be time-inconsistent and therefore would not be credible to

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[1] "credible" and "believable" are used interchangeably in this paper.

the private sector. If the private sector does not believe the policy, the desired effect of announcing the policy will not be realized. This is a stunning result and has led to a large and growing literature on the problem of policy formulation when the government is seen as playing a "game"[2] against forward looking agents.

A standard illustration of the problem of time-inconsistent policy is the question of patents. To stimulate research into discovering new products, it is beneficial to promise patent rights over any discoveries. Once a discovery is made it is then optimal to tax away the monopoly profit that the patent generates[3]. A similar argument is often made over the provision of incentives for resource exploration. Once a discovery is made, the policymaker has an incentive to tax away the economic surplus resulting from the discovery. This insight also applies to the formulation of macroeconomic policy.

The purpose of this paper is to draw out the main ideas in the recent theoretical literature on macroeconomic policy. One of the important directions of research has been the application of game theory to the problem of setting macroeconomic policy. Once the setting of policy is regarded as a repeated strategic interaction between policymakers and the private sector, understanding the issue of "reputation" in policymaking becomes important. The approach taken in this paper leans heavily on the game theoretic developments.

In section 2, the time-consistency problem is illustrated in a

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[2] A game in the context used here and in the literature means any strategic interaction between economic actors.

[3] This of course ignores the impact on future discoveries

simple closed economy model. It is shown why, in a deterministic world with forward looking agents and complete information on the objectives of economic actors and the structure of the economy, the optimal policy rule for a government may not be feasible because it will not be believed by the private sector. Section 3 is addressed to the question: Can the optimal policy be made time-consistent and therefore credible under some circumstances? It is shown how credible threats and repeated interaction between the policymaker and the private sector can lead to the optimal policy being believable. It is also illustrated that the introduction of various types of uncertainty can help to make the optimal policy believable. This section illustrates that in a rational expectations model it may be desirable for a policymaker to introduce noise into the policy implementation process. Section 4 shifts the focus of the paper away from the problem of a single government interacting with a private sector, to a model of competing political parties. The allowance for elections adds an interesting twist to the problem of time-consistency. A conclusion and suggestions for future research are presented in section 5.

2. The Problem of Time-Inconsistent Policies

In this section a simple model is used to illustrate why a government would renege in the future, on a policy rule which is optimal from the point of view of the current period. This analysis draws on the ideas in the work of Barro and Gordon (1983a, 1983b)[4].

Consider a simple model of a closed economy in which wage

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[4] See McKibbin (1987) for an application of a two country version of the model to the question of international policy coordination.

setters are forward looking and have complete knowledge of the government's policy optimization problem. The model will be introduced here although algebraic manipulation will be confined to a technical appendix.

(1.1) $p_t = w_t + \alpha\, q_t$

(1.2) $q_t = m_t - p_t$

(1.3) $\pi_t = p_t - p_{t-1}$

(1.4) $w_{t+1} = {}_tp_{t+1}$

The following notation is used:

p price of output;

w nominal wage;

q real output;

m nominal money supply;

π rate of inflation.

Equation (1.1) is the aggregate supply curve where prices are a markup over unit labour costs. Money market equilibrium is given by equation (1.2). Note that interest rates are dropped to simplify the analysis. Inflation is defined in (1.3). Equation (1.4) contains the assumption that in period t, wage setters choose the wage to be effective in period t+1, based on the expectation of the price in period t+1. Wage setters are assumed to desire to maintain a given real wage. Different assumptions are made about the way in which wage setters form expectations of the future price. Unless otherwise noted, we assume that wage setters have rational expectations so the expectation is conditional on all the information available in period t. In the case with no uncertainty, this is equivalent to assuming perfect foresight.

Now introduce the problem facing the policymaker. For convenience of presentation it is assumed (as in the literature) that the policy variable is the supply of money. However, the general principles apply to any instrument of policy. Assume that the policymaker selects the policy variable (m) to minimise the following quadratic loss function subject to the structure of the economy:

$$(1.5) \quad \text{Min} \quad \Sigma_{t=0}^{\infty} \delta^{t} \{(q_t - q_0)^2 + \mu\pi_t^{\,2}\}$$

subject to (1.1 to 1.4).

It is assumed that the policymaker cares about the squared deviation of output and inflation from some desired level in the current and future periods, where the loss in each period is weighted by $\delta<1$. Note that the policymaker's desired level of output and inflation are q_0 and 0 respectively.

The difference between this model and the Barro-Gordon model is worth highlighting. In Barro and Gordon (1983) the policymaker loses from inflation variability but gains from higher output. Higher output can be achieved by generating unexpected inflation through a Lucas supply function. Here the policy problem is written differently. The policymaker is assumed to minimise a quadratic loss function of output and inflation. The time-consistency problem arises because the policymaker and the wage setters are assumed to have a different desired real wage which implies a different desired level of output.

In the remainder of this section it is assumed that the policymaker chooses a rule for the policy instrument or a sequence of policy settings to be followed forever. In the language of game theory, the interaction between the policymaker and the wage setter is a one shot game.

The timing of the game between the policymaker and the wage setters is crucial. In this particular model, if policy is implemented before the wage is chosen, the issue of time-consistency is irrelevant because the government policy is revealed when it is announced. In this case, the optimal solution [5] will be time-consistent by construction. The optimal control solution for the policymaker will also be the same, independently of who moves first.

To see this, firstly assume that monetary policy is announced and implemented before wage setters select a nominal wage. Wages are based on observed prices in each period, in which case $w_t = p_t$. From equation (1.2) it can be seen that this implies q_t=0, which is less than the policymaker's desired level[6]. The wage setter, by selecting the desired real wage also selects the level of output. The policymaker realises, that whatever policy is chosen, output is given by the wage setter. The policymaker is then left with monetary policy to determine the rate of inflation. Since the desired level of inflation is zero by assumption, the policy maker will choose $m_t = p_{t-1}$. The result is that the policymaker cannot affect output by monetary policy alone, when the wage setter is given the second move. Given this, it is then optimal in every period to select a zero inflation policy. The formal derivation is given in the appendix.

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[5] The optimal control solution is found by optimizing the objective function ignoring the posibility of future reoptimization of the objective function by future governments.

[6] The wage setter desires q=0 whereas the policymaker desires $q=q_0$. Both desire zero inflation.

How is the optimal solution affected when the wage setter has the first move? The optimal control solution remains the same, but it now becomes time-inconsistent. This can be illustrated by assuming that wage setters choose a nominal wage before the policymaker chooses policy, based on the policymaker's announced policy of $m_t = p_{t-1}$. The formal solution is outlined in the Appendix. In the problem now facing the policymaker, the nominal wage is fixed for the period. The policymaker can raise the level of output towards the desired level by undertaking further monetary expansion. The optimal response is to expand monetary policy to the point where the marginal gain on output is equal to the marginal loss on inflation. The tradeoff between inflation and output facing the policymaker before the wage is committed, is different to the tradeoff after the wage is committed. The optimal policy response therefore changes after the wage is committed and the original policy announcement is seen to be time inconsistent.

This illustration of the desire to renege from the announced optimal policy assumes that the wage setters believed the original announcement of zero inflation. If it is assumed that wage setters are forward looking, they understand the incentives of the policymaker and will therefore not be fooled by the policymaker. They will choose a higher wage despite the policy announcement. The result will be that the optimal policy will not give the desired outcome because the wage setter reaction will be different to that perceived by the government.

Can a time-consistent policy be found? A time-consistent policy is derived in the appendix. It is found by the dynamic programming technique of backward recursion where we assume that the optimization is undertaken in each period, taking as given that future governments will also follow the same policy rule. It is characterized by output at the level desired by the wage setters and inflation higher than desired by the policymaker. The economy in this case has an inflationary bias because the wage setters, knowing the government's incentive to inflate away the real wage, will choose a nominal wage which gives a real wage as close as possible to their desired real wage. The wage setter understands that the policymaker chooses policy to set the marginal utility loss from a unit of inflation equal to the marginal utility gain from a unit of output. This is the Barro-Gordon result in a very different model. In the current model it is due to the different desired real wage of the wage setters and the government. In the Barro-Gordon model it is the impact of unanticipated inflation on the aggregate supply function.

It is worth further highlighting the difference between the optimal and the time-consistent policies. In both cases output is at the level desired by the wage setters but inflation is higher in the time-consistent equilibrium. In the case of the time-consistent policy, we have modelled the strategic interaction between the policymaker and the wage setter. The equilibrium of this game is the Nash equilibrium where each player is doing the best it can taking the policies of the other player as given. Both players are worse off in the equilibrium which is sustainable. Some sort of cooperation between the government and the wage setters (either explicit or implicit) could move the economy away from the Nash equilibrium to something such as the optimal equilibrium.

3. Can the Optimal Policy Be Credible ?

There have been several suggestions in the literature of ways to implement the optimal policy so it is believed. One way is to bind the policymaker's hand by constitutional decree enforcing that a particular policy is to be followed. This makes the policy credible and sustainable because it is precommitted. An alternative is to introduce the notion of threats and reputation when there is repeated interaction between players.

(a) Reputation

Many recent papers have raised the issue of reputation and punishment in a situation where players repeatedly interact with each other[7]. Reputation adds more information to the interaction of players in addition to the structure of the economy and the incentives of each player. This information can be past behaviour or threats in response to different moves. In the above analysis we ignored the repetitive nature of the interactions. The notion of reputation is a very useful device, although it does raise the problem that a multiplicity of reputational equilibria can exist, each of which can be sustainable by an appropriate reputational assumption.

Assume first that there is still complete information (i.e. the structure of the economy and the utility functions of the players are known by all participants) but now the game is repeated an infinite number of times (this is called a supergame in the literature). One approach to the technical implementation of reputation is the concept of punishment. For example, suppose the wage setters threaten to

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[7] See for example Barro and Gordon (1983a),Blackburn (1987), Rogoff(1986), Backus and Driffill (1985).

punish the government if it is seen to deviate from the announced policy. The policymaker must then weigh up the longer term cost of the punishment relative to the potential short-run gain from cheating in the current period. The problem facing the policymaker is changed by the threat. The threats can range from destruction of society (hopefully incredible) to a case where the wage setters agree to follow the cooperative (or optimal) equilibrium of zero inflation but threaten to move back to a Nash equilibrium for a long period (or forever) if the policymaker is ever seen to cheat.[8] This second type of threat is that used in Barro and Gordon (1983a) and Canzoneri (1985).

This can be written more formally as a strategy for wage setters:

$$w_{t+1} = 0 \qquad \text{if} \quad m_t = p_{t-1}$$
$$w_{t+1} = {}_tp_{t+1} \quad \text{if} \quad m_t > p_{t-1}$$

It is shown in the appendix that whether or not this type of strategy sustains the optimal policy depends on a number of factors including: the difference between the rate of inflation under government defection and the rate under cooperation; the length of the punishment period; and the rate of time preference of the government. This type of equilibrium is called a "trigger strategy equilibrium" if the threat is actually severe enough to prevent the policymaker from cheating. Note that there is no explicit cooperation required between agents.

It should be pointed out that a problem emerges in this example of complete information if the horizon of the policymaker is assumed

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[8] This strategy is discontinuous. For an interesting example of a continuous strategy of punishment see Swan (1987) who examines this issue in Cagan's model of hyperinflation.

to be less than infinite. In the finite horizon case, the solution to the last period game is for the policymaker to cheat because there can be no subsequent punishment. In the next to last period the wage setter realizes the incentive to cheat and therefore chooses a high wage. This argument can be repeated until the current period so the entire equilibrium path unravels.

The introduction of a finite horizon policymaker raises the issue of institutional arrangements for the conduct of policy. A political regime consisting of short political cycles suggests that, for credibility to be enhanced, it may be important to have an unelected monetary authority with an effectively infinite horizon. This, in conjunction with an appropriate reputational mechanism, could enhance economic stability. A further extension of the role of the political cycle is discussed in section 4.

(b) Uncertainty

To this point we have considered the case of complete information in the interaction of players. The reputation concept would seem to be more appealing in the case of asymmetric information and learning[9]. An interesting implication of information asymmetries between the policymaker and the private sector is illustrated by Canzoneri (1985). In this paper, the policymaker is assumed to have an informational advantage over the private sector. This advantage makes stabilization policy more effective if the private sector is not given all the information. The private sector understands the incentives of the government to transmit false information for stabilization gains and therefore mistrust any attempt at information

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[9] Asymmetric information means that the two players have different information sets. For example the government may have some information that the wage setter does not have.

sharing. The result is a tradeoff facing the policymaker between short run stabilization policy and long run credibility.

The idea of learning and reputation has been further developed in studies by Barro(1986) and Backus and Driffill (1985). In Barro (1986) for example, there are two types of governments. One type of government promises never to inflate and sticks to its promise, while the other type promises the same but will eventually renege. Agents do not know which type of government is in power and therefore learn from the behavior of the government during the term in office. The result is that the inflating type of government will not inflate in early periods in office to fool the agents as to the type of government. However, eventually the government will find it optimal to inflate before losing office.

In the remainder of this section we explore the implications of a stochastic versus a deterministic model by adapting the idea in Currie and Levine (1986) that policy optimization under uncertainty changes the nature of the game between agents. Under full information, the optimal rule is a solution to a one shot deterministic game. Introducing uncertainty converts the one shot game into a repeated stochastic policy game. Each period the policymaker must respond to a realization of the shock as part of the policy rule formulated in the original period. It is therefore in the interest of the policymaker to include in the original policy objective an assessment of the benefits versus the costs of investing in a reputation and to therefore incorporate the comparison of long term gains from avoiding short term reneging. If the costs of reneging are greater than the cost of pursuing the optimal policy, the private sector will believe the policymaker will follow the optimal deterministic policy rule. This is illustrated in the appendix. In this case, the issue of reputation emerges as part of the optimization problem due to the emergence of uncertainty.

To stretch this idea, suppose that the government can credibly introduce random noise into an otherwise deterministic economy. The introduction of noise by the government into the policy process will presumably have a cost associated with it because of losses from variability of policy instruments. However, the gain from introducing noise into an otherwise deterministic system is that the optimal policy rule may become sustainable. The point being made here is that the usual assumption that noise in policymaking is always bad for the economy is not necessarily true because it is possible that the noise leads to a different reputational equilibrium. The loss from noise can be more than offset by the ability to sustain a different reputational equilibrium. In the appendix, I show an example where sustaining the optimal rule at the expense of some variability is better than the deterministic time-consistent rule which is all that can be reached under perfect information. In comparing sustainable rules the introduction of noise improves the outcome for the government. This solution of course depends on assumptions about the type of uncertainty, the horizon of policymakers etc, but it is nonetheless interesting that the policymaker can, in some cases, reach a more desirable outcome with some random changes to the instruments of policy. It is also problematic because if the government can credibly introduce noise into the system why can't it simply credibly commit to the policy in the first place?

4. Competing Political Parties

The importance of the policymaker's time horizon has already been mentioned. This section outlines the implications of competing political parties for the issue of time consistency. In the problem discussed so far, the issue has been whether a government with an infinite horizon would renege on policy promises. It was argued above that a finite horizon changed the nature of the problem and we introduced the possibility of asymmetric information in understanding which party was in power. A new literature [10] has recently emerged, which examines the issue of political parties with difference preferences rotating in power due to a stochastic election process. This literature has been labeled the "partisanship theory" of macroeconomic policymaking. It extends the game to strategic interaction between the private sector and two political parties. The objective function of the government can no longer be assumed to be well defined and stable. The objective function of the government ex-ante is uncertain and ex-poste is only certain for the period of rule. It has very different implications to the "median voter" theory which argues that parties will offer virtually the same policies before an election which appeal to the median voter. The partisanship approach assumes that voters realize that after the election, the winning party will implement policies which favour its traditional constituencies. The parties realise that the electorate understand these motives and therefore convergence of policies is incomplete.

In the discussion above, it was argued that the problem of time

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[10] See Alesina (1985), Alesina and Tabellini (1987), Persson and Svensson (1987) and McKibbin, Roubini and Sachs (1987).

consistency was how to bind future government to follow a particular policy. Lucas and Stokey (1983) pointed out that the future behavior of government could be affected by the current government by leaving the economy in a particular state. For example, by leaving a large government debt, a government can limit the ability of future governments to follow expansionary fiscal policy. The partisanship theory makes the argument even more complicated because a government can regain power at a future date and therefore will be careful to choose policies which may restrict the policy moves of the opposing party when it gains office, but doesn't cause severe problems for its own future government.

As an example of the change in policymaker behavior, it is worth highlighting the results in Persson and Svensson (1987). In this paper, the authors show a case where there are two parties; one desires small fiscal deficits and the other desires larger deficits. Suppose the ruling party, which places large weight on small fiscal deficits, is faced with a high probability of the opposition party gaining power. The ruling party may decide to undertake a fiscal expansion and generate a large budget deficit today in order to restrict the policy option of a future expansionary government by leaving it a large debt. This restricts the extent of expansionism that the new administration can follow. Similarly McKibbin, Roubini and Sachs (1987) give an illustration in which a government that dislikes inflation will nonetheless pursue inflationary policies to restrict the inflationary policies of future expansionary governments.

An extension to this analysis would be to examine the implications of a non-political, infinite horizon monetary authority on the nature of the games pursued using fiscal policy between two political parties and a forward looking private sector. The realization in the most recent literature that political and economic institutions are crucially linked is an important development in the theory of economic policy formulation.

A further extension of the literature is linked to the new work on the problem of international policy coordination [11]. Once the problem is extended to that of a multi-level game in which governments in different countries are interacting with each other as well as with private individuals and firms within each country, the game between a domestic government and its private agents may be significantly effected by the game between countries. An example of the possible importance of this case is the agreement by the German government at the 1978 Bonn Summit to reflate the German economy at a time when the domestic political arguments were overwhelmingly against this course. By committing itself to an external agreement, the government was able to dramatically influence the domestic game.

5. Conclusion

This paper is not intended to be a comprehensive survey of the many recent papers on the issue of reputation and time consistency in policymaking. It has been written with the goal of presenting, in a non-technical way, some of the issues that theoretical economists have been addressing in the last decade in the theory of economic policy formulation. A disturbing feature of many

[11] See the papers in Buiter and Marston (1985) and Blackburn (1987) for a summary. Also see McKibbin and Sachs (1987) for an empirical application of the game theoretic techniques to a empirical model of the world economy.

of the conclusions of this literature is the multiplicity of results possible in the various frameworks. This is a feature of the game theoretic solution concepts which are currently available and indeed it may also be a feature of the "real world". Nonetheless, the recent literature, incorporating the role of political and economic institutions, provides new and potentially useful insights into the theory of economic policy.

APPENDIX

This appendix contains explicit solutions to the different problems discussed in the body of the paper.

(a) Optimal Control Solution

To solve for the optimal control solution it is convenient to rewrite the constraints (1.1) to (1.5) with the targets (q,π) as functions of the state variables (p_{t-1}) and the control variables (m):

(a1) $q_t = 0$

(a2) $\pi_t = m_t - p_{t-1}$

The way the problem has been constructed enables us to select any period t and differentiate (1.5) to find the optimal policy for each period. The policymaker should set:

(a3) $(q_t - q_0)(\partial q/\partial m) = -\pi_t \mu(\partial \pi/\partial m)$

in every period.

Differentiating (a1) and (a2) and substituting into (a3), it can be shown that the optimal closed loop policy is $m_t = p_{t-1}$. This implies $q_t = \pi_t = 0$ in every period. The loss to the policymaker is $q_0{}^2/(1-\delta)$.

(b) Reneging

Now assume that the wage is chosen before policy is selected and the wage setter chooses the nominal wage based on the announcement that the optimal policy will be followed. The policymaker now treats the nominal wage as a state variable inherited from the previous period.

It is convenient to write the model with target variables as a function of state variables (w_t and p_{t-1}) and control variables (m_t).

(a4) $q_t = (m_t - w_t)/(1+\alpha)$

(a5) $\pi_t = \alpha\, m_t/(1+\alpha) + w_t/(1+\alpha) - p_{t-1}$

Assume that the government announces the optimal control policy, which is believed by the wage setters who set $w_t = p_{t-1}$. Differentiating (a4) and (a5) and substituting into the policymaker's first order condition given in (a3) we now find:

(a6) $m_t = p_{t-1} + (1+\alpha)q_0/(1+2\alpha\mu)$

Substituting into (a4) and (a5) and assuming $w_t = p_{t-1}$, gives the results for the target variables:

(a7) $q_t = q_0/(1+\alpha^2\mu)$

(a8) $\pi_t = \alpha q_0/(1+\alpha^2\mu)$

The loss for the policymaker is:

$$\frac{\delta}{1-\delta} \quad \frac{\mu\alpha^2}{1+\mu\alpha^2} \quad q_0{}^2$$

which is less than the optimal control loss. Notice that once the wage setters have pre-committed their wage based on the announced policy, the policymaker has the incentive to renege on the announced policy of zero inflation and to follow an expansionary monetary policy. The policymaker can reduce the output loss[12] at the expense of some loss in inflation relative to the optimal solution. The policymaker now expands monetary policy until the benefit from an extra unit of output is offset by the cost of an additional unit of inflation. The optimal policy is therefore shown to be time

[12] note that the policymaker desires output of q_0.

inconsistent. Once the policy is announced and believed, the policymaker has an incentive to change the policy.

(c) Time-consistent Solution

To aid in solving for the time-consistent equilibrium, the model can again be rewritten with the target variables (q,π) as functions of the control variable (m) and state variable (w). Define the value function as:

(a9) $$V_t = \underset{\{m_t\}}{\mathrm{Max}} -\{(q_t - q_0)^2 + \mu\pi_t^2\} + \delta V_{t+1}$$

subject to:

(a10) $$q_t = (m_t - w_t)/(1+\alpha)$$

(a11) $$\pi_t = \alpha m_t/(1+\alpha) + w_t/(1+\alpha) - p_{t-1}$$

Note that the wage in period t is now a state variable to the policymaker. To find the time-consistent solution to this problem we will us a dynamic programming technique of backward recursion. First we find the solution to the finite horizon problem then take the limit of this problem for the infinite horizon case.

Suppose period T is the final period and $V_{T+1} = 0$. From the optimization of (a9) and as we found for the static game, the policymaker should set:

(a12) $$(q_T - q_0)(\partial q/\partial m) = -\pi_T\mu(\partial\pi/\partial m)$$

Differentiating (a10) and (a11) and substituting into (a12), we find a rule for the control variable m_T in terms of w_T, q_0 and p_{T-1}.

(a13) $$m_T = \{(1-\alpha\mu)w_T + (1+\alpha)q_0 + \alpha(1+\alpha)\mu p_{T-1}\} / (1+\mu\alpha^2)$$

This is the time-consistent, closed loop rule for the control variable (m_T) as a function of the state variables (w_T and p_{T-1}) and the exogenous variable (q_0). The assumption of rational expectations requires that q=0 (i.e. output is q0 less than desired by the policymaker). This can be seen from (4) which has $p_T=w_T$ in equilibrium. Together with (1), this implies $q_T=0$. Substituting $w_T=p_T$ together with the rule for m into (a12), it can be shown that in period T:

(a14) $q_T = 0$

(a15) $\pi_T = q_0/\mu\alpha$

The value function in period T is a function of q_0 and is independent of variables inherited from earlier periods. Each period can therefore be solved independently, taking as given that future governments will be following the policy rules in (a13). In each period the solution will be of the form given in (a14) and (a15). Taking the limit for large T does not change the result.

In this example, with wage setters having rational expectations and a desired level of output different to the policymaker, the time-consistent equilibrium will have an inflationary bias and output will be less than desired by the policymaker. This is the Barro-Gordon result. The private agents know that once they choose a nominal wage the policymaker has the incentive to inflate away the real wage for some output gain. The agents therefore have an incentive to choose a high nominal wage up to the point where they know the policymaker is unwilling to trade off an extra unit loss on inflation for a unit gain on output. The result is an economy with an inflationary bias.

(d) Reputation

To illustrate the idea of reputation assume that the wage setters threaten to follow the zero inflation wage claim if the policymaker has been observed not to cheat, but follow the time-consistent policy forever if cheating is observed. This can be formalized as:

$$w_{t+1} = 0 \qquad \text{if} \quad m_t = p_{t-1}$$

$$w_{t+1} = p_{t+1} \quad \text{if} \quad m_t > p_{t-1}$$

The loss to the policymaker of never reneging can be shown to be equal to:

$$\text{(a16)} \qquad \Sigma_{t=0}^{\infty} \delta^t \{(q_0)^2\} = q_0^2/(1-\delta)$$

If the policymaker reneges in period 0, the loss will be the gain in period 0 plus the loss from reverting to the time-consistent equilibrium:

$$\text{(a17)} \qquad \frac{\mu\alpha^2}{(1 + \mu\alpha^2)} q_0^2 + \frac{\delta}{1-\delta} \frac{1+\mu\alpha^2}{\mu\alpha^2} q_0^2$$

It can be shown that the loss in (a17) is less than the loss in (a16) (i.e. it pays to renege) if :

$$\text{(a18)} \qquad \frac{\delta}{1-\delta} < \frac{\mu\alpha^2}{1+\mu\alpha^2}$$

For different parameters, there is a range where the government will find it beneficial to renege and a range where the government will find it beneficial to follow the optimal policy. As the government discounts the future more heavily (ie as δ approaches 0) the more likely that (a18) will hold and therefore the more likely that the government will find it worth reneging despite the wage setters threat. The length of punishment is also important although here we assume an infinite period of punishment.

(5) Uncertainty

To understand the implications of uncertainty in sustaining the optimal policy, suppose a random shock is added to equation (1.2):

(1.2a) $q_t = m_t - p_t + \epsilon_t$

where ϵ_t is a random variable with mean zero and a variance equal to σ^2.

In calculating the optimal rule, time-consistent rule and reneging rule, we can appeal to the standard result from control theory that with additive shocks, certainty equivalence holds. By assuming in each case the policy rules found above, we can calculate the following movements in the targets variables:

(i) optimal control $\quad q = \epsilon/(1+\alpha)$

$\pi = \alpha\epsilon/(1+\alpha)$

$$\text{expected loss} = V^O = \frac{1}{1-\delta}\left(q_0^{\,2} + \frac{1+\mu\alpha^2}{(1+\alpha)^2}\,\sigma^2\right)$$

(ii) reneging (with punishment) $\quad q = q0/(1+\alpha^2\mu) + \epsilon/(1+\alpha)$

$\pi = \alpha q0/(1+\alpha^2\mu) + \alpha\epsilon/(1+\alpha)$

$$\text{expected loss} = V^R = \left(\frac{\alpha^2\mu}{1+\alpha^2\mu} + \frac{\delta}{1-\delta}\,\frac{1+\alpha^2\mu}{\alpha^2\mu}\right) q_0^{\,2} + \frac{1}{1-\delta}\left(\frac{1+\mu\alpha^2}{(1+\alpha)^2}\,\sigma^2\right)$$

(iii) time-consistent $\quad q = \epsilon/(1+\alpha)$

$\pi = q0/\alpha\mu + \alpha\epsilon/(1+\alpha)$

$$\text{expected loss} = V^T = \frac{1}{1-\delta}\left(\frac{1+\alpha 2\mu}{\alpha 2\mu}\,q_0^{\,2} + \frac{1+\mu\alpha^2}{(1+\alpha)^2}\,\sigma^2\right)$$

The uncertainty converts the one shot game into a repeated game and therefore the government incorporates the incentive to renege (measured by V^O-V^R) into its optimization problem. The wage setters realize that each period the policymaker is evaluating the gain to reneging. It can be shown that if

$$\delta/(1-\delta) > \mu\alpha^2/(1+\mu\alpha^2),$$

there is no incentive to renege as the loss from reneging outweighs the gain. The private sector realize this and the optimal policy is sustainable. The loss in the optimal policy under uncertainty is now larger than in the deterministic case. It can be shown that the loss from sustaining the optimal rule under uncertainty is less than the time-consistent rule under certainty if:

$$\sigma^2 < \frac{(1+\alpha)^2}{\mu\alpha^2(1+\mu\alpha^2)} q_0^2$$

Therefore, if the variance of the noise introduced is sufficiently small and the government sufficiently forward looking it is possible that introducing noise (as a precommittment), the optimal rule can be sustained and the gain from sustaining this rule will outweigh the loss from adding the noise.

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