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# RESEARCH DISCUSSION PAPER

## Monetary Transmission and the Yield Curve in a Small Open Economy

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# Abstract

Long-term nominal interest rates in a number of inflation-targeting small open economies have tended to be highly correlated with those of the United States. This observation has recently lent support to the view that the long end of the yield curve is determined abroad. We set up and estimate a micro-founded two-block small open economy model to study the co-movement of long-term nominal interest rates of different currencies. The expectations hypothesis together with uncovered interest rate parity, which both hold in our model, can account for much of the co-movement of interest rates observed in the data.

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# MONETARY TRANSMISSION AND THE YIELD CURVE IN A SMALL OPEN ECONOMY

**Mariano Kulish and Daniel Rees**

## 1. Introduction

Long-term nominal interest rates in a number of inflation-targeting small open economies, such as Australia, Canada, New Zealand, Norway, Sweden and the United Kingdom, have moved very closely with those of the United States over the past 15 years or so. Figure 1 shows the pattern of interest rate correlations at different points on the yield curve for each country with the US. The pattern is clear: long-term nominal rates are highly correlated with their US counterparts, generally more so than rates at shorter maturities, and more so than with their respective short-term rates (not shown in Figure 1). This pattern has led to the view that long-term nominal interest rates are somehow determined abroad.[1]

Traditionally, the transmission mechanism of monetary policy is understood (among other things) as linking a short-term nominal interest rate to a long-term real interest rate which, in turn, influences aggregate demand. Of course, for a short-term *nominal* rate to influence a long-term *real* rate, it must be that the short-term *nominal* rate influences long-term *nominal* rates.

In this paper we assess whether these reduced-form patterns of correlations are consistent with theory and examine the forces at work in the determination of a small open economy's yield curve. We find that the expectations hypothesis together with uncovered interest rate parity can account for much of the observed co-movement in interest rates of different currencies. In fact, the main contribution of our paper is to uncover a mechanism that can give rise to these observed reduced-form correlations in optimising general equilibrium. As we discuss below, differences in the persistence of domestic and foreign disturbances can bring about the pattern of correlations in Figure 1.

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[1] For the unfiltered data, the pattern of correlation is also increasing along the yield curve and quite similar. In the case of Australia, for example, these correlations are 0.56, 0.76 and 0.85 for 3-month, 5-year and 10-year maturities, respectively. As we discuss below, our conclusions are robust to the filtering of the data.

**Figure 1: Cross Correlations with US Interest Rates**

Hodrick-Prescott (HP)-filtered quarterly data
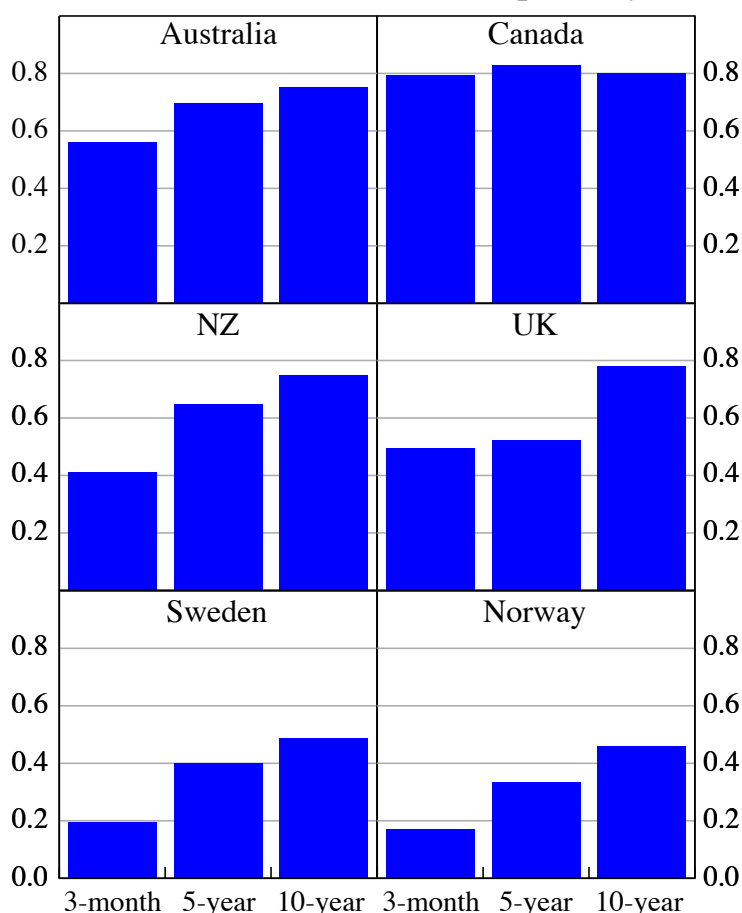


Note:      Australia, Norway, NZ and Sweden cover 1993–2007; Canada covers 1997–2007;
UK covers 1994–2007.

Other papers have tackled the related question of how much domestic and foreign factors influence domestic interest rates. For example, Campbell and Lewis (1998) use an event study to examine how Australian bond yields respond to new information and find that US economic news has a larger effect than domestic news on Australian yields. Tarditi (1996) estimates a reduced-form model of the Australian 10-year bond yield and finds that a one percentage point increase in the US 10-year bond yield is associated with around half a percentage point increase in Australian long-term yields.

There is also a large literature that analyses the yield curve with affine term structure models. These studies typically assume that bond yields are affine functions of unobservable factors and incorporate cross-equation restrictions that eliminate arbitrage opportunities (Knez, Litterman and Scheinkman 1994;

Duffie and Kan 1996; Dai and Singleton 2000; Backus, Foresi and Telmer 2001; Backus and Wright 2007). But while factor models have been relatively successful in matching key statistical properties of the yield curve, factor models are not structural. Recent work addresses this issue by fitting the term structure to macroeconomic factors, either by combining them within unobserved factors, as in Ang and Piazzesi (2003) and Bernanke, Reinhart and Sack (2004), or by incorporating a no-arbitrage model of the term structure within a macroeconomic model as in Rudebusch and Wu (2004), Bekaert, Cho and Moreno (2005) and Hördahl, Tristani and Vestin (2006).

Our focus here is different. We set up a micro-founded two-block model consisting of a small open economy and a large (closed) economy and extend the set of equilibrium conditions in both the large and small economies to allow for an explicit consideration of the co-movement of foreign and domestic interest rates. In our model, the expectations hypothesis links interest rates of different maturities and uncovered interest rate parity links interest rates of different currencies. Short-term nominal rates are set by the monetary authorities on the basis of the fundamentals of their economies. In this respect our analysis resembles that of Evans and Marshall (1998), but unlike them, we study the behaviour of a *small open* economy's yield curve and pay particular attention to its relation to the *large* economy's yield curve. We then estimate the model's parameters and examine its ability to match the co-movement of interest rates of different currencies.

The rest of the paper is organised as follows. Section 2 describes the model and the role of the yield curve in the transmission mechanism of a small open economy. Section 3 discusses the estimation of the model. Section 4 examines the dynamics of the yield curve. Section 5 contrasts the model's moments with their empirical counterparts and Section 6 concludes.

## 2.    The Model

We extend the Galí and Monacelli (2005) small open economy model in two ways. First, we increase the set of equilibrium conditions in both the large and small economies to incorporate interest rates of longer maturities. Second, we add foreign and domestic demand shocks. Instead of working through the details of the derivation, which are in Galí and Monacelli, we discuss the log-linear aggregate equations and the role of the yield curve in the transmission mechanism.

## 2.1 The Large Economy

Variables with a *star superscript* correspond to the large economy, which obeys a standard set of New Keynesian closed economy equations.[2] All variables are expressed in percentage deviations from their steady states.

The aggregate demand schedule links the current level of the foreign output gap, $x_t^*$, to its expected future level, the *ex-ante* short-term real interest rate, foreign total factor productivity, $a_t^*$, and a foreign aggregate demand disturbance, $g_t^*$, as follows:

$$x_t^* = E_t x_{t+1}^* - \sigma^{-1} \left( R_{1,t}^* - E_t \pi_{t+1}^* \right) - \phi_1 (1 - \rho_a^*) a_t^* + \sigma^{-1} \left( 1 - \rho_g^* \right) g_t^* \quad (1)$$

where: $R_{1,t}^*$ is the foreign short-term nominal interest rate; $\pi_t^*$ is the foreign inflation rate; $\sigma$ is strictly positive and governs intertemporal substitution; $\rho_a^*$ is the persistence of $a_t^*$; $\rho_g^*$ is the persistence of $g_t^*$; and $\phi_1$, defined for convenience, is $\frac{1+\varphi}{\sigma+\varphi}$, where the parameter $\varphi > 0$ captures the elasticity of labour supply.

It can be shown that in this model the theory of the term structure implied by optimising behaviour is the expectations hypothesis. Thus, the nominal interest rate at period $t$ associated with a bond that promises to pay one unit of foreign currency at the end of period $t + m - 1$ is determined by

$$R_{m,t}^* = \frac{1}{m} E_t \sum_{j=1}^{m} R_{1,t+j-1}^* \qquad m = 2, 3, 4, .... \quad (2)$$

Firms operate in a monopolistically competitive goods market and are subject to Calvo-price stickiness. Factor markets are competitive and goods are produced with a constant returns-to-scale technology. These assumptions yield the New Phillips curve:

$$\pi_t^* = \kappa x_t^* + \beta E_t \pi_{t+1}^* \quad (3)$$

where: $\kappa \equiv \lambda (\sigma + \varphi)$; $\lambda \equiv (1 - \theta)(1 - \beta\theta)/\theta$; $\theta$ governs the degree of price stickiness; and $\beta$ is the households' discount factor.

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[2]  Ireland (2004) and Woodford (2003) contain detailed discussions of the New Keynesian closed economy model.

The foreign monetary authority is assumed to follow a Taylor-type rule of the form

$$R_{1,t}^* = \rho_r^* R_{1,t-1}^* + \alpha_\pi^* \pi_t^* + \alpha_x^* x_t^* + \varepsilon_{r,t}^* \tag{4}$$

where $\varepsilon_{r,t}^*$ is an independent and identically distributed (*iid*) foreign monetary disturbance with zero mean and standard deviation $\sigma_{\varepsilon_r^*}$.

The potential level of foreign output, $\bar{y}_t^* = \phi_1 a_t^*$, is the level that would prevail in the absence of nominal rigidities. So, in the large economy, the actual level of output, $y_t^*$, and the output gap, $x_t^*$, are such that

$$x_t^* = y_t^* - \phi_1 a_t^*. \tag{5}$$

The technology shock, $a_t^*$, and the demand shock, $g_t^*$, follow autoregressive processes of the form

$$a_t^* = \rho_a^* a_{t-1}^* + \varepsilon_{a,t}^* \tag{6}$$
$$g_t^* = \rho_g^* g_{t-1}^* + \varepsilon_{g,t}^* \tag{7}$$

where: the persistence parameters, $\rho_a^*$ and $\rho_g^*$, are less than unity in absolute value; and the shocks $\varepsilon_{a,t}^*$ and $\varepsilon_{g,t}^*$ are zero-mean *iid* disturbances with standard deviations $\sigma_{\varepsilon_a^*}$ and $\sigma_{\varepsilon_g^*}$, respectively.

## 2.2 The Small Open Economy

The small economy's IS-curve links the output gap, $x_t$, to its expected future value, the one-period nominal interest rate, $R_{1,t}$, the expected rate of *domestically produced goods* inflation, $E_t \pi_{h,t+1}$, the expected growth rate of foreign output, foreign and domestic aggregate demand disturbances, and total factor productivity, $a_t$. Following Galí and Monacelli (2005) the small open economy's IS-curve can be shown to take the form

$$\begin{aligned} x_t = {}& E_t x_{t+1} - \sigma_\alpha^{-1} \left( R_{1,t} - E_t \pi_{h,t+1} \right) + \phi_3 E_t \Delta y_{t+1}^* + \\ & \sigma^{-1} \left( 1 - \rho_g \right) \left( 1 - \phi_2 \right) g_t + \sigma^{-1} \left( 1 - \rho_g^* \right) \phi_3 g_t^* - \phi_4 (1 - \rho_a) a_t \end{aligned} \tag{8}$$

where $\rho_a$ and $\rho_g$ are the persistence parameters of $a_t$ and $g_t$. The parameters $\sigma_\alpha$, $\phi_2$, $\phi_3$, and $\phi_4$ are, in turn, functions of deeper parameters. In particular,

$$
\begin{aligned}
\sigma_\alpha &\equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} \\
\omega &\equiv \sigma\tau+(1-\alpha)(\sigma\iota-1) \\
\phi_2 &\equiv \frac{\sigma_\alpha-\sigma}{\sigma_\alpha+\varphi} \\
\phi_3 &\equiv \alpha(\omega-1)+\phi_2 \\
\phi_4 &\equiv \frac{1+\varphi}{\sigma_\alpha+\varphi}
\end{aligned}
$$

where $\alpha \in [0,1]$ is the share of foreign goods in the consumption basket, and therefore serves as a measure of openness. It is worth noting that for $\alpha = 0$, the small economy's equations reduce to the standard set of closed economy equations discussed above. Thus, the small economy has all of the structural features of the large economy, overlayed, of course, by openness. Indeed, as discussed in Galí and Monacelli (2005), the linearised equations hold around a symmetric steady state. Finally, $\tau$ is the intratemporal elasticity of substitution between foreign and domestically produced goods, while $\iota$ is the elasticity of substitution across varieties of foreign goods.

In equilibrium, the nominal interest rate at $t$, associated with a bond that promises to pay one unit of *domestic* currency at the end of period $t+m-1$, is determined by

$$
R_{m,t} = \frac{1}{m}E_t\sum_{j=1}^{m}R_{1,t+j-1} \qquad m = 2,3,4,.... \tag{9}
$$

The dynamics of *domestically produced* goods price inflation, $\pi_{h,t}$, are governed by an analogous New Phillips curve

$$
\pi_{h,t} = \kappa_\alpha x_t + \beta E_t \pi_{h,t+1} \tag{10}
$$

where $\kappa_\alpha \equiv \lambda\left(\sigma_\alpha+\varphi\right)$.

Monetary policy in the small economy is also assumed to follow a Taylor-type rule of the form

$$
R_{1,t} = \rho_r R_{1,t-1} + \alpha_\pi \pi_t + \alpha_x x_t + \varepsilon_{r,t} \tag{11}
$$

where $\varepsilon_{r,t}$ is an *iid* monetary policy shock with zero mean and standard deviation $\sigma_{\varepsilon_r}$.

The terms of trade, $s_t$, are defined (from the perspective of the large economy) as the price of goods produced in the large economy, $p_t^* + e_t$, relative to the price of small economy goods, $p_{h,t}$. The nominal exchange rate, $e_t$, is defined as the price of foreign currency in terms of the domestic currency. That is, $s_t = p_t^* + e_t - p_{h,t}$. Around a symmetric steady state, the consumer price index of the small economy is a weighted average of the form $p_t = (1 - \alpha)p_{h,t} + \alpha(p_t^* + e_t)$.[3] It is straightforward to show that $p_t = p_{h,t} + \alpha s_t$, which implies that consumer price inflation and domestically produced goods inflation are linked by the expression below:

$$\pi_t = \pi_{h,t} + \alpha \Delta s_t. \tag{12}$$

The real exchange rate, $q_t$, in turn, is defined as $q_t \equiv e_t + p_t^* - p_t$. It follows that changes in the nominal exchange rate, $\Delta e_t$, can be decomposed into changes in the real exchange rate and the differential in consumer price inflation.

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^*. \tag{13}$$

Combining these expressions, it is easy to show that the real exchange rate is proportional to the terms of trade as follows:

$$\Delta q_t = (1 - \alpha) \Delta s_t. \tag{14}$$

Complete international securities markets together with market clearing, imply the following relationship between the terms of trade, $s_t$, and output differentials and demand shock differentials:[4]

$$s_t = \sigma_\alpha (y_t - y_t^*) - \frac{\sigma_\alpha}{\sigma} \left( g_t - g_t^* \right). \tag{15}$$

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[3]  This relationship implies complete and contemporaneous pass-through from the nominal exchange rate to domestic prices. While not realistic, this assumption is used for simplicity. It is not surprising that, in the empirical section later in this paper, the model fails to match the low contemporaneous correlation between the exchange rate and inflation observed in the Australian data.

[4]  Because the demand shock, $g_t$, enters the household's lifetime expected utility – $E_0 \sum_{t=0}^{\infty} \beta^t e^{g_t} \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$ – it follows that demand shocks enter the international risk-sharing condition as in Equation (15).

The presence of the aggregate demand-shocks differential, $g_t - g_t^*$, in Equation (15), alters the small economy's flexible price level of output, relative to Galí and Monacelli (2005). The relationship between the actual level of output, $y_t$, and the output gap, $x_t$, satisfies[5]

$$x_t = y_t - \phi_2 y_t^* - \frac{\phi_2}{\sigma} \left( g_t - g_t^* \right) - \phi_4 a_t. \tag{16}$$

Finally, exogenous domestic processes evolve according to

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{17}$$
$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \tag{18}$$

where the shocks $\varepsilon_{a,t}$, and $\varepsilon_{g,t}$ are *iid* with zero-mean and standard deviations $\sigma_{\varepsilon_a}$ and $\sigma_{\varepsilon_g}$, respectively. The persistence parameters $\rho_a$ and $\rho_g$ are, as before, less than unity in absolute value.

## 2.3 The Transmission Mechanism

The linearised dynamics of the model, as we mentioned above, are valid around a symmetric steady state in which the condition of uncovered interest rate parity holds:

$$R_{1,t} = R_{1,t}^* + E_t \Delta e_{t+1}. \tag{19}$$

Equation (19), however, is not an independent equilibrium condition since it can be recovered from the Euler equations for consumption and the international risk-sharing condition, Equation (15). The expectations hypothesis, Equations (2) and (9), combined with uncovered interest parity, relate foreign and domestic interest rates of equivalent maturities as follows:

$$R_{m,t} = R_{m,t}^* + \frac{1}{m} E_t \sum_{j=1}^{m} \Delta e_{t+j}. \tag{20}$$

This highlights the fact that the expected path of the nominal exchange rate plays a central role to the extent that it governs the degree to which movements in

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[5] One can show that the level of potential output in the small economy is given by $\phi_2 y_t^* + \frac{\phi_2}{\sigma}(g_t - g_t^*) + \phi_4 a_t$. If aggregate demand shocks were absent from our model, the expression for the output gap collapses back to that of Galí and Monacelli (2005).

foreign rates are reflected in domestic rates. In the extreme case in which the small economy is closed, movements in foreign rates would not translate into movements in domestic rates.[6]

While Equations (2) and (9) are independent equilibrium conditions, they are nevertheless redundant for the determination of the equilibrium; that is, the equilibrium has a representation without reference to the equations that determine the yield curve.

However, this is not to say that long-term nominal rates are not central for the transmission of monetary policy, nor that they do not contain important information. As emphasised by Rotemberg and Woodford (1997), in sticky-price models it is the *ex-ante* long-term real interest rate that matters for aggregate demand. In the small open economy version of the model, it also happens to be the *ex-ante* long-term real interest rate that matters for aggregate demand, although the economy's openness alters the relevant measure of the long-term real rate as well as the interest rate sensitivity of aggregate demand. To see this, take the IS-curve for the small economy, Equation (8), set all disturbances to zero for simplicity, and assume that the large economy is in steady state. This implies that

$$x_t = E_t x_{t+1} - \sigma_\alpha^{-1} \left( R_{1,t} - E_t \pi_{h,t+1} \right).$$

Advance the equation one period, take expectations and substitute the resulting expression to obtain

$$x_t = -\sigma_\alpha^{-1} \left( R_{1,t} - E_t \pi_{h,t+1} \right) - \sigma_\alpha^{-1} \left( E_t R_{1,t+1} - E_t \pi_{h,t+2} \right) + E_t x_{t+2}.$$

Repeating this operation *m* times and using Equation (9) we can write

$$x_t = -\sigma_\alpha^{-1} m \left( R_{m,t} - \frac{1}{m} E_t \sum_{j=1}^{m} \pi_{h,t+j} \right) \tag{21}$$

since in a stationary equilibrium, $E_t x_{t+m}$ is approximately zero for large *m*. Equation (21) implies that the current level of the output gap depends on an *ex-ante* long-term real interest rate, measured in domestically produced

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[6]  This would hold if capital markets were closed, which in this model would necessarily be the case if $\alpha = 0$.

goods price inflation, magnified by maturity and scaled by the economy's intertemporal substitution. If the economy were closed, Equation (21) becomes $x_t = -\sigma^{-1}m\left(R_{m,t} - \frac{1}{m}E_t\sum_{j=1}^{m}\pi_{t+j}\right)$, because for $\alpha = 0$, we have that $\sigma_\alpha = \sigma$ and $\pi_{h,t} = \pi_t$. Thus, the sticky-price small open economy model puts long-term nominal interest rates at the very heart of the transmission mechanism in much the same way as the closed economy sticky-price model: the expectations hypothesis implies that monetary policy influences long-term nominal interest rates and nominal rigidities mean that policy will therefore influence real activity.

## 3.    Estimation

For estimation purposes, the discount factor, $\beta$, is set at 0.99 which, at a quarterly frequency, corresponds to a steady-state real rate of interest of 4.1 per cent. The degree of openness, $\alpha$, is set at 0.2, consistent with the value of the share of foreign goods in the Australian consumption basket.[7]

The rest of the model's parameters are estimated with Bayesian techniques, as discussed in Lubik and Schorfheide (2006), An and Schorfheide (2007) and Griffoli (2007).[8] We do so in two steps: in the first, we estimate the large economy's parameters; and in the second, we estimate the remaining small economy's parameters, taking the posterior mean values of the common parameters as given from the first step.[9]

Since our focus is on the cross-correlations of domestic interest rates with their US counterparts, we take the US to be the large economy. We use quarterly HP-filtered data on real US GDP per capita, US consumer price inflation and a

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[7]  In preliminary attempts to estimate the model, we found that $\alpha$ would invariably tend towards zero for a range of prior distributions. Nimark (2007) follows a similar strategy to calibrate these two parameters.

[8]  We used the MATLAB package Dynare for the estimation of the model; the relevant files are available upon request.

[9]  As noted before, because the small economy is open, the definitions of both potential output and CPI inflation are different from those of the large economy. Hence, the choice of prior distributions for the two economies need not be the same.

US 3-month nominal interest rate for the sample period 1993:Q1–2007:Q2.[10] Table 1 summarises results for this first step of the estimation.

| Table 1: Large Economy | | | | | | | |
|---|---|---|---|---|---|---|---|
| Parameters | | Prior mean | Posterior mean | 90 per cent confidence intervals | | Prior distribution | Prior std dev |
| $\sigma^{-1}$ | IS-curve | 0.50 | 0.78 | [0.46 | 1.08] | Gamma | 0.20 |
| $\phi_1$ | | 0.90 | 0.93 | [0.62 | 1.23] | Gamma | 0.20 |
| $\kappa$ | Phillips curve | 0.35 | 0.48 | [0.31 | 0.65] | Gamma | 0.20 |
| $\rho_r^*$ | Taylor rule | 0.90 | 0.89 | [0.87 | 0.92] | Beta | 0.02 |
| $\alpha_\pi^*$ | | 0.25 | 0.35 | [0.19 | 0.50] | Normal | 0.10 |
| $\alpha_x^*$ | | 0.25 | 0.36 | [0.23 | 0.49] | Normal | 0.10 |
| $\rho_a^*$ | Technology | 0.90 | 0.92 | [0.90 | 0.95] | Beta | 0.02 |
| $\rho_g^*$ | Demand | 0.90 | 0.89 | [0.86 | 0.92] | Beta | 0.02 |
| Standard deviations | | | | | | | |
| $\sigma_{\varepsilon_a^*}$. | Technology | 0.008 | 0.006 | [0.004 | 0.008] | Inverse gamma | $\infty$ |
| $\sigma_{\varepsilon_g^*}$. | Demand | 0.015 | 0.016 | [0.012 | 0.020] | Inverse gamma | $\infty$ |
| $\sigma_{\varepsilon_r^*}$. | Monetary policy | 0.003 | 0.001 | [0.001 | 0.002] | Inverse gamma | $\infty$ |

Since the large economy is exogenous to the small economy, we take the smoothed estimates for $g_t^*$ and $E_t y_{t+1}^*$ from the first step of the estimation and use these as additional series in the estimation of the small economy's parameters. This differs from much of the relevant literature on small open economies which typically adopts an unrestricted reduced-form VAR process for foreign variables. Such a reduced-form process may or may not be consistent with the theory at hand. However, to the extent that an arbitrarily imposed reduced-form specification for the dynamics of foreign variables differs from that of the theory, the structural equations for the small economy will be invalid. As noted by Justiniano and Preston (2006), the structural equations of the domestic economy depend on the

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[10] Using the same priors, we have also estimated the model using linearly detrended output and demeaned inflation and interest rates. Although the posterior density changes, our key findings and main conclusions remain the same. In the interest of space, we do not report these results; they are available upon request.

assumption that the large economy is populated with households and firms with identical preferences and technology. Therefore, the assumption of an arbitrary reduced-form process will not generally be consistent with the structural equations of the small economy.

We take the estimated posterior mean parameter values as given from the first step and estimate the remaining small open economy parameters on Australian data. For the small economy we use quarterly HP-filtered data on real GDP per capita, consumer price inflation and a 3-month nominal interest rate for the sample period 1993:Q1–2007:Q2.[11] Table 2 below summarises results for this second step of the estimation.

<div align="center"><strong>Table 2: Small Economy</strong></div>

| Parameters | | Prior mean | Posterior mean | 90 per cent confidence intervals | | Prior distribution | Prior std dev |
|---|---|---|---|---|---|---|---|
| $\omega$ | IS-curve | 2.00 | 2.25 | [1.42 | 3.04] | Gamma | 0.50 |
| $\rho_r$ | Taylor rule | 0.85 | 0.84 | [0.81 | 0.88] | Beta | 0.02 |
| $\alpha_\pi$ | | 0.60 | 0.76 | [0.63 | 0.89] | Normal | 0.10 |
| $\alpha_x$ | | 0.10 | 0.03 | [–0.09 | 0.14] | Normal | 0.10 |
| $\rho_a$ | Technology | 0.90 | 0.91 | [0.88 | 0.93] | Beta | 0.02 |
| $\rho_g$ | Demand | 0.90 | 0.88 | [0.85 | 0.91] | Beta | 0.02 |
| Standard deviations | | | | | | | |
| $\sigma_{\varepsilon_a}$ | Technology | 0.007 | 0.008 | [0.007 | 0.009] | Inverse gamma | $\infty$ |
| $\sigma_{\varepsilon_g}$ | Demand | 0.007 | 0.007 | [0.005 | 0.008] | Inverse gamma | $\infty$ |
| $\sigma_{\varepsilon_r}$ | Monetary policy | 0.001 | 0.002 | [0.001 | 0.002] | Inverse gamma | $\infty$ |

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[11] Appendix A contains a description of the data sources. We take the inflation-targeting period for the estimation of the small economy's parameters because the assumption of a symmetric steady state, which entails relatively similar rates of steady-state inflation, does not appear valid before then.

# 4. The Dynamics of the Yield Curve

Figure 2 shows the impulse responses to a domestic technology shock of the small economy's yield curve for two different parameterisations of persistence: $\rho_a = 0.9$ and $\rho_a = 0.65$.[12] The less persistent the shock, the smaller the impact on longer-term rates. The intuition for this is straightforward: less persistent shocks induce less persistent expected movements of the short-term rate.

**Figure 2: Impulse Responses to a Domestic Technology Shock**
High versus low persistence of technology shocks
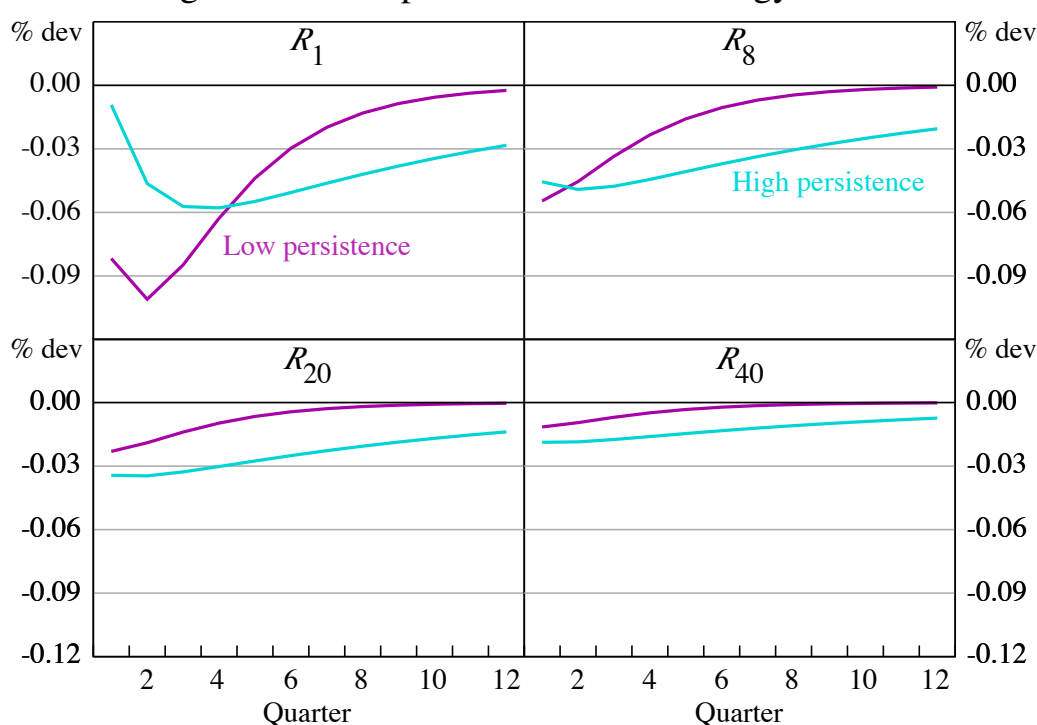


Note: The size of the shock is one standard deviation.

Perhaps less obvious is the fact that a less persistent shock moves shorter-term rates *on impact* by more than a more persistent shock. One way to understand the intuition here is to consider the role of consumption smoothing in response to a positive technology shock. For very persistent shocks, consumption should rise by almost as much as output (since the shock to income is long lasting), hence the output gap is little changed and the monetary policy response can be relatively modest. However, for a temporary shock, much of the extra income will be saved,

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[12] All other parameters are set at the estimated posterior mean values of Tables 1 and 2.

leading to a decline in the output gap and hence monetary policy will respond by cutting short-term interest rates.[13]

The key to understanding the model's ability to reproduce the pattern of correlations in Figure 1 is that less persistent domestic shocks produce a source of variation in the yield curve that is relatively stronger at the short end of the yield curve than at the long end. Other things equal, if the persistence of a domestic shock decreases, the correlation between the short-term rates of the two economies would decrease, while the correlation between their longer-term rates would increase. The correlation at the long end increases because, if the persistence of domestic shocks decreases, foreign shocks – the cause of variability of foreign rates – become a relatively more important source of variation for domestic long-term rates. Also note that domestic shocks, regardless of their persistence, are a source of variation for the domestic yield curve but not one for the foreign yield curve.[14] Thus, the larger the variance of domestic shocks relative to that of foreign shocks, the smaller (in absolute value) the correlation between domestic and foreign interest rates.

Figure 3 shows impulse responses of both domestic and foreign yield curves for two different parameterisations of the persistence of the foreign technology process: $\rho_a^* = 0.9$ and $\rho_a^* = 0.65$. As before, the less persistent the shock, the smaller the impact on both foreign and domestic longer-term rates and the larger the impact on both foreign and domestic short-term rates. Clearly, foreign shocks, unlike domestic ones, constitute a source of variation for both yield curves and therefore drive up the overall level of co-movement between interest rates of all maturities and currencies. But, for the model to be able to produce the upward-sloping pattern of correlations in Figure 1, foreign shocks have to be relatively more persistent than domestic shocks.

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[13] Note that the coefficient on the technology shock in Equation (8) is $(1 - \rho_a)$, so in the extreme case in which $\rho_a = 0$, the contemporaneous impact of the shock would be the highest possible.

[14] We confine our attention to unique rational expectations solutions in which the large economy is exogenous to the small one. See Jääskelä and Kulish (2007) for a discussion of non-uniqueness in this model.

**Figure 3: Impulse Responses to a Foreign Technology Shock**
High versus low persistence of technology shocks
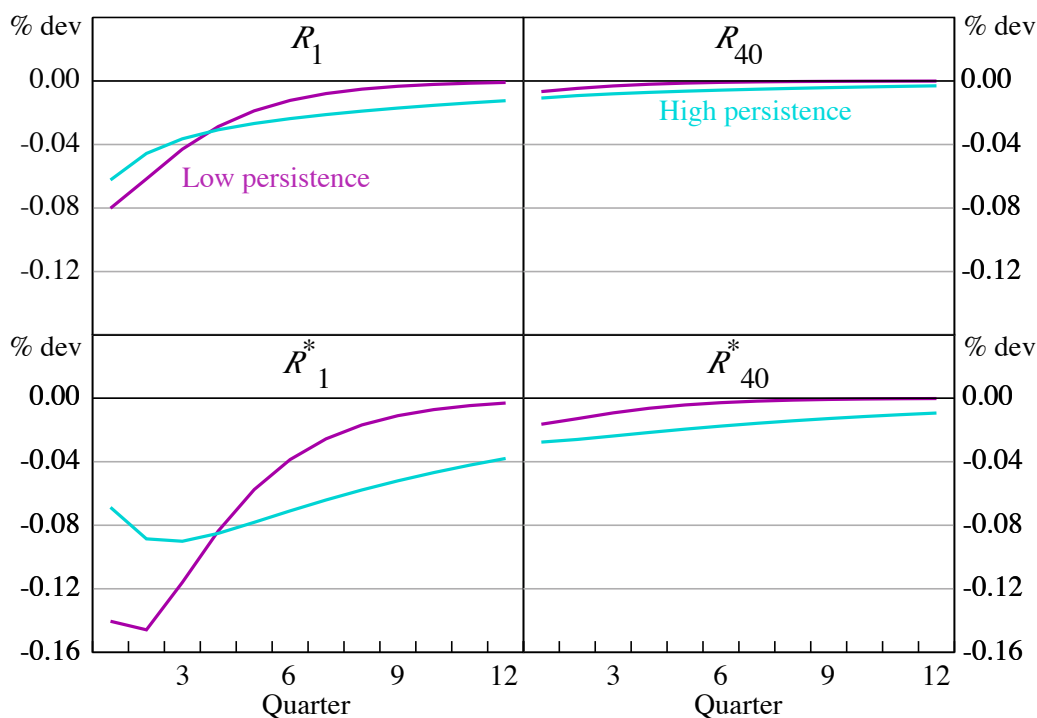


Note:   The size of the shock is one standard deviation.

## 5.   Unconditional Moments

Table 3 compares the theoretical standard deviations of output, inflation, the
nominal exchange rate and nominal interest rates, all computed at the posterior
mean values of the parameters with their empirical counterparts. The model
over-estimates the volatility of output and the short-term interest rate and under-

| | Table 3: Standard Deviations | | | |
|---|---|---|---|---|
| | Model (posterior mean) | 90 per cent confidence interval | | Data (1993:Q1–2007:Q2) |
| $y_t$ | 0.0178 | [0.0146 | 0.0465] | 0.0066 |
| $\pi_t$ | 0.0020 | [0.0017 | 0.0033] | 0.0055 |
| $\Delta e_t$ | 0.0116 | [0.0064 | 0.0182] | 0.0338 |
| $R_{1,t}$ | 0.0023 | [0.0019 | 0.0042] | 0.0017 |
| $R_{8,t}$ | 0.0016 | [0.0014 | 0.0031] | 0.0020 |
| $R_{20,t}$ | 0.0010 | [0.0008 | 0.0019] | 0.0019 |
| $R_{40,t}$ | 0.0006 | [0.0004 | 0.0010] | 0.0018 |

estimates the volatility of inflation, the change in the nominal exchange rate and long-term interest rates. The model's inability to capture the variability of interest rates across the yield curve echoes Shiller's (1979) finding of 'excess volatility'. While the volatility of the short-term interest rate in the model is 35 per cent larger than that of the data, the volatility of the 10-year interest rate in the data is 3 times that of the model.

Table 4 compares the actual correlations of foreign interest rates, domestic interest rates, output, the change in the nominal exchange rate and CPI inflation with their theoretical counterparts at the estimated posterior mean parameter values of Tables 1 and 2. The model has mixed success in confronting these dimensions of the data. On the one hand, it fails to capture the pro-cyclical behaviour of the yield curve and the correlations between changes in the nominal exchange rate with

### Table 4: Contemporaneous Correlations

| | Data (1993:Q1–2007:Q2) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $y_t$ | $\pi_t$ | $\Delta e_t$ | $R_{1,t}$ | $R_{20,t}$ | $R_{40,t}$ | $R^*_{1,t}$ | $R^*_{20,t}$ | $R^*_{40,t}$ |
| $y_t$ | 1.00 | | | | | | | | |
| $\pi_t$ | 0.14 | 1.00 | | | | | | | |
| $\Delta e_t$ | 0.19 | –0.01 | 1.00 | | | | | | |
| $R_{1,t}$ | 0.08 | 0.48 | –0.01 | 1.00 | | | | | |
| $R_{20,t}$ | 0.19 | 0.36 | 0.17 | 0.75 | 1.00 | | | | |
| $R_{40,t}$ | 0.16 | 0.34 | 0.18 | 0.69 | 0.99 | 1.00 | | | |
| $R^*_{1,t}$ | 0.00 | 0.40 | –0.37 | 0.56 | 0.34 | 0.28 | 1.00 | | |
| $R^*_{20,t}$ | 0.19 | 0.30 | –0.19 | 0.49 | 0.70 | 0.66 | 0.66 | 1.00 | |
| $R^*_{40,t}$ | 0.18 | 0.25 | –0.07 | 0.46 | 0.77 | 0.75 | 0.48 | 0.96 | 1.00 |
| | Model (posterior mean) | | | | | | | | |
| | $y_t$ | $\pi_t$ | $\Delta e_t$ | $R_{1,t}$ | $R_{20,t}$ | $R_{40,t}$ | $R^*_{1,t}$ | $R^*_{20,t}$ | $R^*_{40,t}$ |
| $y_t$ | 1.00 | | | | | | | | |
| $\pi_t$ | –0.13 | 1.00 | | | | | | | |
| $\Delta e_t$ | –0.01 | 0.66 | 1.00 | | | | | | |
| $R_{1,t}$ | –0.52 | 0.24 | 0.26 | 1.00 | | | | | |
| $R_{20,t}$ | –0.65 | 0.32 | 0.06 | 0.93 | 1.00 | | | | |
| $R_{40,t}$ | –0.66 | 0.32 | 0.06 | 0.93 | 1.00 | 1.00 | | | |
| $R^*_{1,t}$ | 0.07 | 0.19 | 0.08 | 0.65 | 0.57 | 0.56 | 1.00 | | |
| $R^*_{20,t}$ | 0.06 | 0.26 | 0.17 | 0.67 | 0.59 | 0.58 | 0.98 | 1.00 | |
| $R^*_{40,t}$ | 0.06 | 0.26 | 0.17 | 0.67 | 0.59 | 0.58 | 0.97 | 1.00 | 1.00 |

output, inflation and the short-term interest rate. On the other hand, the signs of the correlations between interest rates of various maturities and currencies, as well as the signs of the correlations between inflation and interest rates, are correctly predicted by the model.

The model's benchmark parameterisation (at the posterior mean) does not generate the upward-sloping pattern of correlations between interest rates of equivalent maturities shown in Figure 1. In particular, the model over-estimates the correlation at the short end of the yield curve and under-estimates it at the long end.

However, there is a set of plausible parameter values capable of producing the upward-sloping pattern of interest rates correlations in Figure 1. In light of the earlier discussion about the role that the persistence of shocks plays in accounting for the moments in Figure 1, we set each of the domestic autoregressive coefficients to the lower bound of their 90 per cent confidence intervals and the foreign autoregressive coefficients to their upper bounds.[15] Then we adjust the standard deviations of the *iid* disturbances as follows: $\sigma_{\varepsilon_a} = 0.0095$, $\sigma_{\varepsilon_g} = 0.013$, $\sigma_{\varepsilon_r} = 0.004$, $\sigma_{\varepsilon_a^*} = 0.013$ and $\sigma_{\varepsilon_g^*} = 0.02$.[16] Notice that this alternative calibration changes only the parameters that govern the exogenous processes, while the deeper model parameters are unchanged. Figure 4 shows the cross-correlations for the two parameterisations of the model. The top panel contains the correlations for the parameters at the posterior mean values of Tables 1 and 2 and the bottom

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[15] We can gauge the plausibility of these assumptions regarding the relative persistence of Australian and US shocks as follows. First, noting that our production function implies that output *per capita* is a function of labour hours *per capita* and productivity, we can back out implied productivity series for Australia and the US. We can then regress HP-filtered productivity in each country on a single lag of itself to produce an estimate of the persistence of the productivity disturbances in each country. The estimated persistence of the productivity disturbance in the US (equivalent to $\rho_a^*$ in Equation (6)) was 0.86 and the estimated persistence of the productivity disturbance in Australia (equivalent to $\rho_a$ in Equation (17)) was 0.67.

[16] The logic behind these changes is as follows. The larger variances of foreign persistent shocks increases the correlations of all rates, but it increases those at the long end of the yield curve relatively more. The larger variance of the domestic monetary policy shock reduces the correlations at the short end, while the larger variances of the persistent domestic shocks reduce the correlations in the middle of the term structure. These values are not unique as it is the relative size of the standard deviations that matter in determining the correlation between foreign and domestic rates.

panel contains the correlations for this alternative parameterisation. For these alternative plausible parameter values, the model matches the pattern of interest rate correlations at different points in the yield curve remarkably well.

**Figure 4: Cross-correlations with US Interest Rates**
1993:Q1–2007:Q2
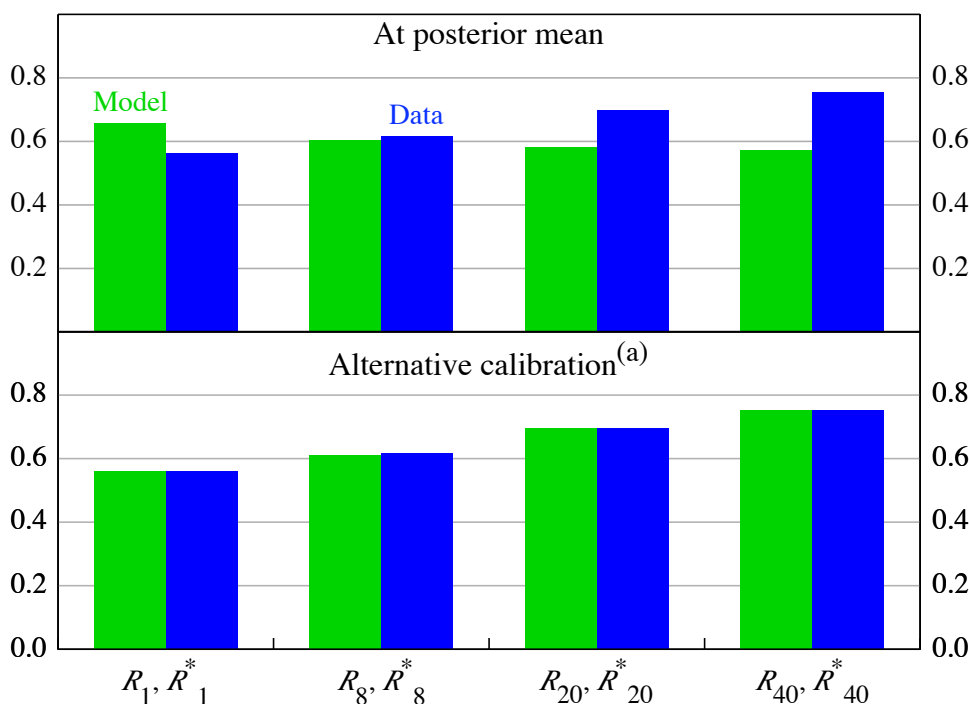


Note:    (a) All parameter values are at posterior means, except $\rho_g^* = 0.92$, $\rho_a^* = 0.95$, $\rho_g = 0.85$, $\rho_a = 0.88$, and standard deviations are as shown in the text.

Table 5 reproduces the contemporaneous correlations in Table 4 for this alternative parameterisation (that is, the one which matches the pattern of interest rate correlations in Figure 1). The most notable differences with respect to the moments computed at the posterior mean are between the correlations of inflation with interest rates and of inflation with output. These parameter values imply that domestic interest rates are somewhat less counter-cyclical, while inflation and output are positively correlated as in the data. However, the correlations between inflation and the yield curve, which are reasonably well-matched to the data in the bottom panel of Table 4, are smaller, though still of the correct sign.

**Table 5: Contemporaneous Correlations**

| | $y_t$ | $\pi_t$ | $\Delta e_t$ | $R_{1,t}$ | $R_{20,t}$ | $R_{40,t}$ | $R^*_{1,t}$ | $R^*_{20,t}$ | $R^*_{40,t}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | **Model (alternative calibration)** | | | | | |
| $y_t$ | 1.00 | | | | | | | | |
| $\pi_t$ | 0.06 | 1.00 | | | | | | | |
| $\Delta e_t$ | −0.03 | 0.59 | 1.00 | | | | | | |
| $R_{1,t}$ | −0.43 | −0.04 | 0.14 | 1.00 | | | | | |
| $R_{20,t}$ | −0.41 | 0.19 | 0.04 | 0.92 | 1.00 | | | | |
| $R_{40,t}$ | −0.37 | 0.19 | 0.05 | 0.90 | 1.00 | 1.00 | | | |
| $R^*_{1,t}$ | 0.15 | 0.11 | 0.04 | 0.56 | 0.68 | 0.73 | 1.00 | | |
| $R^*_{20,t}$ | 0.13 | 0.15 | 0.12 | 0.58 | 0.70 | 0.75 | 0.98 | 1.00 | |
| $R^*_{40,t}$ | 0.13 | 0.15 | 0.12 | 0.58 | 0.69 | 0.75 | 0.98 | 1.00 | 1.00 |

Thus far we have shown that, for plausible parameter values, the model can replicate the correlations between the two yield curves. Hence, in this respect, the theory is consistent with the pattern in Figure 1. However, we can go one step further by analysing not only whether the model *can* match the data, but whether such a match is likely to occur.[17]

To do this, we take 1 000 draws of the parameters from the posterior density and then calculate the moments of interest for each draw. The blue circles in Figure 5 show the resulting combinations of correlation coefficients for a selection of interest rates, inflation and output. Each plot also shows the 90 per cent confidence intervals computed from these 1 000 draws with dashed lines. The empirical correlations are shown as solid red lines.

Consider the top left panel of Figure 5. This shows the joint posterior distributions of the correlation of the Australian and US 3-month interest rate with the correlation of the Australian and US 5-year interest rate. Both of these empirical correlations lie within the model's 90 per cent confidence intervals. More importantly, the intersection of these empirical correlations lies within the mass of the distribution. This implies that, not only can the model independently match

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[17] We cannot determine this with regards to particular parameter values such as those used in the alternative calibration exercise above. To see why, note that while we kept all of the autoregressive coefficients on the disturbance terms within their 90 per cent confidence intervals, it may be that the joint posterior probability of observing that particular *combination* of parameter values is low.

**Figure 5: Joint Correlations between Interest Rates, Inflation and Output**
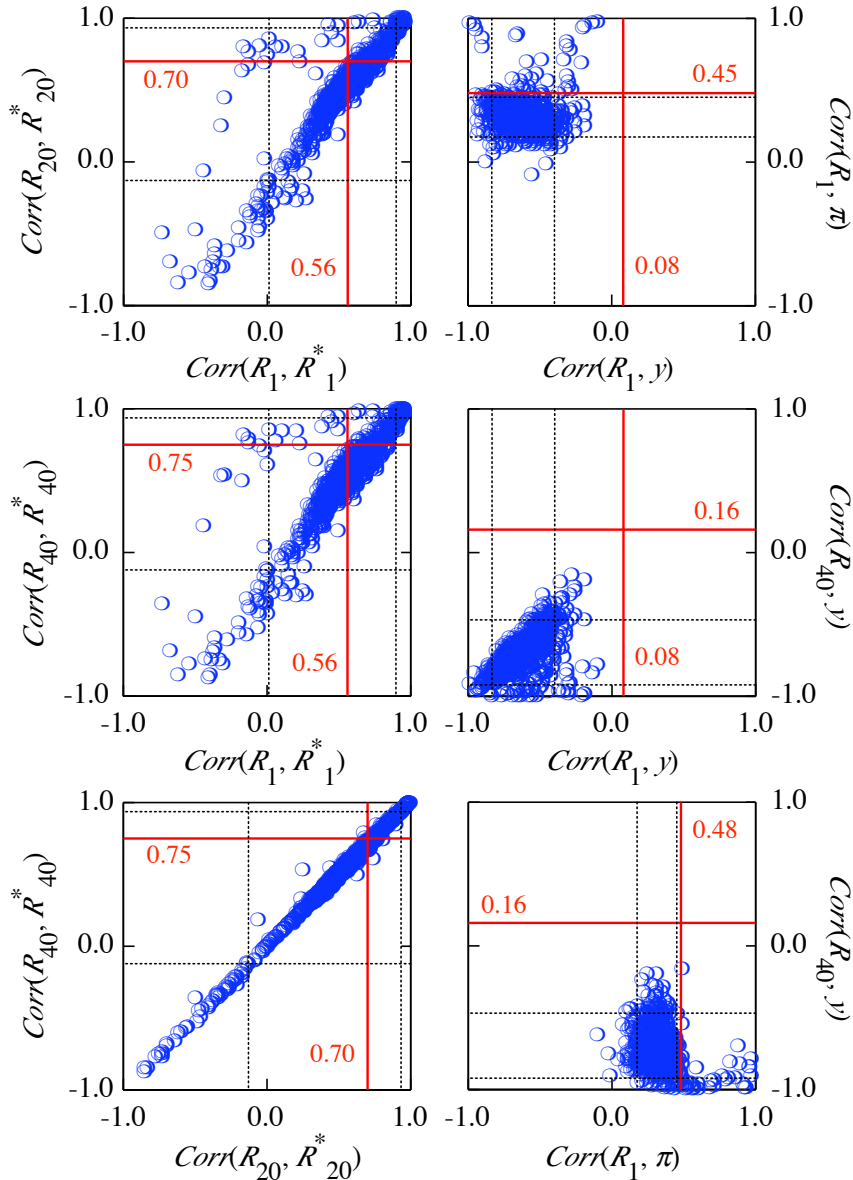


the correlation of Australian and US 3-month and 5-year interest rates, but also that it matches these moments jointly. In other words, the model does not need to sacrifice fit at the short end of the yield curve to match the co-movement at the long end of the yield curve. The results are similar in the remaining two panels on the left side of Figure 5, which show the joint correlations of the 3-month and 10-year, and 5-year and 10-year interest rates.

However, the model is not able to match some of the other correlations. Take, for example, the top-right panel of Figure 5, which shows the joint correlations of the Australian 3-month interest rate with inflation and output. In this case, the

empirical correlations lie outside the 90 per cent confidence intervals, suggesting that the model rarely matches these correlations individually. Moreover, the mass of the joint distribution is nowhere near the intersection of the two correlation coefficients. This implies that there is no set of parameter values that allows the model to match jointly these moments of the data. Clearly, the model seems unable to explain the contemporaneous co-movement between output, inflation and interest rates that we observe in the data.[18]

# 6.    Conclusion

Recently, long-term nominal interest rates in inflation-targeting small open economies, like Australia, Canada, New Zealand, Norway, Sweden and the UK, have moved very closely with those of the US. This observation has led many to the view that the long end of the domestic yield curve is determined abroad, and with it, to a concern that the monetary transmission mechanism of small open inflation-targeting economies may be weaker than it otherwise might be.

In this paper we have set up a fully micro-founded two-block small open economy model to study the co-movement of interest rates across the yield curve of different countries. We have shown that the reduced-form correlations at the short and long end of the domestic and foreign yield curves can be explained by a model in which the expectations hypothesis and uncovered interest rate parity hold. In particular, longer-term domestic interest rates in the model are always linked to the expected future path of the domestic short-term nominal interest rate. Nevertheless, if foreign shocks are more persistent than domestic shocks, then it makes sense that long-term nominal interest rates in the small and large economies are highly correlated, while the correlation between short-term nominal interest rates and long-term nominal interest rates is relatively low. In short, the reduced-form correlations do not imply that long-term nominal interest rates in small open economies are determined abroad.

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[18] Elsewhere in the literature, others have found that backward-looking features (such as habit persistence in consumption or indexation in price- or wage-setting behaviour), can help New Keynesian models match some properties of the data. For simplicity, we have used a model that abstracts from these features and left these extensions for further research.

# Appendix A: Data Descriptions and Sources

**Australian gross domestic product** *per capita***:** Seasonally adjusted quarterly real Australian GDP *per capita* (ABS Cat No 5206.0)

**Australian CPI inflation:** Percentage change in the seasonally adjusted quarterly Australian consumer price index (ABS Cat No 6401.0)

**Australian interest rates:** Quarterly average of monthly constant maturity rates (RBA)

**US gross domestic product** *per capita***:** Seasonally adjusted quarterly real US GDP *per capita* (Bureau of Economic Analysis, National Income and Product Accounts)

**US CPI inflation:** Percentage change in the seasonally adjusted quarterly average US consumer price index (Datastream code: USCP...F)

**US interest rates:** Quarterly average of monthly constant maturity rates (Federal Reserve Bank of St. Louis, Federal Reserve Economic Data)

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