Exchange Rate Policies at the Zero Lower Bound*

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Abstract

Recently, several economies with interest rates close to zero have received large capital inflows while their central banks accumulated large foreign reserves. Concurrently, significant deviations from covered interest parity have appeared. We show that, with limited international arbitrage, a central bank's pursuit of an exchange rate policy at the ZLB can explain these facts. We provide a measure of the costs associated with this policy and show they can be sizable. Changes in external conditions that increase capital inflows are detrimental, even when they are beneficial away from the ZLB. Negative nominal rates and capital controls can reduce the costs.

Keywords: Capital Flows, CIP Deviations, Currency Pegs, Foreign Exchange Interventions, International Reserves, Negative Interest Rates

JEL classification codes: F31, F32, F41

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1 Introduction

In the aftermath of the global financial crisis of 2008, many advanced economies experienced large capital inflows and appreciation pressures on their currencies. To avoid the resulting losses in competitiveness, central banks in these economies implemented policies geared toward containing these appreciations. With interest rates close to their zero lower bound, however, central banks were unable to weaken their currencies by reducing interest rates, and resorted to massive interventions in currency markets. At the same time, historically large deviations from covered interest rate parity have also emerged, with assets denominated in the currencies of these economies displaying a higher return. The case of Switzerland is emblematic in this respect. After experiencing a 35% appreciation of the Swiss franc between 2008 and 2010, the Swiss National Bank (SNB) responded by reducing interest rates to zero and increasing its holdings of foreign reserves up to 100% of GDP between 2011 and 2015. During the same period, the return on Swiss Franc denominated safe assets, converted into US dollars using forward rates, have been consistently higher than returns on comparable US dollar assets (CIP deviations of over 100 basis points). Eventually, in January 2015, the SNB let the exchange rate appreciate, triggering an intense policy debate about the desirability and effectiveness of these interventions.

The goal of this paper is to shed some light on this debate and in particular to address the following questions. First, how can a monetary authority depreciate its currency when it cannot lower interest rates any further? Second, are there costs associated with such policies? And finally, how do external factors, such as the degree of capital mobility, affect the answers to these questions?

To address these questions, we develop a simple monetary model that potentially features nominal interest rates at their zero lower bound (ZLB) and limits to international arbitrage. Our main result is that a central bank can indeed depreciate its currency at the ZLB. However, it needs to intervene in foreign exchange markets, accumulating foreign reserves while triggering capital inflows and deviations from interest rate parity. Such interventions result in losses that are proportional to the stock of reserves and to the deviations from interest rate parity. In addition, the more integrated an economy is to international markets, the larger the required interventions and the resulting losses. Our results help to rationalize the observed movements of gross private and official capital flows, and establish a link between the observed deviations from CIP to the exchange rate policies of advanced economies operating at the ZLB.

To gain some intuition for our results, consider a central bank that wants to achieve a temporarily depreciated nominal exchange rate target. Such a policy, given the domestic nominal interest rate, makes the domestic assets attractive to foreign investors. The increased demand for domestic assets will lead to an increase in domestic asset prices and a reduction in the domestic interest rate. If the market clearing nominal rate remains positive, the domestic and foreign

real rates equalize, and the central bank does not need to intervene to achieve its exchange rate target.

The situation is more complicated if, because of the ZLB, the domestic interest rate cannot fall enough. In this case, domestic assets will pay a higher return than foreign ones in equilibrium, resulting in a deviation from interest rate parity. In the absence of limits to international arbitrage, this would be unsustainable. However, when arbitrage is limited, a potentially large but finite capital inflow would result. To maintain equilibrium, the central bank needs to reverse the inflow by accumulating foreign assets. Therefore, foreign exchange interventions are the instrument through which the central bank achieves its exchange rate target when the ZLB constraint binds. These interventions are costly for the economy as a whole because the central bank takes the opposite side of profitable trades made by foreign investors. At the ZLB, therefore, the central bank faces a dilemma: it has to either give up on its exchange rate target, or intervene in foreign exchange markets and face losses.

Within our framework, deviations from parity that make domestic assets more attractive than foreign are associated with large reserve accumulation by central banks. In addition, these deviations should more likely emerge when interest rates are closer to their lower bound, as they are otherwise unnecessary. In section 4, we provide empirical support for these two key predictions. In particular, we first identify deviations in parity with observed deviations from CIP. We then show that sustained positive deviations in CIP for a currency (which appear mostly after the global financial crisis) are indeed associated, both across countries and over time, with the accumulation of foreign reserves by the central bank issuing that currency. In addition, these deviations arise mostly for currencies with nominal interest rates gravitating around zero.

The above evidence provides an alternative explanation to the "safe haven view" which argues that flight to safety has driven up private capital flows to the advanced economies like Switzerland. If flight to safety was the only driver of these flows, we should observe international investors earning a lower return on Swiss versus international assets. However, the evidence on CIP indicates that investors are earning a higher return on Swiss assets.

We show how data on CIP deviations and foreign reserves can be used to quantify the costs of the foreign exchange interventions. In particular, we found that these costs could have been substantial for the SNB, reaching around 0.8-1% of monthly GDP in January 2015—a result that arises from both the large size of the interventions and the large magnitude of the CIP deviations.

Having established that the costs of foreign exchange interventions can be large in practice, we next use our framework to understand the determinants of these costs. Factors that stimulate capital flows toward the small open economy (SOE), and that are typically thought to be beneficial, increase these costs. For example, a deepening in international capital market integration is beneficial when nominal interest rates are away from zero, but it undermines the efforts of the

central bank to weaken the exchange rate when nominal interest rates are at zero. In this latter case, an increase in the wealth available for international arbitrage translates into more inflows of capital toward the SOE because domestic assets are attractive under the policy pursued by the central bank. In order to sustain its policy, the monetary authority needs to purchase a larger amount of foreign assets, and this magnifies the costs of the interventions.

This property also implies that exchange rate policies at the ZLB are more vulnerable to changes in the beliefs of private agents. Consider, for example, the situation when the ZLB constraint does not bind. If private agents expected a higher appreciation rate of the domestic currency relative to the actual policy, then the central bank can lower the domestic interest rate, accumulate foreign assets, and profit from the mistaken beliefs. When the ZLB constraint binds, however, the central bank cannot decrease the nominal interest rate. Expecting further appreciation, foreign investors increase their demand for assets of the SOE. The central bank's only option for sustaining its exchange rate policy is to accumulate foreign assets.

Policies that hinder capital inflows reduce the costs of carrying out exchange rate interventions at the ZLB. We show that both quantity restrictions and taxes on capital inflows allow the central bank to achieve the exchange rate target without resorting to costly foreign exchange interventions. Our paper also offers a distinct rationale for implementing negative rates. Rather than stimulating aggregate demand management, the role of negative rates in our model is to reduce the arbitrage losses faced by the central bank. Our framework can thus rationalize the behavior of central banks in Switzerland, Denmark, and Sweden, which recently implemented negative nominal interest rates while facing severe appreciation pressures on their currencies.

A remaining question is why a central bank would choose to implement a costly exchange rate target. To this end, we introduce nominal rigidities into our basic model so that equilibrium output might be inefficiently low. At the ZLB, the central bank now faces a trade-off: it can weaken its currency in order to increase output, but this requires costly foreign exchange interventions. If the distortions generated by nominal rigidities are severe enough, the benefits of depreciating the exchange rate dominate its costs, and the central bank finds it optimal to intervene.

Related literature. Our paper is related to the literature on segmented capital markets and exchange rate determination. Backus and Kehoe (1989) derive general conditions under which sterilized official purchases of foreign assets do not affect equilibrium allocations and therefore are irrelevant for the exchange rate determination. An important assumption, in contrast with our paper, is that international arbitrage is perfect. Alvarez, Atkeson and Kehoe (2009) show how asset market segmentation within domestic markets can lead to variable risk premia in exchange rates, and real effects from domestic open market operations. In contrast, we study asset market segmentation within international markets, analyze deviations from covered interest parity and

the real effects from foreign open market operations.

More recently, Gabaix and Maggiori (2015) present a model where capital flows across countries are intermediated by global financial intermediaries that face constraints on their leverage, generating limited international arbitrage (as in ours). Cavallino (2016) and Fanelli and Straub (2015) study the optimality of foreign exchange rate interventions for economies that feature terms of trade externalities (e.g., Costinot, Lorenzoni and Werning 2014). Cavallino (2016) shows, in an open economy New Keynesian model, as in Gali and Monacelli (2005), that foreign exchange interventions are desirable in response to exogenous shifts in the demand for domestic bonds. Fanelli and Straub (2015) show that the deviations from interest parity induced by these interventions generate a cost in the inter-temporal resource constraint of the economy, which is proportional to the size of the deviation.[1] These papers emphasize the role of foreign exchange interventions as an instrument complementing interest rate policy. These papers do not study the restrictions that the ZLB imposes on policies, its implications for covered interest parity deviations, and their potential costs.[2]

The focus on exchange rate policies connects to the various generations of papers on fixed exchange rates and speculative attacks (see, among others, Krugman 1979, Obstfeld 1986, Lahiri and Vegh 2003, and Corsetti and Mackowiak 2006). In that literature, fiscal reasons lead the monetary authority to depreciate the currency when its reserves are depleted. Reserves are also central in our model, but for a different reason. When the nominal interest rate that is consistent with interest rate parity is negative, the accumulation of international reserves becomes necessary to eliminate the resulting excess demand for domestic assets and keep the exchange rate depreciated.

The failure of CIP for certain currencies since 2008 has been documented in detail by Du, Tepper and Verdelhan (2016), who argue that the inability of markets to arbitrage this out may be due to regulatory restrictions on banks implemented after the crisis.[3] We complement this empirical work by providing a theory that explains why CIP deviations appear for some currencies and not others, account for their sign, and account for their connections to foreign reserves accumulation and low interest rates.

The main mechanism at play in our model is related to the one highlighted in New Keynesian models with a ZLB, such as Eggertsson and Woodford (2003), Christiano, Eichenbaum and

[1] A related literature makes a similar point. Calvo (1991) first raised the warning about the potential costs of sterilizations by central banks in emerging markets. Subsequent papers have discussed and estimated the "quasi-fiscal" costs of these operations and similarly identified the costs of sterilization as a loss in the inter-temporal budget constraint of the government, proportional to the interest parity deviations and the size of the accumulated reserves (see Kletzer and Spiegel 2004, Devereux and Yetman 2014, Liu and Spiegel 2015, and references therein).

[2] A related paper, but in a closed economy setting, is Bassetto and Phelan (2015). They also explore how the limits of arbitrage interact with government policy while analyzing speculative runs on interest rate pegs.

[3] For other work on this topic, see Baba and Packer (2009), Ivashina, Scharfstein and Stein (2015), Borio, McCauley, McGuire and Sushko (2016), and Avdjiev, Du, Koch and Shin (2016).

Rebelo (2011), and Werning (2011). In both environments, there is "too much" desired saving in domestic asset markets. In New Keynesian models, excess savings in domestic asset markets is restored by declines in current output, and the cost associated with the ZLB is the recession itself. Here, the reduction in domestic private savings is achieved through the accumulation of foreign assets by the central bank, and the cost arises because the intervention entails a transfer of resources from domestic to foreign agents.

Like our paper, several recent contributions emphasize open economy dimensions of the ZLB (see, among others, Krugman 1998, Cook and Devereux 2013, 2016, Acharya and Bengui 2015, Fornaro 2015, Caballero, Farhi and Gourinchas 2015, Eggertsson, Mehrotra, Singh and Summers 2016, and Corsetti, Kuester and Müller 2016). Svensson (2003) and others have advocated interventions in foreign exchange markets at the ZLB, on the grounds that these interventions would trigger increases in inflation expectations and help achieve a depreciation. Caballero et al. (2015) show how unlimited promises by the government to exchange foreign assets, under perfected arbitrage, can coordinate expectations on a good equilibrium during a liquidity trap. Overall, a distinctive feature of our paper is an explicit modeling of exchange rate policies through reserve accumulation in an environment featuring limited arbitrage.

Our work is also related to the literature that studies unconventional policies when monetary policy is constrained. Correia, Farhi, Nicolini and Teles (2013) and Farhi, Gopinath and Itskhoki (2014) emphasize how schemes of taxes and subsidies can achieve the same outcomes that would prevail in the absence of constraints to monetary policy. Closer to us is the work of Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2012), which studies capital controls as second-best policy instruments to deal with capital flows under fixed exchange rate regimes. In our model with limited international arbitrage, foreign exchange interventions is an alternative, albeit costly, tool to achieve a depreciation at the ZLB.

The structure of the paper is as follows. Section 2 introduces the basic monetary setup and Section 3 studies the implementation of an exchange rate policy. Section 4 presents empirical evidence and measures the costs of foreign exchange rate interventions. 5 examines the determinants of the costs of intervention and Section 6 studies optimal exchange rate interventions. Section 7 concludes.

2 The model

We consider a two-period ($t = 1, 2$), two currency (domestic and foreign), one-good, deterministic SOE, inhabited by a continuum of domestic households, a monetary and a fiscal authority.[4] The

[4]In Appendix B we show how to interpret this two-period environment as an infinite horizon economy in which the exchange rate policy is stationary from date 2 onward. See also Amador et al. (2017), which introduces uncertainty.

SOE trades domestic assets with a continuum of foreign investors and foreign currency assets in the international financial market. We now proceed to describe the economy in detail.

2.1 Exchange rates and interest rates

We denote by s_t the exchange rate in period t (i.e., the amount of domestic currency needed to purchase one unit of foreign currency in period t). We normalize the foreign price level (i.e., the amount of foreign currency needed to buy one unit of the good) to 1 in each period, and we assume that the law of one price holds. As a result, s_t is also the domestic price level, (i.e., the units of domestic currency needed to purchase one unit of the consumption good).

There are three assets available. The first is a domestic nominal bond, which is traded both domestically and internationally. This bond is denominated in domestic currency and pays interest i. Domestic agents can also access the international financial markets and save in a bond denominated in foreign currency, paying interest i^*. The last asset is domestic currency.

While the domestic interest rate will be determined endogenously on the domestic credit market, the foreign rate is exogenously given, in accordance with the SOE assumption.

2.2 Domestic households

Domestic households value consumption of the final good and derive utility from holding real currency balances according to the following utility function:

$$U(c_1, c_2, m) = u(c_1) + \beta u(c_2) + h\left(\frac{m}{s_1}\right), \tag{1}$$

where $u(.)$ is a standard utility function, c_t is household consumption in period t, m is the nominal stock of money held by the household at the end of period 1, and $h(.)$ is an increasing and concave function, also displaying a satiation level $\bar{x}$ (i.e., there exists an $\bar{x}$ s.t. $h(x) = h(\bar{x})$, for all $x \geq \bar{x}$).

Domestic households are endowed with y_1 and y_2 units of the good in the two periods. The domestic households' budget constraints in periods 1 and 2 are

$$y_1 + T_1 = c_1 + \frac{m + a}{s_1} + f \tag{2}$$

$$y_2 + T_2 = c_2 - \frac{m + (1 + i)a}{s_2} - (1 + i^*)f \tag{3}$$

where a and f represent the domestic holdings of domestic and foreign bonds and T_i represents the (real) transfer from the fiscal authority to the households in period i.

We assume that households cannot borrow in international financial markets, $f \geq 0$. This

assumption guarantees that domestic households cannot take full advantage of arbitrage opportunities in capital markets.[5]

The domestic households' problem is thus

$$\max_{m,a,f,c_1,c_2} U(c_1, c_2, m)$$
$$\text{subject to equations (2), (3), and}$$
$$f \geq 0;\ m \geq 0.$$

2.3 Monetary authority

We impose for now that the monetary authority has a given nominal exchange rate objective, which we denote by the pair (s_1, s_2). In general, an exchange rate objective would arise from the desire to achieve a particular inflation or output target. In Section 6, we will study optimal exchange rate policies in a model with wage rigidities. For the moment, however, we simply assume that the monetary authority follows this objective and define equilibrium for the economy given (s_1, s_2). This allows us to transparently illustrate the implementation of an exchange rate and the costs that will arise at the ZLB.

In period 1, the monetary authority issues monetary liabilities M. It uses these resources to purchase foreign and domestic bonds by amounts F and A, respectively, as well as to make a transfer, τ_1, to the fiscal authority.

In the second period, the monetary authority uses the proceeds from these investments to redeem the outstanding monetary liabilities at the exchange rate s_2 and to make a final transfer to the fiscal authority, τ_2.[6]

Just as in the domestic households' case, we assume that the monetary authority cannot borrow in foreign bonds. As a result, the monetary authority faces the following constraints:

$$\frac{M}{s_1} = F + \frac{A}{s_1} + \tau_1$$
$$(1 + i^\star)F + (1 + i)\frac{A}{s_2}M = \frac{M}{s_2} + \tau_2;\ M \geq 0;\ F \geq 0$$

We will sometimes find it useful to analyze the case in which the central bank cannot receive transfers from the fiscal authority in the first period, and cannot issue domestic bonds: [Lack of Fiscal Support] The monetary authority does not receive a positive transfer from the fiscal

[5]The assumption that households cannot borrow from foreigners could be relaxed by assuming a finite borrowing limit, $f \geq -\kappa$, without altering our results.

[6]The assumption of withdrawing currency at the exchange rate s_2 is one of convenience given that our economy lasts only two periods. The analysis is preserved in an infinite horizon economy, assuming that the economy becomes stationary from period 2 onwards (see Appendix B).

authority in the first period and cannot issue interest-paying liabilities: $\tau_1 \geq 0$ and $A \geq 0$.

2.4 Fiscal authority

The fiscal authority makes transfers (T_1, T_2) to households in each period. It also receives transfers from the monetary authority, (τ_1, τ_2), in each period. The fiscal authority issues domestic nominal bonds B in period 1 and redeems them in period 2. The associated budget constraints are

$$\frac{B}{s_1} + \tau_1 = T_1 \tag{4}$$

$$\tau_2 = T_2 + (1+i)\frac{B}{s_2}. \tag{5}$$

Note that we assume that the fiscal authority neither borrows now invests in foreign markets.

Because public debt does not affect equilibrium outcomes due to Ricardian equivalence, we will treat the amount of bonds issued by the fiscal authority, B, as a fixed parameter.

2.5 Foreign investors and the international financial markets

A key assumption is that domestic and foreign markets are not fully integrated. We model this in the simplest possible fashion, that is, by assuming that the only foreign capital that can be invested in the domestic economy is in the hands of a continuum of foreign investors and is limited by a total amount $\bar{w}$, denominated in foreign currency.

We assume that the foreign investors only value consumption in the second period. The investors cannot borrow in any of the financial markets but can purchase both domestic and foreign assets.[7] In period 1, they decide how to allocate their wealth between foreign assets $f^\star$, domestic assets $a^\star$, and domestic currency $m^\star$, whereas in the second period they use the proceeds from their investments to finance their second-period consumption, c^*. The foreign investor's problem is

$$\max_{f^\star, a^\star, m^\star} c^* \quad \text{subject to} \tag{6}$$

$$\bar{w} = f^\star + \frac{a^\star + m^\star}{s_1} \tag{7}$$

$$c^\star = (1+i^\star)f^\star + (1+i)\frac{a^\star}{s_2} + \frac{m^\star}{s_2} \tag{8}$$

$$f^\star \geq 0,\ a^\star \geq 0 \text{ and } m^\star \geq 0. \tag{9}$$

[7] An alternative interpretation is that $\bar{w}$ already represents the total wealth available for investing in period 0, inclusive of any amount that could be borrowed.

Notice that unlike domestic investors, foreign investors do not enjoy a utility flow from holding domestic currency, so as expected, they will choose not to hold domestic currency when the domestic interest rate i is strictly positive.

2.6 Market clearing and the monetary equilibrium

Recall that our objective is to study whether a particular exchange rate policy can be attained as an equilibrium by the monetary authority and to compute the costs of pursuing such a policy. Toward this goal, we now define an equilibrium *for a given exchange rate policy* (s_1, s_2):

Definition 1. A monetary equilibrium, given an exchange rate policy (s_1, s_2), is a consumption profile for households, (c_1, c_2), and asset positions, (a, f, m); second-period consumption for investors, $c^\star$, and their asset positions $(a^\star, f^\star, m^\star)$; money supply, M; transfers from the fiscal to the monetary authority, (τ_1, τ_2); investments by the monetary authority, (A, F); transfers from the fiscal authority to the households, (T_1, T_2); and a domestic interest rate i, such that:

(i) the domestic households make consumption and portfolio choices to maximize utility, subject to their budget and borrowing constraints;

(ii) foreign investors make consumption and portfolio choices to maximize their utility, subject to their budget and borrowing constraints;

(iii) the purchases of assets by the monetary authority, its decision about the money supply and its transfers to the fiscal authority satisfy its budget constraints as well as $F \geq 0$;

(iv) the fiscal authority satisfies its budget constraints;

(v) and the domestic asset market clears for both money and bonds

$$\begin{aligned} m + m^\star &= M \\ a + a^\star + A &= B. \end{aligned}$$

It is helpful to write down, using the market-clearing conditions, the foreign asset position of the SOE in any equilibrium. Using the household budget constraint in the first period, as well as the monetary authority and fiscal authority budget constraints, we obtain the following equality, linking the trade deficit to the net foreign asset position:

$$\underbrace{c_1 - y_1}_{\text{trade deficit}} = \underbrace{\frac{m^\star + a^\star}{s_1}}_{\text{foreign liabilities}} - \underbrace{[f + F]}_{\text{foreign assets}}. \tag{10}$$

Similarly, using the budget constraint in the second period, we obtain the following equality:

$$c_2 - y_2 = (1 + i^\star)(f + F) - \frac{m^\star + (1 + i)a^\star}{s_2}. \tag{11}$$

3 Implementing an exchange rate policy

We now study how the SOE achieves an equilibrium given a policy for the exchange rate (s_1, s_2). We start in Section 3.1 by analyzing foreign reserve accumulation in a real version of the model. The upshot is that these interventions are costly.

We next turn in Section 3.2 to study the monetary equilibria given the exchange rate policy (s_1, s_2). The main result is that a monetary authority that wishes to sustain a given exchange rate policy *has to* engage in these costly interventions when the domestic nominal interest rate hits the ZLB constraint.

Importantly, throughout this Section, we take the exchange rate policy as given, and we focus on the *best implementation*: the one that maximizes the domestic household's welfare. We show that some exchange rate policies reduce welfare, even under the best implementation. Clearly, there are reasons why the Central Bank might choose these exchange rate policies in the first place, and one may worry that, in a more general model where the exchange rate is endogenous, the Central Bank might choose an implementation that is not the best. In Section 6, however, we show that this concern is not valid in our set up. That is, even though the Central Bank optimally chooses an exchange rate policy, it will carry it out using the best implementation described in this section.

3.1 A non-monetary economy

In order to explain in the most transparent way how the accumulation of foreign reserves affects the equilibrium, we begin by considering a version of the model without money, and where the central bank and the fiscal authority are just one single government agency.

We denote by r and $i^\star$ the rates of return on real domestic and foreign bonds. In this environment, the only action of the central bank consists of choosing the amount of foreign reserves F in the first period. Because foreign reserves (plus interest) are rebated back to households in the second period, an increase in F is equivalent to a shift of the domestic endowment from the first to the second period. It is convenient to define the households' endowment after the

monetary authority sets the level of foreign reserves,

$$\tilde{y}_1 = y_1 - F,$$
$$\tilde{y}_2 = y_2 + (1 + i^\star)F.$$

The domestic households maximize utility $u(c_1) + \beta u(c_2)$ subject to the following budget constraints:

$$c_1 = \tilde{y}_1 - f - a$$
$$c_2 = \tilde{y}_2 + (1 + i^\star)f + (1 + r)a,$$

where f and a represent their purchases of foreign and domestic assets, respectively. As in the monetary economy, we impose that they cannot borrow abroad, so $f \geq 0$.

The foreign investors are willing to invest up to the maximum of their wealth, $\bar{w}$, to maximize their returns. That is, their demand of domestic assets a^* satisfies

$$\max_{0 \leq a^\star \leq \bar{w}} a^\star(r - i^\star) = \bar{w}(r - i^\star), \tag{12}$$

where the last equality follows from the maximization in (6).

We assume that the government does not have a position in domestic bonds, so equilibrium in domestic financial markets requires $a^* + a = 0$.

To characterize an equilibrium, note that the first-order conditions of the household imply

$$u'(c_1) = (1 + r)\beta u'(c_2) \tag{13}$$
$$r \geq i^\star, \tag{14}$$

with $f = 0$ if the last inequality strictly holds.

The first condition is the standard Euler equation, while the second condition imposes that the real interest rate at home cannot be below the one abroad. If that were the case, the demand for domestic assets by households would be unbounded. Importantly, the converse is not true because we have assumed that households cannot borrow in foreign currency, $f \geq 0$, and because of the foreign investors' limited wealth.

We can eliminate a in the household's budget constraints and obtain an intertemporal resource constraint for the SOE

$$\tilde{y}_1 - c_1 + \frac{\tilde{y}_2 - c_2}{1 + r} - f\left[\frac{r - i^\star}{1 + r}\right] = 0.$$

From the household optimality condition stated above, we know that $f = 0$ if $r > i^\star$, so it then

follows that the intertemporal budget constraint simplifies to

$$\tilde{y}_1 - c_1 + \frac{\tilde{y}_2 - c_2}{1+r} = 0. \tag{15}$$

There is an additional equilibrium condition constraining the trade deficit that the SOE can run in the first period. Indeed, because $-a = a^\star \leq \bar{w}$, one must have that

$$c_1 = \tilde{y}_1 - f - a \leq \tilde{y}_1 + \bar{w}, \tag{16}$$

where the last inequality follows from the fact that $f \geq 0$. This expression tells us that the first-period consumption of the households and the foreign reserves of the monetary authority cannot exceed the endowment of the SOE and the wealth of foreigners $\bar{w}$. An equilibrium in the non-monetary economy (*non-monetary equilibrium* henceforth) is then fully characterized by conditions (13)-(16).

Before turning to the characterization of the equilibrium, it is useful to define the "first best" consumption allocation,

$$\begin{aligned}(c_1^{fb}, c_2^{fb}) \equiv \arg\max_{(c_1,c_2)} \{u(c_1) + \beta u(c_2)\} \\ \text{subject to:} \\ c_1 + \frac{c_2}{1+i^\star} = y_1 + \frac{y_2}{1+i^\star}.\end{aligned}$$

That is, (c_1^{fb}, c_2^{fb}) represents the equilibrium consumption allocation when the constraint on the first-period trade balance does not bind.

We then have the following proposition.

Proposition 1 (Characterization of non-monetary equilibria)**.** *Non-monetary equilibria given F are characterized as follows:*

(i) If $F \in [0, y_1 + \bar{w} - c_1^{fb}]$, there is a unique non-monetary equilibrium, in which $r = i^\star$, $c_1 = c_1^{fb}$, and $c_2 = c_2^{fb}$.

(ii) If $F \in (y_1 + \bar{w} - c_1^{fb}, y_1 + \bar{w})$, there is a unique non-monetary equilibrium, in which $c_1 = y_1 - F + \bar{w} < c_1^{fb}$, and c_2 solves

$$c_2 = y_2 - (1+r)\bar{w} + (1+i^\star)F,$$

with $r = \frac{u'(c_1)}{\beta u'(c_2)} - 1 > i^\star$.

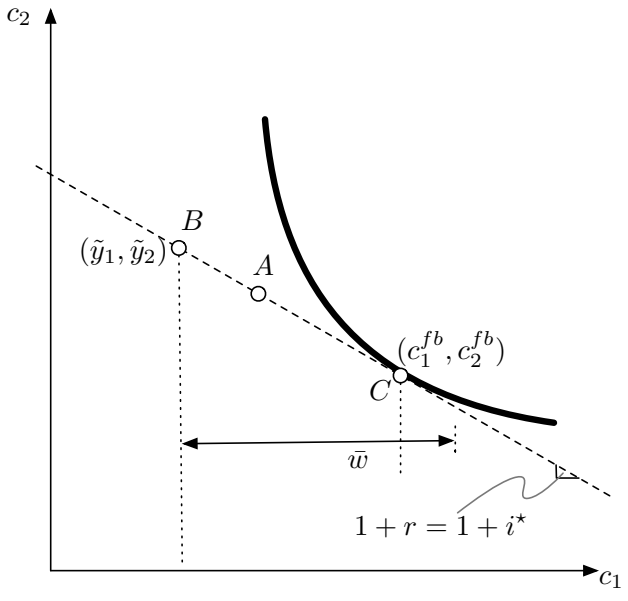
(iii) If $F > y_1 + \bar{w}$, then there is no non-monetary equilibrium.

Proposition 1 tells us that there are only two possible equilibrium outcomes in the real economy, depending on the accumulation of foreign reserves by the central bank. We illustrate these two cases in Figure 1.

Panel (a) in the figure illustrates the first case, when interventions do not move the consumption allocation away from the first best. Point A represents the original endowment of the representative household, while point B is the households' endowment after taking into account the foreign reserves accumulated by the central bank, F. Point C in the figure represents the first best consumption allocation, the one that would arise if the household could freely borrow and lend at the world interest rate $i^\star$. Importantly, point C is feasible for the SOE only if there is sufficient foreign wealth to cover the first-period trade balance, that is, if $y_1 - F - c_1^{fb} < \bar{w}$. This is precisely what happens in case (i) of Proposition 1.

Panel (b) in the figure illustrates the second case, when interventions increase domestic rates r. The accumulation of foreign reserves by the central bank is so large that there is not enough foreign wealth to finance the trade deficit that would arise with the first best consumption allocation. Therefore, the constraint (16) binds, and the consumption allocation is now at point D. Competition for these limited external resources results in a higher domestic real interest rate, which induces the household to consume less in the first period than what it would under the first best. In period 2, the household's consumption equals the endowment minus payments to foreigners, net of the proceeds from the accumulation of foreign reserves by the central bank. We can now characterize the effects that foreign reserves have on the non-monetary equilibrium.

(a) Neutral Interventions



(b) Interventions increasing r

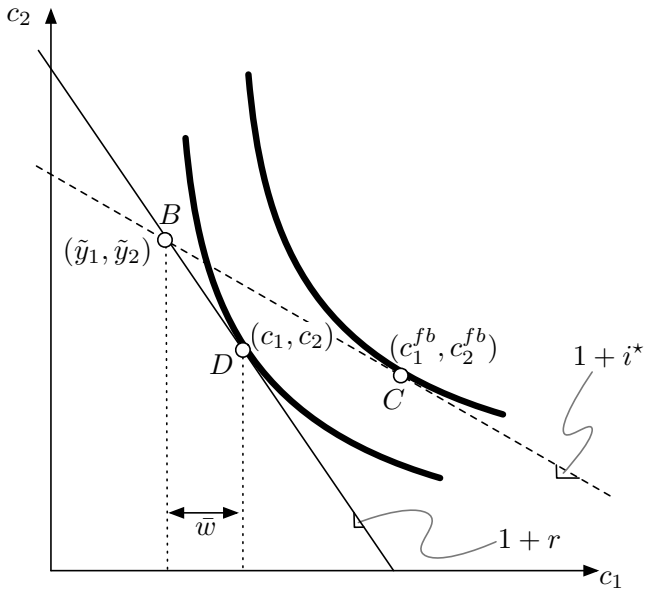


Figure 1: Reserves (F) and non-monetary equilibria

Corollary 1 (Impact of Foreign Reserves)**.** *In the non-monetary equilibrium given F, if $F \leq y_1 + \bar{w} - c_1^{fb}$, foreign reserves have no impact on the domestic interest rate ($r = i^\star$) nor on domestic welfare. If instead $F \in (y_1 + \bar{w} - c_1^{fb}, y_1 + \bar{w})$, the domestic real interest rate r is strictly increasing in F while the welfare of the domestic households is strictly decreasing in F.*

The increase in F reduces $\tilde{y}_1$ and increases $\tilde{y}_2$. When F is small (that is, $F < y_1 + \bar{w} - c_1^{fb}$), these interventions have no effects on the equilibrium because the private sector is able to undo the external position taken by the central bank: enough foreign wealth flows in from the rest of the world to equilibrate the domestic and foreign real rates. When F is large enough (that is, $F > y_1 + \bar{w} - c_1^{fb}$), however, the private sector cannot undo these interventions because the available foreign wealth is not large enough. In this case, the central bank interventions effectively make the SOE "credit constrained" and induce an increase in the domestic real interest rate.

To understand the consequences of this policy on welfare, let us rewrite the intertemporal resource constraint for the SOE, equation (15), as follows:

$$BC \equiv (1+r)(y_1 - c_1) + y_2 - c_2 - F(r - i^\star) = 0. \tag{17}$$

The term $F(r-i^\star)$ captures the losses associated with foreign reserve accumulation by the central bank. These losses appear because the central bank strategy consists of saving abroad, at a low return, while the economy is in effect borrowing at a higher one.

The welfare of the domestic household is given by the maximization of its utility subject to just (17), so we can read the effects on domestic welfare by understanding the effects of F on the budget constraint. Taking first-order conditions (assuming that the equilibrium r is differentiable), we obtain that the marginal effect of F, for $F \in (y_1 + \bar{w} - c_1^{fb}, y_1 + \bar{w})$, is

$$\frac{dBC}{dF} = -(r - i^\star) - \underbrace{(c_1 + F - y_1)}_{\bar{w}} \frac{dr}{dF} < 0.$$

From the above, we can see that there are two effects generated by an increase in F. First, one additional unit of reserves directly increases the budget constraint losses by the interest rate differential, $(r-i^\star) > 0$. But in addition, an increase in F also increases the equilibrium domestic real rate in this region, $dr/dF > 0$; and given that domestic households are net borrowers with respect to endowment point $\tilde{y}_1, \tilde{y}_2$, this induces a negative effect on the budget constraint.[8]

[8] As done in Fanelli and Straub (2015), another way of representing the losses faced by domestic households is to rewrite the intertemporal budget constraint solving out for foreign reserve holdings, using that $a^\star(r-i^\star) = \bar{w}(r-i^\star)$ together with the market-clearing condition, which leads to $y_1 - c_1 + \frac{y_2 - c_2}{1+i^\star} - \bar{w}\left[\frac{1+r}{1+i^\star} - 1\right] = 0$. The first two terms represent the standard intertemporal resource constraint for an economy that could borrow and save freely at rate $i^\star$. But there is an additional term that captures the resource losses faced by domestic households. Note that differently from Fanelli and Straub (2015), in our environment, these losses may arise even absent a central bank intervention if the foreign wealth is not large enough to take the economy to the first best allocation.

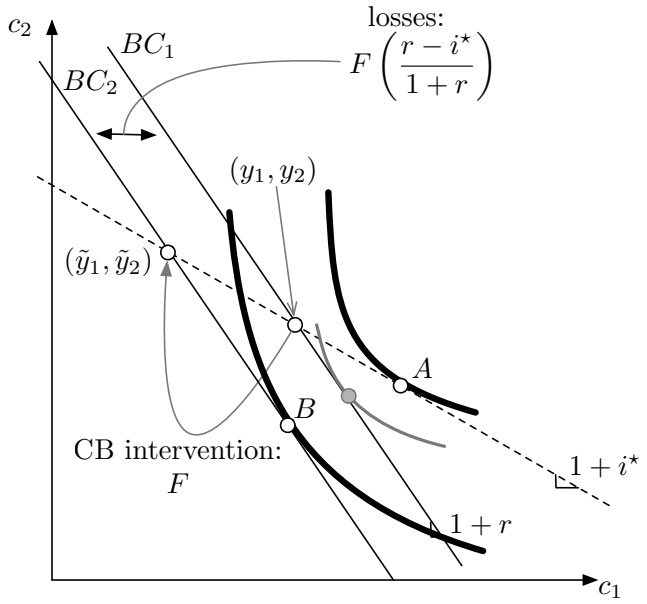


Figure 2: The welfare costs of central bank interventions

Figure 2 graphically illustrates these welfare losses. Without intervention, the equilibrium is denoted by point A, which in this case corresponds to the first best allocation. With a sufficiently large accumulation of foreign reserves, the central bank moves the economy from the income profile $(y_{1,}y_2)$ to $(\tilde{y}_1, \tilde{y}_2)$. In this example, the first best allocation cannot be attained because foreign wealth is not large enough. The intervention leads to an increase in the domestic real rate, which now exceeds $i^\star$, and a new consumption allocation which is now at point B.

We can see from Figure 2 the two effects associated with this intervention of the central bank. The movement from point A to the gray dot in the figure isolates the effect that operates through an increase in the domestic interest rate (which negatively affects the country, given that it is originally a borrower). The movement of the budget set from BC_1 to BC_2 captures the resource costs associated with the central bank interventions.

In this non-monetary world, central bank interventions are never desirable (at best, they have no effect). As a result, it is optimal in this environment for the central bank to always set $F = 0$. However, as we show below, in the monetary economy the central bank may be forced to engage in these costly interventions when its exchange rate objectives conflict with the ZLB constraint on nominal interest rates.

3.2 The implementation of an exchange rate policy

So far, we have seen that the central bank can sustain a wedge between domestic and foreign interest rates by accumulating foreign reserves. In the non-monetary economy, these interventions are never desirable because they entail welfare losses for the domestic household. We now return

to the monetary economy and show that in some cases, the central bank will need to engage in these costly interventions in order to sustain a given exchange rate objective (s_1, s_2).

From the household's optimization problem in any monetary equilibrium given (s_1, s_2), the following conditions must hold:

$$u'(c_1) = \beta(1+i)\frac{s_1}{s_2}u'(c_2) \tag{18}$$

$$(1+i)\frac{s_1}{s_2} \geq (1+i^\star) \tag{19}$$

$$h'\left(\frac{m}{s_1}\right) = \frac{i}{1+i}u'(c_1), \tag{20}$$

and

$$f = 0 \text{ if } (1+i)\frac{s_1}{s_2} > (1+i^\star). \tag{21}$$

Using the budget constraints of the households, together with the market-clearing condition in the money market, we get the following equation:

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - (f+F)\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] + \frac{i}{\frac{s_1}{s_2}(1+i)}\frac{m^\star}{s_2} = 0.$$

Note, however, that $f = 0$ if $1 - \frac{s_2(1+i^\star)}{s_1(1+i)} > 0$. Therefore, the above expression simplifies to

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - F\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] + \frac{i}{\frac{s_1}{s_2}(1+i)}\frac{m^\star}{s_2} = 0.$$

The first three terms in the above expression correspond to the intertemporal resource constraint for the non-monetary economy, equation (17), as the domestic real interest rate in this monetary economy equals $(1+i)\frac{s_1}{s_2}$. The last term, which is peculiar to the monetary economy, captures the potential seigniorage collected from foreigners. Because foreigners do not receive liquidity services from holding money balances, they set $m^\star = 0$, unless the domestic nominal interest rate is 0, implying that $im^\star = 0$. As a result, the intertemporal resource constraint further simplifies to

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - F\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] = 0. \tag{22}$$

The final equilibrium condition revolves around the central bank asset position. Recall from equation (10) that

$$c_1 - y_1 + F = \frac{m^\star + a^\star}{s_1} - f \leq \bar{w},$$

where the last inequality follows from $f \geq 0$ and $m^\star + a^\star \leq s_1\bar{w}$. In addition, if $\frac{1+i}{1+i^\star}\frac{s_1}{s_2} - 1 > 0$, then we know that $m^\star + a^\star = s_1\bar{w}$ and $f = 0$ (i.e., foreigners invest everything in the domestic

assets, and households do not invest in the foreign asset). Therefore, in any monetary equilibrium we must have

$$c_1 \leq y_1 - F + \bar{w}; \text{ with equality if } \frac{1+i}{1+i^\star}\frac{s_1}{s_2} - 1 > 0. \tag{23}$$

In other words, the foreign wealth must finance the trade deficit plus the reserve accumulation of the central bank.

It is then immediate to verify the following result.

Lemma 1. *An allocation (c_1, c_2, F, i, m) is part of a monetary equilibrium if and only if equations (18), (19), (20), (22), and (23) are satisfied.*

Note that equations (18), (19), (22), and (23) are the same equations that characterize a non-monetary equilibrium, equations (13), (14), (15), and (16), with $r = (1+i)\frac{s_1}{s_2} - 1$, $\tilde{y}_1 = y_1 - F$, and $\tilde{y}_2 = y_1 + (1+i^\star)F$. Thus, any monetary equilibrium must deliver an allocation consistent with a non-monetary equilibrium outcome. In addition, equation (20), imposes the restriction that the nominal interest rate must be nonnegative (i.e., the ZLB), a key restriction that will play an important role in what follows.

As a result, there is potentially a continuum of monetary equilibria given the exchange rate objective (s_1, s_2). Each equilibrium differs for the level of foreign reserves F accumulated by the central bank and potentially for the level of the nominal interest rate i, and the consumption allocation (c_1, c_2). For future reference, we denote by $\underline{r}$ the domestic real interest rate in the non-monetary equilibrium associated with $F = 0$. From Proposition 1 we know that $\underline{r} \geq i^\star$.

We can now study how the central bank can implement a given policy for the exchange rate (s_1, s_2) in the monetary economy. We will distinguish between two cases, depending on whether the zero lower bound constraint under the exchange rate policy binds.

3.2.1 Implementation when the ZLB constraint does not bind

We first consider the case in which $(1+\underline{r})\frac{s_2}{s_1} \geq 1$. We have the following result.

Proposition 2 (Implementation away from the ZLB)**.** *If $(1+\underline{r})\frac{s_2}{s_1} \geq 1$ then, for all $F \in [0, y_1 + \bar{w})$, the non-monetary equilibrium given F constitutes a monetary equilibrium outcome. The household's welfare is maximized in the equilibrium with $F = 0$.*

When the central bank does not accumulate foreign reserves, the real interest rate in the non-monetary economy is equal to $\underline{r}$. This real rate, along with the exchange rate policy (s_1, s_2), does not violate the ZLB constraint because, by assumption, the domestic nominal interest rate satisfies $i = (1+\underline{r})\frac{s_2}{s_1} - 1 \geq 0$. Therefore, the allocation $(c_1, c_2, \underline{r})$ for $F = 0$ constitutes a monetary equilibrium outcome. From Corollary 1, we know that the real interest rate is weakly

increasing in F. Thus, all non-monetary equilibria given F, for $F > 0$, will not violate the ZLB constraint on nominal interest rates and will also constitute a monetary equilibrium outcome.

Combining Propositions 1 and 2, we can see that the central bank can implement an exchange rate objective (s_1, s_2) in two distinct ways. First, the Central Bank could implement (s_1, s_2) by adjusting the nominal interest rate in order to guarantee that foreign investors are indifferent between holding domestic or foreign currency assets, that is, that the interest rate parity condition in (19) holds with equality. This is case (i) in Proposition 1. In this first scenario, the accumulation of foreign reserves does not affect the equilibrium outcomes (locally), thus mirroring the classic irrelevance result of Backus and Kehoe (1989).

There is, however, a second way to implement the exchange rate objective (s_1, s_2). This is described in case (ii) of Proposition 1: the central bank could achieve its desired exchange rate policy (s_1, s_2) by accumulating foreign reserves and setting a higher domestic interest rate than the one consistent with interest rate parity.

These results generalize the classic trilemma of international finance to an environment with limits to international arbitrage. The central bank can implement an exchange rate policy by adjusting the nominal interest rate and eliminate arbitrage opportunities in capital markets. In our environment, however, this is not the only option, and the central bank could implement an exchange rate policy (s_1, s_2) while maintaining some degree of monetary independence. To do so, it will need to engage in the costly interventions described in Section 3.1.

In the model described here, though, this trade-off is not operating: given an exchange rate policy (s_1, s_2), the optimal central bank policy would be to not accumulate foreign reserves (a result that follows directly from Proposition 1).[9] However, there is a sense in which this is a stronger result. If a central bank has no fiscal support in the first period, then it may not be feasible for the central bank to engineer a deviation from interest parity. We then have the following proposition.

Proposition 3. *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} \geq 1$ *and that Assumption 2.3 holds. In addition, suppose that* $c_1^{fb} - y_1 + \bar{x} \leq \bar{w}$. *Then all monetary equilibria attain the first best consumption allocation and the same domestic welfare, and the interest rate parity condition* (19) *holds with equality.*

Proposition 3 tells us that a central bank that cannot issue interest rate bearing liabilities and does not receive transfers from the fiscal authority is constrained in its ability to raise the domestic real rate above the foreign one. In order to understand why, suppose that the central bank tries to do so. This leads to an immediate inflow of foreign capital of size $\bar{w}$, which puts

[9] One issue is related to the value of money balances, a consideration that, of course, does not appear in the non-monetary equilibria analysis. However, the equilibrium with $F = 0$ is the monetary equilibrium with the lowest possible nominal interest rate, given the exchange rate policy. And thus, it ends up maximizing total households' utility, inclusive of money balances. Indeed, there is no additional value of raising the domestic interest rate beyond what is necessary to support the exchange rate policy under no reserve accumulation.

downward pressure on the domestic interest rate. To keep the interest rate from falling, the central bank must purchase a large amount of the inflow and accumulate foreign reserves. But the purchasing power of the central bank is limited by its balance sheet because, by Assumption 2.3, the central bank's real liabilities are bounded by the satiation point of money $\bar{x}$. If the external wealth is sufficiently high, the central bank will not be able to sustain a deviation from interest rate parity, and the domestic interest rate will need to adjust. Therefore, it could be challenging for the central bank to gain monetary independence while committing to an exchange rate policy when nominal interest rates are positive.

3.2.2 Implementation when the ZLB constraint binds

The second case we analyze is when $(1+\underline{r})\frac{s_2}{s_1} < 1$. In this case, the non-monetary equilibrium with $F = 0$ cannot arise as a monetary equilibrium outcome because it would lead to a domestic nominal interest rate that violates the ZLB constraint. As a result, the monetary equilibrium will necessarily feature a deviation from interest rate parity, and the domestic real interest rate will need to lie strictly above the foreign one.[10]

So, for there to be a monetary equilibrium, the central bank will need to intervene and accumulate reserves of a magnitude sufficient to increase the real interest rate above the level consistent with interest parity. Let $\bar{r}$ be the highest possible real interest rate in the non-monetary economy (that is, the interest rate associated with the maximum possible intervention). We then have the following result.

Proposition 4 (Implementation at the ZLB)**.** *Suppose that* $1+\underline{r} < \frac{s_1}{s_2} < 1+\bar{r}$*. Then there exists an* $\underline{F} > 0$ *such that for all* $F \in [\underline{F}, y_1 + \bar{w})$*, the non-monetary equilibrium given* F*,* (c_1, c_2, r)*, constitutes a monetary equilibrium outcome. In all these monetary equilibria, the interest rate parity condition* (19) *holds as a strict inequality. The household's welfare is maximized in the equilibrium with* $F = \underline{F}$*.*

Proposition 4 tells us that the central bank is able to sustain the exchange rate policy. However, because of the ZLB, it *has to* engage in the costly interventions described in Section 3.1.

It follows, however, that given an exchange rate policy (s_1, s_2), the optimal central bank policy is to accumulate the minimum amount of foreign reserves necessary to deliver a monetary equilibrium. As a result, the best monetary equilibrium in this case will feature $i = 0$ and

[10] This follows immediately from the following set of inequalities:

$$(1+i)\frac{s_1}{s_2} - 1 \geq \frac{s_1}{s_2} - 1 > \underline{r} \geq i^\star,$$

where the first term is the domestic real rate, the first inequality follows from the ZLB constraint, the second defines the case of interest, and the last one is the restriction that appears in any non-monetary equilibrium.

a violation of the interest parity condition. Differently from the situation in which the ZLB constraint is slack, the central bank can always sustain these exchange rate policies, even without the support of the fiscal authority. This is summarized in the following proposition.

Proposition 5. *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} < 1$ *and that Assumption 2.3 holds. In addition, suppose that* $c_1^{fb} - y_1 + \bar{x} \leq \bar{w}$. *Then the unique monetary equilibrium outcome is the one where* $F = \underline{F}$ *and* $i = 0$.

Proposition 5 tells us that a central bank without fiscal support is able to raise the domestic real rate above the foreign one as long as the nominal interest rate remains at zero. In this case, by sustaining the exchange rate path, the central bank is forced to issue currency to purchase the foreign assets necessary to maintain the domestic rate above the foreign one. The main difference from the case analyzed previously is that now, because of the zero nominal rate, bonds and money are perfect substitutes. Thus, the central bank can expand its balance sheet without limits.

To recap, the analysis above tells us that interest parity deviations in a world with limited arbitrage are sustained by foreign reserve accumulation. We have also shown that central banks need to intervene only when their interest rates are at their ZLB, as interventions are otherwise unnecessary. In the next section, we provide empirical support for these two key predictions of the framework.

4 Empirical evidence

In our framework, the central bank, by accumulating foreign assets, can increase the domestic real interest rate relative to the world real interest rate i.e., open a gap in the interest parity condition. Doing so creates an arbitrage opportunity for foreign investors, which gain at the expense of the central bank, thus these interventions are costly for the domestic economy. Nevertheless, central banks need to carry those interventions out, if they want to achieve an exchange rate objective while their interest rate is constrained by the ZLB. This logic implies that when we observe domestic assets paying a higher return than foreign ones (where returns are made comparable using forward rates) we should also observe large foreign reserve accumulation by the domestic central bank, because the domestic central bank take long positions in the assets with the lower return. The logic also implies that we should observe these gaps when interest rates that are close to zero, because otherwise they are unncessary.

In subsections 4.1 and 4.2 we provide evidence that positive gaps in interest parity (i.e. domestic rates higher than foreign) are indeed associated, both across countries and over time, with an accumulation of reserves by domestic central banks and that these positive deviations arise mostly for currencies with nominal interest rates gravitating around zero. This evidence

suggests that the pursuit of an exchange rate policy at the ZLB might be an important factor in explaining observed deviations from interest parity. In the second part of this section (subsection 4.3), we show how data on deviations from interest parity plus data on reserve accumulation can be combined to quantify the costs of achieving a certain exchange rate objective. In order to make this point concrete, we provide an estimate of the costs of the extensive foreign exchange interventions by the Swiss National Bank (SNB) during the 2010-2015 period.

4.1 Interest parity, foreign reserves, and the ZLB: Recent evidence

As we detail below, we measure deviations between the domestic and world real interest rates using gaps in the covered interest rate parity condition. Because of that, we restrict the analysis to a set of countries for which data on currency forwards are of sufficiently good quality. Our sample borrows mostly from Du et al. (2016) and includes Switzerland, Japan, Denmark, Sweden, Canada, the United Kingdom, Australia, and New Zealand over the 2000-2015 period.

We obtain yearly data on foreign reserve holdings from the *International Financial Statistics* of the IMF,[11] and we scale it by annual gross domestic product obtained from the OECD *National Accounts*. Both foreign reserves and gross domestic product are expressed in U.S. dollars at current prices.

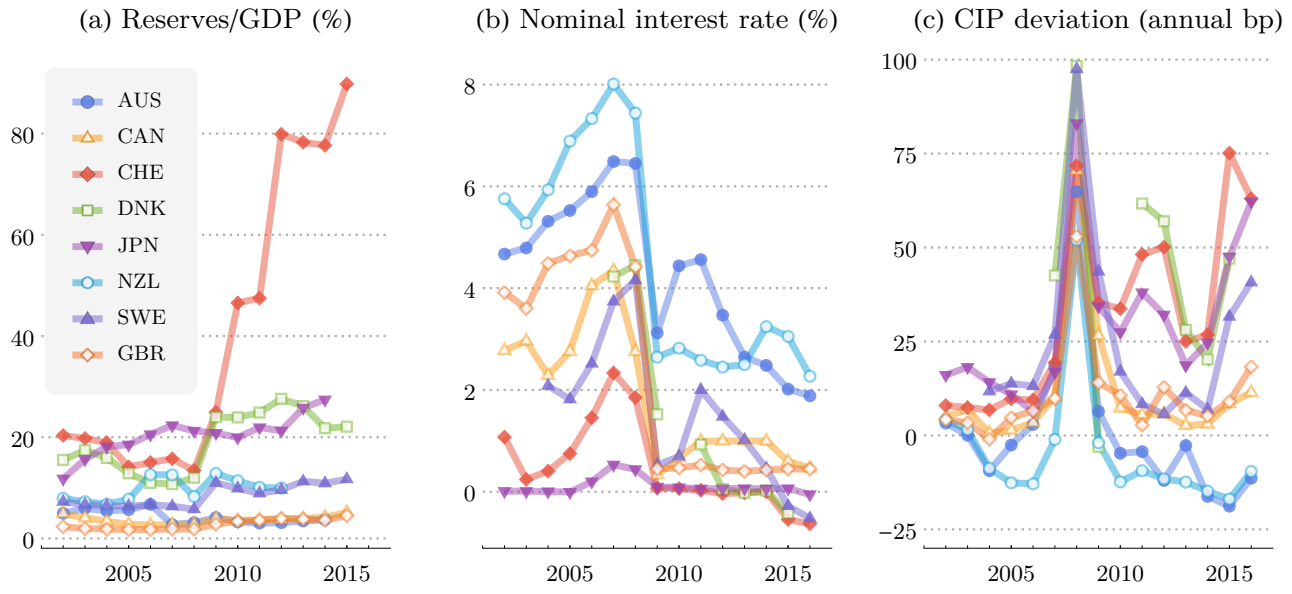


Figure 3: Foreign reserves, interest rates, and CIP gaps

The measurement of deviations between the domestic and world real interest rates is more involved. In our deterministic model, we could proxy these gaps either with deviations from

[11] Total reserves comprise holdings of monetary gold, special drawing rights, reserves of IMF members held by the IMF, and holdings of foreign exchange under the control of monetary authorities. The gold component of these reserves is valued at year-end (December 31) London prices.

the covered interest rate parity (CIP) conditions or using deviations from the uncovered one (UIP). When adding uncertainty, however, the two indicators differ. In Amador et al. (2017) we show that the correct measure to compute the costs of foreign exchange interventions in an environment with uncertainty is the CIP rather than the UIP deviation.[12] For this reason, we use deviations from CIP. Letting $i^{\$}_{t,t+n}$ denote the nominal interest rate in U.S. dollars between time t and time $t+n$, $i^{j}_{t,t+n}$ the corresponding interest rate in currency j, $s^{j,\$}_{t}$ the spot exchange rate of currency j per U.S. dollar, and $f^{j,\$}_{t,t+n}$ the n-periods ahead associated forward contract, we can express deviations from the CIP condition as

$$\text{cip gaps}^{j,\$}_{t,t+n} = i^{j}_{t,t+n} - i^{\$}_{t,t+n} + \frac{1}{n}[\log(s^{j,\$}_{t}) - \log(f^{j,\$}_{t,t+n})].$$

A positive value for this indicator is equivalent, in our model, to a positive gap between the real interest rate in country j and the world real interest rate.

We calculate deviations from the CIP condition at a three-months horizon between major currencies and the U.S. dollar for the period 2000-2015. We map $i^{j}_{t,t+n}$ to the interest rate on an overnight indexed swap (OIS) of three-month duration in currency j, while $i^{\$}_{t,t+n}$ is the respective OIS rate in U.S. dollars with the same duration.[13] The variable $f^{j,\$}_{t,t+n}$ is the three-months forward rate between currency j and the U.S. dollar. All these data are obtained at a daily frequency from Bloomberg, and we use the midpoint between the bid and ask quotes.

Figure 3 plots the three time series (aggregated at a yearly frequency) for the countries considered. The figure shows interesting patterns, both over time and across countries. First, there has been a sizable increase in the ratio of foreign reserve to GDP for advanced economies over this period, which on average went from 9% in 2001 to 25.4% in 2015. This trend was more pronounced for certain economies than for others: central banks in Switzerland, Japan, and Denmark substantially increased their foreign asset positions during the sample, whereas central banks in Australia, Canada and the United Kingdom did not. Second, nominal interest rates have been declining over time: by the end of the period, we have a group of countries with zero or even negative nominal interest rates (Denmark, Switzerland, Japan, and Sweden) and countries with clearly positive nominal rates (Australia, and New Zealand). The third panel in the figure reports yearly average CIP gaps in our sample. Prior to 2007, CIP deviations were on average small and within 20bp from 0 for all the economies, a well-established fact in international finance. During the global financial crisis of 2007-2009, we have observed major deviations from covered interest rate parity for all the currencies in our sample. Interestingly, these deviations

[12] Basically, UIP deviations also contain compensation that lenders may require for holding currency risk, rather than the riskless arbitrage opportunity.

[13] Our results would not change significantly if we were to use the LIBOR rather than the OIS rate when computing deviations from CIP. See Du et al. (2016) for a comparison of CIP deviations computed using the LIBOR and OIS rate.

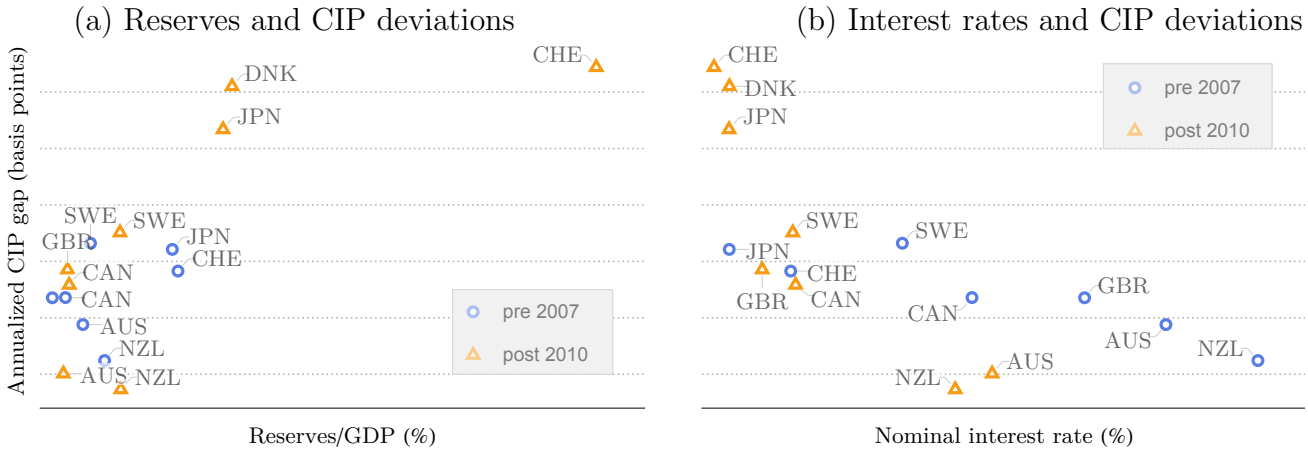


Figure 4: Relation between CIP gaps, reserves, and interest rates

have persisted even after the financial crisis for a group of countries, most notably Switzerland, Denmark, Japan, and Sweden.

We exploit the variation in these three series, both across countries and over time, to verify two main predictions of the model. Because the deviations from CIP during the 2007- 2009 period were extreme and, as discussed by Baba and Packer (2009), Ivashina, Scharfstein and Stein (2015), Du et al. (2016), and Borio, McCauley, McGuire and Sushko (2016), were due to unusually tight limits to arbitrage during the crisis, we exclude this period and split the time dimension of our sample in two subsets: before (2002-2006) and after (2010-2015) the financial crisis. The left panel of Figure 4 plots the average foreign reserve holdings to GDP ratio against the average CIP deviations in these two sub-samples. The plot shows a positive relationship, both across countries and over time, between the level of foreign reserves and the deviations from the CIP (with the appropriate sign). This empirical finding, which to the best of our knowledge has not been previously noted in the literature, is consistent with the mechanism at the heart of our model, whereby the monetary authority is able to sustain a positive gap between the domestic and world real interest rate by accumulating a sufficiently large position in foreign assets. Moreover, this finding helps us understand not only the deviations from CIP per se (any model with limited arbitrage can generate those), but their specific sign. The right panel of Figure 4 plots the nominal interest rate against the average CIP deviations in these two subsamples. We can observe that the CIP gaps are positive for countries-time periods characterized by low nominal interest rates, while they tend to be small when nominal interest rates are positive. This negative relation between CIP gaps and nominal interest rates, also documented by Du et al. (2016), lends support to our result that central banks find it optimal to engage in large foreign exchange interventions when the ZLB constraint on nominal interest rates binds.

4.2 Interest parity, reserves and the ZLB: Switzerland in the 1970s

The experience of Switzerland in the late 1970s provides another interesting episode of exchange rate policy in an environment with very low interest rates.[14] Panel (a) of Figure 5 shows the monthly time series for the Swiss franc against the U.S. dollar for the period 1977-1979, and it shows that the Swiss franc had been steadily appreciating against the U.S. dollar, just as it did in the aftermath of the 2007-2009 crisis.[15] In an effort to prevent the appreciation, the SNB initially reduced the domestic rate, which by the end of 1978 reached levels close to zero (see the shaded area in panel (b) of the figure). At this point, just as it did in 2011, the SNB announced a temporary floor between the Swiss franc and the Deutsche mark, and, to maintain the floor, it engaged in large foreign exchange interventions. Panel (c) of Figure 5 shows monthly times series of foreign reserves (excluding gold, as a fraction of trend GDP), together with deviations from CIP, calculated in exactly the same way as in the previous section.[16]

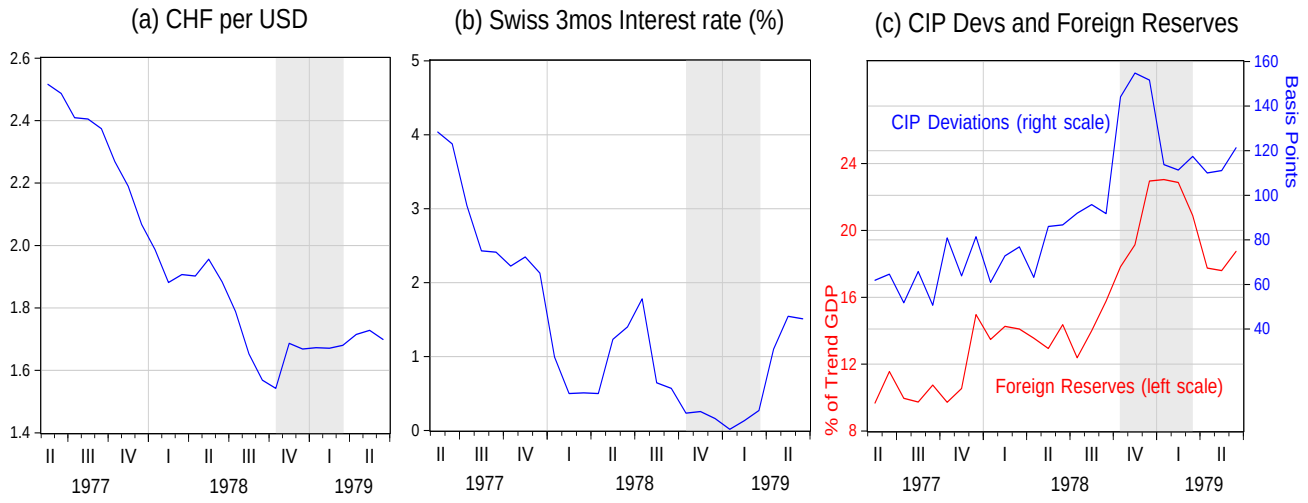


Figure 5: Foreign reserves, interest rates and CIP deviations: Switzerland 1977-1979

Note: The shaded areas represent the months in which the Swiss interest rate was below 0.5%.

The panel shows that the ratio of foreign reserves to GDP increased by over 10% of GDP, and around the same time, the deviations from CIP increased by over 50 basis points. By mid-1979, the international macroeconomic conditions changed substantially, and the SNB was able to avoid appreciation of the currency while maintaining a positive interest rate. As a consequence, both the level of foreign reserves and the deviations from covered parity abated. We should be clear

[14] For an informal description of the macroeconomic environment in Switzerland at the time, see, for example, Jones (2011).

[15] Over the period 1977-1978, the Swiss franc also appreciated 30% against the deutsche mark.

[16] The only difference is our data source, since Bloomberg data are not available for this early period. Three-months nominal interest rates are interbank rates from the OECD *Main Economic Indicators*, and daily spot and three-months forward rates are provided by the SNB.

that we do not claim that CIP deviations only occur with interest rates that are close to zero. During the pre-financial globalization period, with extensive capital controls, deviations from CIP were routinely documented, even between currencies with positive interest rates. The objective of this section is to show that this particular episode of CIP deviations seems closely connected to the reserve accumulation conducted to avoid currency appreciation in an environment with low interest rates.

4.3 The costs of the SNB foreign exchange interventions

In this subsection, we use the Swiss experience to obtain guidance on the size of the potential losses faced by central banks. Starting from 2010, the SNB has intervened massively in foreign exchange markets, either to defend an explicit target for the exchange rate[17] or, more informally, to fight appreciation pressures on the Swiss franc. Our theory provides a simple expression to measure the costs associated with these interventions:

$$\text{losses}_t = \left[\frac{(1+i_t)}{(1+i_t^\star)}\frac{s_t}{s_{t+1}} - 1\right] \times F_t. \tag{24}$$

We can use our data on CIP deviations (on a horizon of three months) and on foreign reserves to provide an approximation for the costs of these foreign exchange interventions.[18]

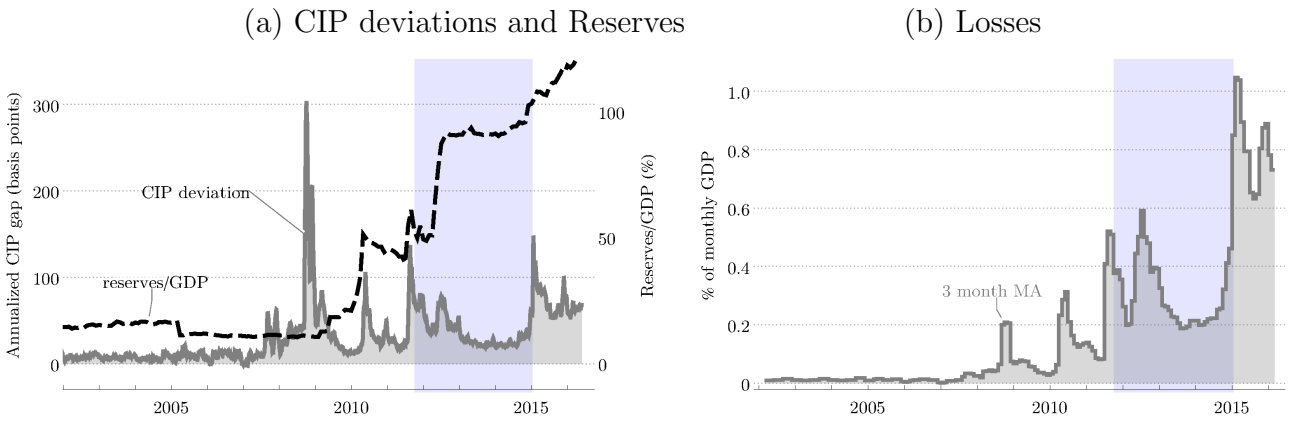


Figure 6: Foreign reserves, CIP deviations, and losses

Panel (a) of Figure 6, we report the monthly three-month CIP deviations between the Swiss franc and the U.S. dollar, along with a monthly series for the stock of foreign reserves as a fraction of Swiss GDP. This plot confirms that the positive relation between foreign reserves and CIP deviations that we documented earlier also holds at a much higher frequency: after the U.S. financial crisis, spikes in the CIP gaps are associated with massive interventions of the SNB.

[17] Between 2011 and 2015, the SNB successfully kept a floor of 1.2 Swiss francs per euro.
[18] See Appendix E.

Panel (b) of Figure 6, we report our corresponding measure of the monthly losses as a fraction of monthly Swiss GDP. The lightly shaded area represents the period in which the SNB maintained an official floor on the franc. As can be seen, throughout this period, the losses were significant. They reach their highest point (around 1% of monthly GDP) around January 2015, the month when the SNB decided to abandon the currency floor vis-à-vis the euro.

5 Capital inflows and foreign exchange interventions

So far we have seen that a central bank that wishes to implement an exchange rate path when nominal interest rates are at zero needs to accumulate foreign reserves. We have also seen that these interventions are costly from the perspective of the SOE. In this section, we study those costs in more detail and discuss how they are affected by changes in the underlying economic environment. The main result of this section will be that factors that raise capital inflows toward the domestic economy increase the costs that the central bank incurs when implementing an exchange rate policy at the ZLB.

5.1 Changes in foreign wealth and the foreign interest rate

We start by considering the effects of increases in foreign wealth, $\bar{w}$, and of reductions in the foreign interest rate $i^\star$ (when the country is a net borrower). Before moving to the ZLB environment, let us first argue that both of these changes unambiguously improve welfare when the ZLB constraint does not bind, that is, when $(1+\underline{r})\frac{s_2}{s_1} \geq 1$.

To see this, note that, away from the ZLB, the best monetary equilibrium given an exchange rate policy (s_1, s_2) sets $F = 0$. As a result, the welfare effects can be read by studying the effects of such changes on the budget constraint of domestic households,

$$y_1 - c_1 + \frac{y_2 - c_2}{1+r} \geq 0,$$

where r is the domestic equilibrium real rate. So, whether increases in $\bar{w}$ or decreases in $i^\star$ are welfare improving or not depend on the effect of these changes on the equilibrium domestic real interest rate. The following Lemma helps to clarify the effects.

Lemma 2. *Consider the non-monetary equilibrium given $F = 0$. Then,*

(i) if $c_1^{fb} > y_1 + \bar{w}$, a marginal increase in $\bar{w}$ strictly decreases the domestic real interest rate, while a marginal decrease in $i^\star$ has no effect.

(ii) if $c_1^{fb} < y_1 + \bar{w}$, a marginal increase in $\bar{w}$ has no effect on the domestic interest rate, while a marginal decrease in $i^\star$ strictly decreases it.

The results of this lemma follow from our characterization of the non-monetary equilibrium. When $F = 0$, if $c_1^{fb} < y_1 + \bar{w}$, then the economy achieves the first best consumption outcome, and the domestic real interest rate will equal $i^\star$. As a result, an increase in $\bar{w}$ would have no effect on the real interest rate in this region, but a reduction in $i^\star$ will reduce the domestic rate one to one, explaining part (ii) of the lemma. However, if $c_1^{fb} > y_1 + \bar{w}$, then the economy is constrained, and the domestic interest rate is the unique value r that solves the following equation:

$$(1+r) = \frac{u'(y_1 + \bar{w})}{\beta u'(y_2 - (1+r)\bar{w})}.$$

In this case, changes in the foreign interest rate have no effects on the equilibrium r. An increase in $\bar{w}$, however, strictly reduces r, a natural outcome of the increase in competition from foreign investors.[19]

It follows that an increase in $\bar{w}$ either has no effect on the domestic real rate or reduces it when $c_1 = y_1 + \bar{w}$, that is, when the country is a net borrower. From the household's budget constraint, an increase in $\bar{w}$ then weakly increases welfare.

A reduction in $i^\star$ has no effect on the domestic real rate when the economy is constrained and reduces the real rate when the economy is at its first best allocation, $c_1 = c_1^{fb}$. If $c_1^{fb} > y_1$, that is, the economy is a net borrower at the first best consumption allocation, then the reduction in $i^\star$ will increase welfare.

We now proceed to show how these beneficial changes become welfare reducing when the economy follows an exchange rate policy at the ZLB. We start by analyzing how a change in foreign wealth $\bar{w}$ affects the costs of foreign reserves accumulation when the central bank is committed to the exchange rate path (s_1, s_2) and $(1 + \underline{r})\frac{s_2}{s_1} < 1$. In this case, the best monetary equilibrium will set the nominal interest rate to zero.

We can characterize domestic welfare as follows:

$$W \equiv \max_{(c_1, c_2)} u(c_1) + \beta u(c_2) + h(\bar{x}) \tag{25}$$

$$\text{subject to}$$

$$y_1 - c_1 + \frac{y_2 - c_2}{s_1/s_2} - \underline{F}\left[1 - \frac{s_2(1+i^\star)}{s_1}\right] = 0,$$

where $\underline{F}$ is the minimum level of foreign reserves necessary for $i = 0$ given (s_2/s_1).[20]

[19] To see this, we can differentiate the equation with respect to $\bar{w}$ and obtain that

$$\frac{dr}{d\bar{w}} = \frac{(1+r)^2 \beta u''(c_2) + u''(c_1)}{\beta u'(c_2) - (1+r)\beta u''(c_2)\bar{w}} < 0.$$

[20] The presence of $h(\overline{x})$ arises from the utility value of money balances. At $i = 0$, domestic households are satiated with respect to money balances, so that $m/s_1 \geq \overline{x}$ and $h(m/s_1) = h(\overline{x})$.

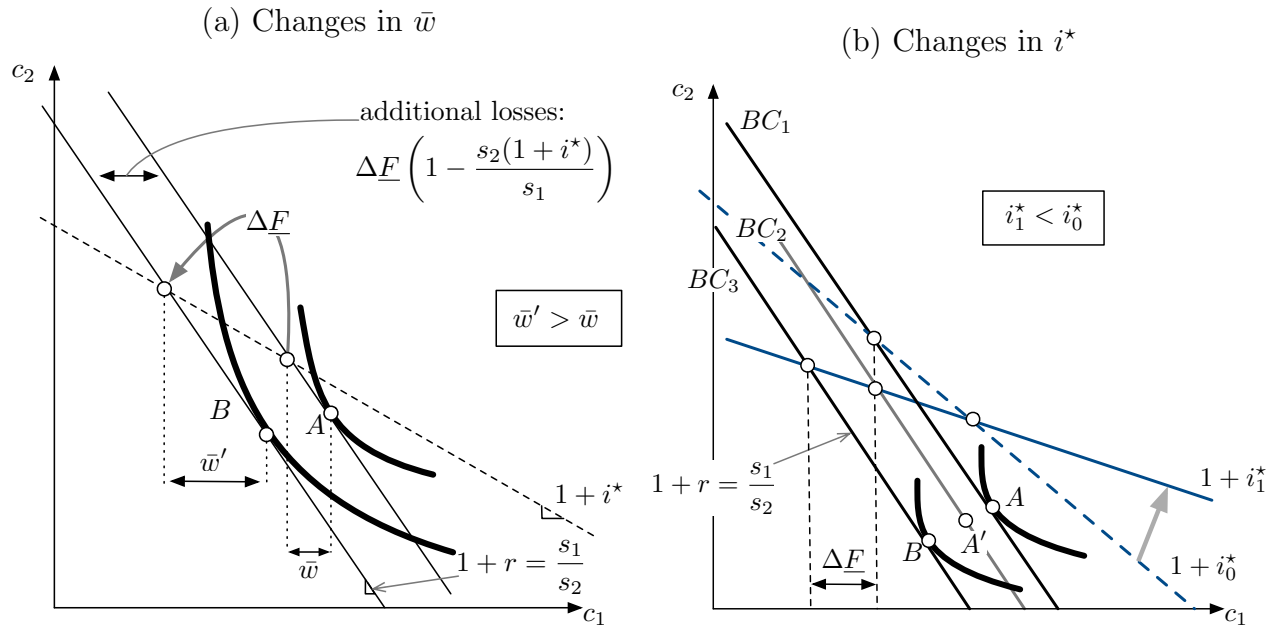


Figure 7: Comparative statics at the ZLB

Note that at the ZLB, the domestic real interest rate equals the rate of appreciation of the currency, which is fixed under the central bank policy. Therefore, a change in the welfare of domestic households purely reflects changes in the arbitrage losses that the central bank sustains when accumulating foreign reserves, the term $\underline{F}\left[1-\frac{s_2(1+i^\star)}{s_1}\right]$ in the resource constraint.

Let us now consider how an increase in the wealth of foreign investors affects this term. In equation (25), the term in the square bracket is independent of $\bar{w}$, so a change in $\bar{w}$ affects welfare only through its effect on $\underline{F}$: if higher $\bar{w}$ leads to higher $\underline{F}$, then welfare unambiguously declines. This is indeed what happens. When foreign wealth increases, the central bank is forced to accumulate more foreign reserves in order to sustain the exchange rate path (s_1, s_2). Because for every penny of foreign reserve accumulated, the central bank incurs a loss, the overall costs of the intervention increase with $\bar{w}$.

Figure 7, panel (a), illustrates this point graphically. Point A depicts the equilibrium consumption under the exchange rate policy for a given level of foreign wealth $\bar{w}$. Suppose now that foreign wealth increases to $\bar{w}'$. Because the interest rate parity condition is violated under the policy, more foreign capital will fly toward the SOE, putting downward pressure on the domestic real interest rate. In order to sustain the path for the exchange rate, the central bank will have to lean against these capital flows and purchase foreign assets, so $\underline{F}$ must increase. Point B in the figure represents the equilibrium that prevails when foreign wealth moves to $\bar{w}'$. The domestic real interest rate at equilibrium B is the same as at A, as the economy is at the ZLB at both points and the domestic real interest rate is pinned down by $\frac{s_1}{s_2}$. Even though the real rate has not changed, welfare at point B is unambiguously lower than at A, as the higher $\bar{w}$ forces the

central bank to intervene more ($\Delta \underline{F} > 0$). The losses generated by this bigger intervention are represented by the parallel shift in the budget lines from point A to point B.

This result shows that a higher degree of capital market integration makes the pursue of an exchange rate objective more costly when the economy is at the ZLB: the central bank has to accumulate more foreign reserves in order to sustain the path (s_1, s_2), and this accumulation leads to resource costs for the SOE.

A similar result occurs when $i^\star$ declines. Suppose, again, that the central bank is sustaining the path (s_1, s_2) at the ZLB, and the foreign interest rate declines. As it was for a change in foreign wealth, the impact of a decline in $i^\star$ on welfare depends on its impact on $\underline{F}\left[1 - \frac{s_2(1+i^\star)}{s_1}\right]$. However, two effects must now be considered. First, for a given (s_1, s_2), the decline in the foreign interest rate implies a larger deviation from interest rate parity, and it leads to an increase in the arbitrage losses made by the central bank for a given $\underline{F}$. Second, the decline in $i^\star$ forces the central bank to accumulate more reserves, and $\underline{F}$ must increase. Both of these forces increase the costs of sustaining the exchange rate path for the central bank.

We illustrate this result in Figure 7, panel (b). We are considering a situation in which the SOE is already at the ZLB, and its consumption lies at point A. The dashed budget line represents the resource constraint using an initial foreign rate equal to $i_0^\star$. We then consider a reduction in the international rate to $i_1^\star < i_0^\star$. The first effect is isolated by the shift of the intertemporal resource constraint from BC_1 to BC_2: the central bank intervention generates bigger resource costs for the SOE because the interest parity deviations, for a given exchange rate path, are larger if the foreign interest rate is $i_1^\star$. However, A' is not an equilibrium. The domestic household would now like to save because its endowment in the second period is not as high as it was before the decrease in the foreign rate, which implies that the domestic asset market is not in equilibrium. As a result, the central bank must increase its foreign reserves, driving the economy to its equilibrium at point B, with an even higher reduction in welfare.[21]

5.2 Expectational mistakes

Capital inflows toward the domestic economy not only may be triggered by changes in underlying fundamentals, but they might also be the results of a change in the beliefs of private agents regarding the appreciation of the domestic currency. In this section we show that when the central bank follows an exchange rate path that conflicts with the ZLB, incorrect beliefs about a future appreciation of the domestic currency necessarily induce an increase in the foreign reserve

[21] There is potentially another effect that we do not consider here. Suppose that the reduction in $i^\star$ allows foreigners to borrow more from the international financial markets. This is equivalent to a larger amount of foreign wealth $\bar{w}$ available for investment in the SOE in the first period. The additional effects generated by this will be similar to the already discussed exogenous increase in $\bar{w}$: it will increase the foreign reserve holdings by the central bank, magnifying the welfare losses.

holdings of the central bank, and they end up increasing the welfare costs of sustaining the exchange rate path (s_1, s_2). This stands in contrast with the case when the ZLB constraint does not bind, as in this latter scenario, a central bank can always exploit these expectational mistakes and strictly increase the welfare of the domestic household.

We introduce the possibility of expectational mistakes as follows. We continue to let (s_1, s_2) denote the actual exchange rate policy, and we maintain the assumption that the central bank will pursue it. Market participants (i.e., domestic households and foreign investors) see the value of s_1 in the first period and believe that the exchange rate in the second period will be $\hat{s}_2$. Expectational mistakes arise when $\hat{s}_2 \neq s_2$. Keeping with our desire to maintain simplicity, we assume that the private agents do not learn about or infer any information from the actions of the central bank. We define an equilibrium under potentially mistaken market beliefs as follows.

Definition 2. An *equilibrium given* (s_1, s_2) *and market beliefs* $\hat{s}_2$ consists of a domestic interest rate i, a consumption profile $(c_1, c_2, \hat{c}_2)$, asset positions for foreign investors $(a^\star, f^\star, m^\star)$, money M, investments by the monetary authority (A, F), transfers from the monetary authority to the fiscal authority, $(\tau_1, \tau_2, \hat{\tau}_2)$, and transfers from the fiscal authority to the households, $(T_1, T_2, \hat{T}_2)$, such that

(i) the allocation $(c_1, \hat{c}_2, a, f, m, a^\star, f^\star, m^\star, \tau_1, \hat{\tau}_2, A, F, T_1, \hat{T}_2)$ with nominal interest rate i constitutes a monetary equilibrium given the exchange rate $s_1, \hat{s}_2$.

(ii) The second-period consumption and transfers, (c_2, τ_2, T_2), satisfy

$$\begin{aligned} c_2 &= y_2 + T_2 + \frac{(1+i)a + m}{s_2} + (1+i^\star)f \\ \tau_2 &= (1+i^\star)F + (1+i)\frac{A}{s_2} - \frac{M}{s_2} \\ T_2 &= \tau_2 - (1+i)\frac{B}{s_2}. \end{aligned}$$

Note that in part (i) we use the beliefs to define the monetary equilibrium. However, in period 2, the realization of the exchange rate will be s_2, and the second-period "true" allocations (c_2, τ_2, T_2) are calculated with respect to the true exchange rate (part (ii) of the definition). We will call $(c_1, \hat{c}_2)$ the *perceived* consumption allocation and (c_1, c_2) the *true* consumption allocation. We will also say that $(1+i)\frac{s_1}{\hat{s}_2}$ is the *perceived* real interest rate, and we call $(1+i)\frac{s_1}{s_2}$ the *true* real rate interest rate. Clearly if $s_2 = \hat{s}_2$, the above definition of equilibrium is identical to our definition of a monetary equilibrium given the exchange rate policy (s_1, s_2).

As explained earlier, we will consider the case in which $s_2 > \hat{s}_2$, that is, when private agents expect the currency to be more appreciated next period relative to the policy that is actually chosen by the central bank. We evaluate welfare by considering the household's utility under the

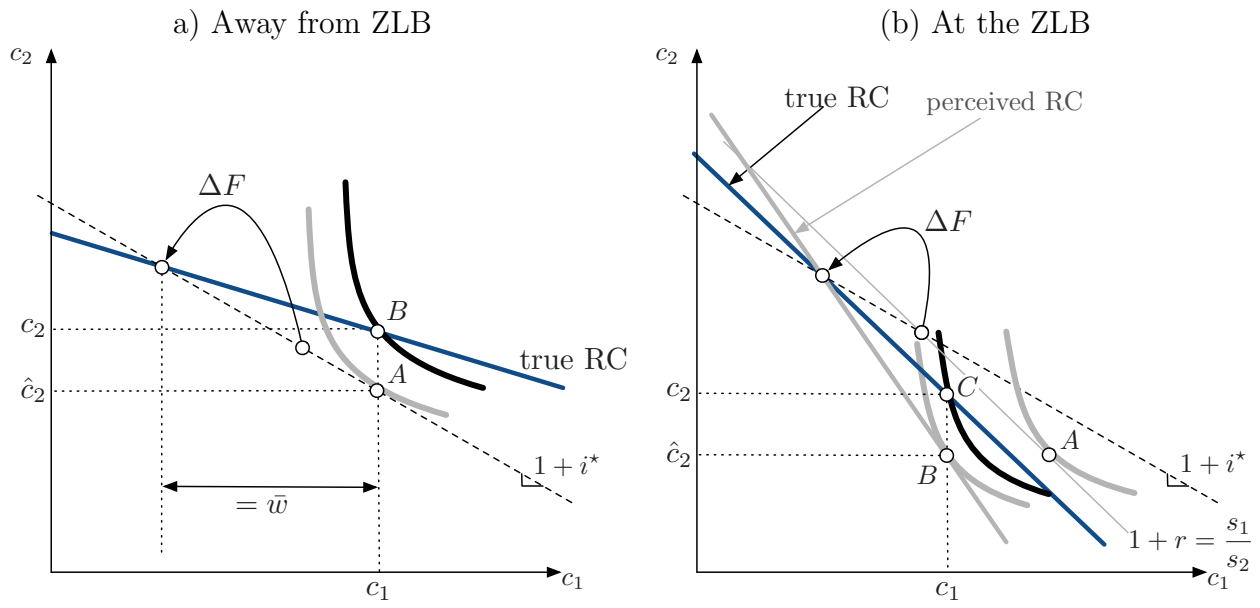


Figure 8: Expectational mistakes

true consumption allocation.

Proposition 6. *Suppose that $s_2 > \hat{s}_2$. Consider the equilibrium with market beliefs $\hat{s}_2$ that maximizes the household's welfare. Then*

(i) if $(1+\underline{r})\frac{\hat{s}_2}{s_1} \geq 1$, the household's welfare strictly decreases with $\hat{s}_2$;

(ii) if $(1+\underline{r})\frac{s_2}{s_1} < 1$, the household's welfare strictly increases with $\hat{s}_2$.

In case (i) of the proposition, the central bank maximizes welfare by reducing the interest rate so that private agents perceive the interest rate parity condition to hold, and by accumulating foreign assets. Ex post, these foreign assets are worth more under the realized exchange rate in period 2 because $s_2 > \hat{s}_2$. These profits are rebated back to the households and increase their effective second-period consumption. As a result, households' welfare, as evaluated by the central bank, increases. An example of this is illustrated in Figure 8, panel (a). Point A represents the rational expectation case, $\hat{s}_2 = s_2$. In our example, the central bank is sustaining the path for the exchange rate with no interventions ($F = 0$). We then consider how the equilibrium changes when $\hat{s}_2 < s_2$. In this case, the central bank reduces the nominal interest rate, such that the "perceived" real rate of return remains the same. Moreover, it accumulates foreign reserves to the point at which all foreign wealth enters the SOE. These two changes do not affect the behavior of private agents, who believe the equilibrium will be at point A. However, ex post, the exchange rate equals s_2, and the consumption of the domestic households will be at point B rather than A, generating a strictly positive welfare gain.

The key reason why the central bank can exploit the mistaken beliefs in case (i) of Proposition 6 is based on its ability to lower the domestic nominal interest rate when the beliefs deviate from the true ones. At the ZLB, this is not possible. As a result, the expectation mistakes cannot be exploited and welfare cannot be increased. We now show an even more negative result: the mistaken beliefs at the ZLB unambiguously generate a reduction in welfare.

Figure 8, panel (b), illustrates case (ii) of the proposition. Point A represents the original equilibrium, where the ZLB binds, and $\hat{s}_2 = s_2$. We then consider how the equilibrium changes if $\hat{s}_2 < s_2$. In this case, the central bank cannot further reduce i, because the interest rate is at zero. As a result, the perceived domestic rate of return is necessarily above the foreign one. Thus, the central bank has to accumulate foreign reserves in order to maintain its desired path for the exchange rate. The perceived equilibrium consumption allocation shifts to point B. However, ex post, the exchange rate remains at s_2, and the realized rate of return is identical to the original. The realized consumption allocation that is attained in equilibrium is given by point C, which dominated by A. Hence, welfare has been reduced.

The results of this section highlight that even if the central bank is committed to its exchange rate policy, if the markets do not believe it, then costs will be associated with defending the policy when the economy operates at the zero bound. In addition, the larger the expectational mistake, the larger the required foreign exchange interventions by the central bank, and the larger the welfare losses. That is, at the ZLB, expectational mistakes are accompanied by costly balance sheet expansions by the central bank. Those expansions could, by themselves, trigger an abandonment of the exchange rate policy if the central bank finds it costly to maintain a large balance sheet.[22] This mechanism opens the door to the possibility of self-fulfilling "appreciation" runs at the ZLB, something that we leave for future work.

5.3 Capital controls and negative nominal interest rates

So far we have shown that capital inflows increase the costs of foreign exchange interventions for a monetary authority that is pursuing an exchange rate policy at the ZLB. An important question, then, is whether the central bank could reduce these costs by imposing capital controls. It is straightforward to extend our framework to tackle such a question. Appendix C provides a full account of the interactions between capital controls and foreign exchange interventions in our setup. In what follows, we provide a summary of our findings and a brief discussion.

In propositions 7 and 8 in Appendix C, we show that capital controls, in the form of either taxes on foreign inflows of capital or quantity restrictions, grant the monetary authority the ability to implement any exchange rate policy for any nominal interest rate without the need to engage in costly foreign exchange interventions. In addition, we show that the monetary

[22] We do not analyze this point here, but we studied it in Amador et al. (2016).

authority, when implementing an exchange rate policy, strictly prefers using capital controls over foreign exchange interventions, making the latter redundant.

This result, however, hinges on the monetary authority's ability to implement capital controls at no cost. In practice, however, monetary authorities face both implementation hurdles and potential costs. First, capital controls often require some form of coordination between the monetary authority and the fiscal authority, whereas foreign exchange interventions can usually be carried out directly by the central bank. Second, for capital controls to be effective with nominal interest rates close to zero, the central bank needs to tax money or near-money financial instruments; otherwise, at the ZLB, the capital inflows would redirect toward these financial instruments. Finally other costs that we do not model might arise, such as reputational considerations or other distortions.

An alternative to capital controls is the policy of imposing negative nominal interest rates. To see why our framework provides a rationale for this intervention, suppose that the monetary authority could impose a tax on money holdings, τ^m.[23] The first-order condition of households with respect to money is modified as follows:

$$h'\left(\frac{m}{s_1}\right) = (i + \tau^m)\frac{\lambda_2}{s_2}.$$

It follows that by setting $i = -\tau^m = (1 + i^*)\frac{s_2}{s_1} - 1$, the monetary authority can implement the exchange rate policy with negative nominal interest rates, rather than by accumulating foreign assets and introducing deviations from interest parity. If the central bank had this option, it could implement any exchange rate policy by varying nominal interest rates, making the accumulation of foreign reserves redundant. Moreover, from Proposition 2 we know that the monetary authority would strictly prefer using negative nominal interest rates in place of foreign exchange interventions. This latter result helps to rationalize the recent behavior of the monetary authorities in Denmark, Switzerland, and Sweden, which experimented with negative nominal interest rates while pursuing exchange rate policies.

6 Optimal exchange rate policy

Until now, we have studied the implementation of a given exchange rate policy (s_1, s_2). In this section, we allow the central bank to choose its exchange rate policy. This approach allows us to verify the robustness of the insights obtained earlier, but now in an environment in which exchange rate policies and foreign exchange interventions are jointly determined. In line with

[23] Buiter and Panigirtzoglou (2003) study how one can overcome the ZLB constraint by imposing a tax on money. Rognlie (2015) analyzes the policy trade-offs of setting negative rates in a model in which storing cash is costly.

the analysis above, we will show that when the ZLB does not bind, the central bank will implement the chosen path for the exchange rate by varying nominal interest rates rather than by accumulating foreign reserves. When the ZLB binds, the central bank may instead find it optimal to incur losses from foreign exchange interventions in order to depreciate its exchange rate.

We extend our basic SOE model to include non tradable (NT) goods, endogenous production, and a nominal rigidity that takes the form of sticky wages. In particular, wages, denoted by p^w, are fixed (and constant) in domestic currency, $p_1^w = p_2^w = \bar{p}^w$. We follow the usual tradition in New Keynesian models of working with a cashless limit, where the value of real money balances in the utility vanishes.

Firms. Tradable and non-tradable goods are produced with a production function that uses labor, l. Taking as given prices and wages, firms in the tradable and non-tradable sector maximize profits

$$\begin{aligned} \Pi_t^T &\equiv \max_{l^T} \left(l_t^T\right)^\alpha - \frac{\bar{p}^w}{s_t} l_t^T, \\ \Pi_t^N &\equiv \max_{l^N} p_t^N (l_t^N)^\alpha - \frac{\bar{p}^w}{s_t} l_t^N, \end{aligned}$$

where l_t^T, l_t^N represent labor demands in each sector, p_t^N is the price of non-tradables expressed in foreign currency and $\bar{p}^w/s_t$ represents the wage in foreign currency. The first-order conditions lead to standard labor demand equations:

$$l_t^N = \left(\frac{\alpha p_t^N s_t}{\bar{p}^w}\right)^{1/(1-\alpha)}, \tag{26}$$

$$l_t^T = \left(\frac{\alpha s_t}{\bar{p}^w}\right)^{1/(1-\alpha)}. \tag{27}$$

Households. Households' preferences over tradable and non-tradable consumption, c^T and c^N, and labor, n, are given by

$$\sum_{t=1,2} \beta^{t-1} \left[\phi \log(c_t^T) + (1-\phi)\log(c_t^N) + \chi \log(1-n_t)\right]. \tag{28}$$

Households solve essentially the same problem as in the previous version of the model. They face a portfolio in domestic and foreign bonds, and in addition they choose the amount of tradable and non-tradable consumption. In line with the sticky wage assumption, we assume that households are off their labor supply, and supply as many hours as firms demand at the given wage. Hence, the household problem consists of choosing $\{c_1^T, c_1^N, c_2^T, c_2^T, f, a\}$ to maximize (28) subject to the

following budget constraints

$$\begin{aligned} c_1^T + c_1^N p_1^N &= \bar{p}^w n_1 + \Pi_1^T + \Pi_1^N + T_1 - \frac{a}{s_1} - f \\ c_2^T + c_2^N p_2^N &= \bar{p}^w n_2 + \Pi_2^T + \Pi_2^N + T_2 + f(1+i^*) + \frac{a(1+i)}{s_2} \end{aligned}$$

and $f \geq 0$. In addition to the first-order conditions (18)-(21), the household problem features an intratemporal Euler equation that equates the relative price of non-tradables to the marginal rate of substitution:

$$p_t^N = \frac{1-\phi}{\phi} \frac{c_t^T}{c_t^N}. \tag{29}$$

In equilibrium, the market for non-tradable goods clears:

$$y_t^N = c_t^N, \tag{30}$$

and households supply labor to meet the labor demand, $n_t = l_t^T + l_t^N$. Notice also that combining (26), (29), and (30) yields a NT employment allocation as a function of the exchange rate and the level of tradable consumption given by

$$\hat{l}^N(c_t^T, s_t) = \left(\frac{1-\phi}{\phi}\right) \frac{\alpha c_t^T s_t}{\bar{p}^w}. \tag{31}$$

Central bank problem. The objective of the central bank is to choose the monetary equilibria that deliver higher welfare.[24] The Central Bank chooses an exchange rate policy (s_1, s_2), in addition to a nominal interest rate i and a foreign asset position F. The key difference with the analysis in the previous section is that now the central bank optimally chooses (s_1, s_2). The optimality conditions for the path of the exchange rate, formally derived in Appendix D, are respectively:

$$\underbrace{\frac{\partial \hat{l}_1^T}{\partial s_1}}_{\text{Keynesian Channel}} \underbrace{\left(\lambda(1+i^\star)\alpha \hat{l}^T(s_1)^{\alpha-1} - \frac{\chi}{1-n_1}\right)}_{\text{Labor Wedge}} + \underbrace{\frac{\partial \hat{l}_1^N}{\partial s_1}}_{\text{Keynesian Channel}} \underbrace{\left(\frac{1-\phi}{c_1^N}\alpha \hat{l}^N(s_1, c_1^T)^{\alpha-1} - \frac{\chi}{1-n_1}\right)}_{\text{Labor Wedge}} \leq \underbrace{\frac{\lambda}{s_2}\bar{w}}_{\text{Intervention Loss}} + \underbrace{\xi c_1^T \beta,}_{\text{Saving Distortion}} \tag{32}$$

[24] Appendix D states the formal definition of monetary equilibrium in this setup, which extends Definition 1 with firms' optimal decision for employment and the market-clearing condition for non-tradables.

and

$$\underbrace{\frac{\partial \hat{l}_2^T}{\partial s_2}}_{\text{Keynesian Channel}} \left(\underbrace{\lambda \alpha \hat{l}^T(s_2)^{\alpha-1} - \frac{\beta \chi}{1-n_2}}_{\text{Labor Wedge}} \right) + \underbrace{\beta \frac{\partial \hat{l}_2^N}{\partial s_2}}_{\text{Keynesian Channel}} \left(\underbrace{\frac{1-\phi}{c_2^N} \alpha \hat{l}^N(s_2, c_2^T)^{\alpha-1} - \frac{\chi}{1-n_2}}_{\text{Labor Wedge}} \right) \leq \underbrace{-\frac{\lambda s_1}{s_2^2} \bar{w}}_{\text{Intervention Loss}} - \underbrace{\xi c_2^T}_{\text{Saving Distortion}} \tag{33}$$

where λ, ξ denote, respectively, the Lagrange multipliers associated with the resource constraint and the domestic Euler equation, and $\hat{l^T}(s_1)$ denote the employment equilibrium function equation given by (27). In a solution in which the central bank intervenes in the asset markets, (33) and (32) hold with equality. The left-hand side of (32) indicates the benefits of depreciating the exchange rate in period 1: by depreciating the exchange rate, the central bank can increase labor demand (the terms labelled Keynesian Channel), and this has positive effects on welfare to the extent that production is inefficiently low (when there are positive labor wedges in the tradable and non-tradable sectors). The right-hand side indicates the potential costs from depreciating the exchange rate, which is composed of the two terms we analyzed in Section 2. The first term represents the intervention losses. Given i and s_2, an increase in s_1 raises the expected appreciation rate of the domestic currency, which opens a wider gap in the interest parity condition. As we have shown in equation (17), this produces losses for the SOE, which are proportional to the foreign wealth of investors. The second term is the loss due to the distortion in the consumption-saving decisions of domestic households. A rise in s_1 increases the real interest rate, and distorts consumption toward the second period.

Equation (33) is analogous to (32), with the key difference that the two terms on the right hand side have the opposite sign. That is, a higher s_2 reduces the real return of domestic bonds and reduces both the intervention losses and the interest rate distortions. Because the right hand side is negative, this indicates that the central bank at the optimum allows for a non-positive labor wedge in the second period, as long as there is also a positive labor wedge in the first period. Putting together (32) and (33) indicates that the central bank trades-off a positive labor wedge in the first period against a negative labor wedge in the second period. While away from the ZLB, the central bank can offset these wedges by cutting down the nominal interest rate, this is not the case at the ZLB. Below, we solve the model numerically and show the role of foreign exchange interventions once the economy hits the ZLB.

Optimal exchange rate policies. We now present a numerical illustration and discuss the optimal policy of the monetary authority. Figure 9 reports key variables in the central bank

solution as a function of the discount factor of the households β.[25] When β increases, households become more patient and reduce their current consumption. In absence of a policy response by the central bank, this shift would depress output in period 1: by reducing their demand for non-tradable consumption, the price of non-tradable goods would drop, leading to a decline in the demand of labor in the non-tradable sector (see equation (31)). The response of the monetary authority to this increase in households' patience is to depreciate the exchange rate; by doing so, the central bank can stimulate labor demand and restore efficient production. Importantly, this policy is achieved initially with a reduction in nominal interest rates and without accumulating foreign assets (see panel (d) and (e) of the figure). This mirrors our results in Proposition 2.

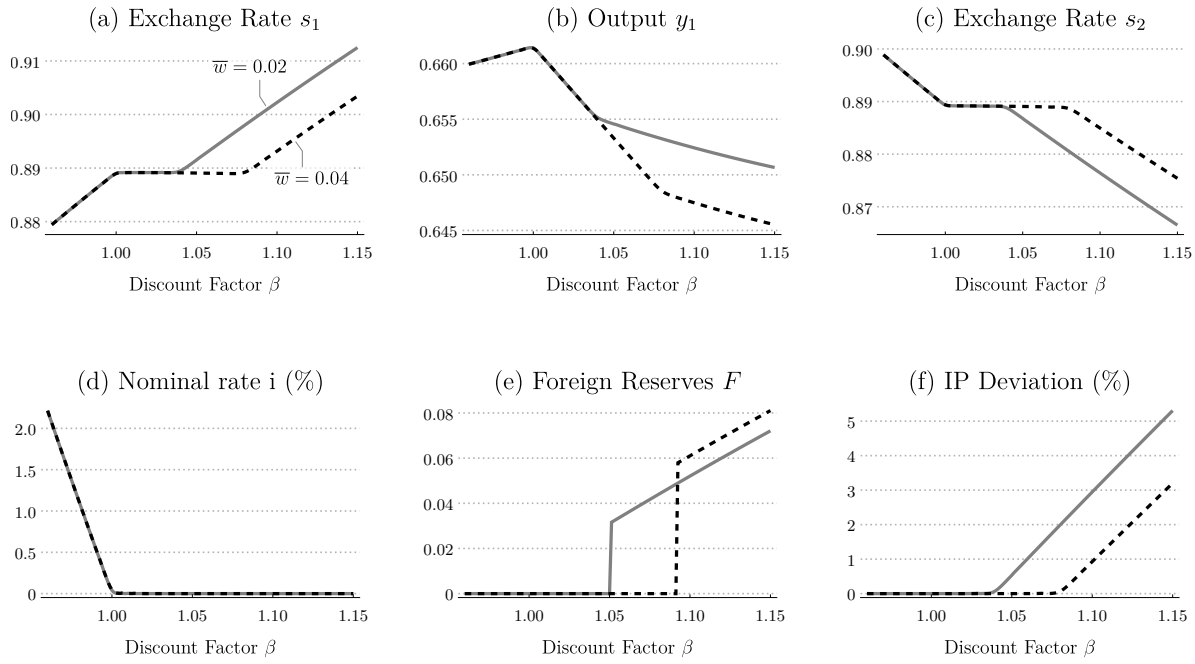


Figure 9: Optimal interventions with endogenous exchange rate policy

Note: Numerical illustration for two different values of foreign wealth for a range of discount factors. Parameter values are as follows $\phi = 0.5, p^w = 1, \chi = 1, \alpha = 0.7, i^\star = 0$ and low and high values for $\bar{w} = \{0.02, 0.04\}$. The discount factor is represented by the x-axis. Output in panel (b) denotes the sum of tradable and non-tradable output expressed in units of tradables.

This response of the central bank, however, is not always feasible. For sufficiently high values of β, the nominal interest rates that would allow the central bank to achieve the desired exchange rate policy is negative. Initially, the central bank sets nominal interest rates at zero and tolerates the output inefficiencies induced by high discounting of the households: we can see from panel

[25] We follow the tradition of closed economy New Keynesian models in generating a binding ZLB with an increase in the households' discount factor (e.g., Eggertsson and Woodford, 2003, Christiano et al., 2011, and Werning, 2011).

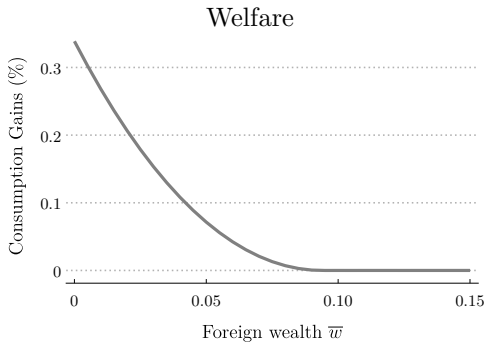


Figure 10: Welfare and foreign wealth under the ZLB

Note: Parameter values are just as in Figure 9 with $\beta = 1.2$. The figure shows the welfare gains generated by the optimal intervention policy relative to a policy where the Central Bank does not accumulate reserves, for different values of foreign wealth. Welfare gains are expressed as percentage increases in permanent consumption of both non-traded and tradable goods.

(b) that output starts dropping as a function of β. Eventually, however, the welfare costs of the recession are so large that the central bank becomes willing to bear the losses from foreign exchange interventions in order to depreciate the exchange rate and moderate the output gaps. The threshold at which the central bank intervenes is higher when the level of foreign wealth is higher, in line with our results that a deviation from interest parity generates a first-order loss proportional to $\bar{w}$. Once the monetary authority intervenes, however, it requires a larger accumulation of foreign assets. This in turn generates a non-monotonic relationship between F and $\bar{w}$, as illustrated in panel (e) of Figure 9.[26]

The lessons learned in the model with an exogenous exchange rate policy carry over to this more general environment. For example, we showed that, when operating at the ZLB, a higher level of foreign wealth unambiguously decreased households' welfare when the exchange rate policy was given. In this new environment, in which the central bank optimally chooses its exchange rate policy, a similar result holds. However, there is a caveat: the central bank may eventually stop intervening and give up on its exchange rate policy if the foreign wealth is large enough. Figure 6 presents a case in which the central bank is operating at the ZLB. As can be seen, higher wealth strictly reduces domestic welfare, up to the point where the central bank stops intervening.

[26]The policies conducted by several developed economies following the global financial crisis have a natural interpretation through the lens of our model. Facing a slump and deflationary pressures, central banks first lowered interest rates before engaging in accumulation of foreign assets to stimulate employment via a weakening of the domestic currency. For the case of Switzerland, in particular, our model suggests that in response to the European Central Bank's quantitative easing, the Swiss National Bank faced larger losses from sustaining a depreciated exchange rate (because of a combination of lower $i^\star$ and higher $\bar{w}$) and hence let the currency appreciate in January 2015.

7 Conclusions

This paper has devloped a simple framework for analyzing the trade-offs that a central bank faces when it implements an exchange rate policy while constrained in its ability to move the nominal interest rate. Consistent with the classic trilemma of international finance, an independent exchange rate policy can be achieved only if international capital mobility is limited. In this case, we have shown that the central bank can achieve the exchange rate objective by using foreign exchange interventions that result in observable deviations from arbitrage in capital markets. These interventions, however, are costly from the point of view of the domestic economy. We characterize how these costs vary with the economic environment: we show that factors that increase capital inflows toward the country raise the costs of these interventions, whereas policies such as capital controls or negative nominal interest rates can reduce these costs. Moreover, the main predictions of our theory are consistent with the behavior of foreign reserves, nominal interest rates, and deviations from the covered interest rate parity conditions for a panel of advanced economies.

The analysis could be extended in several directions. A first interesting question relates to reserve management: given that reserves accumulation is a necessary tool for conducting an exchange rate policy at the zero lower bound, what are the optimality principles that should govern its asset allocations? In Amador et al. (2017) we introduce uncertainty and multiple assets to the model studied in this paper, and we characterize the trade-offs that the monetary authority faces. Moreover, as we have emphasized in our discussion on expectational mistakes, the costs of keeping an exchange rate depreciated at the ZLB increase in private agents' beliefs about the appreciation rate, a mechanism that can potentially generate self-fulfilling exchange rate dynamics. Finally, our theory points toward the role of the central bank's balance sheet in overcoming the ZLB constraint on nominal interest rates, but it is silent about other potential costs associated with these "large" balance sheets. We believe these topics represent exciting avenues for future research.

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Appendix to "Exchange Rate Policies at the Zero Lower Bound"

By Manuel Amador, Javier Bianchi, Luigi Bocola, and Fabrizio Perri

A Omitted proofs

Proof of Proposition 1

Proof. The allocations of the non-monetary equilibrium can be reduced to (c_1, c_2, a, r) which satisfy

$$\begin{aligned} u'(c_1) &= (1+r)\beta u'(c_2) \\ c_2 &= y_2 + (1+r)a + (1+i^\star)F \\ c_1 &= y_1 - F - a \\ r &\geq i^\star \\ -a &\in \underset{0\leq x\leq \bar{w}}{\operatorname{argmax}}\, x(r-i^\star), \end{aligned}$$

given F. Note that we have set $f = 0$ without loss of generality, and we have combined the budget constraints of the households and the government.

Part (i): the case $F \in [0, y_1 + \bar{w} - c_1^{fb}]$. First, we argue that $r = i^\star$. Suppose, by contradiction, that $r > i^\star$. In this case, $\bar{w} = -a$ from the maximization of foreign investors, and $c_1 = y_1 - F + \bar{w} \geq c_1^{fb}$. Using the budget constraints, we arrive at

$$(1+r)(y_1 - c_1) + y_2 - c_2 - F(r - i^\star) = 0.$$

Because $c_1 \geq c_1^{fb}$, this implies that $c_2 \leq c_2^{fb}$. Using the Euler equation of the households, we then arrive at

$$(1+i^\star)\beta u'(c_2^{fb}) = u'(c_1^{fb}) \geq u'(c_1) = (1+r)\beta u'(c_2).$$

That is

$$(1+i^\star)\beta u'(c_2^{fb}) \geq (1+r)\beta u'(c_2),$$

which implies, using $c_2 \leq c_2^{fb}$, that $r \leq i^\star$, a contradiction. Therefore, $r = i^\star$. Now, in this case, the equilibrium conditions are reduced to

$$\begin{aligned} u'(c_1) &= (1+i^\star)\beta u'(c_2) \\ (1+i^\star)(y_1 - c_1) + y_2 - c_2 &= 0, \end{aligned}$$

which are the necessary and sufficient conditions for a solution to the first best problem defined in Section 3.1, which has a unique solution, (c_1^{fb}, c_2^{fb}).

Part (ii): the case $F \in (y_1 + \bar{w} - c_1^{fb}, y + \bar{w})$. We first show that $r > i^\star$. Suppose, by contradiction, that $r = i^\star$. Then the consumption allocation solve:

$$u'(c_1) = (1 + i^\star)\beta u'(c_2)$$

$$(1 + i^\star)(y_1 - c_1) + y_2 - c_2 = 0,$$

which would yield the first best allocation. This allocation, however, can no longer be an equilibrium outcome as $c_1 = c_1^{fb} = y_1 - F - a \leq y_1 - F + \bar{w}$, which puts F outside of the range considered in this case. Therefore, we must have $r > i^\star$, and similar to part (i), $a = -\bar{w}$. It follows that $c_1 = y_1 - F + \bar{w}$, and $c_2 = y_2 - (1 + r)\bar{w} + (1 + i^\star)F$. The range of F guarantees that both c_1 and c_2 are strictly positive.

Part (iii): the case $F > y + \bar{w}$. Following part (ii), we must have that $r > i^\star$ and thus $c_1 = y_1 - F + \bar{w}$, which is negative given that $F > y + \bar{w}$. It follows that there is no non monetary equilibrium in this range. □

Proof of Corollary 1

Proof. From part (ii) of Proposition 1, we know that $c_1 = y_1 - F + \bar{w}$, and $c_2 = y_2 - (1 + r)\bar{w} + (1 + i^\star)F$ when $F \in (y_1 + \bar{w} - c_1^{fb}, y_1 + \bar{w})$. Differentiating both sides of the households' Euler equation and rearranging terms, we obtain

$$\frac{d(1 + r)}{dF} = \frac{-u''(c_1)u'(c_2) - u'(c_1)u''(c_2)(1 + i^\star)}{\beta(u'(c_2))^2} > 0,$$

which shows that the real rate strictly increases in F in this range. The proof that F adversely affects welfare is in the main body of the text. Taken together, they deliver the first part of the corollary.

As to the second part, we know from part (i) of Proposition 1 that if $F \leq y_1 + \bar{w} - c_1^{fb}$, the allocations are independent of F. □

Proof of Lemma 1

The necessary part of the lemma follows from the derivations in the text.

For the sufficiency part, consider an allocation (c_1, c_2, F, i, m) that satisfies (18), (19), (20), (22), and (23). We first need to specify the rest of the equilibrium objects: $a, f, a^\star, f^\star, m^\star, \tau_1, \tau_2, A, T_1, M$,

and T_2.

We let $M = m$, $A = B$, $\tau_1 = -F + (M - A)/s_1$, $\tau_2 = (1 + i^\star)F + (1 + i)A/s_2 - M/s_2$, $T_1 = B/s_1 + \tau_1$, and $T_2 = \tau_2 - (1 + i)B/s_2$. Note that $M \geq 0$, and by construction, the budget constraints of the monetary and fiscal authorities are satisfied.

For the domestic households, we let

$$f = \max\{y_1 - c_1 - F, 0\}$$
$$a = s_1 \min\{y_1 - c_1 - F, 0\}.$$

Note that this implies that the household's budget constraint in the first period, which is:

$$y_1 - F + M/s_1 = c_1 + (m + a)/s_1 + f,$$

is satisfied at the equilibrium choices and that $f \geq 0$.

The household's budget constraint in the second period is then

$$y_2 + (1 + i^\star)F - M/s_2 = c_2 - (m + (1 + i)a)/s_2 - (1 + i^\star)f$$

.Using that $M = m$, and that $y_1 - F + M/s_1 = c_1 + (m + a)/s_1 + f$, we get that

$$(y_1 - c_1) + \frac{y_2 - c_2}{\frac{(1+i)s_1}{s_2}} + \left[\frac{(1 + i^\star)}{\frac{(1+i)s_1}{s_2}} - 1\right](f + F) = 0.$$

But given that (23), it follows that if $\left[\frac{(1+i^\star)s_2}{(1+i)s_1} - 1\right] > 0$, $c_1 = y_1 - F + \bar{w}$ and thus $f = \max\{y_1 - c_1 - F, 0\} = \max\{-\bar{w}, 0\} = 0$. So we have that for the second-period budget constraint to hold, it must be that

$$(y_1 - c_1) + \frac{y_2 - c_2}{\frac{(1+i)s_1}{s_2}} + \left[\frac{(1 + i^\star)}{\frac{(1+i)s_1}{s_2}} - 1\right]F = 0,$$

,which is (22). Thus, the budget constraint for the household in the second period is satisfied.

The household's problem is convex. In addition, (19) holds, and so do the optimality conditions, (18), (20), and (21). It follows that (c_1, c_2, a, f, m) solves the household's problem.

Finally, we let $m^\star = 0$, and $a^\star = -a$. This implies that $(a^\star + m^\star)/s_1 = \max\{c_1 - y_1 - F, 0\} \leq \bar{w}$ and $a^\star \geq 0$. It follows that the constraints for the foreign investors' problem are satisfied with $f^\star = \bar{w} - (a^\star + m^\star)/s_1 \geq 0$ and $c^\star = (1+i^\star)f^\star + (1+i)a^\star/s_2 + m^\star/s_2$. The optimality conditions for the foreign investors are satisfied, as $m^\star = 0$ (which is optimal given that $i \geq 0$) and $a^\star/s_1 = \bar{w}$ if $\left[\frac{(1+i^\star)s_2}{(1+i)s_1} - 1\right] > 0$. So, the foreign investors are maximizing.

Finally, market clearing holds as $a + a^\star + A = B$ and $m + m^\star = M$.

Proof of Proposition 2

Proof. We first prove that for all $F \in [0, y_1 + \bar{w})$, the non-monetary equilibrium given F constitutes a monetary equilibrium outcome. Consider a non-monetary equilibrium for any $F \in [0, y_1 + \bar{w})$, and let r be the associated real rate. Then, we have that $(1+r)\frac{s_2}{s_1} \geq (1+\underline{r})\frac{s_2}{s_1} \geq 1$ where the first inequality follows from Corollary 1 as r is increasing with F and $\underline{r}$ corresponds to the equilibrium real rate when $F = 0$, and the second inequality follows from the statement of this proposition. Finally, to construct the monetary equilibrium, we set $(1 + i) = (1 + r)\frac{s_2}{s_1}$, which satisfies the ZLB constraint. We then let $M = m$ such that $h'(m/s_1) = \frac{i}{1+i}\frac{u'(c_1)}{s_1}$, which has a solution given that $i \geq 0$. We can set $m^\star = 0$, $A = 0$, and $f^\star = \bar{w} - a^\star/s_1$. The asset positions and the consumption allocation for the domestic households remain the same as in the non-monetary equilibrium. We then let τ_1, τ_2, and T_1, T_2 be chosen such that the budget constraints of agents hold.

For the second part, we already know from Corollary 1 that $u(c_1)+\beta u(c_2)$ is weakly decreasing in F in the non-monetary equilibrium. Note that from Proposition 1, we have that an increase in F weakly reduces c_1. Together with the associated increase in r, it follows that, in the associated monetary equilibrium, i increases and c_1 decreases with F. As a result, from $h'(m/s_1) = \frac{i}{1+i}\frac{u'(c_1)}{s_1}$, m decreases with F. Domestic welfare (the sum of utility from consumption plus utility services from money balances) decreases in F, and thus it is maximized when $F = 0$. □

Proof of Proposition 3

Proof. Note first that for $F = 0$, the non-monetary equilibrium is given by part (i) of Proposition 1, and thus $\underline{r} = i^\star$. Towards a contradiction, suppose there is a monetary equilibrium such that $1 + i > (1 + i^\star)\frac{s_2}{s_1} = (1 + \underline{r})\frac{s_2}{s_1} \geq 1$. Then, we must have $a = -\bar{w}$, and $c_1 = y_1 - F + \bar{w} < c_1^{fb}$, where the last inequality follows from part (ii) of Proposition 1. Note also that $i > 0$, which implies that $m^\star = 0$ and $m = M$. The household's optimality condition with respect to money balances then delivers that $\bar{x} > m/s_1$.[27]

Recall that $M/s_1 = F + A/s_1 + \tau_1$. From Assumption 2.3, we have that $M/s_1 \geq F$. Using the previous result, it follows that $\bar{x} > F$, and

$$c_1 = y_1 - F + \bar{w} > y_1 - \bar{x} + \bar{w} \geq c_1^{fb}$$

a contradiction. Hence, for all monetary equilibria, $(1+i) = (1+i^\star)s_2/s_1$, and thus we attain the first best consumption allocation and the same domestic welfare inclusive of the value of money balances. □

[27] From the decision problem of the households, we obtain that money demand satisfies $h'(\frac{m}{s_1}) = \frac{i}{1+i}\frac{u'(c_1)}{s_1}$. When nominal interest rates are positive, we must have $h'(\frac{m}{s_1}) > 0$, which implies $\bar{x} > \frac{m}{s_1}$.

Proof of Proposition 4

Proof. First, note that $(1+i) \geq (1+\underline{r})\frac{s_2}{s_1} \geq (1+i^\star)\frac{s_2}{s_1}$. The fact that $(1+\underline{r})\frac{s_2}{s_1} < 1$ implies that $(1+i) > (1+\underline{r})\frac{s_2}{s_1} \geq (1+i^\star)\frac{s_2}{s_1}$. The non-monetary equilibria given F that can correspond to an outcome of the monetary economy are characterized by part (ii) of Proposition 1.

Let $\underline{F}$ be

$$\frac{u'(y_1 - \underline{F} + \bar{w})}{\beta u'(y_2 - (1+r)\bar{w} + (1+i^\star)\underline{F})} = \frac{s_1}{s_2}.$$

Note that the assumption that $1+\underline{r} < s_1/s_2 < 1+\bar{r}$ guarantees that there exist a finite $\underline{F} > 0$ such that the above equality holds.

We can then verify that the non-monetary equilibrium given $\underline{F}$ can be implemented as a monetary equilibrium with $i = 0$. For $F \in (\underline{F}, y_1 + \bar{w})$, to implement the non-monetary equilibrium given F as a monetary outcome, the nominal interest rate will equal

$$(1+i) = \frac{s_2}{s_1}\frac{u'(y_1 - F - \bar{w})}{\beta u'(\, y_2 + (1+r)\bar{w} + (1+i^\star)F)} = \frac{s_2}{s_1}(1+r) \geq \frac{s_2}{s_1}(1+\underline{r}) > 1,$$

where the first inequality follows from the fact that r is increasing in F. As a result, the nominal interest rate is consistent with the zero lower bound constraint, and we can construct a monetary equilibrium in a fashion similar to that in the proof of Proposition 2.

Again, similar to the proof of Proposition 2, welfare is maximized with $i = 0$ and $F = \underline{F}$ because of Corollary 1, together with the fact that$h(\frac{m}{s_1})$ achieves its maximum when nominal interest rates are at zero. □

Proof of Proposition 5

Proof. From Proposition 4, we know that when $(1+\underline{r})\frac{s_2}{s_1} < 1$, a monetary equilibrium exists only if $F \geq \underline{F}$. Suppose $F > \underline{F}$. Then, the monetary equilibrium must feature $i > 0$. An argument similar to that in the proof of Proposition 3 shows that in this case, the central bank does not have a large enough balance sheet to accumulate the required reserves. Therefore, the unique equilibrium outcome is the one where $F = \underline{F}$ and $i = 0$. □

Proof of Lemma 2

The proof of this lemma is contained in the main body of the text.

Proof of Proposition 6

Proof. An allocation $(c_1, \hat{c}_2, f, F, i)$ satisfies a monetary equilibrium with market beliefs $\hat{s}_2$ if

$$(y_1 - c_1) + \frac{y_2 - \hat{c}_2}{1 + \hat{r}} - F\frac{(\hat{r} - i^\star)}{1 + \hat{r}} = 0 \tag{34}$$

$$\frac{u'(c_1)}{\beta u'(\hat{c}_2)} = (1 + \hat{r}) \geq 1 + i^\star \tag{35}$$

$$\frac{u'(c_1)}{\beta u'(\hat{c}_2)}\frac{\hat{s}_2}{s_1} \geq 1 \tag{36}$$

$$f(\hat{r} - i^\star) = 0 \tag{37}$$

$$(1 + i) = (1 + \hat{r})\frac{\hat{s}_2}{s_1} \tag{38}$$

$$c_1 \leq y_1 - (F + f) + \overline{w}, \tag{39}$$

where the last inequality holds with equality if $\hat{r} > i^\star$, and where $\hat{r}$ represents the perceived real interest rate. For a given F, there is a unique $(c_1, \hat{c}_2, i)$ that solves the above system. In addition, if for a given F, $\hat{r} = i^\star$, then any $f \in [0, y_1 - c_1 - F - \bar{w}]$ is consistent with an equilibrium.

For a given F and its associated perceived consumption allocation $(c_1, \hat{c}_2)$, the actual consumption in the second period, c_2, is given by the "true" resource constraint, which in this case is

$$c_2 = y_2 + T_2 + \frac{(1 + i)a + m}{s_2} + (1 + i^\star)f.$$

Using the budget constraints of the fiscal and monetary authorities, we get that

$$T_2 = \tau_2 - (1 + i)\frac{B}{s_2} = (1 + i^\star)F + (1 + i)\left(\frac{A - B}{s_2}\right) - \frac{M}{s_2}.$$

Substituting back into the previous equation and rearranging terms, we obtain that

$$c_2 = y_2 + (1 + i^\star)(F + f) - \left(\frac{(1 + i)a^\star + (1 + i^\star)m^\star}{s_2}\right) + \frac{i^\star m^\star}{s_2},$$

which can be further simplified to

$$c_2 = y_2 + (1 + i^\star)(F + f) - (1 + i)\frac{s_1}{s_2}\left(\frac{a^\star + m^\star}{s_1}\right),$$

because $\frac{im^\star}{s_2} = 0$. The household's budget constraint in the first period is

$$f + F + c_1 - y_1 = \frac{a^\star + m^\star}{s_1},$$

and substituting out $(a^\star+m^\star)/s_1$, we get that the true consumption in the second period become:

$$\begin{aligned} c_2 &= y_2 + (1+i^\star)(F+f) - (1+i)\frac{s_1}{s_2}(f+F+c_1-y_1) \\ &= y_2 + \left[(1+i^\star) - (1+i)\frac{s_1}{s_2}\right](F+f) + (1+i)\frac{s_1}{s_2}(y_1-c_1)\,. \end{aligned}$$

We can follow the same steps but for the perceived allocation, obtaining that:

$$\hat{c}_2 = y_2 + \left[(1+i^\star) - (1+i)\frac{s_1}{\hat{s}_2}\right](F+f) + (1+i)\frac{s_1}{\hat{s}_2}(y_1-c_1)\,.$$

Taking the difference:

$$c_2 = \hat{c}_2 + \frac{(1+i)s_1}{\hat{s}_2}\left[1-\frac{\hat{s}_2}{s_2}\right](f+F+c_1-y_1).$$

Let us define $H(i,c_1)$ to be

$$H(i,c_1) \equiv h\left(\frac{m}{s_1}\right) \text{ for } m \text{ such that } h'\left(\frac{m}{s_1}\right) = \frac{i}{1+i}\frac{u'(c_1)}{s_1} \tag{40}$$

Note that $H(i,c_1)$ is uniquely defined, and it represents the utility value of money balances in an equilibrium given the nominal interest rate and first period consumption. Note that H is weakly decreasing in i and weakly increasing in c_1.

Then, given an equilibrium allocation with distorted beliefs, $(c_1,c_2,\hat{c}_2,f,F,i)$, the welfare of the household is given by:

$$u(c_1)+\beta u(c_2)+H(i,c_1).$$

We first prove part (i) of the proposition.

Part (i). This is the case in which $(1+\underline{r}) \geq \frac{s_1}{\hat{s}_2}$. The proof is straightforward. Consider an equilibrium with distorted beliefs $\hat{s}_2$. The respective allocation $(c_1,\hat{c}_2,f,F,i)$ must solve (34)-(39). Suppose that $\hat{s}_2$ decreases to $\hat{s}_2' < \hat{s}_2$. Then, $(c_1,\hat{c}_2,F',i')$ where $(1+i') = (1+i)\frac{\hat{s}_2'}{\hat{s}_2}$, $f'=0$ and

$$F' = \begin{cases} F & \text{if } \hat{r} > i^\star \\ c_1 - y_1 - \bar{w} & \text{if } \hat{r} = i^\star \end{cases}$$

also solves (34)-(39) with $\hat{s}_2'$. As long as $(1+\underline{r}) \geq \frac{s_1}{\hat{s}_2'}$, this allocation constitutes an equilibrium with distorted beliefs $\hat{s}_2'$. In addition, note that $i' < i$ and, by construction,

$$f' + F' + c_1 - y_1 = \bar{w}$$

.Hence,

$$c_2' = \hat{c}_2 + (1+i)s_1\left[\frac{1}{\hat{s}_2'} - \frac{1}{s_2}\right]\bar{w} > \hat{c}_2 + (1+i)s_1\left[\frac{1}{\hat{s}_2} - \frac{1}{s_2}\right]\bar{w} \geq \hat{c}_2 + (1+i)s_1\left[\frac{1}{\hat{s}_2} - \frac{1}{s_2}\right](f + F + c_1 - y_1) = c_2$$

where the first inequality follows from $\bar{w} > 0$ and $\hat{s}_2' < \hat{s}_2$ and the second inequality from $(f + F + c_1 - y_1) \leq \bar{w}$. Thus $c_2' > c_2$. It follows then that household welfare with beliefs at $\hat{s}_2'$ is

$$u(c_1) + u(c_2') + H(i', c_1) > u(c_1) + u(c_2) + H(i, c_1)$$

where the inequality follows as c_1 did not change, $c_2' > c_2$ and $i' < i$. Given that the original allocation was arbitrary, households' welfare at the best equilibrium must then strictly increase with a reduction in $\hat{s}_2$.

Part (ii). If $(1 + \underline{r}) < \frac{s_1}{s_2}$, it follows that $(1 + \underline{r}) < \frac{s_1}{\hat{s}_2}$ as $\hat{s}_2 < s_2$. Then by Proposition 4, in all equilibria, we must have that the perceived interest rate parity condition holds with strict inequality, $\hat{r} > i^\star$ and $F \geq \underline{F}$. It then follows that $f = 0$ and $c_1 = y_1 - F + \bar{w}$. Note also that $\hat{c}_2 = y_2 + (1 + i^\star)F - (1+i)\frac{s_1}{\hat{s}_2}\bar{w}$ and $c_2 = \hat{c}_2 + \frac{(1+i)s_1}{\hat{s}_2}\left[1 - \frac{\hat{s}_2}{s_2}\right]\bar{w} = y_2 + (1 + i^\star)F - \frac{(1+i)s_1}{s_2}\bar{w}$.

Households welfare can be written as a function of F:

$$W(F) \equiv u(y_1 - F + \bar{w}) + \beta u\left(y_2 + (1 + i^\star)F - (1+i)\frac{s_1}{s_2}\bar{w}\right) + H\left(i, y_1 - F + \bar{w}\right),$$

where i is the unique value that solves

$$\frac{u'(y_1 - F + \bar{w})}{\beta u'(y_2 + (1 + i^\star)F - (1+i)\frac{s_1}{\hat{s}_2}\bar{w})}\frac{\hat{s}_2}{s_1} = (1 + i).$$

We then have that

$$\begin{aligned} W'(F) &= -u'(c_1) + \beta(1 + i^\star)u'(c_2) - \beta u'(c_2)\bar{w}\frac{s_1}{\hat{s}_2}\frac{di}{dF} + H_i\frac{di}{dF} - H_c \\ &< -u'(c_1) + \beta(1 + i^\star)u'(\hat{c}_2) - \beta u'(c_2)\bar{w}\frac{s_1}{\hat{s}_2}\frac{di}{dF} + H_i\frac{di}{dF} - H_c \end{aligned}$$

where the inequality uses that $c_2 - \hat{c}_2 = \frac{(1+i)s_1}{\hat{s}_2}\left[1 - \frac{\hat{s}_2}{s_2}\right]\bar{w} < 0$ as $s_2 > \hat{s}_2$. Our equilibrium requirements impose that $-u'(c_1) + \beta(1 + i^\star)u'(\hat{c}_2) \leq 0$. Given that i is strictly increasing in F, $H_i \leq 0$, and $H_c \geq 0$, it then follows that $W'(F) < 0$.

As a result, the monetary equilibrium that maximizes households welfare is the one where F is as small as possible, that is, the one where $F = \underline{F}(s_2)$ and $i = 0$. This means that the

maximum welfare level is

$$\bar{w}(\hat{s}_2) = u(y_1 - \underline{F}(\hat{s}_2) + \bar{w}) + \beta u\left(y_2 + (1+i^\star)\underline{F}(\hat{s}_2) - \frac{s_1}{s_2}\bar{w}\right) + H\left(0, y_1 - \underline{F}(\hat{s}_2) + \bar{w}\right)$$

where $\underline{F}(\hat{s}_2)$ is such that

$$\frac{u'(y_1 - \underline{F}(\hat{s}_2) + \bar{w})}{\beta u'\left(y_2 + (1+i^\star)\underline{F}(\hat{s}_2) - \frac{s_1}{\hat{s}_2}\bar{w}\right)}\frac{\hat{s}_2}{s_1} = 1 \tag{41}$$

Then note that

$$\bar{w}'(\hat{s}_2) = \left[-u'(c_1) + \beta(1+i^\star)u'(c_2) - H_c\right]\underline{F}'(\hat{s}_2)$$

.We have already argued that $-u'(c_1)+\beta(1+i^\star)u'(c_2)-H_c < -u'(c_1)+\beta(1+i^\star)u'(\hat{c}_2)-H_c \leq 0$. From equation (41), we have that

$$\underline{F}'(\hat{s}_2) = \frac{\beta u''(\hat{c}_2)\bar{w}\frac{s_1}{\hat{s}_2^2} - \frac{u'(c_1)}{s_1}}{-u''(c_1)\frac{\hat{s}_2}{s_1} - \beta u''(\hat{c}_2)(1+i^\star)} < 0,$$

and thus $\bar{w}'(\hat{s}_2) > 0$. That is, household welfare is strictly increasing in $\hat{s}_2$. □

B Infinite Horizon

In this section we show how the two-period model discussed in the paper can be interpreted as an infinite horizon economy where the exchange rate policy from date 2 onward is stationary. Consider the economy described in Section 2, but now assume that $t = 1, \ldots$. The households choosec_t, a_t, f_t, m_t to maximize their objective function

$$\sum_{t=1}^{\infty}\beta^{t-1}\left[u(c_t) + h\left(\frac{m_t}{s_t}\right)\right],$$

subject to the budget constraints

$$y_t + T_t + \frac{m_{t-1} + a_{t-1}}{s_t} + f_{t-1} = c_t + \frac{m_t + q_t a_t}{s_t} + q_t^\star f_t,$$

where $q_t = 1/(1+i_t)$ and $q_t^\star = 1/(1+i_t^\star)$ are the prices, respectively in local and in foreign currency, of domestic and foreign discount bonds. We assume that the foreign interest rate is constant over time, and $\beta(1+i^\star) = 1$.

The time t budget constraint for the monetary authority is

$$\frac{M_t - M_{t-1} + A_{t-1}}{s_t} + F_{t-1} = \frac{q_t A_t}{s_t} + q_t^\star F_t + \tau_t,$$

while the budget constraint for the government is

$$\frac{q_t B_t}{s_t} + \tau_t = T_t + \frac{B_{t-1}}{s_t}.$$

The decision problem of foreign investors is the same as in the main text, and the market clearing conditions in domestic asset markets are

$$\begin{aligned} m_t + m_t^\star &= M_t \\ a_t + a_t^\star + A_t &= B_t. \end{aligned}$$

An equilibrium given an exchange rate policys_t is defined as in Section 2.

We assume throughout this section that the exchange rate policy that the Central Bank needs to implement from date 2 onward is stationary, $s_{t+1} = \alpha s_t$. For simplicity, we also assume that the endowment is constant from date 2 onward and equal to $\bar{y}$. We can then proove the following Lemma.

Lemma 3. *Consider the infinite horizon economy, and assume that* $(1 + i^\star)\alpha \geq 1$ *for all* $t \geq 2$. *Then, in the best monetary equilibrium we must have:*

(i) $c_t = \bar{c}$ for $t \geq 2$, with

$$\bar{c} = \bar{y} + i^\star \left[(F_1 + f_1) - \frac{a_1^\star + m_1^\star}{s_2} \right]$$

(ii) The welfare of the households from date 1 onward can be written as $\tilde{u}(\bar{c})$.

Proof. From the households' problem we know that in any equilibrium we must have

$$u'(c_t) = \beta(1 + i_t)\frac{s_t}{s_{t+1}} u'(c_{t+1}).$$

Because $(1 + i^\star)\alpha$ by assumption, we have that $(1 + i_t)\frac{s_t}{s_{t+1}} = (1 + i^\star)$ in the best monetary equilibrium. Therefore, the households set a constant consumption between any two periods t and $t + 1$ because $\beta(1 + i^\star) = 1$. The expression for $\bar{c}$ is a standard intertemporal resource constraint that tells us that consumption is equal to income plus the returns on the net foreign asset positions inherited from date 1. This expression is obtained by consolidating the households' budget constraint with that of the fiscal and monetary authority, and using market clearing for domestic assets. To obtain the expression for households' welfare, note that the money demand equation of households implies

$$h'\left(\frac{m_t}{s_t}\right) = u'(c_t)\frac{i_t}{1 + i_t}.$$

Because $s_{t+1} = \alpha s_t$ by assumption, we have that i_t is constant in the best monetary equilibrium. Therefore, from the previous equation we can express $h(m_t/s_t)$ as $\tilde{h}(\bar{c})$. We then have,

$$\sum_{t=2}^{\infty} \beta^{t-2} \left[u(c_t) + h\left(\frac{m_t}{s_t}\right) \right] = \sum_{t=2}^{\infty} \beta^{t-2} \left[u(\bar{c}) + \tilde{h}(\bar{c}) \right] = \tilde{u}(\bar{c}),$$

which proves the second part of the Lemma. □

Given this property, we can now show that there exists a two-period economy whose equilibrium outcome corresponds to the one of the infinite horizon model described in this section. For this purpose, let's fix an exchange rate policy (s_1, s_2) and assume that from date $t \geq 2$ the exchange rate policy is$\alpha = 1/(1+i^\star)$. This assumption guarantees that nominal interest rates from date 2 onward are at zero, and households utility from money is satiated. We can than verify that the allocation $(c_1, \bar{c}, F, f, m_1, a_1^\star, m_1^\star, i_1, q_1)$ that arises in the best monetary equilibrium of the infinite horizon economy can be derived from the following two-period planning problem[28]

$$\max_{c_1,\bar{c},F,f,m_1,m_1^\star,a_1^\star,i_1,q_1} \quad u(c_1) + h\left(\tfrac{m_1}{s_1}\right) + \tilde{u}(\bar{c}) \qquad s.t.$$

$$c_1 = y_1 - q_1^\star(F_1 + f_1) + \tfrac{m_1^\star + q_1 a_1^\star}{s_1}$$

$$\bar{c} = \bar{y} + i^\star \left[(F_1 + f_1) - \tfrac{a_1^\star + m_1^\star}{s_2} \right]$$

$$h'\left(\tfrac{m_1}{s_1}\right) = u'(c_1)(1 - q_1)$$

$$u(c_1) = \beta(1+i)\tfrac{s_1}{s_2}\tilde{u}'(\bar{c})(1-\beta)$$

$$(1 + i_1) \geq (1 + i^\star)\tfrac{s_2}{s_1}$$

$$m_1^\star = 0 \text{ if } i_1 > 0$$

$$\tfrac{a_1^\star + m_1^\star}{s_1} \in \begin{cases} w & \text{if } (1+i_1) > (1+i^\star)\frac{s_2}{s_1} \\ [0, \bar{w}] & \text{if } (1+i_1) = (1+i^\star)\frac{s_2}{s_1} \\ 0 & \text{otherwise.} \end{cases}$$

It is straightforward to verify that this planning problem would also implement the best monetary equilibrium of the two period economy described in Section 2, with $\bar{c} = r^\star c_2$and $\bar{y} = r^\star y_2$. Therefore, all our results carry over to the infinite horizon environment.

[28]The fact that the demand for money is satiated is important for this result. Otherwise, the planner would have an incentive to manipulate households' consumption in order to change its demand for money, and achieve higher welfare than what is achieved under a competitive equilibrium.

C Capital controls

In this section, we study how capital controls interact with foreign exchange interventions in our environment. We consider both price and quantity instruments. We proceed by first characterizing these two types of controls and then comparing the outcomes achieved in the two cases.

C.1 Taxes on inflows

We start by allowing the monetary authority to impose a tax, ϕ, on all capital inflows from foreign investors. We note that at the ZLB the practical implementation of such a policy faces an important hurdle. Taxes must be levied not only on foreign bond holdings, but also on foreign holdings of domestic money. Because bonds and money yield the same pre-tax return at the ZLB, taxing only bonds would have no implications for capital inflows, as the foreign investors would substitute toward money.

With taxes on bond and money inflows, the budget constraint of foreign investors in the second period becomes

$$c_2^\star = (1+i^\star)f^\star + (1-\phi)\left[(1+i)\frac{a^\star}{s_2} + \frac{m^\star}{s_2}\right],$$

while the budget constraint of the monetary authority in the second period changes to account for the revenues from capital inflows:

$$(1+i^\star)F + (1+i)\frac{A}{s_2} + \phi\left[(1+i)\frac{a^\star}{s_2} + \frac{m^\star}{s_2}\right] = \frac{M}{s_2} + \tau_2.$$

A monetary equilibrium is defined as in Definition 1, with the exception that the second-period budget constraints of the foreign investors and the monetary authority are replaced by the ones above. Note that the decision problem of the household is not affected by the tax policy. This implies that the conditions given by equations (18), (19), and (20) are still necessary conditions for a monetary equilibrium and that the households' foreign position, f, must satisfy the same conditions as before. That is,

$$f \geq 0 \text{ and } f\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] = 0. \tag{42}$$

However, the tax on foreign inflows affects the post-tax return on domestic assets held by foreigners. Hence, their demand of assets of the SOE now satisfies

$$\frac{a^\star + m^\star}{s_1} \in \begin{cases} w & \text{if } (1+i)(1-\phi) > (1+i^\star)\frac{s_2}{s_1} \\ [0, \bar{w}] & \text{if } (1+i)(1-\phi) = (1+i^\star)\frac{s_2}{s_1} \\ 0 & \text{otherwise.} \end{cases}$$

The intertemporal resource constraint can be obtained as before. In particular, from the budget constraint of the household and the money market clearing condition, and using the household's optimality condition with respect to f, we obtain

$$(y_1 - c_1) + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - \left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] F + \phi \left[\frac{a^\star + m^\star}{s_1}\right] = 0,$$

where the term $\phi \left[\frac{a^\star + m^\star}{s_1}\right]$ represents the potential income generated from taxing foreign capital inflows.

The trade deficit equation is as before, $c_1 = y_1 - (f + F) + \frac{a^\star + m^\star}{s_1}$. Hence, the intertemporal resource constraint becomes

$$(1-\phi)(y_1 - c_1) + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - \left[(1-\phi) - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] F + \phi f = 0, \tag{43}$$

where we have again used the optimality condition of the households with respect to f. Finally, the optimality conditions on $a^\star + m^\star$ can be restated in terms of c_1, f, and F as

$$c_1 - y_1 + f + F \in \begin{cases} \{\bar{w}\} & \text{if } (1+i)(1-\phi) > (1+i^\star)\frac{s_2}{s_1} \\ [0, \bar{w}] & \text{if } (1+i)(1-\phi) = (1+i^\star)\frac{s_2}{s_1} \\ \{0\} & \text{otherwise.} \end{cases} \tag{44}$$

A version of Lemma 1 applies here: an allocation $(c_1, c_2, f, F, i, m, \phi)$ is part of a monetary equilibrium if and only if equations (18), (19), (20), (42), (43), and (44) holds. Note that these conditions collapse to the ones of Lemma 1 when $\phi = 0$, as expected.[29]

We can now characterize the optimal tax on capital inflows, given an exchange rate objective and a fixed nominal interest rate i. Toward this goal, and for a given s_1, s_2, and i, let us define

[29] Note that, in Lemma 1, f can be dropped from the equilibrium conditions. However, in this case, when $\phi \neq 0$, f needs to be stated because the rate of return on assets is different, depending on whether the savings are originated by households or foreign investors.

c_1^{tax} as

$$c_1^{tax} \equiv \max\{c_1\} \text{ such that}$$
$$c_1 \leq y_1 + \bar{w} \tag{45}$$
$$y_1 - c_1 + \frac{y_2 - c_2^{EE}(c_1)}{1+i^\star} \geq 0 \tag{46}$$

where $c_2^{EE}(c_1) \equiv (u')^{-1}\left(\frac{s_2}{\beta(1+i)s_1}u'(c_1)\right)$.[30]

Let $\phi^{tax} \equiv 1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\left\{1 + \frac{1}{\bar{w}}\left[y_1 - c_1^\star + \frac{y_2 - c_2^{EE}(c_1^{tax})}{1+i^\star}\right]\right\}$. We will see below that ϕ^{tax} corresponds to the optimal tax. Equation (46) implies that $\phi^{tax} \geq 1 - \frac{s_2(1+i^\star)}{s_1(1+i)}$. And in particular, $\phi^{tax} = 1 - \frac{s_2(1+i^\star)}{s_1(1+i)}$ if and only if equation (46) holds with equality. We then have the following proposition,

Proposition 7. *Consider any equilibrium allocation $q = (c_1, c_2, f, F, i, m, \phi)$. There exists another equilibrium allocation $\hat{q} = (\hat{c}_1, \hat{c}_2, \hat{f}, \hat{F}, i, \hat{m}, \hat{\phi})$ with $\hat{c}_1 = c_1^{tax}$, $\hat{c}_2 = c_2^{EE}(c_1^{tax})$, $\hat{\phi} = \phi^{tax}$, $\hat{F} = \max\{y_1 - \hat{c}_1, 0\}$ and $\hat{f} = 0$ such that the utility to the households under $\hat{q}$ is weakly higher than under q.*

Proof. We first argue that in any equilibrium allocation, c_1 must satisfy (45) and (46).

To see this, note that $c_1 \leq y_1 - f - F + \bar{w}$, and thus inequality (45) follows. For inequality (46), note that equation (43) is equivalent to

$$(1-\phi)(y_1 - c_1) + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - \left[(1-\phi) - \frac{s_2(1+i^\star)}{s_1(1+i)}\right]F + \phi f = 0$$
$$(y_1 - c_1) + \frac{y_2 - c_2}{1+i^\star} - \frac{s_1(1+i)}{s_2(1+i^\star)}\left[(1-\phi) - \frac{s_2(1+i^\star)}{s_1(1+i)}\right](c_1 - y_1 + F + f) = 0$$

but $\left[(1-\phi) - \frac{s_2(1+i^\star)}{s_1(1+i)}\right](c_1 - y_1 + F + f) \geq 0$, from (44), and thus $(y_1 - c_1) + \frac{y_2 - c_2}{1+i^\star} \geq 0$. Finally, c_2 must satisfy the Euler equation, (18), which implies that $c_2 = c_2^{EE}(c_1)$. Taken together, all this confirms inequality (46).

Note that $c_2^{tax}(c_1)$ is strictly increasing in c_1. Thus, any equilibrium consumption allocation (c_1, c_2) will be strictly below $(c_1^{tax}, c_2^{EE}(c_1^{tax}))$, as the latter is the highest possible consumption allocation consistent with (45) and (46). That is, $c_1 \leq c_1^{tax}$ and $c_2 \leq c_2^{EE}(c_1^{tax})$.

Now let us check the equilibrium conditions for the $\hat{q}$ allocation. Conditions (18), (19), and (42) are automatically satisfied. We know from the above that equation (43) is equivalent to

$$(y_1 - c_1^{tax}) + \frac{y_2 - c_2^{EE}(c_1^{tax})}{1+i^\star} - \frac{s_1(1+i)}{s_2(1+i^\star)}\left[\left(1-\hat{\phi}\right) - \frac{s_2(1+i^\star)}{s_1(1+i)}\right](c_1^{tax} - y_1 + \hat{F}) = 0$$

[30] Note that the $c_1^{tax}, c_2^{EE}(c_1^{tax})$ does not necessarily correspond to the first best allocation. They coincide only when $(1+i)\frac{s_1}{s_2} = 1 + i^\star$, that is, when the interest parity condition holds from the household's perspective.

and using the definition of $\hat{\phi}$ we get that the above is equivalent to

$$\left[y_1 - c_1^{tax} + \frac{y_2 - c_2^{EE}(c_1^{tax})}{1+i^\star}\right](\bar{w} - c_1^{tax} + y_1 - \hat{F}) = 0$$

But from the solution to the tax problem, we have that either $y_1 - c_1^{tax} + \frac{y_2 - c_2^{EE}(c_1^{tax})}{1+i^\star} = 0$ or $c_1^{tax} = y_1 + \bar{w}$. But not that if $c_1^{tax} = y_1 + \bar{w}$, then $\hat{F} = 0$. And thus the equation is satisfied.

Equation (44) is satisfied as $(1+i)(1-\hat{\phi}) = (1+i^\star)\frac{s_2}{s_1}$ and

$$c_1^{tax} - y_1 + \hat{f} + \hat{F} = c_1^{tax} - y_1 + \max\{y_1 - c_1^{tax}, 0\}.$$

This last either equals 0, when $y_1 \geq c_1^{tax}$ or equals $c_1^{tax} - y_1$, when $c_1^{tax} - y_1 > 0$. But $c_1^{tax} - y_1 \leq \bar{w}$, and thus $c_1^{tax} - y_1 + \hat{f} + \hat{F} \in [0, \bar{w}]$, satisfying the requirement for Equation (44).

Note that at the q allocation, m satisfies $h(m/s_1) = H(i, c_1)$ where H is as defined in equation (40). In addition, we can find an $\hat{m}$ such that $h(\hat{m}/s_1) = H(i, \hat{c}_1)$, which guarantees that (20) holds for the $\hat{q}$ allocation.

As a result, the welfare of the households under the $\hat{q}$ allocation, $u(\hat{c}_1) + \beta u(\hat{c}_2) + H(i, \hat{c}_1)$, is higher than under the q allocation, $u(c_1) + \beta u(c_2) + H(i, c_1)$, as all functions are monotonically increasing in consumption, and $c_1^{tax} \geq c_1$ and $c_2^{EE}(c_1^{tax}) \geq c_2$. □

Proposition 7 tells us that when $y_1 < c_1^{tax}$, the monetary authority can achieve its exchange rate objective without accumulating foreign reserves.[31] When $y_1 > c_1^{tax}$, the monetary authority accumulates reserves, $\hat{F} > 0$, but at the same time, it is optimal to leave foreign investors indifferent between home and foreign assets, $\phi^{tax} = 1 - \frac{s_2(1+i^\star)}{s_1(1+i)}$. In this manner, the foreign exchange interventions by the monetary authority cease to be costly, as the country is able to completely extract the rents made by foreigners in these transactions.

Overall, the introduction of a capital tax instrument grants the monetary authority the ability to implement any exchange rate policy for any nominal interest rate without the need to engage in costly foreign exchange interventions.

Interestingly, the imposition of an optimal capital flow tax restores the comparative statics of the optimal allocation back to their more standard signs: higher foreign wealth, $\bar{w}$, and lower foreign interest rates (when the country is a borrower) weakly increase domestic welfare.

[31] In this case, the monetary authority may be unable to extract all the rents. This occurs if constraint (45) binds before (46) when solving for c_1^{tax}. That is, wealth is sufficiently scarce, and the monetary authority is unable to tax all of the profits that accrue to foreign investors and maintain a competitive equilibrium at the same domestic nominal interest rate.

C.2 Quantity controls

An alternative policy to taxing capital inflows is to place quantity restrictions on these flows. In this section, we consider a situation in which the monetary authority imposes a cap on the amount of foreign wealth that can be invested in domestic assets.

We model quantity restrictions as a direct reduction in the amount of foreign wealth available to be invested at home. That is, the total wealth in the foreign investors problem now becomes a choice variable of the monetary authority, which we denote by $w \in [0, \bar{w}]$. As a result, the definition of an equilibrium allocation is as in Definition 1, with the exception that $\bar{w}$ is replaced by $w \in [0, \bar{w}]$, which is now an equilibrium object.

We already know, from the discussion in Section 5.1, that a marginal decrease in $\bar{w}$ when the economy operates at the ZLB is beneficial. This immediately implies that a quantity restriction on capital inflows can be welfare enhancing. We can, however, say a bit more, and characterize the optimal quantity restriction on capital inflows, given an exchange rate objective and a fixed nominal interest rate i.

Toward this goal, and for a given s_1, s_2, and i, let us define c_1^{qc} as

$$\begin{aligned} c_1^{qc} \equiv & \max\{c_1\} \text{ such that} \\ & \text{equation (45) holds and} \\ & (y_1 - c_1) + \frac{y_2 - c_2^{EE}(c_1)}{(1+i^\star)} = \left[\frac{s_1(1+i)}{s_2(1+i^\star)} - 1\right] \max\{c_1 - y_1, 0\}. \end{aligned} \tag{47}$$

We then have the following proposition

Proposition 8. *Consider any equilibrium allocation $q = (c_1, c_2, F, i, m, w)$ with $w \leq \bar{w}$. Then there exists another equilibrium allocation $\hat{q} = (\hat{c}_1, \hat{c}_2, \hat{F}, i, \hat{m}, \hat{w})$ with $\hat{c}_1 = c_1^{qc}$, $\hat{c}_2 = c_2^{qc}$, $\hat{F} = \max\{y_1 - \hat{c}_1, 0\}$ and $\hat{w} = \max\{\hat{c}_1 - y_1, 0\} \leq \bar{w}$, and such that the household's welfare is higher under $\hat{q}$ than under q.*

Proof. First, let us construct the $\hat{q}$ allocation and argue that it is an equilibrium.

First note that (18) holds, given the construction of $\hat{c}_1$ and $\hat{c}_2 = c_2^{EE}(\hat{c}_1)$. Given that (19) holds in the q allocation , it is therefore also satisfied under the $\hat{q}$ allocation, as i did not change. The resource constraint, (22), is

$$y_1 - \hat{c}_1 + \frac{y_2 - \hat{c}_2}{\frac{s_1}{s_2}(1+i)} - \hat{F}\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] = 0.$$

But $\hat{c}_1 = y_1 - \hat{F} + \hat{w}$ (by construction), and thus

$$\begin{aligned} y_1 - \hat{c}_1 + \frac{y_2 - \hat{c}_2}{\frac{s_1}{s_2}(1+i)} - (y_1 - \hat{c}_1 + \hat{w})\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] &= 0 \\ \left(\frac{s_2(1+i^\star)}{s_1(1+i)}\right)(y_1 - \hat{c}_1) + \frac{y_2 - \hat{c}_2}{\frac{s_1}{s_2}(1+i)} - \hat{w}\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] &= 0 \\ (y_1 - \hat{c}_1) + \frac{y_2 - \hat{c}_2}{(1+i^\star)} &= \hat{w}\left[\frac{s_1(1+i)}{s_2(1+i^\star)} - 1\right] \end{aligned}$$

Using that $\hat{w} = \max\{\hat{c}_1 - y_1, 0\}$, then the above equation is equivalent to (47). It then follows that (22) is satisfied. Note that by construction

$$\hat{c}_1 = y_1 - \hat{F} + \hat{w}$$

so (23) is automatically satisfied. Finally, let $\hat{m}$ be the value such that $h(\hat{m}/s_1) = H(i, \hat{c}_1)$, where H is as defined in equation (40), and it follows that (20) is satisfied at the $\hat{q}$ allocation. As a result, the $\hat{q}$ allocation constitutes a monetary equilibrium with wealth $\hat{w} \leq \bar{w}$ (where this one follows from (45).

Now note that any equilibrium with a wealth constraint w must satisfy (45) and (47). To see this note that (45) must hold in any equilibrium as it is implied by (23) and that $\hat{F} \geq 0$. In addition, because (18), (22), and (23) hold, it follows that, in any equilibrium,

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} = (y_1 - c_1 + w)\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right]$$

and thus (47) holds.

As a result, $c_1 \leq c_1^{qc}$, as c_1^{qc} is the highest consumption that satisfies the necessary equilibrium conditions (45) and (47). From (18), we get that $c_2 \leq c_2^{EE}(c_1^{qc})$. Hence, for any other equilibrium with interest rate i, it must be the case that

$$u(\hat{c}_1) + \beta u(\hat{c}_2) + H(i, \hat{c}_1) \geq u(c_1) + \beta u(c_2) + H(i, c_1),$$

as both u and H are increasing in c. □

Proposition 8 indicates that the monetary authority can improve upon any allocation by restricting capital inflows to the point where there is no longer a need to accumulate foreign assets in order to implement the desired exchange rate policy, $\hat{F} = 0$, or where the capital inflows are completely restricted, $\hat{w} = 0$. That is, given any allocation, the monetary authority can find an alternative allocation that maintains the same nominal interest rate and the same exchange rate policy, but without incurring intervention losses. This ability to insulate the conduct of monetary

policy from potentially large intervention losses is the main benefit of capital controls.[32]

C.3 Price versus quantity controls

Which type of controls achieve higher welfare? Propositions 7 and 8 imply that what can be achieved with the quantity control policy can always be achieved with a price policy. But the reverse is not true.[33]

When $\hat{w} = 0$ under a quantity restriction, the allocation that results is identical to the allocation that could have been achieved with a tax on capital inflows. In both of these cases, the foreign investors obtain no arbitrage profits, either because they are not investing in the country's assets ($\hat{w} = 0$) or because their profits are being completely taxed by the monetary authority.

If under the quantity controls we have $\hat{w} > 0$, however, then the allocation that results with quantity controls could be strictly improved upon by a capital inflow tax as in Proposition 7, because profits are still being been made by the foreign investors, which the monetary authority could, in principle, tax.[34]

D Optimal exchange rate policy

In this appendix, we provide additional details for Section 6.

D.1 Monetary equilibrium in extended model with sticky wages

The definition of a monetary equilibrium is the same as before, modified for the inclusion of nominal wages, as well as for the replacement of money balances in the utility function by a nonnegativity constraint on the nominal interest rate.

Definition 3. A monetary equilibrium, given an exchange rate policy (s_1, s_2), is a consumption profile for households, $(c_1^T, c_2^T, c_1^N c_2^N)$, asset positions, (a, f), employment (l^T, l^N), a consumption for investors, $c^\star$, and their asset positions, $(a^\star, f^\star)$; transfers from the fiscal to the monetary authority, (τ_1, τ_2); investments by the monetary authority, (A, F); transfers from the fiscal authority to the households, (T_1, T_2); and a domestic interest rate i, such that

[32] With log utility, or with constant relative risk aversion in general, we can obtain in closed form that consumption in period 1 and period 2 is decreasing in $\bar{w}$: $c_1 = \left(\frac{y_2+y_1(1+r^*)-\bar{w}(s_1/s_2-(1+r^*))}{\beta s_1/s_2+(1+r^*)}\right), c_2 = \beta(1+r)\left(\frac{y_2+y_1(1+r^*)-\bar{w}(s_1/s_2-(1+r^*))}{\beta s_1/s_2+(1+r^*)}\right)$.

[33] To see this note that (47) implies (46). That is, in both cases, the allocation needs to satisfy equation (45), but with the quantity control policy, the allocation must satisfy the stricter condition (47) rather than (46).

[34] The difference between capital controls based on quantities and those based on taxes contrasts with standard equivalence results (see, e.g., Bianchi, 2011).

(i) the domestic households make consumption and portfolio choices to maximize utility, subject to their budget and borrowing constraints, and the supply of hours equals the labor demand $n_t = l_t^T + l_t^N$;

(ii) foreign investors make consumption and portfolio choices to maximize their utility, subject to their budget and borrowing constraints;

(iii) the purchases of assets by the monetary authority, its decision about the money supply, and its transfers to the fiscal authority satisfy its budget constraints, as well as $F \geq 0$;

(iv) the fiscal authority satisfies its budget constraints;

(v) firms' employment (l_t^T, l_t^N) maximizes profits;

(vi) the domestic market for bonds, employment and non-tradables goods clear

$$\begin{aligned} a + a^\star + A &= B \\ c_t^N &= \left(l_t^N\right)^\alpha \\ n_t &= l_t^T + l_t^N \end{aligned}$$

(vii) and the nominal interest rate is nonnegative, $i \geq 0$.

D.2 Central bank's problem

The central bank's problem consists of choosing $(c_1^T, c_2^T, l_1, l_2, s_1, s_2, i, F)$ to maximize lifetime utility subject to resource constraints, and implementability constraints given by households', firms' and foreign investors' optimality conditions.

The central bank's problem can be written as

$$\max_{\{c_1^T, c_2^T, l_1^T, l_1^N, l_2^T, l_2^N, s_1, s_2, i, F\}} \sum_{t=1,2} \beta^{t-1} \left[\phi \log(c_t^T) + (1-\phi)\log(c_t^N) + \chi \log(1 - l_t^T - l_t^N)\right] \tag{48}$$

subject to

$$
\begin{aligned}
(1+i^\star)y_1 + y_2 - \left(\frac{s_1}{s_2}(1+i) - (1+i^\star)\right)\bar{w} &\geq (1+i^\star)c_1^T + c_2^T \\
c_t^N &= (l_t^N)^\alpha \\
l_t^N &= \frac{1-\phi}{\phi}\frac{\alpha c_t^T s_t}{\bar{p}^w}. \\
l_t^T &= \left(\frac{\alpha s_1}{\bar{p}^w}\right)^{1/(1-\alpha)} \\
s_1(1+i) &\geq s_2(1+i^\star) \\
\frac{c_2^T}{\beta c_1^T} &= \frac{s_1}{s_2}(1+i) \\
y_1^T - c_1^T + \bar{w} &\geq F \geq 0 \\
i &\geq 0
\end{aligned}
$$

with $y_1^T - c_1^T + \bar{w} = F$ if $s_1(1+i) > s_2(1+i^\star)$.

Note that in the above problem it is always feasible to set $F = y_1^T - c_1^T + \bar{w}$, which implies that F (and the complementary slackness condition) can be dropped from the maximization and replaced with just the constraint that $y_1^T - c_1^T + \bar{w} \geq 0$. This is the problem that is solved in Figures 9 and 6.

It is straightforward to see that if the ZLB does not bind, the central bank can achieve the first-best allocations. Replacing the employment in the non-tradable sector by (31) and the employment in the tradable sector by (27), which we replace by $\hat{l}^T(s_t)$, and using market clearing (30), we can write the Lagrangian as follows:

$$
\begin{aligned}
\mathcal{L}(c_1^T, c_1^N, c_2^T, c_2^N, s_1, s_2, i) = {} & \phi \log(c_1^T) + (1-\phi)\log(\hat{l}^N(c_1^T, s_1)^\alpha) + \chi \log(1 - \hat{l}^N(c_1^T, s_1) - \hat{l}^T(s_1)) + \\
& \beta\phi \log(c_2^T) + \beta(1-\phi)\log(\hat{l}^N(c_2^T, s_2)^\alpha) + \beta\chi \log(1 - \hat{l}^N(c_2^T, s_2) - \hat{l}^T(s_2)) \\
& + \lambda \left[(1+i^\star)\left(\hat{l}^T(s_1)^\alpha - c_1^T\right) + \left(\hat{l}^T(s_2)^\alpha - c_2^T\right) - \left(\frac{s_1}{s_2}(1+i) - (1+i^\star)\right)\bar{w} \right] \\
& + \zeta(s_1(1+i) - s_2(1+i^\star)) \\
& + \xi(c_2^T s_2 - s_1(1+i)\beta c_1^T) \\
& + \eta(y_1^T - c_1^T + \bar{w}) \\
& + \nu i.
\end{aligned}
$$

The first-order conditions with respect to c_1^T, c_2^T, s_1, s_2 are as follows:

$$c_1^T :: \quad \frac{\phi}{c_1^T} + \frac{1-\phi}{(\hat{l}_1^N)^\alpha}\alpha(\hat{l}_1^N)^{\alpha-1}\frac{\partial \hat{l}_1^N}{\partial c_1^T} - \frac{\chi\frac{\partial \hat{l}_t^N}{\partial c_1^T}}{1-n_1} - \xi s_1(1+i)\beta - \eta = \lambda(1+i^*) \tag{49}$$

$$c_2^T :: \quad \beta\left[\frac{\phi}{c_2^T} + \frac{1-\phi}{(\hat{l}_2^N)^\alpha}\alpha(\hat{l}_2^N)^{\alpha-1}\frac{\partial \hat{l}_2^N}{\partial c_2^T} - \frac{\chi\frac{\hat{l}_2^N}{\partial c_2^T}}{1-n_2}\right] + \xi s_2 = \lambda \tag{50}$$

$$\begin{aligned} s_1 :: \quad & \lambda(1+i^\star)\alpha(\hat{l}_1^T)^{\alpha-1}\frac{\partial \hat{l}_1^T}{\partial s_1} + \frac{1-\phi}{(l_1^N)^\alpha}\alpha(l_1^N)^{\alpha-1}\frac{\partial \hat{l}_1^N}{\partial s_1} \\ & - \frac{\chi\left(\frac{\partial \hat{l}_1^T}{\partial s_1} + \frac{\partial \hat{l}_1^N}{\partial s_1}\right)}{1-n_1} - \xi c_1^T\beta(1+i) + \zeta(1+i) = \frac{\lambda(1+i)\bar{w}}{s_2} \end{aligned} \tag{51}$$

$$\begin{aligned} s_2 :: \quad & \lambda\alpha(\hat{l}_2^T)^{\alpha-1}\frac{\partial \hat{l}_2^T}{\partial s_2} + \beta\frac{1-\phi}{(l_2^N)^\alpha}(l_2^N)^{\alpha-1}\alpha\frac{\partial \hat{l}_2{}^N}{\partial s_2} - \frac{\beta\chi\left(\frac{\partial \hat{l}_2^T}{\partial s_2} + \frac{\partial \hat{l}_2^N}{\partial s_2}\right)}{1-n_2} \\ & + \xi c_2^T - \zeta(1+i^*) = -\frac{\lambda(1+i)s_1\bar{w}}{s_2^2}. \end{aligned} \tag{52}$$

Consider a state with deviation from the IP condition and binding ZLB, which implies $\nu > 0, i = 0$, and $\zeta = 0$, by complementary slackness conditions. Rearranging (51) and (52), we obtain equations (32) and (33) in the main text.

E Calculating the losses

Our formula in equation (24) is an approximation because the Swiss National Bank holds several assets in the form of foreign reserves that differ by maturity, currency of denomination, and underlying riskiness. The appropriate way to measure the losses would be that of computing CIP deviations for different currencies and at different horizons, and appropriately matching these gaps with the different asset purchases made by the SNB.

Because it is not feasible to compute deviations from CIP at every horizon, we approximate the losses as follows. We will assume that all assets in the SNB balance sheet are three months zero coupon bonds, denominated in U.S. dollars. We observe the monthly market value of the reserves portfolio from the SNB. Let us denote that series by S_t. Let n_t denote the amount of foreign denominated zero coupon bonds purchased by the SNB in period t.

The market value of the foreign reserve portfolio at the end of period t is

$$S_t = q_t^3 n_t + q_t^2 n_{t-1} + q_t^1 n_{t-2},$$

where q_t^i is the international price at time t of a zero-coupon bond that matures in i periods.

The amount of foreign reserves purchased in period t is then

$$F_t = q_t^3 n_t.$$

And thus, we have that the market value of the stock can be written as

$$S_t = F_t + \frac{q_t^2}{q_{t-1}^3} F_{t-1} + \frac{q_t^1}{q_{t-2}^3} F_{t-2}.$$

We know that $q_t^3 = (1 + i_t^\star)^{-1}$, and we approximate the one-period and the two-period-ahead interest rate to be $q_t^2 = (1 + i_t^\star)^{-2/3}$ and $q_t^1 = (1 + i_t^\star)^{-1/3}$.

Using the observed series for S_t, and the three-month international interest rate, $i_t^\star$, we can then solve for the series F_t that solves

$$S_t = F_t + \frac{1 + i_{t-1}^\star}{(1 + i_t^\star)^{2/3}} F_{t-1} + \frac{1 + i_{t-2}^\star}{(1 + i_t^\star)^{-1/3}} F_{t-2},$$

for some starting points with $F_{t_0} = F_{t_0+1} = 0$.

Having computed F_t, we then apply the formula (24) to calculate the monthly losses of the central bank.