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Estimation and Solution of Models with Expectations and Structural Changes

Mariano Kulish and Adrian Pagan

RDP 2012-08

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Abstract

Standard solution methods for linearised models with rational expectations take the structural parameters to be constant. These solutions are fundamental for likelihood-based estimation of such models. Regime changes, such as those associated with either changed rules for economic policy or changes in the technology of production, can generate large changes in the statistical properties of observable variables. In practice, the impact of structural change on estimation is often addressed by selecting a sub-sample of the data for which a time-invariant structure seems valid. In this paper we develop solutions for linearised models in the presence of structural changes using a variety of assumptions relating to agents' beliefs when forming expectations, and whether the structural changes are known in advance. The solutions can be put into state space form and the Kalman filter used for constructing the likelihood function. Structural changes and varying beliefs trigger movements in the reduced-form coefficients and hence model variables follow a time-varying coefficient VAR. We apply the techniques to two examples: a disinflation program and a transitory slowdown in trend growth.

JEL Classification Numbers: C13, C32, C63, E52
Keywords: expectations, structural change, regime change, DSGE, maximum likelihood, Kalman filter

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1. Introduction

Standard solution methods for linear rational expectations models, like Blanchard and Kahn (1980), Binder and Pesaran (1995), Uhlig (1995), King and Watson (1998), Klein (2000), Sims (2002) and Anderson (2010), deal with the case where the parameters of the structural model are constant. These methods are at the heart of likelihood-based estimation of such models. In practice, the magnitude of changes in the properties of observable variables is used to help define sub-samples for which a time-invariant structure seems valid, and estimation is then done with these sub-samples.[1] The analysis of Lubik and Schorfheide (2004), for instance, is based on the assumption that the target inflation rate in the United States – like other structural parameters – stayed constant in the pre-Volcker years, but then possibly shifted in the early 1980s, based on the estimates of steady-state inflation in each of the sub-samples.

Findings of structural instabilities seem to apply to many models of macroeconomic aggregates. While we cannot do justice to the complete literature, one can point to the work of Clarida, Galí and Gertler (2000) who find a significant difference in the way monetary policy was conducted pre- and post-late 1979 in the United States; Ireland (2001) who detects shifts in the discount factor; Inoue and Rossi (2011) who show that the Great Moderation was due to both changes in shock volatilities and policy and private sector parameters; and Stock and Watson (2007) who provide evidence of changes in the variance of shocks to trend inflation.

Our objective in this paper is to develop solutions for linear stochastic models with model-consistent expectations in the presence of structural changes that are possibly foreseen. The solution extends one recently proposed by Cagliarini and Kulish (forthcoming) by providing an econometric representation, namely a state

1 An interesting exception is Cùrdia and Finocchiaro (2005) who estimate a model for Sweden with a monetary regime change.

space form, to which the Kalman filter can be applied to construct the likelihood function of the data. As we show below, the reduced-form solution takes the form of a time-varying coefficients VAR, where movements in the coefficients are governed by the nature of the underlying structural changes.

In the basic case we assume that expectations are formed in such a way as to be consistent with whatever structure (model) holds at each point in the sample. This is analysed in Section 3 by looking at cases where the structural changes are either unknown in advance or where there is some foresight about them. In this scenario a second structure (model) will hold at some given future date and, because agents know what that date is, they factor it into the formation of their expectations before the actual date at which the change occurs. Examples of this latter situation could be a change in inflation targets or an announcement about the introduction of a tax. Section 3 also deals with the situation where beliefs about the structural change can be different from the truth (reality). Thus, if one thinks of a single structural change in the sample, expectations may be based on the first period model for some time into the second period. Of course, eventually it seems reasonable to think that beliefs must centre upon the second period model. We simply specify when beliefs agree with this second model and do not model any learning behaviour.

The particular case in which the structure evolves and agents' beliefs are fully aligned with reality coincides with the problem posed by Cagliarini and Kulish (forthcoming). The generalisation of this paper involves allowing a difference between beliefs and reality to exist for a period of time before, after or during a sequence of structural changes. This generalisation is useful for at least two reasons. First, it is capable of capturing the consequences of structural changes that may go temporarily unnoticed as, for example, happened during the US productivity slowdown of the early 1970s. Second, it may be used to capture the impact of policy announcements which are less than perfectly credible.

Many of the issues we address have long been recognised in the literature. In fact, more than half a century ago Marschak (1953) noticed that, in the case of an anticipated structural change, the purely empirical projections of observed past regularities into the future would not be a reliable guide for decision-making, unless past observations were supplemented by some knowledge of the way the structure was expected to change. Since then the technical apparatus has changed a great deal, but these insights are just as powerful today. In the context of estimating and solving dynamic stochastic models with expectations, some knowledge of the structural changes that might have taken place in-sample allows us to increase the number of observations that are usable in estimation, and therefore has the potential of improving the quality of the estimation. But regardless of what may be the situation in a particular application, this paper is the first to provide the tools to accomplish maximum likelihood estimation of dynamic stochastic economies with structural changes under a variety of assumptions regarding expectations formation.

The paper is organised as follows. The next section reviews the Binder and Pesaran (1997) solution procedure for models with forward-looking expectations. As mentioned previously, Section 3 then extends the solution to situations of structural change and derives the likelihood for the implied model. Section 4 introduces two examples. The first is an anticipated credible disinflation program while the second is a temporary fall in trend growth happening alongside a looser monetary policy. Section 5 concludes and Appendix A provides details of the construction of the log likelihood with the Kalman filter.

2. Solution of Models with Forward-looking Expectations and No Structural Changes

Our solution method is a variant of Binder and Pesaran (1997). Following that paper, a linear rational expectations model of n equations can be written as

$$A_0 y_t = C_0 + A_1 y_{t-1} + B_0 \mathbb{E}_t y_{t+1} + D_0 \varepsilon_t, \tag{1}$$

where y_t is a $n \times 1$ vector of state and jump variables and ε_t is a $l \times 1$ vector of exogenous variables. With no loss of generality we take the latter to be white noise and to have I_l as their covariance matrix. All matrices in Equation (1) conform to the specified dimensions.[2] The formulation can be generalised as in Binder and Pesaran (1997) to allow additional lags of y_t as well as conditional expectations at different horizons and from earlier dates.

If it exists and is unique, the solution to Equation (1) will be a VAR of the form

$$y_t = C + Qy_{t-1} + G\varepsilon_t. \tag{2}$$

Given that this is the solution and $\mathbb{E}_t\varepsilon_{t+1} = 0$ we must have $\mathbb{E}_t y_{t+1} = C + Qy_t$. Substituting this into Equation (1) and re-arranging terms produces

$$y_t = (A_0 - B_0 Q)^{-1}(C_0 + B_0 C + A_1 y_{t-1} + D_0\varepsilon_t). \tag{3}$$

Now $(A_0 - B_0 Q)^{-1} = (I - A_0^{-1} B_0 Q)^{-1} A_0^{-1}$ and defining $\Gamma \equiv A_0^{-1} C_0$, $A \equiv A_0^{-1} A_1$, $B \equiv A_0^{-1} B_0$ and $D \equiv A_0^{-1} D_0$, Equation (3) becomes

$$y_t = (I - BQ)^{-1}(\Gamma + BC + Ay_{t-1} + D\varepsilon_t).$$

But this must equal Equation (2), establishing the equivalences

$$\begin{aligned} (I - BQ)^{-1}(\Gamma + BC) &= C, && (4)\\ (I - BQ)^{-1}A &= Q, && (5)\\ (I - BQ)^{-1}D &= G. && (6) \end{aligned}$$

Equation (5) implies that

$$A - Q + BQ^2 = 0$$

and so determines Q. Equation (4) implies that

$$C = [(I - (I - BQ)^{-1}B]^{-1}(I - BQ)^{-1}\Gamma = (I - F)^{-1}\Lambda,$$

2 We may need to make a distinction between the original shocks of a dynamic stochastic model, e_t, and the shocks ε_t in Equation (1). Often e_t are taken to be serially correlated. This can be captured by writting such a system in the form of Equation (1) with lagged values of the endogenous variables included in y_t. This means that ε_t are the innovations to the shock processes e_t. There may be a numerical advantage to working with e_t rather than ε_t, as that reduces the dimension of y_t and, consequently, all the matrices involved in finding a solution. But there are conceptual advantages in using the system we work with. Our MATLAB function that computes the Binder Pesaran solution, ‘smatsbp.m’, does allow us to work with an e_t that is described by a VAR process.

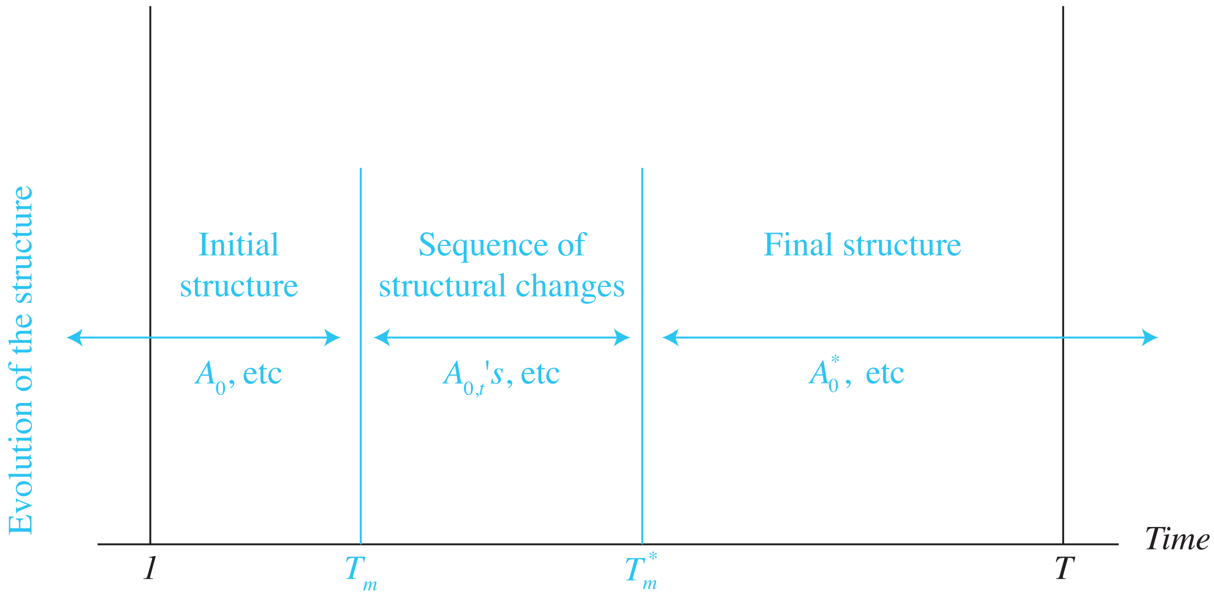
where $\Lambda = (I - BQ)^{-1}\Gamma$, $F = (I - BQ)^{-1}B$. Thus, once Q is found, it is possible to derive C and G, providing the solution to the model.

3. Solutions with Structural Changes

Before we discuss solutions to the different cases, it is useful to introduce some notation. First, there is a sample of data running from $t = 1, 2, ..., T$. Second, we allow for a number of structural changes over the sample period. Hence we begin by assuming that the first structural change is at T_m and the last is at T_m^*. Accordingly, the initial model is replaced by a new one at T_m, following which there may be a sequence of models until T_m^*, when a final model is in place. After T_m^* no further structural changes are assumed to take place (and we will say that the structure has converged). Notice that, given these definitions, if there is just a single structural change then it begins at $T_m = T_m^*$, since the model after the initial one is the final model.

Figure 1 illustrates one possibility. The arrows describe the evolution of the structure. The sequence of structural changes begins in T_m and ends in T_m^*. In Figure 1, just as in our later examples, T_m and T_m^* take place in-sample, although nothing about our solutions requires this to be the case. Further, in practice, one might also have many structural changes in the model parameters (and these could possibly overlap); it suffices to establish the solutions with a single sequence of structural changes.

Figure 1: Timing of Structural Changes



A formal account of the description above follows. Formally it is being assumed that before T_m the structure is stable at Equation (1). Then, during $t = T_m, \ldots, T_m^* - 1$ the structure evolves as

$$A_{0,t}y_t = C_{0,t} + A_{1,t}y_{t-1} + B_{0,t}\mathbb{E}_t y_{t+1} + D_{0,t}\varepsilon_t, \tag{7}$$

subsequently changing over during $t = T_m^*, \ldots, T$ to

$$A_0^* y_t = C_0^* + A_1^* y_{t-1} + B_0^* \mathbb{E}_t y_{t+1} + D_0^* \varepsilon_t. \tag{8}$$

Thereafter, there are no further structural changes and Equation (8) holds into the infinite future.

To be concrete suppose there are two structural changes in the sample. In the first interval (1 to $T_m - 1$) there is a model whose coefficients are $\theta = \{A_0, C_0, A_1, B_0, D_0\}$. In the second interval (T_m to $T_m^* - 1$) these change to $\bar{\theta} = \{\bar{A}_0, \bar{C}_0, \bar{A}_1, \bar{B}_0, \bar{D}_0\}$ and in the final interval (T_m^* to T) to $\theta^* = \{A_0^*, C_0^*, A_1^*, B_0^*, D_0^*\}$. The notation in Equation (7) allows the parameters $A_{0,t}$ etc to vary according to the time period but in the two structural change case $A_{0,t} = \bar{A}_0$ etc from T_m to $T_m^* - 1$ and after that the structure converges to A_0^* etc. In general, when a sequence of structural changes takes place in-sample, the structural matrices are given by $\{\{A_0, C_0, A_1, B_0, D_0\}_{t=1}^{T_m-1}, \{A_{0,t}, C_{0,t}, A_{1,t}, B_{0,t}, D_{0,t}\}_{t=T_m}^{T_m^*-1}, \{A_0^*, C_0^*, A_1^*, B_0^*, D_0^*\}_{t=T_m^*}^{T}\}$.

In the first numerical example of Section 4 we will consider a single structural change as opposed to a sequence of them, and so we will often refer to the interval $t = 1, ..., T_m^* - 1$ as the 'first interval' and $t = T_m^*, ...$ as the 'second interval'. The second of our illustrations in Section 4 refers to two structural changes.

3.1 Regime Shifts with Beliefs Matching Reality

As seen in the solution method for models without structural change, a key element is to replace the forward expectations with a function that is consistent with the existing model and the information agents possess. Thus we need to specify how expectations are to be formed at a point in time and what information is available to agents at that point. We consider two cases. In the first case we take agents' beliefs about the *prevailing* structure to be accurate (i.e. beliefs match reality). The sequence of structural changes given by Equations (7) and (8) are taken to be known *once they occur*. In the second case it is assumed that the sequence of structural changes given by Equations (7) and (8) is *foreseen* from T_m. In particular, from period T_m onwards agents know when all future structural changes occur i.e. at the time of the first structural change they know exactly when future changes will take place.[3]

3.1.1 Structural changes known once they occur

To begin, take the simple case of a single structural change. Up until $T_m - 1 = T_m^* - 1$, agents will assume that the first interval model with coefficients $\theta = \{A_0, B_0, ...\}$ is going to continue indefinitely. Hence the solution is that for the no structural change case i.e. $y_t = C + Qy_{t-1} + D\varepsilon_t$. From T_m^* onwards, agents form expectations with the final model that has coefficients $\theta^* = \{A_0^*, B_0^*, ...\}$ and so the solution will be $y_t = C^* + Q^* y_{t-1} + D^* \varepsilon_t$. So one simply uses the model that holds at any point t to compute the solution for y_t. Clearly, the solution generalises to any number of structural changes.

3 It will be obvious from the solution method that we can handle situations where only some of the future structural changes are known at T_m.

3.1.2 Foreseen structural changes

Now consider what happens if, after the first structural change, agents know when all future changes will take place. In this situation expectations need to be formed which recognise that agents know that different model(s) will hold at some point in the future. In general, from T_m onwards the solution for y_t at any point in time will be a time-varying VAR of the form

$$y_t = C_t + Q_t y_{t-1} + G_t \varepsilon_t. \tag{9}$$

Because the information about future structures (models) is taken to be certain and non-stochastic, it follows that $\mathbb{E}_t y_{t+1} = C_{t+1} + Q_{t+1} y_t$. Then, following the earlier solution method, we would get the equivalent conditions to Equations (4) to (6) as

$$\begin{aligned}
(I - B_t Q_{t+1})^{-1}(\Gamma_t + B_t C_{t+1}) &= C_t && (10)\\
(I - B_t Q_{t+1})^{-1} A_t &= Q_t && (11)\\
(I - B_t Q_{t+1})^{-1} D_t &= G_t, && (12)
\end{aligned}$$

where, as before, $\Gamma_t \equiv A_{0,t}^{-1} C_{0,t}$, $A_t \equiv A_{0,t}^{-1} A_{1,t}$, $B_t \equiv A_{0,t}^{-1} B_{0,t}$ and $D_t \equiv A_{0,t}^{-1} D_{0,t}$. There are two key differences. One is the second condition which now becomes

$$A_t - Q_t + B_t Q_{t+1} Q_t = 0, \tag{13}$$

so that to solve for Q_t we need to use a backward recursion. To do so, we start from the solution of the final structure $Q_{T_m^*} = Q^*$, and choose the sequence $\{Q_t\}_{t=T_m}^{T_m^*-1}$ that satisfies Equation (13). The second difference is the first condition which can now be written as

$$\Lambda_t + F_t C_{t+1} = C_t$$

where $\Lambda_t = \left(I - B_t Q_{t+1}\right)^{-1} \Gamma_t$ and $F_t = \left(I - B_t Q_{t+1}\right)^{-1} B_t$. With Q_t in hand it is possible to solve for C_t through a forward recursion, giving $C_t = \Lambda_t + F_t \Lambda_{t+1} + F_t F_{t+1} \Lambda_{t+2} + \ldots$.

To illustrate, consider the case of two structural changes. From T_m onwards agents know about any future structural changes. Starting with the final interval $T_m^*, \ldots, T$, since the final model is in place from T_m^* onwards one can apply the no structural change solution method to get a VAR structure $y_t = C^* + Q^* y_{t-1} + G^* \varepsilon_t$.

Accordingly, this applies to the last interval and enables us to determine that $Q_{T_m^*} = Q^*$. At $t = T_m^* - 1$ the second interval model with coefficients $\bar{\theta}$ is in place but agents know that the final model holds at T_m^* onwards, so they account for this when forming expectations. Hence one solves for $Q_{T_m^*-1}$ using the backward recursion in Equation (13) but with $A_t = \bar{A}$ etc. Before T_m the data are generated by the initial model with coefficients θ, that is by the first interval VAR structure $y_t = C + Qy_{t-1} + G\varepsilon_t$.

Hence in the interval, $t = T_m, \ldots, T_m^* - 1$, the solution is a time-varying coefficient VAR with the movements in its coefficients being pinned down by the way the structure changes and is expected to change. Notice that the backward recursion implied by Equation (13) makes Q_t a function of Q_{t+1}. This means that the weights used to form expectations at time t are a function of current and future structures (models).

3.1.3 Announcement effects

Announcement effects, such as happens with the introduction of a goods and services tax (GST), the formation of a common currency, etc, can be captured in the set-up above. If there is a single regime shift which is known in advance of when it occurs then the initial model would hold for $t = 1, ..., T_m^* - 1$ and the final model from $t = T_m^*, ..., T$. The date of the break, T_m^*, is the time when the final model is in place. However, agents may now learn about the forthcoming change at, say, T_a. We would choose the sequence $\{Q_t\}_{t=T_a}^{T_m^*-1}$ starting from $Q_{T_m^*} = Q^*$ as before, such that $A - Q_t + BQ_{t+1}Q_t = 0$. Although for $t = T_a, \ldots, T_m^* - 1$, the structure remains constant (i.e $A_{0,t} = A_0, C_{0,t} = C_0$, etc), the announcement itself triggers a drift in the reduced form. In fact, between the announcement date, T_a, and the implementation date, $T_m = T_m^*$, the reduced form drifts from the first interval VAR structure $y_t = C + Qy_{t-1} + G\varepsilon_t$ towards the final interval VAR structure, $y_t = C^* + Q^* y_{t-1} + G^* \varepsilon_t$.

3.2 Regime Shifts Where Beliefs are Different from Reality

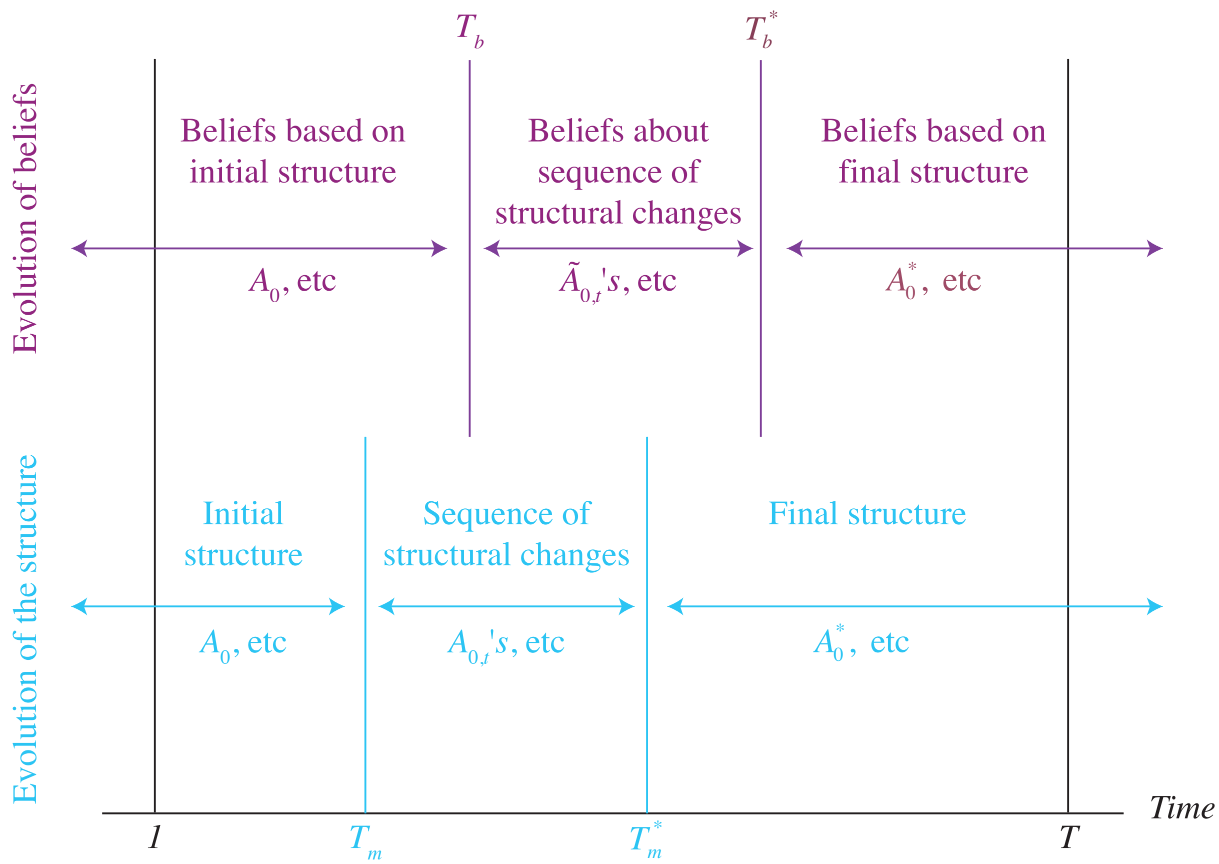
In the analysis above, beliefs agree with reality. When the structural changes are unknown until they occur, expectations are formed at each point in time using the model that pertains to that period of time. When agents foresee the structural changes, and the structural changes do take place, they know both the new and old models and therefore form expectations by weighting the information appropriately at each point in time. In this section we deal with the more general case in which this may not always be true. In doing so we assume agents do eventually use the correct model but there may be a period of time in which they are mistaken about which structure (model) holds. Hence, during that interval, they may form incorrect expectations: expectations are model consistent, but consistency may be with the wrong model for *part* of the sample period.

We introduce notation for the timing of beliefs. We denote by T_b the time when agents update their beliefs about current and future structures and by T_b^* the time when beliefs agree with the final structure. We impose no restrictions between T_m and T_m^* on one hand and T_b and T_b^* on the other, so that beliefs may converge before or after the structure has converged and they may be updated before or after the first structural change.

One possibility is illustrated in Figure 2. The lower arrows describe, as before, the evolution of the structure while the upper arrows now describe the evolution of beliefs. The sequence of structural changes begins in T_m and ends in T_m^*, with beliefs being based on the wrong structure (model) for some time. Beliefs are first updated in period T_b, after the structural changes begin, and converge in period T_b^*, after the structure has converged.

This generalisation allows us to consider situations in which agents do not get the timing of the structural changes right, as well as capturing situations of imperfect credibility in which policy announcements may be carried out as announced, but are not necessarily fully incorporated into expectations formation.

Figure 2: Timing of Structural Changes and Beliefs



We assume the structure evolves as before: that is, before T_m the structure is stable at Equation (1). Then, during $t = T_m, \ldots, T_m^* - 1$, the structure evolves as in Equation (7), subsequently changing for $t \geq T_m^*$ to Equation (8). Agents' beliefs, however, may evolve differently. Before T_b, expectations are based on Equation (1) while after T_b agents believe that the structural coefficients will evolve as follows:

$$\left\{ \tilde{A}_{0,t} \quad \tilde{C}_{0,t} \quad \tilde{A}_{1,t} \quad \tilde{B}_{0,t} \quad \tilde{D}_{0,t} \right\}_{t=T_b}^{T_b^*-1}. \tag{14}$$

Subsequently beliefs change for $t = T_b^*, \ldots, T$ to Equation (8), the final structure. Equation (14) indicates that, in the period up to T_b^*, agents may have inaccurate beliefs about which model is generating the data. In the special case that $A_{0,t} = \tilde{A}_{0,t}$ etc, $T_m = T_b$ and $T_m^* = T_b^*$ beliefs are always accurate and the situation coincides with the one discussed in Section 3.1.2.

In terms of our single structural change example, the period up to T_b^* may have a period of time over which the initial model holds and a further period in which the final model holds. From $\max(T_b^*, T_m^*)$ onwards it is only the final model that generates the data.

Given this departure from the standard rational expectations context, we assume agents combine observed outcomes with their beliefs about the structure to compute the time t conditional expectation, $\tilde{\mathbb{E}}_t y_{t+1}$, where the notation emphasises that expectations are based on Equation (14).[4] In this case, agents use their model beliefs to determine weights to be applied to *observed data* when forming expectations. When agents believe the structure will evolve as in Equation (14), one proceeds as before, starting from $\tilde{Q}_{T_b^*} = Q^*$ to find the sequence $\{\tilde{Q}_t\}_{t=T_b}^{T_b^*-1}$ such that

$$\tilde{A}_t - \tilde{Q}_t + \tilde{B}_t \tilde{Q}_{t+1} \tilde{Q}_t = 0 \tag{15}$$

The solution agents would infer for $t = T_b, ..., T_b^* - 1$ is

$$y_t = \tilde{C}_t + \tilde{Q}_t y_{t-1} + \tilde{G}_t \tilde{\varepsilon}_t, \tag{16}$$

which implies that $\tilde{\mathbb{E}}_t y_{t+1} = \tilde{C}_{t+1} + \tilde{Q}_{t+1} y_t$. However, the actual path of the economy obeys

$$A_{0,t} y_t = C_{0,t} + A_{1,t} y_{t-1} + B_{0,t} \tilde{\mathbb{E}}_t y_{t+1} + D_{0,t} \varepsilon_t. \tag{17}$$

Using Equation (16) it is easy to show that the reduced-form VAR is given by

$$y_t = \hat{C}_t + \hat{Q}_t y_{t-1} + \hat{G}_t \varepsilon_t \tag{18}$$

where

$$\begin{aligned}
\hat{C}_t &= \left(A_{0,t} - B_{0,t}\tilde{Q}_{t+1}\right)^{-1} \left(C_{0,t} + B_{0,t}\tilde{C}_{t+1}\right), \\
\hat{Q}_t &= \left(A_{0,t} - B_{0,t}\tilde{Q}_{t+1}\right)^{-1} A_{1,t}, \\
\hat{G}_t &= \left(A_{0,t} - B_{0,t}\tilde{Q}_{t+1}\right)^{-1} D_{0,t}.
\end{aligned}$$

4 One could alternatively assume that agents utilise their beliefs about the model to produce both the weights *and values for the endogenous variables* themselves when computing expectations, i.e. they project a model consistent path for the endogenous variables which will be incorrect if model beliefs are incorrect. There are other reasonable assumptions as well. For example, we could assume that either only lagged outcomes are observed or that only some subset of the variables are observed at time t. These extensions are left for further research.

The solution in this case also takes the form of a time-varying coefficient VAR with movements in its coefficients being pinned down by the way the structure evolves as well as agents' beliefs about these structural changes.

When the structural changes begin before agents first update their beliefs (i.e. $T_m < T_b$) as is the case in Figure 2, expectations are based on the initial structure in those periods, that is $\tilde{\mathbb{E}}_t y_{t+1} = C + Q y_t$, so the economy in those periods follows

$$A_{0,t} y_t = C_{0,t} + A_{1,t} y_{t-1} + B_{0,t}(C + Q y_t) + D_{0,t} \varepsilon_t.$$

With $\tilde{\mathbb{E}}_t y_{t+1} = \tilde{C}_{t+1} + \tilde{Q}_{t+1} y_t$ in hand, other cases, $T_m > T_b$ or $T_m^* > T_b^*$, are straightforward to compute.

3.3 The Likelihood

As we have discussed above, a set of structural changes and assumptions about beliefs and expectations formation map into a sequence of reduced-form matrices. If the structural changes are unknown until they occur, the solution is computed as in Section 3.1.1. If the structural changes are foreseen, the system follows Equation (9), and in the more general formulation where beliefs may differ from reality, the system follows Equation (18). The derivation of the likelihood is identical in each case since each involves a reduced form. Therefore, with no loss of generality, let the reduced form be given by Equation (9):

$$y_t = C_t + Q_t y_{t-1} + G_t \varepsilon_t.$$

Now assume that we have in hand a sample of data, $\{z_t\}_{t=1}^T$, where z_t is a $n_z \times 1$ vector of observable variables that relate to the model's variables by

$$z_t = H y_t + v_t. \tag{19}$$

In Equation (19), v_t is an *iid* measurement error with $\mathbb{E}(v_t) = 0$ and $\mathbb{E}\left(v_t v_t'\right) = V$. The observation equation, Equation (19), and the state equation, Equation (9), constitute a state space model. Therefore, the Kalman filter can be used to construct the likelihood function for the sample $\{z_t\}_{t=1}^T$, as outlined, for example, in Harvey (1989). Appendix A provides details of the derivation of the log-likelihood in Equation (20).

$$\begin{aligned}\mathscr{L} = &-\left(\frac{n_z T}{2}\right)\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\det\left(H\Sigma_{_{t|t-1}}H' + V\right)\\ &-\frac{1}{2}\sum_{t=1}^{T}u_t'\left(H\Sigma_{_{t|t-1}}H' + V\right)^{-1}u_t\end{aligned} \tag{20}$$

In Equation (20), $u_t = z_t - \mathbb{E}_{t-1}z_t$ is the prediction error,

$$\Sigma_{_{t|t-1}} = \mathbb{E}_{t-1}([y_t - \mathbb{E}_{t-1}y_t][y_t - \mathbb{E}_{t-1}y_t]')$$

is the covariance matrix of the state variables y_t conditional on information at $t-1$, and $cov_{t-1}(z_t) = H\Sigma_{_{t|t-1}}H' + V$.

With Equation (20) in hand, standard likelihood-based tests for parameter stability and detection of date breaks are available.[5]

4. Numerical Examples

4.1 A Credible Disinflation

In this example there is a credible disinflation in the context of the standard New Keynesian model described below in Equations (21)–(27). [6]

$$\begin{aligned}
x_t &= (r-\pi) - (r_t - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t x_{t+1} + (1-\omega)(1-\rho_a)a_t && (21)\\
\pi_t &= \pi + \beta\left(\mathbb{E}_t\pi_{t+1} - \pi\right) + \psi x_t - e_t && (22)\\
r_t &= r + \rho_r\left(r_{t-1} - r\right) + \rho_\pi(\pi_t - \pi) + \rho_g(g_t - g) + \rho_x x_t + \varepsilon_{r,t} && (23)\\
x_t &= \hat{y}_t - \omega a_t && (24)\\
g_t &= g + \hat{y}_t - \hat{y}_{t-1} + \varepsilon_{z,t} && (25)
\end{aligned}$$

5 Under the null hypothesis of no structural change the likelihood ratio statistic, $2(\mathscr{L}(\hat{\theta}_U) - \mathscr{L}(\hat{\theta}_R))$, is asymptotically distributed as a chi-square random variable with $m = dim(\hat{\theta}_U) - dim(\hat{\theta}_R)$ degrees of freedom, where $\hat{\theta}_U$ is the unrestricted maximum likelihood estimate of the vector of structural parameters and $\hat{\theta}_R$ is the restricted maximum likelihood estimate of the vector of structural parameters after imposing the restrictions of no structural change. Detection of structural change is generally done with a recursive likelihood ratio test.

6 See Ireland (2004) for more details.

$$
\begin{aligned}
a_t &= \rho_a a_{t-1} + \varepsilon_{a,t} && (26)\\
e_t &= \rho_e e_{t-1} + \varepsilon_{e,t} && (27)
\end{aligned}
$$

In the equations above, x_t is the output gap defined as the deviation of output from a socially efficient level of output; π_t is the gross rate of inflation, that is $\ln(p_t/p_{t-1})$; r_t is the log of the gross nominal interest rate; g_t is the growth rate of output; $\hat{y}_t$ is the percentage deviation from steady state of the log of the stochastically detrended level of output. The log of total factor productivity follows a unit root with a drift, g. Finally, a_t is a demand shock, e_t is a cost-push shock and $\varepsilon_{z,t}$ is the shock to total factor productivity. The ε's are identically and independently distributed shocks.

We construct a sample of 200 observations from this system with the following characteristics. First, the initial structure (model 1) shown in Table 1 governs the system up to period 159. Second, at the beginning of period 140, the monetary authority announces a disinflation program that involves a lower inflation target ($\pi = 0.0125$) and a more aggressive response to deviations of inflation from this target (ρ_r and ρ_π increase). The response to deviations of growth from trend also increase (ρ_g increases). This new policy will be implemented in period 160. Finally, there are no further structural changes until the end of the sample in period 200. Agents believe the announcement and revise their expectations accordingly. In terms of the sample parameters given earlier, $T = 200$, $T_a = 140$ and $T_m^* = 160$. The parameters of the modified system are then shown in lower panel of Table 1 while data on the observable variables, r_t, π_t and g_t are shown in Figure 3.

In estimation, r_t, π_t, and g_t are taken to be observed without noise, that is $V = 0$. For our choice of observables, ω is unidentified. Moreover, in practice it is typically the case that β is not estimated. For these reasons we set these parameters prior to estimation. The task is then to estimate the values of the remaining 17 parameters, $\left(\sigma_r, \sigma_a, \sigma_e, \sigma_z, \rho_r, \rho_\pi, \rho_g, \rho_x, \psi, \rho_a, \rho_e, g, \pi, \rho_r', \rho_\pi', \rho_g', \pi'\right)$.

The results are given in Table 2. The point estimates obtained with the history of observables in Figure 3 correspond to the MLE column in Table 2. The standard error of the maximum likelihood estimators are computed using the theoretical bootstrap with 250 replications. That is, we generate, at the estimated values of the parameters, 250 histories for the observables and estimate the parameters each time.

Table 1: Parameters of the Simulation

Initial structure			
$\sigma_r = 0.0017$	$\sigma_a = 0.0100$	$\sigma_e = 0.0018$	$\sigma_z = 0.0040$
$\rho_r = 0.7$	$\rho_\pi = 0.3$	$\rho_g = 0.1$	$\rho_x = 0.05$
$\beta = 0.9975$	$\psi = 0.1$	$\omega = 0.1$	$\rho_a = 0.85$
$\rho_e = 0.85$	$g = 0.005$	$\pi = 0.05$	$r = \pi + g - \ln\beta$
Final structure			
$\sigma_r = 0.0017$	$\sigma_a = 0.0100$	$\sigma_e = 0.0018$	$\sigma_z = 0.0040$
$\rho_r' = 1.0$	$\rho_\pi' = 0.8$	$\rho_g' = 0.3$	$\rho_x = 0.05$
$\beta = 0.9975$	$\psi = 0.1$	$\omega = 0.1$	$\rho_a = 0.85$
$\rho_e = 0.85$	$g = 0.005$	$\pi' = 0.0125$	$r' = \pi' + g - \ln\beta$
Announcement and sample size			
$T = 200$	$T_a = 140$	$T_m^* = 160$	

Figure 3: Simulated Data

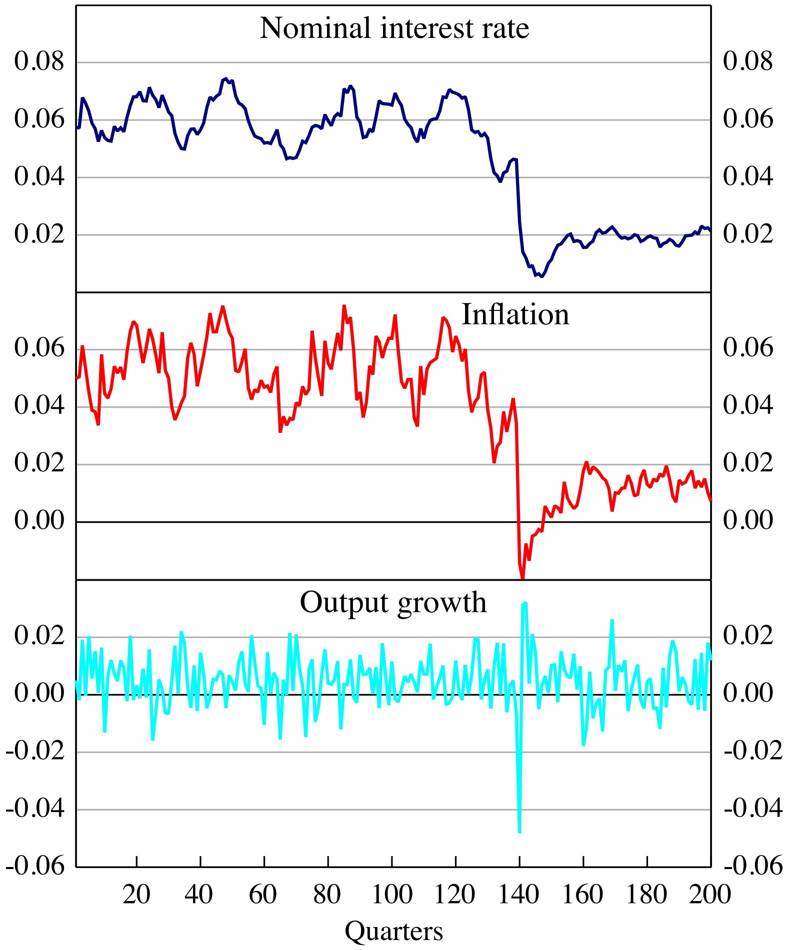


Table 2: Maximum Likelihood Estimation

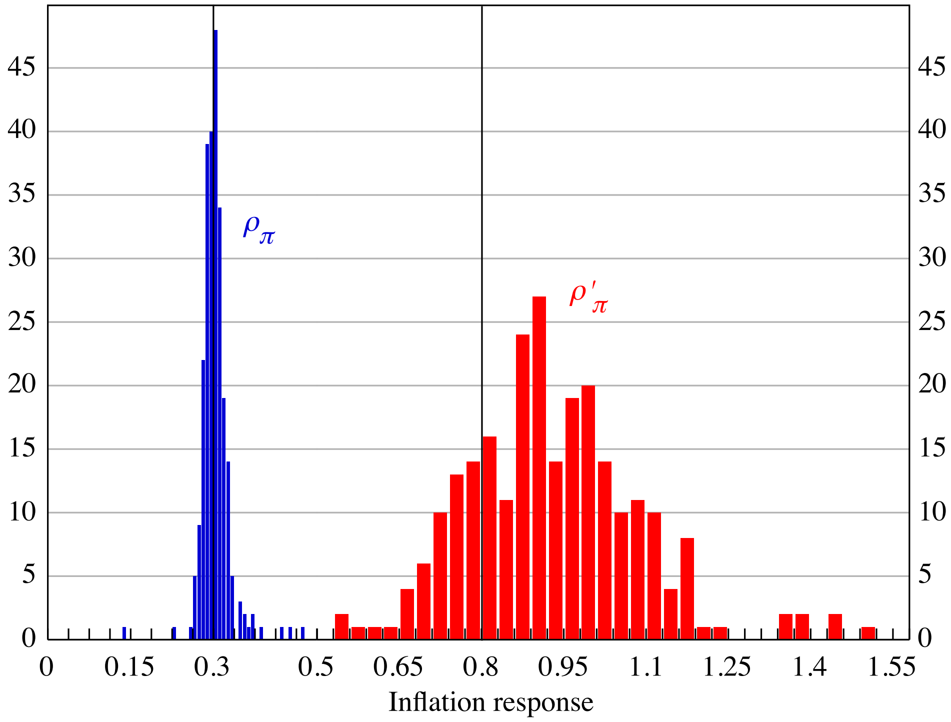
Parameter	True value	MLE	Standard error[a]
σ_r	0.0017	0.0017	0.00060
σ_a	0.0100	0.0106	0.00487
σ_e	0.0018	0.0016	0.00220
σ_z	0.0040	3.0×10^{-5}	0.00331
ρ_r	0.70	0.7007	0.028
ρ_π	0.30	0.2994	0.033
ρ_g	0.10	0.1001	0.068
ρ_x	0.05	0.0384	0.079
ψ	0.10	0.0739	1.749
ρ_a	0.85	0.8195	0.064
ρ_e	0.85	0.8684	0.072
g	0.0050	0.0048	0.0002
π	0.050	0.0501	0.0047
ρ_r'	1.00	1.2964	0.214
ρ_π'	0.80	0.8924	0.1704
ρ_g'	0.30	0.3485	0.1038
π'	0.0125	0.0122	0.0005
$\mathscr{L}$	2 449.62	2 507.24	38.12

Note: (a) Based on 250 replications

There are three distinct sub-samples in the data. The first 139 observations are constructed using the initial structure (model 1), the last 41 observations are found using the final structure (model 2), and the observations during the transition period – 140 to 159 – involve using both model 1 and model 2 weights when forming expectations. The model parameters that change are those of the monetary policy rule, including the target rate of inflation. As one would expect, because there are more observations generated from the initial structure, the parameters of the initial policy rule are estimated more precisely than those of the final structure. In contrast, because the new policy rule penalises deviations from the new inflation target more strongly, the final inflation target is estimated more precisely. This example illustrates an important point – even though there are relatively fewer observations coming from the final structure, not all of its parameters are estimated less precisely.

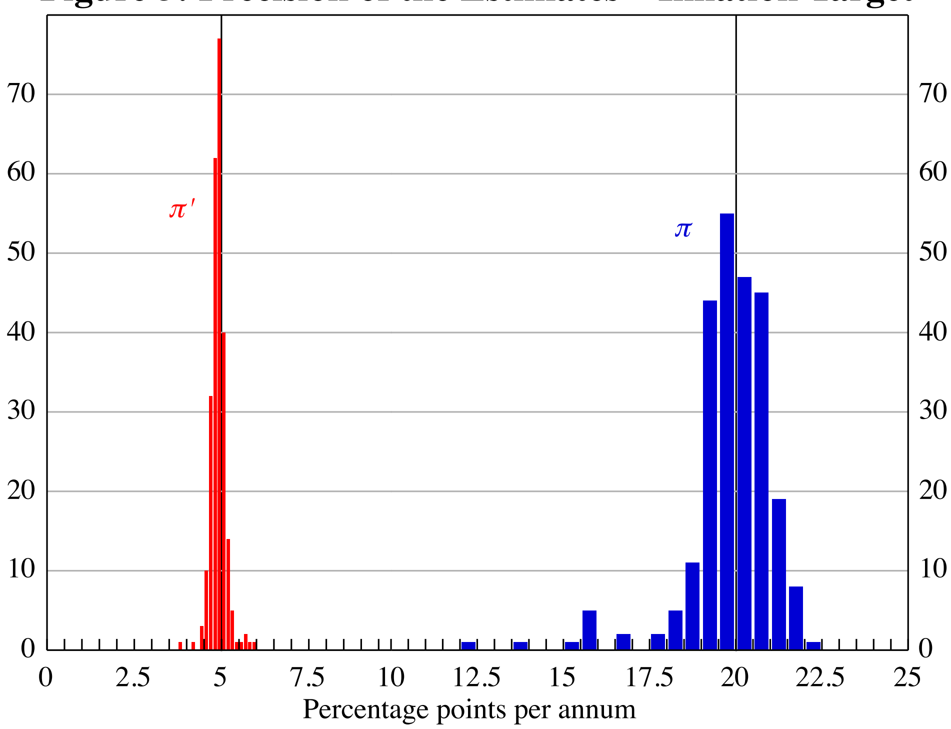
These outcomes are illustrated in Figure 4 which shows distributions of the estimators of the inflation response for both structures, ρ_π and ρ_π', and Figure 5 which shows distributions of the estimators of the inflation targets, π and π'.

Figure 4: Precision of the Estimates – Inflation Response



Notes: Bootstrapped based on 250 replications; vertical lines represent the true values of the parameters

Figure 5: Precision of the Estimates – Inflation Target



Notes: Bootstrapped based on 250 replications; vertical lines represent the true values of the parameters

4.2 A Slowdown in Trend Growth

For this example the monetary policy rule, Equation (23), is replaced with

$$r_t = r_{t-1} + \rho_\pi (\pi_t - \pi_{cb}) + \rho_g (g_t - g_{cb}) + \varepsilon_{r,t}. \tag{28}$$

This specification makes a distinction between the inflation target of the central bank and its estimate of trend growth, π_{cb} and g_{cb}, and those of the private sector, π and g. For the initial and final structures these are the same, that is $\pi_{cb} = \pi$ and $g_{cb} = g$. At $T_m = 32$ there is a structural change: g falls to g' and π_{cb} increases to π'_{cb}. There is another structural change at $T^*_m = 64$ when the parameters revert back to their original values. Unlike the example above, in the period running from $T_m = 32$ to $T^*_m = 64$, expectations are (incorrectly) based on the first model. The reduced-form therefore follows Equation (18). The parameters of this simulation are summarised in Table 3 along with the steady state real interest rate for both structures, rr.

Table 3: Parameters of the Simulation

Initial and final structures		
$\sigma_r =$ 0.001	$\sigma_a =$ 0.0100	$\sigma_e =$ 0.0030
$\rho_r =$ 1.0	$\rho_\pi =$ 0.3	$\rho_g =$ 0.2
$\beta =$ 0.9975	$\psi =$ 0.1	$\omega =$ 0.1
$\rho_e =$ 0.85	$g =$ 0.006	$\pi =$ 0.00625
$\sigma_z =$ 0.0080	$g_{cb} =$ 0.006	$\pi_{cb} =$ 0.00625
$rr = 400(g - \ln\beta) = 3.4$	$\rho_a =$ 0.85	
Temporary structure		
$\sigma_r =$ 0.001	$\sigma_a =$ 0.0100	$\sigma_e =$ 0.0030
$\rho_r =$ 1.0	$\rho_\pi =$ 0.3	$\rho_g =$ 0.2
$\beta =$ 0.9975	$\psi =$ 0.1	$\omega =$ 0.1
$\rho_e =$ 0.85	$g' =$ 0.0015	$\pi =$ 0.00625
$\sigma_z =$ 0.0080	$g_{cb} =$ 0.0060	$\pi'_{cb} =$ 0.02500
rr' $400(g' - \ln\beta) = 1.6$	$\rho_a =$ 0.85	
Timing of breaks and sample size		
$T =$ 160	$T_m =$ 32	$T^*_m =$ 64

While the temporary structure is in place trend growth falls. However, the central bank's view of trend growth does not. At the same time, the central bank runs looser monetary policy in an attempt to offset weaker growth outcomes. This is captured by an increase in the central bank's inflation target to π'_{cb}. Agents' beliefs are never updated and are based on the initial (and final) structure. So in this example, $\tilde{A}_{0,t} = A_0 = A_0^*$, etc and $A_{0,t} = \bar{A}_0$, etc. It is therefore unnecessary to specify T_b and T_b^*. The reduced form therefore follows Equation (18).

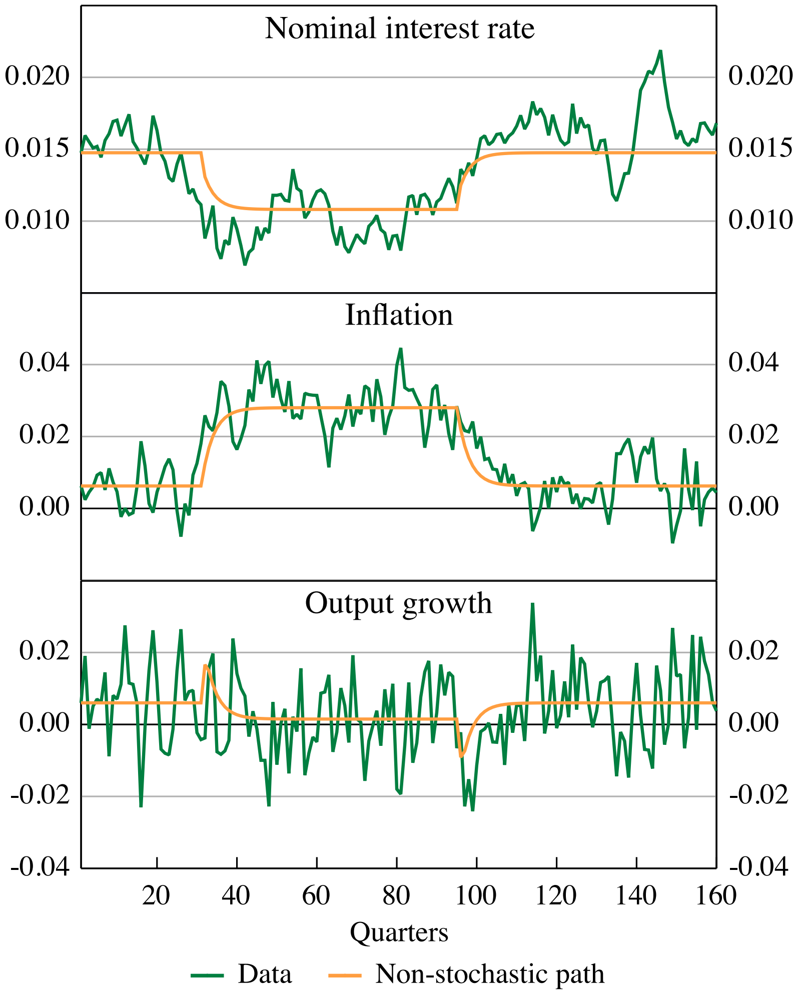
The results are given in Table 4. The point estimates associated with the history of observables shown in Figure 6 correspond to the MLE column in Table 4. The standard error of the maximum likelihood estimator is, as before, computed using the theoretical bootstrap with 250 replications. Figure 6 shows also the non-stochastic path of the simulation which corresponds to the path the economy would have experienced in the presence of structural changes but in the absence of random shocks.

Table 4: Maximum Likelihood Estimation

Parameter	True value	MLE	Standard error[(a)]
σ_r	0.001	0.0011	0.0002
σ_a	0.010	0.0097	0.0030
σ_e	0.003	0.0034	0.0017
σ_z	0.008	1.4×10^{-5}	0.0038
ρ_r	1.0	1.0166	0.0702
ρ_π	0.3	0.3221	0.0569
ρ_g	0.2	0.2111	0.0344
ψ	0.10	0.1022	0.5059
ρ_a	0.85	0.8166	0.0647
ρ_e	0.85	0.8182	0.0826
g	0.0060	0.0054	0.0005
π	0.00625	0.0069	0.0005
g'	0.0015	1.9×10^{-8}	0.0003
π'_{cb}	0.0125	0.0254	0.0008
$\mathscr{L}$	2 109.95	2 115.63	17.82

Note: (a) Based on 250 replications

Figure 6: Observable Data for Estimation



5. Conclusion

In this paper we develop a solution for linear models in which agents use model-consistent expectations but their beliefs about the structure may or may not be consistent with the correct model of the economy, at least for a period of time. The solution in each case takes the form of a time-varying coefficient VAR. This can be put into a state space form and the Kalman filter can be used to construct the likelihood.

In the case of an anticipated structural change, standard estimation methods with observed past regularities would not be a reliable guide, unless the estimation is supplemented by some knowledge of the way the structure was expected to evolve. As we have shown through numerical examples, knowledge of any in-sample structural changes that have taken place can increase the number of observations which are usable and can therefore substantially improve the quality of estimation. Even credible announcements of structural changes that take place out-of-sample would serve to improve the estimates.

Appendix A: The Kalman Filter Equations

Take the state equation

$$y_t = C_t + Q_t y_{t-1} + G_t \varepsilon_t$$

and the observation equation

$$z_t = H y_t + v_t.$$

Define $\mathbb{E}\left(\varepsilon_t \varepsilon_t'\right) = \Omega$, $\mathbb{E}\left(v_t v_t'\right) = V$ and

$$\begin{aligned}
\hat{z}_{t|t-j} &= \mathbb{E}\left(z_t \left| z_{t-j}, \ldots, z_1\right.\right) \\
\hat{y}_{t|t-j} &= \mathbb{E}\left(y_t \left| z_{t-j}, \ldots, z_1\right.\right) \\
\Sigma_{t|t-j} &= \mathbb{E}\left(y_t - \hat{y}_{t|t-j}\right)\left(y_t - \hat{y}_{t|t-j}\right)'.
\end{aligned}$$

The recursion begins from $\hat{y}_{1|0}$ with the unconditional mean of y_1, in our case

$$\mathbb{E}(y_1) = \mu$$

where μ is the steady state under the initial structure, that is $\mu = (I-Q)^{-1} C$ and

$$\Sigma_{1|0} = \mathbb{E}\left(y_1 - \mu\right)\left(y_1 - \mu\right)'$$

implies $vec(\Sigma_{1|0}) = (I - Q \otimes Q)^{-1} vec(G \Omega G')$. Presuming that $\hat{y}_{t|t-1}$ and $\Sigma_{t|t-1}$ are in hand then

$$\hat{z}_{t|t-1} = H \hat{y}_{t|t-1},$$

and the forecast error will be

$$u_t = z_t - \hat{z}_{t|t-1} = H\left(y_t - \hat{y}_{t|t-1}\right) + v_t.$$

The latter implies that

$$\mathbb{E}\left(u_t u_t'\right) = H \Sigma_{t|t-1} H' + V.$$

Next, update the inference on the value of y_t with data up to t as in Hamilton (1994):

$$\begin{aligned}
\hat{y}_{t|t} &= \hat{y}_{t|t-1} \\
&\quad + \left[\mathbb{E}\left(y_t - \hat{y}_{t|t-1}\right)\left(z_t - \hat{z}_{t|t-1}\right)'\right]\left[\mathbb{E}\left(z_t - \hat{z}_{t|t-1}\right)\left(z_t - \hat{z}_{t|t-1}\right)'\right]^{-1} u_t \\
&= \hat{y}_{t|t-1} + \Sigma_{t|t-1} H'\left(H\Sigma_{t|t-1} H' + V\right)^{-1} u_t.
\end{aligned}$$

This follows from

$$\begin{aligned}
\mathbb{E}\left(y_t - \hat{y}_{t|t-1}\right)\left(z_t - \hat{z}_{t|t-1}\right)' &= \mathbb{E}\left(y_t - \hat{y}_{t|t-1}\right)\left(H\left(y_t - \hat{y}_{t|t-1}\right) + v_t\right)' \\
&= \Sigma_{t|t-1} H',
\end{aligned}$$

after using $\mathbb{E}\left(v_t\left(y_t - \hat{y}_{t|t-1}\right)'\right) = 0$. Equation (9) then implies

$$\hat{y}_{t+1|t} = C_{t+1} + Q_{t+1}\hat{y}_{t|t-1} + K_t u_t,$$

where $K_t = Q_{t+1}\Sigma_{t|t-1} H'\left(H\Sigma_{t|t-1} H' + V\right)^{-1}$ is the Kalman gain matrix.

This last expression, combined with Equation (9), implies that

$$\begin{aligned}
y_{t+1} - \hat{y}_{t+1|t} &= C_{t+1} + Q_{t+1} y_t + G_{t+1}\varepsilon_{t+1} \\
&\quad - \left(C_{t+1} + Q_{t+1}\hat{y}_{t|t-1} + Q_{t+1}\Sigma_{t|t-1} H'\left(H\Sigma_{t|t-1} H' + V\right)^{-1} u_t\right) \\
&= Q_{t+1}\left(y_t - \hat{y}_{t|t-1}\right) + G_{t+1}\varepsilon_{t+1} \qquad \text{(A1)} \\
&\quad - Q_{t+1}\Sigma_{t|t-1} H'\left(H\Sigma_{t|t-1} H' + V\right)^{-1} u_t
\end{aligned}$$

The associated recursions for the mean squared error (MSE) matrices are given by,

$$\Sigma_{t+1|t} = G_{t+1}\Omega G'_{t+1} + Q_{t+1}\left(\Sigma_{t|t-1} - \Sigma_{t|t-1} H'\left(H\Sigma_{t|t-1} H' + V\right)^{-1} H\Sigma_{t|t-1}\right) Q'_{t+1}.$$

If the initial state and the innovations are Gaussian, the conditional distribution of z_t is normal with mean $H\hat{y}_{t|t-1}$ and conditional variance $H\Sigma_{t|t-1}H' + V$. The forecast errors, u_t, can then be used to construct the log likelihood function for the sample $\{z_t\}_{t=1}^{T}$ as follows:

$$\begin{aligned}\mathscr{L} = & - \left(\frac{n_z T}{2}\right)\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\det\left(H\Sigma_{t|t-1}H' + V\right) \\ & - \frac{1}{2}\sum_{t=1}^{T}u_t'\left(H\Sigma_{t|t-1}H' + V\right)^{-1}u_t.\end{aligned}$$

This is Equation (20) in the text.

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