A CONTINGENT CLAIM ANALYSIS OF RISK-BASED CAPITAL STANDARDS FOR BANKS

Mark Levonian* and Sarah Kendall**

Research Discussion Paper
9210

September 1992

* Economic Research Department
Reserve Bank of Australia

** Department of Economics
Loyola University of Chicago

The authors acknowledge helpful comments from Allen Berger, Beverly Hirtle, Marjorie Platt, and participants at the 1992 Conference of Economists in Melbourne. Any remaining errors are ours. The views expressed herein are those of the authors and do not necessarily reflect the views of the Reserve Bank of Australia.

ABSTRACT

Many countries are implementing capital adequacy standards developed under the auspices of the Bank for International Settlements (BIS), which explicitly link each bank's minimum capital-to-asset ratio to the riskiness of its operations. In this paper, we use a contingent claim framework to examine the general questions of what goals a risk-based capital framework might be designed to achieve and how risk-based standards might be expected to influence bank behaviour. We identify two related but distinct regulatory policy goals, and derive a capital adequacy rule to achieve each goal: a liability-value (LV) rule designed to limit the contingent liability borne by the deposit guarantor per dollar of deposits; and a failure-probability (FP) rule designed to limit the probability of bank insolvency. We show that an LV rule is likely to push banks toward low-risk, low-capital combinations, whereas an FP rule is likely to encourage high risk and high capital ratios. The results suggest that restrictions on bank asset holdings and on overall financial leverage may be desirable in conjunction with risk-based capital standards.

We then consider the extent to which the BIS standards reflect either an FP or an LV approach to capital regulation. The BIS standards assign risk weights to various types of assets, and establish a minimum ratio of capital to the sum of risk-weighted assets. We find that a BIS-type standard with appropriately chosen weights could be an extremely close approximation to a rule designed to achieve either goal, but that the actual weightings contained in the accord are most consistent with an FP rule. However, we show that the weight assigned to riskless assets should be negative if regulatory goals make an LV rule desirable; we also find that under either LV or FP rules the weight given to risky assets probably should be substantially higher than established in the BIS agreement. The optimal weights also depend on the typical range of risks in bank portfolios. Since this range may vary from one financial system to the next, it may be desirable to retain a degree of national discretion in setting the precise weightings.

TABLE OF CONTENTS

A CONTINGENT CLAIM ANALYSIS OF RISK-BASED CAPITAL STANDARDS FOR BANKS

Mark Levonian and Sarah Kendall

1. INTRODUCTION

Risk-based capital standards are being implemented in Australia and many other countries under the guidance of the Basle-based Bank for International Settlements. These standards require each bank's capital to exceed some proportion of an adjusted asset base. This base is calculated by applying fractional risk weights to the dollar value of various types of assets and converting off-balance-sheet exposures to "credit equivalents". By more directly linking bank capital to the riskiness of bank portfolios, the new standards may give regulators better control over the safety of banking systems. For example, Kim and Santomero (1988) show that risk-based standards can be designed so that the probability of bank failure is bounded by any particular level desired by regulators. Under traditional capital requirements, in contrast, required capital is not linked explicitly to banks' chosen level of asset risk and the probability of failure may be higher than is desirable.

However, the mean-variance framework used by Kim and Santomero has received some criticism. Keeley and Furlong (1990) argue that the model ignores the fact that limitations to shareholder liability, combined with some degree of imperfectly priced government backing of bank liabilities in most countries,[1] cause the economic value of bank liabilities to be sensitive to bank risk-taking. This sensitivity violates one of the fundamental assumptions of the mean-variance model, making it inappropriate for studying bank decision making. To address this issue, several recent papers (Marcus (1984) and Furlong and Keeley (1989), for example) use contingent claim techniques to model the value of a bank and study the bank's portfolio

---

[1] In many countries depositor protection is provided by government or central bank guarantee, while in others the protection is provided through a deposit insurance scheme. In either case, the ultimate backing of the national government is usually implicit if not explicit.

choice problem.

In this paper we use a contingent claim framework to examine risk-based capital standards. Like Kim and Santomero we derive standards that bound the probability of bank insolvency or failure; we refer to these as failure-probability or "FP" rules. We also consider standards designed to limit the contingent liability arising from the government's commitment to protect depositors; we refer to these as liability-value or "LV" rules.[2] Although these two regulatory goals are related, they are not identical. For example, an FP rule treats all bank insolvencies as equally costly, whereas an LV rule considers the size of the expenditure required to protect depositors in each potential insolvency. Hence a policy designed to achieve one goal may differ from a policy designed to achieve the other.

The paper has two main objectives. The first is to characterise FP and LV capital standards and compare bank behaviour under each type of rule. We are particularly interested in whether banks are more likely to choose high-risk, high-capital positions under one rule than under another. The second main objective is to study the feasibility of constructing standards similar to the BIS standards - specifying risk weights to be applied to different classes of assets - that come close to achieving the goals studied here. The analysis is similar to that of Kendall and Levonian (1992), although that paper develops the theoretical model within the context of an explicit deposit insurance system in which banks pay insurance premiums. This paper generalises the results to the case of a possibly implicit deposit guarantee, thus making the model more broadly applicable. The assumptions regarding

---

[2] Related papers include Sharpe (1978), Ronn and Verma (1989), and Duan, Moreau and Sealey (1991). Sharpe introduces the idea of designing risk-based capital standards to hold constant the value of the government liability. Ronn and Verma use the contingent claim model to derive estimates of the capital infusion several banks would have to make in order to achieve that goal. Duan, Moreau and Sealey consider FP and LV rules in the context of risk-based deposit insurance pricing. Holding the riskiness of assets constant across banks, they derive combinations of the capital ratio and the deposit insurance premium that yield a given constant probability of failure, and also derive combinations that yield a constant value of deposit insurance per dollar of deposits. In contrast, we allow the riskiness of assets to vary across banks, and derive combinations of asset risk and the capital ratio that yield constant failure probabilities and values of the deposit guarantee per dollar of deposits.

the riskiness of the environment, and the precise regulatory objectives under which the numerical results are derived, differ substantially from the Kendall and Levonian paper, and additional results are presented and discussed.

We take as a starting point Merton's (1977) model of a bank with government guaranteed deposits, which follows from Merton's (1974) more general model of financially-levered firms. Bank failure, taken here as synonymous with insolvency, occurs if the value of the bank's deposits at the end of the period exceeds the value of its assets. If the bank fails, a government-backed deposit guarantor ensures that depositors are reimbursed in full. The payout by the deposit guarantor is either zero or the difference between deposits and assets, whichever is larger; hence the deposit guarantee can be modelled as an implicit put option on bank assets. Within this model, we derive the guarantor's liability per dollar of bank deposits and an upper bound on the probability of bank failure, expressing both as functions of the bank's capital and the riskiness of its assets.

In order to study bank behaviour we assume that bank management chooses asset risk and capital to maximise the value of equity inclusive of the implicit put option created by the deposit guarantee and net of contributed capital, subject to the constraint that the capital requirement be met. The model suggests that under an LV rule banks will choose low-risk, low-capital portfolios, while under an FP rule they are likely to choose higher risk and higher capital.

We derive a simple version of the BIS/Basle standards by assuming banks can purchase only riskless assets and one type of risky asset. Risk weights are calculated to yield "best fit" approximations to the LV and FP rules. Under a fairly wide range of assumptions, both approximations yield a risk weight for risky assets that is above (in many cases well above) the maximum 100 percent weight specified by the BIS. The FP approximation yields a weight for riskless assets that is close to the zero weight assigned by the Basle standards; under the LV approximation, however, the weight is substantially less than zero.

The paper is organised as follows. Section 2 presents the model of the value of a bank and the corresponding value of the deposit guarantor's liability, and obtains an expression for an upper bound on the probability of failure. Section 3 presents the LV and FP rules and discusses bank behaviour under each. Section 4 derives a simple approximation to each capital standard and compares the results to the Basle standards. The paper is summarised in Section 5, and some directions for future research are discussed.

2. MODEL

Define V_t as the market value of a bank's assets (the discounted value of earnings from those assets) and D_t as the value of the bank's deposits. For simplicity we assume that deposits are the bank's only liabilities, and that they are fully guaranteed and hence riskless. Interest on deposits accrues continuously at the riskfree rate, and is payable at some future date $t=T$. The bank makes decisions regarding asset risk and capital at the present ($t=0$), and is monitored by bank supervisors at date T.[3] At the monitoring date T (and no sooner), the bank is declared insolvent if $D_T > V_T$; in that case, the deposit guarantor pays the depositors D_T and takes control of assets worth V_T, suffering a loss of $D_T - V_T$.

We assume that the value of bank assets follows a constant variance diffusion process:

$$dV = \alpha V dt + \sigma V dz \tag{1}$$

where $\alpha > 0$ is the instantaneous expected rate of return on assets, $\sigma > 0$ is the instantaneous standard deviation of the rate of return, and dz is the differential of a standard Wiener process.

---

[3] We have assumed that the bank's deposits mature at the same point in time that the bank is monitored by authorities. This assumption simplifies the presentation of the model without substantial effect on the conclusions.

2.1 The Contingent Liability of the Deposit Guarantor

Merton (1977) has shown that under the assumptions above the liability associated with the deposit guarantee can be evaluated using the Black-Scholes option pricing equation for put options. The expiration date of the implicit option provided by the deposit guarantor is date T, the next point at which the bank can be declared insolvent by supervisory authorities. The guarantor's liability at $t=0$, which we denote L, is:

$$L = D_0 N(x+\sigma\sqrt{T}) - V_0 N(x) \tag{2}$$

where D_0 is the present value of deposits, V_0 is the present value of assets, $N(\bullet)$ denotes the standard normal cumulative density function, and x is defined as:

$$x \equiv \frac{\ln(D_0 / V_0) - \sigma^2 T/2}{\sigma\sqrt{T}} \tag{3}$$

2.2 The Probability of Failure

As noted above, bank failure occurs in this model if the end-of-period value of assets V_T is less than the end-of-period value of deposits inclusive of continuously-compounded interest, $D_T = e^{rT} D_0$. As shown by Smith (1976, p. 15), the probability of failure, which we denote as p, can be written:

$$p = N\left[\frac{\ln\left(e^{rT} D_0 / V_0\right) - \left(\alpha - \sigma^2/2\right) T}{\sigma\sqrt{T}}\right] \tag{4}$$

Assuming that the expected rate of return on bank assets equals or exceeds the riskfree interest rate, $\alpha \geq r$, the probability of failure satisfies the condition $p \leq N(x+\sigma\sqrt{T})$. Thus, $N(x+\sigma\sqrt{T})$ is an upper bound on the probability of failure; we denote this upper bound as FP:

$$p \leq N(x+\sigma\sqrt{T}) \equiv FP \tag{5}$$

3. TWO RISK-BASED CAPITAL RULES

Within the context of our model, risk-based capital requirements specify a relationship between the capital ratio $c \equiv (V_0 - D_0)/V_0$ and asset risk σ. The exact form of the relationship depends on the regulatory goal being pursued.

3.1 A Liability-Value Rule

One possible regulatory goal that risk-based capital standards could be designed to achieve is to maintain the contingent liability of the deposit guarantor at some fixed level relative to deposits, irrespective of banks' operating decisions. (This goal might be especially attractive in countries such as the United States that meet the cost of the guarantee through a deposit-based assessment on the banking system.) Dividing both sides of equation (2) by D_0 yields the liability per dollar of bank deposits, which we denote LV:

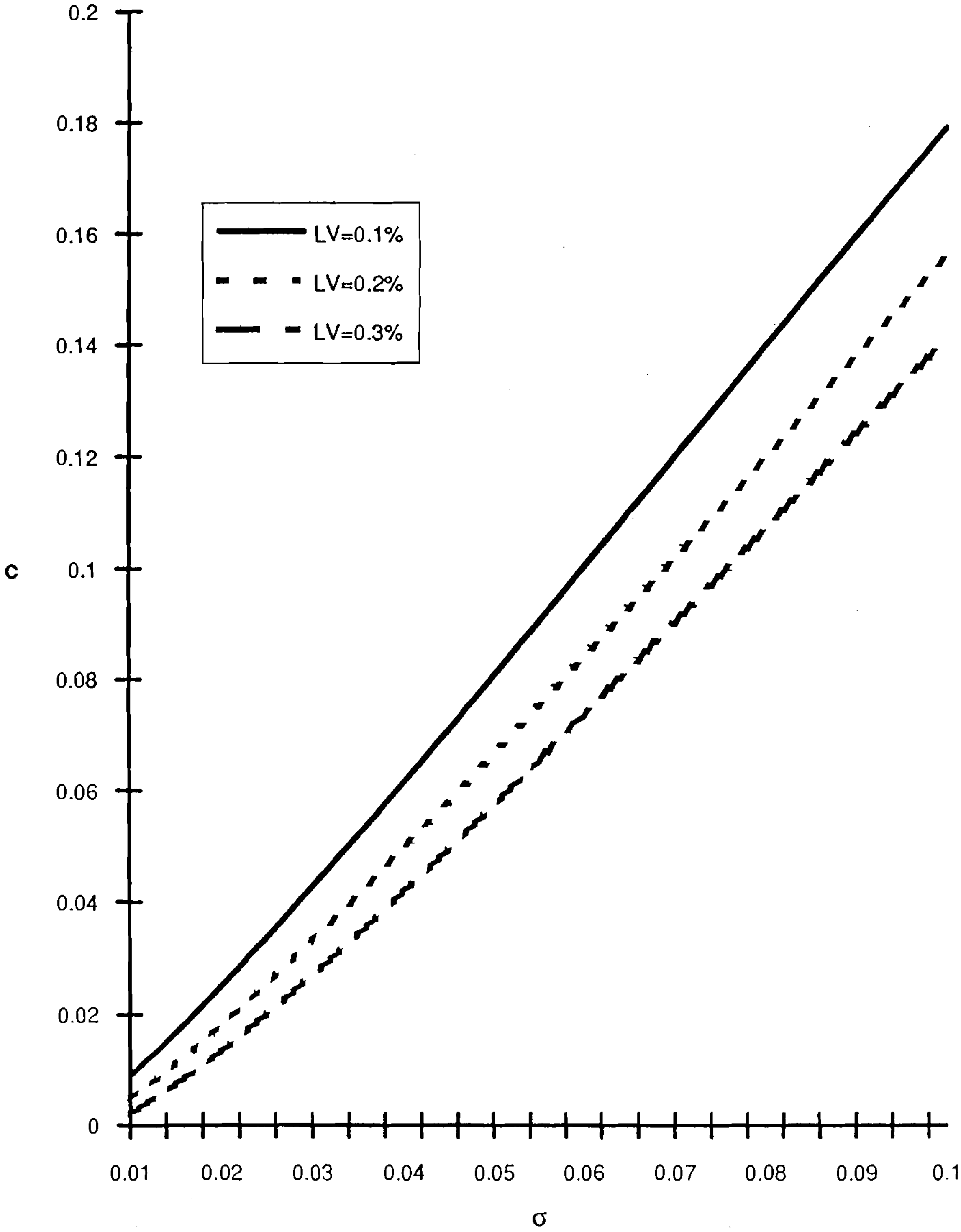
$$LV = N(x+\sigma\sqrt{T}) - \frac{N(x)}{(1-c)} \tag{6}$$

where x is as defined in (3), noting that $D_0/V_0 = 1-c$. Equation (6) shows that the guarantor's liability per dollar of deposits depends only on asset risk σ, the time to the monitoring date T, and the capital ratio c; it is independent of bank size and interest rates.

Level curves $LV(\sigma,c)=LV_0$ give combinations of σ and c that yield the same liability of LV_0 per dollar of deposits. Examples of these curves are shown in Figure 1, with $T=1$. It is easily shown that $LV(\sigma,c)$ is increasing in σ and decreasing in c; in words, the guarantor's liability is greater for higher levels of asset risk and for lower values of the capital ratio. Thus a risk-based capital rule designed to limit the liability imposed by individual banks would allow (σ,c) combinations on or to the left of one of the level curves.

FIGURE 1

LV CAPITAL STANDARDS



A general expression for the slope of the level curves can be obtained by taking the total differential of $LV(\sigma,c)$, setting it equal to zero, and rearranging to obtain $dc/d\sigma$:

$$dLV = n(x+\sigma\sqrt{T})\sqrt{T}\,d\sigma - \frac{N(x)}{(1-c)^2}dc \tag{7}$$

$$\frac{dc}{d\sigma}\bigg|_{dLV=0} = \frac{n(x+\sigma\sqrt{T})(1-c)^2\sqrt{T}}{N(x)} \tag{8}$$

where $n(\bullet)$ is the standard normal density function. For a bank that is on the boundary of allowable combinations of σ and c, expression (8) specifies the increase in capital that would be required for a small increase in asset risk. This positive relationship between risk and capital provides the basis for an LV risk-based capital standard. Of course, (8) only gives the slope of the risk-based capital schedule that holds the liability constant. Policy makers also would have to determine an acceptable level of LV_0 to construct a complete regulatory standard. For a given value of the liability LV_0 and a given monitoring period T, (8) defines the capital ratio as an implicit increasing function of asset risk.

3.2 A Failure-Probability Rule

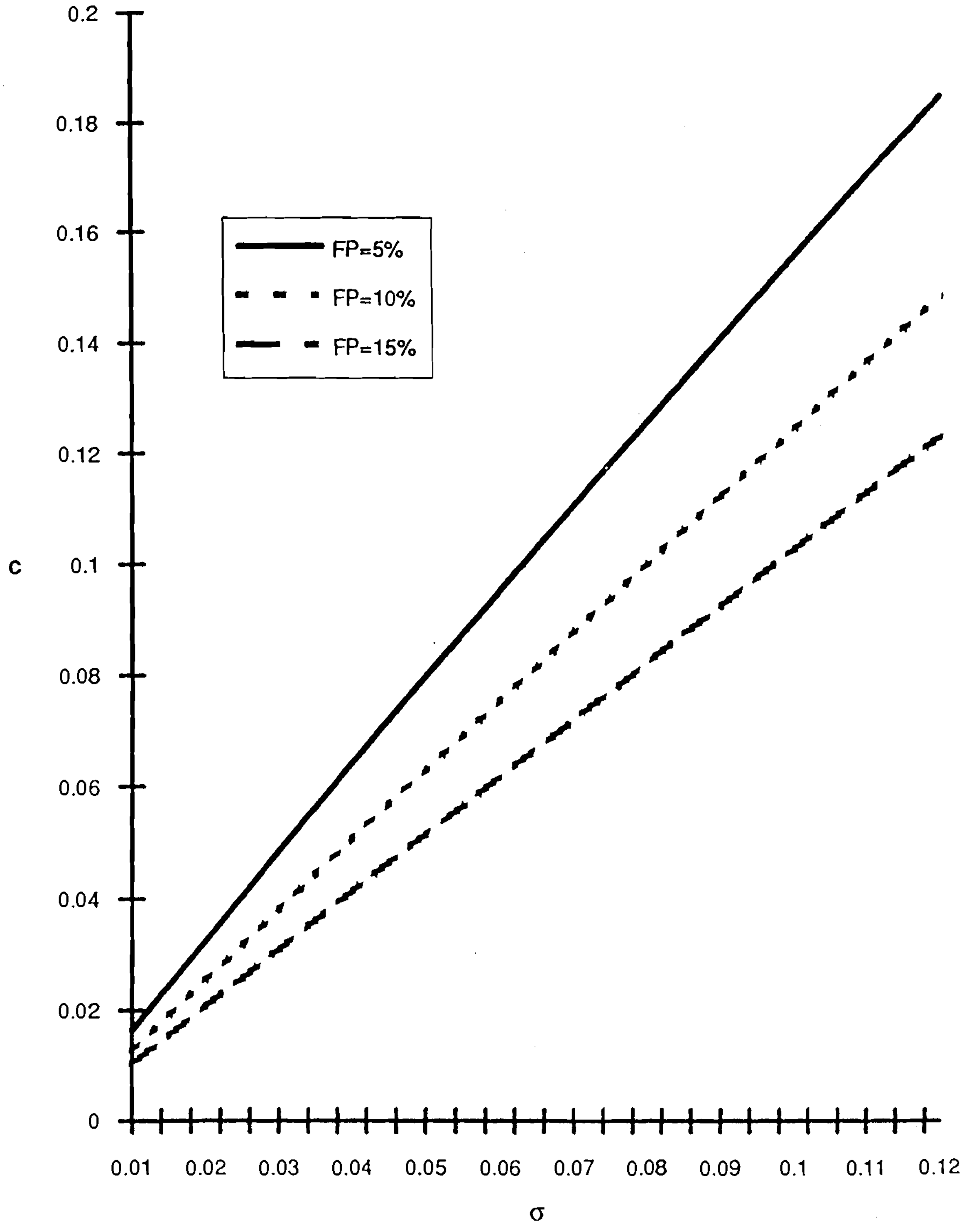
Level curves $FP(\sigma,c)=FP_0$ of the failure probability function give combinations of σ and c that yield a constant probability of failure equal to FP_0. Three examples are plotted in Figure 2, with T again set equal to one. Like the per-dollar contingent liability, $FP(\sigma,c)$ is decreasing in c, and generally is increasing in σ.[4] An FP risk-based capital rule designed to limit the probability of failure would allow (σ,c) combinations on or to the left of one of the constant failure probability curves.

---

[4] In fact, FP may be increasing or decreasing in σ, since for sufficiently insolvent banks the probability of failure is reduced by an increase in asset risk. A sufficient condition for $\partial FP/\partial\sigma$ to be positive is $x<0$, which is satisfied when $0<c<1$. We will restrict our attention to solvent banks, for whom this requirement is always met.

FIGURE 2

FP CAPITAL STANDARDS



To derive the slope of a failure probability locus we take the total differential of $FP(\sigma,c) = N(x+\sigma\sqrt{T})$ and set it equal to zero:

$$dFP = -n(x+\sigma\sqrt{T})\left[\frac{x}{\sigma}d\sigma + \frac{1}{(1-c)\sigma\sqrt{T}}dc\right] \tag{9}$$

$$\frac{dc}{d\sigma}\bigg|_{dFP=0} = -x(1-c)\sqrt{T} \tag{10}$$

For a bank on the boundary of allowable combinations of σ and c, expression (10) defines c implicitly as a function of σ, and hence specifies the increase in capital required for a small increase in asset risk. As with an LV rule, regulators would need to determine an acceptable value of FP_0 to actually implement a policy based on this relationship.

3.3 Comparing the Two Rules

The two risk-based capital standards described above differ in the size of the increase in capital required for any given increase in asset risk. Any combination of σ and c that a bank might select lies on both an LV locus and an FP locus. Comparison of (8) and (10) reveals that a sufficient condition for $dc/d\sigma$ to be greater along the liability-value locus than along the failure-probability locus is $n(x)/N(x)+x>0$.[5]

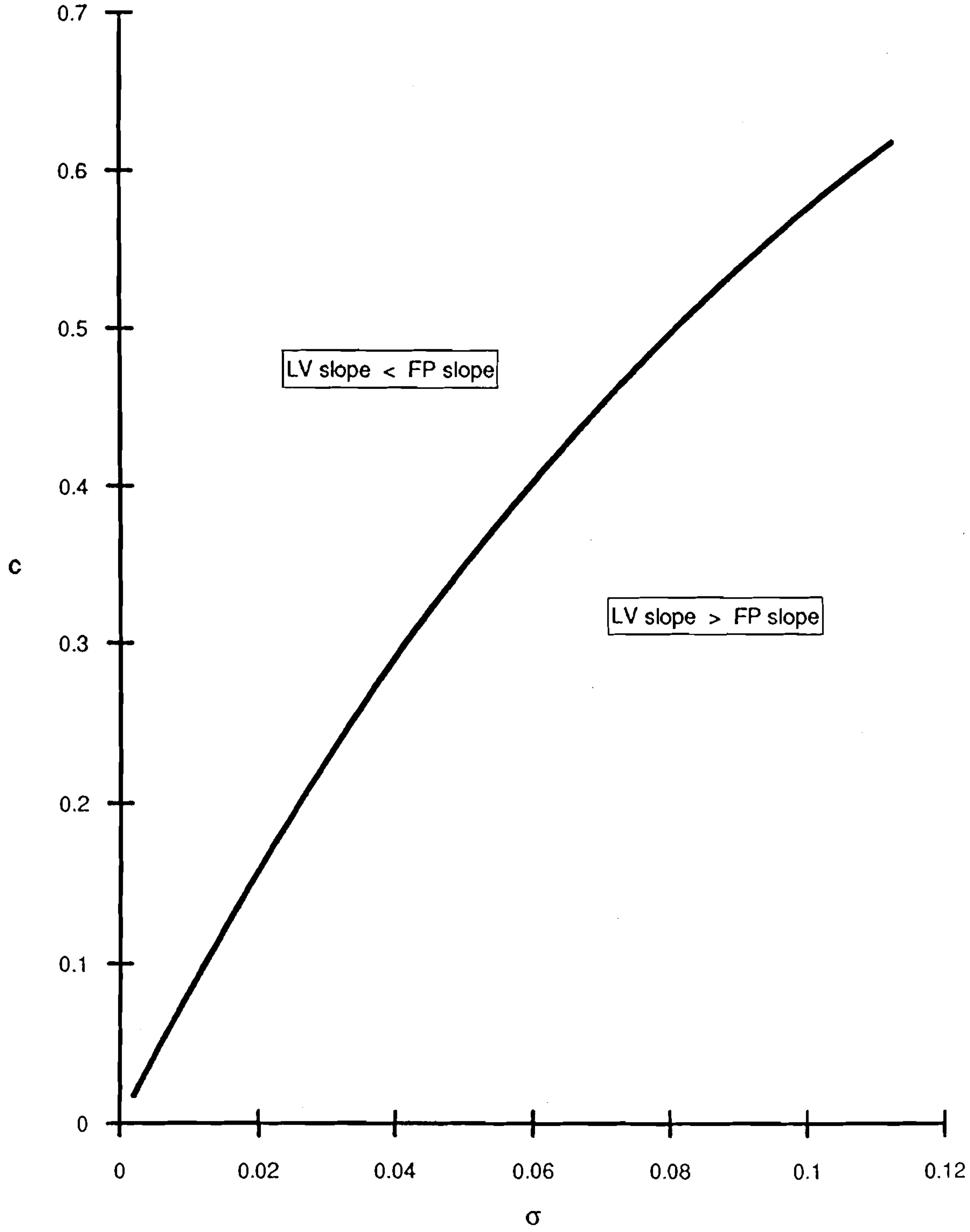
Figure 3 shows combinations of c and σ that make the slopes of the LV and FP loci equal, along the locus labelled $n(x)/N(x)+x=0$. Above this locus LV curves are flatter than FP; the opposite is true below and to the right of the locus. Casual observation suggests that banks generally have capital and asset risk combinations that put them in the lower part of the figure. Thus the LV curve is likely to be steeper than the FP curve, implying that an LV rule would dictate larger capital responses to changes in operating risk than an FP rule. The difference arises because the FP rule weights all insolvency outcomes equally, while the LV rule gives greater weight to outcomes entailing larger losses for the deposit guarantor.

---

[5] The useful fact that $(1-c)n(x+\sigma\sqrt{T}) = n(x)$ makes this relationship between the slopes obvious. We consider only solvent banks, implying that $x<0$; the inequality holds for $-8.64<x<0$. Figure 3 graphs the combinations of σ and c that yield $x=-8.64$.

FIGURE 3

$n(x)/N(x) + x = 0$



To compare bank behaviour under the two rules we assume the bank chooses a combination of capital and risk to maximise the value of equity net of contributed capital (V_o-D_o) but inclusive of the implicit put option (L) generated by the guaranteed deposits, subject to the constraint that the capital requirement be met. We assume that changes in the capital ratio are achieved through substitution of capital for deposits, leaving the value of assets unchanged,[6] and that assets are purchased or originated in competitive markets so that their value is unaffected by the riskiness of the bank. The total value of equity, which we denote E, is given by:

$$E = V_0 - D_0 + L \tag{11}$$

Since contributed capital is $V_0 - D_0$ and the bank maximises E net of contributed capital, the bank's effective objective is to maximise the value of the implicit put option.

If the bank changes its combination of capital and risk, the value of the bank's equity changes according to:

$$dE = \frac{\partial L}{\partial c}dc + \frac{\partial L}{\partial \sigma}d\sigma \tag{12}$$

Differentiating the expression for L given in equation (2), and using the definition of c, dE can be rewritten:

$$dE = V_0\left[(1-c)\sqrt{T}n(x+\sigma\sqrt{T})\,d\sigma - N(x+\sigma\sqrt{T})\,dc\right] \tag{13}$$

Setting dE equal to zero characterises the bank's indifference curves in the (σ,c) plane.[7] Since $N(x+\sigma\sqrt{T}) > 0$ and $(1-c)\sqrt{T}n(x+\sigma\sqrt{T}) > 0$, the net value of bank equity is decreasing in the capital ratio and increasing in asset risk.

---

[6] Furlong and Keeley (1989) discuss the relative merits of modelling bank capital choices with the value of assets constant versus deposits constant within a contingent claim model. They conclude that a stronger logical case can be made for holding assets fixed and allowing deposits to vary.

[7] Since the bank's indifference curves reflect the total value of the deposit guarantee rather than the value per dollar of deposits, they do not coincide with the constant-LV loci.

Thus indifference curves which are downward and to the right in the (σ,c) plane represent higher values of equity.

If the bank's indifference curves are flatter than the boundary implied by the capital standard, the bank will be pushed toward low-risk, low-capital portfolios.[8] Conversely, if the indifference curves are steeper than that boundary, the bank will be pushed toward high-risk, high-capital portfolios. Setting *dE* equal to zero and rearranging yields the slope of the bank's indifference curves:

$$\frac{dc}{d\sigma}\Big|_{dE=0} = \frac{(1-c)\, n(x+\sigma\sqrt{T})\sqrt{T}}{N(x+\sigma\sqrt{T})} \tag{14}$$

Comparing (8) and (14), the indifference curves are flatter than the LV rule boundary if $N(x) < (1-c)N(x+\sigma\sqrt{T})$; we know this inequality always holds, since $N(x+\sigma\sqrt{T}) - N(x)/(1-c) = LV > 0$. Comparing (10) and (14), the bank's indifference curves are flatter than the FP rule boundary if $n(x+\sigma\sqrt{T})/N(x+\sigma\sqrt{T}) + x < 0$. Figure 4 shows the locus of points for which the slopes are just equal. The bank's indifference curves are flatter than the FP locus at points above and to the left, which as noted above are unlikely choices for actual banks. Indeed, these low-risk, high-capital combinations are optimal only if the FP rule requires failure probabilities on the order of 10^{-18} or less. In general the relationship between the slopes is likely to be that the LV curves are steeper than the bank's indifference curves, while the FP curves are flatter:

$$\frac{dc}{d\sigma}\Big|_{dLV=0} > \frac{dc}{d\sigma}\Big|_{dE=0} > \frac{dc}{d\sigma}\Big|_{dFP=0} \tag{15}$$

[8] In practice, banks probably would not seek corner solutions. Contrary to the assumptions of the model, the total value of bank assets is not completely independent of risk. Risky lending is likely to be a source of economic rents for the bank, so that the optimal σ never would be zero. In addition, liquidity considerations probably require a non-zero proportion invested in riskless assets, limiting σ on the upper end. Under such conditions, an interior solution for σ and c probably would exist, and the bank would not jump to either extreme. Gennotte and Pyle (1991) develop a model in this spirit, in which the value of bank assets depends on σ. We do not incorporate this dependence explicitly in our model; under reasonable assumptions about functional forms, it would likely alter the magnitude but not the direction of the effects of risk-based capital standards.

Thus if an LV rule is in place banks will be pushed toward lower-risk, lower-capital portfolios, while an FP rule will push banks toward higher-risk, higher-capital portfolios.

Limiting the probability of bank failure and limiting the liability of the deposit guarantor both are plausible regulatory objectives. Although the two goals are closely related, the contingent claim model suggests that bank behaviour under a policy designed to meet one of these goals may differ markedly from behaviour under a policy designed to meet the other. Moreover, the model suggests that the two regulatory goals may conflict; holding the value of the deposit guarantee liability constant may result in unacceptably high probabilities of failure and vice-versa.

Figure 5 illustrates the potential conflict. Suppose that regulators pursue a goal of holding LV constant at 0.3%. As shown above, under an LV standard banks will be pushed toward positions of lower asset risk and lower capital ratios; in terms of Figure 5, they choose a point such as A rather than B. Although both points are on the same LV locus and hence lead to identical liabilities for the guarantor, the probability of bank failure is higher at A than at B. Similar conclusions follow if regulators pursue a constant-FP goal. The high-risk, high-capital portfolio chosen under an FP rule is on a higher LV locus than the lower-risk portfolios the bank could choose.

It follows that additional restrictions may be desirable under either type of rule. Under an LV rule a minimum ratio of equity to total assets may be needed to prevent failure probabilities from being pushed to unacceptable levels. Similarly, under an FP rule restrictions on the range of permissible assets and activities — perhaps in the form of limits on holdings of certain types of assets — may be necessary to set a maximum on σ and thereby limit the deposit guarantor's liability. Both of these types of supplemental restrictions (leverage ratios and portfolio limits) are already part of the regulatory structure in many countries; our results provide a rationale for their continued use even under comprehensive risk-based capital standards.

FIGURE 4

$n(x+\sigma)/N(x+\sigma) + x = 0$

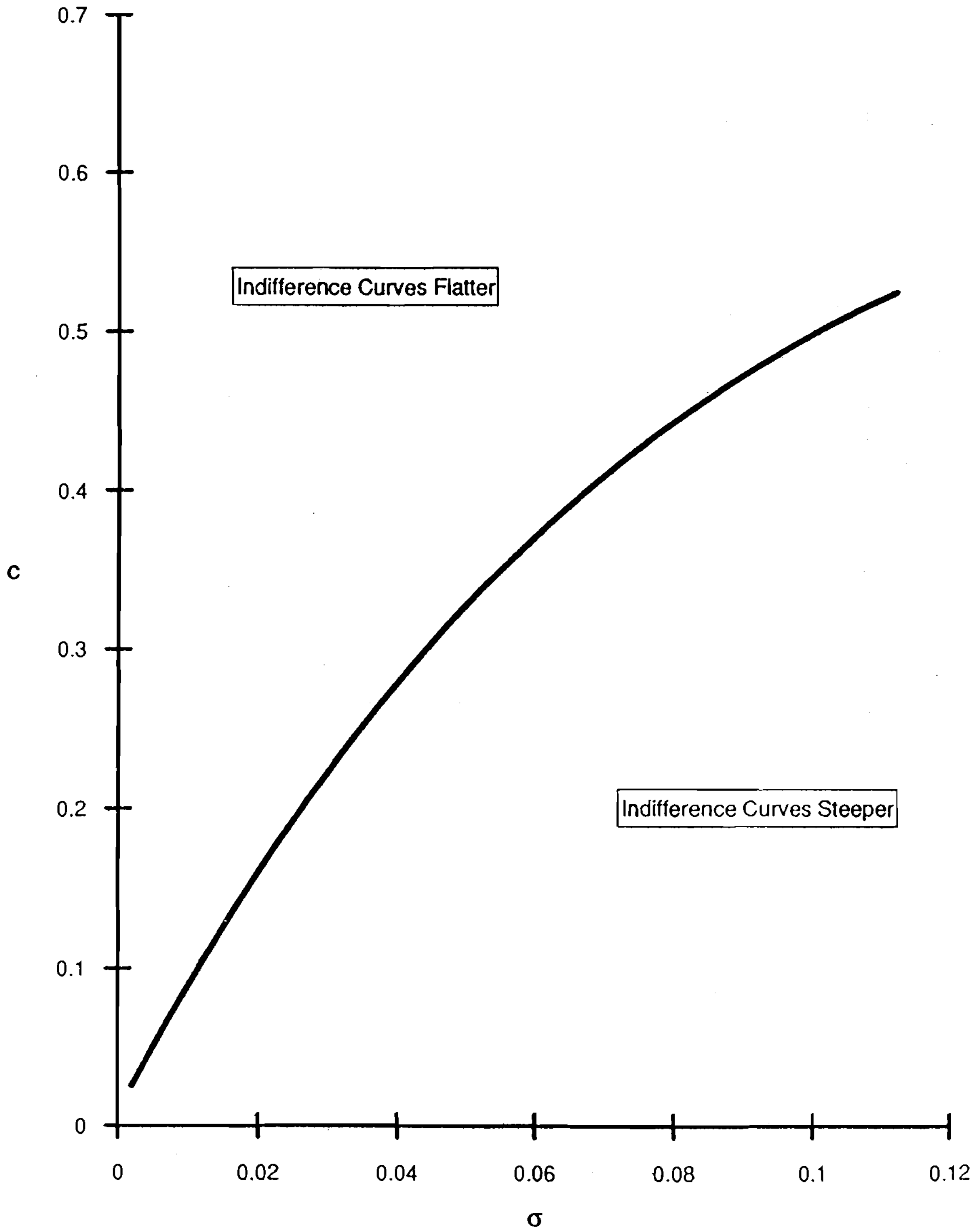
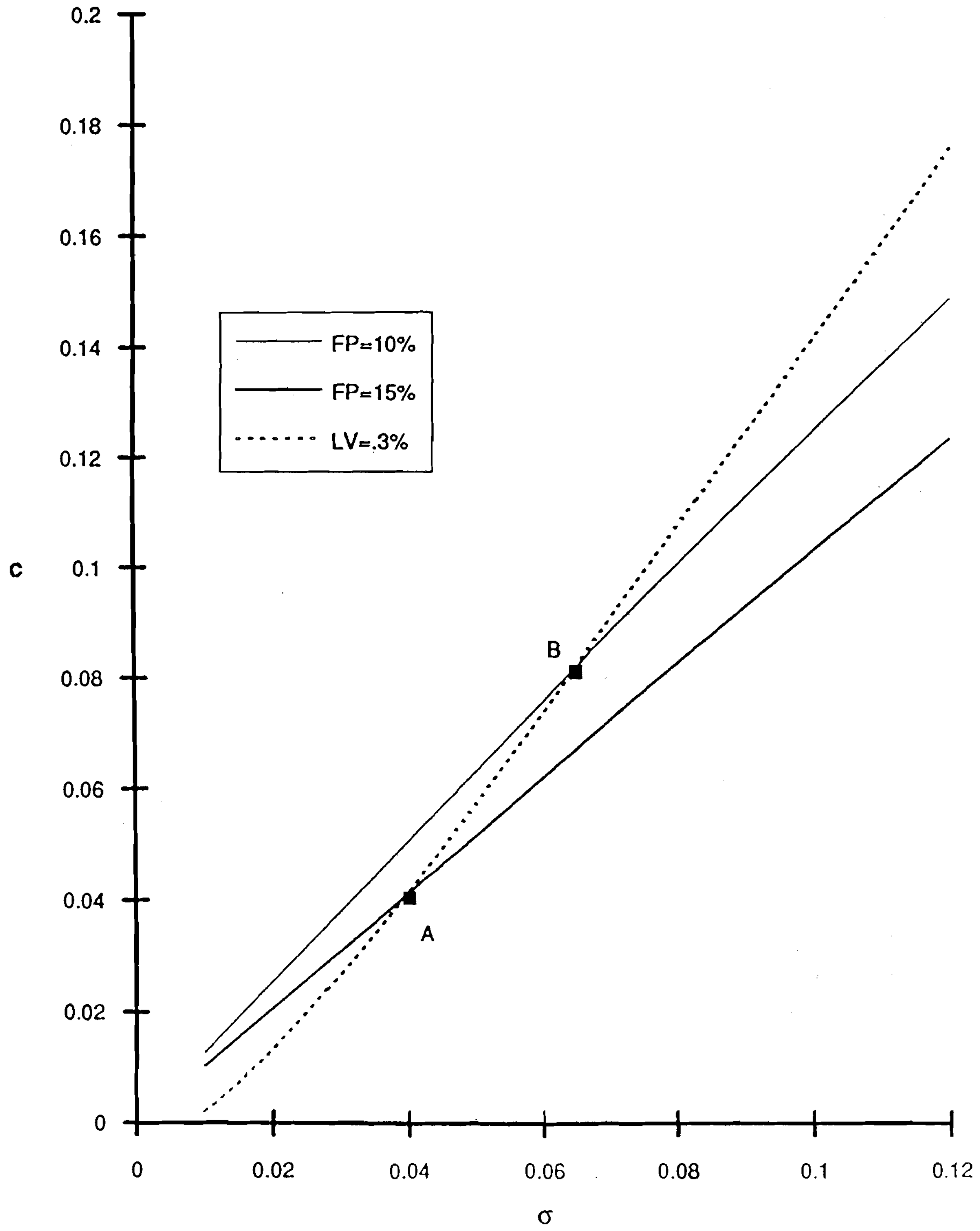


FIGURE 5

POTENTIAL POLICY CONFLICT



4. THE BASLE STANDARDS

In this section, we develop a simple representation of the Basle standards, and compare it to the LV and FP risk-based capital rules analysed above. On the surface, the Basle risk-based capital standards bear little resemblance to either an LV rule or an FP rule. The Basle standards express the minimum capital ratio not as a function of the standard deviation of asset returns but as a predetermined fraction of a weighted sum of assets. Weighted assets are calculated by applying "risk weights" to the dollar value of various types of assets, with the weights ranging from zero to one. Thus, required capital is a linear function of the quantities of assets in different risk categories, and the required capital-to-assets ratio is linear in the proportions of assets in each category.

Nevertheless, a simplified version of the Basle standards can be represented in the (σ,c) plane. Assume that there are only two classes of assets, riskless and risky. Let η be the proportion of risky assets within the bank's total asset portfolio; that is, the bank holds ηV dollars of risky assets and $(1-\eta)V$ of riskless assets. Letting w_1 and w_0 be the risk weights applied to risky and riskless assets respectively, the weighted asset base is calculated as $w_1\eta V + w_0(1-\eta)V$; if c_{rb} is the risk-based capital ratio (the minimum ratio of capital to risk-weighted assets), the bank must satisfy the constraint:

$$V-D \geq c_{rb}\big(w_1\eta V + w_0(1-\eta)V\big) \tag{16}$$

Dividing (16) by V to express the constraint in terms of the capital-to-assets ratio, and collecting terms, the minimum capital ratio for a bank with fraction η in risky assets is:

$$c_{\min} = c_{rb}w_0 + c_{rb}(w_1 - w_0)\eta \tag{17}$$

Thus, a Basle-type standard is linear in the fraction η invested in risky assets.[9]

---

[9] Eichberger (1992) derives an almost identical capital standard, in which the implied minimum capital ratio is a linear function of the proportional investment in risky assets, within an entirely different model. Eichberger's result differs slightly, in that both the

The minimum capital ratio in (17) also can be expressed in terms of σ. Since asset risk for the bank is proportional to the fraction of investment in risky assets, we have $\sigma = \eta\hat{\sigma}$, where $\hat{\sigma}$ denotes the standard deviation of returns to risky assets. (In this formulation, σ depends on the bank's choice of η, whereas $\hat{\sigma}$ is determined by market forces.) Substituting $\eta = \sigma/\hat{\sigma}$ into expression (17), the Basle capital standards restrict the bank's choices to lie on or above the locus:

$$c_{\min} = c_{rb}w_0 + \left[\frac{c_{rb}(w_1 - w_0)}{\hat{\sigma}}\right]\sigma \tag{18}$$

The capital ratio constraint is linear in asset risk σ, and plots as a straight line with slope $c_{rb}(w_1 - w_0)/\hat{\sigma}$ in the (σ,c) plane.

4.1 Linear Forms as Simple Approximations

The linear Basle-type standard in (17) clearly is not equivalent to either an LV standard or an FP standard. Under LV or FP rules, the required capital ratio (defined implicitly by the equations $LV(\sigma,c)=LV_0$ or $FP(\sigma,c)=FP_0$) is a nonlinear function of σ. Nevertheless, the constant LV and FP loci shown in Figures 1 and 2 are nearly linear, which suggests that a simpler Basle-type standard could serve as a good approximation to either regulatory rule. A less complex capital standard might well be desirable, since simplicity would reduce the costs of implementation, enforcement, and compliance.

To develop a suitable approximation, let $c(\sigma)$ represent the minimum capital ratio defined as an implicit function of asset risk, derived using either an LV rule or an FP rule. Assume that most banks choose asset portfolios with risk in the range $[\sigma^*,\sigma^{**}]$, and that regulators apply a squared-loss criterion to derive values of w_1 and w_0 to fit $c_{\min}$ to $c(\sigma)$ as well as is possible within that range. That is, regulators choose w_1 and w_0 to minimise the loss function:

$$\mathcal{L} = \int_{\sigma^*}^{\sigma^{**}} \left(c(\sigma) - c_{\min}\right)^2 \, d\sigma \tag{19}$$

---

proportional investment and the capital ratio are expressed relative to deposits rather than assets.

Minimising $\mathcal{L}$ corresponds to fitting a $c_{\min}$ line to a locus of $c(\sigma)$ values.

An indication of how well the linear risk-based capital schedule approximates either an LV or FP locus can be derived from the loss function $\mathcal{L}$ as defined in equation (19). We define the quantity ρ as:

$$\rho = \frac{\bar{\mathcal{L}} - \mathcal{L}}{\bar{\mathcal{L}}} \tag{20}$$

where

$$\bar{\mathcal{L}} = \int_{\sigma^*}^{\sigma^{**}} (\bar{c} - c_{\min})^2 d\sigma \tag{21}$$

and $\bar{c}$ is the mean of the required capital ratio within the range $[\sigma^*, \sigma^{**}]$:

$$\bar{c} = \frac{1}{\sigma^{**} - \sigma^*} \int_{\sigma^*}^{\sigma^{**}} c(\sigma)\ d\sigma \tag{22}$$

Note that $\rho = 0$ if $\mathcal{L} = \bar{\mathcal{L}}$ (the linear capital standard yields no improvement over simply setting the minimum capital ratio equal to $\bar{c}$ for all banks), $\rho = 1$ if $\mathcal{L} = 0$ (the linear standard fits the target LV or FP locus perfectly), and values of ρ between zero and one represent intermediate degrees of fit. Thus ρ is a "goodness of fit" measure analogous to the R^2 statistic conventionally reported in regression analysis.[10]

What are realistic values of c_{rb}, T, $\hat{\sigma}$, σ^*, and σ^{**}? Under the Basle standards equity capital must be at least four percent of risk-weighted assets, implying $c_{rb}=0.04$. We measure time in years and set $T=1$, corresponding to a one-year monitoring interval. (We discuss the importance of this assumption in subsection 4.5 below.)

Selecting values for the standard deviation of returns on risky assets $\hat{\sigma}$ and the range $[\sigma^*, \sigma^{**}]$ is more complicated. In an analysis of the Australian banking sector using contingent claim methods, Gizycki and Levonian (1992)

---

[10] This measure of fit weights the difference between $c(\sigma)$ and $c_{\min}$ evenly for all values of σ between σ^* and σ^{**}. If a linear standard were implemented, however, the values of σ chosen by banks may or may not be evenly distributed over the $[\sigma^*, \sigma^{**}]$ range used to derive asset weights.

find that the standard deviation has generally been in the range of 0.02 to 0.03 since 1983, but for brief periods has been above 0.05 and as low as 0.01. A number of studies based on U.S. data find values around 0.03, and thus are consistent with the Australian results. However, if deregulation of the banking sector continues, banks may begin to engage in a wider and perhaps riskier range of financial activities, possibly increasing the relevant upper bound on risk.

Accordingly, we consider values of $\hat{\sigma}$ equal to 0.03, 0.05, and 0.10. We consider four combinations we believe cover a range of interest for policy purposes; in each case we set the upper limit $\sigma^{**}=\hat{\sigma}$. The first case, with $\sigma^{*}=0.01$ and $\sigma^{**}=0.03$, corresponds to the average levels of asset risk observed by Gizycki and Levonian for the Australian banking sector. In the second case we again set the lower limit $\sigma^{*}=0.01$, but let $\sigma^{**}=0.05$, corresponding to a slightly wider range of risks that covers most of the range found by Gizycki and Levonian. The third and fourth combinations we interpret as representing environments in which banks can engage in much riskier activities; in both cases $\hat{\sigma}$ (and hence σ^{**}) is equal to 0.10. In the third case the bottom end is $\sigma^{*}=0.05$, so most banks have high operating risk, whereas in the fourth case $\sigma^{*}=0.01$, implying that banks cover a spectrum ranging from very low risk to very high risk.

4.2 Fitting an LV Rule

Table 1 presents the results of minimising the loss function $\mathcal{L}$ to fit a linear approximation to LV capital standards.[11] Three different liability values were used: 0.1 percent of deposits, 0.2 percent of deposits, and 0.3 percent of deposits. Each column of the table presents the results for one of the four combinations of (σ^{*},σ^{**}). One striking implication of Table 1 is that a linear risk-based capital schedule may provide a good approximation to an LV rule; the goodness-of-fit statistic (ρ) is above .995 in every case. A true LV standard would produce only a slight improvement in fit, at substantial cost in terms of complexity.

[11] For all minimisations, the loss function integral was calculated numerically, using rectangular approximation with grid size of 0.001.

The results for w_1 in Table 1 indicate that risky assets should be given a weight of well over 100 percent. This stands in contrast to the 100 percent maximum risk weight under the Basle standards. In the case in which bank risk is low ($1\%<\sigma<3\%$) and the acceptable LV is high, the optimal risk weight on risky assets would still be about 121 percent. If regulators require lower values of the deposit guarantee liability, or if the relevant range of risk for banks is high, then w_1 must

Table 1: Linear Approximation to Liability-Value Standard

		$1\%<\sigma<3\%$	$1\%<\sigma<5\%$	$5\%<\sigma<10\%$	$1\%<\sigma<10\%$
LV=0.1%	w_1	1.4807	2.5271	5.4011	5.1579
	w_0	-0.2137	-0.2702	-0.4598	-0.3557
	ρ	0.999151	0.999002	0.999998	0.999448
LV=0.2%	w_1	1.3136	2.2628	5.0863	4.7143
	w_0	-0.2501	-0.3175	-0.5943	-0.4343
	ρ	0.998400	0.998080	0.999966	0.998677
LV=0.3%	w_1	1.2124	2.0995	4.8852	4.4345
	w_0	-0.2777	-0.3518	-0.6819	-0.4879
	ρ	0.997577	0.997116	0.999917	0.997866

be much larger, up to 540 percent in the case of LV=0.1% and $5\%<\sigma<10\%$.[12]

Table 1 also indicates that riskless assets should receive a fairly large

[12] For the United States, policy probably requires an LV in the range of 0.2 to 0.3 percent of deposits, to correspond to the range of premiums charged for deposit insurance. In that case, if risk remains at the relatively low levels observed in past studies, the Basle risk weight of 100 percent is not far from optimal. However, if structural changes in the industry result in banks engaging in a riskier range of activities, so that risky assets have a standard deviation of returns around $\hat{\sigma}=5\%$, then the highest risk weight should be much higher, at 200 percent or more.

negative weight; w_0 is less than zero in all cases, ranging from about -20 percent to -50 percent. Negative weights would mean that banks could reduce their measured total risk-weighted assets by some fraction of their holdings of riskless assets. This would be a significant departure from the Basle standards, which assign a zero risk weight to riskless assets.[13] The largest negative weights correspond to the cases in which assets are very risky; in these cases, a constant value of the deposit guarantee liability can only be maintained by providing very strong encouragement for banks to hold riskless assets. The w_0 weight is more strongly negative if authorities are willing to tolerate higher LV values, which seems counterintuitive. However, note that in these cases the weight on risky assets also is much lower; as a result, the differential between w_1 and w_0 actually decreases, implying that banks receive less relative reward for holding riskless assets rather than risky assets.

4.3 Fitting an FP Rule

Table 2 presents the weights resulting from minimising $\mathcal{L}$ to construct a linear approximation to FP capital standards. Three different probabilities of bank failure are presented: 5 percent, 10 percent, and 15 percent. As in Table 1, each column presents the results for one of the four combinations of σ^* and σ^{**}. The very high values of ρ (in every case ρ exceeds .999) demonstrate that a simple linear rule could be an extremely close approximation to a theoretically correct but more complicated capital standard. The fit is even better than for the LV rules in Table 1 because the FP loci have less curvature overall.

The calculated values for w_1 indicate lower optimal weights on risky assets than were found in the LV case. For some of the cases, w_1 is actually lower than the weight of 100 percent incorporated in the Basle standards. For example, if asset risk is believed to be relatively low so that the relevant range is $1\% < \sigma < 3\%$, and authorities are willing to tolerate failure probabilities in the neighbourhood of 10 percent or 15 percent, an optimal

---

[13] Using different methods, Avery and Berger (1991) also found that a negative weight on assets in the riskless category would be optimal.

linear approximation calls for weights on risky assets of 95 percent or 78 percent respectively. However, as with an LV rule, w_1 soars if a higher range of bank asset risk is used, to well over 300 percent in the cases where the lowest failure probabilities are desired.

Table 2: Linear Approximation to Failure-Probability Standard

		$1\%<\sigma<3\%$	$1\%<\sigma<5\%$	$5\%<\sigma<10\%$	$1\%<\sigma<10\%$
FP=5%	w_1	1.2002	1.9756	3.6736	3.8259
	w_0	0.0078	0.0163	0.1164	0.0502
	ρ	0.999992	0.999968	0.999944	0.999830
FP=10%	w_1	0.9482	1.5704	3.0227	3.0878
	w_0	0.0030	0.0064	0.0490	0.0207
	ρ	0.999998	0.999992	0.999983	0.999952
FP=15%	w_1	0.7754	1.2904	2.5516	2.5681
	w_0	0.0005	0.0011	0.0116	0.0045
	ρ	0.999999	0.999999	0.999998	0.999996

In contrast to the LV results, under an FP rule the weight given to riskless assets should be positive, although in several cases w_0 is extremely small. For the lower risk cases, w_0 ranges from well under 1 percent to a little more than 1½ percent, fairly close to the Basle weight of zero. Only in the case where all banks are very risky ($5\%<\sigma<10\%$) and the probability of failure must be held to 5 percent does the weight on riskless assets rise above 10 percent; in all other cases, w_0 is 5 percent or less.

Based on the results in Tables 1 and 2, we conclude that the Basle standards come closest to a linear approximation of an FP rule where a probability of bank failure of 10 percent is deemed acceptable, and banks are assumed to have asset risk in the range observed in studies of the U.S. banking industry.

From Table 2, a best-fit linear approximation to an FP rule under these circumstances requires a weight on risky assets of 95 percent, very close to the 100 percent in the Basle Accord, and a weight on riskless assets of 0.3 percent, very close to Basle's zero weight. With those weights, a Basle-type standard approximates an FP rule extremely closely, as indicated by the high value of ρ.

4.4 Accommodating Restrictions on the Risk Weights

Outcomes reached through international negotiations may reflect considerations that are not strictly economic in nature. Thus there may be reasons to accommodate restrictions on some of the parameters of the capital standard. In the calculations presented above, we have taken as given the basic linear structure of the capital standards and the four percent minimum equity capital ratio. However, there may also be some need to accommodate existing risk weights to the extent possible.

Consider the case of maintaining a risk weight of zero on cash-equivalent assets. If the weight on riskless assets w_0 is restricted to zero as under the Basle standards, the linear risk-based capital schedule in (18) becomes:

$$c_{\min} = \left(\frac{c_{rb} w_1}{\hat{\sigma}} \right) \sigma \tag{23}$$

This is a line through the origin in the (σ,c) plane; as indicated in the discussion above, such a line corresponds more closely to an FP rule than an LV rule. Restricting w_0 to zero and deriving w_1 to fit a constant FP rule yields a linear approximation that is slightly steeper than the FP locus. For example, in the case of $FP_0=10$ percent with $c_{rb}=0.04$ and $1\%<\sigma<5\%$, the slope is 1.5768 when w_0 is restricted to be zero, and is 1.5704 when the intercept is not restricted. (Based on the discussion in section 3.3 above, the difference in slopes implies that an approximation with w_0 set to zero is slightly less likely to lead banks to choose high asset risk portfolios than a true FP rule.) The fit to the FP standards is, of course, inferior to the results obtained when w_0 is not restricted.

Alternatively, the Basle weights of $w_1=1$ and $w_0=0$ could be taken as binding restrictions, and the required minimum risk-based capital ratio adjusted to achieve an FP goal. Table 3 shows the results of such calculations, for FP rules with failure probabilities of 5, 10, and 15 percent. In view of the Table 2 results, it is not surprising that the cases in the lower left corner of the table yield values of c_{rb} close to the Basle standard of 4 percent. However, the minimum capital ratio must be raised if higher levels of banking risk need to be covered. For $1\%<\sigma<5\%$, c_{rb} should be in the range of 5 percent to 8 percent; under the highest of the upper bounds on asset risk, capital ratios in the 10 to 15 percent range might be required.

Table 3: Optimal Values of c_{rb} (restricting $w_1=1$ and $w_0=0$)

	$1\%<\sigma<3\%$	$1\%<\sigma<5\%$	$5\%<\sigma<10\%$	$1\%<\sigma<10\%$
FP=5%	0.048	0.080	0.152	0.155
FP=10%	0.038	0.063	0.123	0.124
FP=15%	0.031	0.052	0.103	0.103

Another view of the implications of the standards follows from taking the Basle risk weights as given and determining the FP contour to which the Basle standard most closely corresponds. The results are shown in Table 4, for each of the four risk cases, and for risk-based capital ratios of 0.04 and 0.08. With $c_{rb}=0.04$, the implied upper bound on the probability of bank failure ranges from 9 percent in the lowest risk case to around 36 percent for the higher risk cases. Doubling the risk-based capital ratio to 0.08 reduces the failure probability to a trivial 0.3 percent for the low risk case, although it remains at levels above 20 percent for the highest risk cases.

Table 4: Implied Upper Bound on Probability of Failure (restricting $w_1=1$ and $w_0=0$)

	$1\%<\sigma<3\%$	$1\%<\sigma<5\%$	$5\%<\sigma<10\%$	$1\%<\sigma<10\%$
$c_{rb}=0.04$	0.090	0.214	0.360	0.359
$c_{rb}=0.08$	0.003	0.052	0.219	0.217

It is worth noting that bank supervisors in each country do retain some flexibility under the Basle agreement, especially with regard to the minimum risk-based capital ratio. The Basle standards set a floor; national authorities have the power to require banks to hold more capital, and many banks do in fact hold more than the minimum. The results in this section support this flexibility as a desirable aspect of the regulations; bank supervisors in each country can tailor the standards somewhat to the riskiness of the environment in which their banks operate.

4.5 The Effect of Alternative Assumptions Regarding T

All of the results presented above assume that the monitoring interval T is equal to one year. Obviously this is an arbitrary assumption, and its effect on the computations can be substantial. Straightforward differentiation of equations (5) and (6) with respect to T indicates that FP and LV are increasing in T. Thus a monitoring interval shorter than one year would imply lower values of FP and LV for any given combination of the other parameters, and the opposite for longer monitoring intervals. As a result, with a lower value of T the risk weights needed to achieve any particular targeted levels of either the probability of failure or the value of the deposit guarantor's liability would move toward zero.[14]

On the other hand, the failure probabilities and liability values used in the computations are also expressed on a per-monitoring-period basis. If the monitoring interval is shorter, policy goals almost certainly would demand

---

[14] The results in the tables can be reinterpreted to provide a feel for the effect of alternate assumptions. Note that within the contingent claim model, the only way that T appears is in the term $\sigma\sqrt{T}$; moreover, this is the only way that σ appears. Hence if the monitoring interval is not one year, the computational consequence is effectively to stretch or compress the σ dimension by a factor of $\sqrt{T}$. Any inferences drawn from a case in which σ=0.10, for example, would actually correspond to σ=0.10/$\sqrt{T}$. If the monitoring interval is three months rather than one year so that T=0.25, all of the values of σ, $\hat{\sigma}$, σ^*, and σ^{**} should be doubled; with a corresponding rescaling of column headings, the figures in the tables are still valid. The policy interpretations change, of course: if monitoring intervals are believed to be less than one year, the higher-risk cases (such as the last two columns of the tables) probably represent unrealistically high values of σ, and the results in the first column of each table become much more relevant.

lower target values of *FP* and *LV*; that is, the policy targets probably decline with *T*. For example, if a failure probability of two percent is acceptable over a one-year horizon then the acceptable probability within a three-month interval surely must be lower, perhaps closer to one-half of one percent. As the tables make clear, lower FP and LV targets require higher risk weights (or larger negative weights on riskless assets in the LV case). This works in the opposite direction of the parametric effect of *T* in the expressions for $LV(\sigma,c)$ and $FP(\sigma,c)$. Put differently, a reduction in *T* likely requires that *LV* and *FP* be smaller to meet supervisory goals, but the implicitly more frequent monitoring also reduces failure probabilities and guarantor liabilities in such a way that higher risk weights may not be necessary. The precise degree of offset is unknowable, but the conclusions are surely less sensitive to assumptions regarding *T* than would appear from a simple examination of the impact of *T* on the contingent claim model in isolation.

5. CONCLUSIONS AND AGENDA FOR FUTURE RESEARCH

We have shown within the context of a contingent claim model of banking that risk-based capital standards can be derived to fix the value of the deposit guarantor's liability per dollar of deposits; we call this an LV rule. Alternatively, risk-based capital can place an upper bound on the probability of bank failure during any period, which we refer to as an FP rule. We examine the value-maximising responses of banks facing the constraints imposed by LV and FP risk-based capital standards, assuming that banks maximise the value of equity net of contributed capital. We find that bank behaviour depends on the type of rule imposed. The model suggests that low-risk assets will be chosen under an LV rule, while high-risk assets are likely to be chosen under an FP rule. Thus while the two rules are plausible and are designed to meet related regulatory goals, they have very different implications for bank behaviour.

The LV and FP standards, while desirable in theory, may be too complicated to be feasible in practice. We derive a simple version of the international standards actually being implemented (the BIS/Basle standards) and find that those standards could serve as a good approximation to either an LV rule or an FP rule. However, with either type of rule the "best fit" weight on risky assets is greater than 100 percent under most assumptions; the weight on riskless assets is close to zero for an FP approximation and substantially less than zero for an LV approximation. Judgements regarding the best weightings depend on the range of asset risk believed to be relevant in practice; since this range may vary from one country to the next, capital standards should retain a degree of flexibility to allow national banking authorities to make necessary adjustments.

Further research should explore the sensitivity of the simple linear standards to the number of asset categories. Also, it would be more accurate to allow for non-zero correlation between asset categories, and to treat all assets as risky since even government securities are subject to interest rate risk. For practical applications to banking policy a better sense of the actual variance-covariance matrix for various types of bank assets is needed; estimation of this matrix is the subject of separate work in progress by the authors. Given more precise estimates of the riskiness of various types of assets, it would be interesting to re-examine how close the actual risk weights from the Basle accord come to either of the theoretical possibilities discussed here. Finally, in actual practice regulators probably aim both to prevent failure and to limit the deposit guarantee liability; the authors are exploring the possibility of representing risk-based capital objectives as a weighted average of FP and LV.

REFERENCES

Avery, R.B. and A.N. Berger (1991), "Risk-Based Capital and Deposit Insurance Reform", *Journal of Banking and Finance,* 15: 847-874.

Duan, J-C., A.F. Moreau and C.W. Sealey (1991), "Incentive Compatible Deposit Insurance Pricing and Bank Regulatory Policies," unpublished manuscript.

Eichberger, J. (1992), "Capital Adequacy Versus Reserves as Prudential Control Devices", presented at the Economic Society of Australia Conference of Economists, Melbourne. Unpublished.

Furlong, F.T. and M.C. Keeley (1989), "Capital Regulation and Bank Risk-Taking: A Note", *Journal of Banking and Finance,* 13: 883-891.

Gennotte, G. and D. Pyle (1991), "Capital Controls and Bank Risk", *Journal of Banking and Finance,* 15: 805-824.

Gizycki, M. and M. Levonian (1992), "Australian Banking Risk in the 1980's: Evidence from Share Prices", Reserve Bank of Australia Research Discussion Paper (forthcoming).

Keeley, M.C. and F.T. Furlong (1990), "A Reexamination of Mean-Variance Analysis of Bank Capital Regulation", *Journal of Banking and Finance,* 14: 69-84.

Kendall, S.B. and M.E. Levonian (1992), "Alternative Risk-Based Capital Designs and Their Implications for Bank Behavior and the Deposit Insurance Fund", presented to the Midwest Finance Association, Chicago. Unpublished.

Kim, D. and A.M. Santomero (1988), "Risk in Banking and Capital Regulation", *Journal of Finance,* 43: 1219-1233.

Marcus, A.J. (1984), "Deregulation and Bank Financial Policy", *Journal of Banking and Finance,* 8: 557-565.

Merton, R.C. (1977), "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees", *Journal of Banking and Finance,* 1: 3-11.

Merton, R.C. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *Journal of Finance,* 29: 449-470.

Ronn, E.I. and A.K. Verma (1989), "Risk-Based Capital Adequacy Standards for a Sample of 43 Major Banks", *Journal of Banking and Finance,* 13: 21-29.

Sharpe, W.F. (1978), "Bank Capital Adequacy, Deposit Insurance and Security Values", *Journal of Financial and Quantitative Analysis,* 13: 701-718.

Smith, C.W. (1976), "Option Pricing: A Review", *Journal of Financial Economics,* 3: 3-51.