On the Risk of Leaving the Euro*

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Abstract

Following the recent European sovereign debt crisis, there have been many proposals to leave the Euro for South-European countries. We evaluate this policy change in a standard monetary model with seinorage financing of the deficit. We also evaluate crawling peg policies that can be put in place to temporarily control inflation in the aftermath of exiting. The non-standar part of the model is that we deprat from rational expectations, whle mataining full rationality of agents in sense we make very precise in the paper. The first contribution of the paper is to show that very small departures from rational expectations imply that the resulting inflation rate after exiting can be orders of magnitude higher than in the model with rational expectations. The second contribution of the paper is to provide a framework for policy analysis in models without rational expectations.

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1 Introduction

In this paper we study the inflationary consequences of abandoning a currency union and replacing it by a national currency. The model we use is standard with one exception: we adopt the approach of *Internal Rationality,* which allows for small departures from rational expectations (RE) while maintaining full rationality of agents' behavior.[1] Agents in the model have limited information about the environment they live in. Therefore they cannot predict fully the effects of a policy change involving a currency reform of that magnitude and they forecast inflation by observing inflation behavior itself. In this environment learning is self-referential: agents' expectations influence inflation and inflation influences expectations. As shown by Marcet and Nicolini (2003), this feedback process can substantially amplify the effect of seiniorage on inflation rates.[2]

Following the great world recession of 2008 and the European sovereign debt crisis of 2012, the proposal to leave the Euro and reintroduce a national currency has regained support both in academic and political circles, particularly in some South-European countries. Leaving the euro is supported in Italy by the Five Stars Movement and Lega Norte, who jointly won a majority of the Parliament in the May 2018 elections. In France, Marine Le Pen, leader of the nationalist party Front National and the strongest supporter of "Frexit", got a fifth of the vote in the first round and a third in the ballotage the year before. In Greece, the radical-left party Syriza won the January 2015 elections with the promise to bargain favorable bailout conditions with Europe and, if this was not possible, to leave the EMU.

Leaving the euro would presumably bring about many blessings for these Southern-European economies: individual countries would be free from the fiscal chains of the EU, putting an end to the austerity. In addition re-launching national currencies would allow Central Banks to stimulate the economy. By contrast, remaining in the Euro area amounts to embracing the austerity required by some European rules and the low inflation promoted by the ECB. Bernard Monot, Ms. Le Pen's economic consultant put it this way: "Give us the Banque de France and the Finance Ministry, then France would be out of trouble in three weeks". Alessandro di Battista from "Five Stars movement" said: "We are convinced that, if we are able to take back monetary sovereignty we can raise Italy from the rubble".

An exit from the euro could be done in many different ways. We build our model around a specific exit strategy and other alternatives are briefly discussed. As a summary, the exit

[1]See Adam, Marcet and Nicolini (2016) and Adam, Marcet and Beutel (2017) for applications to stock market volatility, and Adam and Marcet (2011) for a discussion of some theoretical issues.

[2]Marcet and Nicolini (2003) show that such a departure from rational expectations explains key facts relating deficits, seiniorage and inflation during the South-American hyperinflations of the 80's, as well as the policies that were put in place to stop those inflations.

we consider has three elements: *i)* no additional austerity will happen after exiting, hence the deficit process will remain unchanged, *ii)* the country regains its ability to raise seiniorage, *iii)* as a consequence of exiting the ability to access foreign markets for debt will be severely limited. We discuss these assumptions in more detail in the text.

Our first contribution is to show that the interaction of persistent deficits with agents that form expectations using past observations dramatically amplifies the equilibrium inflation rates generated by the model. In particular, a small deviation from RE implies inflation rates that can be several times higher than with RE. The paper quantifies the risk of hyperinflations that may follow a departure of the Euro system, a risk that has been overlooked in the recent debate. We compute the welfare implications of these events and show that they are substantial. We also show that exchange rate policies can substantially ameliorate these welfare consequences. These policies will require external funding and, therefore, an orderly exit. We compute how large these funding requirements ought to be as a function of policy parameters. Our framework can therefore be used to compute the gains of a negotiated exit that includes access to limited funding in case inflation becomes too high.

The reason that hyperinflations emerge is that due to learning there is a region of instability in expectations: if expectations land in this region, inflation can only grow, then expected inflation grows as a response to this higher inflation, leading to higher inflation and so on. The result is that hyperinflations are likely to emerge for a very persistent deficit process as is observed in the data. Only in cases where the deficit is by chance persistently low after exiting, hyperinflations are avoided.

Our second contribution is methodological, as we do a step forward in exploring the use of internal rationality (IR) for policy analysis. Departures from RE are still controversial, specially when they are used for policy analysis. Possibly, the main reason that RE became the dominant paradigm in modelling expectations is that it allowed the analysis of policy reforms in a consistent manner as it addressed the Lucas critique. We claim that by explicitly modelling agents' expectations we can gain a better understanding of policy interventions.

Under IR agents are assumed to hold a coherent belief system about inflation, even if this may not be the true distribution of inflation under the model considered. Therefore, a complete description of the environment requires an explicit assumption about the agents' belief system regarding inflation. Agents understand that the evolution of inflation depends on certain aggregate shocks, since they know that given their beliefs and the beliefs of other agents the model implies a certain mapping between these aggregate shocks and inflation. However, as we will show in detail, given their available information agents are unable to compute this mapping.[3]

[3]A large number of papers assumes agents learn about the exogenous process of the economy. These papers

We are certainly not the first to study policy analysis in models where agents have imperfect knowledge of the model. The paper innovates in two dimensions. The first one is the systematic use of IR in order to explore how expectations may behave after a policy change and how the feedback between inflation and inflation expectations shape the equilibrium dynamics of the model. The second is to use the model with IR to compute the welfare effect of alternative policies using methods that are standard in the realm of rational expectations models.

First and foremost, we study the positive implications of the model for the evolution of equilibrium inflation. We show that small departures from rational expectations - in a sense we make very precise in the paper - imply that the resulting equilibrium inflation rates can reach values order of magnitude higher than in the model with rational expectations. This implies that the policy implications of the rational expectations version of the model are not robust to small deviations on the expectations hypothesis. We go ahead and evaluate policies that use crawling pegs to the exchange rates as a way to temporarily reduce inflation rates. We parameterize those policies and compute the welfare impact they have. We find that early interventions that use shock type policies are preferred over policies that delay the intervention or that follow a gradual approach in reducing inflation.

The use of the IR framework is certainly not standard and it raises several methodological issues that need to be dealt with, particularly in evaluating policy. In order to do so, in Section 2 we discuss a monetary model with heterogeneous agents and incomplete markets. We show that in following our approach, it is perfectly consistent to have fully rational agents that do not know the pricing function for inflation. We then go on to discuss how to chose a reasonable belief system and how it can be made dependent on policy changes. Section 3 where we introduce seigniorage financing and study Learning Equilibria. In Section 4 we assess the quantitative performance of our model and show that the presence of learning translates into recurrent hyperinflationary episodes. In Section 5 we derive testable implications of the belief system and test whether agents can reject their beliefs based on data generated by the model. In Section 6 we present the policy evaluation exercises.

make the implicit assumption that agents are able to figure out the pricing function mapping fundamentals into equilibrium prices, thus we refer to them as "Bayesian/RE learning" models. While this is a consistent assumption to make it still requires that agents know from the outset a lot about how to predict inflation. What we argue in section 2 of the paper is that rational behavior does not imply knowledge of this pricing function.

2 A Model with Heterogeneous Agents

Our basic model equation will be given by a government budget constraint and a money demand equation where higher expected inflation drives down the demand for real balances as in (5).[4] Here we take the simple approach, most commonly used in the literature, of deriving that money demand from an overlapping generations model. However, it is also possible to derive that equation from a model with long-lived agents and we will perform our derivations below having this extension in mind. We consider heterogeneous agents to highlight the fact that an individual agent would not be able to infer the pricing function from observations and her own behavior, although the policy analysis that is the core of the paper will be done with a homogeneous agent model for simplicity.

In addition to the money demand, the budget constraint of the government that chooses to monetize debt will determine equilibrium.

Consider a constant cohort size, overlapping generations model in which each agent lives for two periods. Agents are heterogeneous in their endowments and in their preferences. The endowments of agent $j \in [0, 1]$ born at time t are normalized to 1 when young, common to all agents, and e_t^j when old, and her preferences are given by

$$\ln c_t + \alpha_t^j \ln x_{t+1}$$

Thus agents are heterogeneous in their endowment when old e_t^j and their discount factor α^j. The values of the pair $\{e_t^j, \alpha_t^j\}$ are drawn from some exogenously specified, possibly time varying distribution, at the time each agent is born. In solving their optimal problem, agents know their own values of $\{e_t^j, \alpha_t^j\}$.

We restrict the endowment when old to be smaller than the endowment when young ($e_t^j < 1$ for all j). We assume that agents have a relative preference for consumption when old ($\alpha_t^j \geq 1$ for all j). These assumptions are made to ensure that as long as the return on savings is not too low, young agents would save in equilibrium.

Markets are incomplete in the sense that the only asset agents can hold is fiat money. Thus, at any point in time, there is only one spot market in which agents can exchange goods for money, at a price P_t. When young, agents choose how many units of money to hold for next period, given the price level that prevails at time t. The budget constraint when young is given

[4] Our model will be similar to Marcet and Nicolini (2003), we choose this setup as it has been shown to explain well hyperinflationary episodes. The main difference in the present analysis will be considering various alternatives for the fiscal deficit that reflect more closely deficit in European countries, so we will go away from the iid assumption for fiscal deficit.

by[5]:

$$P_t c_t^j + M_t^j \leq P_t \tag{1}$$

In the following period, they consume their endowment plus whatever they can buy with the money previously held, so their budget constraint when old is:

$$P_{t+1} x_{t+1}^j \leq M_t^j + e_t^j P_{t+1} \tag{2}$$

for all P_{t+1}.

Agents' expectations are possibly heterogeneous as well, hence, the problem of agent of type j born in period t consists in maximizing:

$$\mathbf{E}_t^j [\ln c_t + \alpha_t^j \ln x_{t+1}] \tag{3}$$

by choosing consumption and money holdings, subject to the budget constraints (1) and (2).

Agents are assumed to observe at t the values of variables dated t as well as e_t^j. However agents do not know the value of next period prices level. Hence the expectation is taken with respect to the price level P_{t+1}, which due to the presence of aggregate uncertainty agents can not infer from their observed endowment, more on this later.

Since the budget constraints will hold with equality, once we substitute them in (3), an interior solution requires:

$$\frac{1}{P_t - M_t^j} = \mathbf{E}_t^j \left[\frac{\alpha_t^j}{M_t^j + e_t^j P_{t+1}} \right] \tag{4}$$

which defines implicitly the individual money demand equation for agent j. Importantly, money demand must be measurable with respect to the information set available when young. Since the only source of uncertainty, namely P_{t+1}, appears in the denominator of the right hand side, we cannot solve for the money demand equation in closed form. In order to make progress, we study the linearized version of it, which can be written as[6]:

$$\frac{M_t^j}{P_t} = \phi_t^j \left(1 - \gamma_t^j \mathbf{E}_t^j \frac{P_{t+1}}{P_t} \right)$$

[5] As agents cannot issue money, the constraint $M_t^j \geq 0$ must be imposed. However, the assumption that the endowment in the second period is smaller than in the first period implies this constraint will not be binding as long as the inflation rate is not too high, thus we ignore this constraint in our theoretical analysis. In the numerical section, we impose this constraint on the equilibrium.

[6] The linearization is standard. It is offered for completeness in the Appendix.

which corresponds to the money demand by each agent of generation t, where

$$\phi_t^j = \frac{\alpha_t^j}{1+\alpha_t^j} \text{ and } \gamma_t^j = \frac{e_t^j}{\alpha_t^j}.$$

We have been working on various applications of internal rationality for a few years now. In discussing our work, both in seminars and during the editorial process, we have found a number of researchers in economics holding the view that a rational agent who knows the process for exogenous fundamentals of an asset can not hold separately a view about the prices of that asset. Such "IR-skeptics" sustain that the whole structure of IR is logically inconsistent: rational agents should be able to map their view of asset fundamentals into the value of an asset price.

In the framework of this model we can formalize this view as follows: consider the assumption

Assumption 1 *All agents hold a view about the evolution of the aggregate money supply* M_t^s.

An IR-skeptic would claim that under assumption 1 a rational agent should be able to infer the pricing function that maps realizations of M^s into a price level. The rest of the subsection states that this argument is flawed for a variety of reasons. Therefore, we will conclude that IR is logically consistent.

An IR-skeptic would likely articulate his thoughts using a homogeneous agent version of the model, where $\alpha_t^j = \alpha$ and $e_t^j = e$. In this case, the above money demand is as follows

$$M_t = \phi\left(P_t - \gamma \mathbf{E}_t P_{t+1}\right) \tag{5}$$

Since knowledge of this equation is a consequence of rational behavior it must be that IR agents know this equation. From this it follows that the price level (in a non-bubble solution) satisfies

$$P_t = \frac{1}{\phi}\sum_{s=0}^{\infty} \gamma^s \mathbf{E}_t M_{t+s}^s \tag{6}$$

therefore knowledge of the aggregate money supply M^s plus maximizing behavior by agents indeed determines the price level and, according to an IR-skeptic it is then logically inconsistent to assume (as we will assume below) that agents hold separate expectations about the price level.

However, this argument does not work once we have heterogeneous agents. In this case the only discounted sum an agent can obtain from knowledge of optimizing behavior is

$$P_t = \frac{1}{\phi_t^j}\sum_{s=0}^{\infty} \left(\gamma_t^j\right)^s \mathbf{E}_t^j M_{t+s}^j \tag{7}$$

The key difference is that the money demand in this expression is M^j, with a super-index corresponding to the agent j, not the exogenous supply for money as in (6). In other words, the agent does know that his own optimal decision maps *his* future demands for money to the price level, but optimal behavior does not relate future exogenous values of M^s to price behavior. Hence, there is no contradiction in knowing the behavior of M^s and having a separate belief system for the price level, the first does not map into the second. The optimality condition (7) that agent j knows to hold in a IR equilibrium in no way restricts what agents think about the link between M^s and P.

Since agents with different types will now face a different inference problem, the computation of the right hand side of (7) becomes a much more complicated task. In particular, it requires each agent knowing the inference problem solved by all other agents in the economy so that agent i can figure out $\mathbf{E}_t^j$ for all $j \neq i$. Even if we endow each agent with knowledge of the distribution of types of all other agents in the economy, it is apparent that discovering the mapping from exogenous variables to prices becomes a much more challenging problem.

But an IR-skeptic could bring to the table the following claim "a rational agent could use his rational behavior to infer the relationship aggregate money demand and, in this way, to infer how M^s and P are related". Let's see how this could work. Thus if we add some slight knowledge about how other agents behave, individual optimization and knowledge of exogenous variables maps into a price level.

Let us see how this could work. In the above model, aggregate money demand is:

$$M_t = \int_0^1 \phi^j \left(P_t - \gamma^j \mathbf{E}_t^j P_{t+1} \right) dj. \tag{8}$$

So, *if in addition to knowing how to solve his maximization problem (ie, in addition to being IR) we make the following assumption*

Assumption 2 *All agents know that other agents have similar utility function to their own, up to diversity in γ^j, ϕ^j, E^j. Furthermore, agents know $\overline{\phi} = \int_0^1 \phi^j dj$.*

Under assumption 2 an IR agent could obtain

$$P_t = \int_0^1 \frac{\phi^j \gamma^j}{\overline{\phi}} \mathbf{E}_t^j P_{t+1} dj + \frac{M_t^s}{\overline{\phi}}. \tag{9}$$

Is this enough to map M^s into P?. The answer is no. All that our IR agent could do is to plug the optimality condition (7) into (9) to obtain

$$P_t = \int_0^1 \frac{1}{\overline{\phi}} \mathbf{E}_t^j \sum_{s=0}^{\infty} \left(\gamma^j\right)^{s+1} M_{t+1+s}^j dj + \frac{M_t^s}{\overline{\phi}} \tag{10}$$

so he needs to know, in addition, $\int_0^1 \mathbf{E}_t^j \left(\gamma^{j+1}\right)^s M_{t+1+s}^j dj$ for all t, s and these quantities can not be inferred from the knowledge given under assumption 2.

Let us see under what assumptions the IR-skeptic would be right. Consider

Assumption 3 *Agents have the same system of beliefs, therefore they have homogeneous (although possibly non-RE) expectations $E^j = E^{\mathcal{P}}$.*

Notice that under Assumptions 1 and 2 agents can figure out that

$$M_t^j = \phi^j \left(P_t - \gamma^j \mathbf{E}_t^{\mathcal{P}} P_{t+1}\right) \tag{11}$$

so that

$$\begin{aligned}
\int_0^1 \mathbf{E}_t^j \left(\gamma^j\right)^{s+1} M_{t+1+s}^j dj &= \int_0^1 \mathbf{E}_t^{\mathcal{P}} \left(\gamma^j\right)^{s+1} \phi^j \left(P_{t+s+1} - \gamma^j P_{t+s+2}\right) dj \\
&= \int_0^1 \left(\gamma^j\right)^{s+1} \phi^j \left(\mathbf{E}_t^{\mathcal{P}} P_{t+s+1} - \gamma^j \mathbf{E}_t^{\mathcal{P}} P_{t+s+2}\right) dj \\
&= \int_0^1 \left(\gamma^j\right)^{s+1} \phi^j dj \ \mathbf{E}_t^{\mathcal{P}} P_{t+s+1} - \int_0^1 \left(\gamma^j\right)^{s+2} \phi^j dj \ \mathbf{E}_t^{\mathcal{P}} P_{t+s+2}
\end{aligned}$$

for all j. But assumptions 1-3 still do not allow for the computation of this quantity, in addition we would need to assume

Assumption 4 *Agents know the whole joint distribution of γ, α*

This assumption allows agents to compute the integrals $\int_0^1 \left(\gamma^j\right)^{s+1} \phi^j dj$ and $\int_0^1 \left(\gamma^j\right)^{s+2} \phi^j dj$ in the last equation above. With this knowledge it is possible indeed to map future values of M^s into a price level today.[7]

In other words, it is logically consistent to assume that agents are rational and have price beliefs that do not map M^s into P as we do under Internal Rationality, all we need to assume is agents do not know the distribution of other agents endowments and utilities, and/or that their beliefs are diverse.

Furthermore, in this paper we consider a model where the money supply is not exogenous, but it is determined by the price level. Therefore, just because agents think inflation will be different they will have different beliefs about the money supply. This means that even assumption 1 is not reasonable in our model: in the event of a drastic policy change as the one we consider in the paper, and if government deficits are going to be monetized, how could agents know from the outset the behavior of money supply in the future given their price beliefs?.

[7]Technically, only knowledge of the covariance between e_t^j and α_t^j is required. But this is just an artifact of the linearization of the solution. In general, knowledge of the whole joint distributions of e_t^j and α_t^j are required.

The previous discussion shows how in our model (and arguably in many models) the assumption of RE is logically unrelated to the assumption of optimal agents' behavior. Under incomplete markets and heterogenous agents it is just impossible for consumers to compute the RE equilibrium using only their (incomplete) knowledge of the economy.[8] Therefore agents are still taking saving decisions and filtering information optimally given their beliefs about inflation. This argument, while very compelling, only justifies considering hypothesis for expectations formation that are not necessarily model consistent, as RE imposes. But it does not offer any guidance on how to proceed, so it raises several methodological issues. In what follows, we list three of them and explain how we approach them:

i) *How should expectations be modeled?*

   Having accepted that the model environment does not determine agents' expectations, an explicit *assumption* on the agents' belief system about inflation is needed to fully describe the environment. Thus we treat agents' belief system the same way that utility functions, production functions, or the equilibrium concept are treated in the literature. Being explicit about this modelling choice regarding the agents' system of beliefs has some advantages. First, it highlights the fact that RE is just one assumption about agents' beliefs from among many others. Second, it clarifies that this is the only deviation from the standard paradigm that is now dominant in macroeconomics, agents in our paper are completely rational given this system of beliefs.[9] Third, we can ask questions about how reasonable is this assumption vis a vis the data, observations on agents' expectations and the model itself.

ii) *What is a reasonable assumption about agents' model for inflation?*

   As with any assumption, its usefulness should be judged according to its theoretical and empirical virtues. We start by assuming that the process that governs agents beliefs is the same as the true evolution of inflation under a linearize rational expectations model. However, agents are unsure regarding one parameter in the formulation.[10] The specific assumption we make is that parameter, that governs inflation, is given by a mixture of a transitory and a permanent components. This has the advantage that it coincides

[8] Adam and Marcet (2011) discuss a related issue in the context of stock markets.

[9] The literature on adaptive learning as in, for example, Evans and Honkapohja (2002) and Marcet and Sargent (1989a), was unclear about to which extent agents' expectations were compatible with agents' optimal behavior. That literature tends to emphasize that agents are rational in the limit if the economy converged to RE. IR clarifies this distinction: agents optimize in all periods given their system of beliefs about inflation, we make an explicit assumption about this system of beliefs, and this system is not equal to the behavior of inflation in the model.

[10] To know the value of that parameter, agents would need to be able to compute the equilibrium mapping. As discussed above, they cannot do it unless they posses all the required information.

with RE beliefs for certain parameter values, so it allows to study the robustness of the predictions about leaving the Euro to small deviations from RE. In addition, various papers have shown that survey inflation expectations are well described by a system of beliefs similar to the one we use.[11] More importantly, we perform a series of tests showing that for period lengths of between 10 and 15 years, agents in the model would find hard to reject the hypothesis that their system of beliefs is the correct one under the model generated data. It is in this sense that this system of beliefs is a reasonable one for agents to maintain after exiting. In a way, what happens is that just because agents believe that there is a permanent component in the determination of inflation, then they learn about inflation and expected inflation does become a permanent variable that influences true inflation.

iii) *How do agents' beliefs about inflation change following a policy reform?*

RE ties agents' beliefs and model outcomes in a very specific way: it assumes zero distance between perceived and actual distributions. It seems unlikely that this would be the case immediately after a large change in policy as the one we analyze in this paper. Thus, we assume that after a policy change the belief system is reset but in such a way that it would be difficult for agents to reject their beliefs upon observing the model equilibrium their beliefs generate. Specifically, we allow agents to re-set their prior after exiting, expressing larger uncertainty about the underlying level of inflation, and we discipline that prior so as to be consistent with the rational expectations outcome.

3 Introducing Seigniorage Financing

The model in the previous section highlighted the fact that representative agent models hide valuable insight regarding how expectations must be formed. This implies that inflation expectations may play a role in determining the model outcome even if agents are strictly rational (by which we mean agents are internally rational, IR). We show that, indeed, the dynamics of inflation expectations can play a crucial role in determining the outcome of a policy change.

On purpose, we innovate as little as possible on the front of model building so as to focus on the issue of analyzing the policy change involved in leaving the euro. For this we adapt the model of Marcet and Nicolini (2003) to the case where seigniorage is not iid. This seems a reasonable choice since this model was shown to perform well to explain the dynamics of hyperinflations, it provides policy recommendations in line with the standard view for the right policy in ending hyperinflations, and it is a model where inflation expectations play a key role.

[11] Carvalho, Eusepi, Moench and Preston (2017)

Further research should extend the results of this paper to more involved environments. Here we assume seigniorage is serially correlated and exogenous, future research should study more elaborate cases where seigniorage depends on inflation, capturing the idea that an inflation allows governments to lower the cost of running the government.

In the rest of the paper we shut down heterogeneity considered previously and assume $\alpha^j = \alpha$, $e^j = e$. Results about inflation under heterogeneity would not be substantially different, heterogeneity was only used in the previous section to dismiss criticisms from IR-skeptics. We also introduce seigniorage financing and switch the focus of the analysis to the way aggregate inflation expectations are formed.

In order to consider deviations from rational expectations that are small, we proceed in the following way. First, we compute the stochastic properties that inflation follows in the rational expectations equilibrium. We then endow agents with a system of beliefs regarding the process of inflation - that agents rightly perceive as exogenous to their decisions - that is consistent with the behavior of inflation in the rational expectations equilibrium. But we assume that agents are not completely sure regarding the value of underlying long run inflation in that process. In the background, we can think of this uncertainty as stemming from not knowing the distributions of the money demand of all the agents in the economy, as discussed above.

The system of beliefs for the process of inflation that we endow the agents with is the sum of a transitory and permanent component. Given this system of beliefs, agents rationally use the data generated by the model to update their prior. In particular, given the system of beliefs, agents rationally use the Kalman filter to obtain a more precise estimate of the parameters they are uncertain about.

3.1 Equilibrium Conditions

We carry out the analysis by focusing on three equations: the money demand equation, the government budget constraint, the law of motion for the level of seigniorage. The demand for real balances that arises from (5) can be written as:

$$\frac{M_t^d}{P_t} = \phi \left(1 - \gamma \pi_{t+1}^e\right) \tag{12}$$

where $\pi_{t+1}^e = \mathbb{E}_t^{\mathcal{P}} \left[\frac{P_{t+1}}{P_t}\right]$ denotes the expected gross inflation rate.

The only potential source of uncertainty in this model comes from the level of seigniorage. In particular, the government budget constraint is given by:

$$M_t^s = M_{t-1}^s + d_t P_t \tag{13}$$

where M_t^s is the money supply and d_t denotes exogenous seigniorage, which evolves according to:

$$d_t = (1-\rho)\delta + \rho d_{t-1} + \epsilon_t \tag{14}$$

where ϵ_t denotes an *i.i.d.* perturbation term.

As mentioned in the introduction, this formulation is supposed to capture the feature that upon abandoning a currency union, a country is unable to issue new net debt, it does not default, it keeps primary deficit as before exiting, and must finance government deficit through money printing[12]. Therefore d_t is the real value of the secondary deficit of the government.

This generalizes Marcet and Nicolini (2003) in that it introduces serial correlation of seigniorage, $\rho \neq 0$. We think this feature is important in studying a EMU exit as deficits are in fact highly serially correlated and a proper calibration of this process is crucial for the results.

Expectations are taken using the subjective probability measure $\mathcal{P}$. This probability measure specifies the joint distribution of $\{P_t\}_{t=0}^{\infty}$ at all dates that agents hold and it is fixed at the outset.

In equilibrium we must have $M_t^d = M_t^s = M_t$, which allows us to combine the money demand equation (12) and the government budget constraint (13) to obtain:

$$\pi_t = \frac{\phi - \phi\gamma\pi_t^e}{\phi - \phi\gamma\pi_{t+1}^e - d_t} \tag{15}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the realized gross inflation rate. This equation governs the evolution of inflation in any equilibrium, regardless of how expectations are formed, and we will use it repeatedly.

We start by studying the rational expectations benchmark. Under rational expectations, market prices are assumed to carry only redundant information because agents know the exact mapping from the history of seigniorage levels to prices, $P_t(d^t)$. As usual we denote RE by dropping the superscript $\mathcal{P}$ in the expectation operator and under RE we write:

$$\pi_{t+1}^e = \mathbb{E}_t\left[\frac{P_{t+1}}{P_t}\right] \tag{16}$$

3.2 The Rational Expectations Benchmark

We now study equilibria under RE, restricting attention first to a deterministic environment. We focus on the case with persistence in the level of seigniorage, which embeds the case studied in Marcet and Nicolini (2003).

[12]In the quantitative section, we allow for policy regimes in which the government is able to deplete international reserves to finance its deficit.

In the absence of uncertainty, imposing rational expectations amounts to require $\pi_t^e = \pi_t$ for all t. Plugging this condition into the main equation (15) and rearranging delivers:

$$\pi_{t+1} = (1-\rho)\left(\frac{\phi + \phi\gamma - \delta}{\phi\gamma} - \frac{1}{\gamma\pi_t}\right) + \rho\left(\frac{\phi + \phi\gamma - d_{t-1}}{\phi\gamma} - \frac{1}{\gamma\pi_t}\right) \tag{17}$$

This equation will govern the dynamics of inflation in equilibrium.

The initial position of the economy is given by d_0. Notice that if initial deficit is at the mean $d_0 = \delta$, then under no uncertainty $d_t = \delta$ for all t and the equilibrium will be stationary. In such a case, (17) admits two stationary equilibria, which are obtained as the solutions to the following quadratic equation:

$$\phi\gamma\pi^2 - (\phi + \phi\gamma - \delta)\pi + \phi = 0 \tag{18}$$

One could use this equation to trace out a stationary Laffer Curve, depicting the inflation rates that allow the government to finance the level of seigniorage δ. We use $\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_2}(\delta)\}$ to denote the two roots of (18), where the small root $\boldsymbol{\pi_1}(\delta)$ corresponds to the "good" side of the Laffer Curve.

In the case in which d_0 differs from δ then d_t becomes a state variable of the model solution. Now we define $x_t \equiv (\pi_t, d_t)$ and write the dynamic system composed of (14) and (17) as follows:

$$x_t = \mathbf{G}\left(x_{t-1}\right) \equiv \begin{bmatrix} (1-\rho)\mathbf{F}(\pi_{t-1}, \delta) + \rho\mathbf{F}(\pi_{t-1}, d_{t-1}) \quad (1-\rho)\delta + \rho d_{t-1} \\ \\ (1-\rho)\delta + \rho d_{t-1} \end{bmatrix} \tag{19}$$

where

$$\mathbf{F}(\pi, d) = \frac{\phi + \phi\gamma - d}{\phi\gamma} - \frac{1}{\gamma\pi} \tag{20}$$

In a deterministic environment, d_t will always revert to its long run mean δ. Hence, to characterize equilibria, it suffices to understand the behavior of π_t, conditional on the initial position d_0. To this end, it will prove convenient to ensure that stationary inflation rates are always positive and well-defined, for which we assume the following:

Assumption 5 $\delta \in \mathbf{D} \equiv [0, \phi(1 + \gamma - 2\gamma^{\frac{1}{2}}))$

One can easily check that under this assumption, stationary inflation rates are always within the interval $[1, \gamma^{-1}]$. Moreover, the upper bound of $\mathbf{D}$ can be interpreted as the maximum level of seigniorage that the government can finance, given the primitives of the economy. The following proposition summarizes the behavior of inflation under Rational Expectations:

Proposition 1 *Under Assumption 5, for any $d_0 \in \mathbf{D}$ there exists $\underline{\pi}(d_0)$ such that:*

1. *If $\pi_0 < \underline{\pi}(d_0)$, then $\lim_{t\to\infty} \pi_t = -\infty$*

2. *If $\pi_0 = \underline{\pi}(d_0)$, then $\lim_{t\to\infty} \pi_t = \boldsymbol{\pi}_1(\delta)$*

3. *If $\pi_0 > \underline{\pi}(d_0)$, then $\lim_{t\to\infty} \pi_t = \boldsymbol{\pi}_2(\delta)$*

The proof is relegated to Appendix B. Notice that in the special case $d_0 = \delta$, one can show that $\underline{\pi}(d_0) = \boldsymbol{\pi}_1(\delta)$ and the equilibrium is equivalent to that corresponding to the case with no persistence.

The equilibria characterized in this proposition for the case $d_0 < \delta$ is depicted in Figure 1. The line with circles that starts at $\underline{\pi}(d_0)$ represents the stable path that converges to the low inflation steady state under Rational Expectations. For $\pi_0 \neq \underline{\pi}(d_0)$, the lines with crosses show the inflation paths that either converge to the high inflation steady state or diverge to infinity.

In the remainder of this section, we linearize (19) and introduce a small amount of uncertainty in the seigniorage to learn about the properties of the inflation process around the low inflation steady state.

3.3 Inflation Persistence under Rational Expectations

To learn more about the stochastic properties of inflation in equilibrium, we linearize the main equation (15) around the low inflation steady state and introduce a small amount of uncertainty in the level of seigniorage. The linearization boils down to[13]:

$$\hat{\pi}_t = \frac{\delta}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta}\hat{d}_t \tag{21}$$

$$= \frac{1}{\gamma\boldsymbol{\pi}^2}\hat{\pi}_{t-1} - \frac{\delta}{\phi\gamma\boldsymbol{\pi}}\hat{d}_t \tag{22}$$

$$\hat{d}_t = \rho\hat{d}_{t-1} + \epsilon_t \tag{23}$$

where we are using the notation $\hat{x}_t = \ln x_t - \ln \mathbf{x}$, with bold letters indicating steady state values. We can express inflation recursively as:

$$\hat{\pi}_t = \rho\hat{\pi}_{t-1} + \nu_t \tag{24}$$

where $\nu_t \equiv \delta\epsilon_t/(\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta)$. Hence, around the low inflation steady state, inflation behaves as an AR(1) process that inherits the persistence of seigniorage. Figure 2 displays sample paths of both inflation and seigniorage according to this linearized system.

[13] See Appendix C for details.

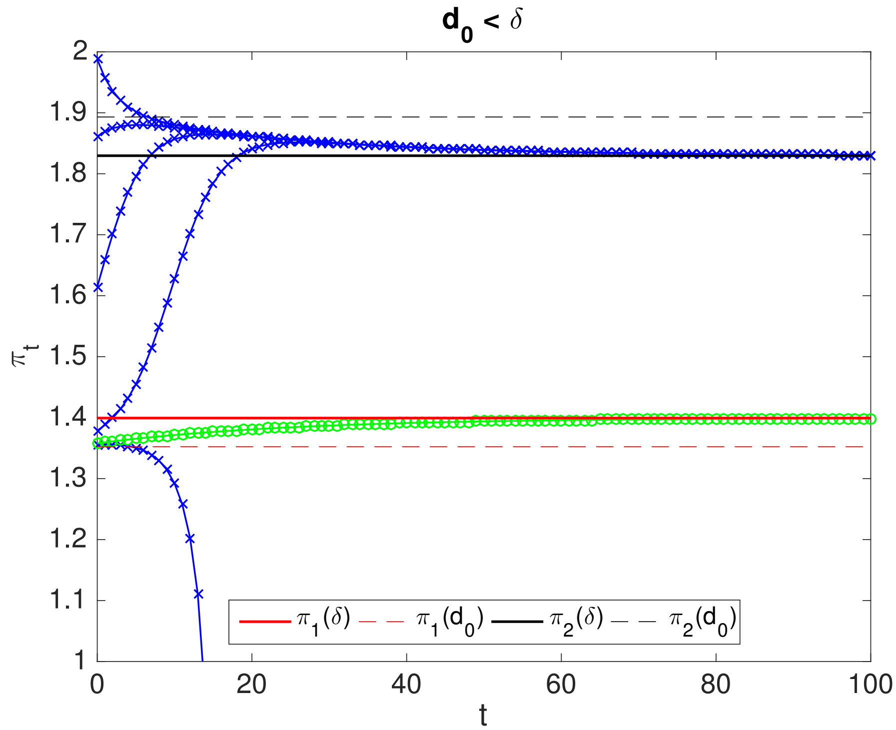

Figure 1: **Inflation paths under Rational Expectations that evolve according to (17).** The horizontal lines correspond to the solutions to the quadratic equation when $d = \delta$ (solid) and when $d = d_0$ (dashed). The circled line that converges to the low inflation steady state starts at $\pi_0 = \underline{\pi}(d_0)$ which is characterized in Proposition 1.

3.4 System of Beliefs about Inflation

We assume that agents hold the following beliefs regarding the inflation process:

$$\begin{aligned} \pi_t &= (1-\rho_\pi)\,\pi_t^\star + \rho_\pi \pi_{t-1} + u_t \\ \pi_t^\star &= \pi_{t-1}^\star + \eta_t \end{aligned} \tag{25}$$

where $u_t \sim N(0, \sigma_u^2)$ and $\eta_t \sim N(0, \sigma_\eta^2)$ are *i.i.d.* and independent of seigniorage d_t. We allow for $\rho_\pi \neq \rho$ although in practice the difference will be small. The intuition behind the proposed belief system (25) is that agents think inflation has a similar behavior as seigniorage, so they think it is an AR(1) process although they are unsure about the long run average level of inflation $\pi_t^\star$ and they express their uncertainty about this long run level by modelling $\pi_t^\star$ as a unit root process.

We choose this process for perceived inflation because it encompasses RE as a special case when $\rho_\pi = \rho$. In the IR equilibrium we assume agents take the serial correlation of inflation to be close to that of seigniorage, thus $\rho_\pi \simeq \rho$.

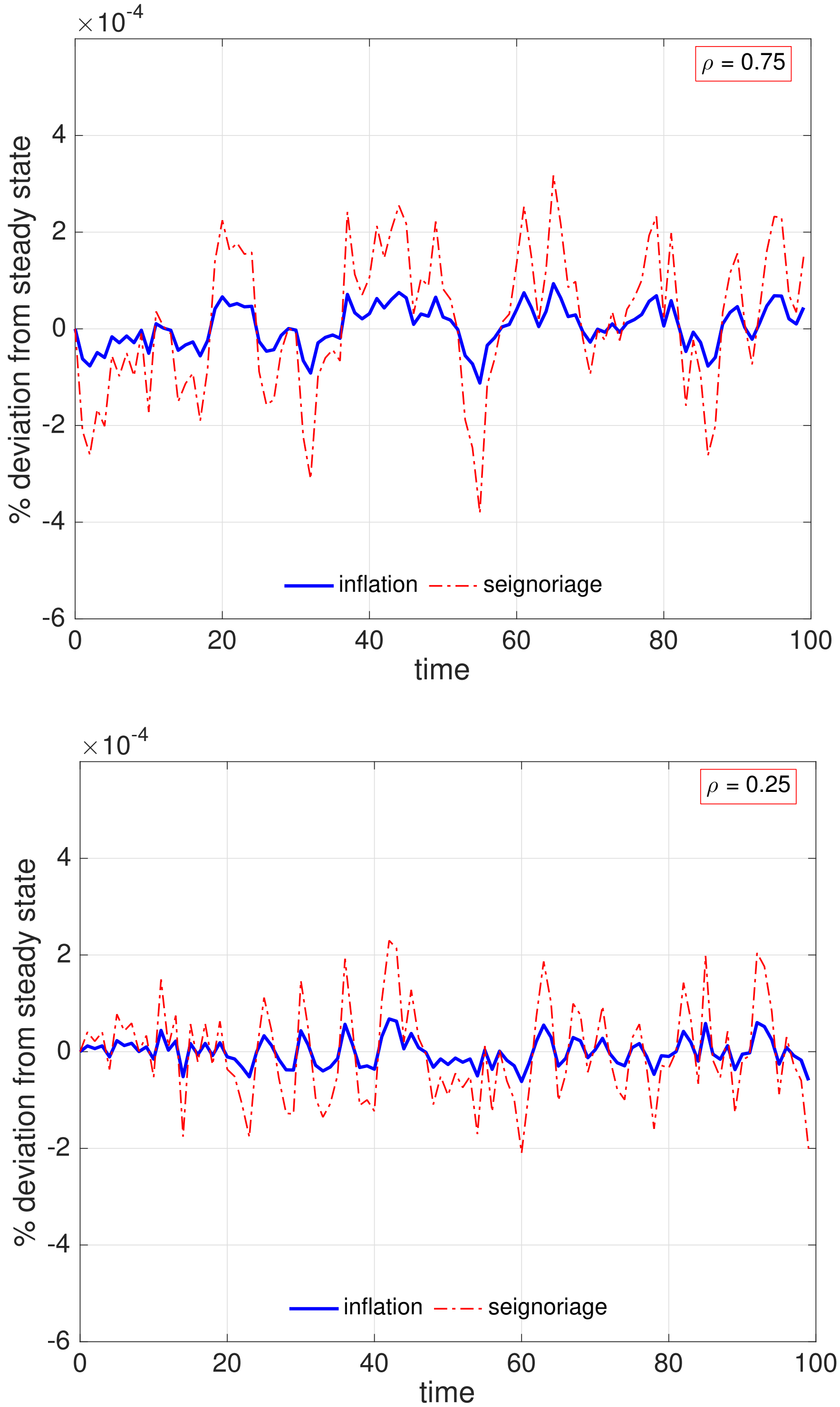

Figure 2: **Sample paths for inflation and seignoriage around the low inflation steady state in the linearized rational expectations equilibrium.**

Agents observe the realizations of inflation but not those of $\pi_t^\star$ and u_t separately. Thus, the learning problem consists of filtering long run inflation $\pi_t^\star$ out of observed inflation π_t. Since agents are rational their filter will involve using Bayes' inference.

We denote the posterior mean of $\pi_t^\star$ entering period t given information available to agents as $\beta_t = E^{\mathcal{P}}(\pi_t^\star \mid \pi^{t-1})$. Agents are endowed with an initial prior belief about $\pi_0^\star$ is normally distributed with mean $\beta_0 = E^{\mathcal{P}}(\pi_0^\star)$ and variance $\sigma_0^2 = E^{\mathcal{P}}(\pi_0^\star - \beta_0)^2$. In most of the paper the prior is assumed to be centered at the low inflation steady state $\beta_0 = \boldsymbol{\pi}_1(\delta)$ with a variance guaranteeing that the gain in the Kalman filter is constant.

Notice that if we make $\sigma_\eta^2 = 0$ then we assign probability 1 to $\beta_0 = \boldsymbol{\pi}_1(\delta)$ so in this case:

$$\pi_t^e = (1 - \rho_\pi)\, \boldsymbol{\pi}_1(\delta) + \rho_\pi \pi_{t-1} \tag{26}$$

which, as long as $\rho_\pi = \rho$, is equivalent to linearized RE equilibrium (24) for small deviations around the low inflation steady state. In that sense, this setup encompasses rational expectations equilibria as a special case.

Under all these assumptions optimal learning then implies that β_t evolves recursively according to:

$$\beta_t = \beta_t + \frac{1}{\alpha}\left(\frac{\pi_t - \rho_\pi \pi_{t-1}}{1-\rho_\pi} - \beta_{t-1}\right) \tag{27}$$

where α denotes the optimal Kalman gain[14].

3.5 Learning Equilibria

The belief system implies that:

$$\pi_{t+1}^e = (1-\rho_\pi)\beta_t + \rho_\pi \pi_t \tag{28}$$

Notice that if we plug this equation into (15) the solution is given by a non-linear equation in π_t so that multiple solutions may arise. To sidestep this problem, we assume that when expectations regarding π_{t+1}^e are formed at period t, agents still do not know the realization of π_t. Therefore, in order to form their expectations regarding future inflation, they forecast inflation two periods ahead using π_{t-1} as follows:

$$\pi_{t+1}^e = \left(1 - \rho_\pi^2\right)\beta_t + \rho_\pi^2 \pi_{t-1} \tag{29}$$

[14] In the quantitative section, and as is customary in models of learning, we will modify (27) to incorporate information regarding $(\pi_t - \rho_\pi \pi_{t-1})/(1-\rho_\pi)$ with a lag, in order to avoid the simultaneity between π_t and β_t.

Using this equation into (15) gives that equilibrium inflation follows

$$\pi_t = \frac{\phi - \phi\gamma((1-\rho_\pi^2)\,\beta_{t-1} + \rho_\pi^2 \pi_{t-2})}{\phi - \phi\gamma((1-\rho_\pi^2)\,\beta_t + \rho_\pi^2 \pi_{t-1}) - d_t} \tag{30}$$

To provide intuition about the behavior of inflation in this case, let us write (30) as

$$H(\beta_t, \beta_{t-1}, \pi_t, \pi_{t-1}, \pi_{t-2}, d_t) = 0$$

and define $h(\beta, \pi, d) \equiv H(\beta, \beta, \pi, \pi, \pi, d)$. The function h is useful to provide an approximation of the behavior of inflation in a situation in which $d_t = \delta$, $\beta_t \approx \beta_{t-1}$ and $\pi_t \approx \pi_{t-1} \approx \pi_{t-2} \approx \beta_{t-1}$. In such a case, (30) boils down to the quadratic equation (18), which implies that the rational expectations stationary inflation rates are also stationary inflation rates under learning. However, notice that the stationary inflation rate $\boldsymbol{\pi_i}(\delta)$ is stable under learning only if:

$$\left.\frac{\partial \pi}{\partial \beta}\right|_{\beta = \boldsymbol{\pi_i}(\delta)} = -\left.\frac{\partial h/\partial \beta}{\partial h/\partial \pi}\right|_{\beta = \boldsymbol{\pi_i}(\delta)} = \frac{\phi\gamma\boldsymbol{\pi_i}(\delta) - \phi\gamma}{\phi - \phi\gamma\boldsymbol{\pi_i}(\delta) - \delta} < 1 \tag{31}$$

As long as the denominator in (31) is positive[15], we can verify that this is indeed the case if and only if:

$$\boldsymbol{\pi_i}(\delta) < \frac{\phi + \phi\gamma - \delta}{2\phi\gamma} \tag{32}$$

Using (18), it is easy to show that this condition is satisfied by the smallest root $\boldsymbol{\pi}_1(\delta)$, but not by the largest $\boldsymbol{\pi}_2(\delta)$. Therefore, as pointed out in Marcet and Sargent (1989b) and Marcet and Nicolini (2003), the low inflation rational expectations Equilibrium is stable under learning, while the high inflation one is unstable under learning.

These dynamics are attractive in explaining periods of relatively large but stable inflation followed by rapid burst in inflation. The reason is that as long as expected inflation is around the stable low inflation equilibrium $\boldsymbol{\pi}_1(\delta)$, inflation itself will remain in that region. However, if a sequence of positive shocks to the deficit increase inflation to the extent that expectations eventually go beyond the value of $\boldsymbol{\pi}_2(\delta)$, the unstable dynamics on the wrong side of the Laffer curve take control and inflation can grow very quickly.

3.5.1 The role of exchange rate regimes

In the simulation, we assume that when inflation expectations go beyond certain upper bound, the government switches its policy regime, and establishes a crawling peg. The role of the ERR

[15]Whenever there is a positive price that clears the money market, the denominator will be positive. We can always extend the model to include the case in which there are reserves that can be depleted to ensure the existence of such a price, in the spirit of Marcet and Nicolini (2003).

is to stop these dynamics and bring inflation down, back to the stable region. The crawling peg implies that inflation will be then determined by the peg, so the money demand equation (12) determines the sequence of money supplies. This implies that new source of financing is required to satisfy the government budget constraint (13). We discuss these external financing needs in detail below.

Once inflation is so stabilized, the government can switch back the policy regime and stop the crawling peg, so the economy is again governed by the money demand (12) and the government budget constraint (13) together with the evolution of the deficit.

3.6 Justifying the System of Beliefs

As is clear from equation (30) in the learning equilibrium π_t is a function of past seigniorage. However, agents think that inflation evolves according to the system of beliefs specified at the beginning of section 3.4, which is obviously different from (30). This is not surprising to the careful reader since, from the very beginning we have said that we depart from RE.

However, our aim is to consider only "reasonable" systems of beliefs. Although IR permits assuming anything you want for the system of beliefs, we do not think it is interesting to consider systems of beliefs that generate inflation processes that render the beliefs to be "obviously wrong". For this, we follow various principles that the belief system should satisfy

1. *Encompass RE*

   In this way, there is a clear sense in which there is a small deviation from RE and that the equilibrium does not deviate too much from the beliefs.

2. *Close to the data*

   If the system of beliefs is close to the data behavior, and to the extent that the equilibrium outcome of our model reproduces the behavior of data, we can expect that agents in the real world can hold this system of beliefs and that this will in fact render the system close to the model behavior.

3. *Close to the model outcome*

   We would like to check that if agents observe the model outcome they can not reject their belief system in a few periods. In this way we can think that the considered system of beliefs is consistent with the model of inflation that we, as economists, consider.

4. *Close to surveys*

The system of beliefs should not be too different from observed surveys of expectations. Since inflation surveys are conducted continuously in many countries it is possible to apply this criterion to inflation.

The system of beliefs specified above turns out to satisfy all these criteria. 1- As explained in section 3.4 the system of beliefs encompasses RE as a special case. 2- various authors have chosen a similar model to explain actual inflation in various countries, for example Stock and Watson (2007). Although they often use a more involved model including time varying volatility, it often has the main features of our system of beliefs, namely serial correlation and a permanent shock to average inflation. 3- We do a full array of tests to analyze how easy it would be for agents to discover that their system of beliefs is not correct, we perform these tests in section 5. This implies that, given the system of beliefs, the equilibrium is such that the agents see their belief system as a reasonable description of the inflation that they observe. The reason that this is likely to happen has been described at the end of section

4- Many authors have fit the above model to inflation surveys, among others Roberts (1997).

4 Quantitative Performance

In this Section we calibrate the model and solve it numerically. We first show how likely hyperinflations are in equilibrium, as a function of the parameter $\frac{1}{\alpha}$. We also show an example of an equilibrium time series, to show the difference between the rational expectations outcome and the outcome with internally rational agents. The example quantifies the amplification effect that expectations can have on the equilibrium inflation rate.

We then show the performance of the tests described in Proposition 2 below, and argue that agents would not reject their beliefs in equilibrium. Finally, we compute the welfare effect of exchange rate policies that can stop the hyperinflations early on, as well as the evolution of the financial assistance required to carry on those policies.

4.1 Calibration

Seigniorage process and money demand

We assume that after exiting, the total deficit, d_t, will follow a process similar to the secondary deficit before exiting. This corresponds to a government that initially keeps austerity at similar levels as it was before exiting, that does not default on the debt, and can roll over the debt at the same interest rate as before. To calibrate the process, we estimate an AR(1) process for Greece's primary balance as a percentage of GDP, though very similar estimates are obtained with data from Italy, Portugal and Spain. We calibrate the values for ρ and σ_ϵ

using the results of that estimation. We also assume that austerity will eventually prevail, so we set the long run value of the deficit, $\bar{\delta}$, to be zero. These assumptions reflect the view that the proponents of exiting the Euro see this path as an alternative to the austerity programs imposed by the monetary union.

But clearly, fiscal policy could differ after exiting. One could entertain alternative hypothesis for the evolution of tax revenues or government spending, or allow for interactions between the real values of expenditures or government debt with the inflation that would follow exit. In that case, one could obtain the implied evolution of d, and describe how inflation behaves according to our model. Assuming the process for the secondary deficit will stay as before and it will have to be financed by monetization seems to us a reasonable benchmark to consider.

The role of preferences and the endowment in the economy boil down to the values of the money demand parameters in (12). These two parameters are the ones that fully determine the shape of the Laffer curve that relates the inflation rate and the amount of revenues it raises. These are key parameters, since the distance between the two solutions discussed in Section 3.5 determine the size of the stable set and therefore the likelihood of a hyperinflation to unravel. To calibrate those parameters, one would ideally observes time series in which the inflation rates are sufficiently high, events that had not occurred in the countries under consideration. Thus, to calibrate those two parameters, we use data from Argentina, following Marcet and Nicolini (2003). Specifically, the Money demand parameters target the inflation rate that maximizes the stationary Laffer curve and the maximum seigniorage as a percentage of GDP for the case of Argentina during the eighties.

Belief system

As explained in the introduction, the key methodological novelty in this paper is how to perform policy analysis under IR. Clearly, unlike in RE models, there are various assumptions one can make about the belief system. We believe it is a virtue of the model, since in fact, we do not know how expectations will be set after exiting. By being explicit about our assumptions and by exploring alternatives, we are forced to express our ignorance about exactly how expectations will react to such a policy change.

We assume agent's belief system is the one specified in section 3.4, with $\rho_\pi = \rho$, which is exactly the case with RE.[16] The point of departure from RE lies in our assumption that agents do not know for sure what the new level of long run inflation will be following exit. That still leaves us with two free parameters, the prior β_0, and the uncertainty regarding the prior, summarized by $1/\alpha$. In all cases below, we assume the initial prior of inflation β_0 to be the low inflation equilibrium in the steady state. The first advantage of this assumption is that by

[16] As it turns out, that is no longer true with IR, due to the feedback between inflation and inflation expectatios. However, as we show in the Appendix D, it is a remarkably good approximation.

setting $1/\alpha = 0$, we obtain the RE equilibrium. The second advantage is that the dynamics are not affected by an asymmetric behavior in the first periods.[17] Notice that due to the seiniorage financing of the deficit, inflation in this countries would be substantially higher that during the years in which the euro was adopted. This recognizes that agents understand that higher inflation following exit is very likely.

The only remaining free parameter is therefore $1/\alpha$. We proceed by showing the results for values that are slightly larger than zero, which as we mentioned before, delivers the RE equilibrium. The size of $1/\alpha$, reflects the uncertainty that agents have on their prior and can therefore be interpreted as the distance from the RE beliefs.

Exchange Rate Rules

We assume the government will switch to an Exchange Rate Regime (ERR hereafter) when inflation expectations are above some specified upper bound, β^U. Thus, an ERR is triggered whenever expected inflation exceeds β^U or to restore equilibrium[18]. The value of β^U will be a policy choice that we will analyze below. As explained in Section 3, ERR can avoid too high inflations as long as the government has access to enough financing to satisfy the budget constraint (13). Switching to an ERR leaves several additional policy options. First, the ERR must specify the desired growth rate for the crawling peg, $\overline{\beta}$. In what follows, we always set $\overline{\beta}$ to be the inflation rate in the low inflation steady state, $\boldsymbol{\pi}_1(\delta)$. Second, the ERR must specify how fast the target value for the crawling peg, $\overline{\beta}$, ought to be achieved. We let T be the number of periods after which the crawling peg will effectively be at the long run target $\overline{\beta}$.[19] We will explore several value for T in the policy analysis below. Finally, we must specify a bound $\hat{\beta}$, such that the ERR is abandoned once equilibrium inflation falls below that bound. In what follows, we set $\hat{\beta} = \pi_1(\delta)$, which is the upper bound on the stable set. This choice implies that the ERR is in place till expectations fall back to the stable set, so the model dynamics themselves imply that - absent a new series of negative shocks - the economy converges to the low inflation equilibrium. the ERR

Note that it is possible that a sequence of good shocks brings the deficit d_t to negative territory. In this case, equation (13) implies that the money stock should go down, which may generate a deflation. In such a situation, the most natural policy choice appears to be to save those surpluses in some interest bearing asset. Therefore, in our simulations, we assume that

[17] It is relatively easy to have hyperinflations early on just by assuming that agents starts with higher initial priors.

[18] The two cases in which ERR is required to restore equilibrium are if the money demand becomes negative, or if given the realization of seigniorage, the money demand is too low for an equilibrium to exist.

[19] This variable allows the policy analysis below to consider the trade off between "shock" and "gradualism", using the language in the literature. The choice is also motivates by the experiences of many countries that chose a crawling peg with declining rates to smoothly lower inflation, versus other experinces in which the exchange rate was fixed, setting a devaluation rate of zero, at the moment of switching to the ERR.

Table 1: **Parameters for Baseline Economy**

Parameter	Symbol	Value
Persistence of deficit	ρ	.9584
SD of shocks to deficit	σ_ϵ	.0097
Long run deficit	δ	0
Money Demand Parameters	ϕ	.36
	γ	.39

when $d_t < 0$, then $M_t = M_{t-1}$ and those savings are accumulated in reserves a the Treasury.[20]

Calibration parameters

The baseline parameterization is summarized in Table 1. Below, we will show results for values of $1/\alpha = \{0, .01, .03, .05\}$. In addition, we will make our policy evaluations by solving the model for alternative values of $\{\beta^U, T\}$.

4.2 Probability of Hyperinflations

Table 2 reports the implications of the model for the probability of hyperinflations, given different values of the long run deficit δ and initial deficit d_0. The values for $1/\alpha$ constitute the weight that agents place on past inflation to update their beliefs. Notice that the probability of experiencing a hyperinflation vanishes as the learning equilibrium approaches the rational expectations one (e.g. $1/\alpha \to 0$). This feature is also present in the case without persistence studied in Marcet and Nicolini, since $1/\alpha = 0$ merely keeps expectations constant and ensure the economy stays around the low inflation steady state. In terms of comparative statics higher long run deficit δ and higher initial deficit d_0 both increase the probability of experiencing a hyperinflation, with the largest effect coming from δ.

[20]The extended model that considers asset accumulation is detailed in the appendix.

Table 2: **Probability of n Hyperinflations**: ERR policy $(\beta^U, T) = (150\%, 1)$.

Deficit mean δ $= 0.0\%$, **Initial Deficit** d_0 $= 4.0\%$				
$1/\alpha$	0	1	2	≥ 3
0.00	100.00	0.00	0.00	0.00
0.01	57.10	25.92	12.36	4.62
0.03	22.74	29.40	23.66	24.20
0.05	15.82	25.14	24.40	34.64
Deficit mean δ $= 0.0\%$, **Initial Deficit** d_0 $= 1.0\%$				
$1/\alpha$	0	1	2	≥ 3
0.00	100.00	0.00	0.00	0.00
0.01	62.10	25.46	8.82	3.62
0.03	30.18	30.04	21.20	18.58
0.05	23.78	28.14	22.44	25.64

Figure 3 shows sample paths for inflation in a learning equilibrium. The solid blue line corresponds to the learning equilibrium with a positive constant gain and the solid red line corresponds to an equilibrium with fixed expectations that considers the same realizations of the shocks to seigniorage.

This confirms the intuition we provided in section 3.5: when inflation expectations are too large it is likely that hyperinflationary paths start to appear, these are then stopped by ERR rules, but if average seigniorage is too high these hyperinflations are activated again.

5 Testable Restrictions

In this section we study the conditions under which agents would question their belief system in a learning equilibrium. In order to do this, we consider the implications of the equilibrium conditions and the belief system for the vector $x_t = (e_t, d_t)$, where $e_t \equiv (\pi_t - \rho_\pi \pi_{t-1}) - (\pi_{t-1} - \rho_\pi \pi_{t-2})$, and we evaluate these implications using simulated data.

The following proposition adapts the results in Adam Marcet and Nicolini (2016), section V.II. It lists a set of necessary and sufficient second order conditions for the statement that inflation and seigniorage data are indeed generated by the model.

Proposition 2 *Let d_t be AR(1) with innovation ϵ_t as in (14). There is a belief system as the one described in section 3.4 consistent with the autocovariance function of $\{x_t\}$ if and only if the following restrictions hold:*

1. $\mathbb{E}[x_{t-i} e_t] = 0$ *for all* $i \geq 2$.

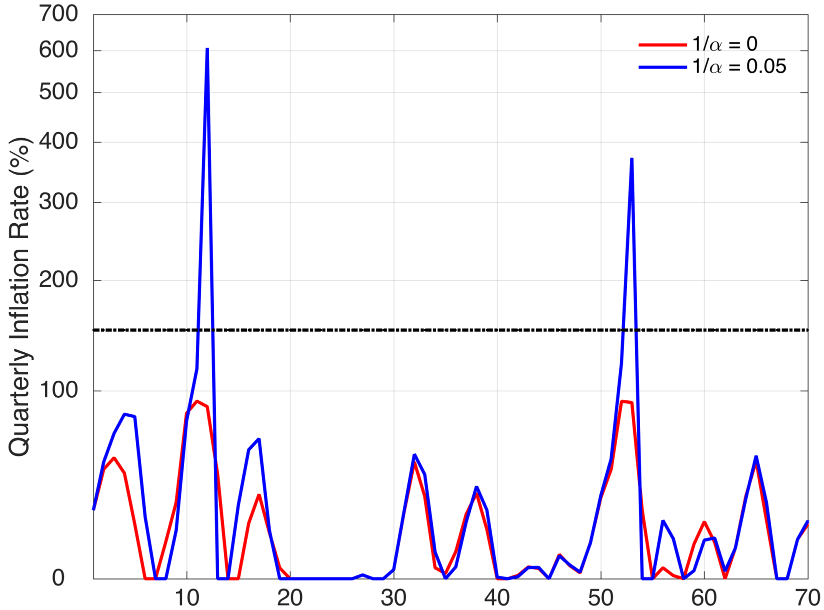

Figure 3: **Sample Path for Inflation in Learning Model.** The parameterization is discussed in the Quantitative Section. The solid blue line corresponds to the learning equilibrium with a positive constant gain and the solid red line corresponds to an equilibrium with fixed expectations that considers the same realizations of the shocks to seigniorage.

2. $\mathbb{E}[(\epsilon_t + \epsilon_{t-1})e_t] = 0$.

3. $\Sigma b^2 + \mathbb{E}[e_t e_{t-1}] < 0$.

4. $\mathbb{E}[e_t] = 0$.

where $\Sigma = \sigma_\epsilon^2$ *and* $b = \mathbb{E}[\epsilon_t e_t]$ *corresponds to the coefficient of a regression of* e_t *on* ϵ_t *in population.*

The proof is presented in the appendix. We test these moment restrictions using the parameterization displayed in Table 1.

If we find that these restrictions can not be rejected in the samples we consider we conclude that agents could be holding the system of beliefs for inflation as stated above in the model at hand. Now we provide tests for these restrictions.

5.1 Statistics

Restrictions 1, 2 and 4 represent first moment restrictions of the form $\mathbf{E}[y_t] = 0$ for $y_t = e_t q_t$, for various $q_t \in \mathbf{R}^n$. In order to test these restrictions, we estimate $\mathbf{E}[y_t]$ through its corresponding sample mean

$$\frac{1}{T}\sum_{t=1}^{T} y_t$$

Using standard arguments, the statistic

$$\hat{Q}_T = T\left(\frac{1}{T}\sum_{t=1}^{T} y_t\right)' \hat{S}^{-1}\left(\frac{1}{T}\sum_{t=1}^{T} y_t\right) \to \chi^2_n \text{ in distribution}$$

as $T \to \infty$ for some $\hat{S}$ that is a consistent estimator of the asymptotic variance of $\frac{1}{T}\sum_{t=1}^{T} y_t$. We use[21]

$$\hat{S} = T \cdot \mathbf{E}\left[(\bar{y}_t - \mathbf{y})(\bar{y}_t - \mathbf{y}))'\right] = \sum_{\nu=-\infty}^{\infty} \Gamma_\nu = \Gamma_{-1} + \Gamma_0 + \Gamma_1$$

In order to test Restriction 3, we use a one-sided test of the form $H_0 : \alpha < 0$, where α is set to satisfy

$$\mathbf{E}\left[(\epsilon_t b + e_{t-1})e_t - \alpha\right] = 0$$

GMM sets the estimate of b to the OLS coefficient of a regression of e_t on ϵ_t and the estimate of α precisely to $b'\Sigma b + E(e_{t-1}e_t)$.

5.2 Rejection Frequencies

Observe that for all restrictions, the null hypotheses implies that the data $\{x_t\}$ was generated by the belief system. Therefore, the belief system can be evaluated by checking whether the rejection frequencies exceed a predetermined significance level. If agents are using the wrong model of inflation, they should expect that as the sample size increases, rejection frequencies also increase for at least some of the restrictions being tested.

We calculate rejection frequencies in two different ways. We first use the theoretical asymptotic distribution of $\hat{Q}_T$. In the case of Restrictions 1, 2 and 4, asymptotic theory implies that $\hat{Q}_T \to \chi^2_n$ as the sample size increases. In testing Restriction 1, we use as many as three lags of ϵ_t and we always include a constant term in the instrument vector q_t[22]. In the case of Restriction 3, the asymptotic properties of the GMM estimator of α imply that under the null hypothesis it will be normally distributed and centered at 0.

[21] Here we exploit the MA(1) property of e_t and use the fact that beyond the the first lead and lag, these auto-covariances matrices must be equal to zero.

[22] Notice that by including a constant, Restriction 4 is embedded in the joint hypothesis testing performed for Restriction 1.

Rejection Frequencies using the Asymptotic Distribution. Table 3 displays the results of testing the restrictions of Proposition 1 using the asymptotic theoretical distribution of the statistics described in the previous paragraphs. Since Restriction 1 required some discretionary choice regarding the set of instruments, we single out its results.

The results indicate that agents will find difficult to reject their beliefs based on the observation of realized inflation in a span of 10 years (40 periods) when the signal to noise ratio of their beliefs is higher, since this implies a higher stationary $1/\alpha$, in this case, 0.05. On the contrary, the results show that for values closer to the RE equilibrium, the rejections frequencies are higher than 10%, particularly for restrictions 2 and 3. This emphasizes the notion that hyperinflations add a persistent component that has, due to the formation of expectations, a life on its own. Thus, for values of $1/\alpha$ that generate several hyperinflations, agents are less likely to reject the belief system, which makes hyperinflations themselves more likely.

Table 3: **Rejection Frequencies at the** 5% **level for** $(\beta^U, T) = (150\%, 1)$
This Table reports rejection frequencies obtained from testing restrictions 1-4 using simulated data. The set of instruments for Restriction 1 includes up to three lags of x_t (Restrictions 1a-1c).

	T		
	40	60	100
Deficit mean δ $= 0.0\%$, and $1/\alpha = 0.01$			
Restriction 1a	7.2 %	9.9 %	13.0 %
Restriction 1b	13.4 %	15.5 %	23.7 %
Restriction 1c	15.8 %	18.1 %	26.1 %
Restriction 2	26.8 %	31.1 %	38.4 %
Restriction 3	19.0 %	13.5 %	11.1 %
Restriction 4	0.0 %	0.0 %	0.0 %
Deficit mean δ $= 0.0\%$, and $1/\alpha = 0.05$			
Restriction 1a	3.5 %	4.0 %	4.8 %
Restriction 1b	5.5 %	5.2 %	6.2 %
Restriction 1c	7.3 %	6.4 %	7.0 %
Restriction 2	10.8 %	10.9 %	12.2 %
Restriction 3	7.5 %	4.1 %	2.3 %
Restriction 4	0.0 %	0.0 %	0.0 %

6 Policy analysis

In this section, we first evaluate the welfare consequences of the hyperinflationary equilibria. We first present the computations when the policy parameters $\{\beta^U = 150\%, T = 1\}$, which represent the value of quarterly expected inflation above which the ERR is triggered and the number of periods that it takes for the crawling peg to achieve its target. We then evaluate policies that imply an earlier intervention (lower value for β^U) or a more gradual intervention (larger values for T).

Before doing so, however, it is important to notice that in this model, the value of the monetary system does depend on how different the endowments are. As it is well known, a monetary policy that maintains the quantity of money constant so equilibrium inflation is zero implements a Pareto optimum allocation in which consumption is given by

$$c = \frac{1+e}{1+\alpha} \text{ and } x = \frac{\alpha}{1+\alpha}(e+1).$$

Recall that we assumed that $e < \alpha$ so a monetary equilibrium exists. On the other hand, as it is also well known, this economy has a non-monetary equilibrium, which is equivalent to autarky, where consumption is given by

$$c = 1 \text{ and } x = e.$$

The consumption equivalent of the monetary system, that we define as $\Delta - 1$, is therefore given by

$$\ln \frac{1+e}{1+\alpha} + \alpha \ln \frac{\alpha}{1+\alpha}(e+1) = \ln \Delta - \alpha \ln \Delta e, \tag{33}$$

which is bounded as long as $e > 0$. Note also that Δ is decreasing on e, and it approaches one as $e \to \alpha$.

At any equilibrium in which inflation is different from zero in some or all the periods, but finite, will imply a utility for the different generations higher than in autarky, but lower than at the first best. Therefore, the welfare cost of the hyperinflationary equilibria is bounded above by $\Delta - 1$. Thus, the choice of the parameters e and α imply the value to society of having a monetary system, which in itself puts a bound on the welfare costs of a monetary system that does not work so well. Using the values of the benchmark calibration in Table 1 in equation (33) delivers a value of Δ of roughly 0.10, or 10 percent of total consumption. In what follows, we present results for that benchmark calibration and for an alternative one that implies a substantially higher value of the monetary system.

6.1 Welfare costs of hyperinflations

We start by computing the compensating variation of eliminating all inflationary dynamics that arise due to learning. Specifically, for any given realization of the deficit for 200 periods, we compute the utility attained in equilibrium when setting $1/\alpha = 0$, which corresponds to the RE equilibrium. Then, for the same realization, we compute the utility attained in the equilibrium when setting $1/\alpha$ equal to 0.01, 0.03 and 0.05 respectively. Then, for each value of $1/\alpha$, we compute the percentage of consumption that agents would be willing to forgo under the RE equilibrium to avoid the inflation rates that arise for positive values of $1/\alpha$. We repeat this exercise for 10.000 different simulations and compute the average.[23]

We perform two different computations. In each case we compute the welfare change for the full sample, and also restricting to the 10 periods leading to the hyperinflation and subsequent adoption of the ERR. The results are depicted in Table 4. The first column of the table reports the value used for the parameter $1/\alpha$. The second column indicates the consumption equivalent for the 200 periods, while the third column reports the computations for the 10 periods leading to the first hyperinflation in each simulation. The first measure is the one standard in the literature, and as it can be seen, the numbers are sizable. For example, when $1/\alpha = 0.05$, the cost of the hyperinflations is around 0.4% of consumption in each of the 200 periods. This corresponds to 4% of the total gain of having a monetary system. The second measure is higher by construction. Again, when $1/\alpha = 0.05$, the cost of the hyperinflations is around 1,8% of each quarter consumption, which amounts to 20% of the total value of a monetary system. We will use these computations of the welfare costs just in the 10 quarters prior to the switch to the ERR as a benchmark to discuss the financing needs of the government during the ERR.

Table 4: **Consumption Equivalent Welfare Change (in %)**.
ERR policy is $(\beta^U, T) = (150\%, 1)$. We compute the welfare change using the full sample and restricting attention to the 10 quarters leading to the first hyper in each simulated path. The two calibrations correspond to economies in which the value of the monetary system is 10% and 30%.

$1/\alpha$	Gains from Money: 10%		Gains from Money: 30%	
	Full	10q	Full	10q
0.05	0.42	1.82	0.80	2.86
0.03	0.35	1.42	0.66	2.25
0.01	0.18	1.17	0.34	1.87

The second and third columns just discussed correspond to the benchmark calibration pre-

[23] Notice that this procedure implies that we compute welfare using the true distribution of prices, rather than the distribution agents believe is the true distribution.

sented in Table 1. The value of Δ that solves equation (33) for this calibration is around 10% of total consumption. This number appears rather low to us for modern economies. Thus, as we mentioned above, we repeat the computations for the same parameter values, except that we increase the endowment in the first period and reduce he endowment in the second period, while maintaining total output constant, and such that the value of Δ that solves equation (33) now becomes 30% of total output. The resulting numbers are reported in columns four an five of the same Table 4. The numbers are substantially larger in this case.

The numbers discussed so far correspond to the case in which the maximum tolerated inflation expectation is $\beta^U = 150\%$, which corresponds to Cagan's definition of hyperinflation (notice that actual inflation can in equilibrium much higher than that as Figure 3). In addition, we have so far considered only ERR that set the crawling peg to the desired low inflation rate on impact, rather than allowing for a more gradual policy that achieves that low target after a certain number of periods.

We now compute the welfare cost for alternative values of those policy parameters. In particular, we set $\beta^U = 100\%$ and allow for values of T all the way up to 4. As we now show, this earlier intervention implies sizeable welfare gains. On the other hand, gradual policies that delay the convergence of inflation to its desired target do convey substantial costs. The results are reported in Figure 5. The first line in the table considers alternative values for the policy parameter T. The first column in the table reports the two considered values for β^U and the three values for $1/\alpha$. The top three numbers in the second column correspond to numbers already reported in Figure 4.

Table 5: **Consumption Equivalent Welfare Change (in %)**.
Full sample comparison, welfare gain from Monetary system is 30%.

$1/\alpha$	$T = 1$	$T = 2$	$T = 3$	$T = 4$
$\beta^U = 150\%$				
0.05	0.80	1.16	1.29	1.47
0.03	0.66	0.89	1.00	1.15
0.01	0.34	0.43	0.49	0.55
$\beta^U = 100\%$				
0.05	0.15	0.34	0.42	0.50
0.03	0.04	0.19	0.26	0.34
0.01	0.01	0.11	0.15	0.22

As it can be seen in the table, the costs of a gradual policy that takes four periods in bringing inflation down are sizable. For $1/\alpha = 0.05$, the cost increases by more than 50% when

$\beta^U = 150\%$, and it more than triples when $\beta^U = 100\%$. In addition, the benefits of an early intervention are very large. For instance, when $T = 1$ and $1/\alpha = 0.05$, the welfare cost goes down to 0.07% of consumption when the ERR is adopted earlier, from 0.42% of consumption when the ERR is adopted later.

We would like to emphasize that none of these results are qualitatively surprising: earlier interventions imply lower inflation rates, so welfare ought to be higher. Similarly, gradual policies imply higher equilibrium inflation rates so welfare ought to be smaller. The value of these computations to us is that provides magnitudes that can be compared to the costs of these ERR: the external financing required to satisfy the government budget constraint while the ERR is in place. To that issue we turn next.

6.2 Financing requirements to stop the hyperinflations

As we mentioned in Section 3, the hyperinflations are stopped by a policy regime switch that adopts a crawling peg. But in doing so, the government will need external financing to satisfy the budget constraint. We now show how large and how persistent these funding requirements are according to the model. In Table 6 we show the results for the case in which $\alpha = 0.05$. The first column in the table reports the values considered for β^U and for T. The first row reports the accumulated balance of an account that is set to zero at the moment the ERR is implemented and that uses a real interest rate equal to zero.[24] We did 10.000 simulations and we report the median value for all the simulations. For example, the number -2.2 at the top of the second column, means that the quarter in which the ERR is adopted, the median external funds required to satisfy the government budget constraint is 2.2% of yearly GDP. The number 2.5% at the top of the second column implies that by the second quarter, the government has accumulated assets. The rest of the table can be read in a similar fashion. We report the balance in the account for one period more than the policy variable T.

[24]The period is a quarter, so a risk free interest rate would be very close to zero. One could easily impute a non zero interst rate, but given the magnitude of the numbers, the table would barely change.

Table 6: **Cumulative Median Change in Reserves ($1/\alpha = .05$, in % of yearly GDP)**

ERR T	Periods After Intervention 1	2	3	4	5
$\beta^U = 150\%$					
1	−2.2	2.5	–	–	–
2	−1.0	0.6	3.0	–	–
3	−0.9	−0.5	0.8	1.4	–
4	−0.8	−0.9	−0.2	0.5	0.7
$\beta^U = 100\%$					
1	−2.3	1.0	–	–	–
2	−1.0	0.3	1.6	–	–
3	−0.7	0.3	1.2	1.8	–
4	−0.5	0.4	1.2	1.7	1.9

A remarkable feature of the Table is that the external financing is a very temporary phenomenon. In all cases, the government would be able to pay the debt at the latest one period after T, and in some cases even before. In all cases, the government will end up with additional resources after paying the debt. This may seem a surprise, but as the theoretical analysis in Section 3 implies, the hyperinflations are not only bad for welfare: they are also bad for tax purposes. The reason is that the hyperinflations are the result of unstable dynamics that appear on the wrong side of the Laffer curve. Along these dynamics, the inflation tax is decreasing with inflation. At the same time, real money balances are shrinking. Once the ERR is put in place both processes revert. In particular, real money balances grow substantially, which means that nominal money is growing at a rate higher than the inflation rate.[25]

Three important messages arise from the Table, when combined withe the computations in the previous sub-section. First, it shows that early interventions can be a win-win scenario. As Figure 5 shows, there are substantial welfare gains of an early intervention and set $\beta^U = 100\%$ rather than at 150%. In addition, Table 6 shows that the external financing is essentially the same. Thus, all those gains of this early intervention are net gains. Second, it shows that shock policies can be more demanding in terms of external financing, since they require a lower negative balance. This is the case when comparing $T = 1$ with $T = 2$. For this choice there seems to be a trade off: a policy that brings inflation down quickly is better, but it may call for larger external support. However, for higher values of T, the change in the numbers are

[25]Interstingly, this process of reserve accumualtion has been a common features in the stabilizationplans in Latin America in the 80s, even for those that did not adjust the deficit, so that they eventually experienced a new burst in inflation.

very small, suggesting that if external financing is an issue, one may have to adopt gradual policies, but ideally ones that bring inflation down in two or three quarters. Third, notice that the worst case in terms of the requirement for the external finance is the case with $T = 1$, and $\beta^U = 100\%$. In that case, the needs amount to 2.3% of an yearly GDP. We want to compare this number with the welfare cost of the hyperinflation in Table 4 when computed as a fraction of total consumption in the 10 quarters leading to the hyperinflation and the ERR. The welfare cost reported there ranges from 1.8% of GDP to 2.9% depending on what we assume is the welfare gain of the monetary system. These numbers, accumulated over 10 quarters (two years and a half) represent a range between 4.5% and 7.2% of the GDP of one year. These welfare gains represent money on the table. To grab a good share of that a very short loan of a smaller magnitude is required, a loan that very quickly can be paid in full and that in all cases leaves the government with some extra revenue. These ERR appear to be very close to a free lunch.

7 Conclusions

Some countries have been recently confronted with the following policy decision: is it worthwhile to leave the EMU? In this paper we analyses this policy decision when it involves no government debt default and the government can not increase its debt level. In this situation leaving the euro with the desire to regain control of the CB is just an illusion: the money supply will be driven by the need to finance the deficit

We find that the resource to deficit monetization carry very high risks. Given the current levels of government deficit exiting the euro is likely to lead to hyperinflations. As is well accepted in policy circles, and as justified by the setup in Marcet and Nicolini (2003), one can stop hyperinflations with a ERR *and* lower average seigniorage. Therefore, to the extent that a hyperinflation is a very costly outcome that should be avoided, countries exiting the euro should find ways to reduce deficits anyway, even outside the EMU.

Another outcome of the paper has been to show how the framework of IR can be used to perform policy analysis. We just state clearly an assumption on the system of beliefs, we justify its validity using various criteria spelled out in the text, and we calibrate the parameters of the belief system. Indeed, the effects of the policy decision will depend on the agents' expectations in the model: if agents have RE leaving the euro implies a modest inflation, if agents learn about underlying inflation as a consequence of their "near-rational" belief system, rational behavior will lead to hyperinflations. That the policy outcome depends on how expectations are formed is an advantage of our approach, as it highlights that this is a key element in policy decisions, since policy makers never really know how agents' expectations will react to a policy change.

This paper hardly exhausts the effects of leaving the euro. Exiting countries could have an outright debt default; but this alternative has additional costs that should be factored in, other papers have attempted to measure these costs. Exiting countries may actually not loose access to debt markets, but even if they have access to debt markets, given their very high current debt levels it is unlikely they can keep increasing their debt levels much. Exiting countries may hope that a devaluation brings some growth, this would be beneficial in itself and it would decrease deficit as a percentage of GDP, but past experience shows that post-devaluation growth is not always to be had, most of its effect comes through devaluating internal salaries. This would say, in our model, that lowering the civil servant salaries is a convenient way to implement austerity and indeed lowering the probability of a hyperinflation, but this is austerity in disguise anyway. Such lowering of the deficit thanks to inflation can be modelled in by making d_t to depdn on equilibrium inflation, a substantial complication that goes way beyond the current setup. Another supposed advantage of devaluations is to bring some growth due to lower real wages and increased international competitiveness of local labor, our model also captures this as a way to lower seigniorage as a percentage of GDP. However, we are silent about other costs and benefits, namely, that lower wages reduce welfare while higher gDP increases welfare. We have also yet to explore different alternatives for the re-set of beliefs after the euro, consider various paths for the deficit, calibrate to other countries etc.

In any case the remaining issues can be incorporated in future research that goes vastly beyond the scope of this paper.

A Linearization of the Money Demand Equation

We linearize the individual money demand around a fictitious steady state with no aggregate uncertainty. In this appendix, we use the notation $m_t^j \equiv M_t^j/P_t$ and $\pi_{t+1} \equiv P_{t+1}/P_t$. The linearization boils down to:

$$\left(\frac{1}{1-\tilde{m}^j}\right)^2 (m_t^j - \tilde{m}^j) = -\left(\frac{\alpha_t^j}{m_t^j + e_{t+1}^j \tilde{\pi}_{t+1}}\right)^2 \frac{1}{\alpha_t^j} \left\{\mathbf{E}_t^j[m_t^j - \tilde{m}^j] + e_{t+1}^j \mathbf{E}_t^j[\pi_t^j - \tilde{\pi}^j]\right\}$$

where the tilde variables represent steady state variables. In steady state we must have that

$$\frac{1}{1-\tilde{m}^j} = \frac{\alpha_t^j}{\tilde{m}^j + e_{t+1}^j \tilde{\pi}}$$

which also implies that

$$\frac{1+\alpha_t^j}{\alpha_t^j}\tilde{m}^j + \frac{e_{t+1}^j}{\alpha_t^j}\tilde{\pi} = 1$$

Using these two expressions above and rearranging we obtain:

$$m_t^j = \frac{\alpha_t^j}{1+\alpha_t^j}\left\{1 - \frac{e_{t+1}^j}{\alpha_t^j}\mathbf{E}_t^j[\pi_{t+1}]\right\}$$

which corresponds to the expression in the main text.

B Proof of Proposition 1

If $d_0 = \delta$, it is straightforward that $\underline{\pi}(d_0) = \boldsymbol{\pi}_1(\delta)$. Hence, the goal is to show that $\underline{\pi}(d_0)$ exists when $d_0 \neq \delta$. The logic consists in showing that, given d_0, one can find a value for π_0 so as to be exactly at $\boldsymbol{\pi_1}(d_t)$ in period t, where $\boldsymbol{\pi_1}(d_t)$ denotes the solution to the quadratic equation (18) given the level of seigniorage d_t. Notice that such value is well defined for all d_t as long as $d_0 \in \mathbf{D}$.

Lemma 1 *If $d_0 \neq \delta$, there exists a monotone and bounded sequence of initial conditions $\{\beta_t\}$ such that for all t, $\pi_0 = \beta_t$ implies $\pi_t = \boldsymbol{\pi_1}(d_t)$.*

This result indicates that $\lim_{t\to\infty} \beta_t$ is well defined. We denote this limit by $\underline{\pi}(d_0)$, making explicit its dependence on the initial value of seigniorage d_0. Furthermore, it also indicates that if $\pi_0 = \underline{\pi}(d_0)$, then $\lim_{t\to\infty} \pi_t = \lim_{t\to\infty} \boldsymbol{\pi_1}(d_t) = \boldsymbol{\pi_1}(\delta)$. Hence, the second statement of Proposition 1 can be viewed as a corollary of Lemma 1. Although the two cases $d_0 < \delta$ and

$d_0 > \delta$ need to be considered separately, the proof is analogous so we present only the one corresponding to $d_0 < \delta$.

Proof of Lemma 1. The proof is by induction. For the initial step, observe that

$$\pi_0 = \boldsymbol{\pi_1}(\delta) \implies \pi_1 = (1-\rho)\boldsymbol{\pi_1}(\delta) + \rho\mathbf{F}(\boldsymbol{\pi_1}(\delta), d_0) > \boldsymbol{\pi_1}(\delta) > \boldsymbol{\pi_1}(d_1)$$

where the first inequality follows from the following property about F:

$$\mathbf{F}(\pi, d) > \pi \iff \pi \in (\boldsymbol{\pi_1}(d), \boldsymbol{\pi_2}(d))$$

and the second follows from the fact that $d < \delta$ implies $\boldsymbol{\pi_1}(d) < \boldsymbol{\pi_1}(\delta)$. On the other hand, we also have that:

$$\pi_0 = \boldsymbol{\pi_1}(d_0) \implies \pi_1 = (1-\rho)\mathbf{F}(\boldsymbol{\pi_1}(d_0), \delta) + \rho\boldsymbol{\pi_1}(d_0) < \boldsymbol{\pi_1}(d_0) < \boldsymbol{\pi_1}(d_1)$$

Since $\mathbf{F}$ is continuous and monotone, it follows that there exists a unique $\beta_1 \in (\boldsymbol{\pi_1}(d_0), \boldsymbol{\pi_1}(\delta))$ such that if $\pi_0 = \beta_1$ then $\boldsymbol{\pi_1} = \boldsymbol{\pi_1}(d_1)$.

For the inductive step, suppose there exists β_t such that $\pi_0 = \beta_t$ implies $\pi_t = \boldsymbol{\pi_1}(d_t)$. Then it follows that

$$\pi_t = \boldsymbol{\pi_1}(d_t) \implies \pi_{t+1} = (1-\rho)\mathbf{F}(\boldsymbol{\pi_1}(d_t), d_t) + \rho\boldsymbol{\pi_1}(d_t) < \boldsymbol{\pi_1}(d_t) < \boldsymbol{\pi_1}(d_{t+1})$$

and we have again that

$$\pi_t = \boldsymbol{\pi_1}(\delta) \implies \pi_{t+1} = (1-\rho)\boldsymbol{\pi_1}(\delta) + \rho\mathbf{F}(\boldsymbol{\pi_1}(\delta), d_t) > \boldsymbol{\pi_1}(\delta) > \boldsymbol{\pi_1}(d_{t+1})$$

Hence there exists $\beta_{t+1} \in (\beta_t, \boldsymbol{\pi_1}(\delta))$ such that $\pi_0 = \beta_{t+1}$ implies $\pi_{t+1} = \boldsymbol{\pi_1}(d_{t+1})$. This also implies that $\beta_{t+1} > \beta_t$ for all t, so the sequence is monotone and bounded. ■

To complete the proof of the proposition, take an arbitrary sequence $\{\pi_t\}$ that evolves according to (17) and suppose $\pi_0 > \underline{\pi}(d_0)$. Two cases need to be considered. First, if $\pi_0 \geq \max\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_1}(d_0)\}$, then (17) and the properties of F imply that statement 3 is satisfied. Second, if $\pi_0 \in (\underline{\pi}(d_0), \max\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_1}(d_0)\})$, then the proof of Lemma 1 indicates there exists a period t in which $\pi_t > \max\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_1}(d_t)\}$ and therefore (17) and the properties of F again tell us that statement 3 holds. The proof of the first statement, when $\pi_0 < \underline{\pi}(d_0)$, follows exactly the same steps.

C Linearization of Equation 15

To linearize Equation 15, we treat π_t, π_t^e and π_{t+1}^e as different variables. Thus we obtain:

$$\hat{\pi}_t = \frac{\delta}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta}\hat{d}_t - \frac{\phi\gamma\boldsymbol{\pi}_1(\delta)}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta)}\hat{\pi}_t^e + \frac{\phi\gamma\boldsymbol{\pi}_1(\delta)}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta}\hat{\pi}_{t+1}^e$$

Rational Expectations implies that, around the low inflation steady state, $\hat{\pi}_t^e = 0$ for all t. Therefore, the last two terms in the right hand side cancel out and we obtain the equation in the main text.

D Estimating Inflation Persistence

To calculate inflation paths we use the following system

$$\begin{aligned} d_t &= (1-\rho)\delta + \rho d_{t-1} + \epsilon_t && (34) \\ \pi_t &= \frac{\phi - \phi\gamma(\rho_\pi^2 \pi_{t-2} + (1-\rho_\pi^2)\,\beta_{t-1})}{\phi - \phi\gamma(\rho_\pi^2 \pi_{t-1} + (1-\rho_\pi^2)\,\beta_t) - d_t} && (35) \\ &= \alpha_{t-1} + 1 && (36) \\ \beta_t &= \beta_{t-1} + \frac{1}{\alpha}\left(\frac{\pi_{t-1} - \rho_\pi \pi_{t-2}}{1-\rho_\pi} - \beta_{t-1}\right) && (37) \end{aligned}$$

with the initial condition $\{\pi_{-1}, d_0, \alpha_0, \beta_0\} = \{\boldsymbol{\pi}_1, \delta, \overline{\alpha}, \boldsymbol{\pi}_1\}$, where $\boldsymbol{\pi}_1$ is the low inflation steady state in the RE equilibrium, and $\overline{\alpha}$ is just a positive integer. We assume that the variance of the i.i.d shock ϵ_t is small enough so that the system remains stable.

We generate N samples of length T for each variable defined in (34)-(37). Let $\hat{\rho}_\pi(N,T)$ be the bootstrap estimate of the persistence of inflation, which is a function of all primitives of the model, including ρ_π and ρ. We allow $\rho_\pi \neq \rho_\delta$ and restrict attention to positive autocorrelation coefficients. Figure 4 displays $\hat{\rho}_\pi(N,T)$ as a function of ρ_π. The figure shows that for any ρ, there is a unique ρ_π such that $\rho_\pi = \hat{\rho}_\pi(N,T)$. We are interested in those fixed points because they correspond to the case in which agents are able to estimate ρ_π using past data. Since we proved that in a RE equilibrium, $\rho = \rho_\pi$, considering $\rho_\pi \neq \rho_\delta$ implies that we are allowing the persistence of inflation to be biased in a learning equilibrium. The nature of that bias is portrayed in Figure 5, which plots the fixed point $\rho_\pi^\star = \rho_\pi = \hat{\rho}_\pi(N,T)$ as a function of ρ. The figure indicates that the bias tends to be negative for low persistence of seigniorage and positive for high persistence of seigniorage

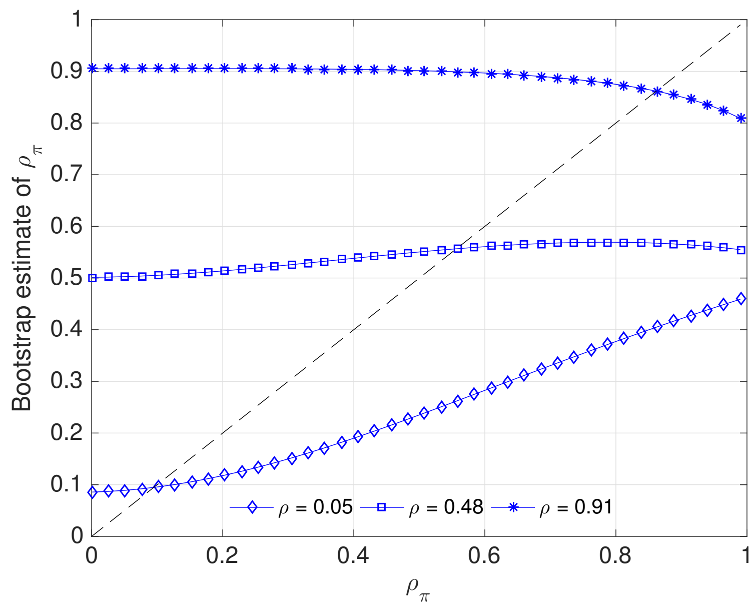

Figure 4: **Inflation persistence: Bootstrap estimate vs True Parameter:** The points in which the solid lines cross the dashed line represent the fixed points $\rho_\pi = \hat{\rho}_\pi(N, T)$. For this simulation we set $N = 2500$ and $T = 2500$.

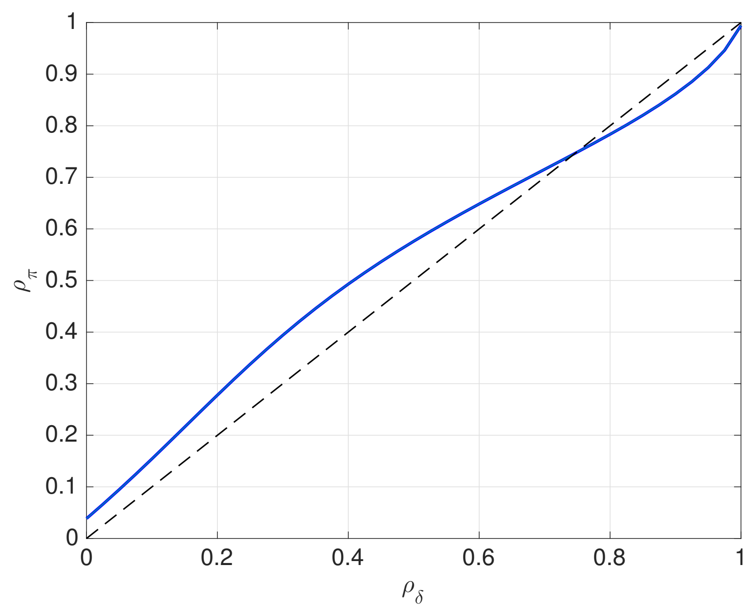

Figure 5: **Inflation persistence vs. seigniorage Persistence:** The solid line represent the fixed points portrayed in Figure 4 for different values of ρ. For this simulation we set $N = 2500$ and $T = 2500$.

E Proof of Proposition 2

The proof considers the general formulation in which agents use information about the innovations to the deficit to adjust their expectations regarding future inflation.

Restriction 1. Note that, according to the belief system:

$$\begin{aligned} e_t &= \pi_t^* + \psi\epsilon_t + u_t - \left(\pi_{t-1}^* + \psi\epsilon_{t-1} + u_{t-1}\right) \\ &= \eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1} \end{aligned}$$

whereas

$$d_t - \rho d_{t-1} = (1-\rho)\delta + \epsilon_t$$

so that Restriction 1 holds for $i \geq 2$.

Restriction 2. To prove Restriction 2, observe that:

$$\mathbb{E}\left[(d_t - \rho d_{t-1})e_t\right] = \mathbb{E}\left[((1-\rho)\delta + \epsilon_t)(\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1})\right] = \psi\sigma_\epsilon^2$$

while

$$\mathbb{E}\left[(d_{t-1} - \rho d_{t-2})e_t\right] = \mathbb{E}\left[((1-\rho)\delta + \epsilon_{t-1})(\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1})\right] = -\psi\sigma_\epsilon^2$$

so that Restriction 2 also holds.

Restriction 3. Note that:

$$\begin{aligned} \mathbb{E}\left[e_t e_{t-1}\right] &= \mathbb{E}\left[(\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1})(\eta_{t-1} + \psi\epsilon_{t-1} - \psi\epsilon_{t-2} + u_{t-1} - u_{t-2})\right] \\ &= -\psi^2\sigma_\epsilon^2 - \sigma_u^2 \end{aligned}$$

Now consider the projection of $d_t - \rho d_{t-1}$ on e_t. The projection is given by

$$\frac{\text{Cov}\left[e_t, d_t - \rho d_{t-1}\right]}{\text{Var}\left[d_t - \rho d_{t-1}\right]}(d_t - \rho d_{t-1})$$

where

$$\begin{aligned}\text{Cov}\left[e_t, d_t - \rho d_{t-1}\right] &= \text{Cov}\left[\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1}, \epsilon_t\right] = \psi\sigma_\epsilon^2 \\ \text{Var}\left[d_t - \rho d_{t-1}\right] &= \sigma_\epsilon^2\end{aligned}$$

so that the projection is given by $\psi\epsilon_t$, and the variance of the projection is $\psi^2\sigma_\epsilon^2$. Plugging these results into the right hand side of Restriction 3 delivers:

$$\mathbb{E}\left[e_t, e_{t-1}\right] + b'^2\sigma_\epsilon^2 - \sigma_u^2 + \psi^2\sigma_\epsilon^2 = -\sigma_u^2 < 0$$

Restriction 4. Since all the perturbation terms have zero expectation, Restriction 4 follows immediately.

F Model with Assets

The exogenous process is given by sequences $(\hat{d}_t, \hat{b}_t)$ such that $\widehat{b}_0 = 0$, $\widehat{d}_0 = \delta$ and

$$\widehat{d}_t = (1-\rho)\widehat{d}_{t-1} + \rho\delta + \xi_t$$

Thus, the true deficit evolves as before, and there is an initial value of assets set to zero. Consider then the beginning of period t, given $\widehat{b}_{t-1}$, after the draw of $\widehat{d}_t$. Define the new variable:

$$d_t = \max\{0, \widehat{d}_t - (1+r)\widehat{b}_{t-1}\}$$

which is interpreted as the part of the deficit that can't be financed with existing assets. Define also:

$$b_{t-1} = \max\{(1+r)\widehat{b}_{t-1} - \widehat{d}_t, 0\}$$

which corresponds to the stock of assets left after the deficit is financed. Notice that these definitions imply that:

$$\begin{aligned}d_t - b_{t-1} &= \max\{\widehat{d}_t - (1+r)\widehat{b}_{t-1}, 0\} - \max\{(1+r)\widehat{b}_{t-1} - \widehat{d}_t, 0\} \\ &= \max\{\widehat{d}_t - (1+r)\widehat{b}_{t-1}, 0\} - \min\{\widehat{d}_t - (1+r)\widehat{b}_{t-1}, 0\} \\ &= \widehat{d}_t - (1+r)\widehat{b}_{t-1}\end{aligned}$$

Now, we define a monetary policy rule that avoids deflation. The idea behind this policy rule is that all money printing is used to finance what is left of the deficit once assets are depleted. In that case, there are no additional resources created. However, money printing can exceed what is needed to finance the deficit and in that case we assume the extra resources are accumulated as assets. This amounts to set the law of motion for assets as follows:

$$\hat{b}_t = \max\{b_{t-1}, b_{t-1} + (m_t - m_{t-1} - d_t)\} \tag{38}$$

where m_t denotes real money balances. In principle, total assets next period should be equal to end of period assets b_{t-1} which corresponds to the first term in the max operator. However, if at zero inflation, real balances were more than enough to cover the deficit after assets were depleted, then those extra resources are indeed printed and used to purchase additional assets.

To understand the logic of this policy rule, notice that the money demand equation dictates that:

$$\phi(P_t - P_{t-1}) = (M_t - M_{t-1}) + \phi\gamma(P^e_{t+1} - P^e_t)$$

So, in order to avoid deflation we must have that

$$M_t - M_{t-1} \geq \phi\gamma(P^e_t - P^e_{t+1}) \tag{39}$$

Deflation becomes an issue only if prices are expected to move down. In that case, the government must print money to avoid deflation. If the money printed exceeds the amount of the deficit to be financed, then it is used to purchase assets. In other words, if expectations are such that:

$$\phi\gamma(P^e_t - P^e_{t+1}) > (d_t - b_{t-1})P_t$$

then according to the government budget constraint, no asset accumulation implies:

$$M_t - M_{t-1} = (d_t - b_{t-1})Pt$$

so that (39) is violated and deflation ensues. In such a case, assets need to be accumulated to ensure that $\phi(P_t - P_{t-1}) = 0$. Since that implies

$$M_t - M_{t-1} > (d_t - b_{t-1})Pt$$

The excess money printing is used to purchase assets. In other words, the modified government budget constraint is given by

$$M_t - M_{t-1} + (b_{t-1} - d_t)Pt = b_t P_t$$

which, expressed in real terms, it can be written as:

$$m_t - \frac{m_{t-1}}{\pi_t} + b_{t-1} - d_t = b_t$$

Evaluated at zero realized inflation, the left hand side of the previous equation corresponds to the second argument in the max operator of (38). Since it is increasing in π_t, if at zero inflation it is still positive, the excess resources printed are used to purchase extra assets.

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