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A State-space Approach to Australian GDP Measurement

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Richard Finlay

RDP 2014-12

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Abstract

We use state-space methods to construct new estimates of Australian gross domestic product (GDP) growth from the published national accounts estimates of expenditure, income and production. Across a range of specifications, our measures are substantially less volatile than headline GDP growth. We conclude that much of the quarter-to-quarter volatility in Australian GDP growth reflects measurement error rather than true shifts in the level of economic activity.

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1. Introduction

The level and growth of real economic activity is of great interest to economic policymakers as well as the general public. Increases in activity are typically associated with rising living standards. And economic activity influences other economic outcomes, such as inflation and unemployment.

But measuring economic activity is difficult. In Australia, a key measure of activity, gross domestic product (GDP), is measured using three different approaches, based on expenditure (GDP(E)), income (GDP(I)) and production (GDP(P)).[1] Conceptually, the three measures should be equal, but in practice the measures differ because they are constructed from different data sources and have varying degrees of measurement error.[2] It is important to emphasise that the term measurement error does not imply any failure on the part of statistical agencies. It is a statistical term that refers to the inherent errors that occur when one infers aggregate quantities from a sample of observations.

In this paper, we use state-space methods to combine the three Australian Bureau of Statistics (ABS) measures of GDP into an estimate of aggregate economic growth. In contrast to existing approaches, our method allows us to capture three salient features of GDP measurement. First, GDP(E), GDP(I) and GDP(P) should

1 GDP(E) is calculated as the sum of all expenditure by resident households, businesses and governments on final production, plus exports and the change in inventories, less imports; it is available at a quarterly frequency in both nominal and real terms. GDP(I) measures the income received for providing labour and capital services as inputs to production, adjusted for indirect taxes and subsidies, and is available in nominal terms; an estimate of real GDP(I) is obtained by dividing nominal GDP(I) by the GDP(E) deflator. GDP(P) measures the value of production in the economy as the difference between the value of outputs and the value of intermediate inputs consumed in production, and is available at a quarterly frequency in real terms and annually in nominal terms. For more detail on the data construction methods, see ABS (2007, 2011, 2012). The Australian Bureau of Statistics is one of only a few statistical agencies in the world to compile and publish all three measures of GDP.

2 See, for example, Bishop, Gill and Lancaster (2013) for a recent discussion of measurement error associated with the various GDP estimates.

be equal. Second, all three of these quantities are measured with some degree of error. Third, because of overlap between the data sources that feed into the three published estimates of GDP, these measurement errors are likely to be correlated. Once we account for these features of the data, we generate an estimate of economic activity which is smoother than suggested by conventional measures of GDP. This suggests that many large quarterly fluctuations in the rate of economic growth reflect errors in measurement rather than fundamental shifts in the pace of economic activity.

In Australia, the most common alternative to our approach is to take a simple average of the three measures, known as GDP(A).[3] The ABS considers this to be the most reliable estimate of final output, in part because independent errors in the underlying measures are often offsetting (Aspden 1990; ABS 2011). More broadly, the literature on model averaging suggests that if one possesses a set of estimates for some quantity being measured, then a combination of the estimates tends to perform better than any individual estimate.[4]

While using a simple average of the three GDP measures as an estimate for actual GDP is simple and transparent, it does not fully exploit all available information. For example, if one measure of GDP is particularly noisy, so that any given observation is likely to be quite different from actual GDP, then it may make sense to place less weight on that measure and more weight on the remaining two. The technique we explore in this paper provides one way of achieving this: it uses the time series properties of the three GDP measures to construct a composite GDP measure that more fully exploits the available information.

Our paper builds on the existing literature on GDP measurement. Most directly, it represents an application to Australian data of the techniques derived by Aruoba *et al* (2013), who construct a state-space measure of US GDP.[5] The Australian dimension of our study is of interest for two reasons, aside from our natural curiosity as Australian researchers. First, whereas the US statistical authorities

3 In many other countries a single measure of GDP is typically used.

4 See, for example, Timmermann (2006) for an overview of the literature, or Laplace (1818) for an early application of model averaging.

5 In unpublished work using Australian national accounts data, Scutella (1996) also explored the possibility of extracting underlying economic growth from the noisy expenditure, income and production measures.

only construct income and expenditure measures of GDP at a quarterly frequency, the ABS also publishes a production measure. We show that the methods of Aruoba *et al* (2013) extend to this environment. Second, the Australian economy differs in several respects from that of the United States in ways that may make GDP measurement more challenging. In particular, Australia is a smaller, more trade-exposed economy with a large resource sector. Our results support the idea that these variations in economic structure translate into a different pattern of GDP measurement errors in Australia.

Our work is also related to research evaluating the relative merits of expenditure, income and production as measures of economic activity. The primary focus of the research to date has been on the US economy, for which the most widely reported measure of output is derived from the expenditure side of the accounts. Despite this, a common finding is that expenditure-side estimates of output in the United States suffer from more severe measurement issues than income-side estimates. For example, Nalewaik (2010) cites the imprecise source data for personal consumption expenditure on services as a likely source of noise in the US GDP(E) estimates. In contrast, movements in many US GDP(I) components can be estimated reliably using tax data. Estimates of US GDP(I) tend to be less variable than GDP(E), while also being more highly correlated with other indicators of economic conditions (Fixler and Grimm 2006; Nalewaik 2010, 2011). Further, in the United States, GDP(E) tends to be revised towards GDP(I) over time.

Research using Australian national accounts data favours the use of the production-side rather than expenditure- or income-side estimates (Aspden 1990; ABS 2012; Bishop *et al* 2013). The relatively large share of resources in Australian GDP makes measures of output particularly responsive to trade data. Timing differences in imports and exports and variability in trade prices can introduce noise into estimates of expenditure and income (ABS 2012). In addition, GDP(I) and GDP(E) are reliant on the ABS register of businesses, which is typically updated with a delay. Bishop *et al* (2013) found that GDP(P) tends to be revised less than the other two measures and is as reliable in real time as GDP(A). These factors provide a case for applying a larger weight on GDP(P) in model averaging.

While it is useful to know the relative merits of expenditure, income and production measures of economic activity, using just one measure is unlikely to be optimal. The techniques that we use in this paper allow information from all three measures of GDP to be combined, and allow more weight to be placed on the more reliable measures.

2. Estimating GDP Growth

We treat GDP growth as an unobserved variable that follows a first-order autoregressive (AR(1)) process:

$$\Delta y_t = \mu(1-\rho) + \rho \Delta y_{t-1} + \varepsilon_{G,t} \tag{1}$$

where Δy_t represents the growth rate of real GDP, μ is the mean growth rate of GDP, ρ indicates persistence and $\varepsilon_{G,t}$ is a normally distributed innovation. It is common to model GDP growth as an AR(1) process, as growth rates are typically assumed to be homoscedastic and moderately persistent.

We then assume that the three observed GDP measures – GDP(E), GDP(I) and GDP(P) – provide noisy readings of actual GDP. For example, in our model the growth rate of GDP(E) is equal to the growth rate of actual GDP plus a measurement error term:

$$\Delta y_t^E = \Delta y_t + \varepsilon_{E,t}.$$

Stacking the three observed measures in matrix form gives us our measurement equation:

$$\begin{bmatrix} \Delta y_t^E \\ \Delta y_t^I \\ \Delta y_t^P \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Delta y_t + \begin{bmatrix} \varepsilon_{E,t} \\ \varepsilon_{I,t} \\ \varepsilon_{P,t} \end{bmatrix} \tag{2}$$

where Δy_t^E, Δy_t^I, Δy_t^P represent growth in GDP(E), GDP(I) and GDP(P), and $\varepsilon_{E,t}$, $\varepsilon_{I,t}$ and $\varepsilon_{P,t}$ represent their measurement errors.

Using this basic framework, we estimate three models that differ in their treatment of the observable variables, the shocks to GDP and the measurement errors.

2.1 Model 1: No Correlation

Our first model assumes that all stochastic terms are independent, that is, $[\varepsilon_{G,t}, \varepsilon_{E,t}, \varepsilon_{I,t}, \varepsilon_{P,t}] \sim N(0, \Sigma)$ where

$$\Sigma = \begin{bmatrix} \sigma_G^2 & & & \\ 0 & \sigma_E^2 & & \\ 0 & 0 & \sigma_I^2 & \\ 0 & 0 & 0 & \sigma_P^2 \end{bmatrix}.$$

2.2 Model 2: Correlation

Next we allow for correlation between the various GDP measures, that is, for $[\varepsilon_{G,t}, \varepsilon_{E,t}, \varepsilon_{I,t}, \varepsilon_{P,t}] \sim N(0, \Sigma)$ where

$$\Sigma = \begin{bmatrix} \sigma_G^2 & & & \\ \sigma_{GE} & \sigma_E^2 & & \\ \sigma_{GI} & \sigma_{EI} & \sigma_I^2 & \\ \sigma_{GP} & \sigma_{EP} & \sigma_{IP} & \sigma_P^2 \end{bmatrix}.$$

This model allows the errors in the three observable GDP measures to be interrelated, and for the size of the shock to actual GDP to affect the measurement error in the observed measures of GDP. For example, large innovations in actual GDP may be associated with less precise estimates of GDP(E), GDP(I) and/or GDP(P) than is the case for small innovations.

As shown in Appendix A, however, in order to identify the model we must place at least one restriction on the Σ matrix.[6] In line with Aruoba *et al* (2013) we impose this restriction by requiring that:

$$\zeta = \frac{\mathbb{V}\mathrm{ar}(\Delta y_t)}{\mathbb{V}\mathrm{ar}(\Delta y_t^E)} = \frac{\frac{1}{1-\rho^2}\sigma_G^2}{\frac{1}{1-\rho^2}\sigma_G^2 + 2\sigma_{GE} + \sigma_E^2} = 0.5. \qquad (3)$$

6 The model is unidentified in the sense that with an unrestricted Σ, different model parameters can give rise to identical distributions for the observable quantities.

That is, we assume that the variance of actual GDP growth is equal to half of the variance of the observed GDP(E) growth series. Although intuitively appealing, the restriction is arbitrary, and any number of alternative restrictions would also suffice.[7]

2.3 Model 3: Unemployment

Our third model includes an additional observable variable that depends on GDP growth but whose measurement error is unrelated to that of the other observable variables: the quarterly change in the unemployment rate.[8] That is, we replace Equation (2) with

$$\begin{bmatrix} \Delta y_t^E \\ \Delta y_t^I \\ \Delta y_t^P \\ \Delta U_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \kappa \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ \lambda \end{bmatrix} \Delta y_t + \begin{bmatrix} \varepsilon_{E,t} \\ \varepsilon_{I,t} \\ \varepsilon_{P,t} \\ \varepsilon_{U,t} \end{bmatrix} \tag{4}$$

where ΔU_t is the change in the unemployment rate. In this case we assume that $[\varepsilon_{G,t}, \varepsilon_{E,t}, \varepsilon_{I,t}, \varepsilon_{P,t}, \varepsilon_{U,t}] \sim N(0, \Sigma)$ with

$$\Sigma = \begin{bmatrix} \sigma_G^2 & & & & \\ \sigma_{GE} & \sigma_E^2 & & & \\ \sigma_{GI} & \sigma_{EI} & \sigma_I^2 & & \\ \sigma_{GP} & \sigma_{EP} & \sigma_{IP} & \sigma_P^2 & \\ \sigma_{GU} & 0 & 0 & 0 & \sigma_U^2 \end{bmatrix}.$$

That is, we impose three restrictions on the Σ matrix – zero correlation between the measurement error of the change in the unemployment rate ($\varepsilon_{U,t}$) and the

7 We experimented with alternative values of ζ, and with applying the restriction to GDP(P) instead; all produced very similar results.

8 The unemployment rate is estimated in a monthly survey of households, known as the Labour Force Survey (LFS). Measurement errors associated with these surveys are likely to be largely unrelated to errors in the quarterly GDP series, which are mainly derived from surveys of businesses and governments, although data from the LFS does feed into GDP(I). Relaxing the assumption that the correlation between measurement errors in GDP(I) and the unemployment rate is zero produces very similar results. We also estimated a model including the growth rate of employment rather than the change in the unemployment rate. Once again, this exercise produced very similar results.

measurement error for growth of GDP(E), GDP(I) and GDP(P) ($\varepsilon_{E,t}$, $\varepsilon_{I,t}$ and $\varepsilon_{P,t}$). By a similar argument to that put forward in Appendix A, the model is identified (in fact the model is over-identified).

3. Estimation

We follow the approach of Aruoba *et al* (2013) and estimate the models within a Bayesian framework. We work with Model 3 in this section; Models 1 and 2 are nested in Model 3 and can be recovered by setting appropriate parameters to zero.

First we express our model in state-space form. Let $s_t = [\Delta y_t, \varepsilon_{E,t}, \varepsilon_{I,t}, \varepsilon_{P,t}, \varepsilon_{U,t}]'$, $m_t = [\Delta y_t^E, \Delta y_t^I, \Delta y_t^P, \Delta U_t]'$, $M = [\mu(1-\rho), 0, 0, 0, 0]'$, $K = [0, 0, 0, \kappa]$, $\varepsilon_t = [\varepsilon_{G,t}, \varepsilon_{E,t}, \varepsilon_{I,t}, \varepsilon_{P,t}, \varepsilon_{U,t}]'$,

$$A = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \lambda & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then we can then express Model 3 as

$$\begin{aligned} s_t &= M + As_{t-1} + \varepsilon_t \\ m_t &= K + Cs_t. \end{aligned}$$

For ease of notation, we collect the parameters in the vector $\Theta = (\mu,\ \rho,\ \kappa,\ \lambda,\ \sigma_G^2,\ \sigma_{GE},\ \sigma_E^2,\ \sigma_{GI},\ \sigma_{EI},\ \sigma_I^2, \sigma_{GP},\ \sigma_{EP},\ \sigma_{IP},\ \sigma_P^2,\ \sigma_{GU},\ \sigma_U^2)$.

We use the Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm to estimate model parameters.[9] We first maximise the posterior distribution of Θ given the observed data

$$p(\Theta|m_{1:T}) \propto p(m_{1:T}|\Theta)p(\Theta)$$

where $p(m_{1:T}|\Theta)$ is the density of the observable data given the model parameters and $p(\Theta)$ is the density of the priors over the parameter draw. This gives us an initial estimate of Θ, denoted Θ^0. We use the inverse Hessian at the maximum to

9 See An and Schorfheide (2007) for a description of these techniques.

obtain an estimate of the covariance matrix of Θ, Σ_0. Θ^0 and Σ_0 are then used to initiate the MCMC algorithm: at each iteration i we draw a proposed parameter vector $\Theta^* \sim N(\Theta^{i-1}, c\Sigma_{i-1})$. Here c is a scaling parameter set to achieve an acceptance rate of around 25 per cent, where we accept Θ^* as Θ^i with probability

$$\min\left(1, \frac{p(m_{1:T}|\Theta^*)p(\Theta^*)}{p(m_{1:T}|\Theta^{i-1})p(\Theta^{i-1})}\right)$$

and set $\Theta^i = \Theta^{i-1}$ otherwise. We set $p(\Theta^*) = 0$ if Θ^* is not a valid draw, for example if it implies a covariance matrix that is not positive definite.

In order to sample Θ^* from the $N(\Theta^{i-1}, c\Sigma_{i-1})$ distribution we need to evaluate $p(m_{1:T}|\Theta)$. To do this we use the standard Kalman filter and simulation smoother, as described in Durbin and Koopman (2012). We take 50 000 draws from the posterior distribution and discard the first 25 000.

3.1 Priors

Our prior for the mean growth rate of GDP, μ, follows a normal distribution with mean 0.80 and standard deviation 10.[10] The mean of this prior corresponds to the average quarterly growth rate of GDP over our sample while the standard deviation is extremely large relative to the volatility of the GDP series, indicating that this prior places only a very weak restriction on the range of potential values. For the persistence of shocks to GDP growth, ρ, we use a beta prior with mean 0.50 and standard deviation 0.20. The prior restricts the value of this parameter to lie between 0 and 1, consistent with GDP growth being a stationary series.[11]

For the variances of the shocks to GDP and the measurement errors, we impose inverse-gamma priors with mean 2 and standard deviation 4. These priors ensure that the variances of all shocks are greater than 0. Finally, for the covariance terms, the priors follow a normal distribution with mean 0 and standard deviation 5.

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10 Our estimation procedure assumes that the trend growth rate of GDP has been constant over our sample. To test whether this assumption is reasonable, we ran Bai-Perron tests for a break in the mean growth rate of GDP(A) using an AR(1) model over the sample 1980:Q1–2013:Q2. These tests did not point to any evidence of a break in the mean growth rate of GDP(A) over our sample.

11 Imposing a normally distributed prior with a mean of zero produces almost identical results.

In all cases, our priors are loose, ensuring that we place a large weight on information from the data, but rule out unreasonable parameter values.

3.2 Data

Our data span 1980:Q1–2013:Q2. The starting date reflects the fact that, while Australian national accounts data are available on a quarterly basis from 1959:Q3, the quality of the underlying data sources has changed over time, so that the pattern of measurement errors in the early years of each GDP series may be unrepresentative of their current performance. The GDP and unemployment rate data that we use in our estimation are all seasonally adjusted by the ABS.

4. Results

4.1 Model 1: No Correlation

In Model 1 we assume that shocks to GDP and the measurement errors are independent of each other. Table 1 shows the parameter estimates.

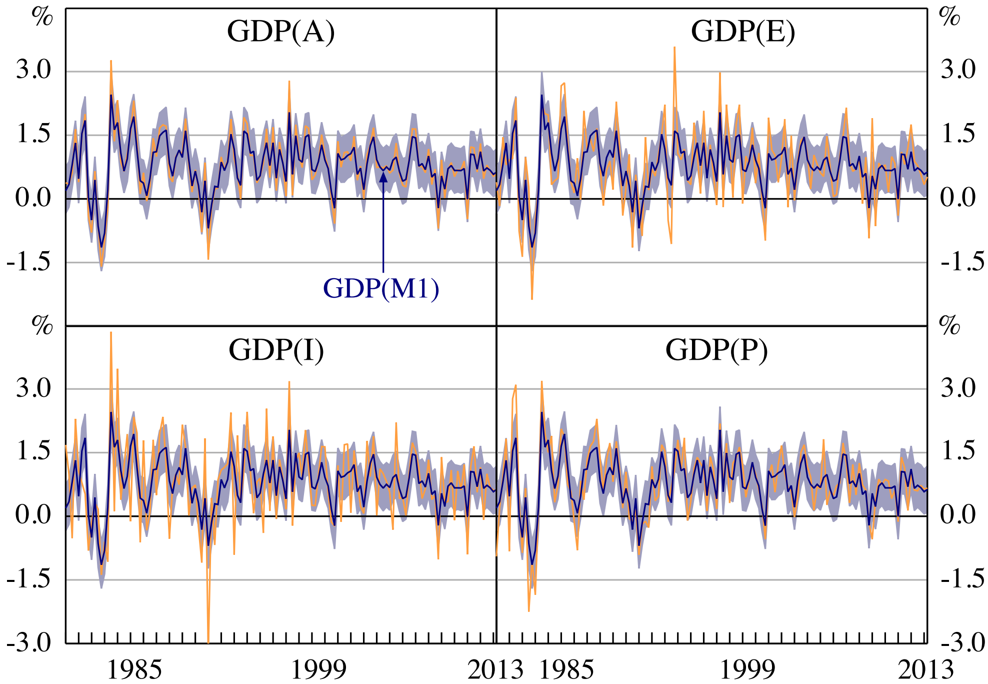
Table 1: Prior and Posterior Distributions – Model 1

		Prior		Posterior			
Parameter	**Distribution**	Mean	Std dev	Mode	Median	5%	95%
GDP equation							
μ	Normal	0.80	10	0.79	0.78	0.62	0.95
ρ	Beta	0.50	0.2	0.37	0.40	0.24	0.56
Exogenous processes							
σ_G^2	Inv Gamma	2	4	0.38	0.39	0.29	0.52
σ_E^2	Inv Gamma	2	4	0.43	0.44	0.34	0.57
σ_I^2	Inv Gamma	2	4	0.68	0.71	0.56	0.89
σ_P^2	Inv Gamma	2	4	0.29	0.31	0.24	0.42
Marginal data density –516.61							

The median estimate of μ is 0.78, which is close to the average growth rate of GDP(A) over the sample. The estimate of ρ is 0.40. This implies that the GDP growth process has relatively little persistence, although the parameter is larger than estimates from an AR(1) model of GDP(A) growth over our sample. Innovations to GDP growth are estimated to be similar in size to the measurement errors in the expenditure and production equations, and smaller than the average measurement errors in the income equation.

Using the posterior distribution of the model's parameter values, we can recover an estimate of 'true' GDP growth over the sample. We call this series derived from Model 1 GDP(M1). Figure 1 compares this estimate to the published quarterly growth rates of GDP(A), GDP(E), GDP(I) and GDP(P). GDP(M1) is highly correlated with GDP(A), but it is less volatile.[12] That is, our model suggests that some extreme readings of GDP(A) are likely to represent measurement error in one or more of the individual measures of GDP. Our methodology also allows us to construct confidence bands around the GDP growth estimates, which are also shown in Figure 1. These are generally wide. For example, in the June quarter of 2013, our model's median estimate of GDP growth was 0.6 per cent, and the 95 per cent confidence bands spanned 0.1–1.2 per cent.

Figure 1: GDP Growth – Comparison with GDP(M1)



Note: Shaded areas show 95 per cent confidence intervals around GDP(M1)
Sources: ABS; Authors' calculations

4.2 Model 2: Correlation

In Model 2, we allow for correlation between innovations to GDP and the measurement errors. Table 2 presents the parameter estimates.

12 Table 5 contains descriptive statistics for all of the estimated GDP series.

Table 2: Prior and Posterior Distributions – Model 2

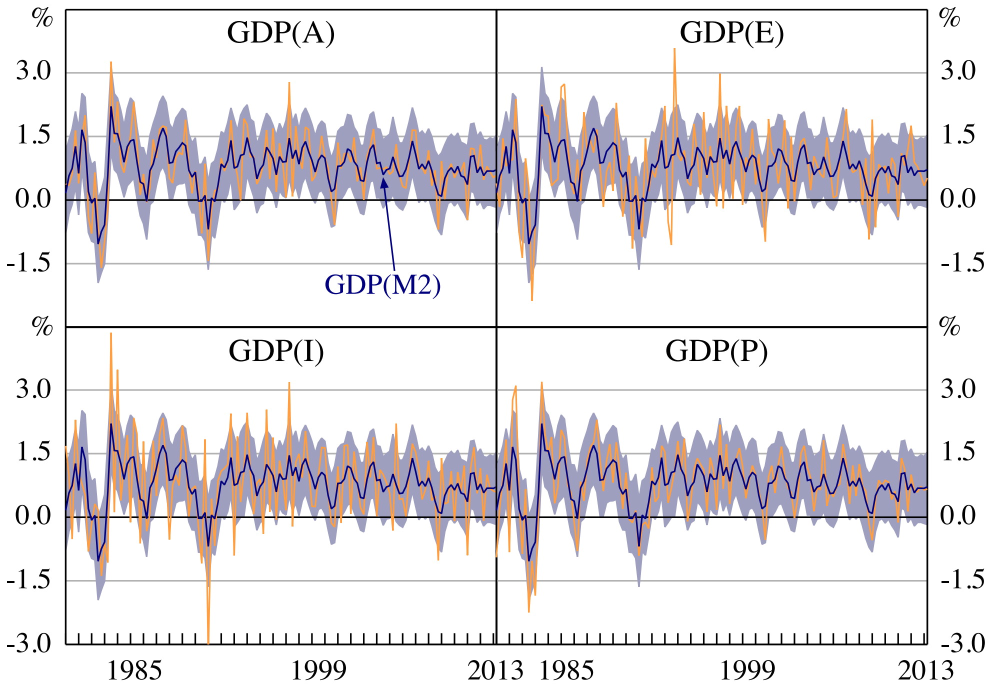
Parameter	**Distribution**	**Prior** Mean	Std dev	**Posterior** Mode	Median	5%	95%
GDP equation							
μ	Normal	0.80	10	0.79	0.79	0.63	0.94
ρ	Beta	0.50	0.2	0.47	0.46	0.26	0.67
Exogenous processes							
σ_G^2	Inv Gamma	2	4	0.39	0.43	0.31	0.56
σ_{GE}	Normal	0	10	–0.25	–0.27	–0.48	–0.09
σ_{GI}	Normal	0	10	–0.17	–0.18	–0.46	0.10
σ_{GP}	Normal	0	10	–0.13	–0.16	–0.35	–0.02
σ_E^2	Inv Gamma	2	4	0.84	0.91	0.56	1.35
σ_{EI}	Normal	0	10	0.37	0.39	0.10	0.74
σ_{EP}	Normal	0	10	0.31	0.36	0.11	0.66
σ_I^2	Inv Gamma	2	4	0.92	0.98	0.58	1.46
σ_{IP}	Normal	0	10	0.17	0.20	–0.07	0.52
σ_P^2	Inv Gamma	2	4	0.45	0.52	0.29	0.88
Marginal data density –514.53							

The estimated parameters of the GDP process, μ and ρ, are similar to those in Model 1. However, the variance of innovations to GDP and the measurement errors are larger. This is most notable in the expenditure equation, where the variance of the measurement errors is now similar in magnitude to the income equation. This is not an artefact of the restriction imposed in Equation (3); varying the restriction, or applying it to GDP(P) rather than GDP(E), leaves the value of σ_E^2 largely unchanged. In contrast, the variance of the measurement errors in the production equation remains around the same size as for the estimated GDP innovations.

The covariances between the measurement errors are positive, and generally statistically significant. This is consistent with the fact that information from some surveys feed into more than one measure of GDP. In contrast, covariances between innovations to GDP and the measurement errors are generally negative and statistically significant. This suggests that the characteristics of measurement errors vary over the business cycle, perhaps because the types of challenges the ABS faces in measuring GDP growth vary across the business cycle. In general, the fact that the covariances of innovations to GDP and measurement errors are statistically significant highlights the importance of controlling for these correlations when evaluating the pace of economic growth.

Figure 2 shows the plot of GDP derived from Model 2, GDP(M2). This measure is considerably smoother than GDP(A). This reflects the fact that when we allow for correlation between shocks some large changes in multiple measures are attributed to measurement error rather than treated as signal.

Figure 2: GDP Growth – Comparison with GDP(M2)



Note: Shaded areas show 95 per cent confidence intervals around GDP(M2)
Sources: ABS; Authors' calculations

4.3 Model 3: Unemployment

In Model 3 we include the quarterly change in the unemployment rate as an additional observable variable. Table 3 presents the parameter estimates.

The estimated mean parameter for the GDP process is similar to the previous models, although GDP growth has more persistence than in Models 1 and 2. The coefficients in the unemployment equation suggest that a 1 percentage point increase in the rate of quarterly GDP growth lowers the unemployment rate by around 0.6 percentage points, which is slightly above existing Okun's law estimates for Australia (Borland 2011).

The parameter estimates for the shock processes differ from the previous models' in two respects. First, the variance of GDP innovations is much smaller when we include the unemployment rate as an observable variable in the model. Second,

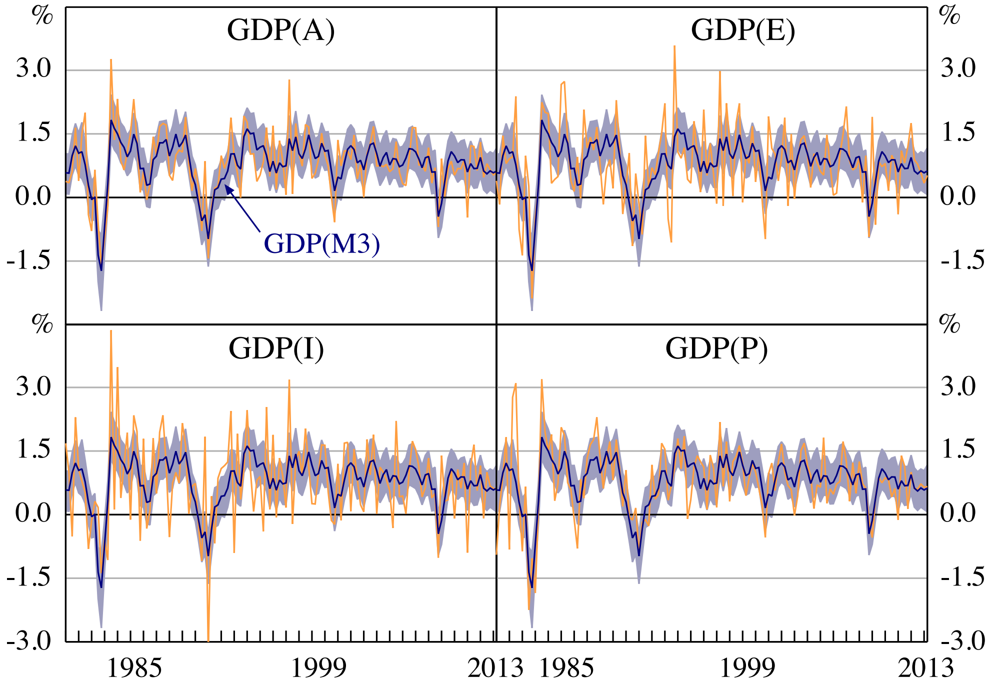
Table 3: Prior and Posterior Distributions – Model 3

Parameter	Distribution	Prior Mean	Prior Std dev	Posterior Mode	Posterior Median	Posterior 5%	Posterior 95%
GDP equation							
μ	Normal	0.80	10	0.79	0.79	0.59	1.01
ρ	Beta	0.50	0.2	0.65	0.62	0.46	0.77
Unemployment equation							
κ	Normal	0	10	0.54	0.51	0.38	0.82
λ	Normal	–0.50	10	–0.69	–0.64	–1.03	–0.50
Exogenous processes							
σ_G^2	Inv Gamma	2	4	0.24	0.28	0.19	0.46
σ_{GE}	Normal	0	10	0.00	–0.01	–0.11	0.05
σ_{GI}	Normal	0	10	–0.03	–0.04	–0.14	0.03
σ_{GP}	Normal	0	10	–0.04	–0.05	–0.14	0.01
σ_{GU}	Normal	0	10	0.12	0.12	0.07	0.21
σ_E^2	Inv Gamma	2	4	0.63	0.70	0.54	0.88
σ_{EI}	Normal	0	10	0.25	0.23	0.10	0.40
σ_{EP}	Normal	0	10	0.19	0.22	0.11	0.34
σ_I^2	Inv Gamma	2	4	0.80	0.87	0.69	1.10
σ_{IP}	Normal	0	10	0.09	0.11	0.00	0.24
σ_P^2	Inv Gamma	2	4	0.42	0.47	0.36	0.60
σ_U^2	Inv Gamma	0.3	4	0.08	0.08	0.05	0.21
Marginal data density –510.43							

the negative correlation between GDP innovations and measurement errors in the GDP(M2) measurement equations largely disappears. However, the covariances between the measurement errors remain positive and statistically significant.

Figure 3 compares this model's estimate of GDP growth, GDP(M3), to the published figures. Overall, the results for Model 3 are similar to those of Model 2 as, once again, our measure of GDP is smoother than GDP(A). The greatest difference lies in the recessions of the early 1980s and 1990s and the slowdown associated with the global financial crisis in the late 2000s. The inclusion of the unemployment rate, which increased in all three episodes, lowers Model 3's estimate of GDP growth relative to the estimates in Models 1 and 2 that do not include the unemployment rate.

Figure 3: GDP Growth – Comparison with GDP(M3)



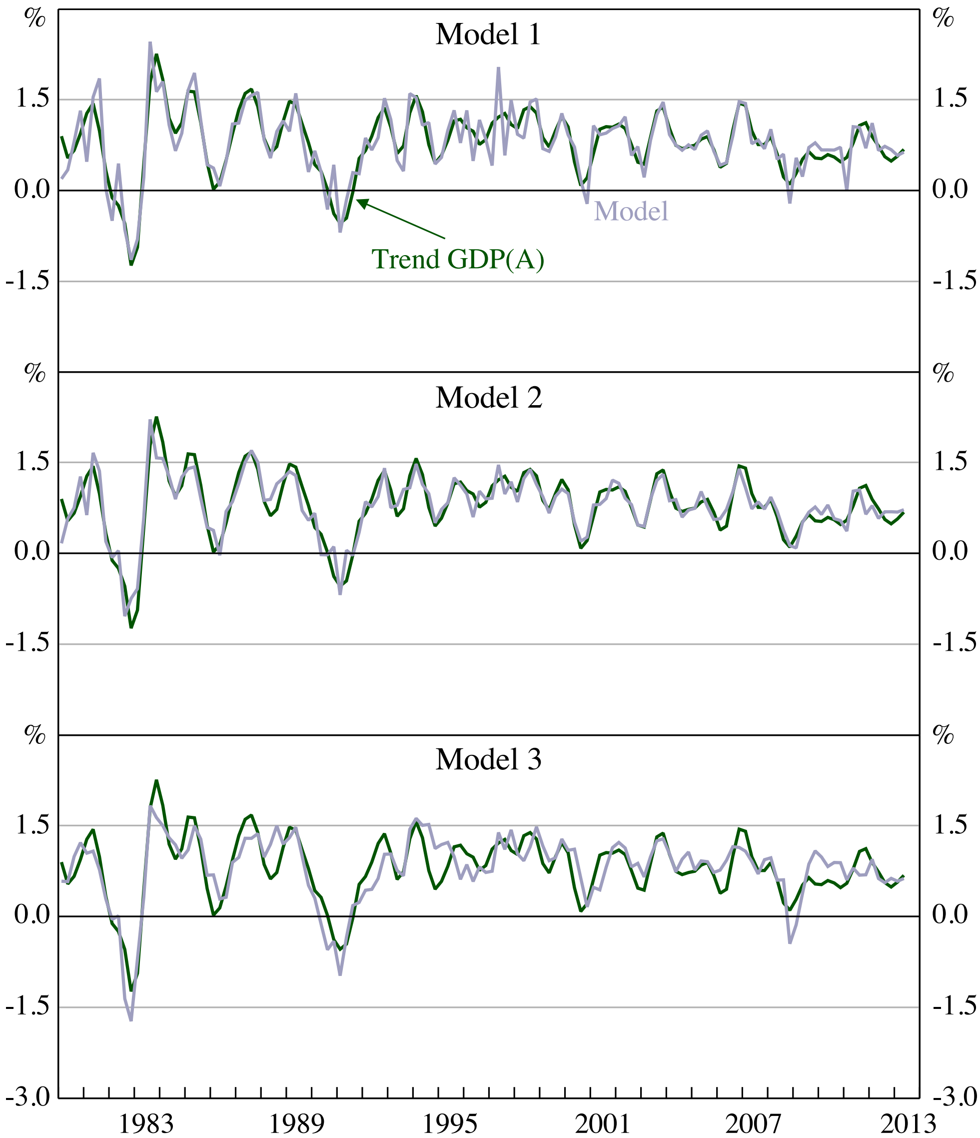
Note: Shaded areas show 95 per cent confidence intervals around GDP(M3)
Sources: ABS; Authors' calculations

4.4 How Do Our Measures Compare with the Published Trend Measure of GDP?

Our methodology provides measures of GDP growth that incorporate information about the degree of noise generated by measurement error in the published estimates. The result is a smoother measure. The ABS also produces a smoother measure of output growth, constructed by applying a Henderson moving average to GDP(A). The ABS publish the resulting measure, known as 'trend' GDP, at a quarterly frequency. Figure 4 compares the ABS trend GDP with the measures introduced in this paper.

The histories of the series are generally quite similar, which is encouraging. Trend GDP(A) has a disadvantage relative to our method, however, in that it suffers from end-point problems. The Henderson trends used by the ABS apply moving averages to past and future observations in a series. As the series approaches its end point, there are fewer observations upon which to calculate these averages. While the ABS takes steps to ameliorate this issue, recent trend GDP data remain subject to substantial revision as new data are received.[13] In Section 5, we demonstrate that the techniques presented in this paper appear to be less affected by end point problems and so should provide users with a better indication of output growth in real time.

Figure 4: Comparison of Models with Trend GDP Growth



Sources: ABS; Authors' calculations

13 Of course, seasonally adjusted series may also feature end point problems if there are changes in seasonal patterns over time.

4.5 What are the Relative Contributions of GDP(E), GDP(I) and GDP(P)?

At its core, our methodology represents an alternative way of combining the information in the three existing ABS measures of GDP growth. One might wonder how our models weight each of the three measures and the extent to which this differs from the simple average used to construct GDP(A). We answer these questions in two ways: first we examine Kalman gains; and second we find the weighted average of GDP(E), GDP(I) and GDP(P) that is closest to our measure of GDP growth.

4.5.1 Kalman gains

Kalman gains govern the extent to which our models adjust their estimates of the rate of GDP growth in light of new observations of GDP(E), GDP(I) or GDP(P). If the Kalman gain on a particular measure of GDP is large then the model extracts a large amount of signal from new data on that measure. For example, the model will interpret a large increase in the growth rate of an observed GDP measure that has a large Kalman gain as a signal that GDP growth has increased. In contrast, it will consider a similar increase in the growth rate of an observed GDP measure that has a small Kalman gain as being more likely to reflect measurement error.

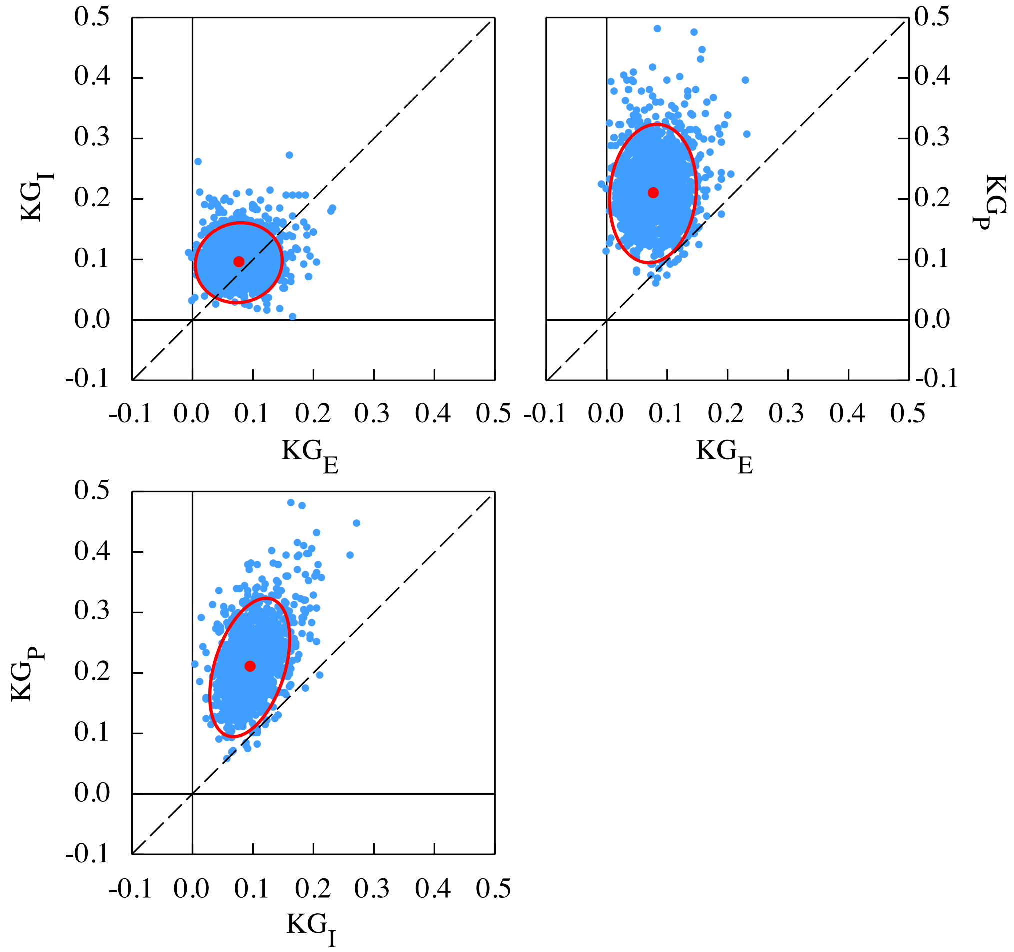
For each draw from the posterior distribution of model parameters, we can recover an estimate of the Kalman gain for each observable variable. Figure 5 summarises these Kalman gains for Model 3.[14] In the figure, each blue dot compares the Kalman gains of two measures of GDP for an individual draw from the posterior distribution. The red dot and circle represent the posterior median and 90 per cent probability interval for each pair. Intuitively, if most dots lie to the left of the dashed 45 degree line, then the Kalman gain for the observed GDP measure on the vertical axis is greater than that of the measure on the horizontal axis, and vice versa. A mass of dots surrounding the dashed 45 degree line indicates that the model puts roughly equal weight on the two observed measures of GDP.

Figure 5 confirms that the model places more weight on GDP(P) than on the other two measures. It also places roughly equal weight on GDP(E) and GDP(I). This is consistent with the fact that the estimated measurement errors in the

14 The distributions for the other models are similar.

production equation are considerably smaller than those in the expenditure and income equations.

Figure 5: Kalman Gain Pairs – Model 3



4.5.2 Closest convex combination

The second way in which we gauge the relative importance of the three observed measures of GDP is by calculating the fixed-weight combinations of the three measures that come closest to replicating our measures of GDP growth. That is, we calculate the values of α_1 and α_2 that solve:[15]

$$[\alpha_1^*, \alpha_2^*] = \arg\min_{\alpha_1,\alpha_2} \sum_{t=1}^{T} \Big[\alpha_1 GDP(E)_t + \alpha_2 GDP(I)_t + (1-\alpha_1-\alpha_2) GDP(P)_t - GDP_{M,t} \Big]^2 .$$

Table 4 shows the weights for each model.

Table 4: Closest Convex Combination

Weight on	Model 1	Model 2	Model 3
GDP(E)	0.30	0.09	0.22
GDP(I)	0.21	0.26	0.24
GDP(P)	0.49	0.65	0.54

Consistent with the results in the Kalman gain section, it appears that our models extract more information from GDP(P) than from the other two measures of GDP. However, there is some discrepancy in the relative weight attached to the other two measures. Model 1 places relatively more weight on GDP(E) than GDP(I), while Model 2 does the reverse and Model 3 places roughly equal weight on the two measures.

4.6 GDP Behaviour during Slowdowns

Although our measures of GDP exhibit similar cycles to GDP(A), the quarterly growth rates differ. These differences are most relevant around business cycle turning points, when distinguishing signal from noise in GDP growth is of greatest importance. In this section, we discuss the behaviour of our models during the Australian economy's two most recent slowdowns, which occurred in 2000–2001 and 2008–2009.

In both of these episodes, GDP(A) indicates that the Australian economy experienced a large contraction in economic activity, followed by a strong recovery in the subsequent quarter. In the earlier episode, the economy returned rapidly to

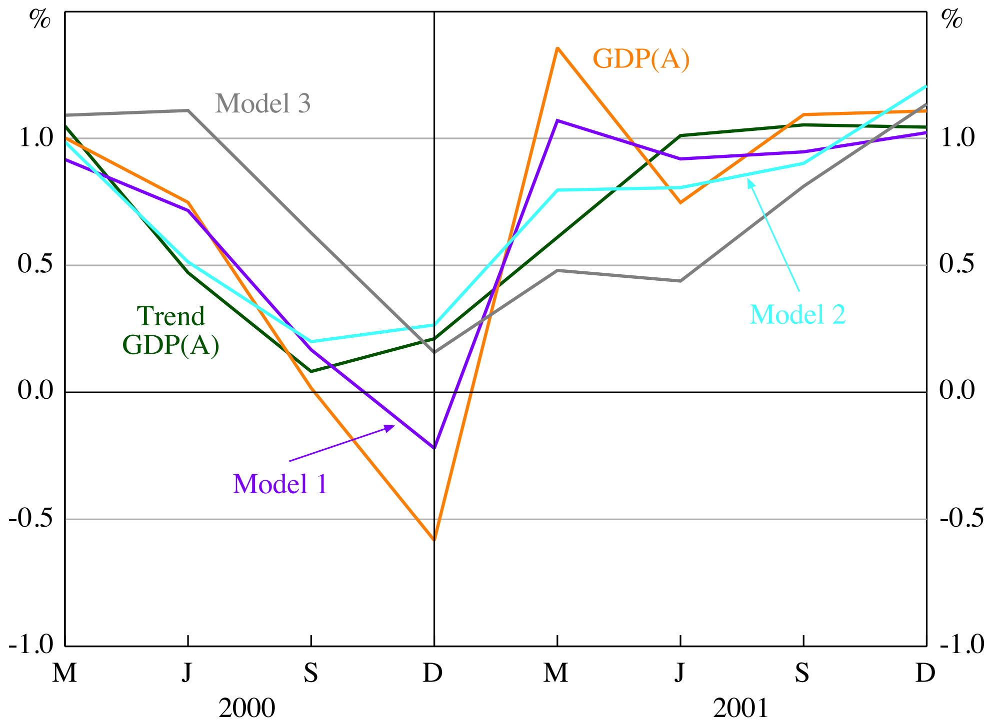
15 Note that constructing a measure of GDP using these weights will not recover GDP(M1), GDP(M2) or GDP(M3) because the Kalman filter does more than a simple contemporaneous averaging of the GDP measures in its extraction of actual GDP growth.

trend growth. In contrast, in the third quarter of the 2008–2009 episode, GDP growth slowed again, with GDP(A) expanding by a mere 0.1 per cent in the June quarter of 2009.

In the presence of measurement error, large changes in economic activity make policymaking difficult. Did the strong GDP growth recorded in the March quarters of 2001 and 2009 accurately signal that the economy had recovered from the declines of previous quarters? Or was it merely statistical noise that concealed ongoing economic weakness?

Our models suggest that neither the slowdown of 2000–2001 nor the subsequent recovery was as dramatic as the GDP(A) outcome suggests (Figure 6). Models 2 and 3 suggest that the economy experienced a period of two to three quarters of substantially below-average growth, but did not actually contract. Model 1 displays a similar quarterly pattern to GDP(A), but with less extreme movements. All of the models suggest that by early 2001 growth in economic activity had begun to recover.

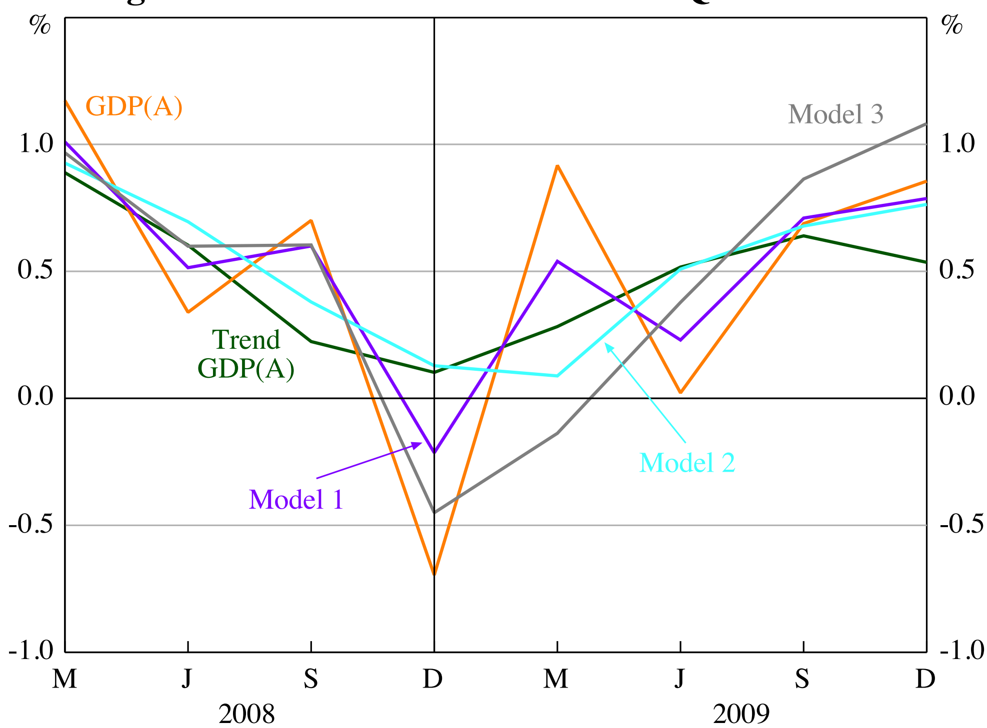
Figure 6: GDP Growth – December Quarter 2000



Sources: ABS; Authors' calculations

In contrast, according to our models, the slowdown of 2008–2009 was more prolonged than indicated by GDP(A) (Figure 7). Model 2 suggests that the economy experienced at least three quarters of growth substantially below average. And Model 3 records two consecutive quarters of negative growth in the December quarter of 2008 and March quarter of 2009. This is consistent with the beliefs of policymakers at the time that the Australian economy was in recession in early 2009 (Stevens 2009). All three measures assign a large proportion of the recovery in GDP(A) growth in the March quarter of 2009 to measurement error. This is consistent with the fact that the increase in GDP growth in that quarter was primarily observable in GDP(E) and GDP(I), to which the models apply relatively less weight.

Figure 7: GDP Growth – December Quarter 2008



Sources: ABS; Authors' calculations

4.7 Is Australian GDP Measurement Different?

A natural benchmark against which to compare our results is Aruoba, Diebold, Nalewaik, Schorfheide and Song (2013), who conduct a similar exercise using US data. Our results differ from theirs in two important respects.

First, across a number of specifications, Aruoba *et al* find that the average size of measurement errors in US GDP(I) is smaller than in US GDP(E). Consequently, their model places more weight on income than expenditure in constructing a measure of US GDP growth. In contrast, we find that in the Australian data measurement errors on the income side of the accounts tend to be a little larger than on the expenditure side.

A second difference is that Aruoba *et al* find that innovations to US GDP are on average larger than measurement errors. In contrast, we find larger relative measurement errors in Australian GDP data.

While it is hard to reach firm conclusions as to the differences between Aruoba *et al* (2013) and our work, we find it plausible that they could reflect differences in the structure of the US and Australian economies. Relative to the United States, Australia is a smaller and more open economy, and commodity exports are relatively more important. Given that commodity prices are typically more volatile than manufacturing or services prices, commodity exporters tend to experience greater volatility in nominal GDP – the quantity of output multiplied by its price – than other economies. This nominal volatility makes real GDP measurement on the income side of the national accounts particularly challenging, because of the need to determine appropriate deflators to apply to volatile nominal GDP flows. Similar challenges apply when measuring expenditure, in particular export and import volumes. To the extent that commodity prices and exchange rates are observable, it should be possible to deflate export and import values accurately. However, if prices and exchange rates are volatile, imposing appropriate deflators is more difficult, creating the possibility of additional measurement error. The volatility of Australian export prices could go some way to explaining the relatively large measurement errors that we report for Australian GDP(E) and GDP(I).

5. Comparison with GDP(A)

It is natural to compare the performance of our models against GDP(A). We first describe the statistical properties of our GDP measures, and we then examine whether our GDP measures are better able to explain and forecast unemployment and inflation.

5.1 How Volatile is GDP Growth?

Visual inspection suggested that our measures of GDP growth smooth out some of the volatility in the published ABS series. A statistical analysis of the alternative GDP measures confirms this conjecture.

Table 5 compares moments of the published GDP series to those of our models. The mean of our models is similar to those of the published series. However, other moments of the distributions differ. All three of our constructed measures are considerably less volatile than the ABS series, with the standard deviation of GDP growth being around one-third lower in our series than in GDP(A).

Table 5: Descriptive Statistics

	ABS series				**GDP(M) series**		
Parameter	GDP(A)	GDP(E)	GDP(I)	GDP(P)	Model 1	Model 2	Model 3
Moments							
Mean	0.79	0.80	0.78	0.80	0.79	0.79	0.79
σ	0.76	0.93	1.03	0.84	0.57	0.56	0.49
ρ_1	0.21	–0.03	–0.19	0.31	0.47	0.79	0.68
Results from an AR(1) regression							
RSE	0.74	0.93	1.02	0.80	0.51	0.34	0.36
R^2	0.04	0.00	0.04	0.10	0.22	0.63	0.47

Notes: The sample period is 1980:Q1–2013:Q2; model-based statistics are for the posterior median estimate of true GDP; σ = standard deviation, ρ_1 is the first-order correlation coefficient, RSE = residual standard error from a fitted AR(1) model

Our measures of GDP growth are also more persistent, with the correlation coefficients on our measures of GDP growth far larger than on the ABS series. As a consequence, our measures of GDP growth are also more predictable; an estimated AR(1) model of our constructed GDP series produces a far closer fit than it does for standard measures of GDP.

5.2 Real-time Performance

In order to produce timely estimates, statistical agencies publish GDP before all information sources are available. They then revise these preliminary estimates as more information comes to light.[16] Bishop *et al* (2013) find that initial estimates of Australian GDP often differ substantially from later, more informed estimates.

16 See Bishop *et al* (2013) for a discussion of the revisions process.

Knowing this, users may prefer measures that are less subject to revision, as long as those measures are close approximations to 'true' output growth.

We use real-time estimates of GDP(E), GDP(I) and GDP(P) to construct a history of real-time model estimates from 2001:Q1 to 2013:Q2.[17] We evaluate the real-time performance of our models using two common metrics: 'mean absolute revision' and 'mean revision'. Mean absolute revision measures the average size of revisions regardless of sign. Mean revision is the average of revisions and can be interpreted as a tendency for GDP to be revised in a particular direction, that is, whether it is biased. Table 6 presents these statistics for GDP(A) and our models over the period 2001:Q1 to 2009:Q3. Final GDP is defined as the estimate after four years, consistent with Bishop *et al* (2013).

Table 6: Revisions to GDP
Percentage points

Measure	GDP(A)	Model 1	Model 2	Model 3
Mean absolute revision	0.29	0.20	0.19	0.18
Mean revision	0.13	0.11	0.10	0.05

Notes: The sample period is 2001:Q1–2009:Q3; revisions are calculated as the difference between each measure's growth estimate after four years and its initial growth estimate

Our models are more reliable than GDP(A) in real time, with the mean absolute revisions statistically smaller than for GDP(A) at the 5 per cent level. These differences are economically meaningful as well; revisions to our models are around a third smaller than revisions to GDP(A). Consistent with Bishop *et al* (2013), over this sample period there has been a slight upward tendency to revisions, and this is evident across the measures, although slightly less so for the model estimates.

In addition, our models' GDP growth estimates are also easier to forecast in real time. The root mean squared errors for out-of-sample AR(1) forecasts are 0.39 percentage points for Model 1 and 0.35 percentage points for Models 2 and 3, compared with 0.52 percentage points for GDP(A). This suggests that contemporaneous estimates from our models may provide a better indication of future outturns than GDP(A).

17 Due to the time required for estimation, we re-estimate the model every four quarters using real-time data. We use these parameter estimates, combined with real-time national accounts data, to produce estimates for the subsequent three quarters.

Table 7 shows that our models also converge to their final values more quickly than GDP(A). The performance of Model 1 is particularly noteworthy as this model is highly correlated with GDP(A).

Table 7: Error Relative to 'Final' Estimate
Mean absolute error, percentage points

Measure	GDP(A)	Model 1	Model 2	Model 3
Initial	0.29	0.20	0.19	0.18
One year	0.25	0.17	0.16	0.11
Two years	0.22	0.15	0.12	0.08
Three years	0.17	0.10	0.09	0.06

Notes: The sample period is 2001:Q1–2009:Q3; errors are calculated as the difference between each measure's growth estimate after four years and its growth estimate at the specified horizon

5.3 Explaining Macroeconomic Relationships

In this section we examine whether our measures of GDP display a closer relationship with unemployment and inflation than GDP(A).

5.3.1 Unemployment

Macroeconomic theories typically predict a close relationship between output growth and unemployment, a relationship known as 'Okun's law'. Figure 8 illustrates the Okun's law relationship for Australia for GDP(A) and Models 1 and 2.[18] As theory would suggest, for all three measures, lower output growth is associated with an increase in the unemployment rate. But the relationship between changes in unemployment and changes in GDP appears stronger for Models 1 and 2 than for GDP(A), represented by steeper fitted lines and higher adjusted R^2s.

We confirm this result more formally by examining the in-sample fit of our models and GDP(A). To do this, we estimate the specification:

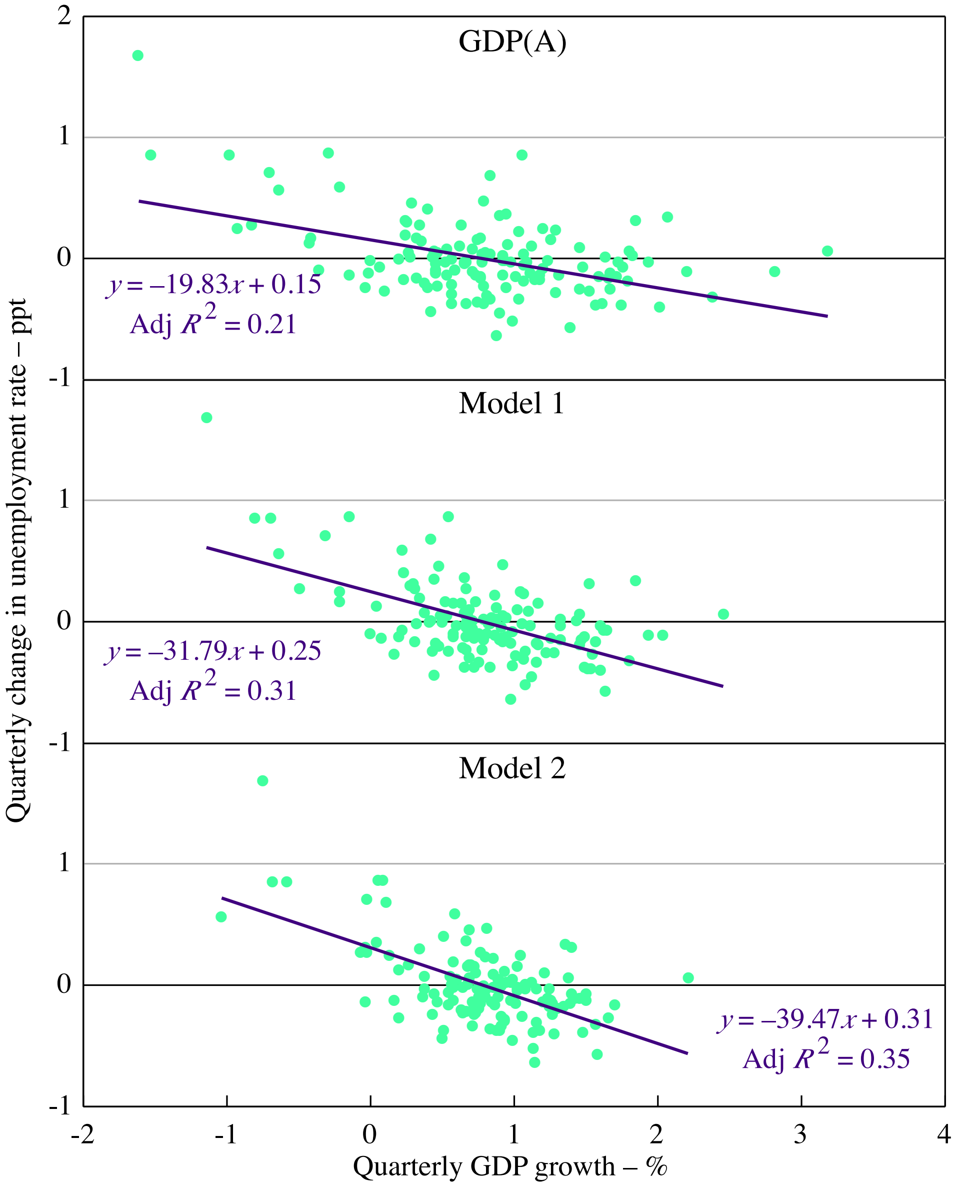
$$\Delta U_t = \alpha + \gamma \Delta U_{t-1} + \sum_{i=0}^{2} \beta_i \Delta y_{t-i} + \varepsilon_t \qquad (5)$$

18 We exclude Model 3 because it is identified using the unemployment rate.

where Δy_{t-i} is the quarterly growth rate of a measure of GDP in quarter $t-i$. The long-run response of unemployment to changes in output, known as 'Okun's coefficient', can be approximated by:

$$C = \Sigma_{i=0}^{2} \beta_i / (1-\gamma).$$

Figure 8: Okun's Law



Sources: ABS; Authors' calculations

Table 8 presents our results. The estimated coefficients are mostly statistically significant and are of the expected sign. The regressions including Models 1 and 2 appear to fit the data better than those including GDP(A), as shown by the adjusted

R^2s, although the difference is not statistically significant. The coefficients on the lagged changes in unemployment are smaller in the regressions with our model measures, suggesting that our measures contribute more information than GDP(A) or a random walk. Finally, the coefficients on contemporaneous and lagged values of output growth are larger for our models than for GDP(A), reflected in larger Okun's coefficients. This may indicate attenuation bias in the regressions including GDP(A), caused by the presence of measurement error.

Table 8: Unemployment Rate – Okun's Law

Parameter	GDP(A)	Model 1	Model 2
α	0.23**	0.32**	0.34**
ΔU_{t-1}	0.36**	0.29**	0.25**
Δy_t	–0.12**	–0.17**	–0.11*
Δy_{t-1}	–0.13**	–0.14**	–0.27**
Δy_{t-2}	–0.04	–0.08*	–0.05
Implied Okun's coefficient	–0.45	–0.56	–0.59
Adj R^2	0.54	0.57	0.60

Notes: The sample period is 1980:Q4–2013:Q2; ** and * represent significance at the 1 and 5 per cent levels, respectively; the models were estimated using robust (White 1980) standard errors

Despite the better in-sample fit, we find that our models do not improve real-time forecasting. Table 9 presents root mean squared errors for one-step-ahead forecasts of the unemployment rate, incorporating either GDP(A), Model 1 or Model 2. We find that over the sample period, root mean squared errors were broadly similar across the measures, suggesting that the models are similarly useful at forecasting unemployment.[19]

Table 9: Unemployment Rate – Real-time Forecast Errors

Measure	GDP(A)	Model 1	Model 2
Root mean squared error	0.20	0.20	0.20
Mean error	0.02	0.03	0.04

Note: The sample period is 2001:Q1–2013:Q2

19 The similar forecasting performance of the models may be the result of timing issues. In Table 9 we use the timing of the RBA's *Statement on Monetary Policy* to simulate the RBA staff's forecasting experience, where applicable. At that time, GDP for the previous quarter is not available because the ABS release the national accounts with a delay of a little over two months. The forecast specification of Equation (5) includes lags that are a fair way down the lag structure and, therefore, lack forecasting power.

5.3.2 *Inflation*

We now examine whether our model measures of GDP are useful for explaining inflation. We estimate an expectations-augmented mark-up model of inflation, following Norman and Richards (2010). The model includes terms for bond market inflation expectations ($E(\pi)$), unit labour costs (ulc), import prices (mp) and the output gap (gap). The dependent variable is the rate of inflation (π). Like Norman and Richards, we choose a polynomial distributed-lag specification as follows:

$$\pi_t = \alpha + \beta E_{t-1}(\pi_{t+s}) + \sum_{j=0}^{9} \lambda_j \Delta ulc_{t-j} + \sum_{k=1}^{12} \gamma_k \Delta mp_{t-k} + \varphi gap_{t-1} + \varepsilon_t. \quad (6)$$

Our model measures of GDP enter the mark-up model through the output gap, where potential GDP growth is derived using a Hodrick-Prescott (HP) filter.[20]

Table 10 presents the results. Our GDP measures seem to fit about as well as GDP(A). The coefficients on the output gap terms for all of the measures are statistically significant and of a similar magnitude.[21] The models appear to fit the data similarly well, with the adjusted R^2 for each model around 0.6. Overall, our GDP measures appear about as useful as GDP(A) in explaining inflation.

Table 10: Inflation Rate – Mark-up Model

Parameter	GDP(A)	Model 1	Model 2	Model 3
α	0.00**	0.00**	0.00**	0.00**
$E_{t-1}(\pi_{t+s})$	0.16**	0.17**	0.17**	0.16**
Δulc_{t-j}	0.36**	0.35**	0.34**	0.33**
Δmp_{t-k}	0.12**	0.12**	0.11**	0.12**
gap_{t-1}	0.07**	0.08**	0.08**	0.07**
Adj R^2	0.60	0.60	0.60	0.60

Notes: The sample period is 1990:Q1–2013:Q2; ** represents significance at the 1 per cent level; where multiple lags are included, coefficients are the sum of the lags

20 The output from the estimation procedures outlined in Section 3 is in one-quarter changes. The HP filter is run over a levels index constructed using these quarterly growth rates. We introduce levels versions of our model measures in Appendix B.

21 One explanation for the similarity of coefficients is that, since our measure of the output gap is based on the accumulation of quarterly growth rates, quarter-to-quarter errors in GDP(A) will wash out to some degree, leaving all gap estimates quite similar.

6. Conclusion

In this paper we have constructed several new measures of Australian GDP growth that use state-space methods to extract a measure of underlying economic activity from the noisy published measures of expenditure, income and production. Although our measures are highly correlated with published GDP growth, they are noticeably less volatile and easier to forecast. Moreover, they explain variations in inflation and unemployment as well as or slightly better than the published GDP growth measures. Our measures also perform well in real-time.

Appendix A: Identification

We use the results contained in Appendix A of Aruoba *et al* (2013) and Section 4 of Komunjer and Ng (2011) to prove that the model presented in Section 2.2 is identified with a single parameter restriction. In particular, and ignoring the constants, the model can be written as

$$\begin{aligned} s_{t+1} &= As_t + B\varepsilon_{t+1} \\ m_{t+1} &= Cs_t + D\varepsilon_{t+1} \end{aligned}$$

where $s_t = \Delta y_t$, $m_t = [\Delta y_t^E, \Delta y_t^I, \Delta y_t^P]'$, $A = \rho$, $B = [1,0,0,0]$, $C = [\rho,\rho,\rho]'$ and

$$D = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_G^2 & & & \\ \sigma_{GE} & \sigma_E^2 & & \\ \sigma_{GI} & \sigma_{EI} & \sigma_I^2 & \\ \sigma_{GP} & \sigma_{EP} & \sigma_{IP} & \sigma_P^2 \end{bmatrix}$$

where $\varepsilon_t = [\varepsilon_{G,t}, \varepsilon_{E,t}, \varepsilon_{I,t}, \varepsilon_{P,t}] \sim N(0,\Sigma)$.

Assuming that $0 \leq \rho < 1$ and that Σ is positive definite, and noting that the rows of D are linearly independent, ensures that Assumptions 1, 2 and 4-NS of Komunjer and Ng (2011) are satisfied, while Appendix A of Aruoba *et al* (2013) shows that Assumption 5-NS of Komunjer and Ng is satisfied. Then by Proposition 1-NS of Komunjer and Ng, two models (with the second model indexed by a $*$ subscript) are observationally equivalent if and only if $\rho_* = \rho$ and

$$p_*CC' + D\Sigma_*D' = pCC' + D\Sigma D' \tag{A1}$$

$$p_*\rho C' + B\Sigma_*D' = p\rho C' + B\Sigma D' \tag{A2}$$

$$\sigma_{G*}^2 = \sigma_G^2 + (p_* - p)(1-\rho^2) \tag{A3}$$

where p solves $p = p\rho^2 + \sigma_G^2 - (p\rho C' + B\Sigma D')(pCC' + D\Sigma D')^{-1}(p\rho C + D\Sigma B')$.

If $p_* = p$ then the above equations imply that $\Sigma_* = \Sigma$ and the models are identical. If $p_* \neq p$ we can write p_* as $p_* = p + \delta$ for some $\delta \neq 0$, in which case

Equation (A3) becomes $\sigma_{G*}^2 = \sigma_G^2 + \delta(1-\rho^2)$. From Equation (A2) we have

$$\begin{aligned}
B\Sigma D' &= \begin{bmatrix} \sigma_G^2 + \sigma_{GE} & \sigma_G^2 + \sigma_{GI} & \sigma_G^2 + \sigma_{GP} \end{bmatrix} \\
&= B\Sigma_* D' + \delta\rho C' \\
&= \begin{bmatrix} \sigma_G^2 + \sigma_{GE*} + \delta & \sigma_G^2 + \sigma_{GI*} + \delta & \sigma_G^2 + \sigma_{GP*} + \delta \end{bmatrix}
\end{aligned}$$

so that $\sigma_{GE*} = \sigma_{GE} - \delta$, $\sigma_{GI*} = \sigma_{GI} - \delta$ and $\sigma_{GP*} = \sigma_{GP} - \delta$. Finally,

$$D\Sigma D' = \begin{bmatrix} \sigma_G^2 + 2\sigma_{GE} + \sigma_E^2 & \sigma_G^2 + \sigma_{GE} + \sigma_{GI} + \sigma_{EI} & \sigma_G^2 + \sigma_{GE} + \sigma_{GP} + \sigma_{EP} \\ \sigma_G^2 + \sigma_{GE} + \sigma_{GI} + \sigma_{EI} & \sigma_G^2 + 2\sigma_{GI} + \sigma_I^2 & \sigma_G^2 + \sigma_{GI} + \sigma_{GP} + \sigma_{IP} \\ \sigma_G^2 + \sigma_{GE} + \sigma_{GP} + \sigma_{EP} & \sigma_G^2 + \sigma_{GI} + \sigma_{GP} + \sigma_{IP} & \sigma_G^2 + 2\sigma_{GP} + \sigma_P^2 \end{bmatrix}$$

so that from Equation (A1) we have

$$\begin{aligned}
0 &= D\Sigma_* D' - D\Sigma D' + \delta CC' \\
&= \begin{bmatrix} \sigma_{E*}^2 - \sigma_E^2 - \delta & \sigma_{EI*} - \sigma_{EI} - \delta & \sigma_{EP*} - \sigma_{EP} - \delta \\ \sigma_{EI*} - \sigma_{EI} - \delta & \sigma_{I*}^2 - \sigma_I^2 - \delta & \sigma_{IP*} - \sigma_{IP} - \delta \\ \sigma_{EP*} - \sigma_{EP} - \delta & \sigma_{IP*} - \sigma_{IP} - \delta & \sigma_{P*}^2 - \sigma_P^2 - \delta \end{bmatrix}
\end{aligned}$$

so that $\sigma_{E*}^2 = \sigma_E^2 + \delta$, $\sigma_{EI*} = \sigma_{EI} + \delta$, $\sigma_{EP*} = \sigma_{EP} + \delta$, $\sigma_{I*}^2 = \sigma_I^2 + \delta$, $\sigma_{IP*} = \sigma_{IP} + \delta$ and $\sigma_{P*}^2 = \sigma_P^2 + \delta$. Hence the 'star' model is observationally equivalent to the 'non-star' model if and only if

$$\Sigma_* = \begin{bmatrix} \sigma_G^2 + \delta(1-\rho^2) & & & \\ \sigma_{GE} - \delta & \sigma_E^2 + \delta & & \\ \sigma_{GI} - \delta & \sigma_{EI} + \delta & \sigma_I^2 + \delta & \\ \sigma_{GP} - \delta & \sigma_{EP} + \delta & \sigma_{IP} + \delta & \sigma_P^2 + \delta \end{bmatrix}$$

for some δ. As such, we need to place at least one restriction on Σ to ensure an identified model.

Appendix B: Levels or Differences?

Our baseline models relate the growth rate of GDP to its expenditure, income and production measures. We focus on growth rates because it is the growth rate of GDP (and its implications for the output gap), rather than its level in dollars, that is of day-to-day interest to policymakers. However, even if we are most interested in GDP growth, the level of GDP may still contain useful information, particularly as the three ABS measures should be cointegrated. To examine whether accounting for the level of GDP affects our results, we modified our models to allow for cointegration between GDP and the expenditure, income and production measures. To do this, we augment our basic measurement equation to include an error correction term:

$$\Delta y_t^j = \Delta y_t - \eta_j \left(y_{t-1}^j - y_{t-1} \right) + \varepsilon_{j,t} \tag{B1}$$

for $j \in \{E, I, P\}$ where y_t^j and y_t are the log-levels of a measure of GDP and the unobserved 'true' measure of GDP. The parameter η_j tells us how quickly the levels of a measure of GDP and its 'true' value converge.

The state-space form and estimation procedure of this model is similar to our baseline models, although we adjust the Kalman filter to account for the fact that the level of GDP is non-stationary. We do this using the methods of Koopman and Durbin (2003).

Table B1 shows the results for Model 2 estimated in error correction form (ECM). In most cases, the parameter estimates are similar to those in the equivalent model estimated in growth rates. However, the variances of some of the shocks are smaller in the ECM model. And the negative relationship between innovations to GDP and the measurement errors ceases to be statistically significant. The error correction term in the GDP(I) equation is larger than those in the equations for the other two observed measures of GDP. That is, deviations of the level of GDP(I) from the true GDP appear to close more rapidly than those of other measures. Most of these deviations, however, are likely to have occurred in the early years of the sample. The ABS reconciliation process ensures that for all years since 1994, and before the latest financial year, the sums of GDP(E), GDP(I) and GDP(P) over

Table B1: Prior and Posterior Distributions – ECM Model 2

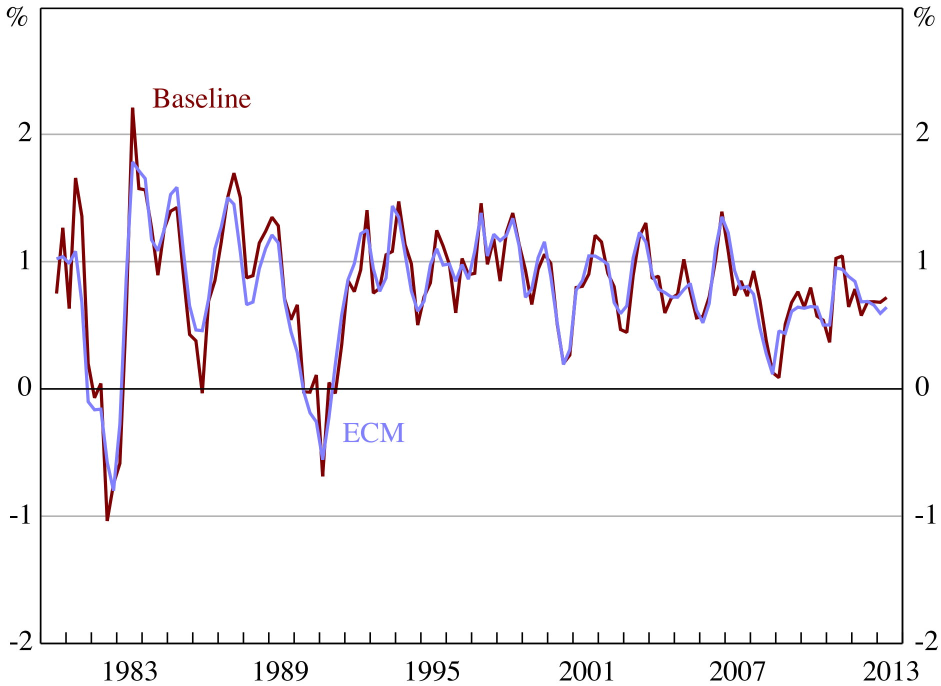
		Prior		Posterior			
Parameter	**Distribution**	Mean	Std dev	Mode	Median	5%	95%
GDP equation							
μ	Normal	0.80	10	0.73	0.74	0.56	0.91
ρ	Beta	0.50	0.2	0.55	0.54	0.36	0.73
Exogenous processes							
σ_G^2	Inv Gamma	2	4	0.25	0.28	0.17	0.39
σ_{GE}	Normal	0	10	–0.10	–0.09	–0.22	0.03
σ_{GI}	Normal	0	10	–0.11	–0.07	–0.28	0.12
σ_{GP}	Normal	0	10	–0.15	–0.14	–0.26	–0.03
σ_E^2	Inv Gamma	2	4	0.57	0.58	0.39	0.81
σ_{EI}	Normal	0	10	0.18	0.19	0.02	0.38
σ_{EP}	Normal	0	10	0.24	0.24	0.08	0.42
σ_I^2	Inv Gamma	2	4	0.46	0.56	0.36	0.80
σ_{IP}	Normal	0	10	0.15	0.15	–0.03	–0.34
σ_P^2	Inv Gamma	2	4	0.48	0.50	0.34	0.69
Error correction terms							
η_E	Beta	0.5	0.2	0.37	0.43	0.27	0.70
η_I	Beta	0.5	0.2	0.87	0.70	0.38	0.92
η_P	Beta	0.5	0.2	0.23	0.24	0.15	0.33
Marginal data density 249.22							

a financial year are equal.[22] Hence, it is unlikely that any of these series could deviate from true GDP for a substantial period of time.

Figure B1 compares the GDP series from the ECM model with the equivalent series derived from our baseline model. Compared with the baseline model, the ECM model smooths some of the peaks and troughs in GDP growth. This is particularly noticeable in the 1980s and early 1990s. The 1986 growth slowdown is hardly noticeable in the ECM series while the early 1980s and 1990s recessions are shallower but more prolonged.

22 The ABS reconcile the measures of GDP using annual supply-use tables. Due to lags in compiling the tables, the measures of GDP are not equal for the most recent financial year (or two years in the case of the June quarter).

Figure B1: GDP Growth – Baseline and ECM Models



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