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The Power of Unconventional Monetary Policy in a Liquidity Trap*

Masayuki Inui† and Sohei Kaihatsu‡

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Abstract

In this study, we examine what unconventional monetary policy measures are effective in escaping from a liquidity trap. We develop a heterogeneous agent New Keynesian model with uninsurable income uncertainty and a borrowing constraint. We show that adverse effects of income uncertainty deteriorate in the liquidity trap, which crucially undermines the transmission mechanism of unconventional monetary policy through an increase in precautionary savings. We then draw the following implications: (1) decreasing risk premiums by quantitative easing (QE) is more effective than forward guidance (FG) in the liquidity trap; (2) when the liquidity trap becomes deeper, central banks should conduct QE with sufficiently rapid pace of asset purchases; and (3) the combination of QE and FG yields synergy effects that strengthen the power to escape from the liquidity trap through mitigating precautionary saving motives.

JEL classification: E21, E31, E52, E58

Keywords: unconventional monetary policy; liquidity trap; uninsurable income uncertainty; incomplete market; quantitative easing; forward guidance

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1. Introduction

After the global financial turmoil, major central banks have aggressively implemented unconventional monetary policy measures to avoid or escape from a liquidity trap, in which the zero lower bound imposes a binding constraint on nominal interest rates. Facing a risk of falling into a liquidity trap, households would suffer from the adverse effects of deflationary recession heterogeneously because the abilities to insure themselves against idiosyncratic income risk are likely to vary across households. In other words, households face uninsurable income uncertainty, which may attenuate the transmission mechanism of unconventional monetary policy.

This study examines what unconventional monetary policy measures are effective in escaping from a liquidity trap. To this end, we develop a heterogeneous agent New Keynesian model with uninsurable income uncertainty and a borrowing constraint. We take up two types of unconventional monetary policy measures: forward guidance (FG) and quantitative easing (QE). Specifically, we refer to a commitment to keep interest rates low for longer than expected as FG, and to a long-term bond purchase program for the purpose of decreasing risk premiums as QE.

Under the assumptions of uninsurable income uncertainty and a borrowing constraint, households have an incentive to maintain a buffer stock of precautionary savings due to the fear of hitting a borrowing constraint in the future. As indicated by McKay *et al*. (2015), the demand for precautionary savings reduces the effectiveness of FG by discouraging households from taking full advantage of the opportunity for intertemporal substitution toward present consumption. However, it remains unclear how uninsurable income uncertainty and a borrowing constraint alter the transmission mechanism of unconventional monetary policy in a liquidity trap.

To examine a liquidity trap, we introduce a shock to the natural rate of interest that makes the economy hit the zero lower bound on nominal interest rates, thereby causing a deflationary recession. This deflationary shock has a diminishing effect on demand and fuels fear of hitting a borrowing constraint in the future. Consequently, households increase their precautionary savings to prepare for future possible losses due to the deflationary recession. Income uncertainty exacerbated by the deflationary recession may reduce the effectiveness of FG, which is problematic because the central bank would lose its power to affect the real economy by commitment.

There are three major findings in this paper. First, in the presence of income uncertainty, decreasing risk premiums by QE is more effective than FG in the liquidity trap. The deflationary shock exerts persistent downward pressure on household wage income until the natural rate of interest increases back to the steady-state level. Then, households accumulate precautionary savings at the expense of their present consumption. In addition, firms decrease their investment as a response to a rise in firm-level uncertainty about future return. The negative reactions of both households and firms lead to a severer deflation and fuel fear of hitting a borrowing constraint in the future. Those heightened fears further attenuate the transmission mechanism of FG since rising precautionary saving motives preclude intertemporal substitution. In contrast, QE can mitigate precautionary saving motives through the increases in asset prices, which alleviates households' fears of hitting a borrowing constraint in the future. In addition, QE can also decrease firm-level uncertainty, thereby increasing investment.

Second, when the liquidity trap becomes deeper, the central bank should conduct more active policy responses. The precautionary saving motives become further entrenched in the deep liquidity trap. In the presence of income uncertainty, accelerating the pace of asset purchases yields a considerably stronger amplification of policy effects even if the total amount of asset purchases remains unchanged. Therefore, for a smooth recovery from the deep liquidity trap, the central bank should conduct QE with a sufficiently rapid pace of asset purchase to adequately decrease precautionary saving motives. If, instead, the central bank conducts QE at a considerably slow pace of asset purchase, then the effect of decreasing precautionary savings does not emerge and deflationary pressure persists.

Third, even if the severer deflationary pressure remains, QE with a sufficiently rapid pace of asset purchase can restore the effectiveness of FG. The combination of QE and FG creates synergy effects that strengthen the power to escape from the liquidity trap. Our simulations reveal that the immediate impact of QE increases household wealth by raising asset prices. Consequently, households can take better advantage of the intertemporal substitution toward present consumption without the fear of hitting a borrowing constraint in the future. QE also decreases firm-level uncertainty through decreasing entrepreneurial leverage ratio by both raising asset prices and stimulating aggregate demand, leading to increasing investment. Thus, the impaired transmission mechanism of FG can be made effective in escaping from the liquidity trap.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model. Section 4 discusses how unconventional monetary policy measures work to escape from the liquidity trap. Section 5 presents the conclusions.

2. Literature

To the best of our knowledge, this study is the first to examine how uninsurable income uncertainty and a borrowing constraint affect the transmission mechanism of QE in a general equilibrium framework with an incomplete market. Most previous studies on unconventional monetary policy have been based on the complete market model, in which all risks are insurable. For example, Cúrdia and Woodford (2011), Gertler and Karadi (2011, 2013), and Chen *et al.* (2012) develop general equilibrium models to examine various QEs by the Federal Reserve, based on the assumption of market completeness. That is, agents in their models are supposed to be able to insure themselves against all risks. As McKay *et al*. (2015) show, in the incomplete market model with price stickiness, households have an incentive to accumulate a buffer stock of precautionary savings when they expect their wage income to decline. Kaplan *et al.* (2016) report that in their incomplete market model, intertemporal substitution effects of interest rate shock become small whereas the indirect transmission mechanism through income effects and portfolio reallocation effects become substantial. These features do not appear in the complete market model. McKay *et al*. (2015) discuss that households' precautionary saving motives may impair the effectiveness of FG since households give up taking full advantage of the intertemporal substitution toward current consumption.

This study examines the interplay between FG and QE under the incomplete market. Eggertsson and Woodford (2003) mention that FG is effective in escaping from prolonged deflation. However, McKay *et al*. (2015) report that market incompleteness reduces the effectiveness of FG because uninsurable income uncertainty poses concern for households to take full advantage of intertemporal substitution toward current consumption. Such distortions in households' consumption/saving decisions affect the interplay between FG and QE. We examine how the interplay between these two measures works based on the assumption of the incomplete market. In contrast to the complete market model, the dynamics of wealth distribution matter in the incomplete market model. In this case, the redistribution of wealth can affect the transmission mechanism of unconventional monetary policy.

This study also examines the effects of unconventional monetary policy on firms' investment behaviors in response to varying business uncertainty. Christiano *et al.* (2014) calculate a model-based measure of uncertainty. They report that firm-level uncertainty is an important factor for the business cycle; that is, risk shocks create firm-level uncertainty and decrease investment. We derive and compute a model-based measure of uncertainty for our model. Then, we examine how the effects of market incompleteness manifest as firm-level uncertainty and, therefore, how firms' investment decisions are affected by this uncertainty.

3. Model

We build a heterogeneous agent New Keynesian model based on McKay and Reis (2016), which incorporate market incompleteness into the canonical New Keynesian model, such as those of Christiano *et al.* (2005) and Smets and Wouters (2007).

Two types of households are populated in the economy. The first group has access to a complete set of insurance. The second group does not have access to such an insurance market and faces a borrowing constraint. Entrepreneurs obtain loans from financial intermediaries for the purchase of capital and rent it out, assuming asymmetric information between borrowers and lenders and costly monitoring. Capital goods producers build new capital goods and sell it. Intermediate goods producers hire labor and rent capital to produce intermediate goods. Final goods producers transform intermediate goods into consumption goods. The central bank conducts QE and FG. Each agent's behavior is described in turn.

3.1. Households

3.1.1 Households with Perfect Insurance

Some households have access to a complete set of insurance; thus, they are perfectly self-insured against all risks. The representative household chooses consumption and labor supply to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_h^s \left[\frac{\left(c_{t+s}^h\right)^{1-\sigma_h}}{1-\sigma_h} - \psi_1 \frac{{n_t^h}^{1+\psi_2}}{1+\psi_2} \right], \tag{1}$$

where c_t^h denotes consumption, n_t^h denotes labor supply, $\beta_h \in (0,1)$ is the individual discount factor, $\sigma_h > 0$ denotes the parameter of relative risk aversion, ψ_1 represents the relative willingness to work, and $\psi_2 \geq 0$ is the Frisch elasticity of labor supply.

The flow budget constraint is given by

$$\begin{aligned} &\left(1+\xi_t^k\left(k_t^h\right)\right) Q_t^k k_t^h + \left(1+\xi_t^b\left(b_t^h\right)\right) Q_t^{B,G} b_t^h + c_t^h + D_t^h = \frac{\left(1+i_t^B\right)}{\pi_t} Q_{t-1}^{B,G} b_{t-1}^h \\ &+T_t^F + T_t^E + T_t^{FI} + \frac{\left(1+R_t^k\right)}{\pi_t} Q_{t-1}^k k_{t-1}^h + (1-\tau_t) w_t s_t^h n_t^h + \frac{(1+i_t)}{\pi_t} D_{t-1}^h, \end{aligned} \tag{2}$$

where ξ_t^k and ξ_t^b denote transaction costs for capital and bond holdings, respectively, which will be discussed in more details later. k_t^h, b_t^h, and D_t^h denote capital, long-term bonds, and short-term bonds owned by the households with perfect insurance, respectively. For each type of asset, it pays the respective market returns R_t^k, i_t^B, and i_t. Q_t^k and $Q_t^{B,G}$ are the prices of capital stock and of long-term bond, respectively. T_t^F and T_t^E are transfers from intermediate firms and entrepreneurs, respectively. The financial intermediary transfers its profits as dividends T_t^{FI}. τ_t denotes a proportional labor income tax. The wage $w_t s_t^h$ is the product of the average wage w_t and the individual skill s_t^h. This skill can be rewritten as an average of the individual skills of all households with perfect insurance $\bar{s}$ because they are perfectly self-insured.

Following Andrés *et al.* (2004) and Gertler and Karadi (2013), we assume quadratic transaction costs specified as $\xi_t^k = 1/2\,\kappa\left(k_t^h - k_{ss}^h\right)^2$ and $\xi_t^b = 1/2\,\kappa\left(b_t^h - b_{ss}^h\right)^2$ where k_{ss}^h and b_{ss}^h are the steady-state levels of capital holdings and of bond holdings, respectively. We also assume, following Woodford (2001), that long-term bonds purchased at time t are perpetuities that pay an exponentially decaying coupon ρ^s at time $t+s+1$ for $\rho \in (0,1]$.

Maximizing (1) subject to (2), we obtain the following optimality conditions:

$$1 = \mathbb{E}_t\left[\beta_h \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{(1+i_{t+1})}{\pi_{t+1}}\right], \tag{3}$$

$$\left(1 + \xi_t^{b\prime}\left(b_t^h\right)\right) = \mathbb{E}_t\left[\beta_h \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{Q_{t+1}^{B,G}}{Q_t^{B,G}} \frac{(1+i_{t+1}^B)}{\pi_{t+1}}\right], \tag{4}$$

$$\left(1 + \xi_t^{k\prime}\left(k_t^h\right)\right) = \mathbb{E}_t\left[\beta_h \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{Q_{t+1}^{k}}{Q_t^{k}} \frac{\left(1+R_{t+1}^k\right)}{\pi_{t+1}}\right], \tag{5}$$

$$\beta_h \psi_1 n_{h,t}^{\psi_2} = \lambda_{h,t}(1-\tau_t) w_t s_t^h, \tag{6}$$

$$\lambda_{h,t} = \left(c_t^h\right)^{-\sigma_h}, \tag{7}$$

where $\lambda_{h,t}$ denotes the Lagrange multiplier on the flow budget constraint (2). Note that households with perfect insurance have the following stochastic discount factor:

$$\Lambda_{t,t+1} = \beta_h \frac{\lambda_{h,t+1}}{\lambda_{h,t}}. \tag{8}$$

We assume that households with perfect insurance own firm sector. The stochastic discount factor (8) is used in the firm sector.

3.1.2 Households with Imperfect Insurance

Remaining households do not have access to a complete set of insurance and face a borrowing constraint. Then, they cannot perfectly insure themselves against idiosyncratic risk to their skill s_t^ν. There is a measure ν of households indexed by $\nu \in [0, \bar{\nu}]$, where $\bar{\nu}$ denotes a ratio of households with imperfect insurance to those with perfect insurance. Following Krusell and Smith (1998), we assume heterogeneous discount factors $\beta_\nu \leq \beta_h$ to match the skewed wealth distribution consistent with income inequality in the data. This assumption can generate the very skewed wealth distribution in the model. As shown later, it would allow QE to obtain rooms to work through by directly affecting the wealth distribution. Following McKay and Reis (2016), we assume that each household's skill s_t^ν follows a three-point grid Markov chain: low, middle, and high skills. That is, each of them has a draw of skill determining wage as $w_t s_t^\nu$ every period. Due to this, each household in this group faces idiosyncratic income risk and then macro-level income inequality emerges in the economy.

Each household with imperfect insurance chooses consumption and labor supply to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_\nu^s \left[\frac{(c_{t+s}^\nu)^{1-\sigma_h}}{1-\sigma_h} - \psi_1 \frac{n_t^{\nu\,1+\psi_2}}{1+\psi_2} \right], \tag{9}$$

where c_t^ν denotes consumption, n_t^ν denotes labor supply, and $\beta_\nu \in (0,1)$ is the individual discount factor. Following Andrés *et al.* (2004), we assume the market segmentation in financial markets, i.e., households with imperfect insurance do not have shares in the firms or short-term bonds, and they only trade in long-term bonds b_t^ν.[1] The assumptions just introduced above would appear to be rather *ad hoc*. However, in this study, we mainly focus on how uninsurable income uncertainty affects the transmission mechanism of unconventional monetary policy. So we introduce these additional assumptions for breaking Wallace's (1981) irrelevance result of QE. Then, QE decreases risk premiums, as the recent empirical studies indicate (see for example, Gagnon *et al.*, 2010; Krishnamurthy and Vissig-Jorgensson, 2011). Additionally, this form of market segmentation excludes portfolio reallocation channel for households with imperfect insurance and allows us to directly examine how QE affects their fear of hitting the borrowing constraint through increasing household wealth.

The flow budget constraint and the borrowing constraint are given by

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[1] Mankiw and Zeldes (1991) reported the presence of limited participation in the sense that most households do not have any stocks in the U.S. McKay and Reis (2016) also restricted some households to own equity in their model.

$$Q_t^{B,G} b_t^\nu + c_t^\nu = \frac{(1+i_t^B)}{\pi_t} Q_{t-1}^{B,G} b_{t-1}^\nu + (1-\tau_t) w_t s_t^\nu n_t^\nu, \tag{10}$$

$$b_t^\nu \geq 0. \tag{11}$$

We can derive optimality conditions that are analogous to those for households with perfect insurance. Households in this group face uninsurable uncertainty, and thus, their optimal decision making considers an expectation over idiosyncratic income risk as well as over aggregate uncertainty. We apply the solution method of Reiter (2009, 2010) that allows for a nonlinear relation between individual decisions and individual states. The optimal decision rule for consumption can be derived as a nonlinear function of individual wealth. Thus, even though optimality conditions are analogous to those for households with perfect insurance, an expectation over idiosyncratic income risk can lead to heterogeneous consumption/saving decisions.

3.2. Capital Goods Producers

Capital goods producers combine depreciated capital and investment to produce new capital subject to investment adjustment costs. The new capital replaces depreciated capital and adds to capital shock. They choose the quantity of investment I_t to maximize expected profits

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t+s} \left[Q_{t+s+1}^k \{ K_{t+s+1} - (1-\delta) K_{t+s} \} - P_{t+s} I_{t+s} \right], \tag{12}$$

subject to

$$K_{t+1} = (1-\delta) K_t + \left[1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \tag{13}$$

where δ is the rate of depreciation on capital. The quadratic term implies the adjustment costs and χ is a relevant parameter. After transformation, the first-order condition becomes

$$
\begin{aligned}
1 = Q_t^k &\left[1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \chi\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right] \\
&+ \mathbb{E}_t\left[\Lambda_{t+1} Q_{t+1}^k \chi\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right].
\end{aligned}
\tag{14}
$$

3.3. Intermediate Goods Producers

There is a unit continuum of intermediate goods producers, each combining rented capital $k_t(j)$ and hired labor $l_t(j)$ to produce variety j intermediate goods $y_t(j)$ using a standard Cobb-Douglass technology

$$
y_t(j) = a_t k_t(j)^{\alpha} l_t(j)^{1-\alpha}, \tag{15}
$$

where a_t denotes total factor productivity (aggregate productivity shock). Intermediate goods producers set prices subject to nominal price rigidities *á la* Calvo (1983), with probability of price revision θ. We assume that households with perfect insurance own intermediate goods producers. Intermediate goods producers use the stochastic discount factor of the household with perfect insurance $\Lambda_{t,t+s}$ to choose optimal price level $p_t(j)^*$ at a revision date.

Thus, the intermediate goods producer chooses $p_t{}^*$ and $\{y_s(j), k_s(j), l_s(j)\}_{s=t}^{\infty}$ to maximize

$$
\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}(1-\theta)^{t+s-t}\left[\frac{p_t{}^*}{p_{t+s}} y_{t+s}(j) - w_{t+s} l_{t+s}(j) - (r_{t+s} + \delta) k_{t+s}(j) - \Psi\right], \tag{16}
$$

subject to

$$
y_{t+s}(j) = \left(\frac{p_t{}^*}{p_{t+s}}\right)^{\mu/1-\mu} y_{t+s}, \tag{17}
$$

$$
y_{t+s}(j) = a_{t+s} k(j)_{t+s}^{\alpha} l(j)_{t+s}^{1-\alpha}, \tag{18}
$$

where μ is the desired markup. In equation (16), Ψ denotes a fixed cost. The optimality conditions are given by

$$(r_t + \delta) = M_t \alpha a_t k(j)_t^{\alpha-1} l(j)_t^{1-\alpha}, \tag{19}$$

$$w_t = M_t(1-\alpha) a_t k(j)_t^{\alpha} l(j)_t^{-\alpha}, \tag{20}$$

$$\begin{aligned} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}(1-\theta)^{t+s-t} \left[\frac{\mu}{1-\mu} \left(\frac{p_t{}^*}{p_{t+s}} \right)^{1/(1-\mu)-1} \frac{y_{t+s}}{p_{t+s}} \right] \\ = \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}(1-\theta)^{t+s-t} \left[M_{t+s} \frac{\mu}{1-\mu} \left(\frac{p_t{}^*}{p_{t+s}} \right)^{\mu/(1-\mu)-1} \frac{y_{t+s}}{p_{t+s}} \right], \end{aligned} \tag{21}$$

where M_{t+s} is the Lagrange multiplier and

$$\frac{p_t{}^*}{p_t} = \frac{p_t \mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s}(1-\theta)^{t+s-t} M_{t+s} \mu \left(\frac{p_t}{p_{t+s}} \right)^{\mu/(1-\mu)-1} \frac{y_{t+s}}{p_{t+s}} \right]}{p_t \mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s}(1-\theta)^{t+s-t} \left(\frac{p_t}{p_{t+s}} \right)^{\mu/(1-\mu)-1} \frac{y_{t+s}}{p_{t+s}} \right]}.$$

The price index is

$$p_t = \left(\int_0^1 p_t(j)^{1/1-\mu} dj \right)^{1-\mu}.$$

Using the law of motion for an aggregate price index, we have

$$p_t = \left((1-\theta) \int_0^1 p_t(j)^{1/1-\mu} dj + \theta (p_t{}^*)^{1/1-\mu} \right)^{1-\mu}.$$

3.4. Final Goods Producers

A representative competitive final goods producer combines intermediated goods $y_t(j)$ to produce homogeneous final goods y_t according to the following production technology

$$y_t = \left(\int_0^1 y_t(j)^{1/\mu} dj \right)^{\mu}. \tag{22}$$

Given the final goods price p_t, cost minimization together with zero profits imply that

$$y_t(j) = \left(\frac{p_t(j)}{p_t}\right)^{\mu/1-\mu} y_t, \tag{23}$$

$$p_t = \left(\int_0^1 p_t(j)^{1/1-\mu} dj\right)^{1-\mu}. \tag{24}$$

3.5. Entrepreneurs and Financial Intermediaries

The entrepreneurial sector is populated by a large number of entrepreneurs. We denote an entrepreneur with net worth N as an N-type entrepreneur. The density of N-type entrepreneurs is given by $f_t(N)$, and the aggregate net worth is $N_t = \int_0^\infty N f_t(N) dN$.

Each entrepreneur buys a specific amount of physical capital from capital producers and rents it out to intermediate goods producers for a competitive market rental rate denoted by r_{t+1}^k. To finance the capital used in production, each entrepreneur obtains a loan $B_{N,t+1}^E$. Following Bernanke *et al.* (1999) and Christiano *et al.* (2014), in the financial contract, we assume asymmetric information between borrowers and lenders and costly monitoring. After purchasing capital $K_{N,t+1}$, each entrepreneur faces an idiosyncratic productivity shock ω_t, which is independent and identically distributed (i.i.d.) across agents and periods, following a log normal distribution with a mean of unity. These shocks affect entrepreneurs' individual productivity, which determines the effective stock of capital with which the entrepreneur can work. Thus, N-type entrepreneurs earn the rate of return on capital $\omega_t R_{t+1}^k$, where

$$1 + R_{t+1}^k = \mathbb{E}_t \left[\frac{\pi_{t+1}\left\{(1-\delta)Q_{t+1}^k + r_{t+1}^k\right\}}{Q_t^k}\right].$$

To prevent households from having to bear the costs of defaulting, households transfer their savings consequently to financial intermediaries that hold a fully diversified portfolio and intermediate them to the entrepreneurs. Here, let us denote Z_{t+1} as the gross interest rate on debt and denote $\bar{\omega}_{N,t+1}$ the value of ω below which default occurs. Then, we have the following relation:

$$\overline{\omega}_{N,t+1}\left(1+R_{t+1}^{k}\right)Q_t^k K_{N,t+1}=Z_{t+1}B_{N,t+1}^{E}.$$

We assume that the financial intermediaries can invest in entrepreneurial loans or long-term government bonds b_{t+1}^{FI}.[2] Each entrepreneur chooses $K_{N,t+1}$ and $\overline{\omega}_{N,t+1}$ to maximize

$$\mathbb{E}_t\left\{\left\{1-\Gamma_t\left(\overline{\omega}_{N,t+1}\right)\right\}\left(1+R_{t+1}^{k}\right)Q_t^k K_{N,t+1}\right\}, \tag{25}$$

subject to the participation constraint of the financial intermediaries

$$\begin{aligned}\mathbb{E}_t&\left[\left\{1-F\left(\overline{\omega}_{N,t+1}\right)\right\}Z_{t+1}B_{N,t+1}^{E}+(1-\mu^e)\int_0^{\overline{\omega}_{N,t+1}}\omega\left(1+R_{t+1}^{k}\right)Q_t^k K_{N,t+1}\,dF(\omega)\right]\\ &\geq \mathbb{E}_t\left[(1+i_{t+1}^{B})Q_t^B b_{t+1}^{FI}\right],\end{aligned} \tag{26}$$

with

$$\Gamma_t\left(\overline{\omega}_{N,t+1}\right)=\left\{1-F\left(\overline{\omega}_{N,t+1}\right)\right\}\overline{\omega}_{N,t+1}+G_t\left(\overline{\omega}_{N,t+1}\right),$$

$$G_t\left(\overline{\omega}_{N,t+1}\right)=\int_0^{\overline{\omega}_{N,t+1}}\omega\,dF_t(\omega),$$

where $1-\Gamma_t\left(\overline{\omega}_{N,t+1}\right)$ denotes the share of average entrepreneurial earnings received by entrepreneurs, $F\left(\overline{\omega}_{N,t+1}\right)$ denotes a probability of default, and μ^e denotes a monitoring cost.

The optimality condition is given by

$$\begin{aligned}\mathbb{E}_t&\left\{\left\{1-\Gamma_t\left(\overline{\omega}_{N,t+1}\right)\right\}\frac{1+R_{t+1}^{k}}{1+i_{t+1}^{B}}\right.\\ &+\frac{\Gamma_t'\left(\overline{\omega}_{N,t+1}\right)}{\Gamma_t'\left(\overline{\omega}_{N,t+1}\right)-\mu'\left(\overline{\omega}_{N,t+1}\right)}\left[\frac{1+\mathrm{R}_{t+1}^{k}}{1+i_{t+1}^{B}}\left\{\Gamma_t\left(\overline{\omega}_{N,t+1}\right)\right.\right.\\ &\left.\left.\left.-\mu^e G_t\left(\overline{\omega}_{N,t+1}\right)\right\}-1\right]\right\}=0.\end{aligned} \tag{27}$$

---

[2] In equilibrium, the financial intermediaries invest only in entrepreneurial loans because the participation constraint of the financial intermediaries binds.

Using optimality condition (27), we can derive equilibrium external finance premium, which depends on the size of entrepreneurial net worth (or alternatively, the entrepreneurial leverage ratio). Aggregate capital purchased by all entrepreneurs in period t is

$$K_{t+1} = \int_0^\infty K_{t+1}^N f_t(N) dN.$$

Following Christiano *et al.* (2014), we assume that each entrepreneur keeps only a fraction γ_t of their wealth and the remaining fraction $1-\gamma_t$ is transferred to households. At the same time, each entrepreneur receives a transfer W_t^e from households. Then, aggregate entrepreneurial net worth is given by

$$N_{t+1} = \frac{\gamma_t}{\pi_t}\{1 - \Gamma_{t-1}(\bar{\omega}_{N,t})\}(1 + R_{t+1}^k) Q_t^k K_{t-1} + W_t^e.$$

Christiano *et al.* (2014) indicates that cyclical variation in the cross-sectional standard deviation of firm-level stock returns affects firms' optimal decision making. Following Christiano *et al.* (2014), we compute the measure of uncertainty as the standard deviation of the nonbankrupt entrepreneurial return $R_t^e(\omega_t)$ on equity in a cross-section. Therefore, we define the following form of the measure of uncertainty:

$$std\left(R_t^e(\omega_t) \middle| \omega_t > \bar{\omega}_{N,t}\right) = (1 + R_t^k) Lev_{t-1} \sqrt{var\left(\omega_t - \bar{\omega}_{N,t} \middle| \omega_t > \bar{\omega}_{N,t}\right)},$$

where

$$R_t^e(\omega_t) = max\{0, [\omega_t - \bar{\omega}_{N,t}] R_t^k L_{t-1}\},$$

$$Lev_t = \frac{Q_t^k K_{t+1}}{N_t}.$$

Here, Lev_t denotes the leverage ratio of *N*-type entrepreneurs. Christiano *et al.* (2014) reports that greater uncertainty decreases investment.[3]

[3] We can rewrite the measure of firm-level uncertainty as an increasing function of the probability of default, which implies that an adverse shock can increase the firm-level uncertainty through raising entrepreneurial leverage ratio by impairing entrepreneurial net worth.

3.6. Central Bank

The central bank has two policy measures: QE and FG. QE is designed to be a policy measure that affects both the real economy and inflation by decreasing risk premiums. FG works by affecting the expected future path of short-term interest rates.

3.6.1 Quantitative Easing

We assume that the central bank purchases quantities of long-term bonds following a simple autoregressive rule:

$$Q_t^B b_t^{CB} = \rho_{QE} Q_{t-1}^B b_{t-1}^{CB} + \varepsilon_t^{QE}, \tag{28}$$

where $\rho_{QE} \in (0,1)$ is an autoregressive parameter and b_t^{CB} denotes the long-term bond held by the central bank through the asset purchase program, and ε_t^{QE} is an exogenous shock (QE shock). The long-term bond purchase program is financed by interest-bearing reserves that pay the risk-free interest rate i_t.[4] The balance sheet of the central bank is given by

$$Q_t^B b_t^{CB} = D_t. \tag{29}$$

The central bank designs a whole asset purchase program by determining the amount of each period's asset purchases. As our model setup breaks Wallace's (1981) irrelevance result, QE affects the real economy and inflation through a reduction in risk premiums.

As we will discuss later, under the assumptions of complete market and perfect foresight, households foreseeing the future path of asset purchases can perfectly smooth their consumption through an intertemporal substitution channel. Thus, total amount of asset purchases only matters. However, in our incomplete market model, even under the perfect foresight of QE, the policy design (e.g., the pace of asset purchase) also matters because households facing uninsurable income uncertainty and

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[4] We assume that interest-bearing reserves and short-term bonds are perfect substitutes. It then implies that whether QE is financed by interest-bearing reserves or by selling the short-term bonds does not matter.

the borrowing constraint cannot flexibly smooth their consumption due to a fear of hitting the budget constraint in the future. Policy effects can differ depending on how QE affects the future paths of their wealth and wage income because these determine their consumption/saving choices.

3.6.2 Forward Guidance and Deflationary Shock

The central bank influences the future path of the long-term real interest rate using FG. Following Eggertsson and Woodford (2003), the central bank pursues an inflation target, which is interpreted as a commitment to achieve:

$$\pi_t = \pi^*, \tag{30}$$

where π^* denotes a target inflation rate. For simplicity, we assume $\pi^* = 0$ in this paper.

Since we mainly focus on the situation that the economy faces the liquidity trap, we assume that the nominal interest rate hits the zero lower bound, i.e., $i_t = 0$, where $(1 + i_t) = (1 + r_t)/(1 + \mathbb{E}_t\pi_{t+1})$. The liquidity trap emerges if a certain type of adverse shock hits the economy in which the zero lower bound is binding. In this paper, following Eggertsson and Woodford (2003), we introduce a temporary shock that depresses the natural rate of interest, which causes a deflationary recession in the liquidity trap, to examine how the unconventional monetary policy measures work under the situation.

Following McKay *et al*. (2015), we assume that the interest rate rule is given by an exogenous rule for the real interest rate that tracks the natural rate of interest r_t^n with some error:

$$r_t = i_t - \mathbb{E}_t\pi_{t+1} = r_t^n + \varepsilon_{t-s}^{FG}, \tag{31}$$

where ε_{t-s}^{FG} is FG shock, which is an s-period ahead shock to the real interest rate. Note that the FG shock is assumed to be known in period $t - s$. We can interpret FG shock as the central bank's commitment to keep the interest rate at low level for an extended period. We assume that the natural rate of interest follows

$$r_t^n = r_{ss}^n + \varepsilon_t^D, \tag{32}$$

where r_{ss}^n denotes the steady-state level of the natural rate of interest and ε_t^D denotes a deflationary shock. The real interest rate gap is defined as

$$\hat{r}_t = r_t - r_t^n,$$

A positive value of $\hat{r}_t$ represents a monetary tightening. When the zero lower bound is binding, the central bank cannot reduce $\hat{r}_t$ to zero by adjusting the nominal interest rate. Accordingly, the deflationary shock under the zero lower bound has a tightening effect on the economy, leading to a deflationary recession in the liquidity trap.

3.7. Government

In each period, the government issues a government bond B_t. The government bond market clears if

$$B_t = \int_0^{\overline{\nu}} b_t(\nu) d\nu + b_t^h + b_t^{CB}. \tag{33}$$

The nominal interest rate on the government bond is given by

$$1 + i_t^B = \frac{\pi_t\left(1 + \rho Q_t^{B,G}\right)}{Q_{t-1}^{B,G}},$$

where ρ denotes a maturity parameter.[5] Every period, a fraction $1 - \rho$ of bonds matures, whereas the remaining fraction ρ is repaid in later periods. The government levies a proportional labor income tax to finance interest payments on this bond. Thus, the government faces a flow budget constraint

$$\frac{(1 + i_t^B)}{\pi_t} Q_{t-1}^{B,G} B_{t-1} = Q_t^{B,G} B_t + \tau_t w_t \left[\int_0^{\overline{\nu}} s_t(\nu) n_t(\nu) d\nu + \bar{s} n_t\right] + T_t^{CB}, \tag{34}$$

where T_t^{CB} denotes a transfer from the central bank.

[5] Following Andrés *et al.* (2004), we assume that there is no secondary market for long-term government bonds and thus anyone cannot obtain a capital gain or loss through trading in existing long-term bonds.

3.8. Shocks

Four types of aggregate shocks hit the economy: aggregate productivity shock a_t, QE shock ε_t^{QE}, FG shock ε_{t+s}^{FG}, and deflationary shock ε_t^{D}. We assume that these aggregate shocks follow AR(1) processes. In addition, each household faces a stochastic idiosyncratic skill s_t^{ν}. Following McKay and Reis (2016), we assume that each household's skill s_t^{ν} follows a three-point grid Markov chain with the structure

$$\begin{array}{c} L \\ M \\ H \end{array}\begin{pmatrix} 1-p & p & 0 \\ p & 1-2p & p \\ 0 & p & 1-p \end{pmatrix},$$

where $p \in [0,1]$ and L, M, H denote low, middle, and high skill realizations, respectively. Since we assume a small value of p, heterogeneous income profiles among households emerge in a persistent way.

3.9. Equilibrium and Resource Constraint

In equilibrium, all agents maximize their objective functions subject to their constraints such that:

(a) households with perfect insurance maximize (1) subject to (2);

(b) households with imperfect insurance maximize (9) subject to (10) and (11);

(c) capital goods firms maximize (12) subject to (13);

(d) intermediate goods firms maximize (16) subject to (17) and (18);

(e) final goods firms follow (22), (23), and (24);

(f) entrepreneurs maximize (25) subject to (26);

(g) central bank conducts QE and FG following (28) and from (30) to (32);

(h) government behaves according to (34).

The resource constraint is given by

$$Y_t = C_t + I_t + \mu^e F(\bar{\omega}_{t+1}) \frac{\left(1+R_t^k\right) Q_{t-1}^k K_t}{\pi_t},$$

where aggregate consumption C_t is given by

$$C_t = \int_0^{\bar{\nu}} c_t(\nu) d\nu + c_t^h,$$

The labor market clears if

$$L_t = \int_0^1 l_t(j) dj = \int_0^{\bar{\nu}} s_t(\nu) n_t(\nu) d\nu + \bar{s} n_t.$$

The market clearing condition for government bonds is given by (33).

4. Model Analysis

In this section, we present some numerical experiments to illustrate how an unconventional monetary policy works in the liquidity trap. To obtain intuition, our experiments start with cases without the liquidity trap. We first explain the calibration and the solution algorithm of our model and then turn to numerical simulations.

4.1. Calibration

Our calibration is summarized in Table 1. For the depreciation rate, the capital share, the gross markup, and Calvo price stickiness, we set standard values. Thus, we only explain several parameters. For the Frisch elasticity of labor supply parameters, we choose the calibrated values of Chetty (2012). Following McKay and Reis (2016), we set a discount factor for the households to match the fact that 83.4% of wealth is held by the top 20% in the U.S., which is shown in Diaz-Gimenez *et al.* (2011). Household skill is described using a three-point grid Markov chain, which is the same as that in McKay and Reis (2016). Calibration for entrepreneurs and financial intermediaries corresponds to those in Christiano *et al.* (2014) and Bernanke *et al.* (1999). Following Gertler and Karadi (2013), transaction costs of financial assets are set to 1.00. Calibration of the skill process corresponds to that in McKay *et al.* (2015), which calibrates the skill process using Panel Study of Income Dynamics data with a three-point grid Markov chain.

4.2. Computation

We compute our incomplete market model and simulate various policy experiments by employing a solution algorithm proposed by Reiter (2009, 2010).

To solve the incomplete market model, a Krusell and Smith (1998)-type solution algorithm is one possible approach. They extend earlier incomplete market models, e.g., those of Hugget (1993) and Aiyagari (1994), to examine how an aggregate shock affects the economy. However, Den Haan (2010) reports that the algorithm of Reiter (2009) is faster and more accurate than other algorithms such as that of Krussel and Smith (1998) as well as Den Haan and Rendahl (2010).

Reiter (2009, 2010) proposes the algorithm to approximate wealth distribution using a histogram with a large number of bins. The mass of households in each bin works as a state variable. This algorithm enables us to more flexibly deal with various forms of wealth distribution.

4.3. Simulating Unconventional Monetary Policy Measures under Incomplete Market

To obtain intuition, we begin with several simulations designed to show how unconventional monetary policy measures work in our model without the liquidity trap. We consider the impulse responses of key variables to the unconventional monetary policy measures: QE and FG. We define QE as purchases of long-term government bonds, which work through a reduction in risk premiums. We assume that the government issues government bonds so that the average maturity is 10 years; thus, QE is designed to buy 10-year bonds. FG denotes the central bank's commitment to a 50-basis-point decrease in the real interest rate for a single quarter two years ahead.

We first compare the responses of key variables to QE and FG. We then simulate the responses to QE with a greater degree of income uncertainty and, for comparison, those with a lesser degree of income uncertainty. In addition, we compare the QE with slower-paced bond purchases. We calculate the precautionary savings as a difference between the bond holdings of households with imperfect insurance and those if they have access to a complete set of insurance.

Figure 1 illustrates the responses of key variables to QE and FG. The total size of QE through the long-term bond purchases is set to match its initial effect on inflation with that of FG. The solid lines report the responses to QE, and the dotted lines illustrate responses to FG.

Figure 1 shows that QE can work more effectively than FG by decreasing concern for households to run down their wealth in the presence of uninsurable income uncertainty. Both QE and FG increase the output and inflation rate. However, while FG leads to future decreases in both output and inflation, QE does not cause such negative effects on the economy.

FG increases precautionary savings. As McKay *et al.* (2015) discuss, under the incomplete market model, FG redistributes wealth away from households with lower labor skill realization to households with higher labor skill realization.[6] Due to this, output decreases through lowering aggregate demand until the wealth distribution converges back to steady state. Households expect that wage income will decrease in the future. Since they cannot fully insure themselves against such risks, the resulting heterogeneous income profiles pose concern for households to run down their wealth and they then prefer saving to consumption.

In contrast, QE decreases the precautionary savings under the above setup of QE. QE has immediate effects to increase household wealth by raising asset prices, which decrease precautionary saving motives through lowering the probability of hitting the borrowing constraint in the future. The decreasing precautionary savings weakens the negative effects on intertemporal substitution. Such contrast leads to differences in macroeconomic responses to policy implementation.

Figure 2 compares the responses of our model to QE with greater income uncertainty and those with lesser income uncertainty. We define the degree of income uncertainty as a fraction of households that is not perfectly self-insured. Total amount of asset purchase is the same as above. The solid lines illustrate the responses to QE with greater income uncertainty. The dotted lines indicate the responses to equivalent amount of asset purchase with lesser income uncertainty. There are the following two observations.

[6] Since households with higher skill have lower marginal propensities to consume, aggregate demand decreases until the wealth distribution converges back to the steady state. This decreases wage income growth. Then, uninsurable income uncertainty weakens the transmission of FG through an increase in precautionary savings.

First, the effectiveness of QE with greater income uncertainty increases more than that with lesser income uncertainty, in the sense that QE in the former case yields larger increases in both inflation and output. This policy measure increases both investment and labor demand by lowering financing cost; thus, wage inflation pressure arises. It would attenuate precautionary savings, and the inflation rate would rise through an increase in consumption. Stronger policy effect under greater income uncertainty seems unintuitive at first, but can be interpreted as follows. As the degree of income uncertainty is greater, the resulting heterogeneous income profiles pose more concern to run down their wealth since households face greater fear of hitting the borrowing constraint in the future. This implies that with greater income uncertainty, QE has more room to be able to decrease the precautionary savings by alleviating those fears.[7]

Second, QE decreases firm-level uncertainty more significantly under greater income uncertainty, through decreasing entrepreneurial leverage ratio by both raising asset prices and stimulating aggregate demand. This indicates that QE can obtain more room to mitigate firm-level uncertainty as the economy faces greater income uncertainty. Greater income uncertainty creates larger firm-level uncertainty because adverse effects of wealth redistribution endogenously arise. Entrepreneurial net worth decreases because the entrepreneurial income falls with decrease in economic activity, which is mainly caused by the strengthened precautionary saving motives. It follows that the standard deviation of the entrepreneurial return on equity increases. As Christiano *et al.* (2014) reports, it gives firms an incentive to decrease investment.

We now examine how the policy design of QE affects its transmission mechanism in our model. In this simulation, we particularly focus on the pace of asset purchase.[8] In Figure 3, the panels are the responses to a long-term bond purchase program. We consider two cases for comparison. Although total amount of asset purchase is the

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[7] As the degree of income uncertainty becomes larger, the generated wealth distribution becomes more skewed, which would amplify household concern for the resulting heterogeneous income profiles.

[8] As a matter of course, QE with larger total amount of asset purchase yields larger policy effects. In this study, we only focus on differences in purchase pace at equivalent amount of total asset purchase.

same in both cases, they differ in the pace of purchase.[9] The solid line indicates QE with a rapid pace of asset purchase, while the dotted line indicates those with a slower pace of asset purchase.

In the presence of uninsurable income uncertainty, as discussed below, the pace of asset purchase matters for the effectiveness of QE. Figure 3 illustrates that the rapid pace of purchase allows QE to yield larger increases in both output and inflation than at the slower pace of purchase. Thus accelerating purchase pace yields considerably stronger amplification of the policy effects.

In the complete market model, all households are perfectly insured. Under the perfect foresight about the central bank's behavior, only total amount of asset purchase matters. In our incomplete market model, if households have insufficient amount of wealth, they decrease current consumption to increase precautionary savings. This implies that early fulfillment of precautionary saving motives is crucial to stimulating current consumption. The immediate impact of QE would abate concern to run down their wealth. This effect would amplify the transmission mechanism of QE, thereby strengthening the positive reaction of aggregate demand to accommodative policy.

As indicated in the middle right panel of Figure 3, such amplification effects propagate to firm-level uncertainty, through decreasing entrepreneurial leverage ratio by both raising asset prices and stimulating aggregate demand. A rapid pace of asset purchase further decreases firms' concerns with respect to increasing investment.

Next we examine the relation between QE and FG. We mainly discuss that QE can restore the impaired transmission mechanism of FG. We first run three simulations: (1) the independent effects of only QE; (2) the independent effects of only FG; and (3) the combined effects of QE and FG.[10] Then, we calculate two responses: (a) the arithmetic sum of the responses in simulations (1) and (2); and (b) the responses

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9 Cumulative total purchase size is set to approximately 2.5% of GDP, which is the same as that in Gertler and Karadi (2013) that evaluate QE2 of the Federal Reserve in 2010–11. To discuss how purchase pace affects the effectiveness of QE, we consider hypothetical scenarios about the pace of asset purchase.

10 We use the same setup for QE as a rapid pace of asset purchase in Figure 3.

in simulation (3). We call them "QE + FG" and "QE × FG," respectively.

Figure 4 reports the responses to QE + FG (dotted lines) and the responses to QE × FG (solid lines). This comparison enables us to examine whether the synergy effects emerge in the combination of QE and FG. If the responses of QE × FG were larger than those of QE + FG, then the synergy effects would exist as discussed below. We find the following insight from this simulation.

QE restores the impaired transmission mechanism of FG. The recovered effectiveness of FG yields the synergy effects such that increases in both output and inflation of QE × FG are larger than that of QE + FG. The immediate impacts of QE are to ease financing conditions for firms and decrease households' fears of hitting the borrowing constraint in the future. Households can take better advantage of intertemporal substitution; thus, the effectiveness of FG recovers. Strengthened aggregate demand decreases firm-level uncertainty; in addition, a reduction in the external finance premium becomes larger and more persistent in QE × FG. Note that this synergy effect arises only if the central bank implements FG after the introduction of QE and in the presence of uninsurable income uncertainty.

We turn now to consider how QE restores the impaired effectiveness of FG. To do this, we run the same simulations as in Figure 4 for various degrees of income uncertainty. We plot the peak gaps in responses between the key variables of QE × FG and those of QE + FG in Figure 5 as the horizon of various degrees of income uncertainty.[11] As the bottom right panel indicates, case 1 (solid lines) is the case that the central bank implements QE so that purchase pace is equivalent to "Slower case" in figure 3, while case 2 (dotted lines) is the case that purchase pace of QE is slower than that of case 1. The simulation results provide the following two implications.

First, synergy effects are stronger as the degree of income uncertainty increases. With greater income uncertainty, the economy faces more potential concern for

[11] We define the synergy effects on variables as the gap between a response to QE × FG and a response to QE + FG. Figure 5 plots these gaps when the absolute values of the responses are maximized as the horizon of various degrees of income uncertainty.

households to run down their wealth and for firms to increase investment. Since increase in precautionary savings mitigates the effectiveness of FG but strengthens that of QE, the economy benefits from QE's immediate impact. If larger amount of precautionary savings decrease by QE, households can take better advantage of intertemporal substitution; thus, the effectiveness of FG recovers. Note that with no income uncertainty, the responses to QE + FG are identical to those to QE × FG.

Second, the synergy effects of this policy mix depend on the pace of asset purchase. That is, if the central bank sufficiently accelerates the pace of asset purchase, then this synergy effect works well. However, if QE is pursued through a gradual pace of asset purchase, then the effectiveness of FG only modestly recovers, which in turn leads to a limited synergy effects.

4.4. Simulating Deflationary Shock with Uninsurable Income Uncertainty

In this subsection, we examine how uninsurable income uncertainty amplifies the adverse effect of the deflationary shock, which is a shock to the natural rate of interest that makes the economy hit the zero lower bound on nominal interest rates and causes a deflationary recession in the liquidity trap.

Figure 6 reports the responses to the deflationary shock. The solid lines indicate the responses to the shock under greater income uncertainty, and the dotted lines depict responses to the shock under lesser income uncertainty.

Figure 6 indicates that under greater income uncertainty, the deflationary shock causes a severe liquidity trap compared with the case of lesser income uncertainty. Precautionary savings accumulate further under greater income uncertainty. The fall in the natural rate of interest decreases corporate earnings by increasing financing cost. Then, households accumulate more precautionary savings in preparation for a future decrease in wage income.[12] Firms decrease investment because firm-level uncertainty

[12] Persistent downward pressure on wage income endogenously arises and creates severer concern for the resulting heterogeneous income profile. This would be able to strengthen the adverse effects of interaction between income dynamics and inequalities in wealth and consumption.

rises, which reflects an increase in firms' concern to suffer from both a future surge in financing cost and a future decrease in aggregate demand. Prolonged deflationary pressure then arises since aggregate demand continues to decrease for at least as long as the underlying deflationary shock works.

4.5. Simulating Unconventional Monetary Policy Measures in the Liquidity Trap

We now examine how the unconventional monetary policy measures work to escape from the liquidity trap. We begin with the responses of the key variable to FG: the commitment to a 50-basis-point decrease in the real interest rate for a single quarter two years ahead. We next simulate the responses to QE with various paces of asset purchase. In addition, we examine how the combination of QE and FG yields the synergy effects under the deep liquidity trap.

Figure 7 shows the responses of macroeconomic variables to FG under the liquidity trap. The solid lines denote responses to FG, and the dotted lines denote responses to the case of no policy reaction.

Top panels show that FG increases both output and inflation only modestly. The deflationary shock amplifies households' expectation that wage income will decrease, which poses severe concern for households to run down their wealth. Households give up taking full advantage of the intertemporal substitution toward current consumption. Then, FG loses its effectiveness to escape from the liquidity trap.

In Figure 8, we report the peak responses to FG under the liquidity trap as the horizon of the initial impact of deflationary shock on the natural rate of interest. The solid lines denote the peak responses with FG, and the dotted lines denote the peak responses with no policy reaction.

Figure 8 indicates that FG is not effective enough to escape from the liquidity trap. The dotted lines indicate that even if the size of deflationary shock becomes smaller, the effectiveness of FG does not recover. Even a smaller deflationary shock can increase precautionary savings and thereby impair the intertemporal substitution channel such that FG does not work well. As shown in the bottom right panel, this propagates to firm-level uncertainty and induces concern to increase investment as a response to FG.

Figure 9 reports the responses to QE under the liquidity trap. We consider two cases that are identical to the simulations in Figure 6. The solid lines depict the responses to QE with rapid purchase pace, and the dotted lines denote those to QE with slower purchase pace. The marked lines are the responses of no policy reaction. We obtain the following two findings.

First, if the liquidity trap becomes deeper, more aggressive policy responses are required. With a sufficiently rapid pace of asset purchase, QE effectively works to escape from the liquidity trap. The precautionary savings decrease to levels where households do not have a fear of hitting the borrowing constraint in the future. Firm-level uncertainty then decreases; thus, investment increases. Both output and inflation increase so that the economy succeeds in escaping from the liquidity trap. In contrast, the slower pace of QE does not decrease adequately the precautionary savings, and thus, sufficiently large increases in both output and inflation do not emerge.

Second, the central bank should conduct QE soon after the deflationary pressure arises. The underlying transmission mechanism of the deflationary shock continues to decrease the wage income, which strengthens the negative reaction of both households and firms. After the deflationary shock considerably decreases wage income and their own wealth, households further accumulate precautionary savings, even if the central bank conducts QE. This results in greater firm-level uncertainty and decreased investment.

In Figure 10, we plot the peak responses of key variables to QE as the horizon of pace of asset purchase. In this simulation, we assume that the total amount of asset purchases remain unchanged.

Figure 10 shows that an insufficient pace of asset purchases fails to escape from the liquidity trap even if the central bank sets a huge amount of asset purchases. While deflationary shock decreases wage income, QE immediately increases wealth. However, the slower pace of QE insufficiently reduces precautionary savings because wage income decreases at a pace that exceeds QE's effect of increasing wealth. Then, households do not benefit from the effects of QE; instead, households continue to increase the buffer stock of precautionary savings. The slower pace of QE also does not decrease firm-level uncertainty; investment remains at low levels.

Figure 11 reports responses of key variables to the combination of FG and QE. The solid lines indicate the responses to the combination of FG and QE, and the dotted lines depict the responses to only QE. The marked lines indicate the responses of no policy reaction. The pace and total amount of asset purchases are to the same as those of the simulations in Figure 10. FG is defined as the same commitment to the simulations in Figure 7.

The combination of QE and FG strengthens power to escape from the liquidity trap by restoring the impaired transmission mechanism of FG. The immediate impact of QE is to increase household wealth and wage income. Then, this effect allows households to take better advantage of intertemporal substitution. The effectiveness of FG then recovers to yield a larger synergy effects on real economic activity and inflation, thus allowing the economy to escape more easily from the liquidity trap. The important point here is that the combination of QE and FG would allow us to save total amount of asset purchases necessary to succeed in escaping from the liquidity trap.

5. Concluding Remarks

In this study, we develop a general equilibrium model with uninsurable income uncertainty and a borrowing constraint. We then examine what unconventional monetary policy measures are effective in escaping from the liquidity trap. The model analyses produced the following three major findings.

First, under uninsurable income uncertainty, decreasing risk premiums by QE is more effective than FG in the liquidity trap. The rising precautionary saving motives in the liquidity trap reduce the effectiveness of FG. Meanwhile, QE can mitigate precautionary savings and, thus, weaken the adverse effects of the liquidity trap.

Second, the central bank should engage in more aggressive policy responses to escape from the deep liquidity trap. With uninsurable income uncertainty, accelerating the pace of asset purchases would foster a recovery by eliminating the heightened fear of hitting the borrowing constraint and the heightened firm-level uncertainty. In contrast, QE with a slower pace of purchase cannot sufficiently decrease such uncertainty, leading to a more persistent liquidity trap.

Third, even if severe deflationary pressure exists, QE with a sufficiently rapid pace of asset purchases can restore the effectiveness of FG through a decrease in precautionary savings. The synergy effects resulting from the combination of QE and FG strengthen the power for escaping from the liquidity trap.

Note that in this paper, the central bank is assumed to be perfectly credible in achieving its policy goal. However, if we consider the case of imperfect commitment, the degree of central bank credibility would also matter because it critically affects the transmission mechanism of monetary policy measures. In this case, the interplay between QE and FG can be more complicated because an immediate action of QE may affect the degree of credibility, which is likely to change the effectiveness of FG. In addition, our model does not consider uncertainty about monetary transmission mechanism, which may affect the desirable policy combination in the liquidity trap. Future work can take up these issues.

References

Aiyagari, S. Rao (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving,"*Quarterly Journal of Economics*, Vol.109, pp.659-684.

Andrés, Javier J., David López-Salido, and Edward Nelson (2004), "Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium," *Journal of Money, Credit and Banking*, Vol.36(4), pp.665-690.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," In *Handbook of Macroeconomics*, Vol. 1C, edited by John B. Taylor, and Michael Woodford, pp.1341-1393. Amsterdam: North-Holland.

Calvo, Guillermo A. (1983), "Staggered Prices in a Utility-maximizing Framework," *Journal of Monetary Economics*, Vol.12(3), pp.383-398.

Chen, Han, Vasco Cúrdia, and Andrea Ferrero (2012), "The Macroeconomic Effects of Large-scale Asset Purchase Programmes" *Economic Journal*, Vol.122, pp.289-315.

Chetty, Raj (2012), "Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply," *Econometrica,* Vol.80(3), pp.969-1018.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy,* Vol. 113(1), pp.1-45.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno (2014), "Risk Shocks," *American Economic Review,* Vol.104(1), pp.27-65.

Cúrdia, Vasco, and Michael Woodford (2011), "The Central-Bank Balance Sheet as an Instrument of Monetary Policy," *Journal of Monetary Economics*, Vol.58(1), pp. 54-79.

Del Negro, Marco, Marc Giannoni, and Christina Patterson (2015), "The Forward Guidance Puzzle," Federal Reserve Bank of New York Staff Reports, No.574.

Den Haan, Wouter J. (2010), "Comparison of Solutions to the Incomplete Market Model with Aggregate Uncertainty," *Journal of Economic Dynamics and Control*, Vol.34 (1), pp. 4-27.

Den Haan, Wouter J., and Pontus Rendahl (2010), "Solving the incomplete markets model with aggregate uncertainty using explicit aggregation," *Journal of Economic*

Dynamics and Control, Vol.34 (1), pp.69-78.

Díaz-Giménez, Javier, Andy Glover, and José-Víctor Ríos-Rull (2011), "Facts on the distributions of earnings, income, and wealth in the United States: 2007 update," *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol.34(1), pp.2-31.

Eggertsson, Gauti B., and Michael Woodford (2003), "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, No.1, pp. 139-211.

Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack (2010), "Large-scale Asset Purchases by the Federal Reserve: Did They Work?," Federal Reserve Bank of New York Staff Reports, No.441.

Gertler, Mark, and Peter Karadi (2011), "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, Vol.58(1), pp.17-34.

Gertler, Mark, and Peter Karadi (2013), "QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool," *International Journal of Central Banking*, Vol.9(1), pp.5-53.

Huggett, Mark (1993), "The Risk-Free Rate in Heterogeneous-Agent Incomplete Insurance Economies," *Journal of Economic Dynamics and Control*, Vol.17(5), pp.953-969.

Kaplan Greg, Benjamin Moll, and Giovanni L. Violante (2016), "Monetary Policy According to HANK," NBER Working Paper Series, No.21897.

Krishnamurthy, Arvind, and Annete Vissing-Jorgensen (2011), "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, No.2, pp.215-265.

Krugman, Paul (1998), "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, No.2, pp.137-205.

Krusell, Per, and Anthony A. Smith Jr. (1998), "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, Vol. 106(5), pp.867-896.

Mankiw, N. Gregory, and Stephan P. Zeldes (1991), "The Consumption of Stockholders and Non-stockholders," *Journal of Financial Economics*, Vol.29(1), pp.97-112.

McKay, Alisdair, and Ricardo Reis (2016), "The Role of Automatic Stabilizers in the U.S.

Business Cycle," *Econometrica*, Vol.84(1), pp.141-194.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson (2015), "The Power of Forward Guidance Revisited," *American Economic Review*, Vol.106(10), pp.3133-3158.

Reiter, Michael (2009), "Solving Heterogeneous-agent Models by Projection and Perturbation," *Journal of Economic Dynamics & Control*, Vol.33(3), pp.649-665.

Reiter, Michael (2010), "Approximate and Almost-exact Aggregation in Dynamic Stochastic Heterogeneous-agent Models," Institute for Advanced Studies, Economic Series 258.

Smets, Frank, and Rafael Wouters (2007), "Shocks and Frictions in U.S. Business Cycles; a Bayesian DSGE Approach," *American Economic Review*, Vol.97(3), pp.586-606.

Wallace, Neil (1981), "A Modigliani-Miller Theorem for Open-Market Operations," *American Economic Review*, Vol.71(3), pp.267-274.

Woodford, Michael (2001), "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, Vol.33(3), pp.669-728.

Table 1. Calibration of the Parameters

Symbol	Parameter	Value
β_h	Discount factor of perfect insurance households	0.989
α	Capital share	0.232
μ	Desired gross markup	1.1
$\bar{\nu}$	Imperfect ins. households per perfect ins. households	0.9
β_v	Discount factor of imperfect ins. households	0.979
$\bar{s}$	Skill level of perfect ins. households	3.72
θ	Calvo price stickiness	0.286
ψ_1	Relative willingness to work	21.6
ψ_2	Frisch elasticity of labor supply	2
δ	Capital depreciation rate	0.025
χ	Adjustment costs for investment	12
σ	Risk aversion	1
Ψ	Fixed cost	0.573
σ^e	St. Dev. of entrepreneurial idiosyncratic productivity	0.26
$1-\gamma$	Fraction of entrepreneurial net worth transferred to households	$1-0.985$
W^e	Transfer received by entrepreneurs	0.005
μ^e	Monitoring cost	0.12
κ	Transaction cost of financial asset	1
p	Transition probability	0.03

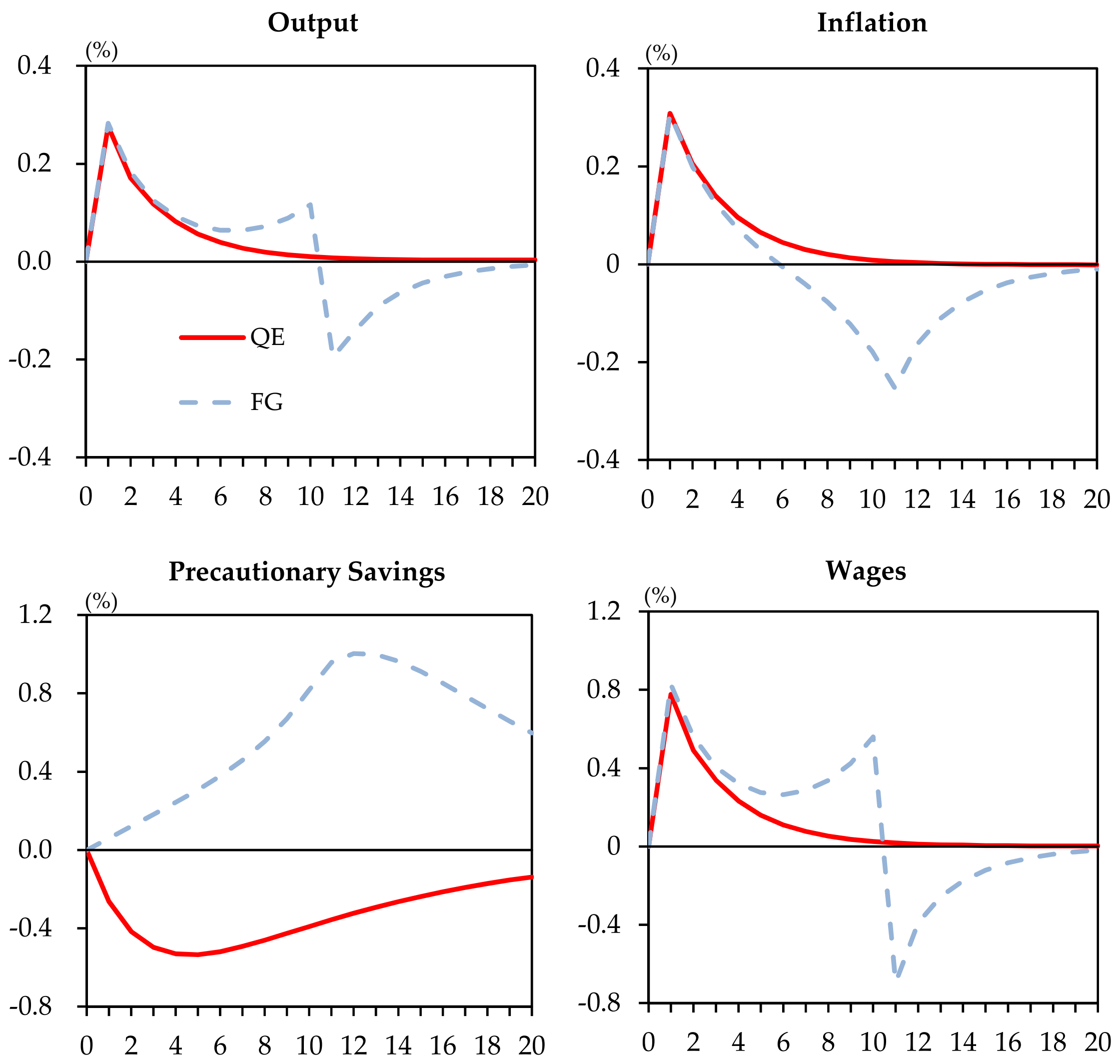

Figure 1. Responses to the long-term bond purchases (QE) and the forward guidance (FG)

Note: FG is designed as the central bank's commitment to a 50-basis-point decrease in the real interest rate for a single quarter two years ahead. QE is calibrated to match its initial effect on inflation with that of FG.

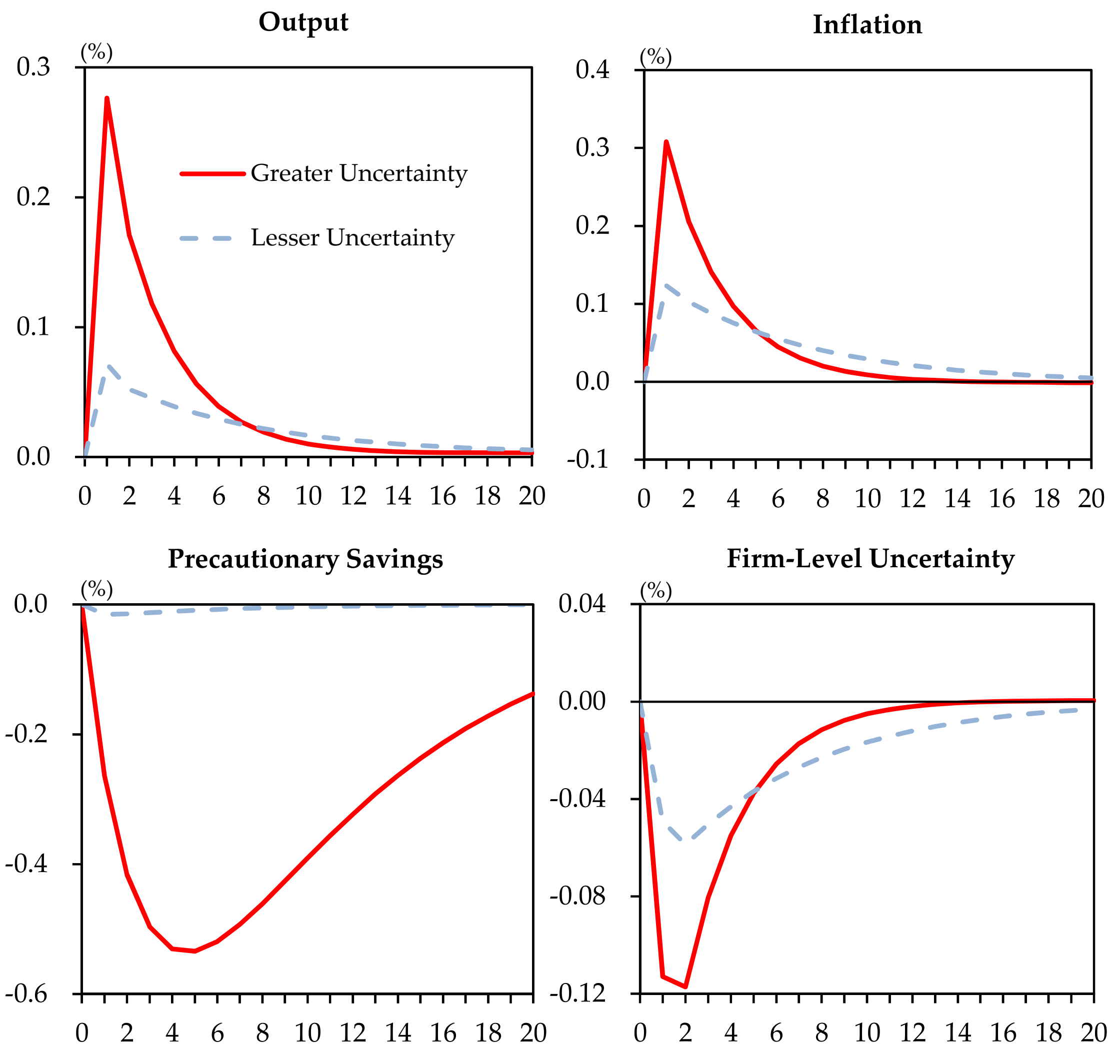

Figure 2. Responses to QE under greater income uncertainty and under lesser income uncertainty

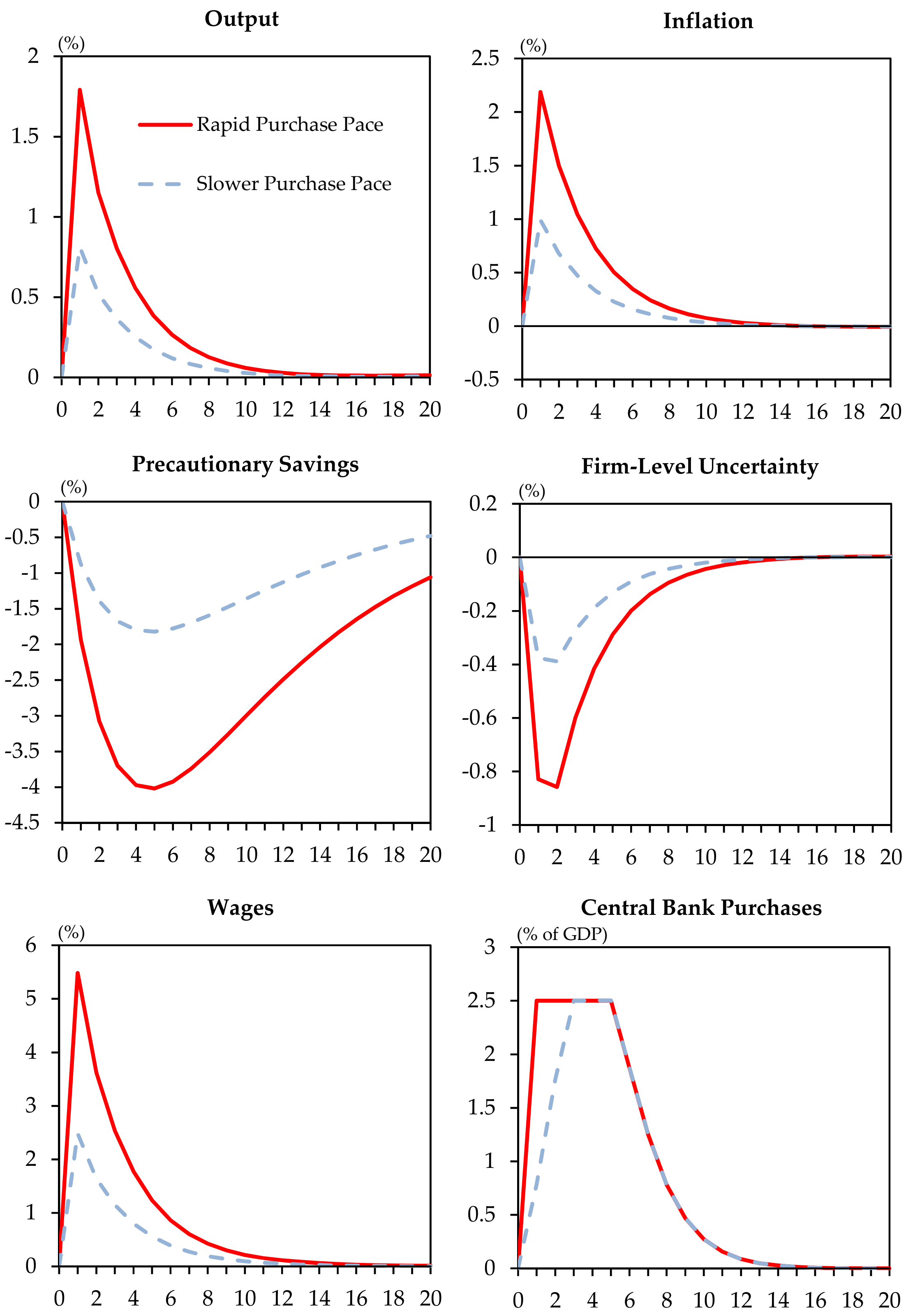

Figure 3. Responses to QE with rapid purchase pace and with slower purchase pace.

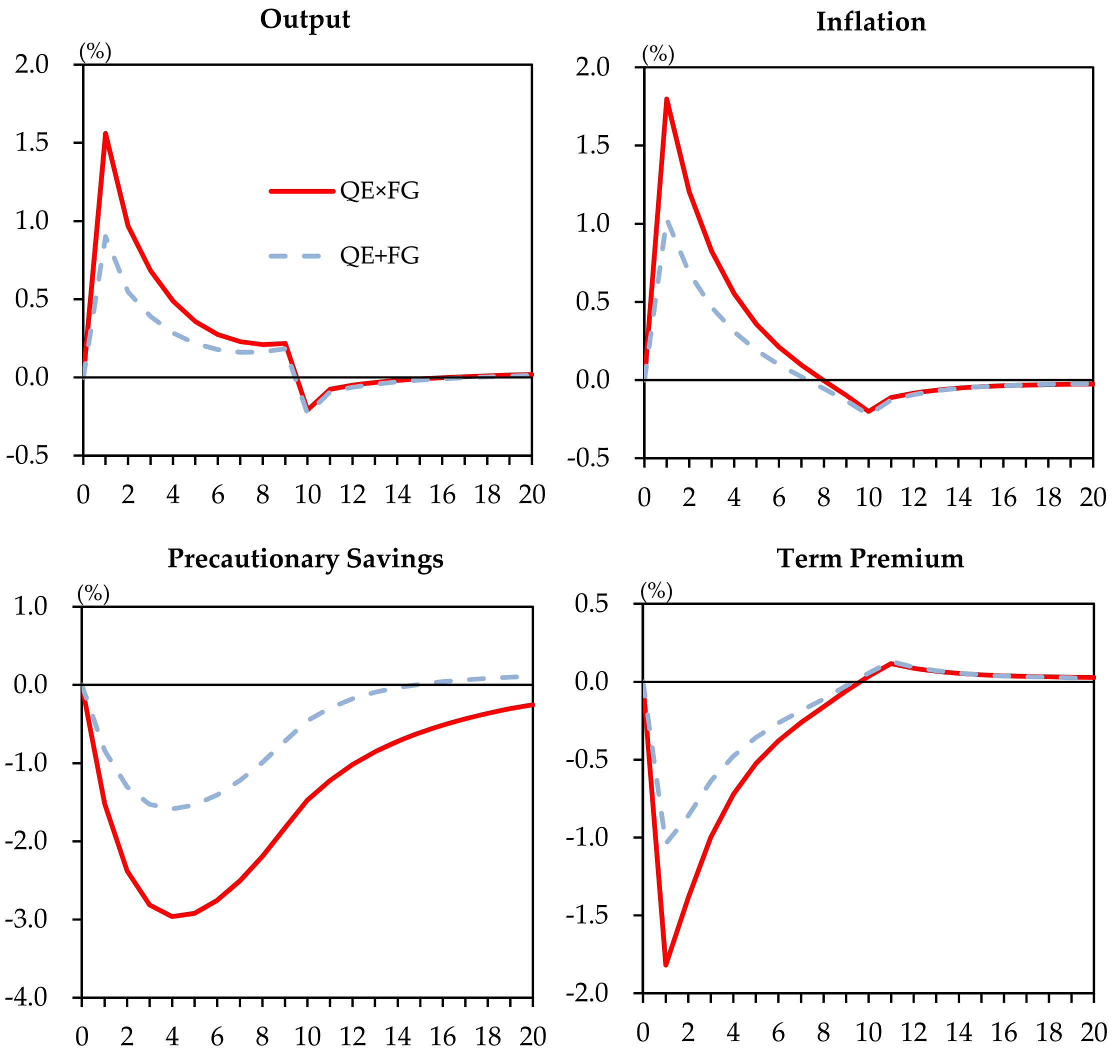

Figure 4. Responses to the combination of QE and FG (QE × FG) and the arithmetic sum of independent policy effects of QE and FG (QE + FG)

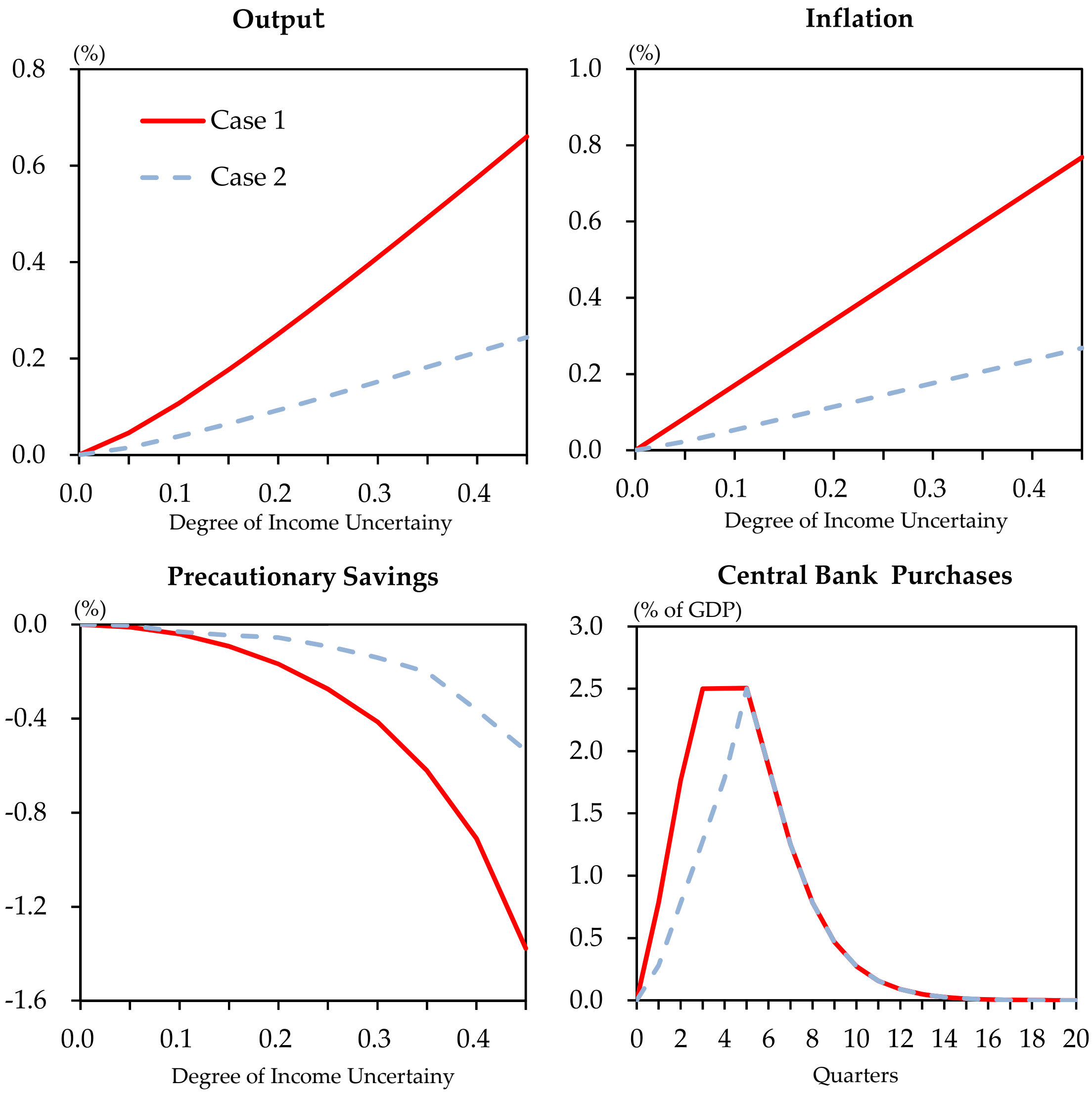

Figure 5. Peak of synergy effects to the combination of QE and FG (QE × FG) with different degrees of income uncertainty

Note: "Synergy effect" is defined as the gap between a response to QE × FG and a response to QE + FG. These figures plot synergy effects for key variables when the absolute values of the impulse responses are maximized as the horizon of various degrees of income uncertainty.

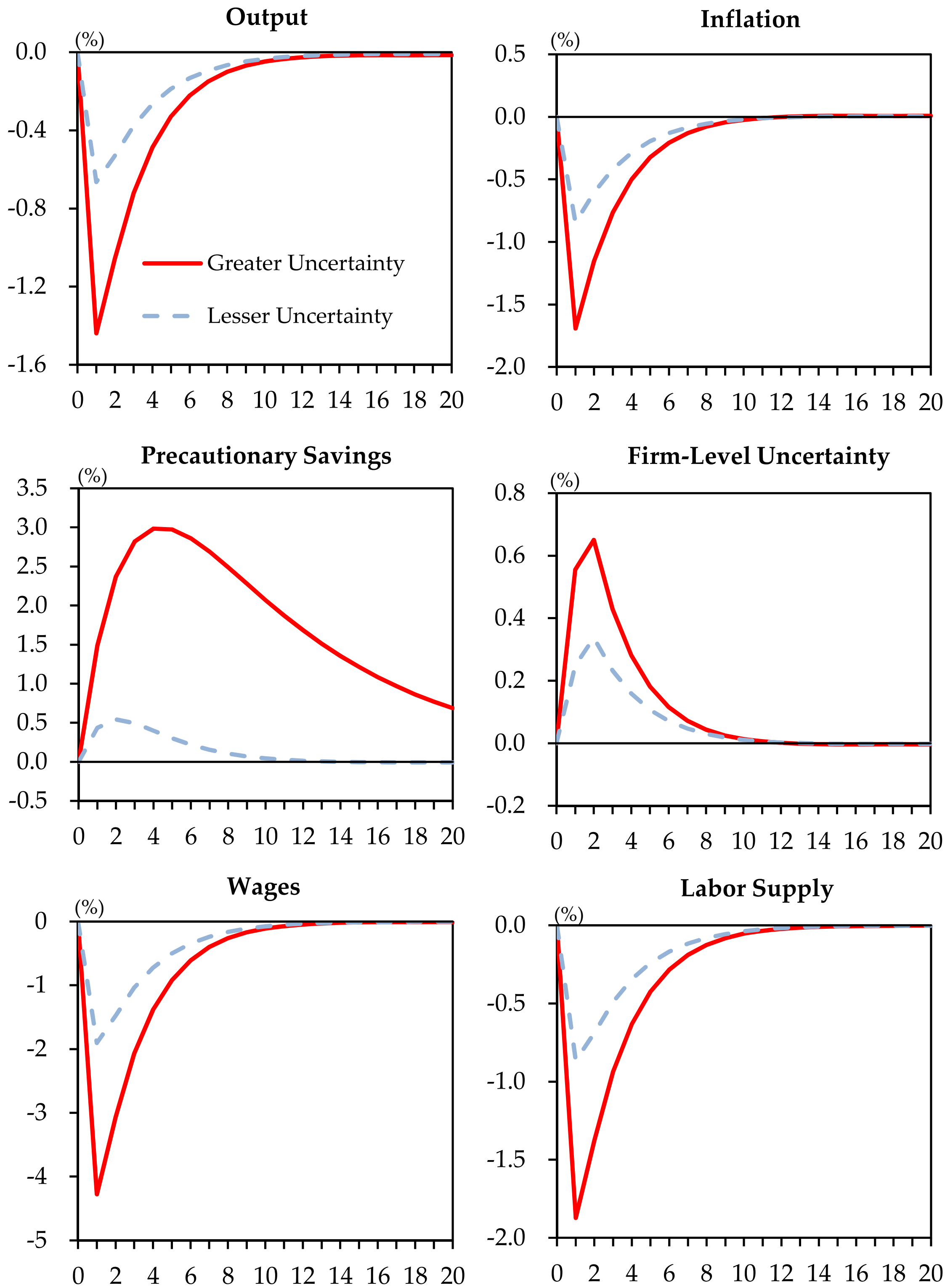

Figure 6. Responses to deflationary shocks under greater income uncertainty and under lesser income uncertainty

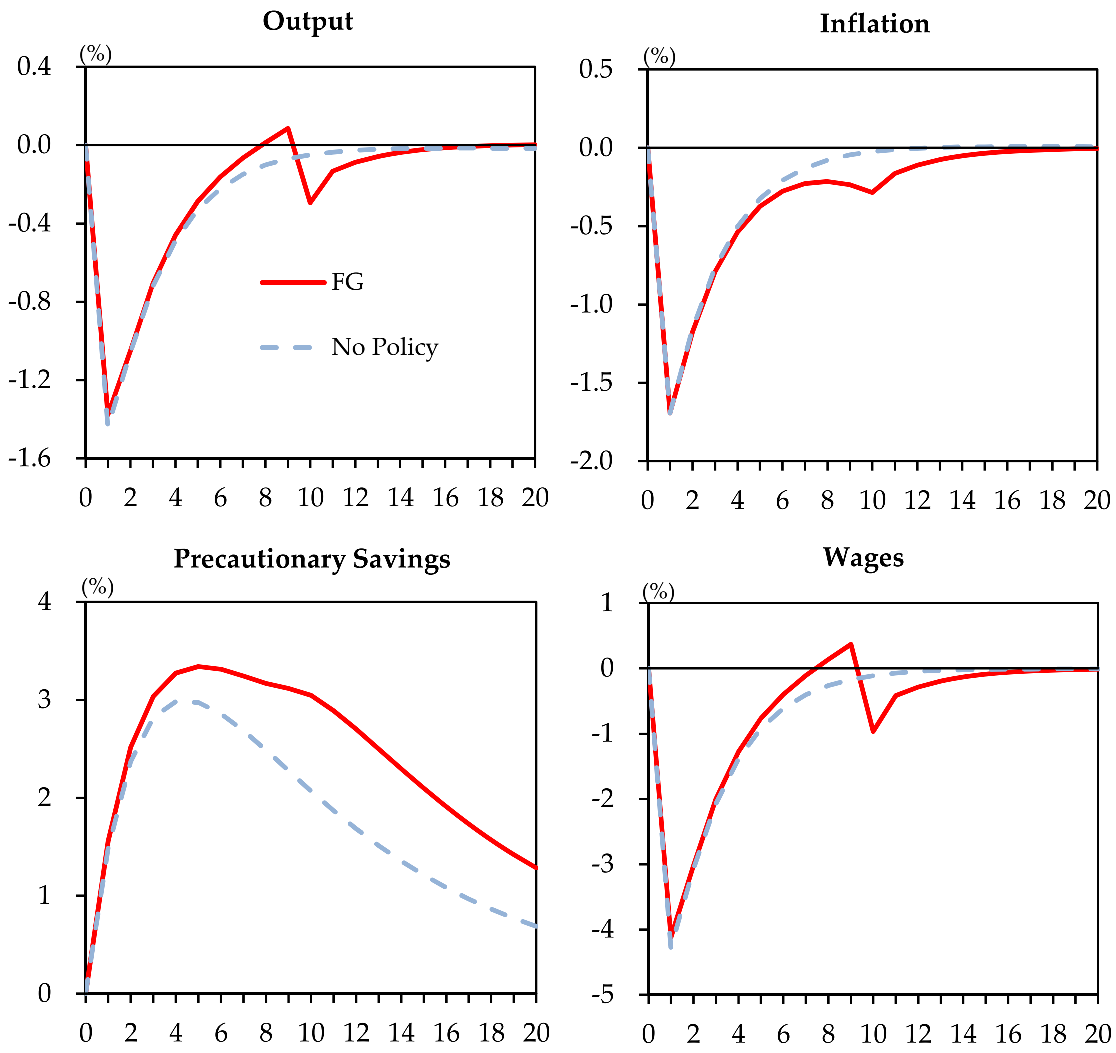

Figure 7. Responses to FG and no policy reaction under the liquidity trap

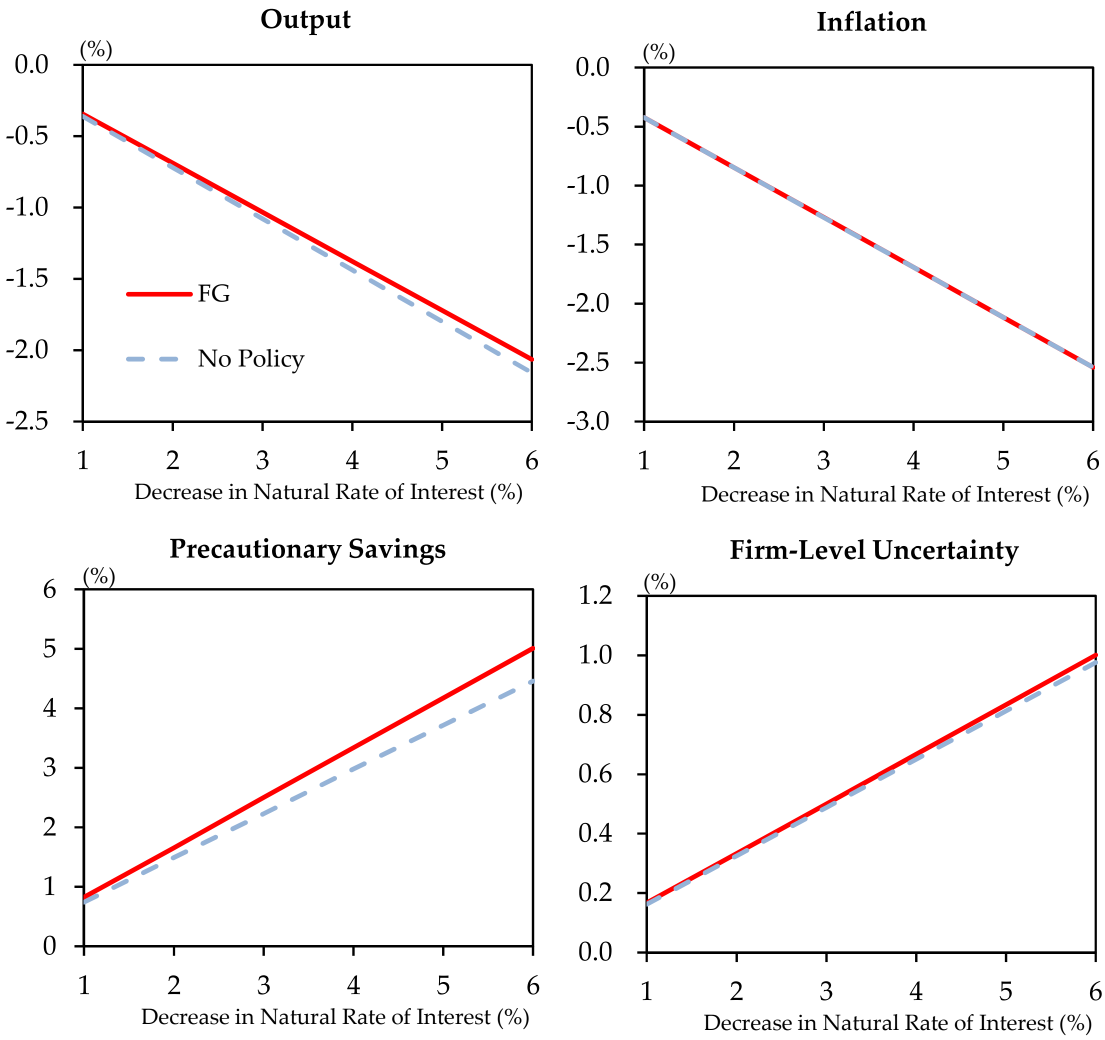

Figure 8. Peak responses to FG and no policy reaction at different size of deflationary shock
Note: These figures plot key variables when the absolute values of the impulse responses are maximized as the horizon of various degrees of income uncertainty.

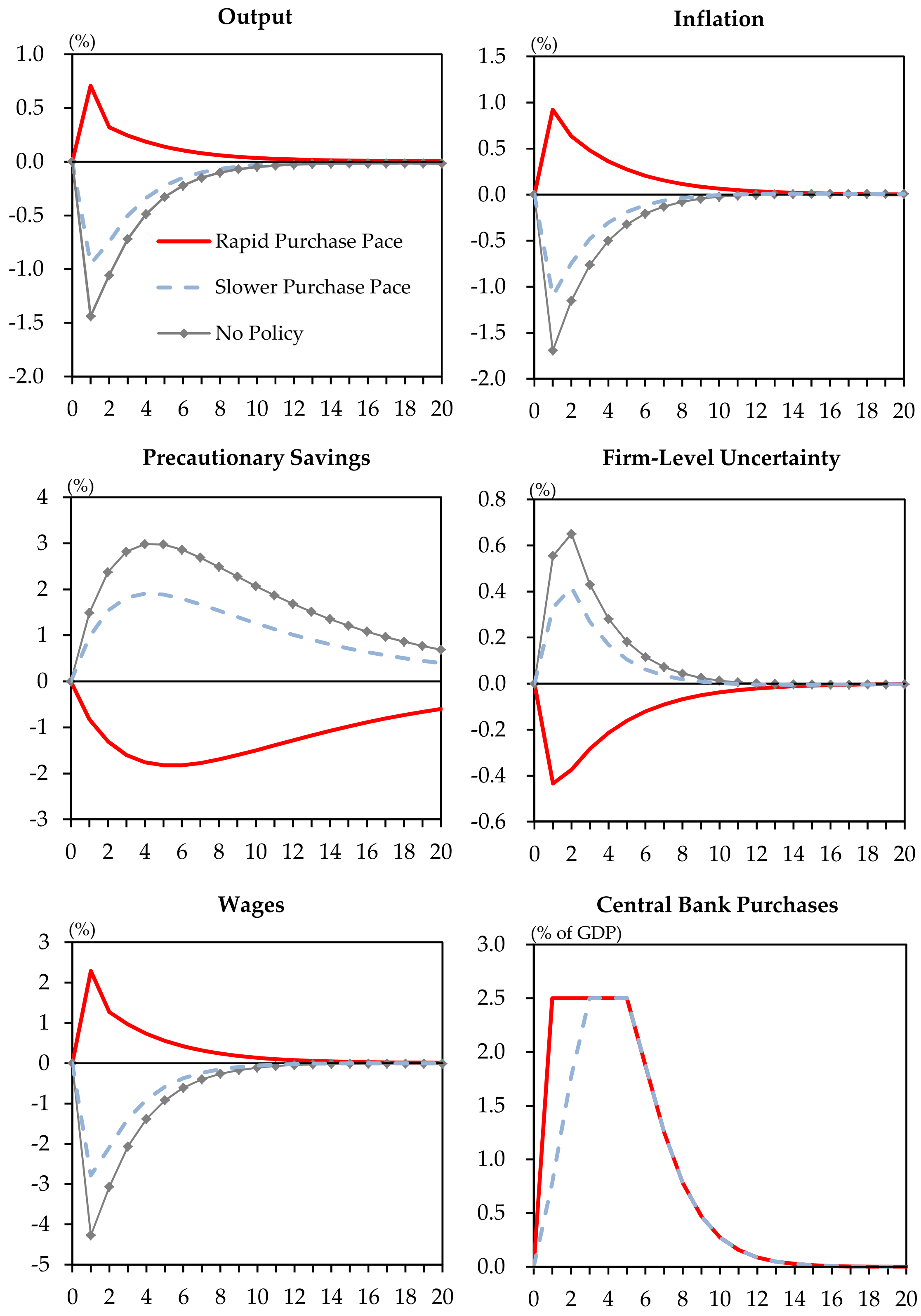

Figure 9. Responses to QE at different purchase paces under the liquidity trap

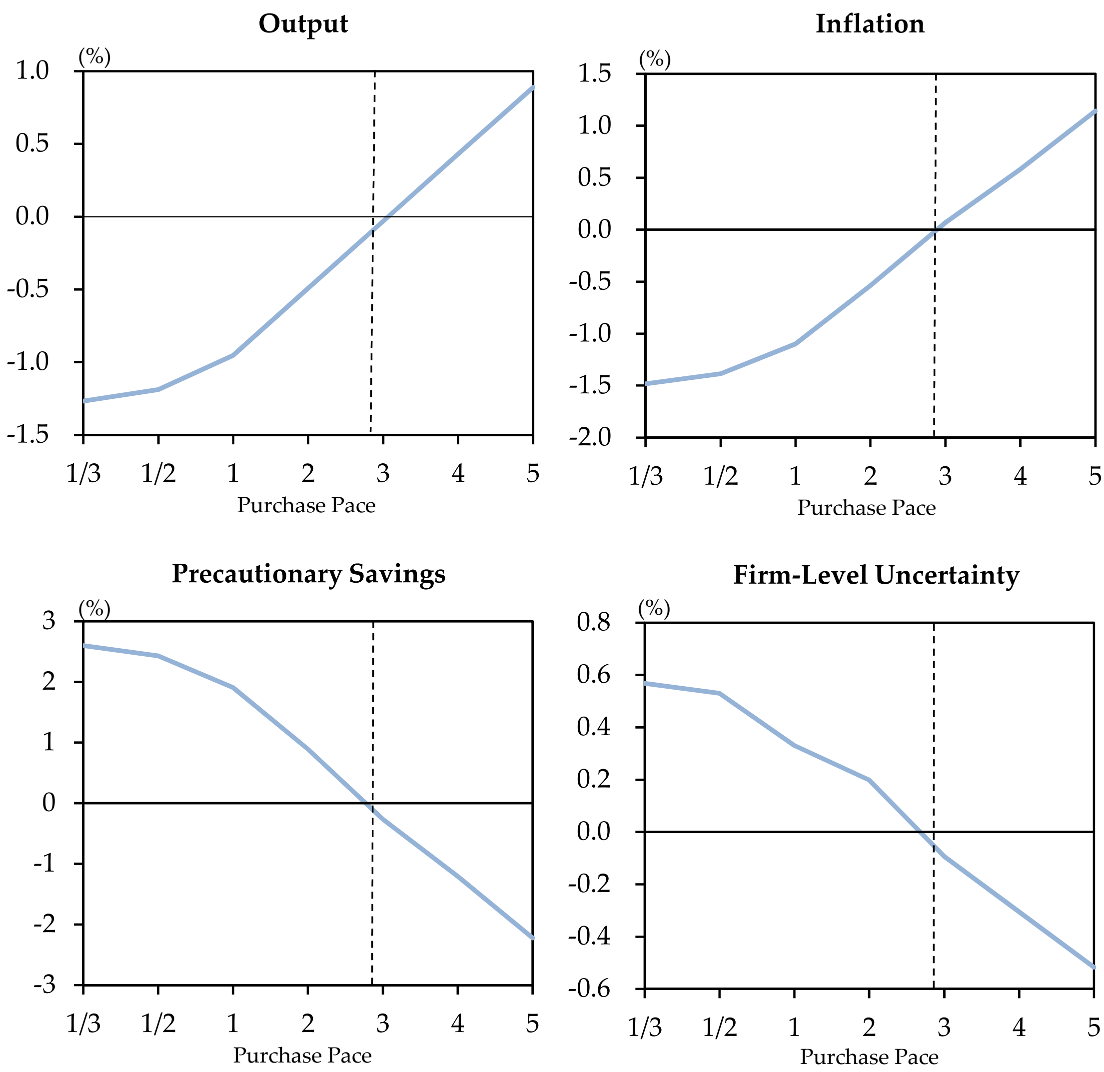

Figure 10. Peak responses to QE at different purchase paces under the liquidity trap

Note: Dotted lines indicate the purchase pace that allows the economy to escape from the liquidity trap when the central bank conducts the long-term bond purchase program. We set a purchase pace of “1” in this figure as the baseline purchase pace, which is equivalent purchase pace to the “Slower Purchase Pace” in the simulation of Figure 9.

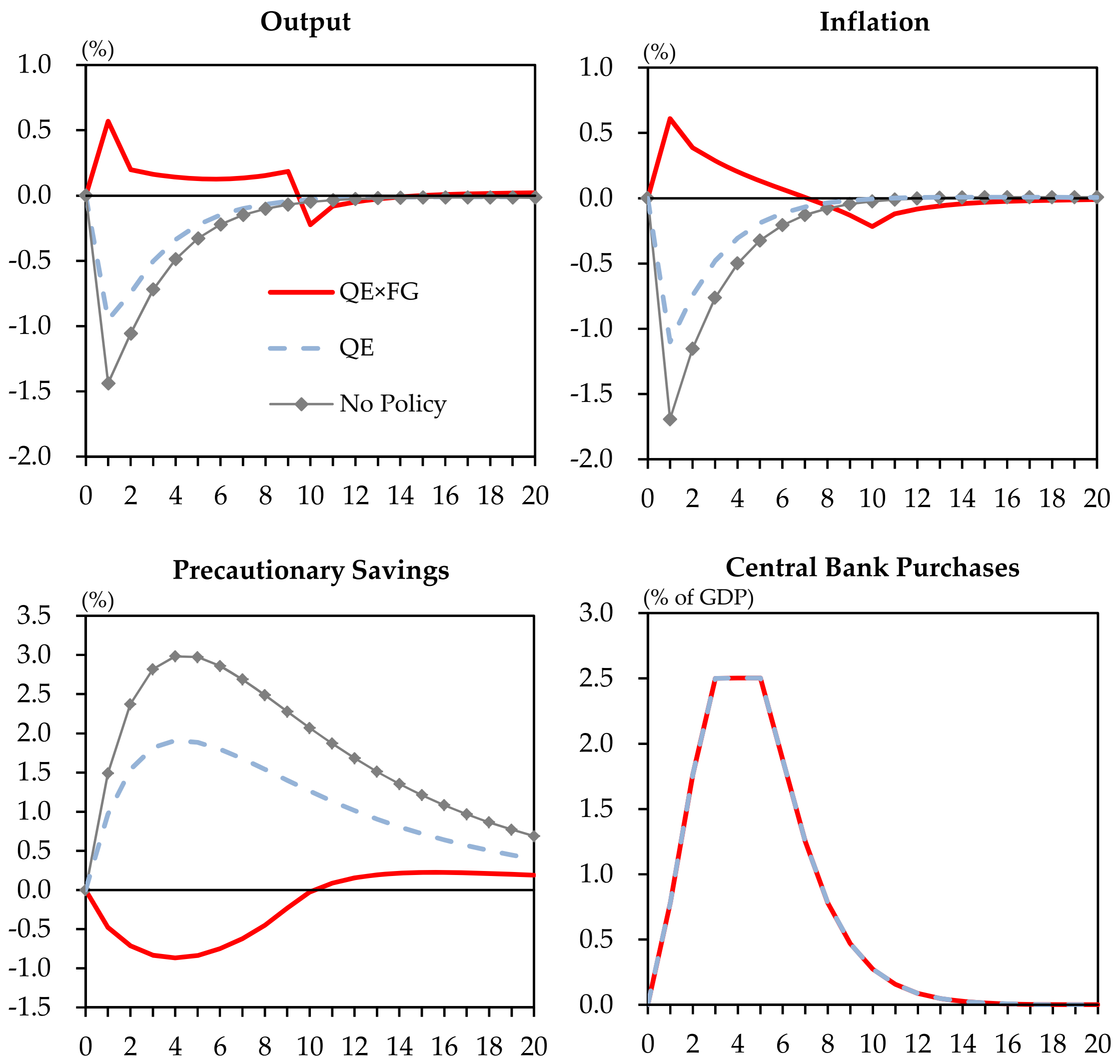

Figure 11. Responses to the independent effects of only QE (QE) and the combination of QE and FG (QE × FG) under the liquidity trap.