RESERVE BANK OF AUSTRALIA

Optimal Monetary Policy when Expectations are Rational, Fixed, Learned, or Anything in Between

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What are the research questions?

1. What is the optimal policy prescription with a general form of expectation formation?

2. How could economic outcomes in Australia have been improved by following these optimal policy prescriptions?

Why is this important?

- Optimal policy crucially depends on how expectations are formed.
  - Deriving optimal policy with a general form of expectation formation makes the solution more generally applicable.

- It is important to evaluate how policy outcomes could be improved.

- RBA Review recommendation 4: Institute regular reviews of the monetary policy framework and tools.
  - Should do this in the most rigorous way possible.

What we do? (Part 1)

- Adapt New Keynesian DSGE to allow for a general form of expectation formation
  - Nests fixed, learned, and rational (full or bounded) expectations
  - Allow for ZLB and imperfect information

- Analytically derive optimal policy solution
  - Pre-emptive Weighted Average Inflation Target (P-WAIT)
  - Use solution to extract four qualitative lessons for policymakers

What we do? (Part 2)

- Incorporate general expectation formation into RBA's DSGE
  - Assess how policy in Australia could have been improved by incorporating the lessons from our optimal policy solution

- Monetary policy in Australia has historically been:
  - Insufficiently pre-emptive (i.e. moved too late)
  - Too smooth (i.e. moved too slowly)

- Other than during 2015–2020 and Covid, the welfare gains from more pre-emption and less smoothing would historically have been small.

What we DON'T do?

- We are not proposing a new form of behaviour!

- Rather than being micro-founded, we design our expectation formation to nest several of the micro-founded behaviours in the literature.

- The advantage is that instead of taking a stand on a particular behaviour, we can model how optimal policy changes as me move between different types of beliefs.
  - This affords us more flexibility in quantitative exercises and allows us to assess the likelihood of different belief formation.
  - And it allows us to draw out some general lessons for policymakers based on the characteristics engendered by different beliefs.

What behaviours do we nest/approximate?

- Full information rational expectations (standard NK DSGE)
- NK DSGE with price indexation and/or habit formation
- Boundedly rational expectations
  - Myopia / over-discounting (Gabaix 2020)
  - Incomplete knowledge / level-k reasoning (Angeletos & Lian 2018, Farhi and Werning 2019; Evans, Gibbs & McGough 2023)
  - THANK (McKay, Nakamura and Steinsson 2017)
  - OLG (Del Negro, Giannoni & Patterson 2023)
- Learning
  - Eusepi & Preston 2018; Molnár & Santoro 2014.
- Fixed expectations

Representative agent model

- Phillips curve:

$$\pi_t = \beta \widehat{\mathbb{E}}_t \pi_{t+1} + \kappa x_t + u_t$$

  - Output gap: $x_t = y_t - y_t^e$
  - Cost-push shock (e.g. markup): u_t
  - General expectations: $\widehat{\mathbb{E}}_t$ (e.g. Evans and Honkapohja 2001)

- Misses some dynamics under learning (Eusepi and Preston 2018), but approximation permits tractable and useful closed-form policy rules.
  - Problem mitigated by assuming rational expectations for nominal interest rates. So IS curve is not a binding constraint.

General expectations formation

- Mix of rational and learned expectations:

$$\widehat{\mathbb{E}}_t \pi_T = \lambda \mathbb{E}_t \pi_T + (1 - \lambda) \mathbb{E}_t^l \pi_T$$

- Constant gain learners update expectations based on forecast errors (with assumed persistence ρ):

$$\mathbb{E}_t^l \pi_T = \rho^{T-t} \omega_{t-1}$$

$$\omega_t = \rho \omega_{t-1} + \rho g (\pi_t - \omega_{t-1})$$

This setup nests/approx. several other models

$$\lambda = 1$$
$$\widehat{\mathbb{E}}_t \pi_{t+1} = \mathbb{E}_t \pi_{t+1}$$

Standard full information rational expectations

$$\lambda = 0$$
$$\widehat{\mathbb{E}}_t \pi_{t+1} = \rho \omega_{t-1} = \rho^2 g \sum_{k=0}^{\infty} [\rho(1-g)]^k \pi_{t-1-k}$$

Learning

$$g = 0$$
$$\widehat{\mathbb{E}}_t \pi_{t+1} = \lambda \mathbb{E}_t \pi_{t+1}$$

Myopia/over-discounting, incomplete knowledge/level-k reasoning, OLG

$$g = 1$$
$$\widehat{\mathbb{E}}_t \pi_{t+1} = \lambda \mathbb{E}_t \pi_{t+1} + (1-\lambda)\rho^2 \pi_{t-1}$$

Hybrid NK Phillips curve

Optimal policy problem

- CB wants to minimise $L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha x_t^2)$ subject to constraints
  - We assume full commitment (from a timeless perspective)
  - We derive targeting rules, not instrument rules

Optimal policy with cost-push shocks only

- IS curve irrelevant – CB can set output gap exactly
  - Set π, x s.t. constraints: (i) Phillips curve; (ii) learners' inflation expectations
- Optimal policy:

$$\frac{\alpha}{\kappa}x_t = -\frac{1}{1-\lambda L}\pi_t + g\rho\frac{1}{1-\lambda L}\mu_t^{\omega}$$

Where:

- $\frac{1}{1-\lambda L}z_t = z_t + \lambda z_{t-1} + \lambda^2 z_{t-2} + \cdots$
- μ_t^{ω} is Lagrange multiplier on learners' inflation expectations constraint

Optimal policy is pre-emptive

$$\frac{\alpha}{\kappa}x_t = -\frac{1}{1-\lambda L}\pi_t + g\rho\frac{1}{1-\lambda L}\mu_t^{\omega}$$

- Solving for μ_t^{ω}:

$$\mu_t^{\omega} = \beta^2(1-\lambda)\rho\mathbb{E}_t\left\{\frac{1}{1-\beta\rho(1-g)L^{-1}}\frac{\alpha}{\kappa}x_{t+1}\right\}$$

- Cost of policy being constrained by learners is a function of expected future output gaps.
- Doing more on policy today, reduces expected future gaps by reducing drift in learned expectations.
- **Optimal policy is "Pre-emptive".**

Example

Start with $\lambda = 0$, $g = 0$ and a positive cost-push shock. Optimal policy:

$$\frac{\alpha}{\kappa} x_t = -\pi_t$$

With $g > 0$ the positive cost-push shock causes $\mu_t^\omega < 0$ (expectations drift).

$$\frac{\alpha}{\kappa} x_t = -\pi_t + g\rho\mu_t^\omega$$

$$\mu_t^\omega = \beta^2 \rho \mathbb{E}_t \left\{ \frac{1}{1 - \beta\rho(1-g)L^{-1}} \frac{\alpha}{\kappa} x_{t+1} \right\}$$

Larger (negative) output gap today reduces future drift.

– Optimal "pre-emptive" policy suffers welfare cost today to achieve better trade-off in the future.

Optimal policy is backward-looking

$$\frac{\alpha}{\kappa} x_t = -\frac{1}{1-\lambda L}\pi_t + g\rho \frac{1}{1-\lambda L}\mu_t^{\omega}$$

- With some rational expectations ($\lambda > 0$), optimal policy responds to past inflation gaps and past Lagrange multipliers.
- Benefit is that a commitment to enact policy this way in the future changes expectations today in a way that alleviates current trade-offs.
  - Will be costly when enacting the commitment, but benefit today outweighs cost tomorrow.
- Optimal commitment is a weighted average inflation target (WAIT).

$$\frac{1}{1-\lambda L} z_t = z_t + \lambda z_{t-1} + \lambda^2 z_{t-2} + \cdots$$

Four lessons for policymakers

1. Committing to overshoot in the future alleviates current trade-offs (buy now, pay (less) later).
   - Forward-looking agents bring the benefits forward.

2. When responding to cost-push inflation, more contractionary policy now gets the economy back to normal faster (brake into the corner, accelerate out).
   - Reduces drift in learned expectations.

- Combined, optimal policy in response to cost-push shocks is a Pre-emptive Weighted Average Inflation Target (P-WAIT).

$$\frac{\alpha}{\kappa} x_t = -\frac{1}{1-\lambda L}\pi_t + g\rho\frac{1}{1-\lambda L}\mu_t^{\omega}$$

IS curve is a binding constraint

- If CB cannot set the output gap exactly where it wants, the IS curve becomes a binding constraint.

$$x_t = (1-\beta)\widehat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} x_{T+1} - \frac{1}{\sigma} \sum_{T=t}^{\infty} \beta^{T-t} \left(\mathbb{E}_t i_T - \widehat{\mathbb{E}}_t \pi_{T+1} - \mathbb{E}_t r_T^n \right)$$

IS curve under subjective beliefs (Preston 2005) but with rational rate expectations.

- We consider imperfect information and ZLB as reasons why CB can't set output gap exactly.
  - Similar lessons apply for policy lags or a desire for rate smoothing.

Optimal policy with binding IS curve

- For simplicity, assume prices fully fixed, $\kappa = 0$ (general case is in paper)
  - CB wants to stabilise output gap $(x_t = 0)$.

- Might think optimal to get as close to zero as possible, but optimal policy requires the following moving target:

$$x_t^* \equiv (1-\Theta)\left(\frac{\lambda}{\beta}(1-\beta)+1\right)\Delta_{t-1} + \Theta \mathbb{E}_t \frac{1-\beta\rho(1-g)}{1-\beta\rho(1-g)L^{-1}}\Delta_{t+1}$$

$\Delta_t = x_t^* - x_t,\ \Theta = \frac{(1-\lambda)g\rho^2 K}{1+(1-\lambda)g\rho^2 K},\ K$ is a function of structural parameters

- Moving target is a weighted average of the past difference between desired and actual gap (Δ_{t-1}) and expected future differences.
  - Look familiar?

Optimal policy with a binding IS curve

$$x_t^* \equiv (1-\Theta)\left(\frac{\lambda}{\beta}(1-\beta)+1\right)\Delta_{t-1} + \Theta\mathbb{E}_t \frac{1-\beta\rho(1-g)}{1-\beta\rho(1-g)L^{-1}}\Delta_{t+1}$$

$$\Delta_t = x_t^* - x_t, \quad \Theta = \frac{(1-\lambda)g\rho^2 K}{1+(1-\lambda)g\rho^2 K}$$

- Committing to make up for past target misses changes the rational expectations and alleviates the constraint.
  - Even when $\lambda = 0$, channel works via rational nominal rate expectations (1).
  - More rationality makes channel stronger.
- At ZLB, committing to make-up for an inability to lower rates today brings the benefits forward.

Optimal policy with a binding IS curve

$$x_t^* \equiv (1-\Theta)\left(\frac{\lambda}{\beta}(1-\beta)+1\right)\Delta_{t-1} + \Theta\mathbb{E}_t \frac{1-\beta\rho(1-g)}{1-\beta\rho(1-g)L^{-1}}\Delta_{t+1}$$

$$\Delta_t = x_t^* - x_t, \;\; \Theta = \frac{(1-\lambda)g\rho^2 K}{1+(1-\lambda)g\rho^2 K}$$

- As long as some learning ($\lambda < 1, g > 0$), optimal to move learners' expectations **pre-emptively** so that constraint binds less in the future.
- When approaching ZLB, optimal to be more expansionary (than if ZLB didn't exist) so that learners' income expectations are higher if hit ZLB.

Four lessons for policymakers

1. Committing to overshoot in the future alleviates current trade-offs (buy now, pay (less) later).
   – Forward-looking agents bring the benefits forward.

2. When responding to cost-push inflation, more contractionary policy now gets the economy back to normal faster (brake into the corner, accelerate out).
   – Reduces drift in learned expectations.

3. Committing to make up for any periods when policy is constrained brings the benefits forward (making up for lost time).

4. Expansionary policy now makes the ZLB constraint less binding in the future (use it or lose it).

Existing literature

The individual parts of what we show are not new:

- Make-up commitments (smoothing) is optimal with forward-looking expectations; PLT is optimal under rational expectations.
  - Clarida, Eggertsson, Gali, Gertler, Woodford…
- Aggressive/pre-emptive policy is optimal to prevent learned expectations from drifting.
  - Evans, Giannoni, Honkapohja, Molnár, Santoro…
- Do more of both when faced with the ZLB.
  - Eusepi, Gibbs, Preston, …
- Our contribution is to combine these insights within a general expectations framework and solve for the optimal policy rule.

General expectations in the RBA DSGE

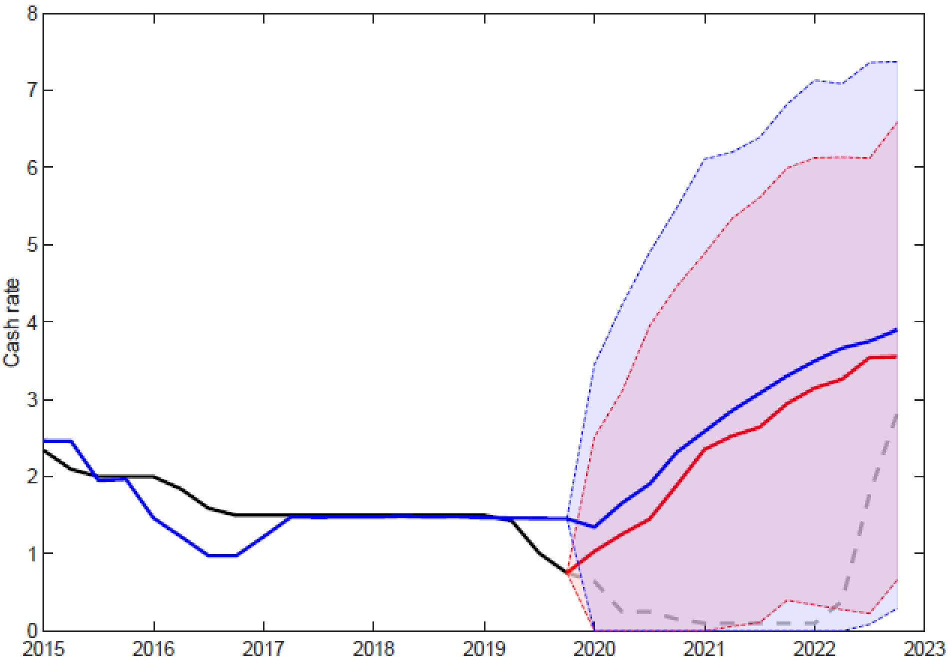
- Want to find out which, if any, of the micro-founded expectation behaviours are supported by the data.
  - Can compare to previous work looking at survey measures of expectations.
- Estimated rationality share: 40% (90% HPDI: 32% – 56%)
  - Higher than households/unions, lower than near-rational professional.
- Gain parameter for learners: 0.18 (90% HPDI: 0.14 – 0.21)
  - Faster learning than households, slower than unions.
- Data rejects full RE, full learning and bounded rationality (with no learning).

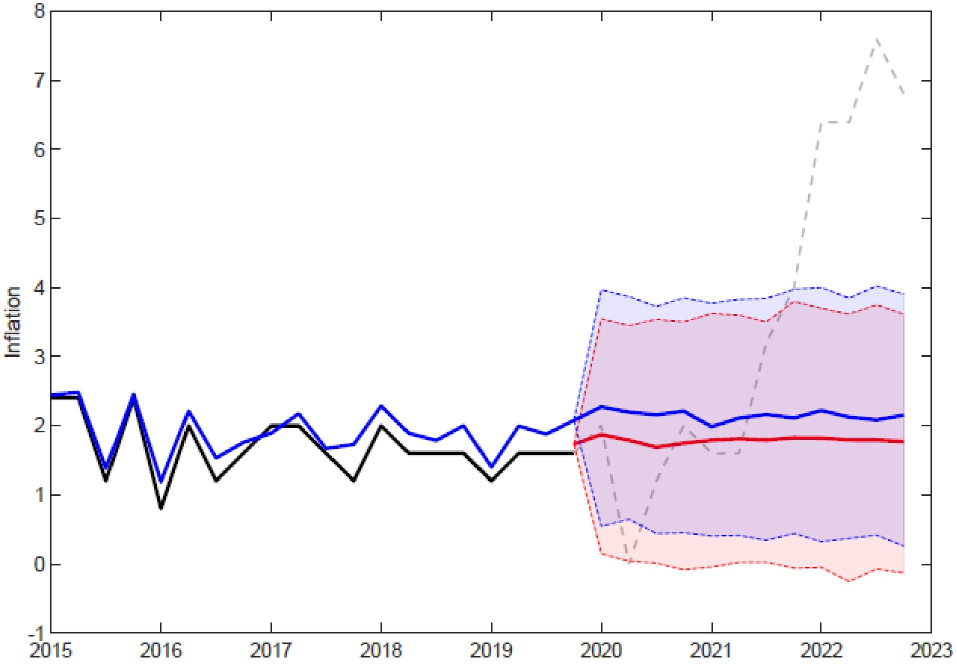
Approximating P-WAIT in the RBA DSGE

$$\begin{aligned} i_t &= \phi_\pi \eta_t + \phi_h h_t + \phi_{\Delta_h}(h_t - h_{t-2}) + \epsilon_t \\ \eta_t &= \eta_{t-1} + (1-\rho_i)(\pi_t - \eta_{t-1}). \end{aligned}$$

- Historical estimate: $\phi_\pi = 1.18$ and $\rho_i = 0.91$
- Optimise ϕ_π and ρ_i in loss function that puts *half* as much weight on inflation variance as hours worked variance:
  - $\phi_\pi = 1.65$ and $\rho_i = 0.47$
- Alternative loss function with *twice* as much weight on inflation variance:
  - $\phi_\pi = 2.98$ and $\rho_i = 0.64$
- Historical policy is too smooth (ρ_i too high) and insufficiently pre-emptive (ϕ_π too low)!

Historical vs optimal policy path





Notes: Solid black lines are the actual outcomes. Dashed black lines are the actual outcomes after we stop our exercise (i.e. during the forecasting period). The red line are forecasted values from the estimated rule. The blue line is the CO rule (both backward-looking and forecasted).

Key contributions

- Generalise expectation formation in NK DSGE (with ZLB or imperfect information) and analytically derive optimal policy.
- Distil into four key lessons for policymakers.
- Adapt expectation formation in RBA's DSGE model.
- Apply lessons to updated model:
  - In general, policy should be more aggressive and smooth less.
  - Largest welfare gain in pre-Covid period when rates held too high.
  - Stronger state-based forward guidance (i.e. more smoothing) leads to higher inflation during Covid **and** faster lift-off in response to subsequent inflationary shocks. (HAVE SPARE SLIDES IF QUESTIONS)

SPARES

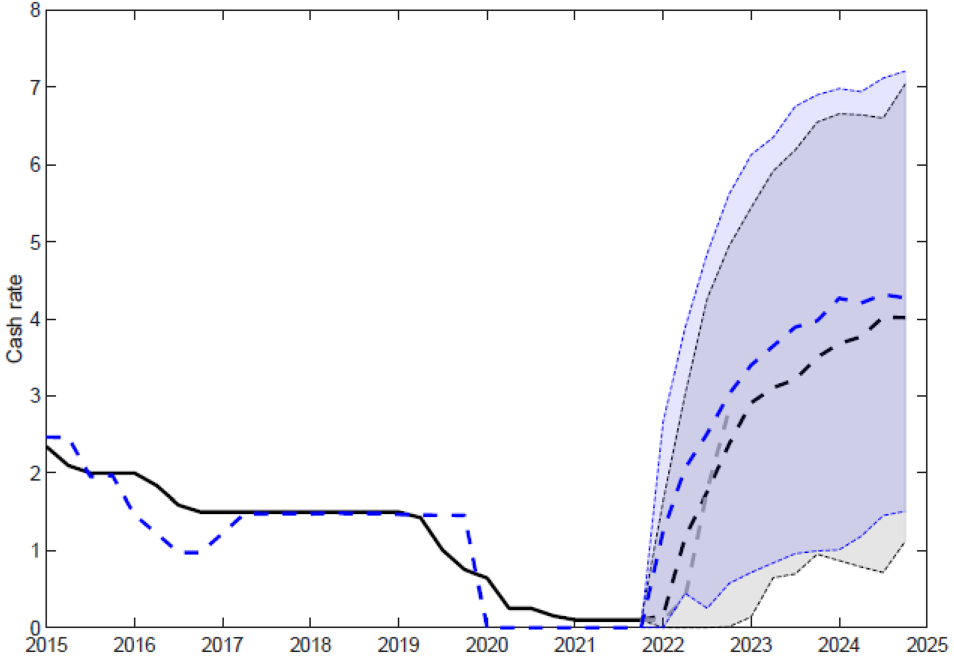
Covid and the ZLB

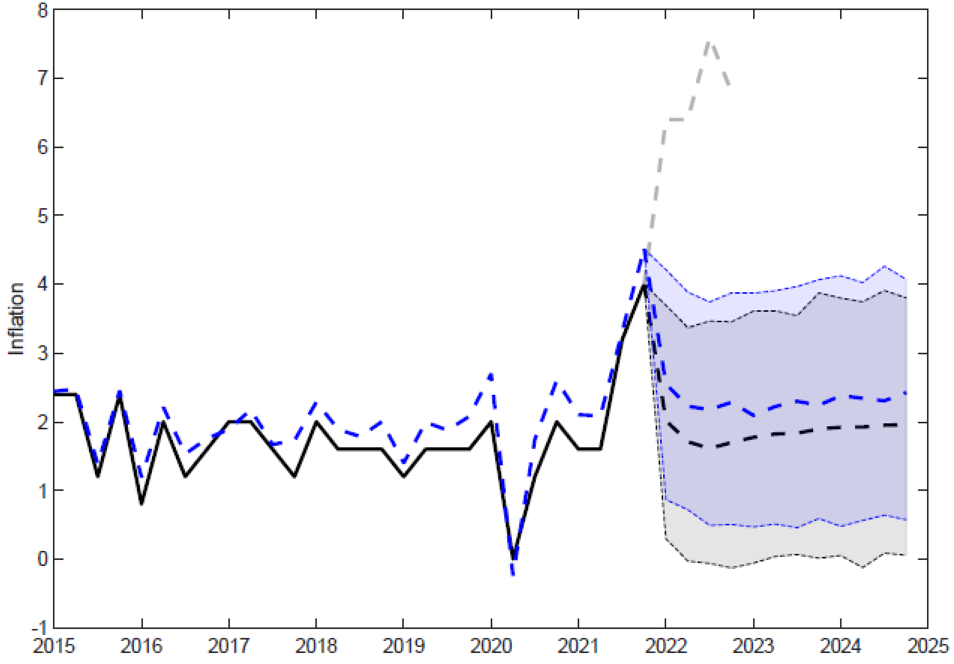
- Lessons say when confronted with ZLB, policy should be both more smooth (Lesson 3) and more aggressive (Lesson 4).

- Exercise 1: Respond aggressively when Covid first spread globally (hitting ZLB at beginning of 2020).
  - Inflation 30bps higher going into Covid, but swamped by size of shock.

- Exercise 2: Increase smoothing (0.975)…

Smoothing = make-up commitments





Notes: Solid black lines are the actual outcomes. Dashed black lines are the forecasted outcomes after we stop our exercise. Dashed grey lines are actual outcomes. Dashed blue lines are the counter-factual policy rule.

How much does welfare increase?

- Can’t just look at policy rule, because historical deviations from the rule could be welfare improving (e.g. real-time data, liaison).
- But most of the action occurs through the expectations channel.
- Can isolate expectations channel by having different rules in model, but constructing policy shocks so that historical cash rate is exactly the same.

Welfare loss per quarter