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Currency Misalignments and Optimal Monetary Policy: A Re-examination

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Abstract

This paper examines optimal monetary policy in an open-economy two-country model with sticky prices. Currency misalignments are shown to be inefficient and lower world welfare. Also, optimal policy must target not only inflation and the output gap, but also the currency misalignment. However, the interest rate reaction function that supports this targeting rule involves only the CPI inflation rate. This result illustrates how examination of 'instrument rules' may hide important trade-offs facing policy-makers that are incorporated in 'targeting rules'. The model is a modified version of Clarida, Galí and Gertler's (2002). The key change is to allow pricing to market or local-currency pricing and consider the policy implications of currency misalignments. Besides highlighting the importance of the currency misalignment, this model also gives a rationale for targeting CPI inflation, rather than producer price inflation as in Clarida, Galí and Gertler.

JEL Classification Numbers: E52, F41

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1. Introduction

Exchange rates among the large economies have fluctuated dramatically over the past 30 years. The euro/US dollar exchange rate has experienced swings of greater than 60 per cent, and even the Canadian dollar/US dollar has risen and fallen by more than 35 per cent in the past decade, but inflation rates in these economies have differed by only a few percentage points per year. Should these exchange rate movements be a concern for policy-makers? Would it not be better for policy-makers to focus on output and inflation and ignore a freely floating exchange rate that settles at a market-determined level?

It is widely understood that purchasing power parity does not hold in the short run. Empirical evidence points to the possibility of 'local-currency pricing' (LCP) or 'pricing to market'.[1] That is, exporting firms may price discriminate among markets and/or set prices in the buyers' currencies. A currency could be overvalued if the consumer price level is higher at home than abroad when compared in a common currency, or undervalued if the relative price level is lower at home. Currency misalignments can be very large even in advanced economies.

There is frequent public discussion of the importance of controlling currency misalignments. For example, on 3 November 2008, Robert Rubin (former US Secretary of the Treasury) and Jared Bernstein (of the Economic Policy Institute) co-authored an op-ed piece in the *New York Times* that argued, 'Public policy … has been seriously deficient [because of] false choices, grounded in ideology' (Rubin and Bernstein 2008). One of the principles they argue that all should agree upon is 'we need to work with other countries toward equilibrium exchange rates'. Yet there is little support in the modern New Keynesian literature on monetary

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1 Many studies have found evidence of violations of the law of one price for consumer prices. Two prominent studies are Engel (1999) and Atkeson and Burstein (2008). The literature is voluminous – these two papers contain many relevant citations.

policy for the notion that central banks should target exchange rates. Specifically, if policy-makers are already optimally responding to inflation and the output gap, is there any reason to pay attention to exchange rate misalignments?

The answer is yes. In a simple, familiar framework, this paper draws out the implications for monetary policy when currency misalignments are possible. Currency misalignments lead to inefficient allocations for reasons that are analogous to the problems with inflation in a world of staggered price setting. When there are currency misalignments, households in the home and foreign countries may pay different prices for an identical good. A basic tenet of economics is that violations of the law of one price are inefficient – if the good's marginal cost is the same irrespective of where the good is sold, it is not efficient to sell the good at different prices. The key finding of this paper is that currency misalignments lead to a reduction in world welfare and that optimal monetary policy trades off this currency misalignment with inflation and output goals.

These currency misalignments arise even when foreign exchange markets are efficient. That is, the currency misalignment distortion that is a concern to policy-makers arises in the goods market – from price setting – and not in the foreign exchange market. In the model of this paper, the foreign exchange rate is determined in an efficient currency market as a function of fundamental economic variables.

The literature has indeed previously considered models with LCP. Some of these models are much richer than the model considered here. To understand the contribution of this paper, it is helpful to place it relative to four sets of papers:

1. Clarida *et al* (2002, hereafter referred to as CGG) develop what is probably the canonical model for open-economy monetary policy analysis in the New Keynesian framework. Their paper assumes that firms set prices in the producer's currency (PCP, for 'producer-currency pricing'). Their two-country model also assumes that home and foreign households have identical preferences. These two assumptions lead to the conclusion that purchasing

power parity (PPP) holds at all times – the consumption real exchange rate is constant.[2]

This paper introduces LCP into CGG's model. Simple rules for monetary policy are derived that are similar to CGG's. While the model is not rich relative to sophisticated models in the literature (models that introduce capital, working capital, capacity utilisation, habits in preferences, etc), the simple model is helpful for developing intuition because the model can be solved analytically, an explicit second-order approximation to the policy-maker's loss function can be derived, as can explicit 'target criteria' for policy and explicit interest rate reaction functions. As I shall quickly explain, a lot can be learned from these relationships in the simple model.

The paper also allows home and foreign households to have different preferences. They can exhibit a home bias in preferences – a larger weight on goods produced in a household's country of residence.[3] This generalisation does not change the optimal target criteria at all in the CGG framework, but as I now explain, is helpful in developing a realistic LCP model.

2. Devereux and Engel (2003) explicitly examine optimal monetary policy in a two-country framework with LCP. Corsetti and Pesenti (2005) extend the analysis in several directions. However, neither of these studies is suited toward answering the question posed above: is currency misalignment a separate concern of monetary policy, or will the optimal exchange rate behaviour be achieved through a policy that considers inflation and the output gap?

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[2] Benigno and Benigno (2003, 2006) are important contributions that use models similar to CGG's but consider optimal policy when the optimal subsidies to deal with monopoly distortions are not present in steady state.

[3] De Paoli (2009) allows for home bias in preferences in a small open economy model. There is home bias in the sense that while the country is small, the limit of the ratio of the expenditure share on home goods to the population share is not equal to one. Faia and Monacelli (2008) examine optimal monetary policy in a small open economy model with home bias, using a Ramsey-style analysis. Pappa (2004) considers a two-country model with home bias. However, the second-order approximation to the welfare function is expressed in terms of deviations of consumption from its efficient level, rather than in terms of the output gap, so the analysis is not strictly comparable to that here. See Woodford (2003) for a discussion of why it makes sense to approximate in terms of the output gap rather than consumption.

These models have a couple of crucial assumptions that make them unsuited to answering this question. First, like CGG, they assume identical preferences in both countries. This assumption (as shown below) leads to the outcome that currency misalignments are the only source of CPI inflation differences between the two countries in the LCP framework. Eliminating inflation differences eliminates currency misalignments and *vice versa*.[4]

Second, price stickiness is the only distortion in the economy in these papers. In contrast, CGG introduce ‘cost-push shocks’, so that policy-makers face a trade-off between the goals for inflation and the output gap. In Devereux and Engel, the optimal monetary policy under LCP sets inflation to zero in each country, thus eliminating any currency misalignment.

By introducing home bias in preferences, the tight link between relative inflation rates and currency misalignments is broken. A more realistic model for inflation results when relative CPI inflation rates depend not only on currency misalignments, but also on the internal relative price of imported to domestically produced goods. Moreover, we follow CGG in allowing for cost-push shocks.[5]

This paper also derives optimal policy in a framework that is consonant with the bulk of New Keynesian models of monetary policy analysis. Devereux and Engel (2003) and Corsetti and Pesenti (2005) assume price setting is synchronised, with prices set one period in advance.[6] Here the standard Calvo price-setting technology is adopted, which allows for asynchronised price setting. This change is important, because it emphasises the point that the cost of inflation under sticky prices is misaligned relative prices. Also, the

4 See Duarte and Obstfeld (2008), who emphasise this point.

5 The contribution of Sutherland (2005) merits attention. His two-country model allows for imperfect pass-through, and for differences in home and foreign preferences. His model is static, and he derives a welfare function in which the variance of the exchange rate appears. However, the other terms in the welfare function are prices, so it is not clear how this function relates to standard quadratic approximations that involve output gaps and inflation levels. Moreover, Sutherland does not derive optimal monetary policy in his framework.

6 A sophisticated extension of this work is the recent paper by Corsetti, Dedola and Leduc (forthcoming). That paper extends earlier work in several dimensions, including staggered price setting. But it does not directly address the issue of whether currency misalignments belong in the targeting rule along with output gaps and inflation.

previous papers assumed that the money supply was the instrument of monetary policy. This paper follows CGG and most of the modern literature in assuming that the policy-makers directly control the nominal interest rate in each country.[7]

3. Monacelli (2005) has considered optimal monetary policy under LCP in a simple small country model.[8] But a small country model is not capable of addressing the global misallocation of resources arising from violations of the law of one price. In such a model, import prices are exogenous for the home country, and the welfare of the rest of the world is ignored. Hence, such a framework is not designed to consider the problems of currency misalignments.

4. There are many papers that numerically solve very rich open economy models and examine optimal policy. Some of these papers allow for LCP. Many of those papers are in the framework of a small open economy, and so do not specifically account for the global misallocation of resources that occurs with currency misalignments.[9] Moreover, many use *ad hoc* welfare criteria for the policy-maker or approximations that are not strictly derived from household welfare.[10]

   Some papers have considered whether it is beneficial to augment the interest rate reaction function of central banks with an exchange rate variable.[11] They ask the question: if the Taylor rule has the interest rate reacting to inflation and the output gap, is there any gain from adding the exchange rate?

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[7] While the model of this paper adheres strictly to the set-up of CGG, changing only the assumptions of identical preferences and LCP instead of PCP price setting, the model is very similar to that of Benigno's (2004). Woodford (forthcoming) also considers the LCP version of CGG (though not for optimal monetary policy analysis) and makes the connection to Benigno's paper.

[8] See also Leith and Wren-Lewis (2007) who examine a small open economy model with non-traded goods (but with PCP for export pricing).

[9] See, for example, Kollmann (2002), Smets and Wouters (2002), Ambler, Dib and Rebei (2004) and Adolfson *et al* (2008).

[10] For example, Smets and Wouters (2002), Ambler *et al* (2004) and Adolfson *et al* (2008).

[11] In a small open economy, see Kollmann (2002) and Leitemo and Söderström (2005). In a two-country model, see Wang (forthcoming).

> Typically these studies find little or no evidence of welfare gains from adding the exchange rate to the Taylor rule.

The question posed this way is misleading. To understand this point, it is helpful first to return to the optimal policy analysis in CGG. That paper finds (under their assumption of PCP) that optimal monetary policy can be characterised by a pair of 'target criteria' or 'targeting rules': $\tilde{y}_t + \xi\pi_{Ht} = 0$ and $\tilde{y}_t^* + \xi\pi_{Ft}^* = 0$. In these equations, $\tilde{y}_t$ refers to the output gap of the home country – the percentage difference between the actual output level and its efficient level. π_{Ht} is producer price inflation in the home country. Analogously, $\tilde{y}_t^*$ is the foreign output gap and π_{Ft}^* is foreign producer price inflation.[12] These equations describe the optimal trade-off between the output gap and inflation for the policy-maker. It will be desirable to allow inflation to be positive if the output gap is negative, for example. CGG then derive optimal interest rate rules that will deliver these optimal policy trade-offs. They find that the optimal interest rate reaction functions (assuming discretionary policy) are: $r_t = \overline{rr}_t + b\pi_{Ht}$ and $r_t^* = \overline{rr}_t^* + b\pi_{Ft}^*$. r_t is the home nominal interest rate, and $\overline{rr}_t$ is the 'Wicksellian' or efficient real interest rate. The response of the interest rate to inflation, b, is a function of model parameters.[13]

The key point to be made here is that CGG's model shows that optimal policy must trade off the inflation and output goals of the central bank. But the optimal interest rate reaction function does *not* necessarily include the output gap. That is, adding the output gap to the interest rate rule that already includes inflation will not improve welfare. Focusing on the 'instrument rule' does not reveal the role of the output gap that is apparent in the 'targeting rule' in the terminology of Svensson (1999, 2002).

An analogous situation arises in the LCP model concerning currency misalignments. We can characterise the 'target criteria' in this model with two rules, as in the CGG model. The first is $\tilde{y}_t + \tilde{y}_t^* + \xi(\pi_t + \pi_t^*) = 0$. This rule, at first glance, appears to be simply the sum of the two 'target criteria' in the CGG model. It is, except that the inflation rates that appear in this trade-off (π_t and π_t^*) are based on the CPI, rather than the producer price index (PPI) as in CGG's model.

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12 ξ is a preference parameter defined below.

13 Specifically, I show below that $b = \rho + (1-\rho)\sigma\xi$; parameters are defined below.

The second target criterion is $\frac{1}{\sigma}\tilde{q}_t + \xi(\pi_t - \pi_t^*) = 0$, where q_t is the real exchange rate (defined as foreign prices relative to home prices expressed in a common currency) and $\tilde{q}_t$ is the deviation of the real exchange rate from its efficient level. (The parameters in this equation are defined below.) The important point is that the trade-off described here relates real exchange rates and relative CPI inflation rates. For example, even if inflation is low in the home country relative to the foreign country, optimal policy may, under some circumstances, still call for a tightening of the monetary policy stance in the home country if the home currency is sufficiently undervalued.

Like CGG, the optimal interest rate rules that support these targeting rules can be derived. These interest rate reaction functions are $r_t = \overline{rr}_t + b\pi_t$ and $r_t^* = \overline{rr}_t^* + b\pi_t^*$. They are identical to the ones derived in CGG (the parameter *b* is the same), except they target CPI inflation rather than PPI inflation as in CGG. The conclusion is that while the target criteria include currency misalignments, the currency misalignment is not in the optimal interest rate reaction function. If we focus on only the latter, we miss this trade-off the policy-maker faces.

Previous studies have found little welfare gain from adding an exchange rate variable to the Taylor rule. Properly speaking, these studies examine the effects of simple targeting rules under commitment. My results describe the welfare function, the target criteria and the optimal interest rate reaction functions under discretionary policy-making. But the results here suggest that even if there is no role for the currency misalignment in a simple targeting rule, exchange rate concerns may still be important in terms of welfare. This point is brought out in the context of a relatively simple model that can be solved analytically (with approximations), but is obscured in larger models that are solved numerically.[14]

The paper proceeds in two steps. After setting out the objectives of households and firms, the production functions, and the market structure, a global loss function for cooperative monetary policy-makers is derived. The period loss function can be derived without making any assumptions about how goods prices or wages are

[14] Coenen *et al* (2008) examine optimal monetary policy in a two-country model that exhibits incomplete pass-through. However, the numerical analysis does not allow the reader to see explicitly the role of currency misalignments.

set.[15] I find that in addition to squares and cross-products of home and foreign output gaps, and the cross-sectional dispersion of goods prices within each country, the loss also depends on the squared currency misalignment. This loss function evaluates the welfare costs arising because firms set different prices in the home and foreign countries (assuming that the costs of selling the good in both countries are identical), and does not depend on whether the price differences arise from local-currency price stickiness, from price discrimination, or for some other reason.

The paper then follows CGG and assumes a Calvo mechanism for price setting. However, allowance is made for the possibility of LCP. As in CGG, optimal policy under discretion is derived. Both targeting rules and instrument rules are obtained. Only optimal cooperative policy is considered. The goal is to quantify the global loss from currency misalignments, which can be seen by deriving the loss function for a policy-maker that aims to maximise the sum of utilities of home and foreign households. Practically speaking, international agreements that prohibit currency manipulation may mean that the currency misalignment can only be addressed in a cooperative environment.[16] That is, it seems likely that if central banks are going to move toward policies that explicitly target exchange rates, they will do so cooperatively.

2. The Model

The model is nearly identical to CGG's. It is based on two countries of equal size, while CGG allow the population of the countries to be different. Since the population size plays no real role in their analysis, the model here is simplified along this dimension. But two significant generalisations are made. The first is to allow for different preferences in the two countries. Home agents may derive greater utility from goods produced in the home country. Home households put a weight of $\frac{\nu}{2}$ on home goods and $1-\frac{\nu}{2}$ on foreign goods (and *vice versa* for foreign

[15] Except that I do assume that all households (which are identical) set the same wage. As I note later, this rules out a model of staggered wage setting such as in Erceg, Henderson and Levin (2000), though a generalisation to encompass that case would be straightforward.

[16] For example, under the rules of the World Trade Organization (WTO), countries may not deliberately devalue their currencies.

households.) This is a popular assumption in the open economy macroeconomics literature, and can be considered as a shortcut for modelling ‘openness’. That is, a less open economy puts less weight on consumption of imported goods, and in the limit the economy becomes closed if it imports no goods. The second major change, as already noted, is to allow for goods to be sold at different prices in the home and foreign countries.

The model assumes two countries, each inhabited with a continuum of households, normalised to a total of one in each country. Households have utility over consumption of goods and disutility from provision of labour services. In each country, there is a continuum of goods produced, each by a monopolist. Households supply labour to firms located within their own country, and get utility from all goods produced in both countries. Each household is a monopolistic supplier of a unique type of labour to firms within its country. Trade in a complete set of nominally denominated contingent claims is assumed.

Monopolistic firms produce output using only labour, subject to technology shocks.

In this section, no assumptions are made about how wages are set by monopolistic households or prices are set by monopolistic firms. In particular, prices and wages may be sticky, and there may be LCP or PCP for firms. The period loss function is derived for the policy-maker, which expresses the loss (relative to the efficient outcome) in terms of within-country and international price misalignments and output gaps. This loss function applies under various assumptions about how prices are actually set, and so is more general than the policy rules subsequently derived which depend on the specific nature of price and wage setting.

All households within a country are identical. It is assumed that in each period their labour supplies are identical. This assumption rules out staggered wage setting as in Erceg *et al* (2000), because in that model there will be dispersion in labour input across households that arises from the dispersion in wages. The set-up is consistent with sticky wages, but not wage dispersion. However, it is entirely straightforward to generalise the loss functions derived to allow for wage dispersion following the steps in Erceg *et al*. This is not done so that the model is more directly comparable to CGG.

2.1 Households

The representative household, *h*, in the home country maximises

$$U_t(h) = E_t\left\{\sum_{j=0}^{\infty}\beta^j\left[\frac{1}{1-\sigma}C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi}N_{t+j}(h)^{1+\phi}\right]\right\},\ \sigma>0,\phi\geq 0 \tag{1}$$

$C_t(h)$ is the consumption aggregate. We assume Cobb-Douglas preferences:

$$C_t(h) = \left(C_{Ht}(h)\right)^{\frac{\nu}{2}}\left(C_{Ft}(h)\right)^{1-\frac{\nu}{2}},\ 0\leq\nu\leq 2\,. \tag{2}$$

If $\nu = 1$, home and foreign preferences are identical as in CGG. There is home bias in preferences when $\nu > 1$.

In turn, $C_{Ht}(h)$ and $C_{Ft}(h)$ are CES aggregates over a continuum of goods produced in each country:

$$C_{Ht}(h) = \left(\int_0^1 C_{Ht}\left(h,f\right)^{\frac{\xi-1}{\xi}}df\right)^{\frac{\xi}{\xi-1}} \text{ and } C_{Ft}(h) = \left(\int_0^1 C_{Ft}\left(h,f\right)^{\frac{\xi-1}{\xi}}df\right)^{\frac{\xi}{\xi-1}}. \tag{3}$$

$N_t(h)$ is an aggregate of the labour services that the household sells to each of a continuum of firms located in the home country:

$$N_t(h) = \int_0^1 N_t(h,f)df\ . \tag{4}$$

Households receive wage income, $W_t(h)N_t(h)$, and aggregate profits from home firms, Γ_t. They pay lump-sum taxes each period, T_t. Each household can trade in a complete market in contingent claims (arbitrarily) denominated in the home currency. The budget constraint is given by:

$$P_tC_t(h) + \sum_{s^t\in\Omega_t} Z(\nabla^{t+1}\,|\,\nabla^t)D(h,\nabla^{t+1}) = W_t(h)N_t(h) + \Gamma_t - T_t + D(h,\nabla^t)\,, \tag{5}$$

where $D(h,\nabla^t)$ represents household h's pay-offs on state-contingent claims for state ∇^t. $Z(\nabla^{t+1}\,|\,\nabla^t)$ is the price of a claim that pays one dollar in state ∇^{t+1}, conditional on state ∇^t occurring at time *t*.

In this equation, P_t is the exact price index for consumption, given by:

$$P_t = k^{-1} P_{Ht}^{\nu/2} P_{Ft}^{1-(\nu/2)},\; k = (1-(\nu/2))^{1-(\nu/2)} (\nu/2)^{\nu/2}\,. \tag{6}$$

P_{Ht} is the home-currency price of the home aggregate good and P_{Ft} is the home-currency price of the foreign aggregate good. Equation (6) follows from cost minimisation. Also, from cost minimisation, P_{Ht} and P_{Ft} are the usual CES aggregates over prices of individual varieties, *f*:

$$P_{Ht} = \left(\int_0^1 P_{Ht}(f)^{1-\xi}\, df\right)^{\frac{1}{1-\xi}}, \text{ and } P_{Ft} = \left(\int_0^1 P_{Ft}(f)^{1-\xi}\, df\right)^{\frac{1}{1-\xi}}. \tag{7}$$

Foreign households have analogous preferences and face an analogous budget constraint.

Because all home households are identical, we can drop the index for the household and use the fact that aggregate per capita consumption of each good is equal to the consumption of each good by each household. The first-order conditions for consumption are given by:

$$P_{Ht} C_{Ht} = \frac{\nu}{2} P_t C_t\,, \tag{8}$$

$$P_{Ft} C_{Ft} = \left(1-\frac{\nu}{2}\right) P_t C_t\,, \tag{9}$$

$$C_{Ht}(f) = \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} C_{Ht} \quad \text{and} \quad C_{Ft}(f) = \left(\frac{P_{Ft}(f)}{P_{Ft}}\right)^{-\xi} C_{Ft}\,, \tag{10}$$

$$\beta\left(C(\nabla^{t+1}) / C(\nabla^t)\right)^{-\sigma} (P_t / P_{t+1}) = \ddot{Z}(\nabla^{t+1}\,|\,\nabla^t)\,. \tag{11}$$

In Equation (11), an index for the state at time *t* is made explicit for the purpose of clarity. $\ddot{Z}(\nabla^{t+1} | \nabla^t)$ is the normalised price of the state-contingent claim. That is, it is defined as $Z(\nabla^{t+1} | \nabla^t)$ divided by the probability of state ∇^{t+1} conditional on state ∇^t.

Note that the sum of $Z(\nabla^{t+1} | \nabla^t)$ across all possible states at time $t + 1$ must equal $1/R_t$, where R_t denotes the gross nominal yield on a one-period non-state-contingent bond. Therefore, taking a probability-weighted sum across all states of Equation (11), we have the familiar Euler equation:

$$\beta R_t E_t \left[\left(C(\nabla^{t+1}) / C(\nabla^t) \right)^{-\sigma} (P_t / P_{t+1}) \right] = 1. \tag{12}$$

Analogous equations hold for foreign households. Since contingent claims are (arbitrarily) denominated in home currency, the first-order condition for foreign households that is analogous to Equation (11) is:

$$\beta \left(C^*(\nabla^{t+1}) / C^*(\nabla^t) \right)^{-\sigma} (E_t P_t^* / E_{t+1} P_{t+1}^*) = \ddot{Z}(\nabla^{t+1} | \nabla^t). \tag{13}$$

Following the unfortunate notation of CGG, E_t is the nominal exchange rate, defined as the home-currency price of foreign currency, and should not be mistaken for the conditional expectations operator. As noted above, at this stage the labour input of all households is assumed to be the same, so $N_t = N_t(h)$.

2.2 Firms

Each home good, $Y_t(f)$ is made by firm *f* according to a production function that is linear in the labour input. These are given by:

$$Y_t(f) = A_t N_t(f). \tag{14}$$

Note that the productivity shock, A_t, is common to all firms in the home country. $N_t(f)$ is a CES composite of individual home-country household labour, given by:

$$N_t(f)=\left(\int_0^1 N_t(h,f)^{\frac{\eta_t-1}{\eta_t}}\,dh\right)^{\frac{\eta_t}{\eta_t-1}}, \tag{15}$$

where the technology parameter, η_t, is stochastic and common to all home firms.

Profits are given by:

$$\Gamma_t(f)=P_{Ht}(f)C_{Ht}(f)+E_tP^*_{Ht}(f)C^*_{Ht}(f)-(1-\tau_t)W_tN_t(f). \tag{16}$$

In this equation, $P_{Ht}(f)$ is the home-currency price of the good when it is sold in the home country and $P^*_{Ht}(f)$ is the foreign-currency price of the good when it is sold in the foreign country. $C_{Ht}(f)$ is aggregate sales of the good in the home country:

$$C_{Ht}(f)=\int_0^1 C_{Ht}(h,f)\,dh. \tag{17}$$

Sales of the same good in the foreign country, $C^*_{Ht}(f)$, is defined analogously. It follows that $Y_t(f)=C_{Ht}(f)+C^*_{Ht}(f)$. The subsidy for using labour is τ_t.

There are analogous equations for $Y^*_t(f)$, with the foreign productivity shock given by A^*_t, the foreign technology parameter shock given by η^*_t, and foreign subsidy given by τ^*_t.

2.3 Equilibrium

Goods market-clearing conditions in the home and foreign countries are given by:

$$Y_t=C_{Ht}+C^*_{Ht}=\frac{\nu}{2}\frac{P_tC_t}{P_{Ht}}+\left(1-\frac{\nu}{2}\right)\frac{P^*_tC^*_t}{P^*_{Ht}}=k^{-1}\left(\frac{\nu}{2}S_t^{1-(\nu/2)}C_t+\left(1-\frac{\nu}{2}\right)(S^*_t)^{-\nu/2}C^*_t\right), \tag{18}$$

$$Y^*_t=C_{Ft}+C^*_{Ft}=\left(1-\frac{\nu}{2}\right)\frac{P_tC_t}{P_{Ft}}+\frac{\nu}{2}\frac{P^*_tC^*_t}{P^*_{Ft}}=k^{-1}\left(\frac{\nu}{2}(S^*_t)^{1-(\nu/2)}C^*_t+\left(1-\frac{\nu}{2}\right)S_t^{-\nu/2}C_t\right). \tag{19}$$

S_t and S_t^* are used to represent the price of imported goods relative to locally produced goods in the home and foreign countries, respectively (the inverse of the standard definition for the terms of trade):

$$S_t = P_{Ft} / P_{Ht} , \tag{20}$$

$$S_t^* = P_{Ht}^* / P_{Ft}^* . \tag{21}$$

Equations (11) and (13) provide the familiar condition that arises in open economy models with a complete set of state-contingent claims when PPP does not hold:

$$\left(\frac{C_t}{C_t^*}\right)^{\sigma} = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}} (S_t^*)^{-\nu/2} S_t^{(\nu/2)-1} . \tag{22}$$

This condition equates the marginal rate value of a dollar for home and foreign households, in terms of its purchasing power over aggregate consumption in each country.

Total employment is determined by output in each industry:

$$N_t = \int_0^1 N_t(f)df = A_t^{-1} \int_0^1 Y_t(f)df = A_t^{-1}\left(C_{Ht}V_{Ht} + C_{Ht}^* V_{Ht}^*\right), \tag{23}$$

where

$$V_{Ht} \equiv \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} dfc \text{, and } V_{Ht}^* \equiv \int_0^1 \left(\frac{P_{Ht}^*(f)}{P_{Ht}^*}\right)^{-\xi} df \ . \tag{24}$$

3. Log-linearised Model

This section presents some log-linear approximations to the models presented above. The full set of log-linearised equations appears in Appendix A. The approach to the optimal policy decision is to consider a second-order approximation of the welfare function around the efficient steady state. The derivation of the loss function itself requires a second-order approximation of the utility function itself, but in the course of the derivation will actually require second-order approximations to some of the equations of the model. However, for many purposes, the first-order approximations are useful: the constraints in the optimisation problem need only be approximated to the first order; the optimality conditions for monetary policy – the ‘target criteria’ – are linear; and, the dynamics under the optimal policy can be analysed in the linearised model.

With regard to notation, lower case letters refer to the log of the corresponding upper case letter less its deviation from steady state.

If firms set the same price for home and foreign consumers, then $s_t^* = -s_t$. To a first order, $s_t^* = -s_t$ is assumed even if firms set different prices in the two countries. That is, for the aggregate price indices, $p_{Ft} - p_{Ht} = p_{Ft}^* - p_{Ht}^*$, so relative prices are the same in the home and foreign countries. This relationship will turn out to hold in the Calvo pricing model under LCP, when the frequency of price adjustment is identical in the two countries.

The log of the deviation from the law of one price is defined as:

$$\Delta_t \equiv e_t + p_{Ht}^* - p_{Ht}. \tag{25}$$

Because $s_t^* = -s_t$, the deviation of the law of one price is the same for both goods: $\Delta_t \equiv e_t + p_{Ft}^* - p_{Ft}$.

The market-clearing conditions, (18) and (19), are approximated as:

$$y_t = \frac{\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t + \frac{2-\nu}{2} c_t^*, \tag{26}$$

$$y_t^* = \frac{-\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t^* + \frac{2-\nu}{2} c_t. \tag{27}$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for home and foreign households, Equation (22), is given by:

$$\sigma c_t - \sigma c_t^* = \Delta_t + (\nu - 1) s_t . \tag{28}$$

c_t, c_t^*, and s_t can be expressed in terms of y_t, y_t^* and Δ_t:

$$c_t = \frac{D + \nu - 1}{2D} y_t + \frac{D - (\nu - 1)}{2D} y_t^* + \frac{\nu(2 - \nu)}{2D} \Delta_t \tag{29}$$

$$c_t^* = \frac{D + \nu - 1}{2D} y_t^* + \frac{D - (\nu - 1)}{2D} y_t - \frac{\nu(2 - \nu)}{2D} \Delta_t \tag{30}$$

$$s_t = \frac{\sigma}{D}(y_t - y_t^*) - \frac{(\nu - 1)}{D} \Delta_t , \tag{31}$$

where $D \equiv \sigma\nu(2 - \nu) + (\nu - 1)^2$.

The model is closed and solutions for the endogenous variables can be derived once policy rules are determined. I now turn to consideration of optimal monetary policy.

4. Loss Functions and Optimal Policy

The loss function is derived for the cooperative monetary policy problem, which is the relevant criterion for evaluating world welfare. It is based on a second-order approximation to households' utility functions. Loss is measured relative to the efficient allocations.

The policy-maker wishes to minimise

$$-E_t \sum_{j=0}^{\infty} \beta^j X_{t+j} . \tag{32}$$

This loss function is derived from household's utility, given in Equation (1). The discount factor, β, is the household's, and the per-period loss, X_{t+j}, represents the difference between the utility of the market-determined levels of consumption and leisure and the maximum utility achievable under efficient allocations.

The aim is to highlight the global inefficiency that arises from currency misalignments. For that reason, the difficult issues involved with deriving the loss function for a non-cooperative policy-maker and defining a non-cooperative policy game are set aside.

It is worth noting for future work that there are three technical problems that arise in the case of non-cooperative policy. One problem is discussed in detail in Devereux and Engel (2003). To consider policies in the non-cooperative framework requires an examination of the effects of one country changing its policies, holding the other countries' policies constant. In a complete markets world, to evaluate all possible alternative policies, it is necessary to calculate prices of state-contingent claims under alternative policies. In particular, Equation (22), which was derived assuming equal initial home and foreign wealth, cannot be assumed to hold under all alternative policies that the competitive policy-maker considers. Policies can change state-contingent prices and therefore change the wealth distribution. It is not uncommon for studies of optimal policy in open economies to treat Equation (22) as if it were independent of the policy choices, but it is not. There is a special case in which it holds in all states, which is the set-up in CGG. When the law of one price holds, when home and foreign households have identical preferences and there are no preference shocks, and when preferences over home and foreign aggregates are Cobb-Douglas, Equation (22) holds in all states. This well-known outcome arises because in all states of the world, the terms of trade change in such a way as to leave home/foreign wealth unchanged.

Even if this technical challenge could be overcome,[17] there are a couple of other technical challenges that appear in this framework that did not plague CGG. First, in the LCP model, it is not the case that $S_t^{*-1} = S_t$, that is, the relative price of foreign to home goods is not the same in both countries. This relationship holds up

[17] The Appendix of Devereux and Engel (2003) demonstrates how this problem can be handled (available at http://www.restud.com/uploads/suppmat/app0017.pdf).

to a first-order log-linear approximation in the LCP model (as long as we assume equal speeds of price adjustment for all goods,[18] but only up to a first-order approximation. Complex first-order relative price terms (s_t and s_t^*) appear in the objective function of the non-cooperative policy-maker. However, those wash out in the objective function under cooperation. Second, CGG neatly dichotomise the choice variables in their model – the home policy-maker sets home PPI inflation and the home output gap taking the foreign policy choices as given, and *vice versa* for the foreign policy-maker. Such a neat dichotomy is not possible in the model with currency misalignments – we cannot just assign the exchange rate to one of the policy-makers.

In a sense, all of these technical problems are related to the real world reason why it is more reasonable to examine policy in the cooperative framework when currency misalignments are possible. The non-cooperative model assumes that central banks are willing and able to manipulate currencies to achieve better outcomes. However, in practice both WTO rules and implicit rules of neighbourliness prohibit this type of policy. Major central banks have typically been unwilling to announce explicit targets for exchange rates without full cooperation of their partners.

Even if the cooperative policy analysis is not a realistic description of actual policy decision-making, the welfare function is a measure of what could be achieved under cooperation.

Appendix B shows the steps for deriving the loss function when there are no currency misalignments, but with home bias in preferences. It is worth pointing out one aspect of the derivation. In closed economy models with no investment or government, consumption equals output. That is an exact relationship, and therefore the deviation of consumption from the efficient level equals the deviation of output from the efficient level to any order of approximation: $\tilde{c}_t = \tilde{y}_t$. In the open economy, the relationship is not as simple. When preferences of home and foreign agents are identical, and markets are complete, then the consumption aggregates in home and foreign are always equal (up to a constant of proportionality equal to relative wealth). But that is not true when preferences are not the same. Equation (22) shows that $C_t = C_t^*$ does not hold under complete

[18] See Benigno (2004) and Woodford (forthcoming) on this point.

markets, even if the law of one price holds for both goods. Because of this, $\tilde{c}_t + \tilde{c}_t^*$ does not equal $\tilde{y}_t + \tilde{y}_t^*$, except to a first-order approximation. Since a second-order approximation of the utility function is being used, the effect of different preferences (or the effects of the terms of trade) needs to be taken into account when translating consumption gaps into output gaps.

The period loss of the policy-maker under no currency misalignments is $-X_t$, where:

$$X_t = \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p^*t}^2\right). \quad (33)$$

This depends on the squared output gap in each country, as well as the squared difference in the output gaps. The terms $\sigma_{p_H t}^2$ and $\sigma_{p_F^* t}^2$ represent the cross-sectional variance of prices of home goods and foreign goods, respectively. (Recall $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$.)

Appendix B also shows the derivation of the loss function in the more general case in which currency misalignments are possible. Two aspects of the derivation merit attention. First, in examining the first-order dynamics of the model, the first-order approximation $s_t^* = -s_t$ can be used. That is an exact equation when the law of one price holds, but it is not necessary that this relationship hold to a second-order approximation. The derivation of the loss function must take this into account. The second point to note is that, as is standard in this class of models, price dispersion leads to inefficient use of labour. But, to a second-order approximation, this loss depends only on the cross-section variances of p_{Ht}, p_{Ht}^*, p_{Ft}, and p_{Ft}^*, and not their co-movements (which would play a role in a third-order approximation.)

In the case of currency misalignments, X_t is given by:

$$\begin{aligned} X_t = {} & \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ & - \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_H^* t}^2 + \frac{\nu}{2}\sigma_{p_F^* t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right]. \end{aligned} \quad (34)$$

It is important to recognise that this loss function and the loss function derived previously (Equation (33)) do not depend on how prices are set – indeed whether prices are sticky or not. The loss function of Equation (34) generalises Equation (33) to the case in which there are deviations from the law of one price, so that $\Delta_t \neq 0$. This can be seen by directly comparing the two equations. In Equation (34), $\sigma^2_{p_H t}$ is the cross-sectional variance of home goods prices in the home country, $\sigma^2_{p_H{}^* t}$ is the cross-sectional variance of home goods prices in the foreign country, etc. If there is no currency misalignment, then $\Delta_t = 0$, so $p_{Ht}(f) = p^*_{Ht}(f) + e_t$ and $p_{Ft}(f) = p^*_{Ft}(f) + e_t$ for each firm *f*. In that case, $\sigma^2_{p_H t} = \sigma^2_{p_H{}^* t}$ and $\sigma^2_{p_F t} = \sigma^2_{p_F{}^* t}$ because the exchange rate does not affect the cross-sectional variance of prices. If we have $\Delta_t = 0$, $\sigma^2_{p_H t} = \sigma^2_{p_H{}^* t}$, and $\sigma^2_{p_F t} = \sigma^2_{p_F{}^* t}$, then (34) reduces to (33).

Why does the currency misalignment appear in the loss function? That is, if both home and foreign output gaps are zero, and all inflation rates are zero, what problem does a misaligned currency cause?[19] From Equation (31), if the currency is misaligned, then internal relative prices (s_t) must also differ from their efficient level if the output gap is zero. The home and foreign countries could achieve full employment, but the distribution of the output between home and foreign households is inefficient. For example, suppose $\Delta_t > 0$, which from Equation (31) implies $\tilde{s}_t < 0$ if both output gaps are eliminated. On the one hand, $\Delta_t > 0$ tends to lead to overall consumption at home to be high relative to foreign consumption (Equations (29) and (30)). That occurs because financial markets make payments to home residents when their currency is weak. But home residents have a bias for home goods. That would lead to overproduction in the home country, were it not for relative price adjustments – which is why $\tilde{s}_t < 0$.

It is worth highlighting the fact that the loss functions are derived without specific assumptions about price setting not to give a false patina of generality to the result, but to emphasise that the loss in welfare arises not specifically from price stickiness but from prices that do not deliver the efficient allocations. Of course it is the specific assumptions of nominal price and wage setting that give rise to the internal and external price misalignments in this model, and indeed monetary

[19] Optimal inflation targets are zero in this model because we have assumed zero inflation in steady state.

policy would be ineffective if there were no nominal price or wage stickiness. But one could imagine a number of mechanisms that give rise to deviations from the law of one price, because the literature has produced a number of models based both on nominal stickiness and real factors. In the next section, the CGG model is modified in the simplest way – allowing LCP instead of PCP – to examine further the implications of currency misalignments.

5. Price and Wage Setting

This section introduces the models of price and wage setting. Following CGG, wages are set flexibly by monopolistic suppliers of labour, but goods prices are sticky.

Wages adjust continuously, but households exploit their monopoly power by setting a wage that incorporates a mark-up over their utility cost of work.

Government is assumed to have only limited fiscal instruments. The government can set a constant output subsidy rate for monopolistic firms, which will achieve an efficient allocation in the non-stochastic steady state. But unfortunately, the mark-up charged by workers is time-varying because the elasticity of demand for their labour services is assumed to follow a stochastic process. These shocks are sometimes labeled ‘cost-push’ shocks, and give rise to the well-known trade-off in CGG’s work between controlling inflation and achieving a zero output gap.

Households are monopolistic suppliers of their unique form of labour services. Household *h* faces demand for its labour services given by:

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\eta_t} N_t\,, \tag{35}$$

where

$$W_t = \left(\int_0^1 W_t(h)^{1-\eta_t}\, dh\right)^{\frac{1}{1-\eta_t}}. \tag{36}$$

The first-order condition for household *h*'s choice of labour supply is given by:

$$\frac{W_t(h)}{P_t} = (1+\mu_t^W)(C_t(h))^{\sigma}(N_t(h))^{\phi}, \text{ where } \mu_t^W \equiv \frac{1}{\eta_t - 1}. \tag{37}$$

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units).

Because all households are identical, $W_t = W_t(h)$ and $N_t = N_t(h)$.

Since all households are identical, Equation (37) implies:

$$W_t / P_{Ht} = (1+\mu_t^W) C_t^{\sigma} N_t^{\phi} S_t^{1-(\nu/2)}. \tag{38}$$

Three different scenarios for firm behaviour are considered. In the first, prices can be adjusted freely. In the second – the PCP scenario that CGG analyse – firms set prices in their own country's currency and face a Calvo pricing technology. In the third, when firms are allowed to change prices according to the Calvo pricing rule, they set a price in their own currency for sales in their own country and a price in the other country's currency for exports. This is the LCP scenario.

The following notation is adopted. For any variable Z_t:

- $\dot{Z}_t$ is the value under flexible prices.

- $\bar{Z}_t$ is the value of variables under globally efficient allocations. In other words, this is the value for variables if prices were flexible, and optimal subsidies to monopolistic suppliers of labour and monopolistic producers of goods were in place. This includes a time-varying subsidy to suppliers of labour to offset the time-varying mark-up in wages in Equation (37).

- $\tilde{Z}_t$ is the gap: the value of the variable under PCP or LCP relative to $\bar{Z}_t$

We will treat the PCP and LCP cases separately, so there will be no need to use notation to distinguish variables under PCP versus LCP.

5.1 Flexible Prices

Home firms maximise profits given by Equation (16), subject to the demand curve (10). They optimally set prices as a mark-up over marginal cost:

$$\dot{P}_{Ht}(f) = \dot{E}_t \dot{P}^*_{Ht}(f) = (1-\tau_t)(1+\mu^P)\dot{W}_t / A_t \text{, where } \mu^P \equiv \frac{1}{\xi - 1}. \tag{39}$$

When optimal subsidies are in place:

$$\overline{P}_{Ht}(f) = \overline{E}_t \overline{P}^*_{Ht}(f) = \overline{W}_t / A_t. \tag{40}$$

From Equations (37), (39) and (40), it is apparent that the optimal subsidy satisfies

$$(1-\tau_t)(1+\mu^P)(1+\mu^W_t) = 1. \tag{41}$$

Note that from Equation (39) all flexible price firms are identical and set the same price. Because the demand functions of foreign residents have the same elasticity of demand for home goods as home residents, firms set the same price for sale abroad:

$$E_t \dot{P}^*_{Ht} = \dot{P}_{Ht} \text{ and } E_t \overline{P}^*_{Ht} = \overline{P}_{Ht}. \tag{42}$$

Equation (41), combined with Equation (38), implies:

$$\dot{P}_{Ht} = \dot{E}_t \dot{P}^*_{Ht} = \dot{W}_t / ((1+\mu^W)A_t) \text{ and } \dot{P}^*_{Ft} = \dot{E}^{-1}_t \dot{P}_{Ft} = \dot{W}^*_t / ((1+\mu^{*W})A^*_t). \tag{43}$$

So, it can be concluded that:

$$\dot{S}^{*-1}_t = \dot{S}_t. \tag{44}$$

Because $\dot{P}_{Ht}(f)$ is identical for all firms, Equation (23) collapses to

$$\dot{Y}_t = A_t \dot{N}_t. \tag{45}$$

5.2 PCP

A standard Calvo pricing technology is assumed. A given firm may reset its prices with probability $1-\theta$ each period. Assume that when the firm resets its price, it will be able to reset its prices for sales in both markets. The PCP firm sets both prices in its own currency – that is, the home firm sets both $P_{Ht}(f)$ and $P_{Ht}^{**}(f) \equiv E_t P_{Ht}^{*}(f)$ in home currency. (As will become apparent, the firm optimally chooses the same price for both markets, $P_{Ht}(f) = P_{Ht}^{**}(f)$.)

The firm's objective is to maximise its value. Its value is equal to the value of its entire stream of dividends, valued at state-contingent prices. Given Equation (11), it is apparent that the firm that selects its prices at time *t*, chooses its reset prices, $P_{Ht}^{0}(f)$ and $P_{Ht}^{0**}(f)$, to maximise

$$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[P_{Ht}^{0}(f) C_{Ht+j}(f) + P_{Ht}^{0**} C_{Ht+j}^{*}(f) - (1-\tau_t) W_{t+j} N_{t+j}(f) \right], \qquad (46)$$

subject to the sequence of demand curves given by Equation (10) and the corresponding foreign demand equation for home goods. In this equation, define

$$Q_{t,t+j} \equiv \beta^j \left(C_{t+j} / C_t \right)^{-\sigma} (P_t / P_{t+j}) . \qquad (47)$$

The solution for the optimal price for the home firm for sale in the home country is given by:

$$P_{Ht}^{0}(z) = \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} P_{Ht+j}^{\xi} C_{Ht+j} / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} P_{Ht+j}^{\xi} C_{Ht+j}} . \qquad (48)$$

For sale in the foreign market, the optimal home currency price is:

$$P_{Ht}^{0**}(z) = \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} (E_{t+j} P_{Ht+j}^*)^{\xi} C_{Ht+j}^* / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (E_{t+j} P_{Ht+j}^*)^{\xi} C_{Ht+j}^*} . \qquad (49)$$

Under the Calvo price-setting mechanism, a fraction θ of prices remain unchanged from the previous period. From Equation (7), we can write:

$$P_{Ht} = \left[\theta (P_{Ht-1})^{1-\xi} + (1-\theta)(P_{Ht}^0)^{1-\xi} \right]^{1/(1-\xi)}, \qquad (50)$$

$$P_{Ht}^* = \left[\theta (P_{Ht-1}^*)^{1-\xi} + (1-\theta)(P_{Ht}^{0*})^{1-\xi} \right]^{1/(1-\xi)}. \qquad (51)$$

Taking Equations (48), (49), (50) and (51), it can be seen that the law of one price holds under PCP. That is, $P_{Ht}(f) = E_t P_{Ht}^*(f)$ for all *f*, hence $P_{Ht} = E_t P_{Ht}^*$. Hence, under PCP we have $S_t^{*-1} = S_t$.

5.3 LCP

The same environment as for the PCP case holds, with the sole exception that the firm sets its price for export in the importer's currency rather than its own currency when it is allowed to reset prices. The home firm, for example, sets $P_{Ht}^*(f)$ in foreign currency. The firm that can reset its price at time *t* chooses its reset prices, $P_{Ht}^0(f)$ and $P_{Ht}^{0*}(f)$, to maximise

$$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[P_{Ht}^0(z) C_{Ht+j}(f) + E_t P_{Ht}^{0*} C_{Ht+j}^*(f) - (1-\tau_t) W_{t+j} N_{t+j}(f) \right]. \qquad (52)$$

The solution for $P_{Ht}^{0}(z)$ is identical to Equation (48), while for export prices,

$$P_{Ht}^{0*}(z)=\frac{E_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}(1-\tau_t)W_{t+j}(P_{Ht+j}^{*})^{\xi}C_{Ht+j}^{*}/A_{t+j}}{E_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}E_{t+j}(P_{Ht+j}^{*})^{\xi}C_{Ht+j}^{*}}. \tag{53}$$

Equations (50) and (51) hold in the LCP case as well. However, the law of one price does not hold.

5.4 Subsidies

As in CGG, assume that subsidies to monopolists are not set at their optimal level except in steady state. That is, instead of the efficient subsidy given in Equation (41), we have:

$$(1-\tau)(1+\mu^{P})(1+\mu^{W})=1. \tag{54}$$

Here, μ^{W} is the steady-state level of μ_t^{W}. The time subscript on the subsidy rate τ has been dropped because it is not time-varying.

6. Log-linearised Phillips Curves

Under PCP, a New Keynesian Phillips curve for an open economy can be derived:

$$\pi_{Ht}=\delta(\tilde{w}_t-\tilde{p}_{Ht})+\beta E_t\pi_{Ht+1},$$

or:

$$\pi_{Ht}=\delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t+\frac{\sigma(D-1)}{2D}\tilde{y}_t^{*}\right]+\beta E_t\pi_{Ht+1}+u_t, \tag{55}$$

where $u_t=\delta\mu_t^{W}$.

Similarly for foreign producer price inflation:

$$\pi_{Ft}^{*} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^{*} + \frac{\sigma(D-1)}{2D}\tilde{y}_t\right] + \beta E_t \pi_{Ft+1}^{*} + u_t^{*}. \tag{56}$$

In the LCP model, the law of one price deviation is not zero, so that:

$$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^{*} + \frac{D-(\nu-1)}{2D}\Delta_t\right] + \beta E_t \pi_{Ht+1} + u_t, \tag{57}$$

$$\pi_{Ft}^{*} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^{*} + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{\nu-1-D}{2D}\Delta_t\right] + \beta E_t \pi_{Ft+1}^{*} + u_t^{*}. \tag{58}$$

There are also price adjustment equations for the local prices of imported goods:

$$\pi_{Ht}^{*} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^{*} - \left(\frac{D+\nu-1}{2D}\right)\Delta_t\right] + \beta E_t \pi_{Ht+1}^{*} + u_t, \tag{59}$$

$$\pi_{Ft} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^{*} + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{D+\nu-1}{2D}\Delta_t\right] + \beta E_t \pi_{Ft+1} + u_t^{*}. \tag{60}$$

Equations (57)–(58) and (59)–(60) imply that $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^{*} - \pi_{Ht}^{*}$. Assuming a symmetric initial condition, so that $p_{F0} - p_{H0} = p_{F0}^{*} - p_{H0}^{*}$, leads to the conclusion that $s_t^{*} = -s_t$ as noted above. That is, the relative price of foreign to home goods is the same in both countries. It is worth emphasising that this is true in general for a first-order approximation.

The efficient allocations cannot be obtained with monetary policy alone because of the sticky-price externality, and because the policy-maker is assumed to not have access to fiscal instruments aside from setting a constant subsidy rate to firms.

The policy-maker has home and foreign nominal interest rates as instruments. As is standard in the literature, the policy-maker can be modelled as directly choosing output gaps, inflation levels, and (in the LCP case) deviations from the law of one price, subject to constraints. From the first-order conditions, the optimal choice of nominal interest rates can be backed out using a log-linearised version of Equation (12) and its foreign counterpart, given by:

$$r_t - E_t \pi_{t+1} = \sigma(E_t c_{t+1} - c_t)\,, \tag{61}$$

$$r_t^* - E_t \pi_{t+1}^* = \sigma(E_t c_{t+1}^* - c_t^*)\,. \tag{62}$$

In these equations, π_t and π_t^* refer to home and foreign consumer price inflation, respectively:

$$\pi_t = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft}\,, \tag{63}$$

$$\pi_t^* = \frac{\nu}{2}\pi_{Ft}^* + \frac{2-\nu}{2}\pi_{Ht}^*\,. \tag{64}$$

7. Optimal Policy under PCP

As is familiar in the New Keynesian models with Calvo price adjustment, the loss function can be rewritten in the form:

$$-E_t \sum_{j=0}^{\infty} \beta^j X_{t+j} = -E_t \sum_{j=0}^{\infty} \beta^j \Psi_{t+j}$$

where:

$$\Psi_t \propto \frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2 + (\tilde{y}_t^*)^2) - \frac{\xi}{2\delta}\left((\pi_{Ht})^2 + (\pi_{Ft}^*)^2\right). \tag{65}$$

This loss function extends the one derived in CGG to the case of home bias in preferences, or non-traded goods (that is, $\nu \geq 1$ rather than $\nu = 1$).

The policy-maker chooses values for $\tilde{y}_t$, $\tilde{y}_t^*, \pi_{Ht}$ and π_{Ft}^* to minimise the loss, subject to the Phillips curves (55) and (56). Under discretion, the policy-maker takes past values of $\tilde{y}_t$, $\tilde{y}_t^*, \pi_{Ht}$ and π_{Ft}^* as given, and also does not make plans for future values of these variables understanding that future incarnations of the policy-maker can alter any given plan. The policy-maker at time *t* cannot influence $E_t\pi_{Ht+1}$ and $E_t\pi^*_{Ft+1}$ because future inflation levels are chosen by future policy-makers and there are no endogenous state variables that can limit the paths of future inflation levels. Hence, the policy-maker's problem is essentially a static one – to maximise Equation (65) subject to Equations (55) and (56), taking $E_t\pi_{Ht+1}$ and $E_t\pi^*_{Ft+1}$ as given.

Even though home bias in consumption has been introduced, the optimal policy rules are the same as in CGG. The first-order conditions are given by:

$$\tilde{y}_t + \tilde{y}^*_t + \xi(\pi_{Ht} + \pi^*_{Ft}) = 0, \tag{66}$$

$$\tilde{y}_t - \tilde{y}^*_t + \xi(\pi_{Ht} - \pi^*_{Ft}) = 0. \tag{67}$$

These two 'target criteria' can be rewritten as:

$$\tilde{y}_t + \xi\pi_{Ht} = 0, \text{ and } \tilde{y}^*_t + \xi\pi^*_{Ft} = 0. \tag{68}$$

The criteria given in Equation (68) are identical to those that arise in the closed economy version of this model. There is a trade-off between the goals of eliminating the output gap and driving inflation to zero, and the elasticity of substitution among goods produced in the country determines the weights given to output gaps and inflation.

It is worth emphasising that Equation (68) indicates the optimal policy entails a trade-off between the output gap and the *producer price* inflation level. In a closed economy with no intermediate goods, there is no distinction between producer and consumer prices. But in an open economy there is an important distinction. The policies described in Equation (68) imply that policy-makers should not give any weight to inflation of imported goods. In conjunction with the Phillips curves, (55) and (56), Equation (68) allows us to solve for the home and foreign output gaps and π_{Ht} and π^*_{Ft} as functions of current and expected future cost-push shocks, u_t and u^*_t. With the output gap determined by optimal policy, the terms of trade must adjust to ensure goods market clearing. But the terms of trade adjust freely in the PCP world, because nominal exchange rate changes translate directly into import price changes. In essence, the import sector is like a flexible-price sector, so policy-makers can ignore inflation in that sector, as in Aoki (2001).

8. Optimal Policy under LCP

The transformed loss function, $-\Psi_t$, can be written as:

$$\Psi_t \propto \frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2 + (\tilde{y}_t^*)^2) - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2$$
$$-\frac{\xi}{2\delta}\left(\frac{\nu}{2}(\pi_{Ht})^2 + \frac{2-\nu}{2}(\pi_{Ft})^2 + \frac{\nu}{2}(\pi_{Ft}^*)^2 + \frac{2-\nu}{2}(\pi_{Ht}^*)^2\right). \quad (69)$$

The loss function is similar to the one under PCP. The main point to highlight is that squared deviations from the law of one price matter for welfare, as well as output gaps and inflation rates. Deviations from the law of one price are distortionary and are a separate source of loss in the LCP model.

The policy-maker under discretion seeks to minimise the loss subject to the constraints of the Phillips curves, (57)–(58) and (59)–(60). There is an additional constraint in the LCP model. Note that $\pi_{Ft} - \pi_{Ht} = s_t - s_{t-1}$. But Equation (31) implies:

$$s_t - s_{t-1} = \frac{\sigma}{D}\left(y_t - y_t^* - (y_{t-1} - y_{t-1}^*)\right) - \frac{(\nu-1)}{D}(\Delta_t - \Delta_{t-1}). \quad (70)$$

Using Equation (70) in conjunction with Equations (57) and (60), the following constraint can be derived:

$$\frac{\sigma}{D}\left(y_t - y_t^* - (y_{t-1} - y_{t-1}^*)\right) - \frac{(\nu-1)}{D}(\Delta_t - \Delta_{t-1}) =$$
$$\delta\left[\left(\frac{\sigma}{D}+\phi\right)(\tilde{y}_t - \tilde{y}_t^*) - \frac{(\nu-1)}{2D}\Delta_t\right] + \beta E_t(\pi_{H\,t+1} - \pi_{Ft+1}) + u_t - u_t^*. \quad (71)$$

This constraint arises in the LCP model but not in the PCP model, precisely because import prices are sticky and subject to a Calvo price adjustment mechanism, rather than free to respond via nominal exchange rate changes.

Another contrast with the PCP model is that there are four sticky prices in the LCP model, so non-zero inflation rates for each of the four matter for welfare. Indeed, note that Ψ_t can be rewritten as:

$$\Psi_t \propto \frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t-\tilde{y}_t^*)^2-\left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2+(\tilde{y}_t^*)^2)-\left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2$$
$$-\frac{\xi}{2\delta}\left((\pi_t)^2+(\pi_t^*)^2+\frac{\nu(2-\nu)}{2}(s_t-s_{t-1})^2\right). \tag{72}$$

Under this formulation, the loss function is seen to depend on the aggregate CPI inflation rates, π_t and π_t^*, and the change in the terms of trade, s_t-s_{t-1}, rather than the four individual inflation rates given in Equation (69). This formulation is particularly useful when considering a simplification below, under which s_t-s_{t-1} is independent of policy.

The LCP optimisation problem under discretion becomes very complex and difficult because of the additional constraint given by Equation (71). That is because there now are endogenous state variables – the choices of home output gap relative to the foreign output gap, and the deviation from the law of one price puts constraints on the evolution of future output gaps, inflation rates and deviations from the law of one price. In the LCP case, the dynamic game between current and future policy-makers is non-trivial.

But inspection of Equation (71) reveals a special case in which the policy decision under uncertainty can be settled under the same simple conditions as in the PCP model. When $\phi=0$ – utility is linear in labour – Equation (71) simplifies considerably. Indeed, it can be rewritten as:

$$s_t-s_{t-1}=-\delta\tilde{s}_t+\beta E_t(s_{t+1}-s_t)+u_t-u_t^*. \tag{73}$$

With $\phi = 0$, we have $\tilde{s}_t = s_t - \overline{s}_t = s_t - (a_t - a_t^*)$. So we can write (73) as a second-order expectational difference equation:

$$s_t = \frac{1}{1+\delta+\beta} s_{t-1} + \frac{\beta}{1+\delta+\beta} E_t s_{t+1} + \frac{1}{1+\delta+\beta} \vartheta_t, \tag{74}$$

where $\vartheta_t = \delta(a_t - a_{t*}) + u_t - u_t^* = \delta \dot{s}_t$.

The point here is that Equation (74) determines the evolution of s_t independent of policy choices. So while s_t is a state variable, it is not endogenous for the policy-maker. One nice thing about considering this special case is that the parameter ϕ does not appear in either the target criteria or the optimal interest rate rule in the CGG model, so the targeting and instrument rules under PCP can be compared directly to the LCP model. We also note that Devereux and Engel (2003) make the same assumption on preferences.

The first-order conditions for the policy-maker can be derived independently of any assumption about the stochastic process for s_t. Equation (71) is replaced by Equation (31), expressed in ‘gap’ form, as a constraint on the choice of optimal values by the policy-maker.

In fact, in this case the policy problem can be simplified further by using the version of the loss function given by Equation (72). A useful way to rewrite (72) when $\phi = 0$ is:

$$\begin{aligned} \Psi_t \propto & \frac{-\sigma}{4D}(\tilde{y}_t^R)^2 - \frac{\sigma}{4}(\tilde{y}_t^W)^2 - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ & -\frac{\xi}{4\delta}\left((\pi_t^R)^2 + (\pi_t^W)^2 + \nu(2-\nu)(s_t - s_{t-1})^2\right) \end{aligned} \tag{75}$$

where the R superscript represents home relative to foreign. That is, $\pi_t^R = \pi_t - \pi_t^*$, $u_t^R = u_t - u_t^*$, etc. Likewise, the W superscript refers to the sum of home and foreign variables: $\pi_t^W = \pi_t + \pi_t^*$, $u_t^W = u_t + u_t^*$, etc. Since $s_t - s_{t-1}$ is independent of policy, the policy-maker’s problem can be expressed as choosing relative and world output gaps, $\tilde{y}_t^R$ and $\tilde{y}_t^W$, relative and world CPI inflation rates, π_t^R and π_t^W, and the currency misalignment, Δ_t to maximise Equation (75) subject to the ‘gap’ version of Equation (31) and the linear combination of the Phillips curves that provide equations for CPI inflation in each country (which are derived here under the assumption that $\phi = 0$):

$$\tilde{s}_t = \frac{\sigma}{D}(\tilde{y}_t - \tilde{y}_t^*) - \frac{(\nu - 1)}{D}\Delta_t, \tag{76}$$

$$\pi_t^R = \delta\left[\frac{\sigma(\nu-1)}{D}\tilde{y}_t^R + \frac{\sigma\nu(2-\nu)}{D}\Delta_t\right] + \beta E_t \pi_{t+1}^R + (\nu - 1)u_t^R, \tag{77}$$

$$\pi_t^W = \delta\sigma\tilde{y}_t^W + \beta E_t \pi_{t+1}^W + u_t^W. \tag{78}$$

The first condition seems quite similar to the first condition in the PCP case, Equation (66):

$$\tilde{y}_t^W + \xi\pi_t^W = 0. \tag{79}$$

This condition calls for a trade-off between the world output gap and the world inflation rate, just as in the PCP case. But there is a key difference – here in the LCP model, it is the CPI, not the PPI, inflation rates that enter into the policy-maker's trade-off.

The second condition can be written as:

$$\frac{1}{\sigma}\tilde{q}_t + \xi\pi_t^R = 0. \tag{80}$$

Here, q_t is the consumption real exchange rate, defined as:

$$q_t = e_t + p_t^* - p_t. \tag{81}$$

$\tilde{q}_t$ is the deviation of the real exchange rate from its efficient level, where the following relationship has been used:

$$\frac{1}{\sigma}\tilde{q}_t = \frac{\nu - 1}{D}\tilde{y}_t^R + \frac{\nu(2-\nu)}{D}\Delta_t. \tag{82}$$

Equation (80) represents the second of the target criteria as a trade-off between misaligned real exchange rates and relative CPI inflation rates. In the LCP model, where exchange rate misalignments are possible, Equation (82) highlights that optimal policy involves trading off relative output gaps, relative CPI inflation rates, and the currency misalignment.

8.1 Optimal Policy under PCP versus LCP

It is helpful to compare the target criteria under PCP (Conditions (66) and (67)) and LCP (Conditions (79) and (80)).

First, compare Equation (66) to (79). Both involve a trade-off between the world output gap and world inflation. But under PCP, producer price inflation appears in the trade-off. However, world producer price inflation is equal to world consumer price inflation under PCP. To see this,

$$\pi_t^W = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft} + \frac{\nu}{2}\pi_{Ft}^* + \frac{2-\nu}{2}\pi_{Ht}^* = \pi_{Ht} + \pi_{Ft}^* . \tag{83}$$

The second equality holds because the relative prices are equal in home and foreign under PCP (and, for that matter, to a first-order approximation under LCP) so $\pi_{Ht}^* = \pi_{Ft}^* + \pi_{Ht} - \pi_{Ft}$.

This trade-off is the exact analogy to the closed economy trade-off between the output gap and inflation, and the intuition of that trade-off is well understood. On the one hand, with asynchronised price setting, inflation leads to misalignment of relative prices, so any non-zero level of inflation is distortionary. On the other hand, because the monopoly power of labour is time-varying due to the time-varying elasticity of labour demand, output levels can be inefficiently low or high even when inflation is zero. Conditions (66) or (79) describe the terms of that trade-off. Inflation is more costly the higher is the elasticity of substitution among varieties of goods, ξ, because a higher elasticity will imply greater resource misallocation when there is inflation.

The difference in optimal policy under PCP versus LCP comes in the comparison of Condition (67) with (80). Under PCP, optimal policy trades off home relative to foreign output gaps with home relative to foreign PPI inflation. Under LCP, the trade-off is between the real exchange rate and home relative to foreign CPI inflation.

First, it is helpful to consider Equation (80) when the two economies are closed, so that $\nu = 2$. Using (82), under this condition, $D = 1$, and (80) reduces to $\tilde{y}_t^R + \xi \pi_t^R = 0$. Of course, when $\nu = 2$, there is no difference between PPI and CPI inflation, and so in this special case the optimal policies under LCP and PCP are identical. That is nothing more than reassuring, since the distinction between PCP and LCP should not matter when the economies are closed.

When $\nu \neq 2$, understanding these conditions is more subtle. It helps to consider the case of no home bias in preferences, so $\nu = 1$. Imagine that inflation rates were zero, so that there is no misallocation of labour within each country. Further, imagine that the world output gap is zero. There are still two possible distortions. First, relative home to foreign output may not be at the efficient level. Second, even if output levels are efficient, the allocation of output to home and foreign households may be inefficient if there are currency misalignments.

When $\nu = 1$, it follows from Equations (26) and (27) that relative output levels are determined only by the terms of trade. We have $\tilde{y}_t^R = \tilde{s}_t$. On the other hand, from Equation (28) when $\nu = 1$, relative consumption is misaligned when there are currency misalignments, $\tilde{c}_t^R = \frac{1}{\sigma} \Delta_t$. Moreover, when $\nu = 1$, the deviation of the real exchange rate from its inefficient level is entirely due to the currency misalignment: $\tilde{q}_t = \Delta_t$.

Under PCP, the law of one price holds continuously, so there is no currency misalignment. In that case, $\Delta_t = 0$, and relative home to foreign consumption is efficient. In that case, policy can influence the terms of trade in order to achieve the optimal trade-off between relative output gaps and relative inflation, as expressed in Equation (67). Policy can control the terms of trade under PCP because the terms of trade can adjust instantaneously and completely through nominal exchange rate adjustment. That is, $s_t = p_{Ft} - p_{Ht} = e_t + p_{Ft}^* - p_{Ht}$. While p_{Ft}^* and p_{Ht} do not adjust freely, movements in the nominal exchange rate e_t are unrestricted, so the terms of trade adjust freely.

Under LCP, the nominal exchange rate does not directly influence the consumer prices of home to foreign goods in either country. For example, in the home country, $s_t = p_{Ft} - p_{Ht}$. Because prices are set in local currencies, neither p_{Ft} and p_{Ht} adjust freely to shocks. In fact, as I have shown, when utility is linear in labour ($\phi = 0$) monetary policy has no control over the internal relative prices.

But under LCP, there are currency misalignments, and monetary policy can influence those. Recall from (25) $\Delta_t \equiv e_t + p^*_{Ht} - p_{Ht} = e_t + p^*_{Ft} - p_{Ft}$, so the currency misalignment can adjust instantaneously with nominal exchange rate movements. Because policy cannot influence the relative output distortion (when $\nu = 1$) but can influence the relative consumption distortion, the optimal policy puts full weight on the currency misalignment. When $\nu = 1$, Equation (80) can be written as $\frac{1}{\sigma}\Delta_t + \xi\pi^R_t = 0$. When $\Delta_t > 0$, so that the home currency is undervalued, and $\pi_t - \pi^*_t > 0$, so home CPI inflation exceeds foreign, the implications for policy are obvious. Home monetary policy must tighten relative to foreign. But the more interesting case to consider is when home inflation is running high, so that $\pi_t - \pi^*_t > 0$, but the currency is overvalued, so that $\Delta_t < 0$. Then Equation (80) implies that the goals of maintaining low inflation and a correctly valued currency are in conflict. Policies that improve the inflation situation may exacerbate the currency misalignment (implying an even larger correction of *e* at some future date). Equation (80) parameterises the trade-off.

9. The Interest Rate Reaction Functions

An interest rate rule that will support the optimal policies given by Equations (79) and (80) can be derived.

Substituting Equation (80) into Equations (57)–(58) and (59)–(60), and using the definitions of CPI inflation given in Equations (63) and (64) implies:

$$\pi^R_t = \frac{\beta}{1+\delta\sigma\xi}E_t\pi^R_{t+1} + \frac{\nu-1}{1+\delta\sigma\xi}u^R_t. \tag{84}$$

Assuming that u_t and u_t^* are each AR(1) processes, independently distributed, with a serial correlation coefficient given by ρ. Under these assumptions, Equation (84) can be solved as[20]:

$$\pi_t^R = \frac{\nu - 1}{1 + \delta\sigma\xi - \beta\rho} u_t^R . \tag{85}$$

Similarly, under the optimal monetary policy, the solution for 'world' inflation is:

$$\pi_t^W = \frac{\beta}{1 + \delta\sigma\xi} E_t \pi_{t+1}^W + \frac{1}{1 + \delta\sigma\xi} u_t^W . \tag{86}$$

Under the assumption that the mark-up shocks follow AR(1) processes, it follows that:

$$\pi_t^W = \frac{1}{1 + \delta\sigma\xi - \beta\rho} u_t^W . \tag{87}$$

Substituting these equations into the Euler equations for the home and foreign country, given by Equations (61) and (62), making use of the consumption Equations (29) and (30), it can be shown that:

$$r_t^R = (\rho + (1 - \rho)\sigma\xi)\pi_t^R + \frac{\sigma(\nu - 1)}{D}(E_t \overline{y}_{t+1}^R - \overline{y}_t^R) = (\rho + (1 - \rho)\sigma\xi)\pi_t^R + \overline{rr}_t^R , \tag{88}$$

$$r_t^W = (\rho + (1 - \rho)\sigma\xi)\pi_t^W + \sigma(E_t \overline{y}_{t+1}^W - \overline{y}_t^W) = (\rho + (1 - \rho)\sigma\xi)\pi_t^W + \overline{rr}_t^W . \tag{89}$$

Here, $\overline{rr}_t$ represents the real interest rate in the efficient economy. Equations (88) and (89) can be used to write:

$$r_t = (\rho + (1 - \rho)\sigma\xi)\pi_t + \overline{rr}_t , \tag{90}$$

$$r_t^* = (\rho + (1 - \rho)\sigma\xi)\pi_t^* + \overline{rr}_t^* . \tag{91}$$

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[20] See Appendix B for the complete solutions of the model under optimal policy.

Surprisingly, these interest rate reaction functions are identical to the ones derived in CGG for the PCP model, except that the inflation term that appears on the right-hand side of each equation is CPI inflation here, while it is PPI inflation in CGG.

This finding starkly highlights the difference between monetary rules expressed as ‘target criteria’ (or targeting rules) and monetary rules expressed as interest rate reaction functions (or instrument rules). The optimal interest rate reaction functions presented in Equations (90) and (91) appear to give no role for using monetary policy to respond to deviations from the law of one price. However, the ‘target criteria’ show the optimal trade-off does give weight to the law of one price deviation. The key to understanding this apparent conflict is that the reaction functions, such as (90) and (91) are not only setting inflation rates. By setting home relative to foreign interest rates, they are also prescribing a relationship between home and foreign output gaps, the law of one price deviation, and home relative to foreign inflation rates.

If central bankers really did mechanically follow an interest rate rule, their optimal policy rules would have the nominal interest rate responding only to CPI inflation. In practice, however, central bankers set the interest rates to achieve their targets. We have shown that the optimal target criteria involves trade-offs among the goals of achieving zero inflation, driving the output gaps to zero, and eliminating the law of one price gap.

The optimal policy indeed is not successful in eliminating the currency misalignment. Nor does policy drive inflation to zero or eliminate the output gap. Monetary policy-makers do not have sufficient control over the economy to achieve the efficient outcome. Appendix B.4 displays the solutions for inflation, the output gap, and the currency misalignment under optimal policy.

10. Conclusions

Policy-makers do not in general adhere to simple interest rate reaction functions. Instead, as Svensson (1999, 2002) has argued, they set targets for key economic variables. It has generally been believed, especially in light of CGG, that the key trade-offs in an open economy are the same as in a closed economy. That is, policy-makers should target a linear combination of inflation and the output gap. This paper shows that in fact, with a model that is rich enough to allow for currency misalignments, the trade-offs should involve not only inflation and the output gap but also the exchange rate misalignment. However, the interest rate reaction function that supports this policy has the nominal interest rate reacting only to CPI inflation.

The paper derives the policy-maker's loss function when there is home bias in consumption and deviations from the law of one price. The loss function does depend on the structure of the model, of course, but not on the specific nature of price setting. Currency misalignments may arise in some approaches for reasons other than LCP. For example, there may be nominal wage stickiness but imperfect pass-through that arises from strategic behaviour by firms as in the models of Atkeson and Burstein (2007, 2008) or Corsetti *et al* (forthcoming). Future work can still make use of the loss function derived here, or at least of the steps used in deriving the loss function.

The objective of this paper is to introduce LCP into a familiar and popular framework for monetary policy analysis. But the CGG model does not produce empirical outcomes that are especially plausible, even with the addition of LCP. In the model, if output gaps are eliminated, the reason a currency misalignment is inefficient is because consumption goods are misallocated between home and foreign households. In a richer framework, even with LCP as the source of currency misalignments, there are other potential misallocations that can occur when exchange rates are out of line. For example, an overvalued currency would lead to a movement in resources away from the traded sector to the non-traded sector. Or if countries import oil from the outside, a misaligned currency affects the real cost of oil in one importing country relative to the other.

Future work should also consider the difficult issue of policy-making in this environment when there is not cooperation. A separate but related issue is a closer examination of who bears the burden of currency misalignments. If a currency is overvalued, does it hurt consumers in one country more than another, both under optimal and sub-optimal policy?

While rich models that can be estimated and analysed numerically offer valuable insights, they seem to provide inadequate guidance to policy-makers for how they should react to exchange rate movements. Perhaps more basic work on simple models such as the one presented here, in concert with quantitative exploration of more detailed models, can be productive.

Appendix A: Log-linearised Model

The log-linear approximations to the models presented above are presented in this Appendix.

Use is made of the first-order approximation $s_t^* = -s_t$, as explained in the text. That relationship is obvious in the flexible-price and PCP models, but will require some explanation in the LCP case. That explanation is postponed until later.

In this Appendix, all of the equations of the log-linearised model are presented, but those that are used in the derivation of the loss function (which do not involve price setting or wage setting) are separated from those that are not.

Equations used for derivation of loss functions

The log of the deviation from the law of one price is defined as:

$$\Delta_t \equiv e_t + p_{Ht}^* - p_{Ht} \tag{A1}$$

In the flexible-price and PCP models, $\Delta_t = 0$. In the LCP model, because $s_t^* = -s_t$, the law of one price deviation is the same for both goods: $\Delta_t \equiv e_t + p_{Ft}^* - p_{Ft}$.

In all three models, to a first order, $\ln(V_{Ht}) = \ln(V_{Ht}^*) = \ln(V_{Ft}) = \ln(V_{Ft}^*) = 0$. That allows Equation (23) and its foreign counterpart to be approximated as:

$$n_t = y_t - a_t, \quad \text{and} \tag{A2}$$

$$n_t^* = y_t^* - a_t^*. \tag{A3}$$

The market-clearing conditions, (18) and (19) are approximated as:

$$y_t = \frac{\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t + \frac{2-\nu}{2} c_t^*, \tag{A4}$$

$$y_t^* = \frac{-\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t^* + \frac{2-\nu}{2} c_t. \tag{A5}$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for home and foreign households, Equation (22), is given by:

$$\sigma c_t - \sigma c_t^* = \Delta_t + (\nu - 1)s_t . \tag{A6}$$

For use later, it is helpful to use Equations (A4)–(A6) to express c_t, c_t^*, s_t, in terms of y_t and y_t^* and the exogenous disturbances, a_t, a_t^*, μ_t^W, and μ_t^{*W} :

$$c_t = \frac{D + \nu - 1}{2D} y_t + \frac{D - (\nu - 1)}{2D} y_t^* + \frac{\nu(2-\nu)}{2D} \Delta_t , \tag{A7}$$

$$c_t^* = \frac{D + \nu - 1}{2D} y_t^* + \frac{D - (\nu - 1)}{2D} y_t - \frac{\nu(2-\nu)}{2D} \Delta_t , \tag{A8}$$

$$s_t = \frac{\sigma}{D}(y_t - y_t^*) - \frac{(\nu - 1)}{D} \Delta_t , \tag{A9}$$

where $D \equiv \sigma\nu(2-\nu) + (\nu - 1)^2$.

Under a globally efficient allocation, the marginal rate of substitution between leisure and aggregate consumption should equal the marginal product of labour times the price of output relative to consumption prices. To see the derivation more cleanly, insert the shadow real wages in the efficient allocation, $\overline{w}_t - \overline{p}_{Ht}$ and $\overline{w}_t^* - \overline{p}_{Ft}^*$ into Equations (A10) and (A11) below. So, the efficient allocation would be achieved in a model with flexible wages and optimal subsidies. These equations then can be understood intuitively by looking at the wage-setting equations below (Equations (A14)–(A15), and (A16)–(A17)) assuming the optimal subsidy is in place. But they do not depend on a particular model of wage setting, and are just the standard efficiency condition equating the marginal rate of substitution between leisure and aggregate consumption to the marginal rate of transformation.

$$a_t = \overline{w}_t - \overline{p}_{Ht} = \left(\frac{\sigma(1+D)}{2D} + \phi \right) \overline{y}_t + \frac{\sigma(D-1)}{2D} \overline{y}_t^* - \phi a_t , \tag{A10}$$

$$a_t^* = \overline{w}_t^* - \overline{p}_{Ft}^* = \left(\frac{\sigma(1+D)}{2D} + \phi \right) \overline{y}_t^* + \frac{\sigma(D-1)}{2D} \overline{y}_t - \phi a_t^* . \tag{A11}$$

Equations of wage and price setting

The real home and foreign product wages, from Equation (38), are given by:

$$w_t - p_{Ht} = \sigma c_t + \phi n_t + \frac{2-\nu}{2} s_t + \mu_t^W \, , \tag{A12}$$

$$w_t^* - p_{Ft}^* = \sigma c_t^* + \phi n_t^* - \left(\frac{2-\nu}{2}\right) s_t + \mu_t^{*W} \, . \tag{A13}$$

$w_t - p_{Ht}$ and $w_t^* - p_{Ft}^*$ can be expressed in terms of y_t and y_t^* and the exogenous disturbances, a_t, a_t^*, μ_t^W, and μ_t^{*W}:

$$w_t - p_{Ht} = \left(\frac{\sigma(1+D)}{2D} + \phi\right) y_t + \frac{\sigma(D-1)}{2D} y_t^* + \frac{D-(\nu-1)}{2D} \Delta_t - \phi a_t + \mu_t^W \, , \tag{A14}$$

$$w_t^* - p_{Ft}^* = \left(\frac{\sigma(1+D)}{2D} + \phi\right) y_t^* + \frac{\sigma(D-1)}{2D} y_t + \frac{\nu-1-D}{2D} \Delta_t - \phi a_t^* + \mu_t^{*W} \, . \tag{A15}$$

A.1 Flexible Prices

The values of all the real variables under flexible prices can be solved by using Equations (A2), (A3), (A7), (A8), (A9), (A14) and (A15), as well as the price-setting conditions, from (43):

$$\dot{w}_t - \dot{p}_{Ht} = a_t \, , \tag{A16}$$

$$\dot{w}_t^* - \dot{p}_{Ft}^* = a_t^* \, . \tag{A17}$$

A.2 PCP

Log-linearisation of Equations (49) and (50) leads to the familiar New Keynesian Phillips curve for an open economy:

$$\pi_{Ht} = \delta(w_t - p_{Ht} - a_t) + \beta E_t \pi_{Ht+1} \, , \tag{A18}$$

where $\delta = (1-\theta)(1-\beta\theta)/\theta$.

This equation can be rewritten as:

$$\pi_{Ht} = \delta(\tilde{w}_t - \tilde{p}_{Ht}) + \beta E_t \pi_{Ht+1},$$

or, using (A14) and (A10):

$$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^*\right] + \beta E_t \pi_{Ht+1} + u_t, \qquad \text{(A19)}$$

where $u_t = \delta\mu_t^W$.

Similarly for foreign producer price inflation:

$$\pi_{Ft}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t\right] + \beta E_t \pi_{Ft+1}^* + u_t^*. \qquad \text{(A20)}$$

A.3 LCP

Equation (A18) holds in the LCP model as well. But in the LCP model, the law of one price deviation is not zero. It follows that:

$$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^* + \frac{D-(\nu-1)}{2D}\Delta_t\right] + \beta E_t \pi_{Ht+1} + u_t, \quad \text{(A21)}$$

$$\pi_{Ft}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{\nu-1-D}{2D}\Delta_t\right] + \beta E_t \pi_{Ft+1}^* + u_t^*. \qquad \text{(A22)}$$

In addition, Equations (18) and (51) imply:

$$\pi_{Ht}^* = \delta(w_t - p_{Ht}^* - e_t - a_t) + \beta E_t \pi_{Ht+1}^* = \delta(w_t - p_{Ht} - \Delta_t - a_t) + \beta E_t \pi_{Ht+1}^*. \quad \text{(A23)}$$

This can be rewritten as

$$\pi^*_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}^*_t - \left(\frac{D+\nu-1}{2D}\right)\Delta_t\right] + \beta E_t \pi^*_{Ht+1} + u_t . \quad \text{(A24)}$$

Similarly:

$$\pi_{Ft} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}^*_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{D+\nu-1}{2D}\Delta_t\right] + \beta E_t \pi_{Ft+1} + u^*_t . \quad \text{(A25)}$$

From Equations (A7)–(A8) and (A14)–(A15), it can be seen that $\pi_{Ft} - \pi_{Ht} = \pi^*_{Ft} - \pi^*_{Ht}$. Assuming a symmetric initial condition leads to the conclusion that $s^*_t = -s_t$ as noted above. That is, the relative price of foreign to home goods is the same in both countries. I emphasise that this is true in general for a first-order approximation.

Appendix B: Welfare Functions and Other Derivations

B.1 Derivation of Welfare Function in Clarida-Galí-Gertler Model with Home Bias in Preferences

The object is to rewrite the welfare function, which is defined in terms of home and foreign consumption and labour effort, into terms of the squared output gap and squared inflation. The joint welfare function of home and foreign households is derived, since cooperative monetary policy is being examined.

Most of the derivation requires only first-order approximations of the equations of the model, but in a few places, second-order approximations are needed. If the approximation is first-order, the notation '$+o\left(\left\|a^2\right\|\right)$' is used to indicate that there are second-order and higher terms left out, and if the approximation is second-order, '$+o\left(\left\|a^3\right\|\right)$' is used. ($a$ is notation for the log of the productivity shock.)

From Equation (1) in the text, the period utility of the planner is given by:

$$\upsilon_t \equiv \frac{1}{1-\sigma}(C_t^{1-\sigma} + C_t^{*1-\sigma}) - \frac{1}{1+\phi}(N_{t+j}^{1+\phi} + N_{t+j}^{*1+\phi}). \qquad \text{(B1)}$$

Take a second-order log approximation around the non-stochastic steady state. Allocations are assumed to be efficient in steady state, so $C^{1-\sigma} = C^{*1-\sigma} = N^{1+\phi} = N^{*1+\phi}$. The fact that $C^{1-\sigma} = N^{1+\phi}$ follows from the fact that in steady state $C = N$ from market clearing and symmetry, and $C^{-\sigma} = N^{\phi}$ from the condition that the marginal rate of substitution between leisure and consumption equals one in an efficient non-stochastic steady state.

It follows that:

$$\begin{aligned}\upsilon_t &= 2\left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right)C^{1-\sigma} + C^{1-\sigma}\left(c_t + c_t^*\right) \\ &+ \frac{1-\sigma}{2}C^{1-\sigma}\left((c_t)^2 + (c_t^*)^2\right) \\ &- C^{1-\sigma}\left(n_t + n_t^*\right) - \frac{1+\phi}{2}C^{1-\sigma}\left((n_t)^2 + (n_t^*)^2\right) + o\left(\left\|a^3\right\|\right).\end{aligned} \qquad \text{(B2)}$$

Since maximising an affine transformation of Equation (B2) is equivalent, it is convenient to simplify that equation to get:

$$\upsilon_t = c_t + c_t^* - n_t - n_t^* + \frac{1-\sigma}{2}\left(c_t^2 + c_t^{*2}\right) - \frac{1+\phi}{2}\left(n_t^2 + n_t^{*2}\right) + o\left(\left\|a^3\right\|\right). \tag{B3}$$

Utility is maximised when consumption and employment take on their efficient values:

$$\upsilon_t^{\max} = \overline{c}_t + \overline{c}_t^* - \overline{n}_t - \overline{n}_t^* + \frac{1-\sigma}{2}\left(\overline{c}_t^2 + \overline{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\overline{n}_t^2 + \overline{n}_t^{*2}\right) + o\left(\left\|a^3\right\|\right). \tag{B4}$$

In general, this maximum may not be attainable because of distortions. Writing $x_t = \overline{x}_t + \tilde{x}_t$, where $\tilde{x}_t \equiv x_t - \overline{x}_t$, it follows that:

$$\begin{aligned}\upsilon_t = &\left[\overline{c}_t + \overline{c}_t^* - \overline{n}_t - \overline{n}_t^* + \frac{1-\sigma}{2}\left(\overline{c}_t^2 + \overline{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\overline{n}_t^2 + \overline{n}_t^{*2}\right)\right] \\ &+\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2} + 2\overline{c}_t\tilde{c}_t + 2\overline{c}_t^*\tilde{c}_t^*\right) \\ &-\frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2} + 2\overline{n}_t\tilde{n}_t + 2\overline{n}_t^*\tilde{n}_t^*\right) + o\left(\left\|a^3\right\|\right)\end{aligned} \tag{B5}$$

or

$$\begin{aligned}\upsilon_t - \upsilon_t^{\max} = &\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) \\ &+(1-\sigma)\left(\overline{c}_t\tilde{c}_t + \overline{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\overline{n}_t\tilde{n}_t + \overline{n}_t^*\tilde{n}_t^*\right) + o\left(\left\|a^3\right\|\right).\end{aligned} \tag{B6}$$

The object is to write (B6) as a function of squared output gaps and squared inflation if possible. A second-order approximation of $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$ is needed. But for the rest of the terms, since they are squares and products, the first-order approximations that have already been derived will be sufficient.

Recalling that $\Delta_t = 0$ in the PCP model, Equations (A7)–(A8) can be written as:

$$c_t = c_y y_t + (1 - c_y) y_t^* + o\left(\left\|a^2\right\|\right), \tag{B7}$$

$$c_t^* = (1 - c_y) y_t + c_y y_t^* + o\left(\left\|a^2\right\|\right), \tag{B8}$$

where $c_y \equiv \dfrac{D + \nu - 1}{2D}$.

It follows from (B7) and (B8) that:

$$\overline{c}_t = c_y \overline{y}_t + (1 - c_y) \overline{y}_t^* + o\left(\left\|a^2\right\|\right), \tag{B9}$$

$$\overline{c}_t^* = (1 - c_y) \overline{y}_t + c_y \overline{y}_t^* + o\left(\left\|a^2\right\|\right), \tag{B10}$$

$$\tilde{c}_t = c_y \tilde{y}_t + (1 - c_y) \tilde{y}_t^* + o\left(\left\|a^2\right\|\right), \tag{B11}$$

$$\tilde{c}_t^* = (1 - c_y) \tilde{y}_t + c_y \tilde{y}_t^* + o\left(\left\|a^2\right\|\right). \tag{B12}$$

Next, it is easy to show that:

$$\tilde{n}_t = \tilde{y}_t + o\left(\left\|a^2\right\|\right), \quad \text{and} \tag{B13}$$

$$\tilde{n}_t^* = \tilde{y}_t^* + o\left(\left\|a^2\right\|\right). \tag{B14}$$

These follow as in Equations (A2)–(A3) because $n_t = y_t - a_t + o\left(\left\|a^2\right\|\right)$ and $\overline{n}_t = \overline{y}_t - a_t$ (and similarly in the foreign country).

Expressions for $\overline{n}_t$ and $\overline{n}_t^*$ are required. From Equations (A10)–(A11):

$$a_t = \left(\frac{\sigma(1+D)}{2D} + \phi \right) \overline{y}_t + \frac{\sigma(D-1)}{2D} \overline{y}_t^* - \phi a_t ,$$

$$a_t^* = \left(\frac{\sigma(1+D)}{2D} + \phi \right) \overline{y}_t^* + \frac{\sigma(D-1)}{2D} \overline{y}_t - \phi a_t^* .$$

Using $a_t = \overline{y}_t - \overline{n}_t$ and $a_t^* = \overline{y}_t^* - \overline{n}_t^*$, these can be written as

$$\overline{n}_t = \frac{1-\sigma}{1+\phi}\left[(\nu-1)c_y + \frac{2-\nu}{2} \right]\overline{y}_t + \frac{1-\sigma}{1+\phi}\left[(1-\nu)c_y + \frac{\nu}{2} \right]\overline{y}_t^* + o\left(\left\|a^2\right\|\right), \text{ and} \quad \text{(B15)}$$

$$\overline{n}_t^* = \frac{1-\sigma}{1+\phi}\left[(\nu-1)c_y + \frac{2-\nu}{2} \right]\overline{y}_t^* + \frac{1-\sigma}{1+\phi}\left[(1-\nu)c_y + \frac{\nu}{2} \right]\overline{y}_t + o\left(\left\|a^2\right\|\right). \quad \text{(B16)}$$

Turning attention back to the loss function in Equation (B6), focus first on the terms $\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\overline{c}_t\tilde{c}_t + \overline{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\overline{n}_t\tilde{n}_t + \overline{n}_t^*\tilde{n}_t^*\right)$. As noted above, these involve only squares and cross-products of $\tilde{c}_t$, $\tilde{c}_t^*$, $\overline{c}_t$, $\overline{c}_t^*$, $\tilde{n}_t$, $\tilde{n}_t^*$, $\overline{n}_t$, and $\overline{n}_t^*$. Equations (B9)–(B16) can be substituted into this expression. It is useful to provide a few lines of algebra since it is a bit messy:

$$\frac{1-\sigma}{2}\left(\tilde{c}_t^2+\tilde{c}_t^{*2}\right)-\frac{1+\phi}{2}\left(\tilde{n}_t^2+\tilde{n}_t^{*2}\right)+(1-\sigma)\left(\overline{c}_t\tilde{c}_t+\overline{c}_t^*\tilde{c}_t^*\right)-(1+\phi)\left(\overline{n}_t\tilde{n}_t+\overline{n}_t^*\tilde{n}_t^*\right)$$
$$=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)$$
$$+(1-\sigma)\left(2c_y^2-2c_y\right)\left(\overline{y}_t-\overline{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+(1-\sigma)\left(\overline{y}_t\tilde{y}_t+\overline{y}_t^*\tilde{y}_t^*\right)$$
$$-(1-\sigma)\left(\frac{\nu}{2}\overline{c}_t+\left(\frac{2-\nu}{2}\right)\overline{c}_t^*\right)\tilde{y}_t-(1-\sigma)\left(\frac{\nu}{2}\overline{c}_t^*+\left(\frac{2-\nu}{2}\right)\overline{c}_t\right)\tilde{y}_t^*$$
$$=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)$$
$$+2(1-\sigma)\left(c_y^2-c_y\right)\left(\overline{y}_t-\overline{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+(1-\sigma)\left(\overline{y}_t\tilde{y}_t+\overline{y}_t^*\tilde{y}_t^*\right)$$
$$-(1-\sigma)\left[\left((\nu-1)c_y-\frac{\nu}{2}\right)\left(\overline{y}_t-\overline{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+\overline{y}_t\tilde{y}_t+\overline{y}_t^*\tilde{y}_t^*\right]$$
$$=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)$$
$$+(1-\sigma)\left(2(c_y^2-c_y)-(\nu-1)c_y+\frac{\nu}{2}\right)\left(\overline{y}_t-\overline{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+o\left(\left\|a^3\right\|\right). \tag{B17}$$

Now return to the $\tilde{c}_t+\tilde{c}_t^*-\tilde{n}_t-\tilde{n}_t^*$ term in Equation (B6) and conduct a second-order approximation. Start with Equation (18), dropping the k^{-1} term because it will not affect the approximation, and noting that in the PCP model, $S_t^*=S_t^{-1}$:

$$Y_t=\frac{\nu}{2}S_t^{(2-\nu)/2}C_t+\left(\frac{2-\nu}{2}\right)S_t^{\nu/2}C_t^* \tag{B18}$$

Then use Equation (22), but using the fact that $S_t^*=S_t^{-1}$ and there are no deviations from the law of one price:

$$C_t^*=C_tS_t^{\frac{1-\nu}{\sigma}}. \tag{B19}$$

Substitute in to get:

$$Y_t=\frac{\gamma}{2}S_t^{\frac{2-\nu}{2}}C_t+\left(\frac{2-\nu}{2}\right)S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}}C_t. \tag{B20}$$

Solve for C_t:

$$C_t = Y_t \left(\frac{\gamma}{2} S_t^{\frac{2-\nu}{2}} + \left(\frac{2-\nu}{2} \right) S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}} \right)^{-1} \text{ or} \tag{B21}$$

$$c_t = y_t - \ln\left(\frac{\nu}{2} e^{\left(\frac{2-\nu}{2}\right)s_t} + \left(\frac{2-\nu}{2} \right) e^{\left(\frac{\nu}{2}+\frac{1-\nu}{\sigma}\right)s_t} \right). \tag{B22}$$

Take first and second derivatives, evaluated at the non-stochastic steady state:

$$\frac{\partial c_t}{\partial s_t}\Big|_{s=0} = \left(\frac{\nu-2}{2} \right)\left(\nu + \frac{1-\nu}{\sigma} \right), \tag{B23}$$

$$\frac{\partial^2 c_t}{\partial s_t^2}\Big|_{s=0} = \left(\frac{\nu-2}{2} \right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma} \right)^2. \tag{B24}$$

Then this second-order approximation is obtained:

$$c_t = y_t + \left(\frac{\nu-2}{2} \right)\left(\nu + \frac{1-\nu}{\sigma} \right)s_t + \frac{1}{2}\left(\frac{\nu-2}{2} \right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma} \right)^2 s_t^2 + o\left(\left\|a^3\right\|\right). \tag{B25}$$

Symmetrically,

$$c_t^* = y_t^* - \left(\frac{\nu-2}{2} \right)\left(\nu + \frac{1-\nu}{\sigma} \right)s_t + \frac{1}{2}\left(\frac{\nu-2}{2} \right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma} \right)^2 s_t^2 + o\left(\left\|a^3\right\|\right). \tag{B26}$$

Since only $\tilde{c}_t + \tilde{c}_t^*$ is of interest, these can be added together to get:

$$c_t + c_t^* = y_t + y_t^* + \left(\frac{\nu-2}{2} \right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma} \right)^2 s_t^2 + o\left(\left\|a^3\right\|\right). \tag{B27}$$

Now take a first-order approximation for s_t to substitute out for s_t^2. Equation (A9) implies:

$$s_t = \frac{\sigma^2}{D^2}(y_t^* - y_t)^2 + o\left(\left\|a^3\right\|\right). \tag{B28}$$

Substituting into Equation (B28) leads to:

$$c_t + c_t^* = y_t + y_t^* + \Omega(y_t^* - y_t)^2 + o\left(\left\|a^3\right\|\right), \tag{B29}$$

where $\Omega = \frac{\nu(\nu-2)}{4}\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2$.

Evaluating (B29) at flexible prices:

$$\overline{c}_t + \overline{c}_t^* = \overline{y}_t + \overline{y}_t^* + \Omega(\overline{y}_t^* - \overline{y}_t)^2 + o\left(\left\|a^3\right\|\right). \tag{B30}$$

It follows from the fact that $\tilde{c}_t + \tilde{c}_t^* = c_t + c_t^* - (\overline{c}_t + \overline{c}_t^*)$ that

$$\begin{aligned}\tilde{c}_t + \tilde{c}_t^* = \tilde{y}_t + \tilde{y}_t^* + \Omega(&\tilde{y}_t^2 + 2\overline{y}_t\tilde{y}_t + \tilde{y}_t^{*2} + 2\overline{y}_t^*\tilde{y}_t^* - 2\overline{y}_t\tilde{y}_t^* - 2\overline{y}_t^*\tilde{y}_t - 2\tilde{y}_t\tilde{y}_t^*) \\ &+o\left(\left\|a^3\right\|\right).\end{aligned} \tag{B31}$$

See Section B.3 below for the second-order approximations for $\tilde{n}_t$ and $\tilde{n}_t^*$:

$$\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\sigma_{pt}^2 + o\left(\left\|a^3\right\|\right), \tag{B32}$$

$$\tilde{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2}\sigma_{p*t}^2 + o\left(\left\|a^3\right\|\right). \tag{B33}$$

Substitute expressions (B31)–(B33) along with (B17) into the loss function (B6):

$$\upsilon_t - \upsilon_t^{\max} = \left[\Omega + (1-\sigma)(c_y^2 - c_y) \right](\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma + \phi}{2} \right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2} \right)$$
$$-\frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p^*t}^2 \right) + 2\Omega\left(\overline{y}_t - \overline{y}_t^* \right)\left(\tilde{y}_t - \tilde{y}_t^* \right) \qquad \text{(B34)}$$
$$+(1-\sigma)\left(2(c_y^2 - c_y) - (\nu - 1)c_y + \frac{\nu}{2} \right)\left(\overline{y}_t - \overline{y}_t^* \right)\left(\tilde{y}_t - \tilde{y}_t^* \right) + o\left(\left\| a^3 \right\| \right).$$

Some tedious algebra demonstrates that

$$2\Omega + (1-\sigma)\left(2(c_y^2 - c_y) - (\nu - 1)c_y + \frac{\nu}{2} \right) = 0. \qquad \text{(B35)}$$

So, finally it is possible to write:

$$\upsilon_t - \upsilon_t^{\max} = \left[\Omega + (1-s)(c_y^2 - c_y) \right](\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\rho + \phi}{2} \right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2} \right) - \frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p^*t}^2 \right)$$
$$= \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D} \right)(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma + \phi}{2} \right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2} \right) \qquad \text{(B36)}$$
$$-\frac{\xi}{2}\left(\sigma_{p_H t}^2 + \sigma_{p_F^* t}^2 \right) + o\left(\left\| a^3 \right\| \right).$$

This expression reduces to CGG's when there is no home bias ($\gamma = 1$). To see this from their expression at the top of p 903, multiply their utility by 2 (since they take average utility), and set their γ equal to ½ (so their country sizes are equal).

B.2 Derivation of Welfare Function under LCP with Home Bias in Preferences

The second-order approximation to welfare in terms of logs of consumption and employment of course does not change, so Equation (B6) still holds. As before, the derivation is broken down into two parts. First-order approximations to structural equations are used to derive an approximation to the quadratic term

$$\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2} \right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2} \right) + (1-\sigma)\left(\overline{c}_t \tilde{c}_t + \overline{c}_t^* \tilde{c}_t^* \right) - (1+\phi)\left(\overline{n}_t \tilde{n}_t + \overline{n}_t^* \tilde{n}_t^* \right).$$

Then second-order approximations to the structural equations are used to derive an expression for $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$.

The quadratic term involves squares and cross-products of $\tilde{c}_t$, $\tilde{c}_t^*$, $\bar{c}_t$, $\bar{c}_t^*$, $\tilde{n}_t$, $\tilde{n}_t^*$, $\bar{n}_t$, and $\bar{n}_t^*$. Expressions (B9)–(B10) still provide first-order approximations for $\bar{c}_t$ and $\bar{c}_t^*$; Equations (B13)–(B14) are first-order approximations for $\tilde{n}_t$ and $\tilde{n}_t^*$; and Equations (B15)–(B16) are first-order approximations for $\bar{n}_t$ and $\bar{n}_t^*$. But Equations (A7)–(A8) and (B11)–(B12) are required to derive:

$$\tilde{c}_t = c_y \tilde{y}_t + (1-c_y)\tilde{y}_t^* + \frac{\nu(2-\nu)}{2D}\Delta_t + o\left(\left\|a^2\right\|\right), \tag{B37}$$

$$\tilde{c}_t^* = (1-c_y)\tilde{y}_t + c_y \tilde{y}_t^* - \frac{\nu(2-\nu)}{2D}\Delta_t + o\left(\left\|a^2\right\|\right). \tag{B38}$$

With these equations, the derivation as in Equation (B17) can be followed. After tedious algebra, the same result is achieved, with the addition of the terms $\frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}\Delta_t^2$ and $\frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}\Delta_t(y_t - y_t^*)$. Note that the last term involves output levels, not output gaps. That is:

$$\begin{aligned}
&\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right) \\
&= \left(\frac{1-\sigma}{2}\right)\left(2c_y^2 - 2c_y + 1\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - (1-\sigma)\left(2c_y^2 - 2c_y\right)\tilde{y}_t\tilde{y}_t^* \\
&\quad - \left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) + (1-\sigma)\left(2(c_y^2 - c_y) - (\nu-1)c_y + \frac{\nu}{2}\right)\left(\bar{y}_t - \bar{y}_t^*\right)\left(\tilde{y}_t - \tilde{y}_t^*\right) \\
&\quad + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}\Delta_t^2 + \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}\Delta_t(y_t - y_t^*) + o\left(\left\|a^2\right\|\right).
\end{aligned} \tag{B39}$$

The derivation of $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$ is similar to the PCP model. However, one tedious aspect of the derivation is that the equality $S_t^* = S_t^{-1}$, that holds under PCP and flexible prices, cannot be used. The equilibrium conditions are expressed for home output, and its foreign equivalent, from Equations (18) and (19):

$$Y_t = \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right)(S_t^*)^{-\nu/2} C_t^*, \tag{B40}$$

$$Y_t^* = \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t. \tag{B41}$$

Directly taking second-order approximations of these equations around the efficient non-stochastic steady state implies:

$$\begin{aligned} y_t + \frac{1}{2} y_t^2 = {} & \frac{\nu}{2} c_t + \left(\frac{2-\nu}{2}\right) c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* \\ & + \frac{1}{2}\left\{ \frac{\nu}{2} c_t^2 + \left(\frac{2-\nu}{2}\right) c_t^{*2} + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^2 + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^{*2} \right\} \\ & + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* + o\left(\left\| a^3 \right\|\right), \end{aligned} \tag{B42}$$

$$\begin{aligned} y_t^* + \frac{1}{2} y_t^{*2} = {} & \frac{\nu}{2} c_t^* + \left(\frac{2-\nu}{2}\right) c_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t \\ & + \frac{1}{2}\left\{ \frac{\nu}{2} c_t^{*2} + \left(\frac{2-\nu}{2}\right) c_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^{*2} + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^2 \right\} \\ & + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t + o\left(\left\| a^3 \right\|\right). \end{aligned} \tag{B43}$$

Note that in a second-order approximation, $s_t = -s_t^*$ cannot be imposed. However, $s_t^2 = s_t^{*2}$ can be imposed. Then adding Equations (B42) and (B43) together, leads to:

$$y_t + y_t^* + \frac{1}{2}(y_t^2 + y_t^{*2}) = c_t + c_t^* + \frac{1}{2}\left\{ c_t^2 + c_t^{*2} + \nu\left(\frac{2-\nu}{2}\right) s_t^2 \right\} + o\left(\left\| a^3 \right\|\right). \tag{B44}$$

Next, Equations (A7), (A8) and (A9) can be used to get approximations for c_t^2, c_t^{*2} and s_t^2. These equations are linear approximations for c_t, c_t^* and s_t, but since the goal is to approximate the squares of these variables, that is sufficient. With some algebra, it can be shown that:

$$
\begin{aligned}
c_t^2 + c_t^{*2} + \nu\left(\frac{2-\nu}{2}\right)s_t^2 = y_t^2 + y_t^{*2} &+ \nu\left(\frac{2-\nu}{2}\right)\left(\frac{(1-\nu)(1-\sigma)}{D}\right)^2 (y_t - y_t^*)^2 u \\
&+\nu\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 \\
&+\nu\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t(y_t - y_t^*) + o\left(\left\|a^3\right\|\right).
\end{aligned} \tag{B45}
$$

Then, substituting Equation (B45) into Equation (B44) and rearranging, it follows that:

$$
\begin{aligned}
c_t + c_t^* = y_t + y_t^* &- \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{(1-\nu)(1-\sigma)}{D}\right)^2 (y_t - y_t^*)^2 \\
&- \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t(y_t - y_t^*) + o\left(\left\|a^3\right\|\right).
\end{aligned} \tag{B46}
$$

Note that if $\Delta_t = 0$, Equation (B46) leads to the second-order approximation for $c_t + c_t^*$ from the PCP model.

Then following the derivations as in the PCP model derivation of Equation (B31), it follows that:

$$
\begin{aligned}
\tilde{c}_t + \tilde{c}_t^* = \tilde{y}_t + \tilde{y}_t^* &+ \Omega(\tilde{y}_t^2 + 2\overline{y}_t\tilde{y}_t + \tilde{y}_t^{*2} + 2\overline{y}_t^*\tilde{y}_t^* - 2\overline{y}_t\tilde{y}_t^* - 2\overline{y}_t^*\tilde{y}_t - 2\tilde{y}_t\tilde{y}_t^*) \\
&- \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t(y_t - y_t^*) + o\left(\left\|a^3\right\|\right).
\end{aligned} \tag{B47}
$$

As shown in Section B.3, the following second-order approximation can be made:

$$
\tilde{n}_t + \tilde{n}_t^* = \tilde{y}_t + \tilde{y}_t^* + \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_H^* t}^2 + \frac{\nu}{2}\sigma_{p_F^* t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right] + o\left(\left\|a^3\right\|\right). \tag{B48}
$$

Equation (B47) and (B48), along with Equation (B39), can be substituted into the loss function (B6). Notice the cancellations that occur. The cross-product terms on $\Delta_t(y_t - y_t^*)$ in Equations (B39) and (B47) cancel. The other cross-product terms involving output gaps and efficient levels of output also cancel, just as in the PCP model, when Equation (B35) was used. Hence:

$$\begin{aligned}\upsilon_t - \upsilon_t^{\max} &= \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 \\ &-\left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ &-\frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_H{}^* t}^2 + \frac{\nu}{2}\sigma_{p_F{}^* t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right] + o\left(\left\|a^3\right\|\right).\end{aligned} \tag{B49}$$

B.3 Derivations of Price Dispersion Terms in Loss Functions

In the PCP case, it is true that

$$A_t N_t = A_t \int_0^1 N_t(f)df = Y_t \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df = Y_t V_t, \tag{B50}$$

where $V_t \equiv \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df$. Taking logs:

$$a_t + n_t = y_t + v_t. \tag{B51}$$

It is the case that $v_t \equiv \ln\left(\int_0^1 e^{-\xi \hat{p}_{Ht}(f)} df\right)$,

where we define

$$\hat{p}_{Ht}(f) \equiv p_{Ht}(f) - p_{Ht}. \tag{B52}$$

Following Galí (2008), note that

$$e^{(1-\xi)\hat{p}_{Ht}(f)} = 1 + (1-\xi)\hat{p}_{Ht}(f) + \frac{(1-\xi)^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right). \qquad \text{(B53)}$$

By the definition of the price index P_{Ht}, $\int_0^1 e^{(1-\xi)\hat{p}_{Ht}(f)} df = 1$. Hence, from (B53),

$$\int_0^1 \hat{p}_{Ht}(f) df = \frac{\xi - 1}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right). \qquad \text{(B54)}$$

It is also the case that

$$e^{-\xi\hat{p}_{Ht}(f)} = 1 - \xi\hat{p}_{Ht}(f) + \frac{\xi^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right). \qquad \text{(B55)}$$

It follows, using (B54):

$$\begin{aligned} \int_0^1 e^{-\xi\hat{p}_{Ht}(f)} df &= 1 - \xi\int_0^1 \hat{p}_{Ht}(f) df + \frac{\xi^2}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right) \\ &= 1 + \frac{\xi}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right). \end{aligned} \qquad \text{(B56)}$$

Note the following relationship:

$$\int_0^1 \hat{p}_{Ht}(f)^2 df = \int_0^1 (p_{Ht}(f) - E_f(p_{Ht}(f))^2 df + o\left(\left\|a^3\right\|\right) = \mathrm{var}(p_{Ht}) + o\left(\left\|a^3\right\|\right). \qquad \text{(B57)}$$

Using our notation for variances, $\sigma^2_{p_H t} \equiv \mathrm{var}(p_{Ht})$, and taking the log of (B56) leads to

$$v_t = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\left\|a^3\right\|\right). \qquad \text{(B58)}$$

Substituting this into Equation (B51), and recalling that $\bar{y}_t = \bar{n}_t + a_t$, implies Equation (B32). The derivation of Equation (B33) for the foreign country proceeds identically.

For the LCP model, the following second-order approximation to the equation $Y_t = C_{Ht} + C^*_{Ht}$ is used:

$$y_t = \frac{\nu}{2} c_{Ht} + \left(\frac{2-\nu}{2}\right) c^*_{Ht} + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\left\|a^3\right\|\right). \quad \text{(B59)}$$

In the LCP model, it is possible to write:

$$\begin{aligned} A_t N_t &= A_t \int_0^1 N_t(f)df = C_{Ht}\int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df + C^*_{Ht}\int_0^1 \left(\frac{P^*_{Ht}(f)}{P^*_{Ht}}\right)^{-\xi} df \\ &= C_{Ht}V_{Ht} + C^*_{Ht}V^*_{Ht}, \end{aligned} \quad \text{(B60)}$$

where the definitions of V_{Ht} and V^*_{Ht} are analogous to that of V_t in the PCP model. Taking a second-order log approximation to the Expression (B60):

$$\begin{aligned} a_t + n_t &= \frac{\nu}{2}(c_{Ht} + v_{Ht}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + v^*_{Ht}) \\ &+ \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)\left((c_{Ht} + v_{Ht})^2 + 2(c_{Ht} + v_{Ht})(c^*_{Ht} + v^*_{Ht}) + (c^*_{Ht} + v^*_{Ht})^2\right) \\ &+ o\left(\left\|a^3\right\|\right). \end{aligned} \quad \text{(B61)}$$

The same steps as in the PCP model can be followed to conclude:

$$v_{Ht} = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\left\|a^3\right\|\right), \quad \text{(B62)}$$

$$v^*_{Ht} = \frac{\xi}{2}\sigma^2_{p_H {}^* t} + o\left(\left\|a^3\right\|\right). \quad \text{(B63)}$$

Substituting these expressions into Equation (B61) and cancelling higher-order terms, implies:

$$a_t + n_t = \frac{\nu}{2}(c_{Ht} + \frac{\xi}{2}\sigma^2_{p_Ht}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + \frac{\xi}{2}\sigma^2_{p_H{}^*t}) \\ + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\left\|a^3\right\|\right). \tag{B64}$$

Then using Equation (B59), this implies that:

$$a_t + n_t = y_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_Ht} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H{}^*t}\right) + o\left(\left\|a^3\right\|\right). \tag{B65}$$

Keeping in mind that $\bar{y}_t = \bar{n}_t + a_t$:

$$\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_Ht} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H{}^*t}\right) + o\left(\left\|a^3\right\|\right). \tag{B66}$$

Following analogous steps for the foreign country,

$$\tilde{n}^*_t = \tilde{y}^*_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_F{}^*t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_Ft}\right) + o\left(\left\|a^3\right\|\right). \tag{B67}$$

Adding Equations (B66) and (B67) gives Equation (B48).

Finally, to derive the loss functions for policy-makers (Equation (65) for the PCP model and (69) for the LCP model), note that the loss function is the present expected discounted value of the period loss functions derived here (Equation (B36) for the PCP model and (B49) for the LCP model). That is, the policy-maker seeks to minimise $-E_t\sum_{j=0}^{\infty}\beta^j(u_{t+j} - u^{\max}_{t+j})$.

Following Woodford (2003, Chapter 6), it can be seen that, in the PCP model, if prices are adjusted according to the Calvo price mechanism given by Equation (50) for P_{Ht}, then

$$\sum_{j=0}^{\infty}\beta^j\sigma^2_{p_Ht+j} = \frac{\beta}{(1-\beta\theta)(1-\theta)}\sum_{j=0}^{\infty}\beta^j\pi^2_{Ht+j}. \tag{B68}$$

Analogous relationships hold for P_{Ft}^* in the PCP model, and for P_{Ht}, P_{Ft}^*, P_{Ft}, and P_{Ht}^* in the LCP model. This relationship can then be substituted into the present value loss function, $-E_t\sum_{j=0}^{\infty}\beta^j(u_{t+j}-u_{t+j}^{\max})$, to derive the loss functions of the two models presented in the text.

B.4 Solutions for Endogenous Variables under Optimal Policy Rules

Assume that shocks follow the processes (where the *W* superscript on the wage mark-up shocks have been dropped):

$$a_t = \rho a_{t-1} + \varepsilon_{at}$$

$$a_t^* = \rho a_{t-1}^* + \varepsilon_{a^*t}$$

$$\mu_t = \rho\mu_{t-1} + \varepsilon_{\mu t}$$

$$\mu_t^* = \rho\mu_{t-1}^* + \varepsilon_{\mu^*t}$$

s_t is determined by :

$$s_t = \theta s_{t-1} + \frac{\theta\delta}{1-\beta\theta\rho}\left[(a_t - a_t^*) + (\mu_t - \mu_t^*)\right].$$

The solutions for the variables that appear in the loss function under the optimal policy described in the text are:

$$\pi_t^R = \frac{\delta(\nu-1)}{1+\sigma\xi\delta-\beta\rho}(\mu_t - \mu_t^*)$$

$$\pi_t^W = \frac{\delta}{1+\sigma\xi\delta-\beta\rho}(\mu_t + \mu_t^*)$$

$$\tilde{y}_t^W = -\xi\pi_t^W$$

$$\tilde{y}_t^R = \nu(2-\nu)(s_t - (a_t - a_t^*)) - (\nu - 1)\pi_t^R$$

$$\tilde{y}_t = \frac{1}{2}(\tilde{y}_t^W + \tilde{y}_t^R)$$

$$\tilde{y}_t^* = \frac{1}{2}(\tilde{y}_t^W - \tilde{y}_t^R)$$

$$\Delta_t = -\xi\sigma\pi_t^R - (\nu - 1)(s_t - (a_t - a_t^*))$$

$$\pi_{Ht} = \frac{1}{2}(\pi_t^W + \pi_t^R) - \left(\frac{2-\nu}{2}\right)(s_t - s_{t-1})$$

$$\pi_{Ft} = \frac{1}{2}(\pi_t^W + \pi_t^R) + \frac{\nu}{2}(s_t - s_{t-1})$$

$$\pi_{Ft}^* = \frac{1}{2}(\pi_t^W - \pi_t^R) + \left(\frac{2-\nu}{2}\right)(s_t - s_{t-1})$$

$$\pi_{Ht}^* = \frac{1}{2}(\pi_t^W - \pi_t^R) - \frac{\nu}{2}(s_t - s_{t-1})$$

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