(Un)conventional Policy and the Zero Lower Bound*

Fiorella De Fiore† Oreste Tristani‡

European Central Bank European Central Bank

This version: 1 September 2012

Abstract

We study the optimal combination of interest rate policy and unconventional monetary policy in a model where agency costs generate a spread between deposit and lending rates. We demonstrate that, in the face of adverse financial shocks, measures of the "credit policy" type can be a powerful substitute for interest rate policy: once such unconventional measures have been deployed, it is sub-optimal to lower policy rates further.Thus, credit policy reduces the likelihood of hitting the zero bound constraint.

Keyworks: optimal monetary policy, financial frictions, zero-lower bound, asymmetric information

JEL codes: E44, E52, E61

*All opinions expressed are personal and do not necessarily represent the view of the European Central Bank or the European System of Central Banks. We wish to thank participants in the conferences on "Modelling Monetary Policy," organised by the Bank of England and CCBS, on "Economic Policy and Business Cycle", at Bicocca University, and on "DSGE and Beyond: Expanding the Paradigm in Monetary Policy research?", at the National Bank of Poland. We are also grateful to Paul Fackler for his suggestions on how to improve the speed of our computations.

†Directorate General Research, European Central Bank. Email: fiorella.de_fiore@ecb.int.

‡Directorate General Research, European Central Bank. Email: oreste.tristani@ecb.int.

1 Introduction

In response to the financial and economic crisis of 2008-09, central banks have aggressively cut monetary policy rates, in many cases all the way to the zero lower bound (henceforth ZLB). At the same time, all central banks have implemented so-called "non-standard" or "unconventional" monetary policy measures.

However, standard and non-standard measures have been combined in different ways by different central banks (for a cross-country comparison see e.g. Lenza, Pill and Reichlin, 2010). Taking the expansion of the central banks' balance sheets as an indicator, non-standard measures were implemented in late 2008, after the failure of Lehman Bros., both in the US and in the euro area. As far as standard monetary policy is concerned, the Federal Reserve cut its interest rates to near zero almost at the same time: the Federal funds rate reached 1% at the end of October and the 0.00-0.25% range in December. The European Central Bank, on the contrary, never cut its main policy interest rate to zero. The rate on the main refinancing operations (MRO) was reduced sharply at the end of 2008 and at the beginning of 2009, but it bottomed at 1% in May 2009 without descending further.[1]

The sequencing of standard and non-standard measures implemented by the Federal Reserve can be understood in the light of the ZLB literature which predates the financial crisis (see e.g. Reifschneider and Williams, 2000, Eggertsson and Woodford, 2003, Adam and Billi, 2006, and Nakov, 2008). The tenet of that literature is that standard interest rate policy is the best monetary policy tool in response to shocks leading to a fall in the natural rate of interest. Any other type of policy response should only be considered as a *substitute for standard interest rate policy*, once the latter is no longer available because the ZLB constraint is binding.

Since 2008, however, a number of papers have reconsidered this issue and demonstrated that certain non-standard measures can be an effective response to distortions which prevent the efficient allocation of financial resources–see e.g. Gertler and Karadi (2010), Gertler and Kiyotaki (2010), Del Negro et al. (2010) and Eggertsson and Krugman (2010). Such measures have been described as "credit policy", i.e. measures aimed at offsetting impairments

[1] The rate on the main refinancing operations has been reduced further in 2012, to 0.75%, after the intensification of the sovereign debt crisis.

to the process of credit creation. Cúrdia and Woodford (2011) describe standard and non-standard measures as *complementary* to each other. As such, the two measures could be used contemporaneously. It is not necessary to reach the ZLB, before implementing credit policy.

As a result, it becomes conceivable that under certain circumstances–notably in reaction to financial shocks which impair credit creation–credit policy may be a strictly more efficient tool than policy interest rates. Credit policy could be so effective to become a *substitute for standard policy.* For this type of non-standard measure, the optimal sequencing of policy responses would be the opposite of what was recommended by the pre-crisis ZLB literature: credit policy should be deployed first, while interest rate cuts should only be considered if the scope for credit policy is exhausted.

What is the optimal mix of standard and non-standard policy responses in a dynamic, general equilibrium model? If non-standard measures can be targeted to the prevailing source of financial inefficiency, do they reduce the likelihood that interest rates reach the ZLB? Should interest rate policy be used at all, once unconventional measures have been deployed? Looking forward to the return to normal conditions, the so-called "exit", how long should non-standard policies be optimally kept in place?

Our paper attempts to answer these questions within a simple model, where non-standard measures, while highly stylised, retain some realistic features. The model features both sticky prices and financial frictions in the standard form of asymmetric information and monitoring costs. Under normal conditions, the flow of credit in the economy takes the form of bank intermediation.

In this environments, we model non standard measures as direct central bank intermediation, like in Gertler and Kiyotaki (2010) and Curdia and Woodford (2011). Under normal circumstances central bank intermediation is inefficient, because the central bank has a worse loan monitoring technology than commercial banks. In a crisis, however, we assume that commercial banks monitoring costs increase, for example due to higher costs associated to asset liquidation. If the crisis is sufficiently severe, the central bank becomes a competitive lender and can replace commercial banks in providing loans to firms.

Our main result is that, in the face of adverse financial shocks which reduce banks' monitoring efficiency, non-standard measures can indeed be powerful substitutes of standard interest rate policy. Once non-standard measures have been deployed, the real economy is insulated from further adverse financial developments. There is therefore no reason to lower policy rates further. In an illustrative example, we show that it can be optial for the central bank not to cut rates to zero, and to implement non-standard measures instead. This example is consistent with the mix of standard and non-standard policy actions implemented by the ECB as of 2008.

In general, the exact timing of implementation of standard and non-standard measures depends on the size of the monitoring advantage of commercial banks over the central bank–an object which is difficult to calibrate. Non-standard measures are more likely to be deployed in response to large financial shocks. Non-standard measures are not justified in reaction to demand, or technology shocks.

Concerning "exit", we show that its timing can be significantly affected by some detailed features of the propagation of financial shocks. To develop an intuition for this results, we derive in closed form the target rule which would implement the Ramsey allocation (under the timeless perspective and if the ZLB is ignored) in our simple model. Compared to the model with frictionless financial markets, the target rule implies a stronger mean reversion of the price level. In response to a shock which increases the price level on impact, the price level falls over time and eventually returns to a value lower than its initial level–and viceversa.

In our simple model, financial shocks affect firms' marginal costs and have a cost-push component, but do not affect directly aggregate demand (i.e. consumption). As a result, while typically lowering interest rates on impact to cushion the effects on the real economy, optimal policy also requires a commitment to increasing rates relatively quickly thereafter–notably increasing them long before non-standard measures are reabsorbed.

To test the robustness of this conclusion, we also study the policy implications of a richer model with capital, where financial frictions do affect aggregate demand, and notably investment. In this case, interest rates are optimally increased much more slowly than in the simple model. However it remains true that non-standard measures tend to remain in place long after the policy interest rates has returned to its long run level.

Finally, abstracting from non-standard measures, we revisit the prescription of the simple new Keynesian literature that the likelihood of being at the ZLB and the severity of the ensuing recession can be reduced by an appropriate policy commitment. More specifically the central bank should promise to keep interest rates low in the future for a longer period than optimal in the absence of the ZLB. Such promise, if credible, generates high inflation expectations, reduces the current real interest rate and stimulates the economy. When non-standard measures are ruled out, these prescriptions remain valid in our model.

Our paper is structured as follows. In section 2, we describe the model. In section 3, we outline the procedure we use to solve the model under the ZLB constraint and allowing for non-standard measures. This section also derives a system of log-linear equilibrium conditions, which we use to develop an intuition for our numerical results. We also present here the basic features of the richer model with capital which we analyse to test the robustness of our results. In section 4, we present the welfare analysis. For the benchmark model, we derive a second-order approximation to the welfare function and the first order conditions of the Ramsey allocation. This allows us to derive in closed form the target rule which, absent the ZLB constraint, would implement the Ramsey allocation. Our numerical results are presented in Section 5 and section 6 offers some concluding remarks.

2 The model

The economy is inhabited by a representative infinitely-lived household, wholesale firms owned by risk-neutral entrepreneurs, monopolistically competitive retail firms owned by the households, zero-profit financial intermediaries, a government and a central bank. We describe in turn the problem faced by each class of agents.

2.1 Households

At the beginning of period t, interest is paid on nominal financial assets acquired at time $t-1$. The households, holding an amount W_t of nominal wealth, choose to allocate it among existing nominal assets, namely money M_t, a portfolio of nominal state-contingent bonds A_{t+1} each paying a unit of currency in a particular state in period $t+1$, and one-period deposits denominated in units of currency, D_t, paying back $R_t^d D_t$ at the end of the period.

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period $t+1$ is equal to

$$M_t + P_t w_t h_t + Z_t - P_t c_t + T_t, \tag{1}$$

where h_t is hours worked, w_t is the real wage, Z_t are nominal profits transferred from retail producers to households, and T_t are lump-sum nominal transfers from the government. c_t denote a CES aggregator of a continuum $\eta \in (0,1)$ of differentiated consumption goods produced by retail firms, $c_t = \left[\int_0^1 c_t(\eta)^{\frac{\varepsilon-1}{\varepsilon}} d\eta\right]^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$. P_t is the price of the CES aggregator.

Nominal wealth at the beginning of period $t+1$ is given by

$$W_{t+1} = A_{t+1} + R_t^d D_t + R_t^m \{M_t + P_t w_t h_t + Z_t - P_t c_t - T_t\}, \tag{2}$$

where R_t^m denotes the interest paid on money holdings.

The household's problem is to maximize preferences, defined as

$$E_o \left\{ \sum_0^\infty \beta^t \left[u(c_t) + \kappa(m_t) - v(h_t)\right] \right\}, \tag{3}$$

where $u_c > 0$, $u_{cc} < 0$, $\kappa_m \geq 0, \kappa_{mm} < 0$, $v_h > 0$, $v_{hh} > 0$, and $m_t \equiv M_t/P_t$ denotes real balances. The problem is subject to the budget constraint

$$M_t + D_t + E_t\left[Q_{t,t+1} A_{t+1}\right] \leq W_t, \tag{4}$$

In our model, because external finance needs to be raised before production, financial markets open at the beginning of the period and goods market at the end of the period, as in Lucas and Stokey (1987). One implication of this timing is that real balances affect the equilibrium. In order to relate to the new-Keynesian model with no financial frictions (which is the workhorse model used to analyse monetary policy at the ZLB), we neutralize the effect of the different timing on the equilibrium. We do so by assuming that monetary policy remunerates money holdings at a rate R_t^m that is proportional to the risk-free rate R_t. Define $\Lambda_{m,t} \equiv \frac{R_t - R_t^m}{R_t}$. Under our assumption, $\Lambda_{m,t} = \Lambda_m$ for all t, and money demand satisfies $\kappa_m(m_t) = \frac{\Lambda_m}{1-\Lambda_m} u_c(c_t)$. The households' optimality conditions are then identical to those obtained in the standard New Keynesian model without financial frictions. They are given by $R_t = R_t^d = E_t\left[Q_{t,t+1}\right]^{-1}$ and

$$-\frac{v_h(h_t)}{u_c(c_t)} = w_t, \tag{5}$$

$$u_c(c_t) = \beta R_t E_t \left\{ \frac{u_c(c_{t+1})}{\pi_{t+1}} \right\}, \tag{6}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$. The optimal allocation of expenditure between the different types of goods is given by $c_t(\eta) = \left(\frac{P_t(\eta)}{P_t}\right)^{-\varepsilon} c_t$, where $P_t(\eta)$ is the price of good η.

2.2 Wholesale firms

Wholesale firms, indexed by i, are competitive and owned by infinitely lived entrepreneurs. Each firm i produces the amount $y_{i,t}$ of an homogeneous good, using a linear technology

$$y_{i,t} = \omega_{i,t} l_{i,t}. \tag{7}$$

Here $\omega_{i,t}$ is an iid productivity shock with distribution function Φ and density function ϕ, which is observed at no cost only by firms.

At the beginning of the period, each firm receives an exogenous endowment τ_t, which can be used as internal funds. Since these funds are not sufficient to finance the firm's desired level of production, firms need to raise external finance. Before observing $\omega_{i,t}$, firms sign a contract with a financial intermediary to raise a nominal amount $P_t\left(x_{i,t} - \tau_t\right)$, where

$$x_{i,t} \geq w_t l_{i,t}. \tag{8}$$

Each firm i's demand for labor is derived by maximizing firm's expected profits, subject to the financing constraint (8).

Let $\overline{P}_t$ be the price of the wholesale homogenous good, $\frac{\overline{P}_t}{P_t} = \chi_t^{-1}$ the relative price of wholesale goods to the aggregate price of retail goods, and $(q_t - 1)$ the Lagrange multiplier on the financing constraint. Optimality requires that

$$\begin{aligned} q_t &= \frac{1}{w_t \chi_t} && (9) \\ x_{i,t} &= w_t l_{i,t} && (10) \end{aligned}$$

implying that

$$\mathcal{E}\left(y_t\right) = \chi_t q_t x_t, \tag{11}$$

where $\mathcal{E}[\cdot]$ is the expectation operator at the time of the factor decision.

Equation (11) states that wholesale firms must sell at a mark-up $\chi_t q_t$ over firms' production costs to cover for the presence of credit frictions and for the monopolistic distortion in the retail sector. Notice that all firms are ex-ante identical. Hence, we drop below the subscripts i.

The assumption that firms receive an endowment from the government at the beginning of the period is made for simplicity, in order to reduce the number of state variables and to facilitate the computation of the numerical solution of the model. The absence of accumulation of firms' net worth implies that the persistence of the endogenous variables merely reflects the persistence of the exogenous shocks. Nonetheless, financial frictions provide an important transmission channel in our economy, through the credit constraint faced by firms and the endogenous spread charged by financial intermediaries. As documented in De Fiore and Tristani (2012), up to a linear approximation, the model with and without capital accumulation delivers qualitatively similar responses to both real and financial shocks. Moreover, the characterization of optimal monetary policy is broadly similar in these two cases.

2.3 The financial contract

In writing the financial contract we need to be explicit about what constitutes unconventional policy in our model. We will focus on an interpretation of non-standard measures in which the central bank replaces the private banking sector and does direct intermediation to firms.

Direct lending is closest to the Federal Reserve facilities set up for direct acquisition of high quality private securities (see also Gertler and Kiyotaki, 2010). As in both the Fed and the ECB cases, in our model the central bank lending program is financed though an increase in interest bearing banks' reserves. As a result, non-standard measures lead to a large increase in the central bank's balance sheet.

Direct lending in our model is entirely demand determined: central bank intermediation is chosen endogenously when it can be performed at a lower cost (spread) than private bank intermediation. This has also been a feature of the ECB's provision of liquidity through the "enhanced credit support program", which satisfied liquidity demand completely at a predefined interest rate.[2]

Finally, we design credit policy in such a way that the central bank takes on no credit risk. Together with the assumption that reserves are remunerated, this implies that the expansion of the central bank's balance sheet has no inflationary consequences, nor any implications for government finances.

The financial contract is structured as follows. External finance, $x_t - \tau_t$, takes the form of either bank loans or direct lending from the central bank. Each firm pledges a fraction γ_t of its net worth τ_t as collateral for a financial contract with a commercial bank, and the remaining fraction for a financial contract with the central bank.

[2] Differently from what happens in our model, however, the ECB program operates through banks, rather than being directly aimed at firms.

Firms face the idiosyncratic productivity shock $\omega_{i,t}$, whose realization is observed at no costs only by the entrepreneur. If the realization of the idiosyncratic shock $\omega_{i,t}$ is sufficiently low, the value of firm production is not sufficient to repay the loans and the firm defaults.

The financial intermediaries (banks or the central bank) can monitor ex-post the realization of $\omega_{i,t}$, but a fraction of firm's output is consumed in the monitoring activity. These monitoring costs are associated with legal fees and asset liquidation in case of bankruptcy. We assume that commercial banks are more efficient monitors than the central bank, i.e. $\mu_t^c > \mu_t^b$, where μ_t^c and μ_t^b denote the fraction of the firm output lost in monitoring by the central bank and by commercial banks, respectively.

Define

$$f(\overline{\omega}) \equiv \int_{\overline{\omega}}^{\infty} \omega \Phi(d\omega) - \overline{\omega}\left[1 - \Phi(\overline{\omega})\right] \tag{12}$$

and

$$g(\overline{\omega};\mu) \equiv \int_{0}^{\overline{\omega}} \omega \Phi(d\omega) - \mu\Phi(\overline{\omega}) + \overline{\omega}\left[1 - \Phi(\overline{\omega})\right] \tag{13}$$

as the expected shares of output accruing respectively to entrepreneurs and to the financial intermediary, after stipulating a contract that sets a fixed repayment on one unit of debt at $\overline{P}_t \chi_t q_t \overline{\omega}_t$, when the fraction of output lost in monitoring cost is μ_t. Notice that $f(\overline{\omega}) + g(\overline{\omega};\mu) = 1 - \mu\Phi(\overline{\omega})$.

Commercial banks collect deposits D_t from households. As deposits are the only funds available to finance loans in the economy, $D_t = P_t(x_t - \tau_t)$. Banks use a fraction γ_t of deposits to finance loans to firms, and they deposit the remaining fraction, $1-\gamma_t$, as reserves at the central bank. These reserves are remunerated at the market rate R_t^d and used in turn by the central bank to finance firms. The fraction γ_t of deposits lent by commercial banks is then combined with a fraction γ_t of the firms' internal funds to finance the production of $\gamma_t q_t \chi_t x_t$ units of wholesale goods. The budget constraint for the bank is

$$(1-\gamma_t) R_t^d P_t (x_t - \tau_t) + \gamma_t \overline{P}_t q_t \chi_t g\left(\overline{\omega}_t^b; \mu_t^b\right) x_t \geq R_t^d P_t (x_t - \tau_t).$$

The first term on the LHS is the amount of reserves held at the central bank, gross of their remuneration, in units of currency. The second term on the LHS is the gross nominal return to banks from extending credit of $\gamma_t P_t (x_t - \tau_t)$ units of money to firms. The RHS is the cost of funds for the bank.

The central bank uses all its funds (reserves) to satisfy the demand for credit by firms. Its budget constraint is

$$\left(1-\gamma_t\right)\overline{P}_t q_t \chi_t g\left(\overline{\omega}_t^c;\mu_t^c\right)x_t \geq \left(1-\gamma_t\right)R_t^d P_t\left(x_t-\tau_t\right).$$

The constrain says that the return to the central bank from lending $\left(1-\gamma_t\right)P_t\left(x_t-\tau_t\right)$ units of money to firms must be sufficient to cover for the costs of funds (the remuneration of reserves).

Each firm stipulates a contract with a commercial bank that sets a fixed repayment on each unit of debt of $\overline{P}_t\chi_t q_t\overline{\omega}_t^b$, and a contract with the central bank that sets it at $\overline{P}_t\chi_t q_t\overline{\omega}_t^c$. The firm also chooses optimally the fraction of its net worth to allocate to the two contracts. The informational structure corresponds to a standard costly state verification (CSV) problem (see e.g. Gale and Hellwig (1985)). The problem is

$$\max_{\overline{\omega}_t^b,\overline{\omega}_t^c,x_t,\gamma_t}\left[\gamma_t f(\overline{\omega}_t^b)+\left(1-\gamma_t\right)f(\overline{\omega}_t^c)\right]\ q_t x_t$$

subject to $0\leq\gamma_t\leq 1$ and

$$\begin{aligned}
q_t g\left(\overline{\omega}_t^b;\mu_t^b\right)x_t &\geq R_t^d\left(x_t-\tau_t\right) && (14)\\
q_t g\left(\overline{\omega}_t^c;\mu_t^c\right)x_t &\geq R_t^d\left(x_t-\tau_t\right) && (15)\\
f\left(\overline{\omega}^b\right)+g\left(\overline{\omega}^b;\mu^b\right)+\mu^b\Phi\left(\overline{\omega}^b\right) &\leq 1 && (16)\\
f\left(\overline{\omega}^c\right)+g\left(\overline{\omega}^c;\mu^c\right)+\mu^c\Phi\left(\overline{\omega}^c\right) &\leq 1 && (17)\\
q_t x_t\left[\gamma_t f(\overline{\omega}_t^b)+\left(1-\gamma_t\right)f(\overline{\omega}_t^c)\right] &\geq \tau_t. && (18)
\end{aligned}$$

The optimal contract is the set $\left\{x_t,\overline{\omega}_t^b,\overline{\omega}_t^c,\gamma_t\right\}$ that maximizes the entrepreneur's expected nominal profits from jointly signing the two contracts, subject to the profits of the private bank and those of the central bank being sufficient to cover their respective repayment on deposits, (14) and (15), the feasibility conditions, (16) and (17), the entrepreneur being willing to sign the contract, (18), and the share γ_t being between zero and one.

Notice that the first two constraints hold with equality in equilibrium, implying that all banks make zero-profits. Satisfaction of those two conditions requires that

$$g\left(\overline{\omega}_t^b;\mu_t^b\right)=g\left(\overline{\omega}_t^c;\mu_t^c\right)=1-f\left(\overline{\omega}_t^b\right)-\mu_t^b\Phi\left(\overline{\omega}_t^b\right). \quad (19)$$

The optimality conditions include (19) and

$$
\begin{aligned}
q_t &= \frac{R_t^d}{1-\mu_t^b \Phi\left(\overline{\omega}_t^b\right) - f\left(\overline{\omega}_t^b\right) + \frac{\left[\gamma_t f\left(\overline{\omega}_t^b\right)+\left(1-\gamma_t\right) f\left(\overline{\omega}_t^c\right)\right]\left[f'\left(\overline{\omega}_t^b\right)+\mu_t^b \phi\left(\overline{\omega}_t^b\right)\right]}{\left[\gamma_t f'\left(\overline{\omega}_t^b\right)+\left(1-\gamma_t\right) f'\left(\overline{\omega}_t^c\right)\right]}} \qquad (20)\\
x_t &= \frac{R_t^d}{R_t^d - q_t\left[1-\mu_t^b \Phi\left(\overline{\omega}_t^b\right) - f\left(\overline{\omega}_t^b\right)\right]}\tau_t \\
\lambda_{5t} - \lambda_{6t} &= q_t x_t \left[f(\overline{\omega}_t^c) - f(\overline{\omega}_t^b)\right] \qquad (21)\\
\lambda_{5t}\gamma_t &= 0 \qquad (22)\\
\lambda_{6t}\left(\gamma_t - 1\right) &= 0, \qquad (23)
\end{aligned}
$$

together with $\lambda_{5t} \geq 0$ and $\lambda_{6t} \geq 0$.

Given the solution to the CSV problem, the gross interest rate on loans extended to firms by the commercial bank, R_t^b, and the one extended to firms by the central bank, R_t^c, can be backed up from the debt repayment. They are implicitely given by

$$\overline{P}_t \overline{\omega}_t^j \chi_t q_t x_t = R_t^j P_t \left(x_t - \tau_t\right), \qquad (24)$$

for $j = b, c$.

Define the spread between loan rates and the risk-free rate as $\Lambda_t^j = \frac{R_t^j}{R_t}$. We can now use expressions (24) to relate those spreads to the thresholds for the idiosyncratic productivity shocks, $\overline{\omega}_t^j$,

$$\Lambda_t^j = \frac{\overline{\omega}_t^j}{g(\overline{\omega}_t^j; \mu_t^j)}. \qquad (25)$$

2.4 Entrepreneurs

Entrepreneurs die with probability γ_t. They have linear preferences over the same CES basket of differentiated consumption goods as households, with rate of time preference β^e. This latter is sufficiently high so that the return on internal funds is always larger than the rate of time preference, $\frac{1}{\beta^e} - 1$, and entrepreneurs postpone consumption until the time of death.

As in De Fiore, Teles and Tristani (2011), we assume that the government imposes a tax ν on entrepreneurial consumption. It follows that

$$(1+\nu)\int_0^1 P_t(\eta)\, e_t(\eta)\, d\eta = \overline{P}_t \left[\omega_t - \gamma_t \overline{\omega}_t^b - \left(1-\gamma_t\right)\overline{\omega}_t^c\right] \chi_t q_t x_t,$$

where $e_t(\eta)$ is the firm's consumption of good η. Notice that $\int_0^1 P_t(\eta)\, e_t(\eta) = P_t e_t$,where e_t is the demand of the final consumption good. We can then write

$$(1+\nu)\, e_t = \left[\gamma_t f(\overline{\omega}_t^b) + (1-\gamma_t)\, f(\overline{\omega}_t^c)\right] q_t x_t.$$

We consider the case where ν becomes arbitrarily large. The tax revenue,

$$T_t^e = \frac{\nu_t \left[\gamma_t f(\overline{\omega}_t^b) + (1-\gamma_t)\, f(\overline{\omega}_t^c)\right] q_t x_t}{1+\nu}, \tag{26}$$

approaches the total funds of the entrepreneurs that die and the consumption of the entrepreneurs approaches zero, $e_t \to 0$.

The reason for this assumption is that, with $e_t > 0$, it would be optimal for policy to generate a redistribution of resources between households and entrepreneurs. This would enable to exploit the risk-neutrality of the latter to smooth out consumption of the former. Since risk neutrality of entrepreneurs is a simplifying assumption needed to derive debt as an optimal contract, we eliminate this type of incentives for monetary policy by completely taxing away entrepreneurial consumption. Allowing entrepreneurs to consume would also require arbitrary choices on the weight of entrepreneurs to be given in the social welfare function.

2.5 Government

Revenues from taxes on entrepreneurial consumption are used by the government to finance the transfer τ_t. Funds below (in excess of) τ_t are supplemented through (rebated to) households lump-sum taxes (transfers), T_t^h. The budget constraint of the government is

$$T_t^e = \tau_t - T_t^h. \tag{27}$$

2.6 Retail firms

As in Bernanke, Gertler and Gilchrist (1999), monopolistic competition occurs at the retail level. A continuum of monopolistically competitive retailers buy wholesale output from entrepreneurs in a competitive market and then differentiate it at no cost. Because of product differentiation, each retailer has some market power. Profits, Z_t, are distributed to the households, who own firms in the retail sector.

Output sold by retailer η, $Y_t(\eta)$, is used for households' and entrepreneurs' consumption. Hence, $Y_t(\eta) = c_t(\eta) + e_t(\eta)$. The final good Y_t is a CES composite of individual retail goods $Y_t = \left[\int_0^1 Y_t(\eta)^{\frac{\varepsilon-1}{\varepsilon}}\, d\eta\right]^{\frac{\varepsilon}{\varepsilon-1}}$,with $\varepsilon > 1$.

We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). Let $P_t^*(\eta)$ denote the price for good η set by retailers that can change the price at time t, and $Y_t^*(\eta)$ the demand faced given this price. Then each retailer chooses its price to maximize expected discounted profits. The optimality conditions are given by

$$1 = \theta\pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{\varepsilon}{\varepsilon-1}\frac{\overline{\Theta}_{1,t}}{\overline{\Theta}_{2,t}}\right)^{1-\varepsilon} \tag{28}$$

$$\overline{\Theta}_{1,t} = \frac{1}{\chi_t}Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon}\overline{Q}_{t,t+1}\overline{\Theta}_{1,t+1}\right] \tag{29}$$

$$\overline{\Theta}_{2,t} = Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon-1}\overline{Q}_{t,t+1}\overline{\Theta}_{2,t+1}\right], \tag{30}$$

where $\overline{Q}_{t,t+k} = \beta^k\left[\frac{u_c(c_{t+k})}{u_c(c_t)}\right]$.

Recall that the aggregate retail price level is given by $P_t = \left[\int_0^1 P_t(\eta)^{1-\varepsilon}d\eta\right]^{\frac{1}{1-\varepsilon}}$. Define the relative price of differentiated good η as $p_t(\eta) \equiv \frac{P_t(\eta)}{P_t}$ and divide both sides by P_t to express everything in terms of relative prices, $1 = \int_0^1 (p_t(\eta))^{1-\varepsilon}d\eta$.

Now define the relative price dispersion term as

$$s_t \equiv \int_0^1 (p_t(\eta))^{-\varepsilon}d\eta.$$

This equation can be written in recursive terms as

$$s_t = (1-\theta)\left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{-\frac{\varepsilon}{1-\varepsilon}} + \theta\pi_t^{\varepsilon}s_{t-1}. \tag{31}$$

2.7 Monetary policy

We characterize "standard" monetary policy as one where the central bank uses the nominal interest rate to implement the desired allocation, subject to a non-negativity constraint on the nominal interest rate

$$R_t \geq 0. \tag{32}$$

We define as "non-standard" monetary policy the ability of the central bank to affect allocations by intermediating credit directly. Commercial banks deposit part of their funds (households' deposits) at the central bank as reserves. These latter are remunerated at the risk-free rate R_t^d and used by the central bank to extend direct loans to firms. The rate charged on those loans, R_t^c, reflects the more inefficient monitoring technology available to the central bank ($\mu_t^c > \mu_t^b$) and is set as the optimal solution to a CSV problem.

The central bank also remunerates households' money holdings at a rate R_t^m that is proportional to the risk-free rate.

2.8 Market clearing

Market clearing conditions for money, bonds, labor, loans, wholesale goods and retail goods are given, respectively, by

$$\begin{aligned}
M_t &= M_t^s, && (33)\\
A_t &= 0, && (34)\\
h_t &= l_t, && (35)\\
D_t &= P_t\left(x_t - \tau\right), && (36)\\
y_t &= \int_0^1 Y_t\left(\eta\right) d\eta, && (37)\\
Y_t\left(\eta\right) &= c_t\left(\eta\right) + e_t\left(\eta\right), \text{ for all } \eta. && (38)
\end{aligned}$$

2.9 Equilibrium

An equilibrium is characterized by equations (5), (6), (11), (20)-(23), (26), (27), (28)-(31), (32), and (33)-(38), together with a specification of the path for the policy instrument R_t.

From equations (21)-(23), it can be seen that an equilibrium requires γ_t to take the value of either zero or one. When $\mu_t^b < \mu^c$, it must be that $\gamma_t = 1$. In that case, $\lambda_{5t} = 0$, and

$$\lambda_{6t} = q_t x_t \left[f(\overline{\omega}_t^b) - f(\overline{\omega}_t^c)\right] > 0.$$

The inequality follows from the zero profit conditions, (14) and (15), holding as equalities. Notice that $f'(\overline{\omega}_t^j) < 0$ and $g'(\overline{\omega}_t^j; \mu_t^j) = -f'(\overline{\omega}_t^j) - \mu g'(\overline{\omega}_t^j)$. If an interior solution to the problem exists, it must satisfy $g'\left(\overline{\omega}_{it}^j\right) > 0$. Otherwise, it would be optimal to set $\overline{\omega}_{it}^j = 0$ but banks would not be able to repay depositors. It follows that, if $\mu_t^b < \mu_t^c$, then $\overline{\omega}_t^b < \overline{\omega}_t^c$, and $\lambda_{6t} > 0$. In this case, $\gamma_t = 0$ cannot be an equilibrium because this would imply that $\lambda_{6t} = 0$ and $\lambda_{5t} < 0$.

Instead, when $\mu_t^b > \mu_t^c$, an equilibrium requires that $\gamma_t = 0$. In that case, $\lambda_{6t} = 0$, and $\lambda_{5t} > 0$. Now $\gamma_t = 1$ cannot be an equilibrium because it implies that $\lambda_{5t} = 0$ and $\lambda_{6t} < 0$.

Finally, any value $0 < \gamma_t < 1$ requires that $\lambda_{5t} = \lambda_{6t} = 0$. This can only be a solution to equation (21) when $f(\overline{\omega}_t^c) = f(\overline{\omega}_t^b)$, in which case firms are indifferent between raising credit from commercial banks or from the central bank. In what follows, we assume that whenever firms are indifferent, they choose to go to the commercial bank.

Thus, the firm's optimal choice of γ_t switches among the value of zero and unity, depending on whether $f(\overline{\omega}_t^c) \gtreqless f(\overline{\omega}_t^b)$. As $f(\cdot)$ is monotonic, the solution will be

$$\gamma_t = \begin{cases} 1 \text{ if } \overline{\omega}_t^c \geq \overline{\omega}_t^b \\ 0 \text{ if } \overline{\omega}_t^c < \overline{\omega}_t^b \end{cases}. \tag{39}$$

3 Solution

The ZLB forces us to solve the model using non-linear methods.

To develop an intuition for our results, we linearise the structural equations of the model and focus our attention on the non-linearity introduced by the ZLB. This is consistent with most of the literature on new-Keynesian models (see e.g. Adam and Billi, 2006, and Nakov, 2008).

It has recently been argued, however, that the linearized equations can produce incorrect results when the economy is hit by large shocks (see Braun, Körber and Waki, 2012). Later on in the paper, therefore, when we study an extension to a richer model with capital, we solve the fully non-linear model.

In both cases, the binary choice of γ_t, which cannot be eliminated through linear approximation, entails an additional source of non-linearity. To simplify the solution procedure, we smooth out the two kinks in γ_t through a simple approximation. Specifically, we replace equation (39) with

$$\gamma_t = \Psi\left(\overline{\omega}_t^c - \overline{\omega}_t^b\right) \tag{40}$$

where $\Psi(x) = \frac{1}{2}\frac{e^{(\kappa x)} - e^{(-\kappa x)}}{e^{(\kappa x)} + e^{(-\kappa x)}} + \frac{1}{2}$ and κ is a parameter which can be tuned to improve the accuracy of the approximation at the points of discontinuity.

For the linearization of the simple model we proceed as follows. First, we replace the system of equilibrium conditions with one indexed by j, where $j = b$ denotes an equilibrium where external finance is provided by commercial banks and $j = c$ denotes one where external finance is provided by the central bank. The new system is reported in Appendix A. It builds on the fact that, when $\gamma_t = 0$, equation (20) can be written as

$$q_t = \frac{R_t^d}{1 - \mu^c \Phi\left(\overline{\omega}_t^c\right) + \frac{\mu^c f(\overline{\omega}_t^c)\phi(\overline{\omega}_t^c)}{f'(\overline{\omega}_t^c)}}, \tag{41}$$

and, when $\gamma_t = 1$, as

$$q_t = \frac{R_t^d}{1 - \mu_t^b \Phi\left(\overline{\omega}_t^b\right) + \frac{\mu_t^b f(\overline{\omega}_t^b)\phi(\overline{\omega}_t^b)}{f'(\overline{\omega}_t^b)}}. \tag{42}$$

Second, we log-linearize the new system of equilibrium conditions around a steady state where $\mu^j = \mu^b$ and $\overline{\omega}^j = \overline{\omega}^b$, i.e. where commercial banks are more efficient than the central bank in providing credit to firms. We obtain a linear reduced form of the model.

Third, we solve for the Ramsey problem as one that maximises household's utility, subject to the reduced form system of linearized equilibrium conditions and the two non-linear constraints. The first is the ZLB constraint, (24), and the second is the approximated choice for γ_t. Note that $\overline{\omega}_t^b \gtrless \overline{\omega}_t^c$ is equivalent to $\widehat{\mu}_t^b \gtrless \widehat{\mu}_t^c$. We write the approximated choice for γ_t equivalently as $\gamma_t = \Psi\left(\widehat{\mu}_t^c - \widehat{\mu}_t^b\right)$.

3.1 Log-linearization

We log-linearize the equilibrium conditions around a steady state where $\gamma_t = p_t(\eta) = s_t = 1$, assuming the functional form for utility $u(c_t) - v(h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{h_t^{1+\varphi}}{1+\varphi}$. Define $\widehat{\pi}_{t+1} \equiv \log \pi_{t+1}$, $\widehat{p}_t(\eta) = \log p_t(\eta)$, $\widehat{\mu}_t^b = \log \mu_t^b - \log \mu^b$, $\widehat{\mu}_t^c = \log \mu_t^c - \log \mu^b$, $\widehat{\Lambda}_t^b = \log \Lambda_t^b - \log \Lambda^b$ and $\widehat{\Lambda}_t^c = \log \Lambda_t^c - \log \Lambda^b$.

We define the efficient equilibrium as one where all financial frictions, as well as nominal price stickiness, are absent. We denote variables in such equilibrium with the e superscript. Because financial shocks are absent in such equilibrium, $\widehat{Y}_t^e = \widehat{r}_t^e = 0$, where $\widehat{r}_t^e$ is the efficient real interest rate.

The system of log-linearized equilibrium conditions can be simplified to

$$(\alpha_3 - \alpha_1)\widehat{\Lambda}_t^j = (1+\sigma+\varphi)x_t + (\alpha_2+\alpha_4)\widehat{\mu}_t^j \tag{43}$$

$$x_t = E_t x_{t+1} - \sigma^{-1}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \tag{44}$$

$$\widehat{\pi}_t = \lambda\left[(\sigma+\varphi)x_t + \widehat{R}_t + \alpha_1\widehat{\Lambda}_t^j + \alpha_2\widehat{\mu}_t^j\right] + \beta E_t\widehat{\pi}_{t+1} \tag{45}$$

where

$$j = \begin{cases} b \text{ if } \widehat{\mu}_t^c \geq \widehat{\mu}_t^b \\ c \text{ if } \widehat{\mu}_t^c < \widehat{\mu}_t^b \end{cases}, \tag{46}$$

and where $x_t = \widehat{Y}_t - \widehat{Y}_t^e$ denote the output gap. The coefficients α_1, α_2, α_3 and α_4 are defined in appendix B, and $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$. Notice that α_1 and α_3 can be signed and are always positive. Under our calibration, the coefficients α_2, α_4 and α_5 also take positive values.

To understand condition (46), notice that $j = b$ if $\gamma_t = 1$, or if $\overline{\omega}_t^c \geq \overline{\omega}_t^b$, while $j = c$ if $\gamma_t = 1$, or if $\overline{\omega}_t^c < \overline{\omega}_t^b$. From equations (25), it can be shown that $\frac{\partial \Lambda^\eta}{\partial \overline{\omega}^\eta}$ can be negative either for values of $\overline{\omega}_t^\eta$ close to zero, or for values falling in the right tail of the distribution of ω. Under parameterizations that delivers reasonable default rates, $\overline{\omega}^{\eta}$ always lie in the left tail of the distribution, so that $\frac{\partial \Lambda^{\eta}}{\partial \overline{\omega}^{\eta}} > 0$. At the same time, we know from equation (43) that $\Lambda_t^c \geq \Lambda_t^b$ if $\widehat{\mu}_t^c \geq \widehat{\mu}_t^b$, and $\Lambda_t^c < \Lambda_t^b$ if $\widehat{\mu}_t^c < \widehat{\mu}_t^b$.

Equation (43) shows that the spread between the loan rate and the policy rate $\widehat{\Lambda}_t^j$ increases with the output gap, x_t. A larger demand for retail goods (and thus for wholesale goods to be used as production inputs) tightens the credit constraint of firms, since they need to finance a higher level of debt given the same amount of internal funds. The increased default risk generates a larger spread. The spread is also positively related to the shock to monitoring costs, $\widehat{\mu}_t$. The reason is that intermediaries need to set a higher repayment threshold to cover for increased monitoring costs, which results in larger credit spreads.

Equation (44) is a standard forward-looking IS-curve describing the determinants of the gap between actual output and its efficient level.

Finally, equation (45) represents an extended Phillips curve. The first determinant of inflation in this equation is the output gap. Ceteris paribus, a higher demand for retail goods, and correspondingly for intermediate goods, implies that wholesale firms need to pay a higher real wage to induce workers to supply the required labor services. The second determinant is the nominal interest rate, whose increase also pushes up marginal costs due to the presence of the cost channel. The third term is the credit spread, $\widehat{\Lambda}_t^j$. A higher spread implies a higher cost of external finance for wholesale firms and therefore exerts independent pressure on inflation.

As in De Fiore and Tristani (2012), the credit spread and the nominal interest rate act as endogenous "cost-push" terms in our model. While raising marginal costs and inflation, an increase in either term also exerts downward pressure on economic activity. A higher nominal interest rate determines an output contraction through the ensuing increase in the real interest rate, which induces households to postpone their consumption to the future. An increase in the credit spread contracts activity through the increase in the financial markup q_t and the consequent fall in the real wage.

The shock to monitoring costs acts as an exogenous "cost-push" factor in the New-Phillips curve, as it creates inflationary pressures independently from those exerted by the output gap.

In our model, a positive shock to monitoring costs raises the cost of external finance and depresses economic activity. At the same time, it increases the spread that banks charge over the risk-free rate, and thus firms' marginal costs, which are passed through to higher prices for final consumption goods. In equilibrium, inflation rises in spite of the fall in the output gap.

As a result, this shock does not lead the economy to hit the ZLB under a simple Taylor-type of monetary policy rule. The central bank would react to such a shock by raising the policy instrument.

3.2 Extension: a model with capital

As illustrated above, the shock to monitoring costs acts as a purely "cost-push" factor in our simple model. The higher lending rate does directly affect households's financing conditions: the IS curve is as in the simple new Keynesian model, so there are no effects on aggregate demand.

This feature of the model is probably unrealistic. Specifically, investment was the GDP component which reacted most negatively in the 2008-09 recession. A model with capital is necessary to be able to trace the effects of financial frictions on investment. The pure cost-push nature of the financial shock could also have an impact on our conclusions concerning the optimal mix of standard and non-standard measures. As it will become clear in our numerical results below, policy interest rates are optimally increased very quickly after a very persistent financial shock in our simple model, because the ensuing increase in spreads puts upward pressure on marginal costs. It is important to understand whether this conclusion would be altered in a model where the financial shock also affect investment.

For these reasons, we analyse in this section the robustness of our results to a richer model where financial frictions affect investment. We use a version of the model in De Fiore and Tristani (2011), where the reader can find further details on all features of the model. The key difference compared to the model described above is the presence of competitive firms operating an investment sector. These firms are endowed with a technology which transforms final consumption goods into capital goods. Firms in the investment sector are owned by risk-neutral, infinitely lived entrepreneurs, who make consumption and investment decisions.

Households rent labor and capital services to firms producing intermediate goods, but they do not have access to a technology to produce capital goods. Hence, they purchase capital from competitive firms endowed with such technology, which operate in the investment sector.

Internal funds of firms in the investment sector are not sufficient to finance the desired amount of investment, so entrepreneurs need to raise external finance from the financial intermediary. As in the simpler model, we assume that contracts are stipulated in nominal terms and not contingent on the realization of aggregate uncertainty.

The investment sector is composed of an infinite number of competitive firms, each endowed with a stochastic technology that transforms I units of the final consumption good into ωI units of capital. The random variable ω is i.i.d. across time and across entrepreneurs, with distribution Φ, density ϕ and mean unity. The shock ω is private information, but its realization can be observed by the financial intermediary at the cost of μI units of capital.

The amount of internal funds available to firm i is given by its net worth,

$$n_{i,t} = \left[q_t \left(1-\delta\right) + \rho_t\right] z_{i,t}, \tag{47}$$

where $z_{i,t}$ is the stock of capital owned by firm i at the beginning of period t. The firm's net worth is not sufficient to produce the desired amount of investment goods. Hence, the firm needs to raise external finance.

In analogy to the case of the simpler model, the optimal contract is

$$\begin{aligned} q_t &= \frac{R_t^d}{1-\mu_t^b\Phi\left(\overline{\omega}_t^b\right) - f\left(\overline{\omega}_t^b\right) + \frac{\left[\gamma_t f(\overline{\omega}_t^b)+(1-\gamma_t)f(\overline{\omega}_t^c)\right]\left[f'\left(\overline{\omega}_t^b\right)+\mu_t^b\phi\left(\overline{\omega}_t^b\right)\right]}{\left[\gamma_t f'(\overline{\omega}_t^b)+(1-\gamma_t)f'(\overline{\omega}_t^c)\right]}} & (48) \\ I_t &= \frac{R_t^d}{R_t^d - q_t\left[1-\mu_t^b\Phi\left(\overline{\omega}_t^b\right) - f\left(\overline{\omega}_t^b\right)\right]} n_t & (49) \\ \lambda_{5t} - \lambda_{6t} &= q_t I_t \left[f(\overline{\omega}_t^c) - f(\overline{\omega}_t^b)\right] & (50) \end{aligned}$$

where λ_{5t} and λ_{6t} are as above the multipliers associated with the $0 \leq \gamma_t \leq 1$ constraints.

Entrepreneurs have linear preferences over consumption with rate of time preference β^e, and they die with probability γ. Entrepreneurial consumption is taxed at the rate ζ.

We assume β^e sufficiently high so that the return on internal funds is higher than the preference discount, $\frac{1}{\beta^e} - 1$. It is thus optimal for entrepreneurs to postpone consumption until the time of death.

Entrepreneurial consumption and accumulation of capital are given by

$$e_t = \frac{\left(1-\gamma\right) f\left(\overline{\omega}_t\right) q_t I_t}{1+\zeta},$$

$$z_{t+1} = \gamma f\left(\overline{\omega}_t\right) q_t I_t, \tag{51}$$

We consider the limiting case where ζ is arbitrarely large, so that consumption of the entrepreneurs approaches zero, $e_t \to 0$ and the weight on entrepreneurial consumption in the welfare function becomes irrelevant.

4 Welfare analysis

The welfare criterion in our analysis is the utility of the economy's representative household

$$W_{t_0} = E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^t U_t\right\},$$

where temporary utility is given by $U_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \psi\frac{h_t^{1+\phi}}{1+\phi}$.

For the model with capital, we derive optimal policy directly by maximising households' utility subject to the nonlinear model constraints, including the ZLB constraint and equation (40).

For the simpler model we can instead provide an analytic approximate characterisation of optimal policy using the log-linear model conditions. Specifically, under the functional form for household's utility defined above, appendix C shows that the present discounted value of social welfare can be approximated to second order by

$$W_{t_0} \simeq c^{1-\sigma}\left[\varkappa - \frac{1}{2}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right] + t.i.p., \tag{52}$$

where $t.i.p.$ denotes terms independent of policy,

$$L_t \equiv \kappa_\pi \pi_t^2 + (\sigma+\varphi)\, x_t, \tag{53}$$

$\kappa_\pi = \frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}$ and $\varkappa = \left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right)$.

Define $\widetilde{\sigma} \equiv \sigma + \varphi$, $\widetilde{\lambda} \equiv \lambda\alpha_1\alpha_5$ and $\widetilde{\alpha} \equiv \lambda\left[\widetilde{\sigma} + \alpha_1\alpha_5\left(1+\widetilde{\sigma}\right)\right]$. The planner maximizes (53) subject to the linearized equilibrium condition (44), the New-Phillips curve rewritten as

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \widetilde{\alpha}x_t + \lambda\widehat{R}_t + \left[\widetilde{\lambda}\left(\alpha_2+\alpha_4\right) + \lambda\alpha_2\right]\left[\gamma_t\widehat{\mu}_t^b + \left(1-\gamma_t\right)\widehat{\mu}_t^c\right],$$

the ZLB constraint

$$\widehat{R}_t \geq \ln\beta,$$

and the restriction

$$\gamma_t = \Psi\left(\widehat{\mu}_t^c - \widehat{\mu}_t^b\right). \tag{54}$$

Notice that the social planner does not choose γ_t. Equation (54) is a restriction to the Ramsey problem which ensures that the optimal allocation satisfies the optimality conditions of the CSV problem.

The first-order conditions of the Ramsey problem can be written as

$$
\begin{aligned}
\psi_t &= \frac{(\sigma+\varphi)\,x_t - \beta^{-1}\psi_{t-1} - \lambda^{-1}\widetilde{\alpha}\phi_{t-1}}{\widetilde{\alpha}\lambda^{-1}\sigma^{-1}-1} \\
\phi_t &= -\varepsilon\widehat{\pi}_t + \phi_{t-1} + \sigma^{-1}\frac{\beta+\lambda}{\beta}\psi_{t-1} + \frac{\widetilde{\alpha}\lambda^{-1}\phi_{t-1} + \beta^{-1}\psi_{t-1} - (\sigma+\varphi)\,x_t}{\widetilde{\alpha}\lambda^{-1}-\sigma} \\
0 &= \left(\widehat{R}_t - \ln\beta\right)\phi_t
\end{aligned}
$$

where ψ_t and ϕ_t are the lagrangean multipliers on the Euler equation and the ZLB constraint, respectively (the New-Phillips curve multiplier, ν_t, has been substituted out).

4.1 Target rule without ZLB and non-standard measures

We provide some intuition on what monetary policy ought to do in our model by abstracting from the ZLB constraint and from the possibility that the central bank intervenes with non-standard policy measures. The aim is to disentangle the consequences of the nominal denomination of debt (the "cost channel") and the costly state verification environment (the existence of endogenous credit spreads) for the optimal monetary policy.

Under the assumption that the ZLB constraint can be ignored, and when $\gamma_t = 1$, the optimality conditions of the Ramsey problem can be rewritten in terms of the following target rule

$$\Delta x_t = -\varepsilon\left[1 + \frac{\alpha_1}{\alpha_3-\alpha_1}\left(1+\frac{1}{\sigma+\varphi}\right)\right]\widehat{\pi}_t + \frac{\sigma}{\sigma+\varphi}\varepsilon\left(\widehat{\pi}_t - \frac{\widehat{\pi}_{t-1}}{\beta}\right) + \frac{\lambda}{\beta}x_{t-1} \tag{55}$$

Equation (55) nests the target rule which implements optimal policy in the New Keynesian model, given by $\Delta x_t = -\varepsilon\widehat{\pi}_t$ (see eg Woodford, 2003). In that model, the target rule can be interpreted as the simple prescription to keep contracting the output gap as long as inflation is positive (and viceversa for negative inflation).

The introduction of the cost channel in the model is responsible for the last two terms in equation (55). In fact, when monitoring costs are zero, $\alpha_1 = 0$. To realize the implications of the cost channel for optimal policy, consider the prescription of the target rule in the first period after a shock has hit the economy. Because in steady state $x = \widehat{\pi} = 0$, in the first period $\Delta x_t = -\varepsilon\left(1 - \frac{\sigma}{\sigma+\varphi}\right)\widehat{\pi}_t$. In response to a certain increase in inflation, the last two terms suggest that the initial contraction in the output gap should be smaller than in the model with frictionless financial markets. Intuitively, these terms take into account the cost-push inflationary effects of the increase in the nominal interest rate, which have to be implemented to induce a contraction of the output gap.

Finally, the existence of asymmetric information and credit spreads calls for a more aggressive policy response to current inflation – the coefficient is higher than in the frictionlessK case by the positive amount $\alpha_1/(\alpha_3-\alpha_1)(1+1/(\sigma+\varphi))$. This is necessary to contain any additional inflationary pressures coming from credit spreads.

Equation (55) can also be written differently to highlight its implications on the price level. We then have

$$p_t = p_{t-1} - \frac{1}{\widetilde{\varepsilon}}\left[\beta\frac{\varphi+\sigma}{\sigma}\Delta x_t + \lambda\frac{\varphi+\sigma}{\sigma}(\varepsilon\sigma-1)x_{t-1} + \lambda\widehat{R}_{t-1} + \lambda\alpha_1\widehat{\Lambda}_{t-1} + \beta\mathrm{E}_{t-1}\widehat{\pi}_t\right] \quad (56)$$

where $\widetilde{\varepsilon}$ is a positive reaction coefficient given by $\widetilde{\varepsilon} \equiv \varepsilon\beta\sigma^{-1}\left[\varphi + \frac{\alpha_1}{\alpha_3-\alpha_1}(1+\sigma+\varphi)\right]$.

Note that the NK model would require $p_t = p_{t-1} - (1/\varepsilon)\Delta x_t$. Assuming to start the economy from an initial price level $p_0 = \overline{p}$, this equation says that the economy should always return to that $\overline{p}$ once the output gap is stabilised and $\Delta x_t = 0$. This implies history dependence, in the sense that an inflationary period should be induced after a deflationay shock, so as to ensure a return to the original price level.

In the case of our model, a return to the original price level is not sufficient. Note that all terms inside the square brackets on the right-hand side of equation (56) are positive. This implies that, following again a deflationary shock, some additional upward pressure on the price level must be engineered even after hen the output gap is stabilised and $\Delta x_t = 0$. As a result, prices will remain, as in the NK model, trend stationary, but they will return to a higher price level than the one from when the economy started.

5 Numerical results

We solve the models using nonlinear, deterministic simulation methods. Given initial conditions for pre-determined variables and terminal conditions for non-predetermined variables, the path of all endogenous variables can be found as the solution of a large system of nonlinear equations at all simulation dates.[3]

A more complete solution to the system would include stochastic terms, e.g. using the collocation method as suggested by Judd (1998) or Miranda and Fackler (2002). A stochastic solution would in principle allow for precautionary policy motives, e.g. the possibility to target a slightly positive inflation rate in order to reduce the likelihood of hitting the ZLB. Such precautionary effects, however, have been found to be negligible in the new Keynesian literature.

[3] In practice we use Newton methods as implemented in the Dynare command "simul ".

Since, as we illustrate below, non-standard measures reduce the likelihood of reaching the ZLB, precautionary effects are likely to be small also in our models. There should therefore be no loss of accuracy in our deterministic nonlinear solution, which is much simpler to compute and feasible also for the larger system of our model with capital.[4]

Parameter values are in line with the literature. More specifically, we set the elasticity of intratemporal substitution $\varepsilon = 11$ and the Calvo parameter $\theta = .66$. The discount factor is set as $\beta = 0.995$, to mimic the low interest rates environment which prevailed over the years before the financial crisis. For the utility parameters, we use standard values: $\sigma = 1.0$, $\phi = 0.0$. The contract parameters τ and σ_ω are set consistently with the parametrization used in De Fiore and Tristani (2012), which matches US data on the average annual spread between lending and deposit rates (approximately 2%) and on the quarterly bankruptcy rate (around 1%). These values imply that $\alpha_1 = 4.7$ and $(\alpha_3 - \alpha_1)^{-1} = 0.008$. Consistently with actual financial developments over the past 5 years, we assume very persistent monitoring cost shocks: they have a serial correlation coefficient equal to 0.95.

A new coefficient which we need to calibrate is μ^c, the monitoring cost of central bank lending activities. To gauge a value for this parameter, we draw from the euro area experience during the financial crisis. While the ECB did not provide direct loans to firms, it did intervene to offset impairments in the interbank market. Asymmetric information generated lack of trust between banks concerning each other's ability to repay interbank loans. Many banks were therefore unable to obtain liquidity from other banks at the overnight rate prevailing in the interbank market. These banks chose to borrow directly from the ECB at the rate on the main refinancing operations.

If we interpret the spread between the MRO rate and the overnight rate as due to increased monitoring costs for commercial banks during the crisis, we can conclude that, at some point,

[4]We have tested this conjecture in the case of the simpler model relying on the routines included in the CompEcon toolkit (Miranda and Fackler, 2002). If we define by s_t the state vector, and y_t the vector of jump variables (x_t, π_t and $\widehat{R}_t$), the collocation method amounts to approximating the policy functions through linear combinations of basis functions, θ_j, with coefficient c_j: $y_t = \sum_{j=1}^{n} c_j \theta_j (s_t)$. The coefficients are determined by the requirement that the approximating functions satisfy the dynamic equations exactly at n collocation nodes.

For this solution method we have set the standard deviation of monitoring cost shocks so as to match the unconditional variance of μ as estimated in Levin, Natalucci and Zakrajsek (2004) over the 1997-2003 period. Our results on standard interest rate policy confirm that the stochastic terms are quantitatively negligible.

the ECB became more efficient at monitoring banks' credit worthiness. It was therefore able to provide loans at lower rates.

The spread between MRO and overnight rates, which is essentially zero under normal circumstances, hovered between 50 and 70 basis points during the crisis. This suggests that the ECB intermediation activity only became competitive when the MRO-overnight spread reached 50 basis points. We interpret this spread level as a measure of the ECB's lower monitoring efficiency under normal circumstances. We therefore set μ^c so as to imply a steady state credit spread between ECB loans and banks' loans of 50 basis points.

Figures 1-4 display the impulse responses to a μ_t^b shock under optimal policy in the simple model. As already discussed above, this shock acts like a cost-push shock. On the one hand, it generates an immediate increase in the loan-deposit rate spread, which pushes up firms' marginal costs and thus generates inflationary pressure. On the other hand, the increase in marginal costs generates a persistent increase in the mark-up q_t and persistent downward pressure on wages, hence a reduction in both labour supply and the demand for consumption goods. Hence, the spread moves anti-cyclically.

Figures 1 focuses on the case in which the central bank implements solely standard policy. The shock is such that spreads increase by approximately 70 basis points. Optimal policy requires a cut in interest rates, in spite of the inflationary pressure created by the increase in spreads. The main reason for this policy response is that the financial shock is inefficient, hence the fall in households' consumption is entirely undesirable. The expansion in the monetary policy stance helps smooth the adjustment of households' consumption after the shock, at the cost of producing a short inflationary episode. As already apparent from the target rule, at the end of the adjustment period the price level reverts back to the original level and then crosses it to eventually end up *below* the starting value. The promise of a future fall in the price level keeps expectations of future inflation down. It ensures that only a short inflationary episode follows an inflationary shock, in spite of the impact fall in the policy rate when the shock hits.

If we ignore the ZLB constraint, the nominal rate falls to almost -2%, before returning relatively quickly towards the steady state to check the increase in inflation. Once we impose the ZLB constraint, the nominal rate is zero for two periods. Figure 2 focuses more closely on the effects of the ZLB on standard policy, focusing on case of an adverse financial shock following a pattern closer to the experience of 2008-09. Over this period, spreads increased slightly, then jumped upwards after the bankruptcy of Lehman Brothers and remained elevated for a protracted period. Similarly, in figure 2 the shock to μ_t^b has an AR(2) structure, increasing for 5 quarters before starting its slow return towards the steady state. As a result, private credit spreads progressively increase by up to approximately 1 percentage point. In response to this increase, policy interest rates are cut to 0.25 percent on impact and after 1 period hit the ZLB, where they remain for 3 quarters.

If the ZLB is ignored, policy rates are cut to 0.5 percent on impact, then they turn negative for one quarter and increase again thereafter. Standard policy therefore displays two properties already highlighted in new Keynesian models. First, the initial easing in response to the adverse shock is more aggressive, when it is known that the shock will eventually lead policy rates to zero. The central bank brings forward some of the monetary accommodation that it would implement later, were the ZLB not a constraint on the short term rate. Second, policy rates remain "low for longer" – they increase later than they would in the absence of the ZLB constraint.

Once policy shifts to a tightened phase, interest rates also increase faster than in the case where the ZLB is ignored. Interest rates must be increased quickly back to the steady state to prevent any inflationary consequences from the increase in firms' financing costs. This policy ensures that inflation only fluctuates mildly, even if the output gap (not shown) falls by 2%. Such sharp increase in policy rates turns out not to be a robust feature of our analysis. Interest rates are increased more smoothly in the model with capital.

Figure 3 displays impulse responses to a shock of the same size as in figure 1 when non-standard policy is also available. On impact, the shock continues to justify a fall to zero in the policy rate. Compared to the case without non-standard policy, however, the zero bound is only binding for one period. The response of the policy rate is closer to that which would prevail if the ZLB were not a constraint. The volatility of both the output gap and inflation are much smaller. A lower fall in the future price level is necessary to limit the initially inflationary consequences of the shock.

Non-standard measures – i.e. central bank intermediation – are deployed as soon as the credit spread on banks' loans increases above 50 basis points. Given the persistence of the shock, this is the case for approximately 8 quarters. Non-standard measures are therefore implemented irrespectively of the level of the policy rate. Once the shock hits, the central bank starts providing loans to the economy at the same time as it lowers interest rates to zero. This direct intermediation activity continues long after interest rates have essentially returned to their steady state. Equivalently, standard monetary policy is again tightened quickly few quarters after the shock hits, in spite of the fact that financial market conditions remain impaired.

Figure 4 focuses more closely on the difference introduced by the availability of non-standard measures for the more realistic shock pattern already used in figure 2. When non-standard measures are available, they partially insulate the economy from the effects of the financial shock. At any point in time, figure 4 shows the spreads which would be charged by both commercial banks and the central bank. Whenever the second is lower than the first, non-standard measures are implemented.

When non-standard measures are available, the increase in spreads paid by firms is capped at 50 basis points. As a result, it is no longer optimal to cut policy rates all the way to zero. Rates are reduced to approximately 0.5% for 2 quarters, and can then be increased earlier than they would be if non-standard measures were unavailable. In this sense, non-standard measures are a substitute for the reduction of short term rates to zero. The optimal combination of standard and non-standard measures delivers a superior outcome to the case in which non-standard measures are unavailable: inflation is better stabilised and the output gap is smaller.

With regards to the timing of "exit", non-standard measures remain in place for a long time, notably long after interest rates have return to the steady state. Specifically, central bank intermediation persists for over 4 years, while interest rates are back to steady state after only 1 year.

Figures 5 and 6 present the impulse responses to the financial shock in the model with capital. The shock to μ_t^b is of the same size as in figure 1, but it has a significantly different impact on spreads. The exogenous increase in monitoring costs depresses investment through a sharp rise in the price of capital–see equation (48). The ensuing lower demand for loans (to finance investment) tends to reduce the increase in spreads wihch would otherwise be associated with the increased costs of monitoring. As a result spreads charged by banks go up to about 2.7% on impact, while the spreads charged by the central bank falls below its steady state level.

Investment tanks and drives down output, even if consumption increases slightly (figure 5). The higher price of capital also increases the return on internal funds that, over time, leads to a reaccumulation of net worth and a gradual return to the steady state.

Figure 6 shows the responses of inflation, credit spreads and the policy interest rate. As in figure 4, the spreads offered by both banks and the central bank are shown. Compared to the simple model, non-standard measures last longer–almost 4 years. The increase in the policy interest rate is also much more gradual: the tightening phase lasts approximately 2 years, in contrast to the sharp hike of figure 4.

Nevertheless, it remains true that non-standard measures stay in place longer after the policy rate has returned to the steady state. This suggests that the currently large size of central banks' balance sheets may be a very persistent feature of monetary policy.

6 Conclusions

We have presented a microfounded model with credit market imperfections and nominal price rigidities, which we use to analyse the response of monetary policy to financial shocks in the presence of the ZLB. The model can also sheed light on the role of non-standard policy measures both at the ZLB and away from it.

We find that adverse financial shocks (notably a shock that increases banks' monitoring costs) can lead the economy to the ZLB under optimal policy. Non-standard meaures can also be effective in these situations. When adverse financial shocks impair the efficiency of private banks in intermediating finance, the ability of the central bank to provide direct credit to the economy mitigates the negative consequences of the shock on inflation and real activity. Cutting policy rates to zero may be unnecessary after non-standard measures have been implemented.

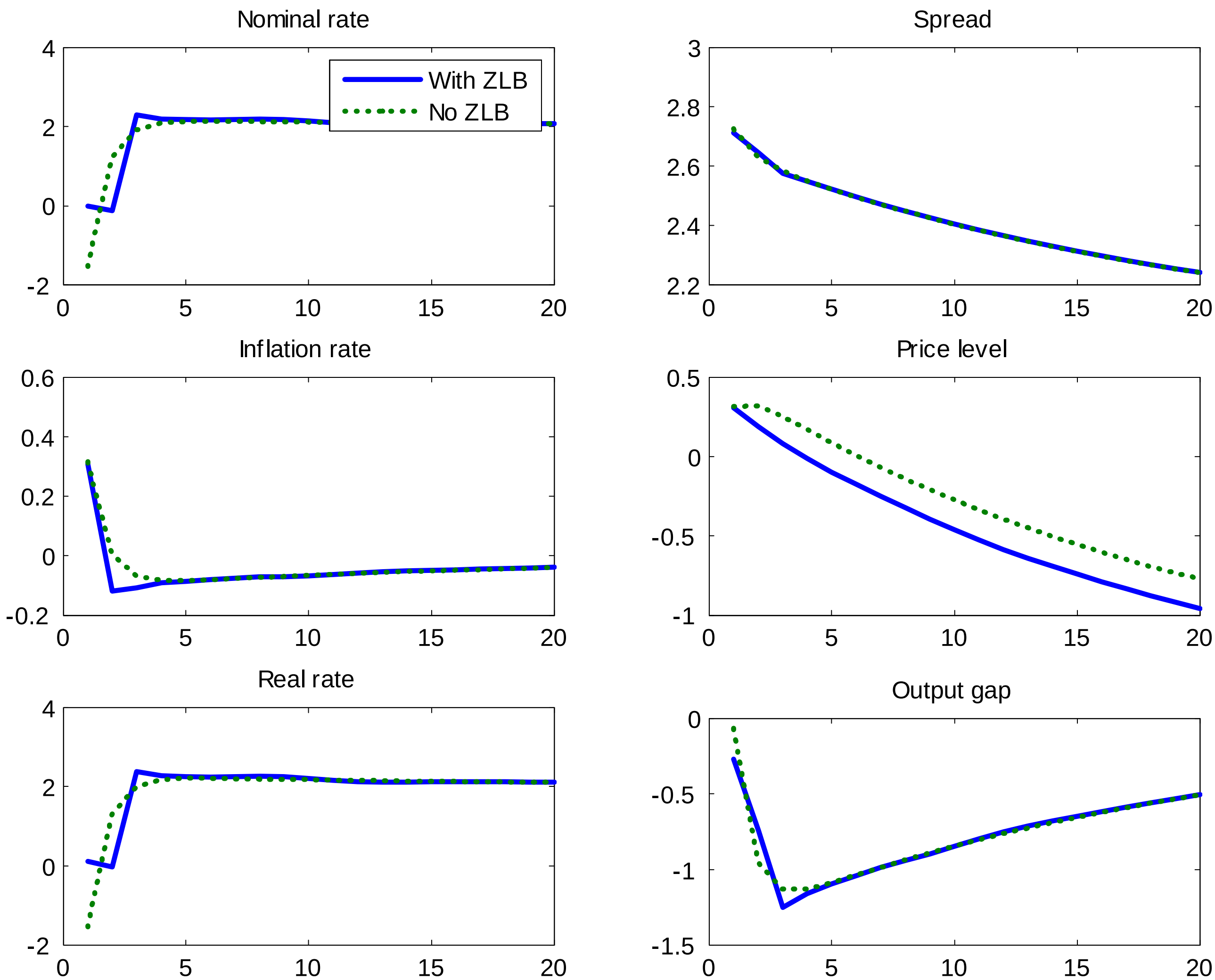

Figure 1: Response to a shock to μ under the optimal monetary policy: with ZLB (solid blue line) and without ZLB (green dotted line). All variables are in levels.

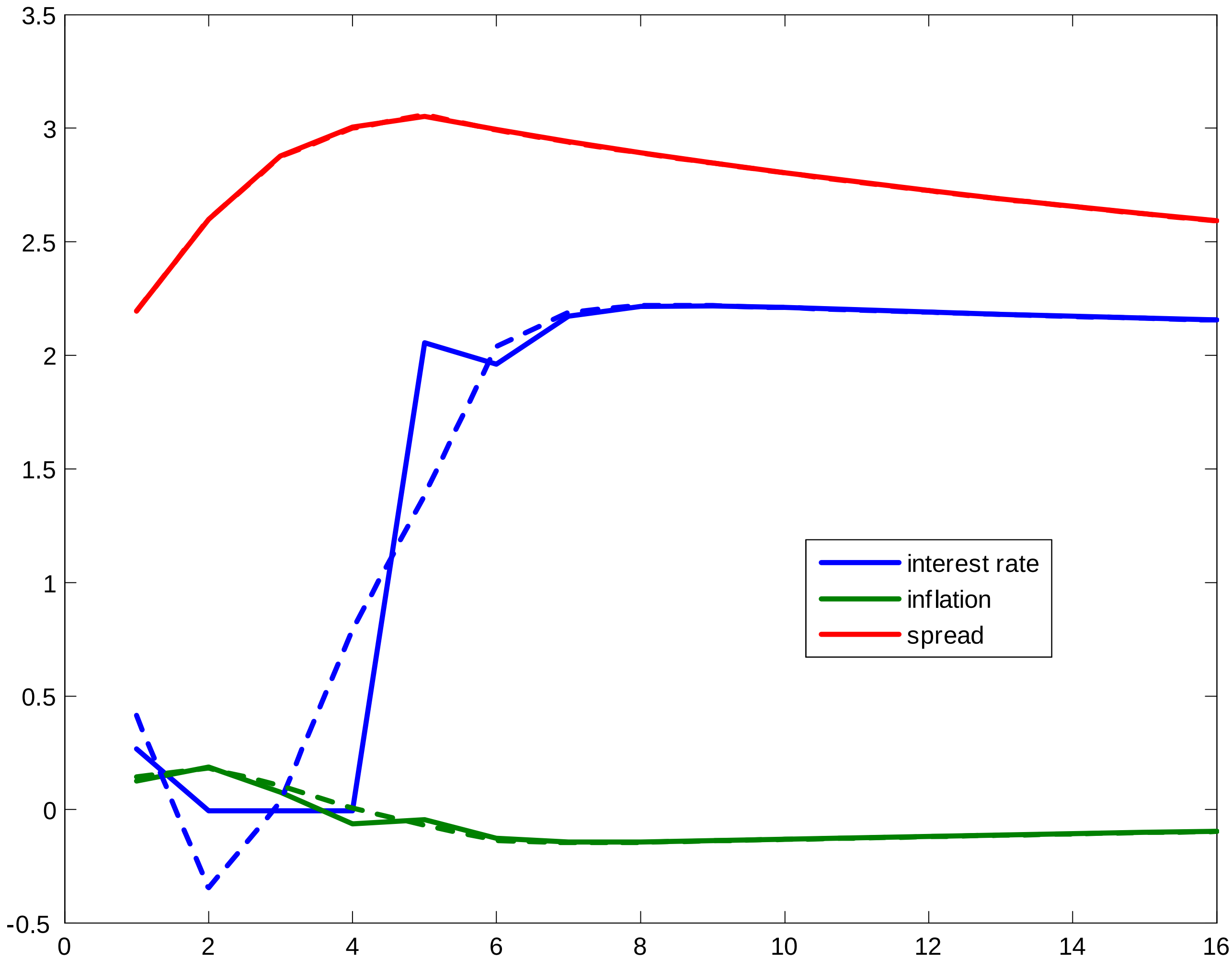

Figure 2: Standard, optimal policy response to a large shock to μ: with ZLB constraint (solid lines) and unconstrained (dahsed lines). All variables are in levels.

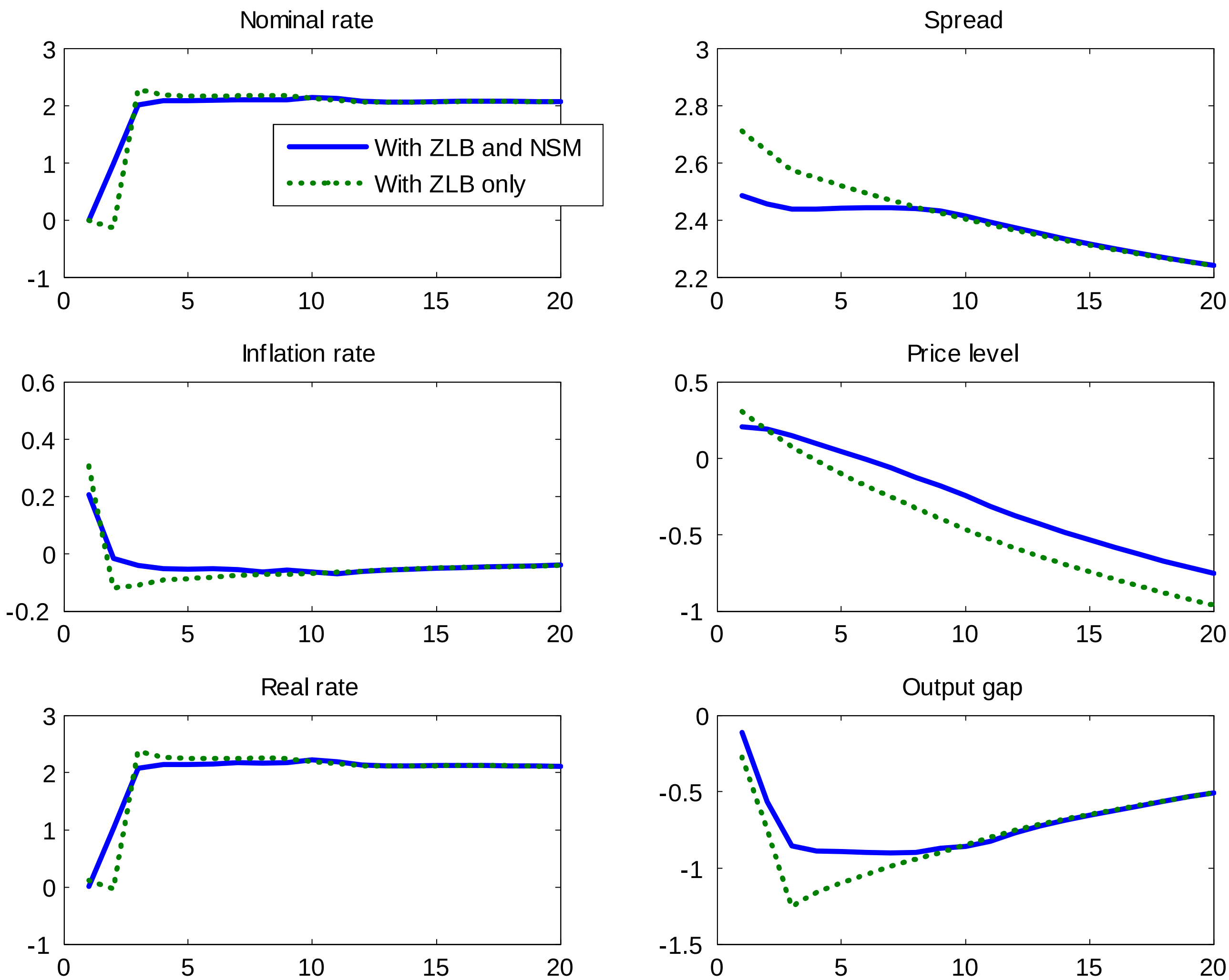

Figure 3: Response to a shock to μ under the optimal monetary policy: with ZLB and non-standard measures (solid blue line) and with ZLB only (green dotted line). All variables are in levels.

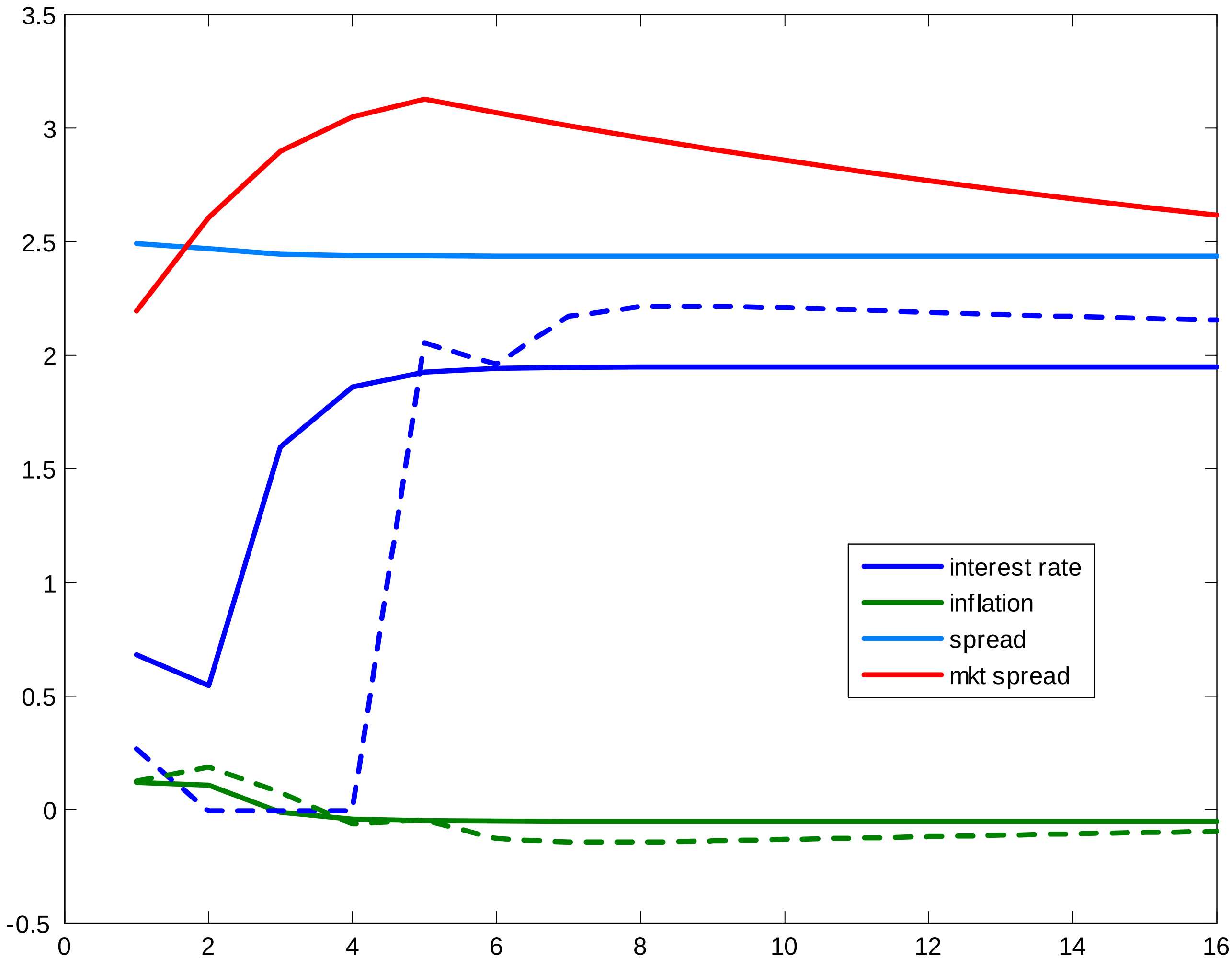

Figure 4: Response to a larger shock to μ under the optimal monetary policy: with non-standard measures (solid lines) and without (dahsed lines). All variables are in levels.

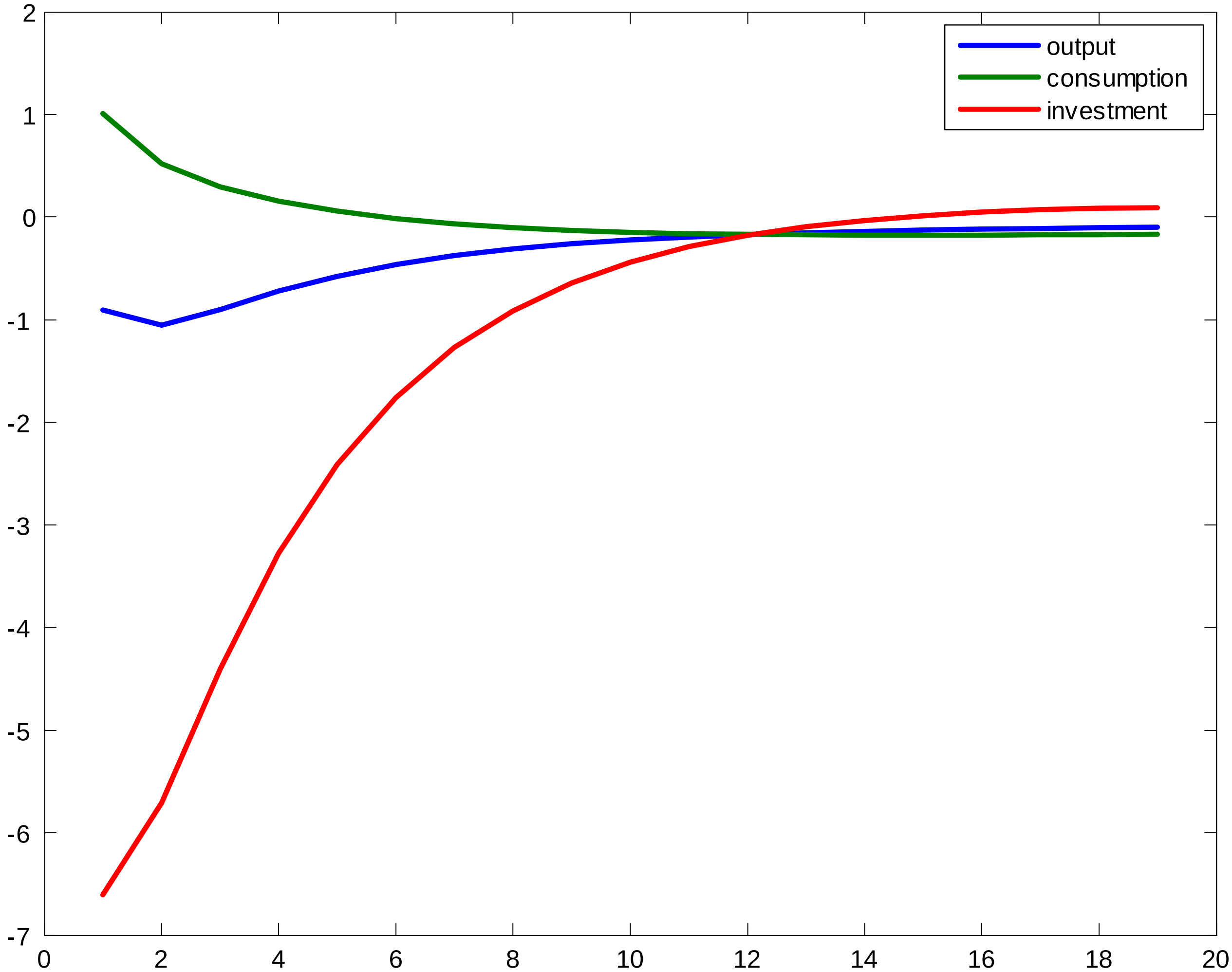

Figure 5: Standard and non-standard impulse response to a μ shock in the model with capital. Percentage changes from the steady state.

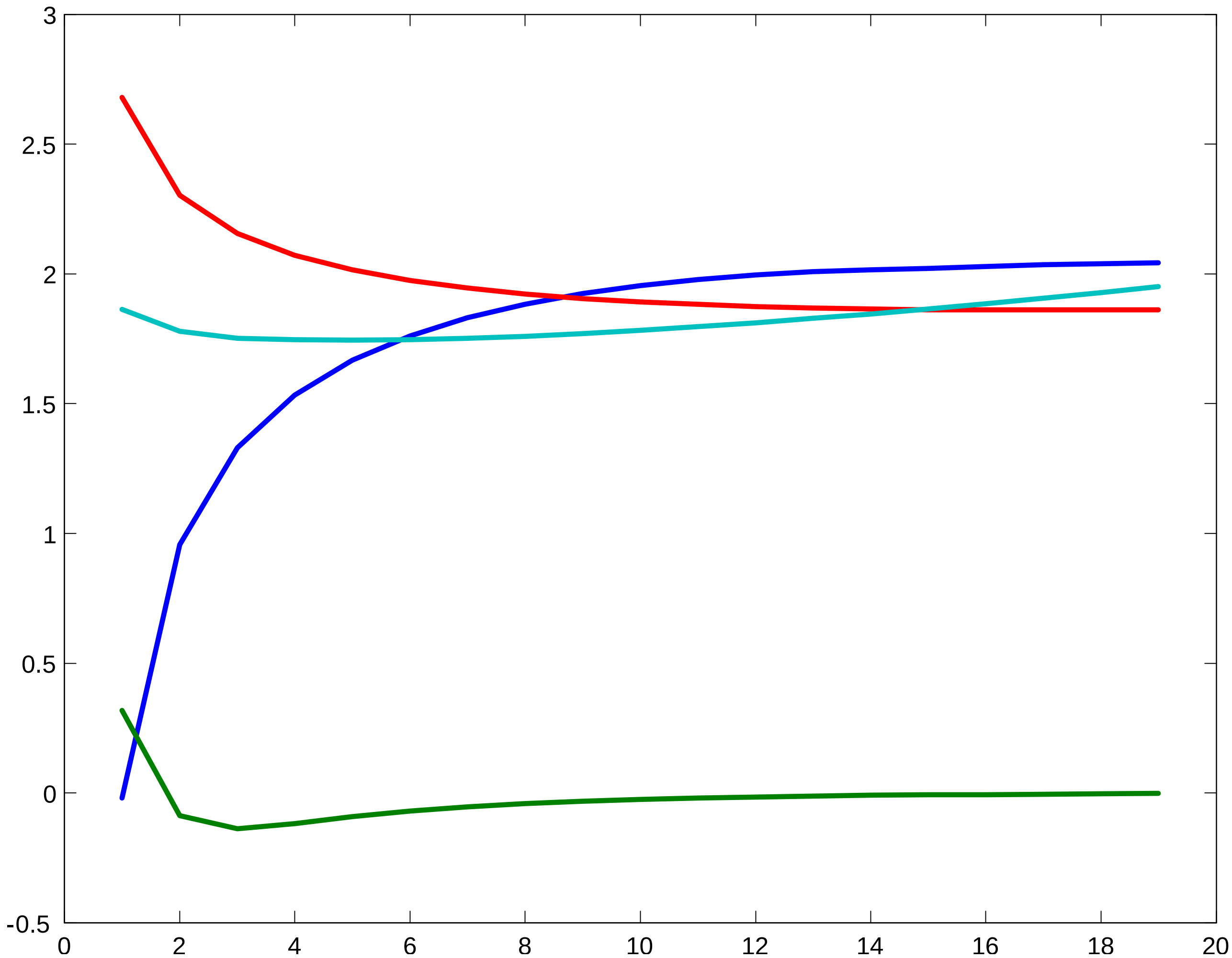

Figure 6: Standard and non-standard impulse response to a μ shock in the model with capital. All variables are in levels.

7 Appendix

A. Competitive equilibrium

When ν becomes arbitrarily large, the equilibrium conditions can be written as

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_t(\eta)^{\frac{\varepsilon-1}{\varepsilon}} d\eta\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
\int_0^1 Y_t(\eta)\, d\eta &= \chi_t q_t x_t \\
\int_0^1 Y_t(\eta)\, d\eta &= s_t c_t \\
h_t &= \chi_t q_t x_t \\
\frac{v_h(h_t)}{U_c(c_t)} &= \frac{1}{q_t \chi_t} \\
u_c(c_t) &= \beta R_t E_t \left\{\frac{u_c(c_{t+1})}{\pi_{t+1}}\right\}
\end{aligned}
$$

$$
\begin{aligned}
q_t &= \frac{R_t^d}{1-\mu_t^j \Phi\left(\overline{\omega}_t^j\right)+\frac{\mu_t^j f(\overline{\omega}_t^j)\phi(\overline{\omega}_t^j)}{f'(\overline{\omega}_t^j)}} \\
x_t &= \frac{R_t^d}{R_t^d - q_t\left[1-\mu_t^j \Phi\left(\overline{\omega}_t^j\right)-f\left(\overline{\omega}_t^j\right)\right]}\tau_t
\end{aligned}
$$

where $f(\cdot)$ and $g(\cdot)$ are given by (12) and (13), together with (28)-(31), a path for the interest rate R_t, and the restrictions

$$R_t \geq 0$$

and

$$j = \begin{cases} b \text{ if } \gamma_t = 1 \text{ or } \overline{\omega}_t^c \geq \overline{\omega}_t^b \\ c \text{ if } \gamma_t = 0 \text{ or } \overline{\omega}_t^c < \overline{\omega}_t^b \end{cases}. \tag{57}$$

The system is complemented by the recursive variables

$$\Lambda_t^j = \frac{\overline{\omega}_t^j}{g(\overline{\omega}_t^j;\mu_t^j)}$$

$$\frac{\nu_t f(\overline{\omega}_t^j) q_t x_t}{1+\nu} = \tau_t - T_t^h$$

$$\frac{D_t}{P_t} = x_t - \tau.$$

B. Coefficients

The coefficients of the system of log-linearized equilibrium conditions are given by

$$
\begin{aligned}
\alpha_1 &= -\frac{q}{R}\frac{\mu\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)}{(1-g_{\overline{\omega}}\Lambda)} \\
\alpha_2 &= \mu\frac{q}{R}\left[\Phi+\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)\frac{\frac{\mu\Phi}{g}}{(1-g_{\overline{\omega}}\Lambda)}-\frac{f\phi}{f_{\overline{\omega}}}\right] \\
\alpha_3 &= -\left(\frac{\mu\frac{f}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)+(f_{\overline{\omega}}+\mu\phi)}{f+\frac{\mu f\phi}{f_{\overline{\omega}}}}\right)\frac{\overline{\omega}}{(1-g_{\overline{\omega}}\Lambda)} \\
\alpha_4 &= \frac{\mu\Phi}{g}\alpha_3+\frac{\mu\frac{f\phi}{f_{\overline{\omega}}}}{f+\frac{\mu f\phi}{f_{\overline{\omega}}}} \\
\alpha_5 &= (\alpha_3-\alpha_1)^{-1}
\end{aligned}
$$

C. Welfare approximation

Welfare is

$$W_{t_0}=E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^t U_t\right\},$$

where households' temporary utility is given by $U_t=u(c_t;\xi_t)-v(h_t)$. This latter can then be approximated as

$$
\begin{aligned}
U_t \simeq\ & U+u_c c\left(\widehat{c}_t+\frac{1}{2}\left(1+\frac{u_{cc}c}{u_c}\right)\widehat{c}_t^2\right)-v_h h\left(\widehat{h}_t+\frac{1}{2}\left(1+\frac{v_{hh}h}{v_h}\right)\widehat{h}_t^2\right)+u_{c\xi}c\widehat{c}_t\widehat{\xi}_t \\
&+u_\xi\left(\widehat{\xi}_t+\frac{1}{2}\left(1+\frac{u_{\xi\xi}}{u_\xi}\right)\widehat{\xi}_t^2\right)
\end{aligned}
$$

where hats denote log-deviations from the deterministic steady state and c and h denote steady state levels.

Under the functional form $U_t=\xi_t\frac{c_t^{1-\sigma}}{1-\sigma}-\psi\frac{h_t^{1+\phi}}{1+\phi}$, and assuming that in steady state $\xi=1$, households' temporary utility can be rewritten as

$$
\begin{aligned}
U_t \simeq\ & \frac{c_t^{1-\sigma}}{1-\sigma}-\psi\frac{h_t^{1+\phi}}{1+\phi}+c^{1-\sigma}\widehat{c}_t-\psi h^{1+\phi}\widehat{h}_t+\frac{1}{2}c^{1-\sigma}(1-\sigma)\widehat{c}_t^2-\frac{1}{2}\psi h^{1+\phi}(1+\phi)\widehat{h}_t^2 \\
&+c^{1-\sigma}\widehat{c}_t\widehat{\xi}_t+\frac{c^{1-\sigma}}{1-\sigma}\left(\widehat{\xi}_t+\frac{1}{2}\widehat{\xi}_t^2\right).
\end{aligned}
$$

We can now express hours and households' consumption as $h_t = \frac{s_t y_t}{A_t}$ so that $\widehat{h}_t = \widehat{s}_t + \widehat{y}_t - \widehat{a}_t$. Using this expression together with $c_t = y_t$, we can write utility as

$$\begin{aligned}\frac{U_t}{c^{1-\sigma}} \simeq\ & \frac{1}{1-\sigma} - \frac{\psi}{1+\phi}\frac{h_t^{1+\phi}}{c_t^{1-\sigma}} + \left(1 - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\right)\widehat{y}_t - \psi\frac{h^{1+\phi}}{c^{1-\sigma}}\widehat{s}_t - \frac{1}{2}\left(\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi) - (1-\sigma)\right)\widehat{y}_t^2 \\ & + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{y}_t\widehat{a}_t + \widehat{\xi}_t\widehat{y}_t - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t\widehat{y}_t + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t\widehat{a}_t - \frac{1}{2}\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t^2 \\ & + \frac{1}{1-\sigma}\left(\widehat{\xi}_t + \frac{1}{2}\widehat{\xi}_t^2\right) + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\widehat{a}_t - \frac{1}{2}\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\psi)\widehat{a}_t^2\end{aligned}$$

or, given that s_t is of second order, as

$$\begin{aligned}\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{\psi}{1+\phi}\frac{h_t^{1+\phi}}{c_t^{1-\sigma}} \quad & + \left(1 - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\right)\widehat{y}_t - \psi\frac{h^{1+\phi}}{c^{1-\sigma}}\widehat{s}_t \\ -\frac{1}{2}\left(\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi) - (1-\sigma)\right)\widehat{y}_t^2 \quad & + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{y}_t\widehat{a}_t + \widehat{\xi}_t\widehat{y}_t + t.i.p.s\end{aligned}$$

Assume a subsidy such that $\frac{\psi h^{1+\phi}}{c^{1-\sigma}} = 1$. Then

$$\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{1}{1+\phi} - \widehat{s}_t - \frac{1}{2}(\varphi+\sigma)\widehat{y}_t^2 + \left[(1+\varphi)\widehat{a}_t + \widehat{\xi}_t\right]\widehat{y}_t + t.i.p.s.$$

Now recall that $\widehat{y}_t^e = \frac{1}{(\sigma+\varphi)}\left[(1+\varphi)a_t + \widehat{\xi}\right]$. Then

$$\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{1}{1+\phi} - \widehat{s}_t - \frac{1}{2}(\sigma+\varphi)\widehat{y}_t^2 + (\sigma+\varphi)\widehat{y}_t^e\widehat{y}_t + t.i.p.s$$

This can be rewritten as

$$\frac{U_t}{c^{1-\sigma}} - \left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right) \simeq -\frac{1}{2}\frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}\widehat{\pi}_t^2 - \frac{1}{2}(\sigma+\varphi)x_t^2 + t.i.p.s.$$

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