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International Business Cycles with Complete Markets

Alexandre Dmitriev and Ivan Roberts

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Abstract

Kehoe and Perri (2002) show that a two-country business cycle model with endogenously incomplete markets helps to resolve the 'international co-movement puzzle' (Baxter 1995) and the 'quantity anomaly' (Backus, Kehoe and Kydland 1992, 1995). We claim that a similar performance can be achieved without resorting to market incompleteness. We show that a model with complete markets driven by productivity shocks alone can account for the 'international co-movement puzzle'. Our model features time non-separable preferences that allow arbitrarily small changes in wealth to affect the supply of labour. It matches the data by predicting (i) positive cross-country correlations of investment and hours worked; and (ii) realistic cross-country correlations of consumption. It reduces the gap between international correlations of output and consumption, but fails to change their order. Unlike models with restricted international markets, ours shows little sensitivity to the parameterisation of the forcing process.

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1. Introduction

Business cycle predictions of two-country models are inconsistent with the data along several dimensions. Kehoe and Perri (2002) describe the 'international co-movement puzzle' (Baxter 1995) and the 'quantity anomaly' (Backus *et al* 1992, 1995) as the '... two major discrepancies between standard international business cycle models with complete markets and the data' (p 907). The essence of the puzzles lies in the models' tendency to predict (i) negative cross-country correlations of investment and employment (the 'international co-movement puzzle'); and (ii) international consumption correlations in excess of output correlations (the 'quantity anomaly'). The opposite of both are observed in the data.

The literature has addressed these puzzles either by restricting the set of assets available in international financial markets (Baxter and Crucini 1995; Kollmann 1996; Kehoe and Perri 2002), by introducing new disturbances such as preference shocks, policy shocks and so on (Stockman and Tesar 1995; Ravn 1997; Wen 2007; Johri and Lahiri 2008), or both (Benigno and Thoenissen 2008). Our approach is different. We deviate from the assumption of time separable Cobb-Douglas preferences. This means that decisions today affect the choices available in the future. Our main result shows that a model with complete markets driven by productivity shocks alone can resolve the 'international co-movement puzzle'.[1] In addition, our model outperforms standard models in accounting for the 'quantity anomaly'.

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1 The importance of this result is emphasised by Baxter (1995, pp 1859–1860) who claims that '... a major challenge to the theory is to develop a model which can explain international comovement in labor input and investment'. In line with the above, Canova and Ubide (1998, p 558) argue that '... the magnitude and the sign of the cross-country investment correlations constitute an important regularity previously under-emphasized by the literature ... For the largest 9 OECD countries the size of pairwise investment correlation ranges between [–0.01, 0.77] with the median value around 0.45. A successful model of the international business cycle must therefore be able to reproduce this important feature of the data ...'.

We consider a one-good model with complete markets, costly capital adjustment and time non-separable preferences that allows arbitrarily small wealth effects on labour supply. Our model matches the data by predicting (i) positive cross-country correlations of investment and hours worked; and (ii) realistic cross-country correlations of consumption. It reduces the gap between international correlations of output and consumption, but fails to match the empirical regularity that the correlations of output are higher than those of consumption. In fact, the empirical performance of our model is remarkably similar to Kehoe and Perri's (2002) model with endogenously incomplete markets. Unlike models with restricted international markets, ours shows little sensitivity to the parameterisation of the forcing process.

Our model has three distinct features. First, it incorporates the preference structure introduced by Greenwood, Hercowitz and Huffman (1988) (henceforth GHH). Second, the model features internal habit formation in consumption. Third, like most international business cycle models, ours incorporates capital adjustment costs.

Introducing the GHH preference structure has two effects. First, in the absence of a wealth effect on labour supply, foreign individuals no longer reduce their hours in response to a positive productivity shock at home. To paraphrase Baxter and Crucini (1995, p 841), inhabitants in the less-productive country no longer 'take a paid vacation'. Since innovations to productivity are contemporaneously correlated across countries, suppressing the wealth effect induces positive employment co-movement. Second, GHH preferences drive a wedge between the consumption behaviour at home and abroad. The response of output in the home country is greater than under standard preferences, leading to a stronger consumption response to productivity shocks. Foreign consumption responds less aggressively, resulting in a more realistic cross-country consumption correlation.

Internal habit formation introduces non-separability of preferences over time. Moreover, under GHH preferences, internal habit formation partially re-introduces the wealth effect on labour supply and intertemporal substitution in leisure. This addresses the concern that GHH preferences remove a potentially important mechanism (Greenwood *et al* 1988). The magnitude of both effects depends on the intensity of habit formation.[2]

Deviation from time separability of preferences alters the consumption dynamics. Habit-forming individuals internalise the negative effect their current consumption has on their future felicity. They aim to smooth not only consumption but also changes in consumption. To see how this transmission mechanism contributes to investment co-movement, consider the aftermath of a positive productivity shock at home. Domestic output jumps whereas consumption's response is hump-shaped: it increases gradually before falling. Consumers need time to adjust their habits, hence they save most of the extra output. This saving can be channelled into either investment or net exports. Since rapid changes in the capital stock are costly, only a fraction of the extra saving ends up being invested at home. Domestic absorption (output less net exports) rises gradually, while domestic output peaks on impact. Net exports increase, making additional resources available abroad. Although the wealth effect compels foreign agents to increase their consumption, habit formation dictates that they do so gradually. As long as the opportunity cost of not investing in the most productive location falls short of the capital adjustment cost at home, investment abroad also rises.

Our model relies on capital adjustment costs as in Hayashi (1982) to deal with excessive volatility of investment in response to productivity shocks. However, in our model sluggish capital adjustment has one more role to play. Capital adjustment costs interact with habit-formation preferences to deliver a hump-shaped response of domestic absorption to productivity disturbances. This feature is responsible for the positive international co-movement of investment.

In our model, the positive co-movement of hours reinforces the mechanism driving the positive cross-correlation of investment. As foreign hours do not fall following an increase in home productivity, there is a much larger response of global output. This creates savings that translate into foreign as well as home investment, augmenting the investment co-movement.

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2 There seems to be little consensus in the empirical literature regarding the strength of the wealth effect on labour supply. On the one hand, Kimball and Shapiro (2008) report empirical evidence that the wealth effect is large, and is balanced by a similarly large substitution effect. On the other hand, Schmitt-Grohé and Uribe (2008) estimate the parameters of the utility function introduced by Jaimovich and Rebelo (2009) and find that the wealth effect is close to zero.

Our work is related to Devereux, Gregory and Smith (1992) who first introduced the GHH preference structure in a two-country business cycle model. Their model predicts a realistic international consumption correlation provided that productivity shocks are uncorrelated. More recently, Raffo (2008) demonstrated that an international real business cycle (RBC) model with GHH preferences and tradable intermediate inputs can be reconciled with the observed countercyclicality of net exports. However, the model fails to account for the 'international co-movement puzzle'. Engel and Wang (2011) augment GHH preferences with consumption as a composite of durables and non-durables. Their two-country, two-sector model accounts for the observed procyclicality and volatility of imports and exports. Dmitriev and Krznar (2012) emphasise the role of time non-separable preferences in the international transmission of productivity shocks. Their model succeeds in matching the cross-country investment correlation at the cost of predicting almost perfectly negatively correlated hours worked. Both habit formation and GHH preferences have been used with relative success in small open economy models (Correia, Neves and Rebelo 1995; Mendoza 1991; Letendre 2004).

The rest of the paper is organised as follows. The next section describes the model economy. Section 3 discusses the parameterisation of the model. Section 4 presents our quantitative results and discusses how each feature of the model is essential for reproducing observed features of the data. Section 4.5 provides an overview of the sensitivity analysis. Section 5 offers some concluding remarks.

2. The Model

The general structure of the model is similar to Backus *et al* (1992) (henceforth BKK).

2.1 The Economies

The world consists of two countries. The same parameters describe technology and preferences in both countries. Each country $j \in J = \{H, F\}$ is populated by a continuum of identical infinitely lived individuals. The two countries produce a single good that can be either consumed or invested. Labour is immobile across countries. In each period t, the world economy experiences an event s_t drawn from the countable set of events, S. Let $s^t = (s_0, s_1, ..., s_t) \in S^t$ denote the history of events from time 0 to time t. The time-0 probability of any given history s^t is denoted by $\pi\left(s^t\right)$.

Consumers Habit-forming agents have their preferences defined over stochastic sequences of consumption, habits, and leisure

$$U=\sum_{t=0}^{\infty}\beta^t\sum_{s^t\in S^t}\pi\left(s^t\right)u\left(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right)\right), \tag{1}$$

where $\beta\in(0,1)$ is the discount factor, $c_j\left(s^t\right)$ denotes household consumption at time t in country j after realisation of history s^t, and $n_j\left(s^t\right)\in[0,1]$ denotes individual labour supply. Time endowment per period is normalised to one. The stock of habits $h_j\left(s^{t-1}\right)$ with which the agent enters period t equals her own consumption in period $t-1$:

$$h_j\left(s^{t-1}\right)=c_j\left(s^{t-1}\right). \tag{2}$$

The instantaneous utility function takes the following form:

$$u(c,h,n)=\frac{1}{1-\sigma}\left(c-bh-\chi\frac{n^{1+\eta}}{1+\eta}\right)^{1-\sigma},$$

where σ is the curvature parameter, χ determines relative importance of leisure, $1-n$, and habit adjusted consumption, $c-bh$. The parameter $b\in[0,1)$ denotes the intensity of habit formation and introduces time non-separability of preferences. The Frisch elasticity of labour supply is given by $1/\eta$.

This specification of preferences has been used by Monacelli and Perotti (2008) to explore the transmission of government spending shocks, and by Boileau and Normandin (2005) to study current account fluctuations in a small open economy model. It nests two well-known special cases. As η tends to infinity, the labour supply becomes inelastic and the preference structure reduces to the internal linear habit formation preferences popularised by Constantinides (1990). When $b=0$ the model features regular GHH preferences.

Producers The households supply labour and capital to firms, which have access to constant returns-to-scale technology. Production is subject to a country-specific exogenous random shock, $z_j\left(s^t\right)$, to total factor productivity (TFP). Output in country j after history s^t is given by

$$y_j\left(s^t\right)=z_j\left(s^t\right)f\left(k_j\left(s^{t-1}\right),n_j\left(s^t\right)\right), \tag{3}$$

where $k_j\left(s^{t-1}\right)$ denotes the capital stock used at time t by the firms in country j. The production function is Cobb-Douglas: $f(k,n)=k^{\alpha}n^{1-\alpha}$. The TFP shocks follow a stationary vector autoregressive process (VAR) in logs:

$$\begin{bmatrix}\log\left(z_H\left(s^t\right)\right)\\ \log\left(z_F\left(s^t\right)\right)\end{bmatrix}=\begin{bmatrix}\rho & \nu\\ \nu & \rho\end{bmatrix}\begin{bmatrix}\log\left(z_H\left(s^{t-1}\right)\right)\\ \log\left(z_F\left(s^{t-1}\right)\right)\end{bmatrix}+\begin{bmatrix}\varepsilon_H\left(s^t\right)\\ \varepsilon_F\left(s^t\right)\end{bmatrix}.$$

Diagonal elements of the transition matrix, ρ, determine the degree of persistence in productivity within each country. When off-diagonal elements, ν, are different from zero, productivity innovations originating in one country spill over national borders. The innovations to the productivity process are zero mean serially independent bivariate normal random variables with the contemporaneous covariance matrix

$$E\left[\varepsilon_t\varepsilon_t'\right]=\sigma_\varepsilon^2\cdot\begin{bmatrix}1 & \rho_\varepsilon\\ \rho_\varepsilon & 1\end{bmatrix}.$$

The capital stock in each economy evolves over time according to the following law of motion:

$$k_j\left(s^t\right)=(1-\delta)k_j\left(s^{t-1}\right)+\phi\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right)k_j\left(s^{t-1}\right), \tag{4}$$

where δ is the depreciation rate of capital. An adjustment cost function ϕ satisfies $\phi\left(\cdot\right)>0$, $\phi'\left(\cdot\right)>0$, and $\phi''\left(\cdot\right)<0$. This formulation has been used by Baxter and Crucini (1995), Baxter and Farr (2005) and Yakhin (2007) in the context of international business cycle models. Since we do not rely on log-linearisation methods for solving the model, we must specify the functional form for capital adjustment costs explicitly. We adopt the following formulation from Boldrin, Christiano and Fisher (2001)

$$\phi\left(x\right)=\frac{\kappa_1}{1-1/\xi}\left(x\right)^{1-1/\xi}+\kappa_2,$$

where $\kappa_1 = \delta^{1/\xi}$, $\kappa_2 = \delta/(1-\xi)$, and ξ is the elasticity of investment with respect to Tobin's q. The restrictions $\phi(\delta) = \delta$ and $\phi'(\delta) = 1$ imposed on the constants κ_1 and κ_2 ensure that incorporation of the adjustment cost does not affect the deterministic steady state of the model.

Asset markets Agents have access to a complete set of state-contingent claims. A claim that sells internationally for $Q\left(s^t, s_{t+1}\right)$ at time t, after realisation of history s^t, entitles the bearer to a unit of the consumption good in the following period provided that the state s_{t+1} is realised. Denoting by $B_j\left(s^t, s_{t+1}\right)$ the quantity of such claims purchased by the residents of country j, their budget constraint can be written as

$$\begin{aligned} & c_j\left(s^t\right) + i_j\left(s^t\right) + \textstyle\sum_{s_{t+1}\in S} Q\left(s^t, s_{t+1}\right) B_j\left(s^t, s_{t+1}\right) \\ = \; & r_j\left(s^t\right) k_j\left(s^{t-1}\right) + w_j\left(s^t\right) n_j\left(s^t\right) + B_j\left(s^{t-1}, s_t\right), \end{aligned} \tag{5}$$

where $w_j\left(s^t\right)$ is the wage and $r_j\left(s^t\right)$ is the rental rate on capital in country j.

Equilibrium The equilibrium in this environment consists of the state-contingent sequences of prices $\left\{\left\{r_j\left(s^t\right), w_j\left(s^t\right)\right\}_{j\in J}, \left\{Q\left(s^t, s_{t+1}\right)\right\}_{s_{t+1}\in S}\right\}_{t=0, s^t\in S^t}^{\infty}$ and allocations $\left\{\left\{c_j\left(s^t\right), i_j\left(s^t\right), n_j\left(s^t\right), k_j\left(s^t\right), \left\{B_j\left(s^t, s_{t+1}\right)\right\}_{s_{t+1}\in S}\right\}_{j\in J}\right\}_{t=0, s^t\in S^t}^{\infty}$ that satisfy the following conditions:

i. Given prices, consumers choose state-contingent sequences $\{c_j(s^t)\}_{t=0}^{\infty}$, $\{n_j(s^t)\}_{t=0}^{\infty}$, $\{i_j(s^t)\}_{t=0}^{\infty}$ and bond holdings $\{B_j(s^t, s_{t+1})\}_{s_{t+1}\in S}$ for all $s^t \in S^t$, to maximise utility subject to the budget constraint and the initial conditions.

ii. Given prices, firms choose $n_j(s^t)$ and $k_j(s^{t-1})$ to maximise profits

$$y_j(s^t) - r_j(s^t) k_j(s^{t-1}) - w_j(s^t) n_j(s^t),$$

subject to technology and the non-negativity constraints $n_j(s^t) \geq 0$ and $k_j(s^{t-1}) \geq 0$.

iii. Asset market clearing requires that for all $t \geq 0$ and for all $s^t \in S^t$,

$$B_H(s^t, s_{t+1}) + B_F(s^t, s_{t+1}) = 0, \text{ for all } s_{t+1} \in S.$$

Optimality conditions An equilibrium allocation in this economy can be computed as the solution to a social planner's problem. In addition to the equations of motion, Equations (2) and (4), the global resource constraint

$$\sum_{j\in J} c_j\left(s^t\right)+\sum_{j\in J} i_j\left(s^t\right)=\sum_{j\in J} z_j\left(s^t\right) f\left(k_j\left(s^{t-1}\right), n_j\left(s^t\right)\right), \tag{6}$$

and the initial conditions and the transversality conditions, the optimal allocations must satisfy the following first order conditions. First, under complete markets, the marginal utilities of consumption of two agents are equalised for each time and state

$$\Lambda_H\left(s^t\right)=\Lambda_F\left(s^t\right). \tag{7}$$

Second, labour supply is controlled by the intratemporal condition

$$-\frac{\Lambda_j\left(s^t\right)}{u_3(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right))}=\frac{1}{z_j\left(s^t\right) f_2(k_j\left(s^{t-1}\right), n_j\left(s^t\right))}, \text{ for } j\in J, \tag{8}$$

where $u_3(\cdot)$ denotes the partial derivative of the utility function with respect to its third argument and $f_2(\cdot)$ denotes the partial derivative of the production function with respect to its second argument. The marginal utility of consumption of agent j after history s^t is

$$\begin{aligned}\Lambda_j\left(s^t\right) &= u_1\left(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)\right)\\ &\quad +\beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1}\mid s^t\right) u_2\left(c_j\left(s^t, s_{t+1}\right), h_j\left(s^t\right), n_j\left(s^t, s_{t+1}\right)\right),\end{aligned}$$

where $\pi\left(s_{t+1}\mid s^t\right)$ denotes the conditional probability of s_{t+1} given s^t, and $\pi\left(s^t\mid s^t\right)=1$. Third, intertemporal choice is governed by the Euler equation given by

$$\Lambda_j\left(s^t\right)=\beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1}\mid s^t\right) \Lambda_j\left(s^t, s_{t+1}\right) R_j\left(s^t, s_{t+1}\right), \text{ for } j\in J, \tag{9}$$

where

$$
\begin{aligned}
R_j\left(s^t, s_{t+1}\right) &= \phi'\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right) z_j\left(s^t, s_{t+1}\right) f_2(k_j\left(s^t\right), n_j\left(s^t, s_{t+1}\right)) \\
&+ \left(1-\delta+\phi\left(\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right) - \phi'\left(\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right) \\
&\times\phi'\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right) / \phi'\left(\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right),
\end{aligned}
$$

is the one-period gross rate of return on capital installed in country j after realisation of history $\left(s^t, s_{t+1}\right)$.

3. Parameter Values and Computation

We solve the benchmark model numerically using the parameter values reported in Table 1. Some of these values are common to the international business cycle literature. The capital income share α, utility curvature σ, and parameters governing the stochastic process for productivity take the values found in Kehoe and Perri (2002). The Frisch elasticity of labour supply $1/\eta$ is set to 1.43, as in Correia *et al* (1995) who incorporate GHH preferences in a small open economy setting. Since we introduce habits to account for investment behavior, we use an estimate from the asset pricing literature. By setting the habit intensity parameter b to 0.73, we follow Jermann (1998), who considers a closed-economy counterpart to our model with inelastic labour supply.[3]

3 Values for habit intensity within the range of 0.69 to 0.9 have been reported to help explain the equity premium puzzle (Constantinides 1990; Jermann 1998; Boldrin *et al* 2001). In the sensitivity analysis, we report results from simulations of the model for the whole range of the habit parameter $b \in [0, 1)$.

Table 1: Parameter Values

Benchmark model	
Preferences	$\beta = 0.989$, $\sigma = 2$, $b = 0.73$, $\eta = 1/1.43$, $\chi = 1.401$
Technology	$\alpha = 0.36$, $\delta = 0.025$
Productivity shocks	$\rho = 0.95$, $\nu = 0$, $\sigma_\varepsilon = 0.007$, $\rho_\varepsilon = 0.25$
Variations	
BKK productivity process	$\rho = 0.906$, $\nu = 0.088$, $\sigma_\varepsilon = 0.00852$, $\rho_\varepsilon = 0.258$
High Frisch elasticity	$1/\eta = 1.8$
Low Frisch elasticity	$1/\eta = 1$
GHH preferences	$b = 0$, $\chi = 5.0388$
Standard preferences	$b = 0$, $\sigma = 1.8254$, $1 - \gamma = 0.6303$
Standard preferences/habits	$\sigma = 1.8095$, $1 - \gamma = 0.6369$

Notes: One period of time corresponds to one quarter. The adjustment cost parameter, ξ, is set to fit the standard deviation of investment relative to the standard deviation of output in the data. The preference parameter χ ($1-\gamma$ under standard preferences) that controls disutility from providing labour is set to ensure that hours worked in the steady state equal 1/3. Other parameters in the variations are the same as in the benchmark model.

Other parameters are calibrated to match long-run averages in the US data as described in Cooley (1997). One period of time corresponds to one quarter. The quarterly depreciation rate δ is set to ensure that the steady-state investment-output ratio is 0.25 and the capital-output ratio is 10. Once δ is set, the discount factor β follows directly from the Euler equation (Equation (9)) in the steady state.[4] The coefficient that controls disutility from providing labour, χ, is set so that the agents spend $1/3$ of their unit time endowment on market activities in the deterministic steady state, n_{ss}. Its value follows from Equation (8) in the deterministic steady state:

$$\chi = (1 - b\beta)(1 - \alpha)\frac{k_{ss}^{\alpha}}{n_{ss}^{\alpha+\eta}}, \tag{10}$$

where $k_{ss} = \left(\frac{\alpha}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\alpha}} n_{ss}$, is the steady-state level of the capital stock.

In the variations we consider Cobb-Douglas preferences (also referred to as ‘standard’ preferences)

$$u(c,n) = \frac{\left[c^{\gamma}(1-n)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma},$$

4 Given the values of α, δ and the steady-state capital-output ratio k_{ss}/y_{ss}, we compute the discount factor as $\beta = (\alpha(y_{ss}/k_{ss}) + 1 - \delta)^{-1}$.

and their time non-separable version

$$u(c,h,n) = \frac{[(c-bh)^{\gamma}(1-n)^{1-\gamma}]^{1-\sigma}}{1-\sigma}.$$

The weight of leisure in the instantaneous utility function, $1-\gamma$, ensures that the steady-state level of hours worked n_{ss} remains at 1/3. The weight depends on the intensity of habits

$$\gamma = \left(1 + \frac{(1-b\beta)(1-n_{ss})w_{ss}}{(1-b)c_{ss}}\right)^{-1}, \tag{11}$$

where $w_{ss} = (1-\alpha)(k_{ss}/n_{ss})^{\alpha}$ is the steady-state real wage. As in Raffo (2008), we keep the Frisch elasticity of labour supply ε_f constant across the models. To accomplish this we adjust the curvature parameter σ to account for variation in γ across models with different levels of habit intensity:

$$\sigma = \frac{(1-\gamma)(1-n_{ss})}{\varepsilon_f n_{ss} - \gamma(1-n_{ss})}.$$

The curvature parameter σ takes the value 1.8095 for $b = 0.73$ and 1.8254 for the time separable case ($b = 0$). In all simulations, the capital adjustment cost parameter ξ is set to match the observation that the standard deviation of investment is 2.88 times higher than that of output.

We solve the model numerically using an Euler equation-based method that does not require linearisation of the optimality conditions. The algorithm replaces conditional expectations in the first order conditions with smooth parametric functions of the current state variables and iterates on the parameter values until the rational expectations equilibrium is achieved (den Haan and Marcet 1990). The details of implementation are described in Appendix B.

4. The Results

4.1 Positive Cross-country Co-movement of Output, Investment and Hours

In this section we show that a model with complete markets driven by TFP shocks can account for the observed pattern of international co-movement of consumption, investment and hours worked. In fact, our model's predictive performance is remarkably similar to that of the model with incomplete markets proposed by Kehoe and Perri (2002).

We start by reviewing the puzzles. The first column of Table 2 displays cross-country correlations in aggregate quantities based on US data and that for the aggregate of 15 European countries. In column 2 we show the averages from 190 pairwise correlations among 20 industrialised countries reported by Ambler, Cardia and Zimmermann (2004). The last column reports predictions of the model with standard Cobb-Douglas preferences (no habits). This model fails along two dimensions. First, it predicts negative cross-country correlations in investment and employment while they are positive in the data. Second, in the data, international correlations of output and consumption are positive and fairly high, with the former exceeding the latter. This standard model predicts almost perfectly correlated consumptions and virtually uncorrelated outputs.

Table 2: International Business Cycles – Cross-country Correlations

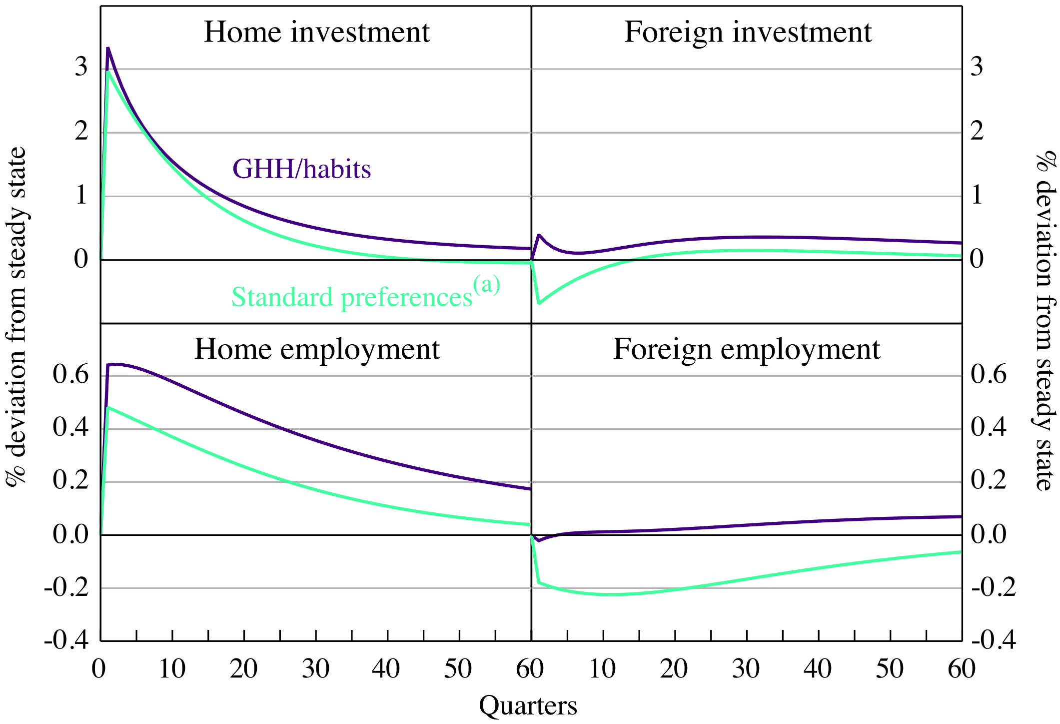
	Data		Model economy with:			
	Europe –US	Average from ACZ	GHH preferences		Cobb-Douglas preferences	
			Habits	No habits	Habits	No habits
Output	0.56	0.28 (0.03)	0.25	0.21	–0.05	0.01
Consumption	0.46	0.15 (0.03)	0.56	0.48	0.92	0.90
Investment	0.43	0.22 (0.04)	0.39	–0.42	0.17	–0.21
Employment	0.31	0.22 (0.03)	0.23	0.21	–0.75	–0.49
Solow residual	0.36	0.21 (0.02)	0.25	0.25	0.25	0.25

Notes: The cross-country correlations in column 1 are calculated from US data and data aggregated over 15 European countries. The sample consists of time series covering the period of 1970:Q1–2008:Q2. The statistics in column 2 are taken from Table 1 in Ambler *et al* (2004) (ACZ), who consider all 190 pairwise cross-country correlations among 20 industrialised countries. Column 2 reports averages of these cross-country correlations and the standard deviations of the average correlations (in parenthesis) for the sample covering 1973:Q1–2000:Q4. Our benchmark model's predictions are reported in the column 'GHH preferences/habits'. The model's statistics are computed from a single simulation of 100 000 periods. All the statistics are based on logged (except for net exports) and HP-filtered data with a smoothing parameter of 1 600.

Our benchmark model, which incorporates GHH preferences and habits, resolves what Baxter (1995) calls the 'international co-movement puzzle'. It matches the data by predicting positive cross-country correlations of investment and hours worked. The benchmark model moves in the right direction in accounting for the 'quantity anomaly' of BKK. It predicts a realistic level of international correlation of output (0.25 vs. 0.28 in the data). Relative to the model with Cobb-Douglas preferences, ours predicts a lower cross-country correlation of consumption (0.56 vs. 0.15 in the data). However, it fails to reverse the order of consumption and output correlations.

Figure 1 provides some additional insights. Following a positive productivity shock at home, foreign investment falls under Cobb-Douglas preferences while it rises in our benchmark model (GHH preferences with habits). What Backus *et al* (1995, p 340) call the tendency to 'make hay where the sun shines' does not apply to investment behaviour in our model. As Figure 1 suggests, the benchmark model predicts positive co-movement of investment even if the productivity innovations are uncorrelated.

Figure 1: **Responses to a One Standard Deviation Positive Productivity Shock in the Home Country**



Note: (a) Cobb-Douglas with no habits

In the aftermath of a positive shock at home, foreign employment falls under both specifications of preferences. The positive co-movement of hours worked reported in Table 2 for our benchmark model is observed because the productivity innovations in the two economies are contemporaneously correlated. In the case of both standard Cobb-Douglas preferences without habits and our benchmark model featuring GHH preferences with habits, the wealth effect tends to decrease foreign hours worked while home employment rises. However, the magnitude of the wealth effect is far smaller under GHH preferences with habits. This explains the reversal of the sign on the cross-country employment correlation.

4.2 Elements of the Model

In this section we discuss the role played by each of the three features of the model. We also describe how all three ingredients are necessary for our result.

4.2.1 Capital adjustment costs

Costly capital adjustment has two roles to play. First, it dampens the unrealistically high volatility of investment relative to output.[5] Second, it complements habits in restricting the immediate response of domestic absorption, $c(s^t) + i(s^t)$, to a productivity shock. In our model, the latter feature is necessary to account for the positive cross-country correlation of investment.

4.2.2 GHH preferences

Under GHH preferences, the marginal rate of substitution between consumption and leisure is independent of consumption. The first order condition for labour, Equation (12), shows that GHH preferences eliminate the wealth effect on labour supply as well as eliminating intertemporal substitution of leisure

$$\chi n(s^t)^\eta = w(s^t). \tag{12}$$

5 Most two-country business cycle models rely on adjustment costs to curb investment volatility. This applies to complete market models (Raffo 2008), as well as models with restricted markets (Baxter and Crucini 1995). A notable exception is Kehoe and Perri (2002), where an endogneously incomplete market framework delivers plausible investment volatility.

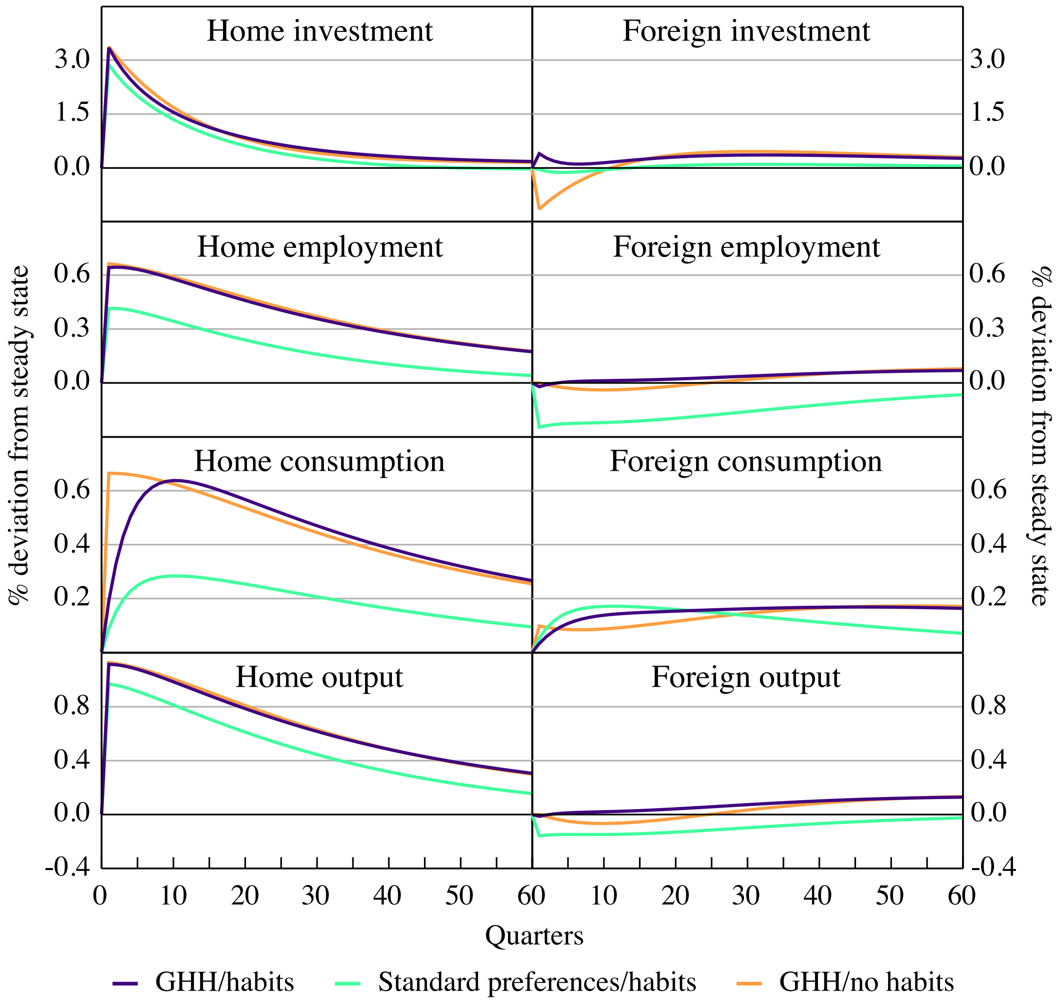
The absence of wealth effects on labour supply is responsible for the positive international correlation in hours worked. To see this, consider the log-linearised version of Equation (12)

$$(\eta + \alpha)\widehat{n} = \widehat{z} + \alpha\widehat{k},$$

where $\widehat{n}$, $\widehat{z}$ and $\widehat{k}$ denote the percentage deviation of the corresponding variable from its steady state. Since $k(s^{t-1})$ is predetermined at t, positively correlated innovations to productivity translate into positive co-movement of hours worked. The latter translates into a positive cross-correlation in output levels.

Figure 2 and Table 2 show that a model featuring GHH preferences but no habits fails to account for the co-movement puzzle. It cannot get the positive investment co-movement right. The tendency to 'make hay where the sun shines' is still present. Investment rapidly moves to the most productive location.

Figure 2: **Alternative Preference Specifications – Responses to a Positive Productivity Shock in the Home Country**



Under GHH preferences with no habits, after a positive productivity shock at home, investment at home rises while foreign investment falls. The fact that innovations to TFP are contemporaneously correlated does little to change this pattern. In fact, Table 2 shows that investment is more negatively correlated under GHH preferences with no habits (column 4) than under Cobb-Douglas preferences (column 6). A similar result is reported in Raffo (2008) who considered a two-country, two-good economy with a GHH preference structure.

4.2.3 Internal habit formation

When we introduce linear internal habits in consumption, the marginal utility of consumption consists of two terms. The first term measures an increase in utility from consuming an extra unit. The second term captures expected discounted future disutility from today's increase in consumption.

$$\Lambda\left(s^t\right) = u_1\left(s^t\right) - b\beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1} \mid s^t\right) u_1\left(s^t, s_{t+1}\right).$$

Habit-forming households strive to smooth not only consumption but also changes in consumption. In the periods following the shock, they increase consumption gradually and allow their habits to adjust. Therefore, on impact, an increase in domestic output compels domestic agents to drastically increase their savings. Since rapid changes in the capital stock at home are costly, the domestic economy responds by increasing net exports. At the same time, foreign agents want to raise their consumption due to the wealth effect. However, habit formation punishes rapid changes in consumption. As a result, world output rises with minor changes to world consumption. An increase in world saving raises investment abroad provided that the adjustment cost at home exceeds the opportunity cost of not investing in the most productive location (i.e. the home country).

The introduction of consumption habits in the Cobb-Douglas class of preferences improves the cross-country correlation of investment. This improvement comes at a cost. The model predicts almost perfectly negatively correlated hours worked. One reason for this failure is the mechanism through which the wealth effect operates on labour supply.

4.3 International Transmission of Productivity Shocks: Combining the Elements

When consumption habits are introduced in the GHH class of preferences, the marginal utility of consumption becomes forward-looking

$$\begin{aligned}\Lambda\left(s^{t}\right) &= \left(c\left(s^{t}\right)-bc\left(s^{t-1}\right)-\chi\frac{n\left(s^{t}\right)^{1+\eta}}{1+\eta}\right)^{-\sigma}\\ &\quad -b\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^{t}\right)\left(c\left(s^{t},s_{t+1}\right)-bc\left(s^{t}\right)-\chi\frac{n\left(s^{t},s_{t+1}\right)^{1+\eta}}{1+\eta}\right)^{-\sigma}.\end{aligned}$$

In contrast, the marginal disutility from work does not become forward-looking:

$$u_3\left(s^{t}\right)=\chi n(s^{t})^{\eta}\left(c\left(s^{t}\right)-bc\left(s^{t-1}\right)-\chi\frac{n\left(s^{t}\right)^{1+\eta}}{1+\eta}\right)^{-\sigma}.$$

The static optimality condition that controls labour supply now reads as

$$\chi n(s^{t})^{\eta}\frac{1}{\Delta(s^{t})}=w(s^{t}), \tag{13}$$

where $\Delta(s^{t})=\left(1-b\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^{t}\right)\frac{u_1\left(s^{t},s_{t+1}\right)}{u_1\left(s^{t}\right)}\right)$. The term $\Delta(s^{t})$ measures the difference between the overall and immediate increase in utility from an extra unit of consumption. Since $\Delta(s^{t})$ depends on current consumption, its presence in Equation (13) re-introduces the wealth effect on labour supply. Its magnitude depends on the intensity of habits, b. Therefore, the wealth effect can be made arbitrarily small by choosing b close to zero. This feature also appears in Jaimovich and Rebelo (2009) who introduce time non-separable preferences consistent with arbitrarily small wealth effects.

The log-linearised version of Equation (13) is given by

$$(\eta+\alpha)\widehat{n}=\widehat{z}+\alpha\widehat{k}+\widehat{\Delta}.$$

As we have already noted, $k(s^{t-1})$ is predetermined at t. The $\Delta(s^{t})$ are equated across countries due to perfect risk-sharing. International co-movement of hours will depend on the covariance of $\widehat{z}$ and $\widehat{\Delta}$ (that is, between the log-linearised productivity shock and the log-linearised $\Delta(s^t)$). Figure 1 shows that under GHH preferences with internal habits, foreign employment falls slightly when home country productivity rises. Leisure is thus a normal good and the terms $\widehat{z}$ and $\widehat{\Delta}$ co-move negatively. Table 2 shows that the cross-country correlation of employment is slightly higher in our benchmark than under GHH preferences with no internal habits (0.23 vs. 0.21). Hence, the wealth effect on labour supply is fairly small for our benchmark value of habit intensity.

The positive co-movement of hours reinforces the mechanism behind the positive co-movement of investment. Since foreign hours do not fall substantially in response to an increase in home productivity, there is a much greater response of global output. This creates additional savings that are channelled into foreign as well as home investment, enhancing the investment co-movement.

4.4 Additional Results

In this section we comment on our model's ability to match the features of within-country business cycles. We discuss how variation in the persistence of shocks and their degree of spillover affects our model's ability to account for international co-movements in the quantity aggregates. This is followed by some sensitivity experiments in Section 4.5.

4.4.1 Within-country business cycles

The within-country business cycle properties of the model featuring GHH preferences with habits are slightly better than those of the model with standard preferences (see Table 3). In particular, the volatilities of consumption, employment and output fit the data more closely. As expected, internal habits induce higher persistence in consumption, which results in a better fit to the data. These improvements are balanced by some deterioration, notably for the within-country correlations with output.

A model with a time separable version of GHH preferences delivers a countercyclical net export to output ratio, matching an important feature of the data. Hence, we replicate the main result of Raffo (2008) in a model without relative price movements or trade in intermediate goods. As in Raffo, this result comes at a cost. The model fails to account for positive international co-movement of investment. Incorporating consumption habits changes the sign of both the investment co-movement and the labour co-movement by reducing the response of domestic absorption to a productivity shock. Similarly to Kehoe and Perri's (2002) model with limited commitment, our benchmark model predicts a positive cross-country investment correlation while it fails to generate countercyclical net exports.

Table 3: Domestic Business Cycles

	Data	**Model economy with:**			
	US	GHH preferences		Cobb-Douglas preferences	
		Habits	No habits	Habits	No habits
Panel A: Volatilities – standard deviation (per cent)					
Output	1.51	1.48	1.49	1.26	1.31
Net exports/output	0.74	0.18	0.40	0.32	0.25
Standard deviations relative to output					
Consumption	0.81	0.44	0.62	0.28	0.36
Investment	2.88	2.88	2.88	2.88	2.88
Employment	0.84	0.58	0.59	0.46	0.48
Solow residual	0.58	0.62	0.62	0.73	0.70
Panel B: Correlations with output					
Consumption	0.86	0.75	0.99	0.67	0.84
Investment	0.94	0.95	0.92	0.97	0.97
Employment	0.88	0.99	0.99	0.92	0.96
Net exports/output	–0.35	0.56	–0.42	0.69	0.40
Solow residual	0.87	0.99	0.99	0.98	0.99
Panel C: Autocorrelations					
Output	0.87	0.74	0.73	0.73	0.73
Consumption	0.88	0.93	0.73	0.93	0.74
Investment	0.90	0.67	0.70	0.68	0.71
Employment	0.92	0.74	0.73	0.74	0.73
Net exports/output	0.86	0.54	0.74	0.76	0.94
Solow residual	0.75	0.71	0.71	0.71	0.71

Notes: Domestic statistics in the Data column correspond to the US time series sample 1970:Q1–2008:Q2. The Solow residuals are constructed using GDP and employment data as described in BKK. The model's statistics are computed from a single simulation of 100 000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1 600.

4.4.2 Can we dispense with near-unit-root shocks without spillovers?

Having incomplete asset markets has helped to improve the empirical fit of international RBC models. Yet a known weakness of incomplete markets models is the sensitivity of their predictions to the parameterisation of the driving process. Baxter and Crucini (1995) illustrate this point in the case where international trade is restricted to non-contingent bonds. They show that the abilities of a complete markets economy and a bond economy to match the data are virtually the same unless TFP shocks follow a near-unit-root process without cross-country spillovers. Kehoe and Perri (2002) report a related result in an environment with endogenously incomplete markets. Their model predicts a negative cross-correlation of employment if the shocks follow the stationary process with mild spillovers used by Backus *et al* (1992).

There seems to be little consensus in the literature regarding the magnitude of spillover or persistence parameters of the TFP process. As Baxter and Farr (2005, p 340) note '[i]t has proved impossible to estimate the parameters of [the productivity process] with much precision'. This raises the question of whether our model allows us to dispense with near-unit-root shocks without spillovers.

Figure 3 and Table 4 show how variation in spillover and persistence parameters affects our model's ability to account for international co-movement in quantity aggregates. The results can be summarised as follows. First, the implications of our model for the quantity anomaly are robust. Cross-country correlations of investment and hours remain positive for commonly used parameter values. Second, the two most sensitive statistics are international correlations of consumption and investment. Third, a higher degree of spillovers leads to a higher cross-correlation of consumption and a lower cross-correlation of investment. Finally, a lower degree of persistence leads to a lower cross-country correlation of consumption and a higher cross-correlation of investment. The intuition for the latter is as follows. The less persistent the shocks are, the less inclined habit-forming individuals are to change their consumption profiles. In the more-productive country this implies higher investment, saving and, therefore, net exports after a positive TFP shock. This increase in net exports, coupled with foreign individuals' reluctance to change consumption, leads to higher investment in the less-productive economy as well. The sensitivity of cross-country correlations of investment and consumption to the persistence parameter highlights the central role of internal habits in getting the investment co-movement right.

Figure 3: International Co-movements – Sensitivity to Parameterisation of the Forcing Process

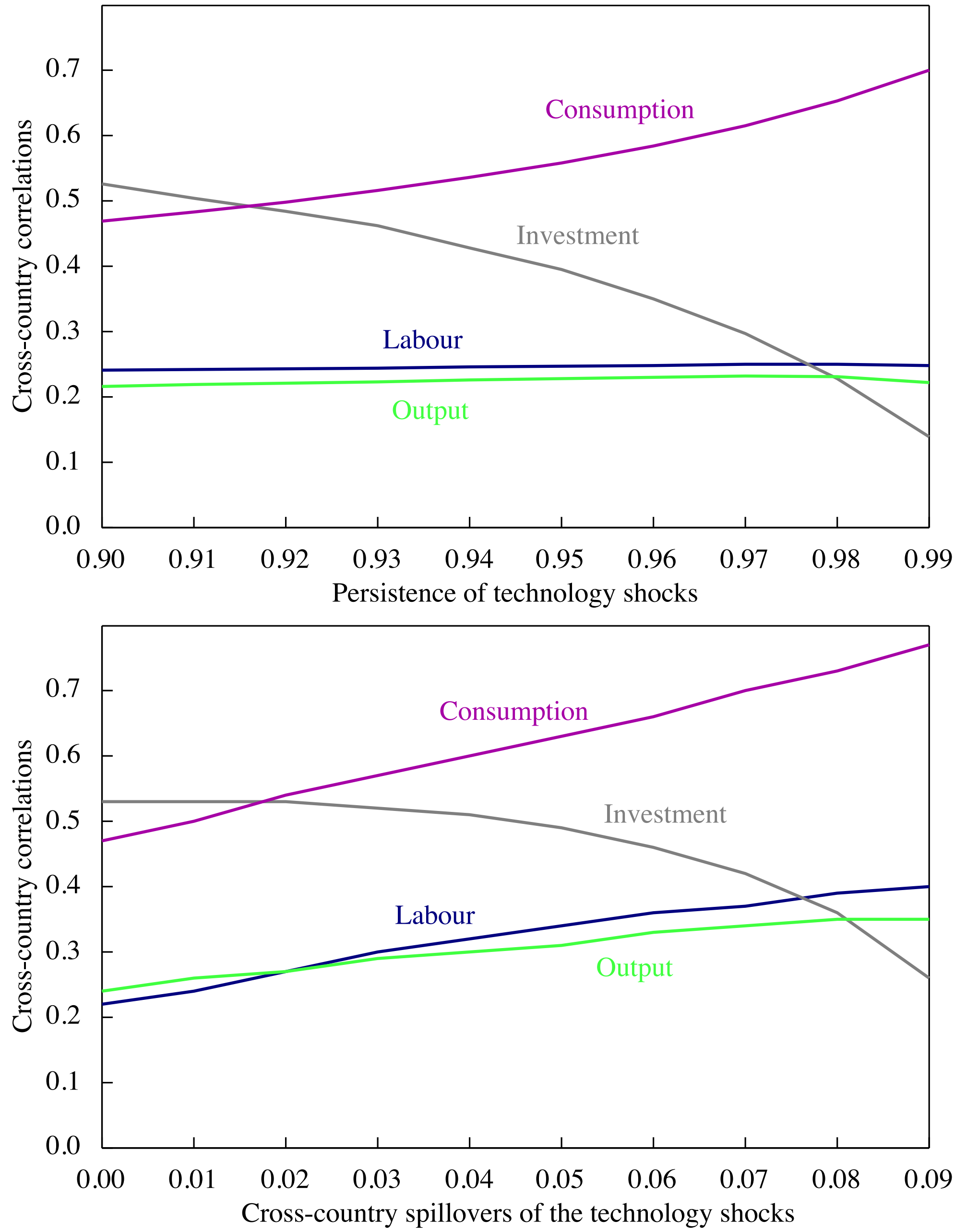


Table 4: Sensitivity Analysis

	Data	**Benchmark model**	Variations: BKK productivity process	Persistence of shocks: Low ($\rho = 0.9$)	Persistence of shocks: High ($\rho = 0.98$)	Frisch elasticity: Low ($\varepsilon_f = 1$)	Frisch elasticity: High ($\varepsilon_f = 1.8$)
Panel A: Volatilities – standard deviation (per cent)							
Output	1.51	1.48	1.83	1.45	1.49	1.36	1.57
Net exports/output	0.74	0.18	0.20	0.24	0.12	0.20	0.17
Standard deviations relative to output							
Consumption	0.81	0.44	0.54	0.39	0.50	0.41	0.46
Investment	2.88	2.88	2.88	2.88	2.88	2.88	2.88
Employment	0.84	0.58	0.61	0.58	0.58	0.49	0.64
Panel B: Correlations with output							
Consumption	0.86	0.75	0.75	0.74	0.76	0.75	0.76
Investment	0.94	0.95	0.91	0.95	0.94	0.95	0.95
Employment	0.88	0.99	0.99	0.99	0.99	0.99	0.99
Net exports/output	–0.35	0.56	0.36	0.59	0.47	0.56	0.56
Panel C: Autocorrelations							
Output	0.87	0.74	0.72	0.71	0.75	0.73	0.74
Consumption	0.88	0.93	0.93	0.93	0.93	0.93	0.93
Investment	0.90	0.67	0.63	0.65	0.67	0.67	0.67
Employment	0.92	0.74	0.72	0.72	0.76	0.74	0.75
Net exports/output	0.86	0.54	0.81	0.56	0.53	0.60	0.49
Panel D: Cross-country correlations							
Output	0.56	0.25	0.35	0.24	0.25	0.25	0.25
Consumption	0.46	0.56	0.79	0.47	0.65	0.63	0.52
Investment	0.43	0.39	0.14	0.53	0.23	0.43	0.36
Employment	0.31	0.23	0.40	0.22	0.23	0.23	0.23

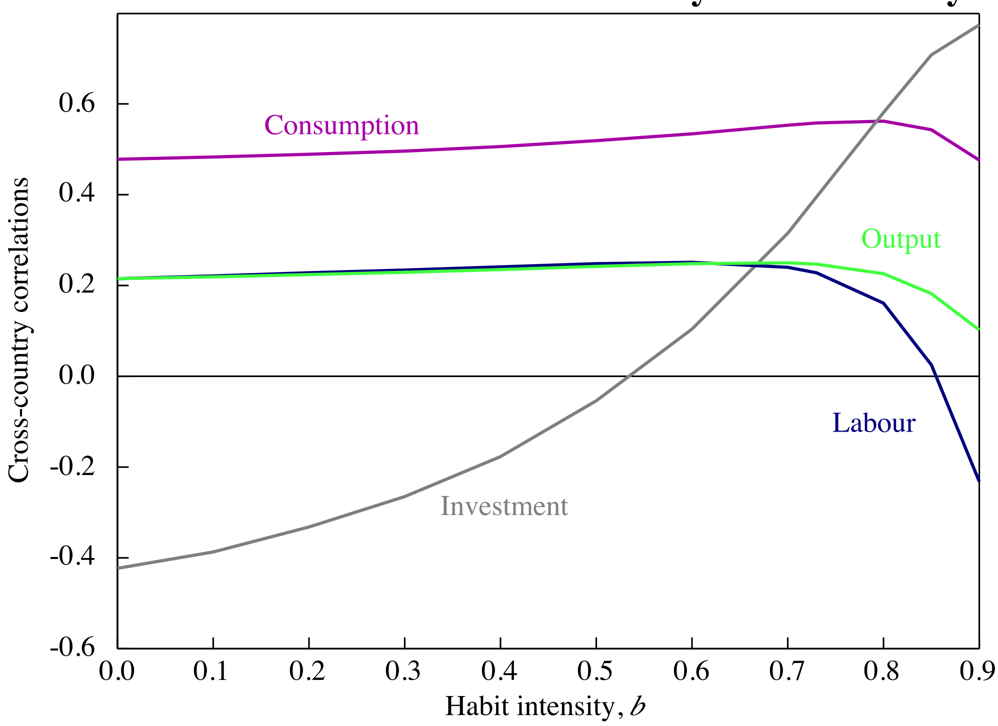
Notes: Domestic statistics in the Data column correspond to the US time series sample 1970:Q1–2008:Q2. The cross-country correlations in this column are calculated from US data and data aggregated over 15 European countries. The model's statistics are computed from a single simulation of 100 000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1 600.

4.5 Sensitivity Experiments

As discussed in Section 3, the benchmark values for two preference parameters are taken from the literature. Here we report how changes in the parameterisation of habit intensity b and the Frisch elasticity of labour supply $1/\eta$ affect our model's predictions for international co-movements.

Habit intensity Figure 4 shows the extent to which cross-correlations of consumption, investment, output and employment depend on the strength of habit formation. There are three key results. First, the cross-country investment correlation is increasing as habit formation intensifies. The larger b is, the more the consumer is interested in smoothing the change rather than the level of consumption. Hence, for small b the model is similar to the model with time separable preferences. Second, the cross-country correlation of hours worked remains positive as long as $b < 0.86$. As b increases further, the positive correlation of TFP innovations fails to compensate for the stronger wealth effect on labour supply. Third, the international correlations of consumption and output levels remain positive for the entire range of b. The gap between the two moments shows almost no dependence on habit intensity.

Figure 4: **International Co-movements – Sensitivity to the Intensity of Habits**



Labour supply elasticity Our results show little sensitivity to variation in the Frisch elasticity of labour supply. In our experiments, we considered the range between 1 and 1.8, which corresponds to the estimates by Chang and Kim (2006) and Fiorito and Zanella (2008). Table 4 illustrates the differences between our benchmark case ($1/\eta = 1.43$), the 'low elasticity' case ($1/\eta = 1$), and the 'high elasticity' case ($1/\eta = 1.8$). Decreasing the labour supply elasticity moderates the volatility of output and hours worked. The rest of the business cycle statistics remain virtually unaffected.

5. Conclusion

We show that a complete markets model driven by productivity shocks alone can help to resolve two major puzzles in the international business cycle literature: the 'international co-movement puzzle' and the 'quantity anomaly'. Instead of restricting international asset markets or introducing new disturbances, we deviate from standard Cobb-Douglas preferences. Our model incorporates a time non-separable version of GHH preferences consistent with a small (but non-zero) wealth effect on labour supply.

The model accounts for positive cross-country co-movement of investment and employment, and delivers a plausible international consumption correlation. In contrast to incomplete market models, ours is robust both to the persistence of shocks and the degree of cross-country spillovers. However, it fails to reverse the inconsistency of the relative magnitude of consumption and output correlations with the data, and predicts that the ratio of net exports to output is procyclical, whereas the ratio is countercyclical in the data.

A limitation of our one-good model is that it abstracts from variations in relative prices. Terms of trade movements provide an extra channel in the international transmission of productivity shocks. Incorporating this channel might improve the model's fit with respect to quantity aggregates. Furthermore, a two-good extension of our model might shed some light on price anomalies, such as the 'Backus-Smith puzzle'. This puzzle highlights the high positive correlation of the real exchange rate with consumption in one country relative to another observed in models featuring complete markets, compared to a correlation of close to zero in the data (see Chari, Kehoe and McGrattan (2002)). We tend to agree with Huang and Liu (2007, p 1288) who conjecture that 'non-time-separable preferences ... may help break the tight link between the real exchange rate and relative consumption'. We leave the testing of this idea for future research.

Appendix A: Data

Data for GDP, consumption, investment and net exports come from OECD *Quarterly National Accounts*. European data cover the following 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. The data are at quarterly frequency, in constant prices, and seasonally adjusted. The sample period is 1970:Q1–2008:Q2. The data are aggregated at the source.

Appendix B: Solving the Model

B.1 The Optimality Conditions

An equilibrium allocation in this economy can be computed as the solution to a social planner's problem. Taking the initial conditions $\left\{k_j(s_0), h_j(s_0), z_j(s_0)\right\}_{j\in J}$ as given, the planner chooses state-contingent plans $\left\{c_j(s^t), i_j(s^t), n_j(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$ for each agent $j \in J = \{1,2\}$ to maximise the expected discounted sum of their weighted utilities

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t\in S^t} \pi(s^t) \sum_{j=J} \omega_j u(c_j(s^t), h_j\left(s^{t-1}\right), n_j(s^t)), \tag{B1}$$

subject to the law of motion for capital

$$k(s^t) = (1-\delta)k\left(s^{t-1}\right) + \phi\left(\frac{i(s^t)}{k\left(s^{t-1}\right)}\right)k\left(s^{t-1}\right), \text{ for } j = J, \tag{B2}$$

the law of motion for habits

$$h_j(s^t) = c_j(s^t), \text{ for } j = J, \tag{B3}$$

as well as the global resource constraint

$$\sum_{j\in J} c_j(s^t) + \sum_{j\in J} i_j(s^t) = \sum_{j\in J} z_j(s^t) f(k_j\left(s^{t-1}\right), n_j(s^t)). \tag{B4}$$

The Lagrangian associated with the planner's problem is given by

$$
\begin{aligned}
L = & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \left[\beta^t \pi\left(s^t\right) \sum_{j=J} \omega_j u(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)) \right. \\
& - \sum_{j \in J} m_j\left(s^t\right) \left(k\left(s^t\right) - (1-\delta) k\left(s^{t-1}\right) - \phi\left(\frac{i\left(s^t\right)}{k\left(s^{t-1}\right)}\right) k\left(s^{t-1}\right) \right) \\
& - \sum_{j \in J} n_j\left(s^t\right) \left(h_j\left(s^t\right) - c_j\left(s^t\right) \right) \\
& \left. - \gamma\left(s^t\right) \left(\sum_{j \in J} c_j\left(s^t\right) + \sum_{j \in J} i_j\left(s^t\right) - \sum_{j \in J} z_j\left(s^t\right) f(k_j\left(s^{t-1}\right), n_j\left(s^t\right)) \right) \right],
\end{aligned}
$$

where $\{m_j\left(s^t\right)\}_{t=0}^{\infty}$, $\{n_j\left(s^t\right)\}_{t=0}^{\infty}$ and $\{\gamma\left(s^t\right)\}_{t=0}^{\infty}$ are the state-contingent paths of Lagrange multipliers associated with constraints in Equations (B2), (B3), and (B4). Equating the gradient of the Lagrangian to zero we obtain

$$
\beta^t \pi\left(s^t\right) \omega_j u_1(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)) + n_j\left(s^t\right) = \gamma\left(s^t\right), \tag{B5}
$$

$$
m_j\left(s^t\right) \phi'\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right) = \gamma\left(s^t\right), \tag{B6}
$$

$$
\begin{aligned}
0 = & \beta^t \pi\left(s^t\right) \omega_j u_3(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)) \\
& + \gamma\left(s^t\right) z_j\left(s^t\right) f_2(k_j\left(s^{t-1}\right), n_j\left(s^t\right))
\end{aligned} \tag{B7}
$$

$$
\begin{aligned}
m_j\left(s^t\right) = & \sum_{s_{t+1} \in S} m_j\left(s^t, s_{t+1}\right) \\
& \times \left(1 - \delta + \phi\left(\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right) - \phi'\left(\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right) \frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)} \right) \\
& + \sum_{s_{t+1} \in S} \gamma\left(s^t, s_{t+1}\right) z_j\left(s^t, s_{t+1}\right) f_1(k_j\left(s^t\right), n_j\left(s^t, s_{t+1}\right)),
\end{aligned} \tag{B8}
$$

$$n_j\left(s^t\right)=\sum_{s_{t+1}\in S}\beta^{t+1}\pi\left(s^t,s_{t+1}\right)\omega_j u_2\left(c_j\left(s^t,s_{t+1}\right),h_j\left(s^t\right),n_j\left(s^t,s_{t+1}\right)\right),\quad \text{(B9)}$$

where $u_1(\cdot)$ is the partial derivative of u with respect to its first argument. We use the same notation to denote the other partial derivatives.

The intertemporal conditions given by Equations (B8) and (B9) can be rearranged as

$$\frac{n_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)}=\sum_{s_{t+1}\in S}\frac{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}{\beta^t\pi\left(s^t\right)}\omega_j u_2(c_j\left(s^t,s_{t+1}\right),h_j\left(s^t\right),n_j\left(s^t,s_{t+1}\right)),\quad \text{(B10)}$$

and

$$\begin{aligned}\frac{m_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)} &= \sum_{s_{t+1}\in S}\frac{m_j\left(s^t,s_{t+1}\right)}{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}\frac{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}{\beta^t\pi\left(s^t\right)}\\ &\times\left(1-\delta+\phi\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)-\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\\ &+\sum_{s_{t+1}\in S}\frac{\gamma\left(s^t,s_{t+1}\right)}{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}\frac{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}{\beta^t\pi\left(s^t\right)} \qquad \text{(B11)}\\ &\times z_j\left(s^t,s_{t+1}\right)f_1(k_j\left(s^t\right),n_j\left(s^t,s_{t+1}\right)).\end{aligned}$$

By denoting $\widetilde{n}_j\left(s^t\right)=\frac{n_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)}$, $\widetilde{m}_j\left(s^t\right)=\frac{m_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)}$, and $\widetilde{\gamma}\left(s^t\right)=\frac{\gamma\left(s^t\right)}{\beta^t\pi\left(s^t\right)}$ we can rewrite Equations (B10) and (B11) as

$$\widetilde{n}_j\left(s^t\right)=\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^t\right)\omega_j u_2(c_j\left(s^t,s_{t+1}\right),h_j\left(s^t\right),n_j\left(s^t,s_{t+1}\right)),\quad \text{(B12)}$$

and

$$
\begin{aligned}
\widetilde{m}_j\left(s^t\right) &= \beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1} \mid s^t\right) \widetilde{m}_j\left(s^t, s_{t+1}\right) \\
&\quad \times \left(1-\delta+\phi\left(\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right)-\phi'\left(\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t, s_{t+1}\right)}{k_j\left(s^t\right)}\right) \\
&\quad +\beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1} \mid s^t\right) \widetilde{\gamma}\left(s^t, s_{t+1}\right) z_j\left(s^t, s_{t+1}\right) f_1(k_j\left(s^t\right), n_j\left(s^t, s_{t+1}\right)).
\end{aligned}
$$

In a similar way, Equations (B5) and (B7) can be rewritten as

$$
\omega_j u_1(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)) + \widetilde{n}_j\left(s^t\right) = \widetilde{\gamma}\left(s^t\right), \tag{B13}
$$

and

$$
\begin{aligned}
0 &= \omega_j u_3(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)) \\
&\quad +\widetilde{\gamma}\left(s^t\right) z_j\left(s^t, s_{t+1}\right) f_2(k_j\left(s^t\right), n_j\left(s^t, s_{t+1}\right))
\end{aligned} \tag{B14}
$$

Let $\Lambda_j\left(s^t\right)$ denote the marginal utility of consumption of agent j after history s^t. Then from Equations (B12) and (B13) it follows that

$$
\begin{aligned}
\Lambda_j\left(s^t\right) &= u_1(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)) \\
&\quad +\beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1} \mid s^t\right) u_2(c_j\left(s^t, s_{t+1}\right), h_j\left(s^t\right), n_j\left(s^t, s_{t+1}\right)),
\end{aligned}
$$

where $\pi\left(s_{t+1} \mid s^t\right)$ denotes the conditional probability of s_{t+1} given s^t, and $\pi\left(s^t \mid s^t\right) = 1$.

Let $R_j\left(s^t,s_{t+1}\right)$ denote the realised one-period gross rate of return on capital in country j after realisation of history $\left(s^t,s_{t+1}\right)$

$$\begin{aligned}R_j\left(s^t,s_{t+1}\right) &= \phi'\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right)z_j\left(s^t,s_{t+1}\right)f_1(k_j\left(s^t\right),n_j\left(s^t,s_{t+1}\right)) \\ &+\left(1-\delta+\phi\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)-\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right) \\ &\times\phi'\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right)/\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right).\end{aligned}$$

Then the first order conditions can be reformulated as

$$\Lambda_1\left(s^t\right)=\left(\frac{\omega_2}{\omega_1}\right)\Lambda_2\left(s^t\right), \tag{B15}$$

$$\Lambda_j\left(s^t\right)=\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^t\right)\Lambda_j\left(s^t,s_{t+1}\right)R_j\left(s^t,s_{t+1}\right),\text{for } j\in J, \tag{B16}$$

$$\begin{aligned}0= &\ u_3(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right)) \\ &+\Lambda_j\left(s^t\right)z_j\left(s^t\right)f_2(k_j\left(s^{t-1}\right),n_j\left(s^t\right)),\end{aligned} \tag{B17}$$

for $j\in J$.

B.2 Optimality Conditions with the Functional Forms

The instantaneous utility function takes the form

$$u(c,h,n)=\frac{1}{1-\sigma}\left(c-bh-\chi\frac{n^{1+\eta}}{1+\eta}\right)^{1-\sigma}.$$

The production function is

$$y=zf(k,n)=zk^{\alpha}n^{1-\alpha}.$$

The capital adjustment cost function is

$$\begin{aligned}
\phi(x) &= \frac{a_1}{1-1/\xi}(x)^{1-1/\xi} + a_2, \\
\phi'(x) &= a_1 x^{-1/\xi} = \left(\frac{\delta}{x}\right)^{1/\xi}
\end{aligned}$$

where the restrictions that $\phi'(\delta) = 1$ and $\phi(\delta) = \delta$ require that $a_1 = \delta^{1/\xi}$ and $a_2 = \frac{\delta}{1-\xi}$. Symmetry between the two economies implies that $\omega_1 = \omega_2$. Incorporating specific functional forms, the optimality conditions can be rewritten as

$$\Lambda_1\left(s^t\right) = \Lambda_2\left(s^t\right),$$

$$\Lambda_j\left(s^t\right) = \beta E_t\left[\Lambda_j\left(s^t\right) R_j\left(s^t\right)\right],$$

$$\begin{aligned}
\Lambda_j\left(s^t\right) &= \left(c_j\left(s^t\right) - bh_j\left(s^{t-1}\right) - \chi\frac{n_j\left(s^t\right)^{1+\eta}}{1+\eta}\right)^{-\sigma} \\
&\quad - b\beta E_t\left[\left(c_j(s^{t+1}) - bh_j(s^t) - \chi\frac{n_j(s^{t+1})^{1+\eta}}{1+\eta}\right)^{-\sigma}\right],
\end{aligned}$$

$$\frac{\Lambda_j\left(s^t\right)}{\chi n_j\left(s^t\right)^{\eta}\left(c_j\left(s^t\right) - bh_j\left(s^{t-1}\right) - \chi\frac{n_j\left(s^t\right)^{1+\eta}}{1+\eta}\right)^{-\sigma}} = \frac{1}{(1-\alpha)z_j\left(s^t\right)k_j\left(s^{t-1}\right)^{\alpha}n_j\left(s^t\right)^{-\alpha}},$$

$$\begin{aligned}
R_{t+1} &= a_1\left(\frac{i_j(s^t)}{k_j(s^{t-1})}\right)^{-1/\xi} \\
&\times\left[\alpha\frac{y_j(s^{t+1})}{k_j(s^t)} + \left(\frac{i_j(s^{t+1})}{k_j(s^t)}\right)^{1/\xi}\left(\frac{1-\delta+a_2}{a_1} + \frac{1}{\xi-1}\left(\frac{i_j(s^{t+1})}{k_j(s^t)}\right)^{1-1/\xi}\right)\right],
\end{aligned}$$

$$c_1\left(s^t\right) + c_2\left(s^t\right) + i_1\left(s^t\right) + i_2\left(s^t\right) = y_1\left(s^t\right) + y_2\left(s^t\right).$$

B.3 Parameter Values for the Benchmark Model

Productivity follows a process similar to that specified by Kehoe and Perri (2002):

$$\begin{bmatrix} \log(z_1(s^t)) \\ \log(z_2(s^t)) \end{bmatrix} = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix} \begin{bmatrix} \log(z_1(s^{t-1})) \\ \log(z_2(s^{t-1})) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(s^t) \\ \varepsilon_2(s^t) \end{bmatrix}.$$

The innovations to the productivity process are zero mean serially independent bivariate normal random variables with the contemporaneous covariance matrix

$$E\left[\varepsilon_t \varepsilon_t'\right] = 0.007^2 \cdot \begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix}.$$

Standard/estimated values are as follows:

- Capital income share $\alpha = 0.36$ and coefficient of relative risk aversion $\sigma = 2$, as in Kehoe and Perri (2002)
- Elasticity of labour supply $1/\eta = 1.43$, that is, $\eta = 1/1.43 = 0.6993$, as in Correia *et al* (1995)
- Intensity of habits $b = 0.73$, as in Jermann (1998).

The calibration targets are: $n_{ss} = 1/3; i_{ss}/y_{ss} = 0.25; k_{ss}/y_{ss} = 10$. The calibrated parameters are as follows:

- Depreciation rate: $\delta = i_{ss}/k_{ss} = (i_{ss}/y_{ss})/(k_{ss}/y_{ss}) = 0.025$
- Discount factor: $\beta = (\alpha(y_{ss}/k_{ss}) + 1 - \delta)^{-1} = (0.36 \cdot 0.1 + 1 - 0.025)^{-1} = 0.989$
- From $1 = \beta R_{ss} = \beta\left(\alpha k_{ss}^{\alpha-1} n_{ss}^{1-\alpha} + 1 - \delta\right)$ it follows that

$$k_{ss} = \left(\frac{\alpha}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\alpha}} n_{ss} = \left(\frac{0.36}{1/0.989 - 1 + 0.025}\right)^{\frac{1}{1-0.36}} (1/3) = 12.108$$

- From the labour supply equation in the non-stochastic steady state $\chi n_{ss}^{\eta} = (1-b\beta)(1-\alpha)k_{ss}^{\alpha}n_{ss}^{-\alpha}$, it follows that the weight of labour in the utility function χ is:

$$\begin{aligned}
\chi &= (1-b\beta)(1-\alpha)\frac{k_{ss}^{\alpha}}{n_{ss}^{\alpha+\eta}} = (1-b\beta)(1-\alpha)\left(\frac{\alpha}{1/\beta-1+\delta}\right)^{\frac{\alpha}{1-\alpha}} n_{ss}^{-\eta} \\
&= (1-0.73\cdot 0.989)(1-0.36)\left(\frac{0.36}{1/0.989-1+0.025}\right)^{\frac{0.36}{1-0.36}} (1/3)^{-(1/1.43)} \\
&= 1.401.
\end{aligned}$$

The other steady-state values are as follows:

$$i_{ss} = \delta k_{ss} = 0.025\cdot 12.108 = 0.3027;$$

$$y_{ss} = k_{ss}^{\alpha}n_{ss}^{1-\alpha} = (12.108)^{0.36}(1/3)^{(1-0.36)} = 1.2149;$$

$$c_{ss} = y_{ss} - i_{ss} = 1.2149 - 0.3027 = 0.9122.$$

B.4 The Numerical Procedure

The model is solved using a variant of the ergodic set methods described by Maliar, Maliar and Judd (2011). The algorithm we use is classified by Judd, Maliar and Maliar (2009) as belonging to the stochastic simulation class of methods. The approach is to replace conditional expectations with smooth parametric approximation functions of the current state variables and a vector of parameters, and then iterate on the parameter values until a rational expectations equilibrium is achieved. The four conditional expectations are parameterised as follows

$$E_t\left[\left(c_{1t+1} - bc_{1t} - \chi\frac{n_{1t+1}^{1+\eta}}{1+\eta}\right)^{-\sigma}\right] = \Psi(\omega_1;\mathbf{x}_t)$$

$$E_t\left[\left(c_{2t+1} - bc_{2t} - \chi\frac{n_{2t+1}^{1+\eta}}{1+\eta}\right)^{-\sigma}\right] = \Psi(\omega_2;\mathbf{x}_t)$$

$$\Lambda_{1t} = \beta E_t\left[\Lambda_{1t+1}R_{1t+1}\right] = \Psi(\omega_3;\mathbf{x}_t)$$

$$k_{2t+1} = \beta E_t \left[\Lambda_{2t+1} R_{2t+1} k_{2t+1} \right] = \Psi(\omega_4; \mathbf{x}_t)$$

where $\mathbf{x}_t = \left[k_{1t}, k_{2t}, c_{1t-1}, c_{2t-1}, z_{1t}, z_{2t} \right]$. From the first order condition for consumption in the home country we have

$$\Lambda_{1t} = \left(c_{1t} - bc_{1t-1} - \chi \frac{n_{1t}^{1+\eta}}{1+\eta} \right)^{-\sigma} - b\beta E_t \left[\left(c_{1t+1} - bc_{1t} - \chi \frac{n_{1t+1}^{1+\eta}}{1+\eta} \right)^{-\sigma} \right].$$

Re-arranging yields

$$\begin{aligned} \left(c_{1t} - bc_{1t-1} - \chi \frac{n_{1t}^{1+\eta}}{1+\eta} \right)^{-\sigma} &= \Lambda_{1t} + b\beta E_t \left[\left(c_{1t+1} - bc_{1t} - \chi \frac{n_{1t+1}^{1+\eta}}{1+\eta} \right)^{-\sigma} \right] \\ &= \Psi(\omega_3; \mathbf{x}_t) + b\beta \Psi(\omega_1; \mathbf{x}_t). \end{aligned}$$

From the first order condition for labour in the country 1

$$\frac{\chi n_{1t}^{\eta} \left(c_{1t} - bc_{1t-1} - \chi \frac{n_{1t}^{1+\eta}}{1+\eta} \right)^{-\sigma}}{\Lambda_{1t}} = (1-\alpha) z_{1t} k_{1t}^{\alpha} n_{1t}^{-\alpha}$$

it follows that

$$\begin{aligned} n_{1t}^{\eta+\alpha} &= \frac{(1-\alpha) z_{1t} k_{1t}^{\alpha}}{\chi} \frac{\Lambda_{1t}}{\left(c_{1t} - bc_{1t-1} - \chi \frac{n_{1t}^{1+\eta}}{1+\eta} \right)^{-\sigma}} \\ &= \frac{(1-\alpha) z_{1t} k_{1t}^{\alpha}}{\chi} \frac{\Psi(\omega_3; \mathbf{x}_t)}{\Psi(\omega_3; \mathbf{x}_t) + b\beta \Psi(\omega_1; \mathbf{x}_t)}. \end{aligned}$$

From the risk-sharing condition:

$$\Lambda_{1t} = \Lambda_{2t}$$

we obtain

$$\begin{aligned} \left(c_{2t} - bc_{2t-1} - \chi \frac{n_{2t}^{1+\eta}}{1+\eta} \right)^{-\sigma} &= \Lambda_{2t} + b\beta E_t \left[\left(c_{2t+1} - bc_{2t} - \chi \frac{n_{2t+1}^{1+\eta}}{1+\eta} \right)^{-\sigma} \right] \\ &= \Psi(\omega_3; \mathbf{x}_t) + b\beta \Psi(\omega_2; \mathbf{x}_t), \end{aligned}$$

and from the country 2 supply equation we get

$$n_{2t}^{\eta+\alpha} = \frac{(1-\alpha)\, z_{2t} k_{2t}^{\alpha}}{\chi} \frac{\Psi(\omega_3;\mathbf{x}_t)}{\Psi(\omega_3;\mathbf{x}_t) + b\beta\Psi(\omega_2;\mathbf{x}_t)}.$$

Current consumption in each country is therefore given by:

$$c_{1t} = [\Psi(\omega_3;\mathbf{x}_t) + b\beta\Psi(\omega_1;\mathbf{x}_t)]^{-\frac{1}{\sigma}} + bc_{1t-1} + \chi\frac{n_{1t}^{1+\eta}}{1+\eta}, \tag{B18}$$

$$c_{2t} = [\Psi(\omega_3;\mathbf{x}_t) + b\beta\Psi(\omega_2;\mathbf{x}_t)]^{-\frac{1}{\sigma}} + bc_{2t-1} + \chi\frac{n_{2t}^{1+\eta}}{1+\eta}. \tag{B19}$$

Labour in each country is given by

$$n_{1t}^{\eta+\alpha} = \frac{(1-\alpha)\, z_{1t} k_{1t}^{\alpha}}{\chi} \frac{\Psi(\omega_3;\mathbf{x}_t)}{\Psi(\omega_3;\mathbf{x}_t) + b\beta\Psi(\omega_1;\mathbf{x}_t)}, \tag{B20}$$

$$n_{2t}^{\eta+\alpha} = \frac{(1-\alpha)\, z_{2t} k_{2t}^{\alpha}}{\chi} \frac{\Psi(\omega_3;\mathbf{x}_t)}{\Psi(\omega_3;\mathbf{x}_t) + b\beta\Psi(\omega_2;\mathbf{x}_t)}. \tag{B21}$$

The algorithm is implemented as follows:[6]

1. Obtain an initial guess for $\omega = [\omega_1, \omega_2, \omega_3, \omega_4]$. We obtain the initial guess using the genetic algorithm and then homotopy. Fix $k_{j0} = k_{ss}$, $h_{j0} = c_{ss}$ and $z_{j0} = 1$ for $j \in J$, and draw a sample of size T of the exogenous stochastic shock $\{z_{1t}, z_{2t}\}_{t=0}^{T}$.

2. Replace the conditional expectations with the parameterised functions $\Psi(\omega_r;\mathbf{x}_t)$, $r = 1...4$. Calculate $\{n_{1t}, n_{2t}, c_{1t}, c_{2t}, h_{1t}, h_{2t}\}_{t=0}^{T}$ using Equations (B18), (B19), (B20) and (B21), and the law of motion for habits, Equation (B3). Calculate $\{y_{1t}, y_{2t}, i_{1t}, i_{2t}, k_{1t}\}_{t=0}^{T}$ using the production function, the law of motion for capital given in Equation (B2) and the global resource constraint, Equation (B4). Similarly compute $\{\Lambda_{1t}, R_{1t}, R_{2t}\}_{t=0}^{T}$.

6 Further details on this class of algorithm are provided by den Haan and Marcet (1990). For a more formal description and related proofs, see Marcet and Marshall (1994).

3. Set

$$Y_t^1(\omega) \equiv \left(c_{1t+1} - bc_{1t} - \chi \frac{n_{1t+1}^{1+\eta}}{1+\eta}\right)^{-\sigma},$$
$$Y_t^2(\omega) \equiv \left(c_{2t+1} - bc_{2t} - \chi \frac{n_{2t+1}^{1+\eta}}{1+\eta}\right)^{-\sigma},$$
$$Y_t^3(\omega) \equiv \beta\left[\Lambda_{1t+1} R_{1t+1}\right],$$
$$Y_t^4(\omega) \equiv \Lambda_{2t+1} R_{2t+1} k_{2t+1}.$$

   and minimise the sum of squared residuals for the equation $Y_t^r(\omega) = \Psi(\omega_r; \mathbf{x}_t(\omega)) + v_t^r$, $r = 1...4$, where v_t^r is the regression error. That is, find

$$G(\omega_r) = \arg\min_{\zeta} \frac{1}{T} \sum_{t=0}^{T} ||Y_t^r(\omega) - \Psi(\zeta; \mathbf{x}_t(\omega))||^2$$

   where ζ is the parameter vector to be estimated.

4. Iterating on w_r, find the fixed point $w_r^* = G(w_r^*)$. Update w_r using the algorithm $\omega_r(\tau+1) = (1-\mu)\omega_r(\tau) + \mu G(\omega_r(\tau))$ for $\mu > 0$, $\omega_r(0)$ given.

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