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## **COMPETITION AND THE PHILLIPS CURVE**

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# COMPETITION AND THE PHILLIPS CURVE

## Abstract

It has been well-documented that the Phillips curve has flattened, making central bankers wary of the reduced effectiveness of monetary policy to achieve price stability. There has also been a growing concern about higher market concentration and the rising profit margins and markup rates. Are these two events merely coincidental or causally related? To address this issue, this paper extends the canonical New Keynesian model to introduce markup-rate changes caused by endogenous entry, by using Homothetic Single Aggregator (HSA), a class of flexible and tractable homothetic demand systems, which contain CES and Translog as special cases. Under Marshall's second law of demand (i.e., the price elasticity of demand goes up with its price) and the third law of demand (i.e., the rate of increase in the price elasticity goes down with its price), a higher entry cost, which leads to market concentration, causes the flattening of the Phillips curve for two reasons. First, market concentration leads to a rise of the markup rate and a decline of the pass-through rate, which structurally reduces the slope of the Phillips curve. Second, an endogenous change in the number of firms generates an endogenous cost-push shock through strategic complementarity. Due to this endogeneity of the cost-push shock, a naive regression of the inflation rate on the real marginal cost leads to the negative omitted variables bias, which is amplified with more concentration. This weakens the estimated relationship between the inflation rate and the marginal cost. These two reasons, one structural and one observational, together can go a long way toward understanding the flattening of the Phillips curve.

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# Competition and the Phillips Curve\*

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## Abstract

It has been well-documented that the Phillips curve has flattened, making central bankers wary of the reduced effectiveness of monetary policy to achieve price stability. There has also been a growing concern about higher market concentration and the rising profit margins and markup rates. Are these two events merely coincidental or causally related? To address this issue, this paper extends the canonical New Keynesian model to introduce markup-rate changes caused by *endogenous entry*, by using *Homothetic Single Aggregator (HSA)*, a class of flexible and tractable homothetic demand systems, which contain CES and Translog as special cases. Under *Marshall's second law of demand* (*i.e.*, the price elasticity of demand goes up with its price) and *the third law of demand* (*i.e.*, the rate of increase in the price elasticity goes down with its price), a higher entry cost, which leads to market concentration, causes the flattening of the Phillips curve for two reasons. First, market concentration leads to a rise of the markup rate and a decline of the pass-through rate, which *structurally* reduces the slope of the Phillips curve. Second, an endogenous change in the number of firms generates an endogenous cost-push shock through strategic complementarity. Due to this endogeneity of the cost-push shock, a *naive* regression of the inflation rate on the real marginal cost leads to the negative omitted variables bias, which is amplified with more concentration. This weakens the *estimated* relationship between the inflation rate and the marginal cost. These two reasons, one structural and one observational, together can go a long way toward understanding the flattening of the Phillips curve.

*Keywords:* New Keynesian Phillips curve, market concentration, monopolistic competition, endogenous entry, HSA, variable markups and pass-through rates, supply side effects of monetary policy, omitted variable bias, cyclicalities of markups, sufficient statistic

*JEL codes:* E31, E52

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# 1 Introduction

One of the major issues central banks have been facing in recent years is the flattening of the Phillips curve. For example, Federal Reserve Vice Chair, Richard Clarida said on Sept. 26, 2019, that “Another key development in recent decades is that price inflation appears less responsive to resource slack. That is, the short-run price Phillips curve – if not the wage Phillips curve – appears to have flattened.” San Francisco Fed President, Mary Daly stated on Aug. 29, 2019, that “As for the Phillips curve... most arguments today center around whether it’s dead or just gravely ill”. New York Fed President, John Williams explained on Feb. 22, 2019, that “The Phillips curve is the connective tissue between the Federal Reserve’s dual mandate goals of maximum employment and price stability. Despite regular declarations of its demise, the Phillips curve has endured. It is useful, both as an empirical basis for forecasting and for monetary policy analysis.” Central banks are wary of it since achieving price stability becomes more challenging due to the reduced impact of monetary policy on inflation rates through real economic activities.

Another significant development that the U.S. economy has experienced in recent years is market concentration. There is a growing concern about the adverse impacts of the resulting increase in profit margins and markups on the macroeconomy. Recent studies provide empirical evidence of increasing market concentration. [Covarrubias, Gutierrez and Philippon \(2019\)](#) show that “After 2000, however, the evidence suggests inefficient concentration, decreasing competition and increasing barriers to entry, as leaders become more entrenched and concentration is associated with lower investment, higher prices and lower productivity growth.” According to [De Loecker, Eeckhout and Unger \(2020\)](#), “In 1980, aggregate markups start to rise from 21% above marginal cost to 61% now. ... We also find an increase in the average profit rate from 1% to 8%. Although there is also an increase in overhead costs, the markup increase is in excess of overhead.” In addition, [Autor, Dorn, Katz, Patterson and Reenen \(2020\)](#) state that “sales concentration is rising across a large set of industries. ... aggregate markups have been rising.”

These two events observed in recent years seem unrelated through the lens of the textbook New Keynesian model (see, for example, [Gali, 2015](#), [Walsh, 2010](#) and [Woodford, 2003](#)). Under the CES demand system extensively used in the literature, market concentration neither leads to the flattening of the Phillips curve nor directly impacts the inflation rate through the Phillips curve. This irrelevance result can be overturned once we depart from CES and adopt more flexible demand systems. The contribution of this paper is to offer a theoretical framework that reveals the role of market concentration on the flattening of the Phillips curve.

To this end, we extend the canonical New Keynesian model under CES monopolistic competition in two respects. First, we incorporate entry and exit as in [Bilbiie, Gironi and Melitz \(2008\)](#) and [Bilbiie, Fujiwara and Gironi \(2014\)](#). Second, we replace CES by *Homothetic Single Aggregator* (HSA), a class of homothetic demand systems, originally proposed in a general

setting by Matsuyama and Ushchev (2017) and subsequently extended to monopolistic competition by Matsuyama and Ushchev (2020b). We use HSA for several reasons:

- HSA contains CES and Translog as special cases.
- It is tractable due to its single aggregator that serves as a sufficient statistic, because it summarizes all the impacts of competitive pressures on the pricing behavior of monopolistically competitive firms, including the markup rate and the pass-through rate.
- HSA is flexible. For example, HSA can accommodate *Marshall's second law of demand* (hereafter, the Second law)<sup>1</sup> as well as what Matsuyama and Ushchev (2023b) call *the third law of demand* (hereafter, the Third law).<sup>2</sup>
- It is straightforward to ensure the uniqueness of equilibrium in spite of endogenous change in the number of firms.

These features of HSA enable us to show how the entry affects the Phillips curve through its impact on two sufficient statistics: the price elasticity and the pass-through rate.<sup>3</sup> To the best of our knowledge, this paper is the first attempt to incorporate the HSA demand system into the New Keynesian framework.

For price adjustment mechanisms, we employ Rotemberg (1982) pricing and Calvo (1983) pricing. We use both of these pricing mechanisms because Rotemberg pricing highlights the crucial role of the Second law and Calvo pricing highlights the role of the Third law in the flattening of the New Keynesian Phillips curve (hereafter, NKPC), as explained in the main text.<sup>4</sup>

Here is our main finding. Under the Second and the Third laws, a higher entry cost, which leads to market concentration, causes the flattening of the Phillips curve for two reasons. First, the slope of the Phillips curve is positively related to the price elasticity under the Rotemberg pricing and the pass-through rate under Calvo pricing. Hence, market concentration, which leads to a decline of the price elasticity under the Second law and of the pass-through rate under the Third law, causes a *structural* flattening of the Phillips curve. This is what we call the *steady-state effect of concentration*. Second, an endogenous change in the number of firms

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<sup>1</sup>The Second law states that the price elasticity of demand goes up with its price, which implies that market concentration leads to a higher markup rate in our setup and that more productive firms have higher markup rates with heterogeneous firms as in Matsuyama and Ushchev (2023b). This is in line with the empirical evidence by Campbell and Hopenhayn (2005), Burstein and Gopinath (2014), De Loecker and Goldberg (2014), Feenstra and Weinstein (2017), Amiti, Itskhoki and Konings (2019) and Baqaee, Farhi and Sangani (2023).

<sup>2</sup>The Third law in its strong (weak) form states that the rate of increase in the price elasticity goes down (does not go up) with its price, which implies that market concentration leads to a lower (does not lead to a higher) pass-through rate in our setup and that more productive firms have lower (do not have higher) pass-through rates with heterogeneous firms in Matsuyama and Ushchev (2023b). This is in line with the empirical evidence by Berman, Martin and Mayer (2012), Amiti, Itskhoki and Konings (2014) and Baqaee, Farhi and Sangani (2023).

<sup>3</sup>We also show in Appendix F that the shape of the Phillips curve is captured by the same two sufficient statistics under HDIA (Homothetic with Direct Implicit Additivity) and HIIA (Homothetic with Indirect Implicit Additivity), the two alternative homothetic demand systems proposed by Matsuyama and Ushchev (2017, 2020a,b). Unlike HSA, however, it is challenging to characterize the impact of the entry on these two sufficient statistics under HDIA and HIIA.

<sup>4</sup>We conjecture that both the Second and the Third laws matter and that results would be a hybrid of Rotemberg and Calvo under more general price adjustment mechanisms.

generates an endogenous cost-push shock through strategic complementarity, which we call *the dynamic effect of endogenous entry*. Due to this endogeneity of the cost-push shock, a *naive* regression of the inflation rate on the real marginal cost leads to the negative omitted variables bias under the Second law, whose magnitude is amplified with more concentration under the Third law (in the case of Rotemberg) and under both the Second and the Third laws (in the case of Calvo). This weakens the *estimated* relationship between the inflation rate and the marginal cost. These two reasons, one structural and one observational, together can go a long way toward understanding the flattening of the Phillips curve.

Our simulation suggests that market concentration generates a sizable reduction in the slope of NKPC. We show that the rise in the markup rate as observed in the data can halve the slope of NKPC. As a result, in impulse responses, the responses of the inflation rate to both technology and monetary policy shocks become smaller as market concentration deepens.

It should be also pointed out that under the Second and the Third laws, the cyclicity of the markup rate is determined by the tension between nominal rigidities and the pass-through rate. Suppose a positive technology shock hits the economy. As is well-known, in a sticky price equilibrium under the CES demand system, the markup rate is procyclical, because the price does not respond in the short run to the decline in the marginal cost. But, under the Second law, a positive technology shock increases the number of firms, which work in the direction of reducing the markup rate and making it countercyclical in a flexible price equilibrium. This explains why the cyclicity of the markup rate in a sticky price equilibrium under the Second and the Third laws is generally ambiguous.<sup>5</sup>

## 1.1 Literature

There is a vast literature on the flattening of the Phillips curve, which can be divided into five categories. First, [Goolsbee and Klenow \(2018\)](#) and [Crump, Eusepi, Giannoni and Sahin \(2019\)](#) attribute this to mis-measurments in the inflation rate and gap measures. Second, [Daly and Hobbijn \(2014\)](#) argue that there has been a structural change in the labor market and that the flattening happened not to the price Phillips curve but to the wage Phillips curve. Third, [McLeay and Tenreyro \(2019\)](#) argue that changes in monetary policy response to inflation have weakened the observed relationship between the gap measures and the inflation rate. Fourth, [Coibion and Gorodnichenko \(2015\)](#) and [Hazell, Herreno, Nakamura and Steinsson \(2022\)](#) state that the declining slope coefficient is due to the declining inflation expectations not being properly taken into account when estimating the Phillips curve. Fifth, [Sbordone \(2010\)](#), [Wang and Werning \(2022\)](#), [Baqaee, Farhi and Sangani \(2021\)](#), [Harding, Linde and Trabandt \(2022\)](#), [L’Huillier et al. \(2022\)](#) and [Rubbo \(2023\)](#) propose models in which the slope of NKPC becomes structurally flatter in line with the empirical evidence in [Del Negro, Lenza, Primiceri and Tambalotti \(2020\)](#). Our paper is related both to the fourth because of the omitted variable bias result, and to the

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<sup>5</sup>There is indeed a disagreement on the cyclicity of the markup rate in the literature as surveyed by [Nekarda and Ramey \(2020\)](#).

Table 1: Monetary policy and competition

	comp.	entry	pref.	nominal friction
Wang and Werning (2022)	oligo.	exo.	CES / Kimball	Calvo
Baqae, Farhi and Sangani (2021)	mono.	exo.	Kimball	Calvo
our paper	mono.	end.	HSA	Rotemberg / Calvo

oligo.: oligopolistic competition, mono.: monopolistic competition, exo.: exogenous entry, end.: endogenous entry

fifth because of the structural flattening result.

It is well-known that in the prototypical New Keynesian model under the CES demand system, competition is irrelevant to the Phillips curve, irrespective of entry and exit. It neither changes the slope of the Phillips curve nor exert any direct impact on the inflation rate. Several recent studies challenge this irrelevance of competition.<sup>6</sup> Wang and Werning (2022) show that in a dynamic oligopoly model with strategic interaction, higher concentration leads to amplified real effects of monetary policy, the Phillips curve with inflation persistence, and the endogenous cost-push shock. Baqae, Farhi and Sangani (2021) demonstrate that higher concentration leads to the flattening of the Phillips curve through two different channels: the real rigidities and the misallocation across heterogeneous firms. They call the second channel the supply side effects of monetary policy, because monetary easing raises aggregate TFP by shifting resources to more efficient firms. Our model also has the supply side effects of monetary policy, but the mechanism is different; it is through the entry of firms rather than through the misallocation channel. By increasing the number of firms, monetary easing lowers the markup rate through the Second law, thereby reducing the positive impact of accommodative monetary policy on the inflation rate. Table 1 highlights the differences among these two and our studies. We use HSA instead of the Kimball (1995) aggregator, because it is more analytically tractable. For example, it is easier to ensure the existence and the uniqueness of equilibrium with endogenous entry.

Our paper is also related to monopolistic competition models with entry and exit. Bilbiie, Ghironi and Melitz (2012, 2019) construct the flexible price model while Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014) set up the sticky price model. Bilbiie, Ghironi and Melitz (2012) shows that the endogenous entry models can account for business cycles as well as the standard RBC models. Bilbiie, Fujiwara and Ghironi (2014) and Bilbiie, Ghironi and Melitz (2019) characterize the optimal policy under entry and exit. Although they offer some robustness checks using a few parametric non-CES demand systems including Translog, the endogeneity of markup and pass-through rates does not play the central role

<sup>6</sup>A few early studies hint the possible structural relationship between competition and the Phillips curve. Means (1936) states that “to a major extent, technology and economic concentration have brought a change in the demand curve faced by the individual producer. ... In the industries dominated by a few big competitors, administered prices would undoubtedly appear and though there might be sufficient competition to make them reasonably fair, they would be likely to be inflexible.” Carlton (1986) states, “The level of industry concentration is strongly correlated with rigid prices. The more concentrated the industry, the longer is the average spell of price rigidity.”



in these four studies. In contrast, our study reveals crucial roles of the endogeneity of markup and pass-through rates with endogenous entry using nonparametric HSA.

Our paper also contributes to the debate on the equivalence/nonequivalence between Calvo and Rotemberg pricing. Under CES, both result in a common formulation of NKPC to the first order approximation, as shown by [Roberts \(1995\)](#). [Nistico \(2007\)](#) shows that these two price settings yield the same welfare losses to a second order of approximation around the efficient steady state, while [Lombardo and Vestin \(2008\)](#) show that they may entail different welfare costs at higher order approximation around the inefficient steady state. [Ascari and Rossi \(2012\)](#) provides that the long-run Phillips curve, *i.e.*, the long-run relationship between inflation and output, and the dynamics in the presence of the trend inflation are different between these two price settings. Some use the equivalence between Calvo and Rotemberg to argue in favor of Rotemberg because of its analytical simplicity and state dependency, while others use it to argue in favor of Calvo because of its consistency with micro evidence of the size and frequency of individual price changes. Our paper demonstrates that the equivalence between Calvo and Rotemberg fails to hold even as the first order approximation under HSA regarding how market concentration affects the slope of the Phillips curve, which suggests both Calvo and Rotemberg offer valuable insights.

The core ingredient of this paper is HSA. Since HSA was proposed by [Matsuyama and Ushchev \(2017\)](#), a growing number of studies have been applying it to explain a variety of issues. [Matsuyama and Ushchev \(2020b\)](#) investigate how the condition for procompetitive vs. anticompetitive entry – the markup rate either goes down or goes up as more firms enter – is related to the condition for excessive vs. insufficient entry. In order to understand how competitive pressures affect selection and sorting of firms with heterogeneous productivity, [Matsuyama and Ushchev \(2023b\)](#) extend [Melitz \(2003\)](#) model to incorporate the endogenous markup embedded in the HSA demand system. [Matsuyama and Ushchev \(2022\)](#) extend [Judd \(1985\)](#) model of endogenous innovation cycles by replacing CES with HSA to show how an increase in market size has the destabilizing effects on the dynamics of innovation. [Grossman, Helpman and Lhuillier \(2021\)](#) discuss whether the policy should promote diversification or re-shoring in the face of insecure supply chains and seek for the policy instruments to achieve efficient supply chains under the Second law. [Baqae et al. \(2023\)](#) study how an increase in market size, say, due to globalization, affects welfare in a monopolistically competition model with heterogeneous markups and endogenous entry. [Trottner \(2023\)](#) extends this line of inquiry to the two-sided market-power by using HSA both in the product and labor markets. [Kasahara and Sugita \(2020\)](#) propose a nonparametric estimation approach of the HSA demand system. Our study is the first to apply the HSA demand system to the New Keynesian model, the workhorse model for policy simulations and forecasting in central banks and international organizations.

## 1.2 Layout

The structure of this article is as follows. Section 2 formally defines the HSA demand system and explains its key properties. Section 3 introduces the New Keynesian model with [Rotemberg \(1982\)](#) pricing under the HSA demand system. Section 4 discusses the relationship between competition and the Phillips curve. Sections 5 and 6 provide the simulation analysis using parametric families of HSA: CES, Translog and Co-PaTh.<sup>7</sup> Section 5 shows how higher entry cost leads to concentration and the flattening of the Phillips curve, while Section 6 investigates impulse responses to technology and monetary policy shocks by different pass-through rate and by different entry cost. Section 7 studies the relationship between competition and the Phillips curve under [Calvo \(1983\)](#) pricing. There, we show simulation results using another parametric family of HSA called PEM.<sup>8</sup> Section 8 shows that the endogenous cost-push shock leads to the negative omitted variables bias, which is amplified with more concentration. Section 9 concludes. All technical materials are in Appendices.

## 2 HSA

This section explains the HSA demand system, originally proposed by [Matsuyama and Ushchev \(2017\)](#), which is in turn restricted by [Matsuyama and Ushchev \(2020b, 2022, 2023b\)](#) to a continuum of varieties ( $\omega \in \Omega_t$ ), gross substitutes, and symmetry to be applied for monopolistic competition.

Consider the single final good, which is produced competitively by assembling differentiated intermediate inputs with the constant-returns-to-scale (CRS) technology, characterized by the unit cost function  $P(\mathbf{p}_t)$ , where  $\mathbf{p}_t$  is the vector of intermediate inputs prices. We call the demand system for intermediate inputs by the final goods producers HSA if the market share for the input variety  $\omega$  depends solely on its single *normalized* price, *i.e.*, its own price divided by the single price aggregator  $A(\mathbf{p}_t)$ , which captures all the cross-price effects in the demand system:

$$\frac{\partial \ln(P(\mathbf{p}_t))}{\partial \ln(p_t(\omega))} = \frac{p_t(\omega) c_t(\omega)}{P(\mathbf{p}_t) C_t} = s\left(\frac{p_t(\omega)}{A(\mathbf{p}_t)}\right), \quad (1)$$

where

$$\int_{\Omega_t} s\left(\frac{p_t(\omega)}{A(\mathbf{p}_t)}\right) d\omega \equiv 1. \quad (2)$$

$s : \mathbb{R}_{++} \rightarrow \mathbb{R}_+$  is the market share function, decreasing in the normalized price  $z_t(\omega) := p_t(\omega) / A(\mathbf{p}_t)$  for  $s(z) > 0$  with  $\lim_{z \rightarrow \bar{z}} s(z) = 0$ . If  $\bar{z} := \inf\{z > 0 | s(z) = 0\} < \infty$ ,  $\bar{z}A(\mathbf{p}_t)$  is the choke price. Because varieties are gross substitutes, the market share is decreasing in  $z$ :  $s'(z) < 0$ .<sup>9</sup>

<sup>7</sup>Co-PaTh, proposed by [Matsuyama and Ushchev \(2020a\)](#), stands for Constant Pass-Through. It satisfies the Second law and the Third law in the weak form, and contains CES as a limit case.

<sup>8</sup>PEM, proposed by [Matsuyama and Ushchev \(2023b\)](#), stands for Power Elasticity of Markup rate. It satisfies the Third law in the strong form and contains Co-PaTh as a limit case, which in turn contains CES as a limit case.

<sup>9</sup>Proposition 1 in [Matsuyama and Ushchev \(2017\)](#) proves the existence of well-defined CRS production technolo-

The single price aggregator  $A_t = A(\mathbf{p}_t)$  is implicitly defined by the adding-up constraint in equation (2). By construction, market shares add up to one. As is evident from equation (2),  $A(\mathbf{p}_t)$  is linear homogenous in  $\mathbf{p}_t$  for fixed  $\Omega_t$ . A larger  $\Omega_t$ , namely, more variety, reduces  $A(\mathbf{p}_t)$ .

By integrating equation (1), one can show that  $A(\mathbf{p}_t)$  and  $P(\mathbf{p}_t)$  are related as follows:

$$\frac{P(\mathbf{p}_t)}{A(\mathbf{p}_t)} = \exp \left\{ \bar{K} - \int_{\Omega_t} \left[ \int_{\frac{p_t(\omega)}{A(\mathbf{p}_t)}}^{\bar{z}} \frac{s(\xi)}{\xi} d\xi \right] d\omega \right\}, \quad (3)$$

where  $\bar{K}$  is a constant.<sup>10</sup>

$A(\mathbf{p}_t)$  is the inverse measure of *competitive pressures* and fully captures the cross-price effects in the demand system. It is different from the unit cost function of the final goods  $P_t = P(\mathbf{p}_t)$ , which is the inverse measure of TFP and captures the *productivity consequences* of price changes. In general, they do not move together:  $A(\mathbf{p}_t) \neq \text{constant} \times P(\mathbf{p}_t)$ . This can be seen by differentiating the adding-up constraint in equation (2):

$$\frac{\partial \ln(A(\mathbf{p}_t))}{\partial \ln(p_t(\omega))} = \frac{\frac{p_t(\omega)}{A(\mathbf{p}_t)} s' \left( \frac{p_t(\omega)}{A(\mathbf{p}_t)} \right)}{\int_{\Omega_t} \frac{p_t(\omega')}{A(\mathbf{p}_t)} s' \left( \frac{p_t(\omega')}{A(\mathbf{p}_t)} \right) d\omega'} \neq s \left( \frac{p_t(\omega)}{A(\mathbf{p}_t)} \right) = \frac{\partial \ln(P(\mathbf{p}_t))}{\partial \ln(p_t(\omega))},$$

which shows that  $A(\mathbf{p}_t) \neq \text{constant} \times P(\mathbf{p}_t)$  unless  $zs'(z)/s(z) < 0$  is constant, in other words,  $s(z)$  is a decreasing power function. This corresponds to CES because plugging  $s(z) = \gamma_{CES} z^{1-\theta}$ , where  $\theta > 1$ , into equations (1) to (3) can verify

$$C_t = Z_C \left[ \int_{\Omega_t} c_t(\omega)^{1-\frac{1}{\theta}} d\omega \right]^{\frac{\theta}{\theta-1}}, \quad (4)$$

$$c_t(\omega) = Z_C^{\theta-1} \left( \frac{p_t(\omega)}{P(\mathbf{p}_t)} \right)^{-\theta} C_t,$$

$$Z_C^{\theta-1} \left( \frac{p_t(\omega)}{P(\mathbf{p}_t)} \right)^{1-\theta} = \frac{p_t(\omega) c_t(\omega)}{P(\mathbf{p}_t) C_t} = s \left( \frac{p_t(\omega)}{A(\mathbf{p}_t)} \right) = \gamma_{CES} \left( \frac{p_t(\omega)}{A(\mathbf{p}_t)} \right)^{1-\theta},$$

where  $Z_C$  is the TFP of the final goods production, and related to  $\bar{K}$  and  $\gamma_{CES}$  as follows:

$$\frac{P(\mathbf{p}_t)}{A(\mathbf{p}_t)} = \exp \left( \bar{K} - \frac{1}{\theta-1} \right) = \frac{\gamma_{CES}^{\frac{1}{\theta-1}}}{Z_C}. \quad (5)$$

**Three price indices** We consider endogenous entry (variety). Even under symmetric situation, individual price is not equal to the aggregate price due to entry effects. In addition, we have two aggregate prices: the final goods price and the single price aggregate. As a result, there are three aggregate prices.

gies that rationalize the HSA demand system as described above.

<sup>10</sup>For the detailed derivation of equation (3), see Matsuyama and Ushchev (2017).

The first one is the final goods price or CPI  $P_t$ , which captures the productivity effects of entry, and is the reference price for consumers, implicitly given by

$$\int_{\Omega_t} \frac{p_t(\omega) c_t(\omega)}{P_t C_t} d\omega \equiv 1.$$

The second is the single price aggregate  $A_t$ , which captures the competitive effects of entry, and is the reference price for firms, implicitly given by

$$\int_{\Omega_t} s \left( \frac{p_t(\omega)}{A_t} \right) d\omega \equiv 1.$$

The third one is the average price index or PPI  $p_t$ :

$$p_t = \int_{\Omega_t} s \left( \frac{p_t(\omega)}{A_t} \right) p_t(\omega) d\omega.$$

Unlike  $P_t$  and  $A_t$ , the average price index  $p_t$  is not affected by entry effects and therefore is the measured price index. Thus, in what follows, we consider the inflation rate in terms of  $p_t$  when evaluating the responsiveness of inflation rates to macroeconomics variables in NKPC.

### 3 New Keynesian model under HSA

Our model is an extension of those in [Bilbiie, Ghironi and Melitz \(2008\)](#) and [Bilbiie, Fujiwara and Ghironi \(2014\)](#) to incorporate the HSA demand system. We consider a closed economy populated by four agents: household, final goods producer, intermediate inputs producer, and the central bank. Intermediate inputs producers are under monopolistic competition and set prices subject to nominal rigidities. They are also subject to endogenous entry but exogenous exit.

**Timing** Time is discrete:  $t = 0, 1, 2, 3, \dots$  There is an unbounded mass of potential entrants in every period. They are subject to one-period time-to-build lag. That is, entrants at time  $t$  only start producing at time  $t + 1$ . Entry is determined endogenously by the free entry condition, but exit is exogenous. All firms face the same probability  $\delta$  of exogenous firm destruction at the end of each period, after production and entry. As a result, a proportion  $\delta$  of new entrants never produces.

#### 3.1 Household

A representative household chooses consumption  $C_t$ , labor supply  $L_t$ , the nominal bond  $B_t$ , and the equity of intermediate inputs producer  $x_t$ , in order to maximize welfare:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (u(C_t) - v(L_t)), \quad (6)$$

subject to the budget constraint:

$$\frac{B_{t+1}}{P_t} + x_{t+1} \int_{\Omega_t + \Omega_{E,t}} \frac{V_t(\omega)}{P_t} d\omega + C_t = (1 + i_{t-1}) \frac{B_t}{P_t} + x_t \int_{\Omega_t} \frac{D_t(\omega) + V_t(\omega)}{P_t} d\omega + \frac{W_t}{P_t} L_t. \quad (7)$$

where  $\beta$  and  $\delta$  denote the subjective discount factor and the probability of exogenous termination.  $W_t$  and  $i_t$  denote the nominal wage and the nominal interest rate.  $V_t(\omega)$  and  $D_t(\omega)$  denote the equity price (the value) and the profit of the intermediate inputs producer  $\omega \in \Omega_t$ .

### 3.2 Final goods producer

Final goods producers are under perfect competition and produce the final good by assembling intermediate inputs using the CRS technology that generates the HSA demand system in equations (1) and (2).

### 3.3 Intermediate inputs producer

Intermediate inputs producer  $\omega$  uses labor  $l_t(\omega)$  to produce its output  $y_t(\omega)$  with the linear technology:

$$y_t(\omega) = Z_{p,t} l_t(\omega), \quad (8)$$

where  $Z_{p,t}$  is the common technology in production. Intermediate inputs producer  $\omega$  chooses the price path  $p_t(\omega)$  to maximize the value of the firm:

$$\frac{V_t(\omega)}{P_t} = \mathbb{E}_t \sum_{i=1}^{\infty} \beta (1 - \delta) \frac{u'(C_{t+i})}{u'(C_t)} \left( \frac{D_{t+i}(\omega)}{P_{t+i}} \right), \quad (9)$$

subject to the HSA demand curve in equation (1):

$$y_t(\omega) = c_t(\omega) = s \left( \frac{p_t(\omega)}{A_t} \right) \frac{P_t C_t}{p_t(\omega)}, \quad (10)$$

and the Rotemberg (1982) quadratic price adjustment cost:<sup>11</sup>

$$\frac{D_t(\omega)}{P_t} = \frac{p_t(\omega)}{P_t} y_t(\omega) - \frac{W_t}{P_t} l_t(\omega) - \frac{\chi}{2} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 \frac{p_t(\omega)}{P_t} y_t(\omega), \quad (11)$$

where  $\chi$  scales the size of the cost.

In Sections 3 to 6, we use Rotemberg (1982) pricing to highlight the role of the Second law, but we will also consider Calvo (1983) pricing in Section 7 to highlight the role of the Third law.

In this model, we depart from CES only in the direction of the Second law. As shown in Proposition 1 in Matsuyama and Ushchev (2020b), this guarantees that the equilibrium is

<sup>11</sup>For simplicity, we follow Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014) to assume that new entrants also incur Rotemberg adjustment costs based on the prices set by incumbents in the previous period. In Appendix B, we derive the NKPC under the scenario where entrants have the flexibility to set prices. Main results carry over under this alternative scenario.

unique and symmetric.

### 3.4 Aggregate conditions and others

**Monetary policy** The central bank sets the nominal interest rate following the simple feedback rule reacting to the PPI inflation rate:

$$(1 + i_t) = (1 + i_{t-1})^{\alpha_i} \left( \frac{p_t}{p_{t-1}} - 1 \right)^{(1-\alpha_i)\alpha_\pi} u_t, \quad (12)$$

where  $\alpha_\pi > 1$  to satisfy the Taylor principle.  $\alpha_i$  represents the policy inertia.  $u_t$  denotes the monetary policy shock.

**Free entry** Entrants pay the entry cost  $W_t/Z_{E,t}f_{E,t}$ , where  $Z_{E,t}$  and  $f_{E,t}$  are the common technology to facilitate entry and the labor demand for entry purpose, respectively. Since the equilibrium is symmetric, entry occurs until the firm value becomes equal to the entry cost, resulting in the free entry condition:

$$\frac{W_t}{P_t} \frac{f_{E,t}}{Z_{E,t}} = \frac{V_t}{P_t}. \quad (13)$$

#### Firm dynamics

$$N_t = (1 - \delta) (N_{t-1} + N_{E,t-1}). \quad (14)$$

$N_t$  is the number of firms (varieties) and the mass of  $\Omega_t$ .  $N_{E,t}$  is the number of entrants and the mass of  $\Omega_{E,t}$ .

**Market clearing** Financial market clearing conditions for the nominal bond and the equity are given by  $B_t = 0$  and  $x_t = 1$ , respectively.

The labor market clearing condition is given by

$$L_t = N_t l_t + N_{E,t} \frac{f_{E,t}}{Z_{E,t}}. \quad (15)$$

The left hand side is the supply of the labor. The first and second terms on the right hand side are the labor demands for production and entry, respectively.

Inserting equation (14) into the budget constraint in equation (7) leads to

$$\frac{V_t}{P_t} N_{E,t} + C_t = \frac{D_t}{P_t} N_t + \frac{W_t}{P_t} L_t.$$

Inserting equations (11), (13) and (15) into the above aggregate accounting equation yields the resource constraint:

$$C_t = \left[ 1 - \frac{\chi}{2} (\pi_t - 1)^2 \right] N_t \frac{p_t}{P_t} y_t = \left[ 1 - \frac{\chi}{2} (\pi_t - 1)^2 \right] Y_t.$$

Output is either consumed or used for the price adjustment costs.

**HSA demand system** As the equilibrium is symmetric, the market share is simply expressed by the inverse of the number of firms:

$$s\left(\frac{p_t}{A_t}\right) = \frac{1}{N_t}. \quad (16)$$

The equation for the reference prices for households and intermediate inputs producers in equation (3) simplifies to

$$\ln\left(\frac{P_t}{A_t}\right) = \bar{K} - \frac{1}{s\left(\frac{p_t}{A_t}\right)} \int_{\Omega_t} \left[ \int_{\frac{p_t(\omega)}{A_t}}^{\bar{z}} \frac{s(\xi)}{\xi} d\xi \right] d\omega, \quad (17)$$

where the second term on RHS captures the love-for-variety effect in the HSA demand system.<sup>12</sup>

### 3.5 Equilibrium

**Definition.** An *equilibrium* in this economy is a collection of sequence of aggregate prices  $\{P_t, A_t, W_t, i_t\}$  and the price of intermediate goods  $\{p_t\}$ ; a collection of sequences of aggregate quantities  $\{Y_t, C_t, L_t\}$  and quantities of intermediate goods  $\{y_t, l_t\}$ ; and a collection of sequences of firm-value functions and profit  $\{V_t, D_t\}$  together with measures of operating firms and entering firms  $\{N_t, N_{E,t}\}$ . These equilibrium objects satisfy the following conditions: households maximize their utility subject to their budget constraints, intermediate-good firms maximize the net present value of their per-period profits, final-good firms maximize profits, all of the feasibility constraints are satisfied.

**Preference** As is customary, we impose the following functional forms to ensure the existence of the steady state:

$$u(C_t) := \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \quad v(L_t) := \frac{L_t^{1+\psi}}{1+\psi}, \quad (18)$$

where  $\sigma$  and  $\psi$  denote the relative risk aversion (inverse of the intertemporal elasticity of substitution) and the inverse of Frisch elasticity, respectively.

In contrast, no specific functional forms for the market share function in the HSA demand system in equation (1) are not necessary to ensure the steady state. Therefore, we will show analytical results without imposing any specific functional forms in Section 4. In numerical analysis provided in Sections 5, 6 and 7, parametric families of the HSA demand system are examined.

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<sup>12</sup>Note that as already shown in equation (5), under CES,  $s(z) = \gamma_{CES} z^{1-\theta}$ , where  $\theta > 1$ , this term is equal to  $1/(\theta - 1)$ .

**De-trending** We de-trend prices by normalizing as follows:

$$w_t := \frac{W_t}{P_t}, d_t := \frac{D_t}{P_t}, v_t := \frac{V_t}{P_t}, z_t := \frac{p_t}{A_t}, \bar{p}_t := \frac{p_t}{P_t}, \pi_t := \frac{p_t}{p_{t-1}}.$$

Notice that both  $P_t$  and  $A_t$ , *i.e.*, the reference prices for households and firms, include entry effects. Since entry effects are not included in the official price statistics, the inflation rate is measured by PPI  $p_t$  throughout this paper. Since the equilibrium is symmetric under Rotemberg pricing, this is equal to the individual prices of intermediate inputs producers.

**System of equations** Appendix A displays the system of nonlinear as well as log-linearly approximated equations in addition to the steady state conditions.

## 4 Competition and the Phillips curve

Now, we are ready to derive NKPC under HSA. By solving the intermediate inputs producers' problem, which is to maximize the value in equation (9) subject to equations (11) to (10),

$$\hat{\pi}_t = \beta(1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{1}{\chi} \left[ -\frac{s'(z)z}{s(z)} \right] (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) + \frac{1}{\chi} \left[ \frac{s'(z)z}{s(z)} - \frac{\frac{s''(z)z}{s'(z)}}{1 - \frac{s(z)}{s'(z)z}} \right] \hat{z}_t, \quad (19)$$

where  $\hat{x}_t := \ln(x_t/x)$  denotes the percentage deviation of  $x_t$  from the steady state value  $x$ .  $\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t$  is the inverse of the markup rate, and, therefore comparable to the real marginal cost measure in the canonical New Keynesian model.<sup>13</sup> From equation (16),  $\hat{z}_t$  can be shown to be positively correlated with  $\hat{N}_t$  as follows:

$$\hat{N}_t = -\overbrace{\frac{s'(z)z}{s(z)}}^{\ominus} \hat{z}_t \quad (20)$$

Then, using this equation, we can further simplify equation (19) to obtain NKPC under HSA:

$$\hat{\pi}_t = \beta(1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \overbrace{\frac{\zeta(z) - 1}{\chi}}^{\text{steady-state effect}} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \overbrace{\frac{1 - \rho(z)}{\chi \rho(z)}}^{\text{dynamic effect}} \hat{N}_t, \quad (21)$$

where

$$\zeta(z) := \frac{\partial \ln(c_t(\omega))}{\partial \ln(p_t)} = 1 - \frac{s'(z)z}{s(z)} > 1, \quad (22)$$

<sup>13</sup>Equation (19) expresses NKPC in terms of the real marginal cost measure. This is because, due to one-period time-to-build lag, there is an endogenous state variable in this model, the number of firms  $\hat{N}_t$ , hence, the real marginal cost term cannot be replaced simply by the contemporaneous output gap. In Appendix C, we consider an alternative specification with entry without one-period time-to-build lag, and derive NKPC in terms of the output gap. Even though the resulting expression is substantially more complicated than equation (19), the main implications do not change.



is the price elasticity function (hence, the markup rate under the flexible price would be  $\mu^f(z) = \zeta(z) / (\zeta(z) - 1)$ ), and

$$\rho(z) := \frac{\partial \ln(p_t)}{\partial \ln(W_t/Z_{P,t})} = \left[ 1 - \frac{d \ln \left( \frac{\zeta(z)}{\zeta(z)-1} \right)}{d \ln(z)} \right]^{-1}, \quad (23)$$

is the pass-through rate that prevails under the flexible price.<sup>14</sup> We call hereafter  $\rho(z)$  the pass-through rate function.

Equation (21) contains the standard textbook NKPC under CES, such as [Gali \(2015\)](#), [Walsh \(2010\)](#) and [Woodford \(2003\)](#) as a special case. It is obtained by setting  $s(z_t) = \gamma_{CES}(z_t)^{1-\theta}$ , where  $\theta$  denotes the constant price elasticity, and  $\zeta(z) = \theta$  and  $\rho(z) = 1$ :

$$\hat{\pi}_t = \beta(1-\delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\theta-1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t).$$

As is well-known, competition does not affect the Phillips curve even with endogenous entry under CES. Market concentration neither changes the slope of the Phillips curve nor exerts any direct impact on the inflation rate. The constant elasticity makes the competitive environment irrelevant to price dynamics.

By extending NKPC from CES to HSA, equation (21) introduces two channels through which competition affects price dynamics. First, the slope of the Phillips curve is no longer constant and determined by the steady state conditions. Thus, changes in the competitive environment will affect the slope of the Phillips curve. We call this *the steady-state effect of concentration*. Second, endogenous changes in  $N_t$  by causing fluctuations in  $z_t$ , affect the inflation rate directly in NKPC, which acts like the endogenous cost-push shock. We call this term in equation (21) *the dynamic effect of endogenous entry*.

Even though HSA endogenizes the price elasticity, the reason why the slope of NKPC in equation (21) depends on the price elasticity is the same with the textbook NKPC under CES. Recall that under CES, the optimal price setting condition is given by

$$(1-\theta) + \theta mc_t - \chi(\pi_t - 1)^2 \pi_t + \mathbb{E}_t \beta(1-\delta) \frac{u'(C_{t+1})}{u'(C_t)} \chi(\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0,$$

where  $mc_t$  denotes the real marginal cost. In response to price increase, the second term shows how much costs decline, which is proportional to the price elasticity, while the third and fourth terms represent losses and gains from price adjustments, which are expressed by inflation rates. Consequently, the higher the the price elasticity, the more the demand declines and the higher the impact of the marginal cost on inflation rates becomes. In other words, with an exogenous

<sup>14</sup>The pass-through rate is closely related to the super-elasticity coined by [Klenow and Willis \(2016\)](#). In Appendix E, NKPC is expressed in terms of the super-elasticity. Building on the insights of [Baqaee, Farhi and Sangani \(2021\)](#) and [Auclert, Rigato, Rognlie and Straub \(2022\)](#), that highlight the usefulness of the pass-through rate in the NKPC framework, we find that expressing the NKPC in terms of the pass-through rate provides a more intuitive understanding of how competition influences the NKPC.

change in the real marginal cost, the incentive to set the price closer to the target level (*i.e.*, the price prevailing under the flexible price) becomes stronger with higher price elasticity. This motive stems from the state dependency in Rotemberg (1982) pricing and therefore is absent in Calvo (1983) pricing.

#### 4.1 Implications of the Second law

Under the Second law,  $\zeta'(z) > 0$  and hence the pass-through is incomplete:  $\rho(z) < 1$ . Therefore, higher cost implies lower markup rates.

More concentration means higher market share  $s(z)$  and lower  $z$ . The Second law,  $\zeta'(z) > 0$ , therefore causes a smaller slope coefficient  $(\zeta(z) - 1) / \chi$  in NKPC as equation (21) shows.

Also, the Second law yields the endogenous cost-push shock, *i.e.*, the dynamics effect of endogenous entry. From equation (23), the markup rate under the flexible price is not constant:

$$\hat{\mu}_t^f = -\frac{1 - \rho(z)}{\rho(z)} \hat{z}_t = -\frac{1 - \rho(z)}{\rho(z)} (\hat{p}_t - \hat{A}_t). \quad (24)$$

The firm reduces its price and the markup rate in response to more competitive pressures, *i.e.*, a lower  $A_t$ , when other firms reduce their prices. This is indeed strategic complementarity in price setting under HSA due to the Second law.

This endogenous cost-push shock has an additional implication to aggregate fluctuations. that If  $\hat{\mu}_t^f = -(\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t)$  and  $\hat{N}_t$  move to the opposite directions (same direction) to a structural shock, its impact on inflation rates is muted (amplified). As the analyses in Section 6 reveal, for example, to a positive technology shock, the number of firms  $\hat{N}_t$  increases through the wealth effect, but the strategic complementarity in price setting reduces the markup rate  $\hat{\mu}_t$  as shown in equation (24). As a result, the dynamic effect of endogenous entry weakens the responses of inflation rates to real economic variables.

This supply side effects of monetary policy resemble those in Baqaee et al. (2021), but our mechanism is different. In their model, monetary easing weakens the price-raising effects of monetary policy by shifting resources to more efficient firms through the misallocation channel. In our model, on the other hand, monetary easing weakens the price-raising effects by increasing the number of firms and lowers the markup rate with the Second law.

#### 4.2 Cyclicity of the markup rate

Table 2 summarizes the cyclicity of the markup rate to the positive technology shock. In a flexible price equilibrium under CES, the markup rate is constant. In a sticky price equilibrium under CES, the markup rate becomes procyclical. This is because the marginal cost decreases but the price does not change, at least, for a short-run. In a flexible price equilibrium under the Second law, the markup rate becomes countercyclical because a positive technology shock increases the number of firms (varieties), which causes the markup rate to decline.

Table 2: Cyclicity of the markup rate to the technology shock

	Flexible price	Sticky price
CES	constant	procyclical
the Second law	countercyclical	procyclical / countercyclical

Table 3: Parametric families of HSA

	Market share	Price elasticity	Pass-through
CES	$s(z) = \gamma_{CES} z^{1-\theta}$	$\zeta(z) = \theta$	$\rho(z) = 1$
Translog	$s(z) = \gamma_{TL} \ln\left(\frac{z}{\bar{z}}\right)$	$\zeta(z) = 1 + \frac{1}{\ln\left(\frac{z}{\bar{z}}\right)}$	$\rho(z) = \frac{1 + \ln\left(\frac{z}{\bar{z}}\right)}{2 + \ln\left(\frac{z}{\bar{z}}\right)}$
Co-PaTh	$s(z) = \gamma_{CP} \theta^{\frac{\rho}{1-\rho}} \left[1 - \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}$	$\zeta(z) = \frac{1}{1 - \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}}$	$\rho(z) = \rho < 1$

This explains why the cyclicity of the markup rate in a sticky price equilibrium under the Second and the Third laws is generally ambiguous and depends on the tension between nominal rigidities and the pass-through rate. This can be seen by rewriting equation (21) in terms of the markup rate under the sticky price:

$$\hat{\mu}_t = \frac{1}{\zeta(z) - 1} \left\{ \overbrace{\chi [\beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} - \pi_t]}^{\text{nominal rigidity}} \overbrace{- \frac{1 - \rho(z)}{\rho(z)} \hat{N}_t}^{\text{incomplete pass-through}} \right\}. \quad (25)$$

As the pass-through rate becomes larger (smaller) and/or prices become stickier (more flexible), the markup rate becomes more procyclical (countercyclical). This could explain the disagreement about the cyclicity of the markup in the literature as surveyed by [Nekarda and Ramey \(2020\)](#).

## 5 Steady state analysis

In the next two sections, we simulate the New Keynesian model under HSA. For this purpose, we use three parametric families of HSA: CES, Translog and Co-PaTh. Co-PaTh is a parametric family of the HSA demand system proposed by [Matsuyama and Ushchev \(2020a\)](#). This parametric family is characterized by the property that under the flexible price equilibrium, the pass-through rate is given by a constant parameter between zero and unity. Table 3 shows the market share function, the price elasticity, and the pass-through rate for each family. We set  $\bar{z} = (\theta / (\theta - 1))^{\rho / (1 - \rho)}$  so that as  $\rho \rightarrow 1$ , Co-PaTh converges to CES. Even though both Translog and Co-PaTh have the choke price, the inequality constraint that the price must be lower than the choke price need not be taken into account. This is because firms are symmetric; The demand for all firms, *i.e.*, the aggregate demand, can never be zero in equilibrium.

Under CES, the price elasticity is constant and, hence, the markup rate is constant and the

Table 4: Calibrated parameters

parameter	definition	value
$\beta$	subjective discount factor	0.99
$\sigma$	relative risk aversion	1
$\delta$	exit rate	0.025
$\psi$	inverse of labor supply elasticity	1
$f_E, Z_E, Z_P$	technologies	1
$\theta$	price elasticity under CES	3.8
$\chi$	Rotemberg adj. cost	77
$\alpha_i$	policy inertia	0.9
$\alpha_\pi$	policy reaction to $\pi$	1.1 or $\infty$
$\rho$	pass-through rate	1, 0.9 or 0.5

pass-through rate is unity. Under both Translog and Co-PaTh, the price elasticity is increasing in  $z$ . Thus, both satisfy the Second law.

Parameters are calibrated as in Table 4. Most of them are taken from [Bilbiie, Fujiwara and Ghironi \(2014\)](#). When the Taylor rule coefficient is infinity, namely,  $\alpha_\pi = \infty$ , the model replicates the flexible price equilibrium. We examine several pass-through rates. We set  $\bar{K}$  so that  $p_t/A_t = z_t = \bar{p}_t = p_t/P_t$  under CES. From equation (5), this can be achieved by setting  $\bar{K} = 1/(\theta - 1)$ .<sup>15</sup>  $\gamma_{TL}$  and  $\gamma_{CP}$  in Table 3 are calibrated so that  $\zeta(z) = \theta$  when  $f_E = 1$ .<sup>16</sup>  $\gamma_{TL}$  and  $\gamma_{CP}$  affect  $z$  in the steady state.

In Figure 1, we replicate higher market concentration, and the rising profit margins and markups reported by [Covarrubias, Gutierrez and Philippon \(2019\)](#), [De Loecker, Eeckhout and Unger \(2020\)](#), and [Autor, Dorn, Katz, Patterson and Reenen \(2020\)](#). The horizontal axis displays the entry cost; the further to the right, the greater the barriers to entry. The green line corresponds to CES, the red line to Translog, and the blue lines to Co-PaTh, when  $\rho = 0.9$  (dotted) and  $\rho = 0.5$  (dashed), respectively. In all cases, an increase in the entry cost leads to market concentration (decline in the number of firms). Under Translog and Co-PaTh, this also leads to higher markup rates and profits.<sup>17</sup> The Second law embedded in Translog and Co-PaTh enables the model to replicate the stylized facts as observed in the data. Notice also that Translog is similar to Co-PaTh with  $\rho = 0.5$ .

Figure 2 illustrates how market concentration affects the slope of the Phillips curve. The horizontal axis shows the entry cost in the upper panel. There, the greater the barriers to entry, the flatter the Phillips curve. The lower panel displays how the slope of the Phillips curve changes with the number of firms. Since the greater the cost of entry, the fewer the number of firms, the lower panel is just like a mirror image of the upper panel. Fewer firms in the market result in the flattening of the Phillips curve.

<sup>15</sup>This is equivalent to setting  $Z_C = \gamma_{CES}^{1/(\theta-1)}$ .

<sup>16</sup>Price elasticity is given by  $\theta$  under CES and thus determined independently from  $\gamma_{CES}$ . We set  $\gamma_{CES}$  at the same value of  $\gamma_{CP}$  when  $\rho = 0.5$ .

<sup>17</sup>The bottom right panel in Figure 1 shows a nonlinear relationship between the entry cost and the profit in the steady state. This is because the entry cost entails wage payments, as shown in equation (13).

Figure 1: Concentration and macroeconomy

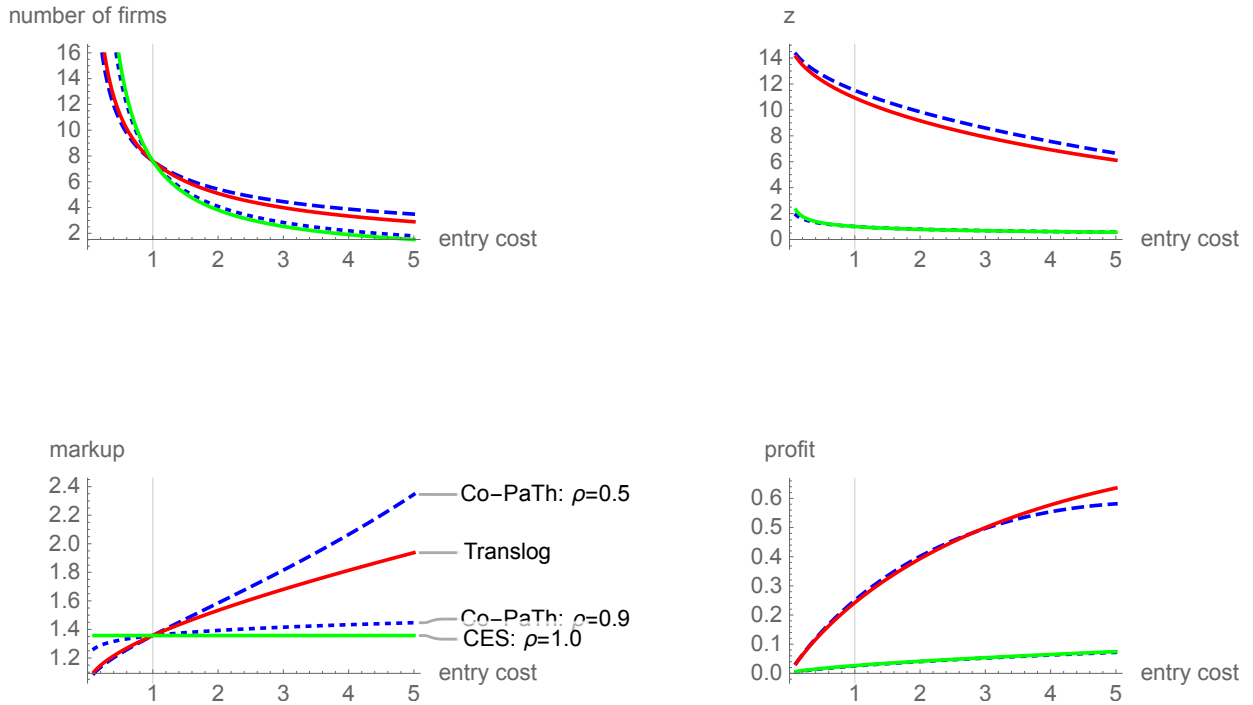
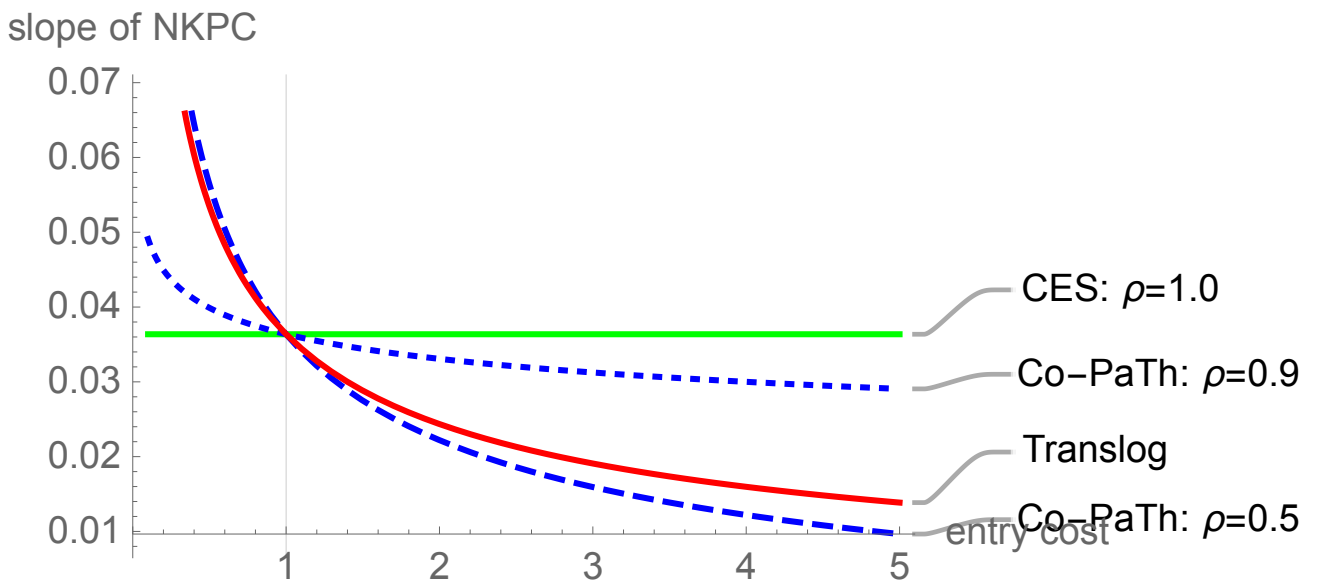


Figure 2: Entry cost, concentration and the slope of the Phillips curve



Fewer firms (higher market share) corresponds to lower  $z$ , as shown in equation (20). Under the Second law,  $\zeta'(z) > 0$ , this leads to lower price elasticity and consequently, the slope of the Phillips curve  $(\zeta(z) - 1) / \chi$  declines with higher market concentration.

The slope of the Phillips declines more to market concentration as the pass-through rate becomes smaller. This is because, as implied in equation (24), the smaller pass-through rate causes the markup rate and therefore the price elasticity to react more strongly to competitive pressures.

De Loecker, Eeckhout and Unger (2020) argue that “aggregate markups start to rise from 21% above marginal cost to 61% now.” This increase in the markup rate of about 40 percentage points suggests that the entry cost would have increased by about 3.5 times in Translog, and by about 2.5 times in Co-PaTh with  $\rho = 0.5$  in Figure 1. According to Figure 2, the accompanying market concentration can halve the slope of the Phillips curve. The Second law is a well-established fact provided by many empirical studies. Thus, it could be said that changes in the competitive environment in recent years have exerted a significant impact on the slope of the Phillips curve.

## 6 Dynamic analysis

This section describes how the economy under the HSA demand system reacts to technology and monetary policy shocks. First, to understand dynamic properties under the Second law, we use Co-PaTh and look at how  $\rho$  affects the responses to the technology shock under both flexible and sticky price. For this purpose, we calibrate parameters so that markup rates are the same at the steady state across preferences,<sup>18</sup> and then compare the impulse responses to technology shocks. Second, to understand the impact of market concentration, we also look at how the entry cost affects the responses to both technology and monetary policy shocks under the sticky price.<sup>19</sup> Under Co-PaTh with  $\rho = 0.5$ , we examine how the entry cost modifies the response of macroeconomic variables to technology and monetary policy shocks.

**Impulse responses to the technology shock by pass-through rate (Figure 3)** The green line corresponds to CES, and the blue lines to Co-PaTh, when  $\rho = 0.9$  (dotted) and  $\rho = 0.5$  (dashed), respectively.<sup>20</sup> The upper panel represents the impulse response under the sticky price model and the lower panel represents the impulse response under the flexible price model.

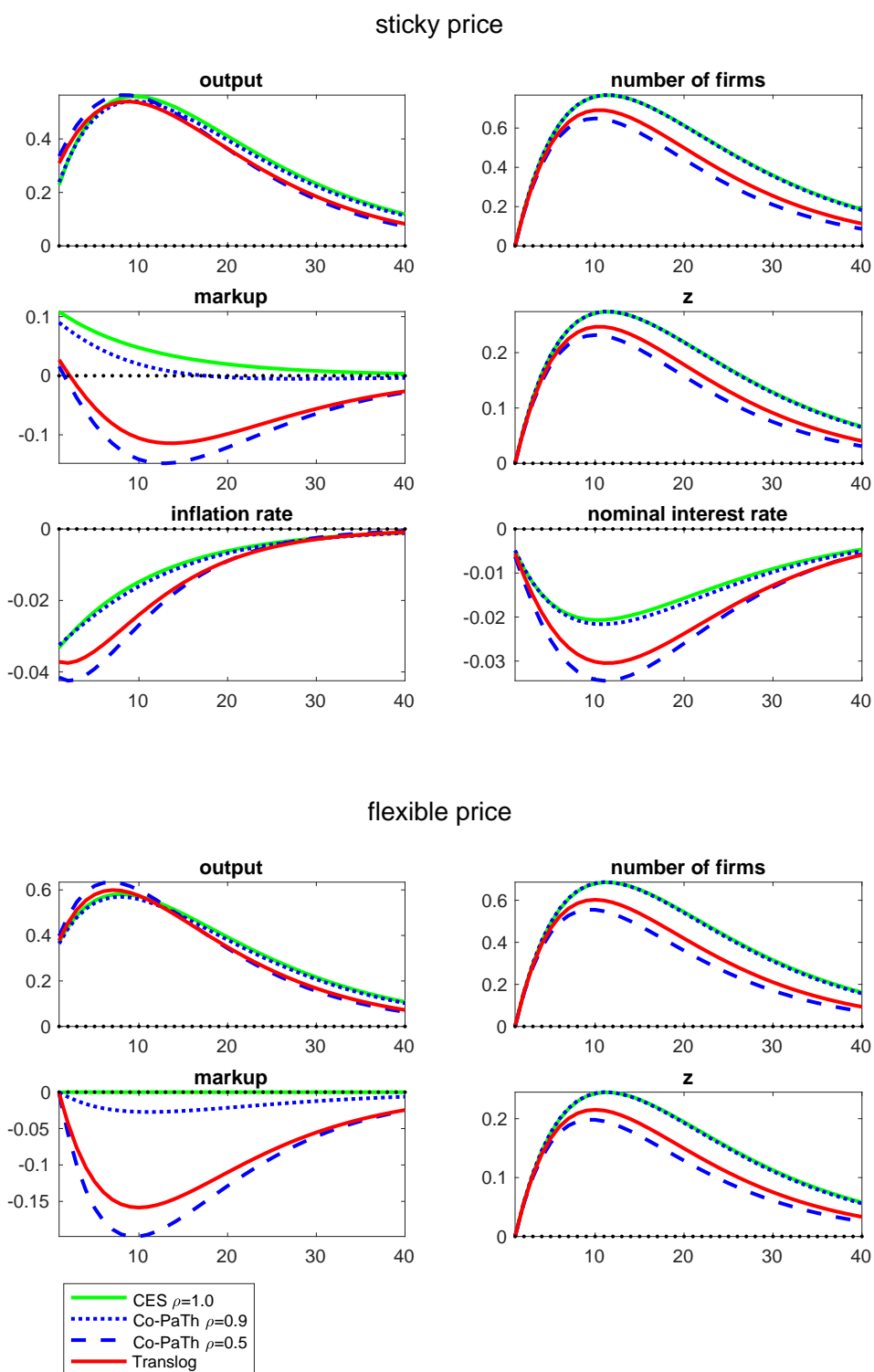
Under both sticky and flexible prices, responses of output and the number of the firms to the technology shocks are qualitatively similar to those obtained in Bilbiie, Ghironi and Melitz (2008, 2012, 2019) and Bilbiie, Fujiwara and Ghironi (2014). A positive technology shock increases consumption and then output by inducing positive wealth effects. Since the shock

<sup>18</sup>This is achieved by calibrating  $\gamma_{TL}$  and  $\gamma_{CP}$  in Table 3 so that  $\zeta(z) = \theta$  in the steady state.

<sup>19</sup>To fix the idea, imagine that we are comparing two countries or two time periods with different entry costs. The aim is to understand how responses to shocks differ under different competitive regimes.

<sup>20</sup>Translog (red line) is similar to Co-PaTh with  $\rho = 0.5$  even in impulse responses.

Figure 3: Impulse responses to the technology shock by pass-through rate



is temporary, it increases savings, *i.e.*, investment in new firms, reflecting the consumption smoothing motive. Then, the number of firms gradually increases. Subsequently, the increase in the number of firms leads to a contraction of market share. As a result, profits become smaller, and the economy returns to its original level.

Now turning to the new features introduced by HSA, let us look at how impulse responses differ by pass-through rate:  $\rho = 0.5, 0.9$ , and 1. Under both sticky and flexible prices, the markup rate declines faster at a lower  $\rho$ . As a result, profits are smaller, and a fewer firms enter after a positive technology shock hits the economy. However, a comparison of the sticky and flexible price cases reveals that the pass-through rate makes a significant difference in the responses of the markup rate. As already pointed out in Section 4.2, in the sticky price and the constant price elasticity, the markup rate is pro-cyclical in response to positive technology shocks. Marginal costs fall from higher efficiency, but prices remain unchanged in the short run, resulting in higher markup rates. On the other hand, in the case of flexible price and the endogenous price elasticity under the Second law, an increase in the number of firms leads to higher price elasticity, resulting in smaller markup rates. This tendency becomes stronger as the pass-through rate becomes smaller. As a result, consistent with equation (25), in the sticky price case, as the pass-through rate becomes smaller, the markup rate moves from procyclical to countercyclical. Cyclicalities of the markup rate depends on the pass-through rate and nominal rigidities. Note also that the dynamic effect of endogenous entry leads to more deflation with smaller pass-through rates, as seen in the panel for inflation rate under sticky price in Figure 3 and implied by equation (21).

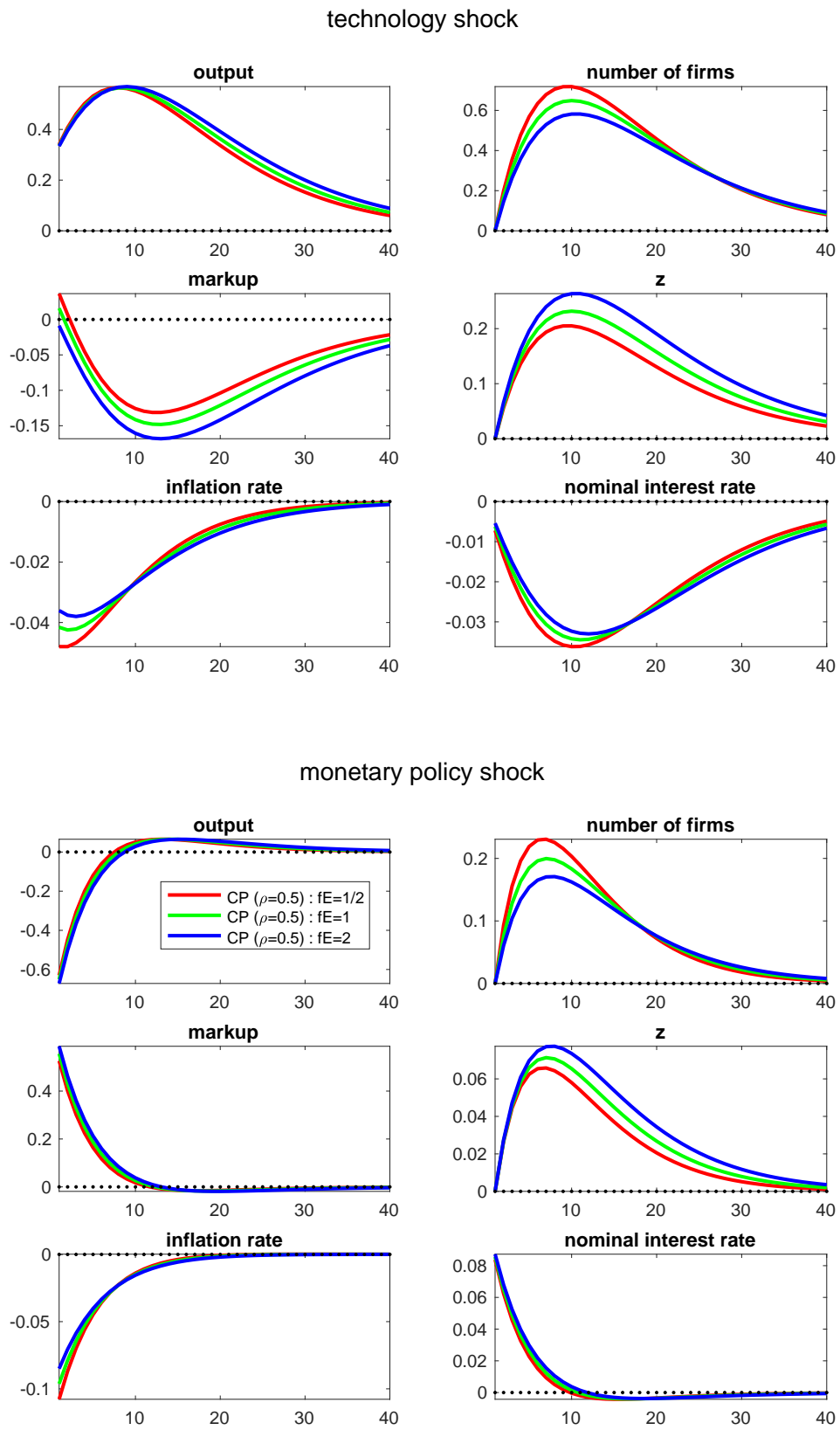
**Impulse responses to technology and monetary policy shocks by entry cost (Figure 4)** Cases shown here are under Co-PaTh with  $\rho = 0.5$  only. The red line indicates the entry cost of 0.5 (half the benchmark), the green line indicates this is 1 (benchmark case), and the blue line indicates this is 2 (twice the benchmark). As the color of the line changes from red to green to blue, the barrier to entry becomes larger.

The responses of the inflation rate to both technology and monetary policy shocks become smaller as the barrier to entry becomes larger. This reflects that the slope of the Phillips curve becomes smaller with higher market concentration, as analyzed in Sections 4 and 5.

The markup rate  $\mu_t$  and  $z_t$  move to the opposite directions to a technology shock. Thus, its impact on inflation is muted by the dynamic effect of endogenous entry. The opposite has occurred for monetary policy shocks. Equation (21) illustrates that the size of the endogenous cost-push shock becomes larger as the pass-through rate become smaller. Taken together, these imply that market concentration yields stronger dynamic effect of endogenous entry and amplify the impact of monetary policy shocks. However, the effect of the flattening of the Phillips curve to weakening monetary policy dominates the dynamic effect of endogenous entry. As a result, the impulse responses of the inflation rate to the monetary policy shock becomes smaller as market concentration develops.



Figure 4: Impulse responses to technology and monetary policy shocks by entry cost



## 7 Calvo pricing

So far, we have looked at the case of the [Rotemberg \(1982\)](#) price adjustment cost as the source of sticky prices, which helped to highlight the central role of the Second law due to its state dependency. We now turn to the case of [Calvo \(1983\)](#) pricing for its time dependency. It turns out that this reveals the importance of the Third law on the flattening of the Phillips curve. Details of the derivation are given in [Appendix D](#).

NKPC under HSA and Calvo pricing is given by

$$\hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} [\rho(\tilde{z}) (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - (1 - \rho(\tilde{z})) \hat{N}_t], \quad (26)$$

where  $1 - \phi$  is the probability that a firm can adjust its price. Under Calvo pricing, unlike Rotemberg pricing, not all firms set the same prices. Therefore, we define the average price under Calvo friction denoted by  $\tilde{p}_t$ , to distinguish it from the case with Rotemberg pricing, where the price of individual firms is equal to the average price. It is implicitly given by

$$\int_{\Omega_t} s \left( \frac{p_t(\omega)}{A_t} \right) d\omega = 1 = N_t s \left( \frac{\tilde{p}_t}{A_t} \right).$$

Accordingly,  $\tilde{z}_t := \tilde{p}_t / A_t$  and  $\tilde{\pi}_t := \tilde{p}_t / \tilde{p}_{t-1}$ .

Under Calvo pricing in [equation \(26\)](#), the slope of the Phillips curve is affected by the pass-through rate. Recall that under Rotemberg pricing in [equation \(21\)](#), the slope is affected by the price elasticity. Here, market concentration leads to the flattening of the Phillips curve under the Third law ([Matsuyama and Ushchev, 2023b](#)). According to the Third law, a higher price leads to a smaller rate of change in the price elasticity.<sup>21</sup> As a result, a higher entry cost leads to less competitive pressures and lowers the pass-through rate. This is because the firm increases its price and the markup rate in response to less competitive pressures, a higher  $A_t$ , when other firms increase their prices. Under this strategic complementarity embedded in HSA, higher concentration results in the flattening of the Phillips curve. This strategic complementarity also induces the dynamic effect of endogenous entry in the same manner as under Rotemberg pricing in [equation \(21\)](#).

**Power elasticity of markup rate (PEM)** Co-PaTh satisfies the Third law only in the weak form: the pass-through rate is constant. PEM, another parametric family of HSA, satisfies the Third law in the strong form: a higher price leads to a smaller rate of change in the price elasticity.<sup>22</sup>

<sup>21</sup>Notice that as evident in [Table 3](#), Translog cannot accommodate the Third law.

<sup>22</sup>For more details, see [Appendix D.3](#) in [Matsuyama and Ushchev \(2023b\)](#).

PEM is defined by the following market share function:

$$s(z) = \exp \left[ \int_{z_0}^z \frac{c}{c - \exp\left(-\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right) \exp\left(\frac{\kappa \zeta^{-\lambda}}{\lambda}\right)} \frac{d\zeta}{\zeta} \right], \quad (27)$$

with either  $\bar{z} = \infty$  and  $c \leq 1$  or  $\bar{z} < \infty$  and  $c = 1$ . By using equation (22), the price elasticity under PEM is given by

$$\zeta(z) = \frac{1}{1 - c \exp\left(\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right) \exp\left(-\frac{\kappa z^{-\lambda}}{\lambda}\right)}, \quad (28)$$

and using equation (23), the pass-through rate under PEM is given by

$$\rho(z) = \frac{1}{1 + \kappa z^{-\lambda}}. \quad (29)$$

Clearly,  $\kappa > 0$  and  $\lambda > 0$  ensure both the Second law and the Third law in the strong form.

Notice that by comparing equations (23) and (29), one can immediately see

$$-\frac{d \ln \left( \frac{\zeta(z)}{\zeta(z)-1} \right)}{d \ln(z)} = \kappa z^{-\lambda},$$

which means that the elasticity of the markup rate under the flexible price is a power function of  $z$ . This is why [Matsuyama and Ushchev \(2023b\)](#) call this family Power Elasticity of Markup Rate.<sup>23</sup> PEM contains Co-PaTh and CES as limit cases; It collapses to Co-PaTh with  $\lambda = 0$ ,  $\kappa = (1 - \rho) / \rho$ ,  $c = 1$  and  $\bar{z} < \infty$ , and to CES with  $\kappa = 0$ ,  $c = 1 - 1/\theta$  and  $\bar{z} = \infty$ . When PEM collapses to CES, the market share function is given by  $s(z) = (z/z_0)^{1-\theta}$ . To be comparable between CES and PEM, we set  $z_0 = \gamma^{\frac{1}{\theta-1}}$ .<sup>24</sup>

In Figure 5, each locus traces the markup rate  $\zeta(\bar{z}) / [\zeta(\bar{z}) - 1]$  and the slope of NKPC  $(1 - \phi) [1 - \phi\beta(1 - \delta)] \rho(\bar{z}) / \phi$  in equation (26) by changing  $\bar{z}$  under several  $\lambda$  with the reset probability  $1 - \phi = 0.25$ .<sup>25</sup> The black dot indicates the slope of the Phillips curve and the steady state markup under CES. There, the price elasticity, and therefore the markup rate, is not affected by competitive pressures, so the markup rate is  $\theta / (\theta - 1)$  and the slope of the Phillips curve is  $(1 - \phi) [1 - \phi\beta(1 - \delta)] / \phi$ . Equation (29) shows that as  $\lambda$  approaches to zero, the pass-through rate hardly moves, *i.e.*, PEM collapses to Co-PaTh. Therefore, variations in the pass-through rate become smaller.  $\lambda$  controls how variable the pass-through rate is to changes in  $z$  as implied in equation (29), while  $\kappa$  determines the level of the pass-through rate.

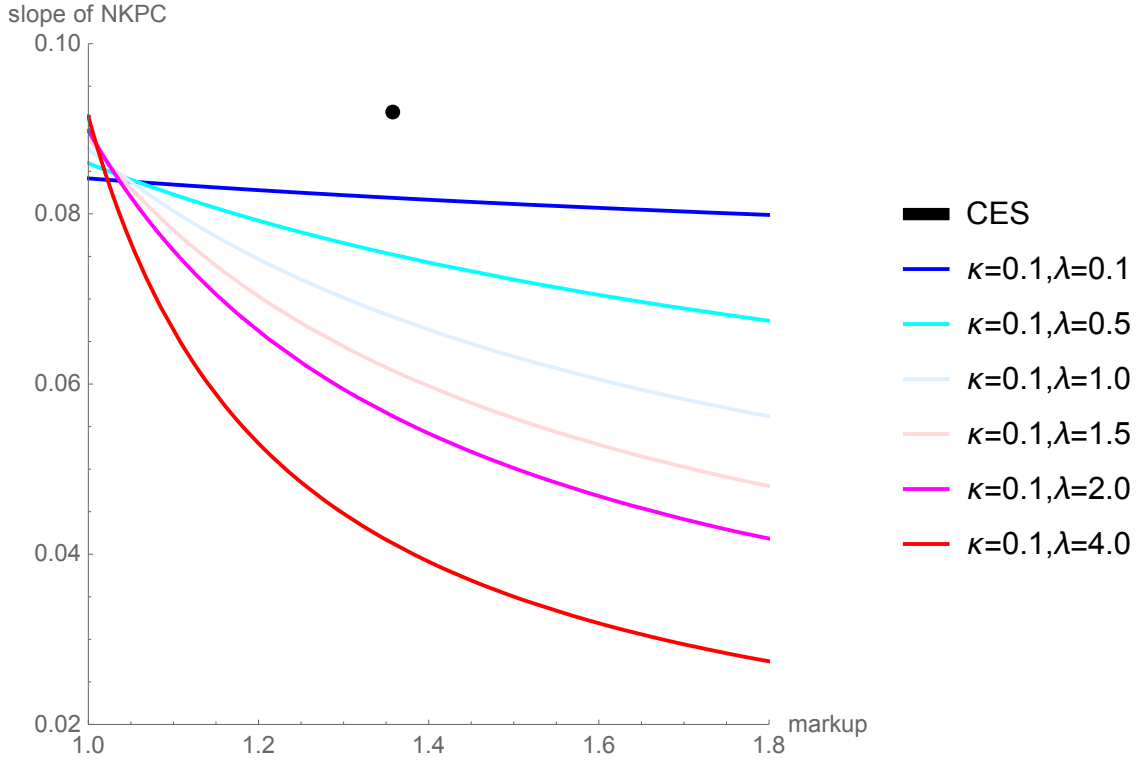
<sup>23</sup>[Matsuyama and Ushchev \(2023b\)](#) also call this family Fréchet Inverse Markup Rate, because its inverse markup rate function is proportional to the familiar Fréchet distribution function:

$$1 - \frac{1}{\zeta(z)} = c \exp\left(\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right) \exp\left(-\frac{\kappa z^{-\lambda}}{\lambda}\right).$$

<sup>24</sup> $\gamma$  is set at a value so that  $\zeta(z) = \theta$  when  $f_E = 1$  and  $\rho = 0.5$ . Notice that  $\gamma$  does not affect the price elasticity and the pass-through rate as evident in equations (28) and (29).

<sup>25</sup>As in Figure 2 in Section 4, it is also possible to consider this as a case in which the entry cost changes.

Figure 5: Markup and the slope of the Phillips curve under Calvo pricing



According to [De Loecker, Eeckhout and Unger \(2020\)](#), the markup rate increased to 21% to 61% in recent years. Although it is difficult to draw quantitative conclusions since there are no estimated values for  $\kappa$  and  $\lambda$ , Figure 5 shows that even under Calvo pricing, changes in the competitive environment among firms can result in a much smaller slope of the Phillips curve through the Third law.

To summarize, the causal impact from market concentration to the flattening of the Phillips curve is summarized by the price elasticity  $\zeta(z)$  under Rotemberg pricing in equation (21), and by the pass-through rate  $\rho(z)$  under Calvo pricing in equation (26). Because they are no longer equivalent even as the first order approximation under HSA, each of them is useful for understanding how concentration affects the slope of the Phillips curve.

## 8 Omitted variable bias

As discussed in Section 4.1, under the Second law, competition affects price setting dynamically through strategic complementarity and yields the endogenous cost-push shock. The imperfect pass-through leads firms to lower its price and the markup rate in response to higher competitive pressures. Thus, a *naive* regression of the inflation rate on the real marginal cost results in the omitted variable bias.

In this section, we first show that under the Second law, the omitted variable bias leads to the underestimation of the slope coefficient of NKPC. Then, we further show that concentration

magnifies this negative omitted variable bias. Under Rotemberg pricing, the Third law matters for the magnitude of the negative bias. Under Calvo pricing, both the Second and the Third laws matter for the magnitude of the negative bias. As a result, market concentration also weakens the observed relationship between the inflation rate and the marginal cost, thereby amplifying the supply side effects of monetary policy.

## 8.1 Rotemberg pricing

NKPC under Rotemberg pricing in equation (21) can be rewritten as

$$\hat{\pi}_t - \beta(1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} = \kappa^R (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1 - \rho(z)}{\chi \rho(z)} \hat{N}_t,$$

where  $\kappa^R$  denotes the true slope of NKPC. An estimated slope from a naive regression of the inflation rate on the real marginal cost without considering the endogenous markup shock is  $\tilde{\kappa}$  in

$$\hat{\pi}_t - \beta(1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} = \tilde{\kappa} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) + \varepsilon_t, \quad (30)$$

where  $\varepsilon_t$  denotes an i.i.d. exogenous cost-push shock.

Even though the inflation expectation and the real marginal cost are correctly measured, the estimated slope is subject to the omitted variable bias, and is different from the true one:

$$\tilde{\kappa} = \kappa^R - \frac{1 - \rho(z)}{\chi \rho(z)} \frac{\text{cov}((\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t), \hat{N}_t)}{\sigma_{\tilde{x}}^2} = \kappa^R + \frac{1 - \rho(z)}{\chi \rho(z)} \frac{\text{cov}(\hat{\mu}_t, \hat{N}_t)}{\sigma_{\tilde{x}}^2}. \quad (31)$$

As discussed in Section 4.2, under the Second law, the markup rate becomes countercyclical, leading to  $\text{cov}(\hat{\mu}_t, \hat{N}_t) < 0$  and therefore the negative omitted variable bias.<sup>26</sup>

This negative omitted variable bias is magnified by concentration. Under the Third law, market concentration results in lower pass-through rate and higher  $[1 - \rho(z)] / \rho(z) / \chi$  in equation (31), leading to larger negative omitted variable bias.

## 8.2 Calvo pricing

NKPC under Calvo pricing in equation (26) can be rewritten as

$$\hat{\pi}_t - \beta(1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} = \kappa^C (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} \frac{1 - \rho(\tilde{z})}{\zeta(\tilde{z}) - 1} \hat{N}_t,$$

where  $\kappa^C$  denotes the true slope of NKPC. A naive estimation of equation (30) leads to the omitted variable bias:

$$\tilde{\kappa} = \kappa^C + \frac{(1 - \phi) [1 - \phi \beta (1 - \psi)]}{\phi} \frac{\rho(\tilde{z}) [1 - \rho(\tilde{z})]}{\zeta(\tilde{z}) - 1} \frac{\text{cov}(\hat{\mu}_t, \hat{N}_t)}{\sigma_{\tilde{x}}^2}. \quad (32)$$

<sup>26</sup>Obviously,  $\text{cov}(\hat{\mu}_t, \hat{N}_t) = 0$  under CES. Under the Second law, we numerically find  $\text{cov}(\hat{\mu}_t, \hat{N}_t) < 0$ .

As in the case with Rotemberg pricing, the Second law leads to the negative omitted variable bias via countercyclical markup and therefore  $\text{cov}(\hat{\mu}_t, \hat{N}_t) < 0$ . Yet, unlike the case with Rotemberg pricing, both the Second and the Third laws magnify the negative omitted variable bias from concentration. Market concentration leads to lower price elasticity under the Second law as well as to lower pass-through rate under the Third law. As a result,  $\rho(\bar{z})[1 - \rho(\bar{z})] / [\zeta(\bar{z}) - 1]$  in equation (32) becomes higher, resulting in larger negative omitted variable bias.<sup>27</sup>

## 9 Conclusion

To understand the causal relationship from market concentration to the flattening of the Phillips curve, this paper extended the canonical New Keynesian model by replacing CES with HSA to allow endogenous entry to cause markup rate and pass-through rate changes. HSA demand system is well-suited for this purpose because the market share function and its single aggregator fully summarize all the information required to understand how market concentration affects the markup rate and the pass-through rate.

It has been shown that a higher entry cost, which leads to market concentration, causes the flattening of the Phillips curve for two reasons. First, market concentration *structurally* reduces the slope of the Phillips curve, by causing a rise of the markup rate (under the Second law) and a decline of the pass-through rate (under the Third law). We demonstrated the former under Rotemberg pricing and the latter under Calvo pricing. Second, due to the endogenous cost-push shock, *i.e.*, the supply side effects of monetary policy, emerging under the Second law, a *naive* regression of the real marginal cost on the inflation rate leads to the negative omitted variables bias, which is amplified by market concentration under the Second and the Third laws. This weakens the *estimated* relationship between the inflation rate and the marginal cost. These two reasons, one structural and one observational, together can go a long way toward understanding the flattening of the Phillips curve.

In this paper, we utilized two canonical pricing mechanisms: Rotemberg and Calvo. Rotemberg pricing captures the effect of the endogenous markup rate due to its state dependency, and Calvo pricing captures the effect of the endogenous pass-through rate due to its time dependency. In a more general pricing mechanism, such as the menu cost, which features both state and time dependencies, we conjecture that these two effects would coexist and that the results would be a hybrid of Rotemberg and Calvo. Of course, a menu cost model would be necessary for assessing numerically the relative importance of the two effects, which is left for future research. Other research topics that could be addressed by applying HSA to the New Keynesian framework may include optimal monetary policy, the wage Phillips curve, and heterogeneous firms.

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<sup>27</sup>As long as the pass-through rate is within the reasonable range, namely between 0.5 and 1, this term becomes smaller as the pass-through rate gets smaller.

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## A New Keynesian model under Rotemberg pricing and HSA

### A.1 Nonlinear system of equations

1. Taylor rule

$$(1 + i_t) = (1 + i_{t-1})^{\alpha_i} (\pi_t - 1)^{(1-\alpha_i)\alpha_\pi} u_t \quad (33)$$

2. Euler equation for bonds

$$C_t^{-\sigma} = \beta \mathbb{E}_t C_{t+1}^{-\sigma} \frac{1 + i_t \bar{p}_{t+1}}{\pi_{t+1} \bar{p}_t} \quad (34)$$

3. NKPC

$$\left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] \frac{s'(z_t) z_t}{s(z_t)} + \left[1 - \frac{s'(z_t) z_t}{s(z_t)}\right] \frac{L_t^\psi C_t^\sigma}{Z_{P,t} \bar{p}_t} - \chi (\pi_t - 1) \pi_t + \beta (1 - \delta) \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \chi (\pi_{t+1} - 1) \pi_{t+1} \frac{s(z_{t+1})}{s(z_t)} \frac{Y_{t+1}}{Y_t} = 0$$

4. Euler equation for equity

$$L_t^\psi C_t^\sigma \frac{f_{E,t}}{Z_{E,t}} = \beta (1 - \delta) \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left\{ \left[1 - \frac{L_{t+1}^\psi C_{t+1}^\sigma}{Z_{P,t+1} \bar{p}_{t+1}} - \frac{\chi}{2} (\pi_{t+1} - 1)^2\right] s(z_{t+1}) Y_{t+1} + L_{t+1}^\psi C_{t+1}^\sigma \frac{f_{E,t+1}}{Z_{E,t+1}} \right\}$$

5. Firm dynamics

$$\frac{1}{s(z_t)} = (1 - \delta) \left[ \frac{1}{s(z_{t-1})} + \frac{Z_{E,t-1}}{f_{E,t-1}} \left( L_{t-1} - \frac{Y_{t-1}}{\bar{p}_{t-1} Z_{t-1}} \right) \right]$$

6.  $P_t/A_t$

$$\ln \left( \frac{P_t}{A_t} \right) = \ln \left( \frac{z_t}{\bar{p}_t} \right) = \bar{K} - \frac{1}{s(z_t)} \left[ \int_{z_t}^{\bar{z}} \frac{s(\xi)}{\xi} d\xi \right] \quad (35)$$

7. Resource constraint

$$C_t = \left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] Y_t \quad (36)$$

### A.2 Steady state conditions

1. Taylor rule

$$\pi = 1$$

2. Euler equation for bonds

$$i = \frac{1 - \beta}{\beta}$$

3. NKPC

$$L^\psi Y^\sigma = \frac{\bar{p}}{1 - \frac{s(z)}{s'(z)z}} Z_P$$

#### 4. Euler equation for equity

$$L = \frac{1}{1-\delta} \frac{f_E}{Z_E} \left[ \delta - \frac{1-\beta(1-\delta)}{\beta} \frac{s'(z)z}{s(z)} \right] \frac{1}{s(z)}$$

#### 5. Firm dynamics

$$Y = -\frac{1-\beta(1-\delta)}{\beta(1-\delta)} Z_P \frac{f_E}{Z_E} \frac{\frac{s'(z)z}{s(z)} \frac{z}{s(z)}}{\exp\left(\bar{K} - \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right)}$$

#### 6. $P/A$

$$\frac{P}{A} = \frac{z}{\bar{p}} = \exp\left(\bar{K} - \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right)$$

For some special cases, we can obtain an analytical expression for the integral,  $\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$ . Under Co-PaTh, the integral is given by the hypergeometric function when  $\nu := \rho / (1 - \rho)$  is integer:

$$\int \frac{\left[1 - \left(\frac{\zeta}{\bar{z}}\right)^{\frac{1}{\nu}}\right]^\nu}{\zeta} d\zeta = \frac{\nu \left[1 - \left(\frac{\zeta}{\bar{z}}\right)^{\frac{1}{\nu}}\right]^{1+\nu}}{1+\nu} \sum_{n=0}^{\infty} \frac{(1)_n (1+\nu)_n}{(2+\nu)_n n!} \left[1 - \left(\frac{\zeta}{\bar{z}}\right)^{\frac{1}{\nu}}\right]^n.$$

Under Translog, the integral is given by

$$\int \frac{\gamma_{TL} \ln\left(\frac{z}{\zeta}\right)}{\zeta} d\zeta = -\frac{\gamma_{TL}}{2} \left[\ln\left(\frac{z}{\zeta}\right)\right]^2.$$

### A.3 Log-linearly approximated system of equations

#### 1. Taylor rule

$$i_t = \alpha_i i_{t-1} + (1 - \alpha_i) \alpha_\pi \hat{\pi}_t + u_t \quad (37)$$

#### 2. Euler equation for bonds

$$\hat{Y}_t = \mathbb{E}_t \hat{Y}_{t+1} - \frac{1}{\sigma} [i_t - (\hat{p}_t - \mathbb{E}_t \hat{p}_{t+1}) - \mathbb{E}_t \hat{\pi}_{t+1}] \quad (38)$$

#### 3. NKPC

$$\hat{\pi}_t = \frac{1}{\chi} \left[ -\frac{s'(z)z}{s(z)} \right] (\psi \hat{L}_t + \sigma \hat{Y}_t - \hat{Z}_{P,t} - \hat{p}_t) + \frac{1}{\chi} \left[ \frac{s'(z)z}{s(z)} - \frac{\frac{s''(z)z}{s'(z)}}{1 - \frac{s(z)}{s'(z)z}} \right] \hat{z}_t + \beta(1-\delta) \mathbb{E}_t \hat{\pi}_{t+1}$$

#### 4. Euler equation for equity

$$\begin{aligned} \psi \hat{L}_t = & \left\{ [1 - \beta(1-\delta)] \frac{s'(z)z}{s(z)} + \beta(1-\delta) \right\} \psi \mathbb{E}_t \hat{L}_{t+1} + [1 - \beta(1-\delta)] \left[ \sigma \frac{s'(z)z}{s(z)} + (1-\sigma) \right] \mathbb{E}_t \hat{Y}_{t+1} \\ & - (\hat{f}_{E,t} - \hat{Z}_{E,t}) + \beta(1-\delta) \mathbb{E}_t (\hat{f}_{E,t+1} - \hat{Z}_{E,t+1}) - [1 - \beta(1-\delta)] \frac{s'(z)z}{s(z)} \mathbb{E}_t \hat{Z}_{P,t+1} \end{aligned}$$

5. Firm dynamics

$$\hat{z}_t = (1 - \delta) \hat{z}_{t-1} - \left\{ \left[ \delta \frac{s(z)}{s'(z)z} - \frac{1 - \beta(1 - \delta)}{\beta} \right] \hat{L}_t - \hat{Y}_t + \hat{p}_t + \hat{Z}_{P,t} \right\} + \delta \frac{s(z)}{s'(z)z} (\hat{f} - \hat{Z}_{E,t})$$

6.  $\hat{P}_t - \hat{A}_t$

$$\hat{P}_t - \hat{A}_t = \hat{z}_t - \hat{p}_t = \left[ 1 + \frac{\int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi}{s(z)} \frac{s'(z)z}{s(z)} \right] \hat{z}_t \quad (39)$$

## B NKPC when entrants set prices flexibly

When entrants set prices flexibly, the adding up condition in equation (2) can be written as follows:

$$1 \equiv \int_{\Omega_t} s \left( \frac{p_t(\omega)}{A_t} \right) d\omega = \sum_{\tau=1}^{\infty} (1-\delta)^\tau N_{E,t-\tau} s \left( \frac{p_{t,t-\tau}}{A_t} \right) = \sum_{\tau=1}^{\infty} (1-\delta)^{\tau-1} [N_{t+1-\tau} - (1-\delta) N_{t-\tau}] s(z_{t,t-\tau}), \quad (40)$$

where  $p_{t,s}$  denotes the price at time  $t$  set by firms that entered at time  $s$ ,<sup>28</sup> and  $z_{t,s} := p_{t,s}/A_t$ .

### B.1 Entrants

Prices set by entrants are given by the Lerner pricing formula:

$$p_{t,t-1} = \frac{s' \left( \frac{p_{t,t-1}}{A_t} \right) \frac{p_{t,t-1}}{A_t} - s \left( \frac{p_{t,t-1}}{A_t} \right) \frac{W_t}{Z_{P,t}}}{s' \left( \frac{p_{t,t-1}}{A_t} \right) \frac{p_{t,t-1}}{A_t}}$$

or

$$z_{t,t-1} = \frac{s'(z_{t,t-1}) z_{t,t-1} - s(z_{t,t-1}) \frac{w_t}{Z_{P,t}} \bar{p}_t}{s'(z_{t,t-1}) z_{t,t-1}} \quad (41)$$

### B.2 Incumbents

From the profit maximization problem shown in Section 3.3, an incumbent firm  $\omega$  which entered at  $s \geq t-2$  set price to satisfy the optimal price setting condition:

$$\begin{aligned} & \left[ 1 - \chi \left( \frac{p_{t,s}(\omega)}{p_{t-1,s}(\omega)} - 1 \right) \frac{p_{t,s}(\omega)}{p_{t-1,s}(\omega)} - \frac{\chi}{2} \left( \frac{p_{t,s}(\omega)}{p_{t-1,s}(\omega)} - 1 \right)^2 \right] s \left( \frac{p_{t,s}(\omega)}{A_t} \right) \\ & + \left[ p_{t,s}(\omega) - \frac{W_t}{Z_{P,t}} - \frac{\chi}{2} \left( \frac{p_{t,s}(\omega)}{p_{t-1,s}(\omega)} - 1 \right)^2 p_{t,s}(\omega) \right] s' \left( \frac{p_{t,s}(\omega)}{A_t} \right) \frac{1}{A_t} \\ & - \left[ p_{t,s}(\omega) - \frac{W_t}{Z_{P,t}} - \frac{\chi}{2} \left( \frac{p_{t,s}(\omega)}{p_{t-1,s}(\omega)} - 1 \right)^2 p_{t,s}(\omega) \right] s \left( \frac{p_{t,s}(\omega)}{A_t} \right) \frac{1}{p_{t,s}(\omega)} \frac{1}{p_{t,s}(\omega)} \\ & + m_{t,t+1} \chi \left( \frac{p_{t+1,s}(\omega)}{p_{t,s}(\omega)} - 1 \right) \frac{p_{t+1,s}(\omega)}{p_{t,s}(\omega)} s \left( \frac{p_{t+1,s}(\omega)}{A_{t+1}} \right) \frac{P_{t+1} Y_{t+1}}{P_t Y_t} = 0. \end{aligned}$$

Under the symmetric equilibrium, this can be rewritten as

$$\begin{aligned} & \left[ 1 - \chi (\pi_{t,s} - 1) \pi_{t,s} - \frac{\chi}{2} (\pi_{t,s} - 1)^2 \right] \frac{p_{t,s}}{A_t} \\ & + \left[ \frac{p_{t,s}}{A_t} - \frac{W_t}{Z_{P,t} A_t} - \frac{\chi}{2} (\pi_{t,s} - 1)^2 \frac{p_{t,s}}{A_t} \right] \frac{s'(z_{t,s}) z_{t,s} - s(z_{t,s})}{s(z_{t,s})} \\ & + m_{t,t+1} \chi (\pi_{t+1,s} - 1) \pi_{t+1,s} \frac{s(z_{t+1,s})}{s(z_{t,s})} \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t} z_{t,s} = 0, \quad (42) \end{aligned}$$

<sup>28</sup>All firms that entered at time  $s$  set the same price  $p_{t,s}$ .

where  $\pi_{t,s} := p_{t,s}/p_{t-1,s}$ .

### B.3 Log-linearization

By log-linearizing equations (40), (41) and (42) around the steady state, we have

$$\frac{1}{\delta} \frac{1}{1-\zeta(z)} \hat{N}_t + \hat{z}_{t,t-1} + (1-\delta) \hat{z}_{t,t-2} + (1-\delta)^2 \hat{z}_{t,t-3} + \dots = 0, \quad (43)$$

$$\hat{z}_{t,t-1} = \rho(z) (\hat{w}_t + \hat{p}_t - \hat{Z}_{P,t}), \quad (44)$$

$$\hat{z}_{t,s} = \frac{\chi}{\zeta(z)-1} \frac{\rho(z)}{1-\rho(z)} [\beta(1-\delta) \hat{\pi}_{t+1,s} - \hat{\pi}_{t,s}] + \frac{\rho(z)}{1-\rho(z)} (\hat{w}_t + \hat{p}_t - \hat{Z}_{P,t}). \quad (45)$$

Inserting equations (44) and (45) into equation (43) leads to

$$\begin{aligned} \sum_{\tau=2}^{\infty} (1-\delta)^{\tau-1} \hat{\pi}_{t,t-\tau} &= \beta(1-\delta) \sum_{\tau=2}^{\infty} (1-\delta)^{\tau-1} \hat{\pi}_{t+1,t-\tau} \\ &\quad + \frac{\zeta(z)-1}{\chi} \left[ \frac{1}{\delta} - \rho(z) \right] (\hat{w}_t + \hat{p}_t - \hat{Z}_{P,t}) \\ &\quad - \frac{1}{\chi} \frac{1-\rho(z)}{\rho(z)} \frac{1}{\delta} \hat{N}_t. \end{aligned}$$

Define the average inflation rate  $\hat{\pi}_t^*$  implicitly to satisfy

$$\sum_{\tau=2}^{\infty} (1-\delta)^{\tau-1} \hat{\pi}_{t,t-\tau} = \sum_{\tau=2}^{\infty} (1-\delta)^{\tau-1} \hat{\pi}_t^* = \frac{1-\delta}{\delta} \hat{\pi}_t^*.$$

Then, NKPC for this average inflation rate is derived as follows:

$$\hat{\pi}_t^* = \beta(1-\delta) \hat{\pi}_{t+1}^* + \frac{\zeta(z)-1}{\chi} \frac{1}{1-\delta} [1-\delta\rho(z)] (\hat{w}_t + \hat{p}_t - \hat{Z}_{P,t}) - \frac{1}{\chi} \frac{1-\rho(z)}{\rho(z)} \frac{1}{1-\delta} \hat{N}_t.$$

Under the Second law, concentration leads to the flattening of the Phillips curve.<sup>29</sup>

**Bilbiie (2021)** shows that if new entrants can adjust prices freely (i.e., no sticky prices), then money becomes neutral, even in the presence of price rigidities among incumbents. This neutrality of money holds when the aggregator is CES in the **Dixit and Stiglitz (1977)** form, where the optimal variety is achieved in a competitive equilibrium, and there is no frictions or no lags in entry. Regarding the former, CES in the **Dixit and Stiglitz (1977)** is a special case of HSA and therefore, this condition does not generally hold in our paper. Regarding the latter, we have one-period time-to-build lag in our model as in **Bilbiie, Ghironi and Melitz (2008)** and **Bilbiie, Fujiwara and Ghironi (2014)**. Thus, monetary policy is not neutral.

<sup>29</sup>As long as  $\delta$  is not be large, main results carry over even under the Third law in the strong form.

## C NKPC in terms of the output gap

To convert the real marginal cost, which is the inverse of the markup rate, into the output gap, we expand upon the static entry model examined in [Bilbiie \(2021\)](#) by integrating HSA. During each period, firms freely enter and exit until their profits dwindle to zero. They are required to incur a fixed cost for production.

### C.1 Households

A representative household maximizes welfare (6) subject to the budget constraint:

$$\frac{B_{t+1}}{P_t} + C_t = (1 + i_{t-1}) \frac{B_t}{P_t} + \int_{\Omega_t} \frac{D_t(\omega)}{P_t} d\omega + \frac{W_t}{P_t} L_t. \quad (46)$$

### C.2 Final goods producer

There is no change from Section 3.2. Final goods producers assemble intermediate inputs using the CRS technology that generates the HSA demand system in equations (1) and (2).

### C.3 Intermediate goods producer

Intermediate goods producer  $\omega$  maximizes the present discounted value of profits:

$$\sum_{i=0}^{\infty} \beta \frac{u'(C_{t+i})}{u'(C_t)} \frac{D_t(\omega)}{P_t},$$

subject to equations (10) and (11) and the production technology:

$$y_t(\omega) = \begin{cases} Z_{P,t} [l_t(\omega) - F_t] & \text{if } l_t(\omega) > F_t \\ 0 & \text{otherwise} \end{cases}, \quad (47)$$

where  $F_t$  is the per-period fixed cost.

### C.4 Aggregate conditions and others

**Monetary policy** The central banks follows the simple feedback rule as in equation (12).

**Free entry and exit** Firms enter and exit freely until profits are reduced to zero after paying a fixed cost for production. Under the symmetric equilibrium, the free entry condition is given by combining equations (11) and (47):

$$\begin{aligned} 0 = \int_{\Omega_t} D_t(\omega) d\omega &= N_t \left[ \frac{p_t}{P_t} y_t - \frac{W_t}{P_t} l_t - \frac{\chi}{2} (\pi_t - 1)^2 \frac{p_t}{P_t} y_t \right] \\ &= \frac{p_t}{P_t} Z_{P,t} (L_t - N_t F_t) - \frac{W_t}{P_t} L_t - \frac{\chi}{2} (\pi_t - 1)^2 \frac{p_t}{P_t} Z_{P,t} (L_t - N_t F_t), \end{aligned}$$



which can be rewritten as

$$L_t = \left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] \mu_t (L_t - N_t F_t).$$

Note that

$$\mu_t := \frac{p_t Z_{P,t}}{W_t} = \frac{\bar{p}_t Z_{P,t}}{w_t}.$$

**Market clearing** The financial market clearing condition is given by  $B_t = 0$ .

Inserting equations (11) and (47) and into equation (46) yields the resource constraint:

$$C_t = \left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] Z_t \bar{p}_t (L_t - F_t N_t) = \left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] Y_t,$$

where

$$Y_t = N_t^{\frac{\theta}{\theta-1}} y_t = Z_t \bar{p}_t (L_t - F_t N_t).$$

### C.5 Nonlinear system of equations

Instantaneous utility is again defined by equation (18). The nonlinear system of equations consist of equations (33), (34), (35), (36),

$$0 = \frac{s'(z_t) z_t}{s(z_t)} + \left[1 - \frac{s'(z_t) z_t}{s(z_t)}\right] \frac{\chi L_t^\varphi C_t^\sigma}{\bar{p}_t Z_{P,t}} - \chi (\pi_t - 1) \pi_t - \frac{\chi}{2} (\pi_t - 1)^2 \frac{s'(z_t) z_t}{s(z_t)} + \beta m_{t,t+1} \chi (\pi_{t+1} - 1) \pi_{t+1} \frac{s(z_{t+1})}{s(z_t)} \frac{Y_{t+1}}{Y_t},$$

$$\left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] \frac{\bar{p}_t Z_{P,t}}{\chi L_t^\varphi C_t^\sigma} \mu_t \left(L_t - \frac{F_t}{s(z_t)}\right) = L_t,$$

and

$$Y_t = Z_{P,t} \bar{p}_t \left(L_t - \frac{F_t}{s(z_t)}\right).$$

### C.6 Log-linearly approximated system of equations

The Log-linearly approximated system of equations consists of equations (37), (38), (39),

$$\hat{\pi}_t = \frac{1}{\chi} \left[-\frac{s'(z) z}{s(z)}\right] (\psi \hat{L}_t + \sigma \hat{Y}_t - \hat{Z}_{P,t} - \hat{p}_t) + \frac{1}{\chi} \left[\frac{s'(z) z}{s(z)} - \frac{\frac{s''(z) z}{s'(z)}}{1 - \frac{s(z)}{s'(z) z}}\right] \hat{z}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \quad (48)$$

$$-\frac{s'(z) z}{s(z)} (\psi \hat{L}_t + \sigma \hat{Y}_t - \hat{Z}_{P,t} - \hat{p}_t) - \hat{L}_t + \hat{F}_t - \frac{s'(z) z}{s(z)} \hat{z}_t = 0,$$

and

$$\hat{Y}_t = \hat{Z}_{P,t} + \hat{p}_t + \frac{s'(z) z - s(z)}{s'(z) z} \hat{L}_t + \frac{s(z)}{s'(z) z} (\hat{F}_t + \hat{N}_t).$$

## C.7 NKPC in terms of output gap

We replace the real marginal cost measure in equation (48), that is,

$$\psi \hat{L}_t + \sigma \hat{Y}_t - \hat{Z}_{P,t} - \hat{p}_t = \hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t = -\hat{\mu}_t,$$

by the output gap. By combining log-linearly approximated equations in Section C.6, we can express the real marginal cost term by the trend output gap.<sup>30</sup>

$$\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t = \frac{1}{1 + \psi} \frac{(\sigma + \psi) - (1 - \sigma) \frac{1}{s(z)} \int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi}{1 - \frac{\zeta(z) - 1}{s(z)} \int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi} \hat{Y}_t.$$

Thus, NKPC in terms of the output gap is given by

$$\hat{\pi}_t = \beta \mathbf{E}_t \hat{\pi}_{t+1} + \kappa(z) \hat{Y}_t - \frac{\zeta(z) - 1}{\chi} \frac{1 - \rho(z)}{\rho(z)} \hat{z}_t,$$

where

$$\kappa(z) := \frac{\zeta(z) - 1}{\chi} \frac{1}{1 + \psi} \frac{(\sigma + \psi) - (1 - \sigma) \frac{1}{s(z)} \int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi}{1 - \frac{\zeta(z) - 1}{s(z)} \int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi}.$$

Concentration leads to the flattening of NKPC:

$$\frac{d\kappa(z)}{dz} > 0,$$

under benchmark calibration.

Note that we replace the real marginal cost term by the trend output gap, not by the flexible-price output gap. Under the Second law, flexible price output does not correspond to the constant markup rate.

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<sup>30</sup>The differences in the log-linearly approximated system of equations between Rotemberg and Calvo are those for NKPC. Therefore, this condition holds for the case with Calvo pricing.

## D NKPC under Calvo pricing and HSA

Some of new entrants cannot change their prices upon entry due to Calvo pricing frictions. They are randomly assigned the prices that the existing firms set in the previous period.

### D.1 Price setting

Intermediate goods producer  $\omega$  maximizes the value of the firm:

$$\frac{V_t(\omega)}{P_t} = \mathbb{E}_0 \sum_{i=0}^{\infty} [\phi\beta(1-\delta)]^i \frac{u'(C_{t+i})}{u'(C_t)} \left( \frac{D_{t+i}(\omega)}{P_{t+i}} \right),$$

where

$$\frac{D_t(\omega)}{P_t} = \frac{p_t(\omega)}{P_t} y_t(\omega) - \frac{W_t}{P_t} l_t(\omega),$$

subject to equations (8) and (10).

The optimal price setting condition is given by

$$\mathbb{E}_0 \sum_{i=0}^{\infty} [\phi\beta(1-\delta)]^i \frac{u'(C_{t+i})}{u'(C_t)} \frac{Y_{t+i}}{A_{t+i}} \left\{ s' \left( \frac{p_t^*}{A_{t+i}} \right) p_t^* - \frac{W_{t+i}}{Z_{P,t+i}} \left[ s' \left( \frac{p_t^*}{A_{t+i}} \right) - s \left( \frac{p_t^*}{A_{t+i}} \right) \frac{A_{t+i}}{p_t^*} \right] \right\} = 0,$$

which can be rewritten as

$$X_{1,t} = X_{2,t},$$

where

$$X_{1,t} := \mathbb{E}_0 \sum_{i=0}^{\infty} [\phi\beta(1-\delta)]^i u'(C_{t+i}) \frac{Y_{t+i}}{A_{t+i}} s' \left( \frac{p_t^*}{A_{t+i}} \right) p_t^*,$$

$$X_{2,t} := \mathbb{E}_0 \sum_{i=0}^{\infty} [\phi\beta(1-\delta)]^i u'(C_{t+i}) \frac{Y_{t+i}}{A_{t+i}} \frac{W_{t+i}}{Z_{P,t+i}} \left[ s' \left( \frac{p_t^*}{A_{t+i}} \right) - s \left( \frac{p_t^*}{A_{t+i}} \right) \frac{A_{t+i}}{p_t^*} \right].$$

#### D.1.1 Log-linearization

Above system can be log-linearized as follows:

$$\hat{X}_{1,t} = \hat{X}_{2,t},$$

$$\hat{X}_{1,t} = u'(C) \frac{Y}{A} s' \left( \frac{p^*}{A} \right) p^* \left[ \frac{1 + \frac{s'' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}}{1 - \phi\beta(1-\delta)} \hat{p}_t^* + \hat{Y}_{1,t} \right],$$

where

$$\hat{Y}_{1,t} = \frac{u''(C_t) C}{u'(C)} \hat{C}_t + \hat{Y}_t - \left[ 1 + \frac{s'' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)} \right] \hat{A}_t + \phi\beta(1-\delta) \mathbb{E}_t \hat{Y}_{1,t+1},$$

and

$$\hat{X}_{2,t} = u'(C) \frac{Y}{A} \frac{W}{Z_p} \left[ s' \left( \frac{p^*}{A} \right) - s \left( \frac{p^*}{A} \right) \frac{A}{p^*} \right] \begin{bmatrix} \frac{s'' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)} \\ \frac{1 - \frac{s \left( \frac{p^*}{A} \right)}{s' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}}{1 - \phi \beta (1 - \delta)} \hat{p}_t^* + \hat{Y}_{2,t} \end{bmatrix},$$

where

$$\hat{Y}_{2,t} = \frac{u''(C_t) C}{u'(C)} \hat{C}_t + \hat{Y}_t + W_t - Z_{p,t} - \frac{\frac{s'' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}}{1 - \frac{s \left( \frac{p^*}{A} \right)}{s' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}} \hat{A}_t + \phi \beta (1 - \delta) \mathbb{E}_t \hat{Y}_{2,t+1}.$$

Combining above five equations yields

$$\begin{aligned} & \frac{1}{1 - \phi \beta (1 - \delta)} \left[ 2 + \frac{\frac{s'' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}}{1 - \frac{s \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}} \right] \hat{p}_t^* = \\ & \hat{W}_t - \hat{Z}_{p,t} + \left[ 1 + \frac{\frac{s'' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}}{1 - \frac{s \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}} \right] \hat{A}_t + \frac{\phi \beta (1 - \delta)}{1 - \phi \beta (1 - \delta)} \left[ 2 + \frac{\frac{s'' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}}{1 - \frac{s \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s' \left( \frac{p^*}{A} \right)}} \right] \mathbb{E}_t \hat{p}_{t+1}^*. \end{aligned} \quad (49)$$

## D.2 Aggregate price

Under the Calvo pricing, the adding-up constraint in equation (2) is given by

$$(1 - \phi) N_t s \left( \frac{p_t^*}{A_t} \right) + \phi \int_0^{N_t} s \left( \frac{p_{t-1}(\omega)}{A_t} \right) d\omega = 1. \quad (50)$$

In the steady state,

$$(1 - \phi) N s \left( \frac{p^*}{A} \right) + \phi N s \left( \frac{\tilde{p}}{A} \right) = 1.$$

Since  $s(\tilde{p}/A) = 1/N$ ,  $s(p^*/A) = 1/N$ . As a result,

$$p^* = \tilde{p}. \quad (51)$$

**Log linearization** Log-linear approximation of equation (50) yields

$$(1 - \phi) \frac{s' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s \left( \frac{p^*}{A} \right)} \hat{p}_t^* = \frac{s' \left( \frac{p^*}{A} \right) \frac{p^*}{A}}{s \left( \frac{p^*}{A} \right)} \hat{A}_t - \hat{N}_t - \phi s' \left( \frac{p^*}{A} \right) \frac{p^*}{A} \int_0^N \hat{p}_{t-1}(\omega) d\omega,$$

where we define:  $\hat{p}_t(\omega) := \ln(p_t(\omega) / \tilde{p})$ .

Log-linear approximation of  $\int_0^{N_t} s(p_t(\omega) / A_t) d\omega = 1$  leads to

$$\int_0^N \hat{p}_t(\omega) d\omega = \frac{1}{s\left(\frac{p^*}{A}\right)} \hat{A}_t - \frac{1}{s'\left(\frac{p^*}{A}\right) \frac{p^*}{A}} \hat{N}_t.$$

By combining above two equations together, we have

$$\hat{p}_t^* = \frac{1}{1-\phi} \hat{A}_t - \frac{\phi}{1-\phi} \hat{A}_{t-1} - \frac{1}{1-\phi} \frac{s\left(\frac{p^*}{A}\right)}{s'\left(\frac{p^*}{A}\right) \frac{p^*}{A}} (\hat{N}_t - \phi \hat{N}_{t-1}). \quad (52)$$

### D.3 NKPC

Inserting equation (52) into equation (49) leads to NKPC in terms of  $A_t$ :

$$\begin{aligned} \hat{\pi}_{A,t} &= \beta(1-\delta) \mathbb{E}_t \hat{\pi}_{A,t+1} \\ &+ \frac{(1-\phi)[1-\phi\beta(1-\delta)]}{\phi} \frac{\frac{s'\left(\frac{p^*}{A}\right) \frac{p^*}{A}}{s\left(\frac{p^*}{A}\right)} - 1}{2\left(\frac{s'\left(\frac{p^*}{A}\right) \frac{p^*}{A}}{s\left(\frac{p^*}{A}\right)} - 1\right) - \frac{s''\left(\frac{p^*}{A}\right) \frac{p^*}{A}}{s'\left(\frac{p^*}{A}\right)}} (\hat{W}_t - \hat{Z}_{P,t} - \hat{A}_t) \\ &+ \frac{s\left(\frac{p^*}{A}\right)}{s'\left(\frac{p^*}{A}\right) \frac{p^*}{A}} \left[ \frac{1}{\phi} (\hat{N}_t - \phi \hat{N}_{t-1}) - \beta(1-\delta) (\mathbb{E}_t \hat{N}_{t+1} - \phi \hat{N}_t) \right]. \end{aligned} \quad (53)$$

Log-linearization of  $N_t s(\tilde{p}_t / A_t) = 1$  yields

$$A_t = \tilde{p}_t + \frac{s\left(\frac{p^*}{A}\right)}{s'\left(\frac{p^*}{A}\right) \frac{p^*}{A}} \hat{N}_t,$$

and

$$\hat{\pi}_{A,t} = \hat{\pi}_t + \frac{s\left(\frac{p^*}{A}\right)}{s'\left(\frac{p^*}{A}\right) \frac{p^*}{A}} (\hat{N}_t - \hat{N}_{t-1}).$$

Inserting these into equation (53) leads to

$$\hat{\pi}_t = \beta(1-\delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1-\phi)[1-\phi\beta(1-\delta)]}{\phi} \rho(\tilde{z}) \left[ (\hat{W}_t - \hat{Z}_{P,t} - \tilde{p}_t) - \frac{1}{\zeta(\tilde{z}) - 1} \frac{1 - \rho(\tilde{z})}{\rho(\tilde{z})} \hat{N}_t \right],$$

where we use equations (22) and (23) and define  $\tilde{z}_t := \tilde{p}_t / A_t = p_t^* / A_t$ . The final equality comes from equation (51).

## E Relationship between the pass-through rate and the super-elasticity

The super-elasticity coined by [Klenow and Willis \(2016\)](#) is the elasticity of the elasticity:

$$\bar{\zeta}(z) := \frac{\zeta'(z)z}{\zeta(z)}.$$

The relationship between the super-elasticity and the pass-through rate is given by

$$\bar{\zeta}(z) = \frac{1 - \rho(z)}{\rho(z)} [\zeta(z) - 1].$$

Using this relationship, NKPCs under Rotemberg and Calvo can be also written in terms of the elasticity and the super-elasticity as follows:

$$\hat{\pi}_t = \beta(1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{p,t} - \hat{p}_t) - \frac{1}{\chi} \frac{\bar{\zeta}(z)}{\zeta(z) - 1} \hat{N}_t,$$

and

$$\hat{\pi}_t = \beta(1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1 - \phi) [1 - \phi\beta(1 - \delta)]}{\phi [\zeta(\bar{z}) - 1 + \bar{\zeta}(\bar{z})]} \left\{ [\zeta(\bar{z}) - 1] (\hat{W}_t - \hat{Z}_{p,t} - \hat{p}_t) - \frac{\bar{\zeta}(\bar{z})}{\zeta(\bar{z}) - 1} \hat{N}_t \right\}.$$

However, similarly to [Baqee, Farhi and Sangani \(2021\)](#) and [Auclert, Rigato, Rognlie and Straub \(2022\)](#), we find that using the pass-through rate, as in equations (21) and (26), helps to capture more intuition on how competition affects NKPC.

## F Alternative homothetic non-CES demand systems: HDIA and HIIA

In this Appendix, we show that the shape of the Phillips curve is captured by the same two sufficient statistics under HDIA and HIIA, the two alternative homothetic demand systems proposed by [Matsuyama and Ushchev \(2017, 2020a,b, 2023a\)](#).

### F1 HDIA

The constant return-to-scale production function,  $Y_t(\mathbf{y}_t)$  is called HDIA when it is defined implicitly by

$$\int_{\Omega_t} \varphi \left( \frac{y_t(\omega)}{Y_t(\mathbf{y}_t)} \right) d\omega \equiv 1,$$

where  $\varphi(\cdot)$  is increasing and concave with  $-[d \ln(\varphi'(\cdot)) / d \ln(\cdot)]^{-1} > 1$ . As shown in [Matsuyama and Ushchev \(2023a\)](#), the demand function can be written as

$$y_t(\omega) = \varphi'^{-1} \left( \frac{p_t(\omega)}{A_{\text{HDIA}}(\mathbf{p}_t)} \right) Y_t, \text{ where } \int_{\Omega_t} \varphi \left( \varphi'^{-1} \left( \frac{p_t(\omega)}{A_{\text{HDIA}}(\mathbf{p}_t)} \right) \right) d\omega \equiv 1, \quad (54)$$

hence the price elasticity of demand becomes a function of  $z_{\text{HDIA},t}(\omega) := p_t(\omega) / A_{\text{HDIA}}(\mathbf{p}_t)$ :

$$\zeta(z_{\text{HDIA},t}(\omega)) = - \frac{\varphi'(\varphi'^{-1}(z_{\text{HDIA},t}(\omega)))}{\varphi'^{-1}(z_{\text{HDIA},t}(\omega)) \varphi''(\varphi'^{-1}(z_{\text{HDIA},t}(\omega)))} > 1.$$

From this and following the same step to derive equation (23), the pass-through rate can also be written as a function of  $z_{\text{HDIA},t}(\omega)$ :

$$\rho(z_{\text{HDIA},t}(\omega)) := \frac{\partial \ln(p_t(\omega))}{\partial \ln(W_t/Z_{P,t})} = \left[ 1 - \frac{d \ln \left( \frac{\zeta(z_{\text{HDIA},t}(\omega))}{\zeta(z_{\text{HDIA},t}(\omega)) - 1} \right)}{d \ln(z_{\text{HDIA},t}(\omega))} \right]^{-1}.$$

### F2 HIIA

The unit cost function,  $P(\mathbf{p}_t)$  is called HIIA when it is defined implicitly by

$$\int_{\Omega_t} \vartheta \left( \frac{p_t(\omega)}{P(\mathbf{p}_t)} \right) d\omega \equiv 1,$$

where  $\vartheta(\cdot)$  is decreasing and convex with  $-d \ln(-\vartheta'(\cdot)) / d \ln(\cdot) > 1$ . As shown in [Matsuyama and Ushchev \(2023a\)](#), the demand function can be written as

$$y_t(\omega) = -\vartheta' \left( \frac{p_t(\omega)}{P(\mathbf{p}_t)} \right) B_{\text{HIIA}}(\mathbf{y}_t), \text{ where } \int_{\Omega_t} \vartheta \left( (-\vartheta')^{-1} \left( \frac{y_t(\omega)}{B_{\text{HIIA}}(\mathbf{y}_t)} \right) \right) d\omega \equiv 1, \quad (55)$$

hence the price elasticity of demand becomes a function of  $z_{\text{HIIA},t}(\omega) := p_t(\omega) / P(\mathbf{p}_t)$ :

$$\zeta(z_{\text{HIIA},t}(\omega)) = - \frac{z_{\text{HIIA},t}(\omega) \vartheta''(z_{\text{HIIA},t}(\omega))}{\vartheta'(z_{\text{HIIA},t}(\omega))} > 1.$$

From this and following the same step to derive equation (23), the pass-through rate can also be written as a function of  $z_{\text{HIIA},t}(\omega)$ :

$$\rho(z_{\text{HIIA},t}(\omega)) := \frac{\partial \ln(p_t(\omega))}{\partial \ln(W_t/Z_{P,t})} = \left[ 1 - \frac{d \ln \left( \frac{\zeta(z_{\text{HIIA},t}(\omega)})}{\zeta(z_{\text{HIIA},t}(\omega)) - 1} \right)}{d \ln(z_{\text{HIIA},t}(\omega))} \right]^{-1}.$$

### E3 NKPC under HDIA and HIIA

By solving the profit maximization problem in Section 3.3 subject to demand functions in equations (54) and (55), respectively, NKPCs under HDIA and HIIA are obtained. Since the price elasticity and the pass-through rate are functions of normalized prices, the same NKPC as in equation (21) is derived by appropriately re-defining  $z_t$ :  $z_{\text{HDIA},t}$  under HDIA, and  $z_{\text{HIIA},t}$  under HIIA, respectively.

Notice, however, that general equilibrium implications, such as entry, productivity, and welfare, can be very different across the three classes. As Sections 5 and 6 show, we solve the model with endogenous entry and exit under monopolistic competition in general equilibrium. This becomes tractable under HSA.