Optimal Size and Composition of Government Spending in Small Open Economies*

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Abstract

This study explores the impact of various government spending policies on a small open economy that belongs to a monetary union. To do so, we consider an environment where households work in both tradable and non-tradable sectors, while smoothing their consumption through an internationally traded risk-free bond. When making hiring decisions, competitive firms face downward nominal wage rigidities and productivity shocks. Public investment increases the stock of public capital, which is an essential input to the production, but requires time-to-build while public consumption affects individual utility. The government subsidizes consumption and finances all its expenditures using income taxes. Within this frictional environment, we derive the optimal size and composition of government expenditures and calibrate it to France. Our numerical findings demonstrate that government spending and consumption subsidy policies significantly enhance economic welfare. Specifically, optimal adjustments in the consumption subsidy play a crucial role in maintaining low unemployment. Moreover, variations in government spending not only amplify the impact of the consumption subsidy on unemployment, but also help households smooth private consumption. This type of intervention results in the highest welfare. Our simulation results indicate that optimal public investment responds more aggressively to shocks. Finally, we find that during a boom-bust cycle, the real exchange rate appreciates initially and then depreciates. All our findings are robust to different productivity shocks, different public capital's time-to-build and share of public capital in the tradable sector.

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1 Introduction

Over time, the proportion of government spending relative to GDP has exhibited significant variation across countries.[1] Following the global financial crisis, there has been a notable shift in the size and composition of government spending within the Eurozone. More precisely, many Eurozone nations have been unable to fully recover their pre-2008 levels of total government spending. Moreover, during the subsequent recovery period, there has been a consistent decrease in public investment as a share of total government spending. In particular, between 2008 and 2013, peripheral countries such as Ireland, Italy, Spain and Greece witnessed a significant decline (an average drop of 37%), while core countries like France experienced a more modest decline of approximately 4%.[2] These empirical facts raise pertinent questions about the implications of shifts in both the size and composition of government spending and their potential economic impact on Eurozone countries.

In the context of an open economy, changes in government spending are intricately linked to real exchange rates, thereby creating a new avenue through which fiscal policy can respond to economic shocks as shown in Froot and Rogoff (1991). An increase in government spending stimulates the economy, while a reduction is detrimental to the economic activity. Recently, Born et al. (2023) have emphasized the pivotal role of changes in government spending as a response to economic shocks in open economies operating under both fixed and flexible exchange rate regimes.[3] Arguably, changes in government spending influence aggregate demand and contribute to short-term economic stabilization. However, it is equally important to note that changes in the composition of government spending, particularly when diverting resources away from public investment, can exert substantial and lasting impacts on overall productivity, aggregate supply and long-term growth.[4]

According to Schmitt-Grohé and Uribe (2012) the combination of downward nominal wage rigidity and a fixed exchange rate results in inefficiencies. Within a currency union, like the Eurozone where there has also seen substantial changes in size and composition of government expenditures, such deviations from first best calls for policy intervention. Previous studies have proposed different optimal policies to address the inefficiencies. These range from the implementation of capital control policies (Schmitt-Grohé and Uribe 2012); use of monetary policy tools, particularly employing

[1]We refer the reader to Figure C.1 in the appendix and recall that total government spending comprises productive government spending (public investment), and non-productive government spending (public consumption).

[2]Jong et al. (2017) documents similar facts.

[3]Similarly, changes in taxes can also affect the real exchange rate and stabilize the economy, namely fiscal devaluation proposed by Keynes (1931). Farhi et al. (2014) provide a formal analysis of fiscal devaluations in a stochastic dynamic general equilibrium New Keynesian open economy environment.

[4]Barro 1990 and Glomm and Ravikumar 1994 highlight these impacts in the closed economy context.

temporary inflation (Schmitt-Grohé and Uribe 2013); combination of wage subsidies at the firm level that is funded by income tax at the household level (Schmitt-Grohé and Uribe 2016) or consumption subsidy/tax (Schmitt-Grohé and Uribe 2012). However, Eurozone countries may not be able to enact such policies. For instance, capital control is not feasible for these countries, and changes in inflation are determined by Eurozone-wide policies rather than individual central banks. Additionally, wage and consumption subsidies face challenges, as expansionary fiscal policy is not endorsed by international institutions. This discouragement, coupled with the requirement of fiscal austerity as a precondition for financial assistance, poses obstacles to the implementation of such subsidies in the Eurozone. Another important aspect to consider is that changes in taxes, in practice, are minor.[5] So far the existing literature, which explores domestic policy options, has overlooked the role of government spending and its composition. We address this gap by specifically examining the optimal size and composition of government spending in the context of a small open economy that belongs to a currency union.

Specifically, in this paper we study how government spending should be adjusted to respond to productivity shocks. We answer this question within the context of a new small open economy model that has two production sectors, fixed exchange rates, downward nominal wage rigidities, and government investment and government consumption programs. More specifically, we build on the frictional framework of Schmitt-Grohé and Uribe (2012) where households and firms face downward nominal wage rigidities (henceforth DNWR). The production sector consists of tradable and non-tradable sectors that are exposed to sector-specific productivity shocks. The government has a central bank and a fiscal authority. The central bank implements a fixed exchange rate policy, while the fiscal authority is in charge of two government spending programs: (i) public investment such as infrastructure (productive spending); and (ii) public consumption such as public goods and services (non-productive spending). The former requires time-to-build and delivers public capital, which directly affects private firms' productivity. Instead, public consumption, through the provision of public goods and support programs, directly affects households' utility. The government also subsidizes consumption in this economy. To finance these public spending programs, the government relies on income taxes.

A key new feature of our model is that changes in government spending affect both aggregate

[5] For example, the mean (variance) of the ratio of VAT tax to GDP for France, Germany, and the average of periphery between 2000 and 2019 respectively are 6.98 (0.02), 6.74 (0.06), and 6.59 (0.04). This highlights that the consumption tax (one of the suggested policy in the literature) trend remains remarkably consistent from 2000 to 2019 for the considered group of countries in the Euro Zone.

demand and supply. An increase in public consumption stimulates current aggregate demand for non-tradable goods. In contrast, an increase in public investment stimulates not only current aggregate demand, but also future aggregate supply. This is the case as it increases private firms' future productivity.[6] Thus, changes in the size and composition of government spending have first order effects on economic activities and the real exchange rate in this small open economy. More precisely, investment in public capital affects the economy's production capacity, long-term employment and output as in Barro (1990). However, because of downward nominal wage rigidities and fixed exchange rates, the economy experiences short-run inefficiencies (involuntary unemployment) as pointed out by Schmitt-Grohé and Uribe (2012), among others. In such environment, nominal wages can not fully adjust when large negative shocks hit the economy. This is the case as wages are bounded from below due to DNWR. When wages reach this lower bound, the economy experiences involuntary unemployment. This suggests that an increase in government spending (fiscal stimulus) could be useful in smoothing out adverse negative shocks.

To assess the short-run effects of diverse government spending policies on economic activity and the real exchange rate, we calibrate the benchmark model to the French economy. We then conduct several experiments across five different policy options. In the first case, we analyze the Ramsey policy, wherein the government simultaneously optimizes the size and composition of its spending as well as the consumption subsidy. The second case considers a decentralized economy, where the government does not adjust the composition of spending nor the consumption subsidy in response to economic shocks. The third policy involves a constant share policy, where the government maintains the composition of spending while optimizing only the consumption subsidy. To delve deeper into the influence of various government spending components on macroeconomic variables, we analyze two policies that adjust government consumption and investment independently, while in both policies the government optimizes the consumption subsidy. In the fourth scenario, the government pursues an optimal policy by adjusting government consumption, while keeping public investment constant. In the fifth policy option, the government optimally adjusts government investment, while maintaining a constant level of government consumption.

Following the approach in Schmitt-Grohé and Uribe (2016), we conduct simulations of the model during boom-bust episodes for all five government policies previously described.[7] In our

[6]The literature suggests that it takes years for public investment to turn into public capital. Hence, public investment will only affect future productivity in production.

[7]In the boom-bust scenario we consider productivity shocks in both tradable and non-tradable sectors.

quantitative analysis, to better isolate the short-run effects of productivity shocks, we assume that the various distinct spending policies result in the same optimal long-run allocation. As a result, when simulating cases two through five, we do not impose their respective distorted steady states.

Our simulations yield these following results: (a) optimal adjustments in the consumption subsidy play a crucial role in maintaining low unemployment; (b) variations in government spending not only amplify the impact of the consumption subsidy on unemployment, but also help households in smooth private consumption, resulting in the highest welfare; (c) the Ramsey policy is associated with a more aggressive adjustment in public investment than in public consumption. These results are robust to alternative assumptions on productivity shocks, time-to-build public capital and share of public capital in the tradable sector.

We also find that during a boom-bust cycle the real exchange rate initially appreciates and then depreciates. It is important to note that when changing the composition of government spending, there are two opposing effects impacting the real exchange rate. On one hand, increased public investment positively affects the non-tradable sector, increasing the relative price of non-tradable goods and causing real exchange rate appreciation –a phenomenon known as the Froot-Rogoff effect. On the other hand, an increase in public investment results in an increase in the stock of public capital. This delivers an outward shift in the economy's productivity and production possibility frontiers. This is a direct consequence of having public capital as an essential input to production. Moreover, when public capital is sufficiently more efficient in the non-tradable sector, an increase in public investment leads to real exchange rate depreciation. This mechanism is known as the Balassa-Samuelson effect. Our simulation results indicate that the Balassa-Samuelson effect dominates.

We also find that under the constant government investment and optimal government consumption and consumption subsidy, the relative price of non-tradable goods exhibits higher volatility relative to the other spending policies. Given the inherent asymmetry of DNWR as there is a lower bound on nominal wages, but not an upper bound. Given this important feature, we examine whether government spending policies and unemployment also exhibit asymmetries over boom-bust episodes. In our simulation results, we find strong asymmetries for government investment, but not government consumption. As long as the government optimizes the consumption subsidy, we do not observe significant unemployment asymmetry. Finally, we note that an increase in consumption subsidies during the bust period is larger than in the boom period.

Finally, our quantitative analyses also reveal that aggressive responses to negative productivity shocks, through reduced public investment, result in the higher social welfare. This is the case as during economic recessions and whenever the government does not make significant adjustments to total expenditure, a sharp decline in government investment can prevent government consumption from falling too rapidly. This offers two key benefits. First, a gradual decline in government consumption has a minimal negative impact on social welfare. This is the case as the government provides goods and services through the non-productive government spending program that directly affects households' utility. The second benefit is that the direct effect of a reduction in government investment on current output is relatively small. This is the case as it takes time for public investment to deliver public capital that can be effectively used for future production. Thus, right after a negative shock, household consumption is reduced the least when we have a decline in government investment. Thus, adjustments not only in the size, but more importantly in the composition of government spending appear to be useful in smoothing adverse technology shocks. These government spending adjustments reduce the variability in households' consumption of non-tradable goods, and subsequently lead to relative improvements in social welfare when compared to adjustments to only consumption subsidy.

The remaining paper is organized as follows. Section 2 describes the economic environment and characterizes the equilibrium, while Section 3 characterizes the optimal composition of government spending and examines some of its properties. Section 4 outlines the calibration. Section 5 presents the quantitative analysis and results. Section 6 offers some conclusions.

Related literature: Our paper makes contributions to various strands of the open economy literature. Firstly, we add to the research that investigates how open economies can counteract negative shocks, especially in the presence of downward nominal wage rigidities. Additionally, our work is aligned with studies that explore the use of fiscal measures in situations where a devaluation by the monetary authority is not a viable option. Finally, we complement the literature that examines the connection between government spending and real exchange rates.

We complement studies that consider small open economy models with downward nominal wage rigidites (DNWR) and unproductive government spending (e.g., see Schmitt-Grohé and Uribe 2012, Schmitt-Grohé and Uribe 2016 and Born et al. 2023, among others). Schmitt-Grohé and Uribe (2016) demonstrate that pegging the nominal exchange rate creates a pecuniary externality, resulting in the failure of the labor market to clear and causing involuntary unemployment. This consequently provides a rationale for policy intervention. Born et al. (2023) extend the model in

Schmitt-Grohé and Uribe (2012) and study how government spending affects economic activities and the real exchange rate in a small open economy where household has preferences are over private and public consumption that finances it through a lump-sum tax. The authors find that the adjustment to government spending shocks is asymmetric. A fiscal expansion appreciates the real exchange rate but does not stimulate output. A fiscal contraction does not alter the exchange rate, but lowers output. Our paper complements this literature by studying the impact of changing the size and composition of the government spending on the real exchange rate, output, and unemployment. Our results highlight the important role of government investment in response to negative technology shocks as well as implications for long-run growth and welfare in a small open economy. These aspects have not been previously considered.

The other literature we relate to was started by the seminal paper by Keynes (1931), who proposed a solution to the stabilization of an economy with fixed exchange rates by introducing the idea of fiscal devaluations. In particular, Keynes (1931) suggested that a uniform ad valorem tariff on all imports plus a uniform subsidy on all exports would have the same impact as an exchange rate devaluation. Since then the literature has studied a variety of tax instruments that can help achieve a fiscal devaluations. Most notably, the seminal work by Farhi et al. (2014) provide a formal analysis of fiscal devaluations in a New Keynesian stochastic dynamic general equilibrium environment where government expenditures are wasteful and unproductive and there is no public infrastructure. These authors find that there are two types of fiscal policies equivalent to an exchange rate devaluation. One such scheme is a uniform increase in import tariff and export subsidy. The second one is a value-added tax increase and a uniform payroll tax reduction. In contrast to these authors, we focus fiscal devaluations using different government spending policies. Other, prominent examples in the literature that study the use of tax instruments to adjust the real exchange are Schmitt-Grohé and Uribe (2012), Schmitt-Grohé and Uribe (2016), Engler et al. (2017), among others. In all these papers do not consider the size and composition of government expenditures in their analysis. Our paper complements this literature by studying other fiscal policies.

Finally, our paper relates to the seminal work of Froot and Rogoff (1991). These authors postulate that increases in government consumption tend to increase the relative price of the non-tradable good, as government consumption is concentrated on the non-tradable good sector. Since then there have been numerous studies estimating such relationship finding mixed results.[8] To

[8] Ricci et al. (2008), using a panel of 48 countries from 1980 to 2004, find that government consumption is highly significant. In contrast, Corsetti and Müller (2006) Monacelli and Perotti (2010), Enders et al. (2011), and Ravn et

provide a theoretical explanation for these disparities, Ravn et al. (2012) propose a model with a deep-habit mechanism, while Bouakez and Eyquem (2015) consider a small open economy model with incomplete and imperfect international financial markets, sticky prices, and a monetary policy that is not overly aggressive. However, recent studies show that the distinction between the various types of government expenditures are also important. This is the case as they have different effects on the real exchange rate. Using frameworks based on the frictionless neoclassical growth model, Galstyan and Lane (2009) show how the composition of government spending influences the long run behavior of the real exchange rate. Within the same spirit, Chatterjee and Mursagulov (2016) show that the effect of government spending on the real exchange rate depends on the sectoral composition of public spending, the underlying financing policy, the sectoral intensity of private capital in production, the relative sectoral productivity of public infrastructure, and the elasticity of substitution in production. Our paper complements this work along several dimensions. First, we allow households obtain utility from government spending, through the provision of public goods and services. In addition, the share of government spending that is invested in public capital is subject to a time-to-build requirement. We also consider an environment with productivity shocks and nominal wage rigidities and a fixed nominal exchange rate. Finally, we study optimal government policies that help mitigate shocks.

2 Model

We build on the small open economy framework of Schmitt-Grohé and Uribe (2016) and Born et al. (2023). In particular, we consider identical households and competitive firms that operate in the tradable and non-tradable sectors. Firms use labor, and public capital (infrastructure) to produce their respective goods. They also face downward nominal wage rigidities (DWNR) when making their hiring decisions. The small open economy also has a government that finances its public goods and services as well as its infrastructure using distortionary income taxes. Finally, the central bank implements a fixed exchange rate policy. Within this environment, we study the design of optimal government expenditures.

Throughout the rest of the paper we assume that the law of one price holds and that that there

al. (2012), among others, find that the real exchange rate depreciates in response to a positive shock to government expenditure. This result holds across different samples and identification schemes.

are no trade costs so that

$$P_t^T = \mathcal{E}_t P_t^{T*}$$

where P_t^T denotes the nominal price (in the home currency) of tradable goods, respectively. In addition, $\mathcal{E}_t$ represents the domestic-currency price of one unit of foreign currency and P_t^{T*} is the price of the tradable good in the world market. Since we are considering a small open economy we take this latter price as given and exogenous. For simplicity, we normalize P_t^{T*} to one. In other words, we use the tradable good as the numeraire. In this case, $P_t^T = \mathcal{E}_t$. Finally, P_t^N represents the nominal price (in the home currency) of the non-tradable goods.

2.1 Households

Infinitely-lived identical households of unit mass discount the future at a rate $\beta \in (0,1)$ and each household is endowed with $\bar{h}$ units of labor that is willing to supply inleastically. Households derive utility from consuming private, c_t, and from public goods and services, g_t^c, as in Born et al. (2023), among others. The private consumption good is a composite of tradable, c_t^T, and non-tradable consumption goods, c_t^N, which is given by

$$c_t = \frac{\left(c_t^T\right)^{\psi}\left(c_t^N\right)^{1-\psi}}{\psi^{\psi}\left(1-\psi\right)^{1-\psi}} \tag{1}$$

where ψ represents the share of tradable goods in the composite private good. The resulting household's lifetime utility is then given by

$$\mathcal{U}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{c_{t+s}^{1-\sigma}}{1-\sigma} + v\left(g_{t+s}^c\right)\right] \tag{2}$$

where $\mathbb{E}$ is the linear expectation operator with respect to an equilibrium distribution of shocks, σ denotes the relative risk aversion associated to consumption of the private good and $v\left(\cdot\right)$ is an increasing and concave function that determines the payoff from consuming public goods and services.

Households earn labor income by allocating their time between the two sectors. To smooth their consumption, households purchase an internationally traded risk-free pure discount bond, which we represent by d_t. This bond pays an interest rate, r_t, when it is held between periods t and $t+1$. This rate is denominated in terms of nominal tradable goods. Thus, the corresponding household's sequential budget constraint in period t is then given by

$$\left(1-s_t^c\right)P_tc_t+\mathcal{E}_td_t+\mathcal{E}_t\mathcal{C}(d_t)\leq\left(1-\tau_t^y\right)\left(W_th_t+\Pi_t^T+\Pi_t^N\right)+\mathcal{E}_td_{t-1}\left(1+r_{t-1}\right) \tag{3}$$

where P_t is the price of the composite good, W_t denotes the nominal wage, h_t is the total labor supplied by the household, τ_t and s_t^c represent the income tax rate and consumption subsidy, respectively. Π_t^T and Π_t^N denote firms' profits in the tradable and non-tradable sectors, respectively. It is important to note, as in in Schmitt-Grohé and Uribe (2016) and Born et al. (2023), in equilibrium, when the economy achieves full employment, we have that $h_t=\bar{h}$. Otherwise, households will face involuntary unemployment, which is equal to $\bar{h}-h_t$. To close the model, we assume, as in Schmitt-Grohé and Uribe (2003), that households pay a quadratic cost when holding the internationally traded risk-free pure discount bond, which we denote by $\mathcal{C}(d_t)$. These holding costs are denominated in terms of the tradable good and are given by

$$\mathcal{C}(d_t)=\frac{\chi}{2}\left(d_t-\bar{d}\right)^2$$

where χ is a cost parameter and $\bar{d}$ represents the long run equilibrium foreign asset level.

Using the previous definitions, we divide both sides of equation (3) by $\mathcal{E}_t$ and the household's budget constraint can be rewritten as follows

$$\left(1-s_t^c\right)\left(c_t^T+p_tc_t^N\right)+d_t+\frac{\chi}{2}\left(d_t-\bar{d}\right)^2=\left(1-\tau_t\right)\left(w_th_t+\pi_t^T+\pi_t^N\right)+\left(1+r_{t-1}\right)d_{t-1} \tag{4}$$

where $p_t=P_t^N/P_t^T$ is the relative price and $w_t\equiv W_t/\mathcal{E}_t$ represents the real wage (in terms of the tradable good). Finally $\pi_t^T\equiv\Pi_t^T/\mathcal{E}_t$ and $\pi_t^N\equiv\Pi_t^N/\mathcal{E}_t$ denote real tradable and non-tradable firms' profits, respectively.

Households choose contingent plans $\left\{c_t^T,c_t^N,d_t\right\}$ in order to maximize their lifetime utility, equation (2), subject to the previous simplified budget constraint, equation (4). These optimal choices imply the following

$$\psi c_t^{-\sigma}\left(\frac{c_t}{c_t^T}\right) = \lambda_t\left(1-s_t^c\right) \tag{5}$$

$$\left(1-\psi\right)c_t^{-\sigma}\left(\frac{c_t}{c_t^N}\right) = \lambda_t p_t\left(1-s_t^c\right) \tag{6}$$

$$1+\chi\left(d_t-\bar{d}\right) = \mathbb{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1+r^*\right)\right] \tag{7}$$

where λ_t denotes the Lagrange multiplier associated with the budget constraint. For simplicity, and throughout the rest of the paper, we assume that $\beta\left(1+r^*\right)=1$ where r^* is the foreign interest rate.

Combining the first order conditions corresponding to tradable and non-tradable goods, equations (5) and (6), we can derive the following expenditure switching condition

$$\frac{c_t^T}{c_t^N}=\frac{\psi}{1-\psi}p_t \tag{8}$$

which implies that the demand for non-tradable goods is strictly decreasing in the relative price of the non-tradable good to the tradable good, and increasing in the tradable good consumption level.

2.2 Firms

The small open economy has tradable and non-tradable sectors. Competitive firms produce tradable and non-tradable goods using labor and public capital, k_t^G. This latter input captures the public infrastructure provided by the government through its investments. The production function for non-tradable goods is given by

$$y_t^N=a_t^N\left(k_t^G\right)^{\phi_N}\left(h_t^N\right)^{\alpha_N} \tag{9}$$

where a_t^N is the total factor productivity in the trabable sector and is subject to exogenous shocks, while $\alpha_N\in(0,1)$ and $\phi_N>0$ represent the labor and public infrastructure shares in the non-tradable sector, respectively.[9] Note that the *effective* total factor productivity in the non-tradable goods sector, $\tilde{a}_t^N$, is given by $\tilde{a}_t^N\equiv a_t^N\left(k_t^G\right)^{\phi_N}$, which explicitly takes into account the impact of government public capital on productivity in the non-tradable sector.

[9]This type of production function is similar to that of Chatterjee and Mursagulov (2016) who, in addition, consider private capital.

Firms in the non-tradable sector maximize profits while taking prices and wages as given. In the absence of any labor market imperfections and denominating profits in terms of the nominal price of the tradable good, this results in the following problem

$$\max_{\{h_t^N\}} p_t y_t^N - w_t h_t^N$$

which implies the following optimal labor hiring decision

$$w_t = \alpha_N p_t a_t^N \left(k_t^G\right)^{\phi_N} \left(h_t^N\right)^{\alpha_N - 1}. \tag{10}$$

Similarly, the production of tradable goods is given by

$$y_t^T = a_t^T \left(k_t^G\right)^{\phi_T} \left(h_t^T\right)^{\alpha_T} \tag{11}$$

where $a_t^T \left(k_t^G\right)^{\phi_T}$ is the total factor productivity in the tradable sector and h_t^T is the labor supplied by households to the tradable sector. Moreover, $\alpha_T \in (0, 1)$ and $\phi_T \in (0, 1)$ denote the labor and public infrastructure shares in the tradable sector, respectively. As in the non-tradable sector, firms maximize profits while taking prices and wages as given. In the absence of any labor market imperfections, the maximization for tradable good producers is given by

$$\max_{\{h_t^T\}} y_t^T - w_t h_t^T$$

which yields the following optimal labor hiring decision

$$w_t = \alpha_T a_t^T \left(k_t^G\right)^{\phi_T} \left(h_t^T\right)^{\alpha_T - 1}. \tag{12}$$

Consistent with the literature, firms in the tradable and non-tradable sectors face exogenous shocks. Innovations to the two different sectoral factor productivities follow an AR(1) process, which is given by

$$\log\left(\frac{a_t^j}{a_{ss}^j}\right) = \rho_j \log\left(\frac{a_{t-1}^j}{a_{ss}^j}\right) + \sigma^j \epsilon_t^j$$

where $\epsilon_t^j \sim N(0, 1)$ with $j \in \{N, T\}$, which capture non-tradable and tradable sectors, respectively. Finally, a_{ss}^j denotes the average productivity in sector j.

2.3 Downward nominal wage rigidity

So far we have not considered any labor market imperfection. However, individual-level data on wage changes as well as survey-based evidence on wage setting show that nominal wage cuts are rare. In a series of seminal papers, Schmitt-Grohé and Uribe have emphasized the importance of this labor market feature in the context of small open economies. Here we follow such tradition. To capture the observed evolution of nominal wages, we introduce labor market frictions as in Schmitt-Grohé and Uribe (2016). Specifically, we assume that there exists downward nominal wage rigidity (DNWR) such that nominal wages in period t, W_t, cannot fall below a certain threshold. In particular, nominal wages have to satisfy the following condition

$$W_t \geq \gamma W_{t-1} \tag{13}$$

where γ is a non-negative parameter governing the degree of DNWR. It is important to note that when equation (13) is binding, the labor market clearing condition does not hold. As a result, the aggregate labor supply is greater than the aggregate labor demand, inducing involuntary unemployment.

It is important to note that we can rewrite the equation governing DNWR as follows

$$w_t \geq \gamma w_{t-1} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \right)^{-1} \tag{14}$$

where $w_t = W_t/\mathcal{E}_t$ and $\mathcal{E}_t/\mathcal{E}_{t-1}$ captures the depreciation in nominal exchange rate. This expression explicitly highlights the role that the evolution of nominal exchange rates can have in the labor market. More precisely, note that if at time t, the nominal exchange rate depreciation is sufficiently high, the DNWR constraint will not be binding and the labor market will deliver full employment. However, if the country has a fixed exchange rate regime, for instance the country belongs to a currency union, we have that $\mathcal{E}_t = \mathcal{E}_{t-1}$, the DNWR constraint becomes

$$w_t \geq \gamma w_{t-1} \tag{15}$$

and the small open economy cannot rely on exchange rate changes to avoid involuntary unemployment whenever the constraint is binding.

When shocks hitting the economy are such that the DNWR constraint is not binding, the labor market clears. In this situation we have that the labor market clears, which implies the following

$$h_t^T + h_t^N = \bar{h}. \tag{16}$$

In contrast, when shocks are large enough whereby the DNWR constraint is binding, the labor market does not clear, in equilibrium, we have that wages satisfy the following condition

$$w_t = \gamma w_{t-1}. \tag{17}$$

Hence, once we introduce labor market imperfections that allow for downward nominal wage rigidity, the resulting equilibrium wage w_t offered by firms is given by

$$w_t = \max\left\{w_t^*, \gamma w_{t-1}\right\} \tag{18}$$

where w_t^* is the solution to equation (16) when DNWR is not binding. In addition, in equilibrium, the aggregate labor input is given by

$$h_t = h_t^T + h_t^N. \tag{19}$$

From now on, as in Schmitt-Grohé and Uribe (2016), among others, we assume that our small open economy has a labor market that faces imperfections that result in DNWR.

2.4 Government

The government finances its spending and maintains a balanced budget by income taxes on households' income. The corresponding government budget constraint is given by

$$p_t g_t + s_t^c \left(c_t^T + p_t c_t^N\right) = \tau_t^y \left(w_t h_t + \pi_t^T + \pi_t^N\right) \tag{20}$$

where g_t denotes total government expenditures. In contrast to most of the literature, government expenditures has two components: one that yields positive payoffs to households' utility, and the other one is useful in productive activities.[10] In particular, total government expenditures are such that $g_t = g_t^c + g_t^k$, with g_t^c and g_t^k represent the government consumption and government investment,

[10] Prominent examples in the literature that study the use of tax instruments to adjust the real exchange are Schmitt-Grohé and Uribe (2012), Schmitt-Grohé and Uribe (2016), Farhi et al. (2014), Engler et al. (2017), among others. These papers do not consider the size and composition of government expenditures in their analysis.

respectively. Note that implicit in this formulation is that both government consumption and government investment are non-tradable goods. This assumption is consistent with Froot and Rogoff (1991), among others.

Government investment contributes to the stock of public productive capital (k_t^G), which evolves as follows

$$k_{t+1}^G = g_{t-T}^k + (1-\delta)\, k_t^G$$

where δ represents the depreciation rate of public capital and T is the number of periods that is needed to transform government investment into public capital. This time lag implies that public investment increases the stock of public capital, but that it requires a time-to-build to be useful in production as in Bouakez et al. (2020), among others. From now on we denote the ratio public investment to total government expenditure as $\theta_t = g_t^k / g_t$.

Finally, to capture the experiences of countries that are part of the European currency union, we assume that the monetary authority follows a fixed nominal exchange rate. This policy regime implies that $\mathcal{E}_t = \overline{\mathcal{E}}$ for all t, where $\overline{\mathcal{E}}$ is a positive constant. This assumption reflects the fact that monetary policy for countries that belong to a monetary union is not solely determined by its central bank.

2.5 Equilibrium

Having described the optimal behavior of households and firms as well as having specified the operating procedures for monetary and fiscal policy, we can now define the equilibrium in this small open economy.

Definition 1 *Given government policies $\left\{\tau_t,\, s_t^c,\, g_t^c,\, g_t^k\right\}$, and the international interest rate $\{r^*\}$, a competitive equilibrium under a fixed exchange rate regime is characterized by a set of goods, labor and prices $\{c_t^T,\, c_t^N,\, \lambda_t,\, d_t,\, h_t^T,\, h_t^N,\, h_t,\, y_t^T,\, y_t^N,\, k_t^G,\, p_t,\, w_t\}$ such that: (i) households' decisions are optimal; (ii) firms hiring decisions maximize their profits; (iii) goods markets clear; (iv) the labor market outcome is characterized by equations (18) and (19); and (v) the government budget constraint is satisfied.*

It is worth noting that in our small open economy, the market for the non-tradable good clears all the time. This is the case as it does not face any frictions nor imperfections. The resulting market clearing condition for non-tradable goods is then given by

$$y_t^N = c_t^N + g_t^c + g_t^k. \tag{21}$$

Moreover, combining the previous market clearing condition, the households' and the government's budget constraints, as well as firms' profits, we can derive the following resource constraint

$$0 = (1 + r^*)\, d_{t-1} + y_t^T - \left(c_t^T + d_t + \frac{\chi}{2}\left(d_t - \bar{d}\right)^2\right) \tag{22}$$

that relates the evolution of the internationally traded risk-free pure discount bond to tradable production and consumption. Finally, the complete set of conditions characterizing the dynamic competitive equilibrium are given by equations (23)-(34). These can be found in Appendix A.

3 Ramsey problem

We now explore the optimal size and composition of government expenditures. In our environment, inefficiencies arise because the labor market faces downward nominal wage rigidities and the monetary authority implements a fixed exchange rate. As we previously highlighted, once the equation governing the downward nominal wage rigidity is binding, involuntary unemployment is observed in equilibrium. Hence, there is room for government intervention. Existing literature on optimal policy often typically neglects the role of government expenditure and its composition.[11] In contrast, we focus on these fiscal policy tools that can enhance the economic behavior and mitigate inefficiencies. We do so as some of the previous tax policy interventions cannot be implemented when a small open economy belongs to a currency and trade union like the Eurozone. Thus, we consider other fiscal tools that could be used within a currency union. Specifically, our research investigates the impact of altering the composition of government expenditure and the size of the government. These aspects of fiscal policy are typically easier to adjust than changing the tax policy. This is consistent with the findings of Vegh and Vuletin (2015), who report that tax policy

[11] For instance, Schmitt-Grohé and Uribe (2012) introduce capital control policies to mitigate the inefficiencies associated with having downward nominal wage rigidities and a fixed exchange rate. An alternative intervention is a combination of firm-level wage subsidies that are financed by household-level income taxes as in Schmitt-Grohé and Uribe (2016). When monetary policy tools are possible, Schmitt-Grohé and Uribe (2013) consider a temporary period of inflation to mitigate the impact of the downward nominal wage rigidity.

in industrial countries is found to be acyclical over the business cycle.[12]

In what follows we focus on a benevolent government that maximizes the households' lifetime utility by choosing the government expenditure and consumption subsidy schemes $\left\{g_t^c, g_t^k, s_t^c\right\}$, while adhering to the complete set of conditions described in the dynamic equilibrium, equations (23) to (34), and the government budget constraint, equation (20). It is important to note that income tax adjusts to meet the government's budget constraint when solving for the optimal policy problem. Therefore, the government budget constraint is not included as one of the constraints facing the policy maker.

Our approach to solving the problem is similar to that of Schmitt-Grohé and Uribe (2012). In particular, we begin by omitting conditions (23) to (25) from the set of constraints in the Ramsey planner's problem. Subsequently, we demonstrate that the solution to this less constrained problem satisfies the omitted constraints. The Lagrangian of the less constrained Ramsey problem can be expressed as follows

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \begin{array}{c} \frac{\left(\frac{\left(c_{t+s}^T\right)^{\psi}\left(y_{t+s}^N - g_{t+s}^c - g_{t+s}^k\right)^{1-\psi}}{\psi^{\psi}(1-\psi)^{1-\psi}}\right)^{1-\sigma}}{1-\sigma} + v\left(g_{t+s}^c\right) + \mu_{t+s}^d \left[\begin{array}{c} (1+r^*)\, d_{t+s-1} + y_{t+s}^T \\ -\left(c_{t+s}^T + d_{t+s} + \frac{\chi}{2}\left(d_{t+s} - \bar{d}\right)^2\right) \end{array} \right] \\ +\mu_{t+s}^{h^T}\left[\alpha_T a_{t+s}^T \left(k_{t+s}^G\right)^{\phi_T} \left(h_{t+s}^T\right)^{\alpha_T - 1} - w_{t+s}\right] \\ +\mu_{t+s}^{h^N}\left[\alpha_N p_{t+s}\left(c_{t+1}^T, y_{t+s}^N, g_{t+s}^k, g_{t+s}^c\right) a_{t+s}^N \left(k_{t+s}^G\right)^{\phi_N} \left(h_{t+s}^N\right)^{\alpha_N - 1} - w_{t+s}\right] \\ +\mu_{t+s}^h\left[h_{t+s} - \left(h_{t+s}^T + h_{t+s}^N\right)\right] + \mu_{t+s}^{k^G}\left[g_{t+s-T}^k + (1-\delta)\, k_{t+s}^G - k_{t+s+1}^G\right] \\ +\mu_{t+s}^{y^T}\left[a_{t+s}^T\left(k_{t+s}^G\right)^{\phi_T}\left(h_{t+s}^T\right)^{\alpha_T} - y_{t+s}^T\right] + \mu_{t+s}^{y^N}\left[a_{t+s}^N\left(k_{t+s}^G\right)^{\phi_N}\left(h_{t+s}^N\right)^{\alpha_N} - y_{t+s}^N\right] \\ +\mu_{t+s}^w\left[\left(1 - h_{t+s}\right)\left(w_{t+s} - \gamma w_{t+s-1}\right)\right] \end{array} \right\}$$

where we have used the fact that

$$p_t\left(c_t^T, y_t^N, g_t^k, g_t^c\right) \equiv \frac{1-\psi}{\psi} \frac{c_t^T}{y_t^N - g_t^c - g_t^k}$$

and μ_t^x denotes the Lagrange multiplier on the equilibrium condition for variable x, with $x \in \left\{d,\, h^T,\, h^N,\, h,\, y^T,\, y^N,\, w,\, k^G\right\}$. It is important to note that given our assumption regarding the interest rate associated to this risk free bond we have that $\mu_t^d > 0$. In contrast, the remaining Lagrange multipliers are greater than or equal to zero. The first-order conditions with respect to c_t^T, d_t, h_t^T, h_t^N, y_t^T, y_t^N, k_{t+1}^G, h_t, w_t, g_t^c, g_t^k, as well as the seven μ_t^x's and associated slackness conditions can be found in Appendix B.

[12]This can be attributed to the fact that implementing legislation requiring changes in the tax code often takes longer than making adjustments to government expenditure.

It can be readily demonstrated that conditions (23) and (25) in the competitive dynamic equilibrium are satisfied when imposing the solution of the less constrained Ramsey problem. Specifically, we calibrate our parameters to ensure the steady-state consumption subsidy, s_t^c, is zero. We set $\lambda_t = \mu_t^d$ to satisfy (25). Then we choose s_t^c to satisfy (23). Additionally, condition (24) is met because we have used the expression for the relative price of the non-tradable good, p_t, from (24) in the Ramsey problem.

Having characterized the optimal size and composition of government expenditures, we explore how output, employment and real exchange rates respond once the small open economy experiences different types of shocks.

4 CALIBRATION

To further explore the properties of our dynamic equilibrium, we need to parameterize our benchmark model, which is calibrated to France. We have chosen France among the different Eurozone core countries due to its extensive time series data for various macroeconomic aggregates. In addition, France is one of the European countries that has had a fixed exchange rate for almost three decades.[13]

To discipline the parameters in our model we use French data at quarterly frequency, unless specified, from the first quarter of 1980 until the last quarter in 2019. Our calibration has two sets of parameters: $\{\beta, r^*, \chi, \psi, \alpha_T, \alpha_N, \phi_T, \phi_N, \rho_T, \rho_N, \sigma_T, \sigma_N, \rho_{TN}\}$ and $\left\{\bar{d}, \kappa, a^T, a^N\right\}$. The parameters in the first set can be directly calibrated using empirical moments or steady state conditions, while the parameters in the second set need to be solved jointly using empirical moments and steady state conditions implied by the model. Below we describe our calibration strategy for the first set of parameters.

Preferences and Private Budget Constraint: To impose symmetry between the payoffs of consuming private and public goods, we assume that the preferences over public consumption are $v\left(g_t^c\right) = \kappa \frac{(g_t^c)^{1-\sigma}}{1-\sigma}$. Moreover, we assume a log utility, which implies that $\sigma = 1$. The home consumption bias, ψ, is chosen to match the average share of the tradable good consumption in

[13]It is worth noting that according to the IMF exchange rate classification, France adopted a *de facto* crawling peg to DM between mid-1979 and 1986, followed by a *de facto* peg to DM from 1987 to 1998. On January 1, 1999, France joined the European Currency Union and adopted the Euro.

the total final household consumption expenditure between 1980 and 2019.[14] In line with the existing literature, we consider services such as health, housing and utility, education, restaurants and hotels, recreation and culture, and services expenditure on transportation, communication, and miscellaneous goods and services as the non-tradable goods in the consumption basket. Goods that are tradable, on the other hand, are assumed to be food, clothing and footwear, furnishings, households equipment, purchase of vehicles spending (in the transportation category) and telephone and telefax equipment (in the communication category), and durable goods, semi-durable goods and non-durable goods. Based on these criteria, the share of the non-tradable good in the consumption, denoted as ψ is equal to 0.59. We also set the foreign interest rate r^* to 0.01. Note that $r_t = r^*$ holds in the steady state and in equilibrium we have that the subjective discount satisfies $\beta(1 + r^*) = 1$. This implies that $\beta = 0.99$.

Finally, we choose an arbitrarily small value for the foreign bond holding cost parameter χ such that the bond holding cost is low and has negligible effects on the dynamics of other macroeconomic aggregates. In particular, we set χ to 0.01. The steady state foreign bond value $\bar{d}$ is calibrated jointly with $\kappa, \bar{a}^T$ and $\bar{a}^N$. A detailed discussion of these parameters can be found in the Other Parameters section, which we discuss below.

Technologies: As is typically assumed in the literature, we set the quarterly depreciation rate equal to 0.025. To determine parameters in production functions and productivity shocks in both tradable and non-tradable sectors, we use French data for the period 1980-2019. The specific data series employed for this analysis include: (i) general government capital stock; (ii) sectoral gross value added measured at current prices, along with the corresponding sectoral price index (implicit deflator); (iii) sectoral hours of work; and (iv) compensation of employees.[15] By using these specific data series from the French economy, we can effectively pin down the parameters in the production functions for both the tradable good and the non-tradable good sectors. Additionally, we can determine the parameters in the AR(1) process that characterizes productivity shocks in both sectors.

[14]The data are taken from the *OECD.Stat* Database. This data is only available at an annual frequency.

[15]We use three data sources for the five data series. The general government capital stock data is sourced from the Investment and Capital Stock Dataset published by the International Monetary Fund (IMF) in 2019. This dataset provides annual data on government capital stock. For sectoral gross value added, price index, and hours of work data, we turn to EUROSTAT as our data source. Specifically, we utilize quarterly data series named "income A*10 industry breakdowns" (referred to as "namq_10_a10" in the EUROSTAT database) to acquire the necessary information. Finally, the compensation of employees data is obtained from the quarterly national accounts of OECD statistics.

To calibrate the parameters in the production functions, namely $\alpha_T, \alpha_N, \phi_T, \phi_N, \rho_T, \rho_N, \sigma_T, \sigma_N$, and ρ_{TN}, we follow a two-step process. First, using information on gross value added and compensation of employees, we calibrate parameters for the labor shares in the tradable and the non-tradable sectors (α_T and α_N). Consistent with our previous analysis, we classify industries into the tradable goods sector and the non-tradable goods sector. The tradable sector includes agriculture, forestry and fishing and manufacturing. The non-tradable sector consists of construction, information and communication, wholesale and retail trade, transport, accommodation and food service activities, financial and insurance activities, real estate activities, professional, scientific and technical activities, administrative and support service activities, public administration, defence, education, human health and social work activities, arts, entertainment and recreation, other service activities, activities of household and extra-territorial organizations and bodies. It is important to note that the first-order conditions derived from our model indicate that α_T and α_N represent the total labor cost to output ratios in the tradable and non-tradable sector, respectively. Using the information on gross value added and labor compensation, we can determine that $\alpha_T = 0.56$ and $\alpha_N = 0.59$.

In the second step we calibrate the parameters ϕ_T, ϕ_N, ρ_T, ρ_N, σ_T, σ_N and ρ_{TN} using the following approach. We compute the productivities in the tradable and non-tradable sector ($\tilde{a}_t^T$ and $\tilde{a}_t^N$) as $y_t^T / \left(h_t^T\right)^{\alpha_T}$ and $y_t^N / \left(h_t^N\right)^{\alpha_N}$, respectively.[16] To obtain these sectoral productivities, we utilize data from the EUROSTAT database. The sectoral productivities for the tradable and non-tradable sectors are constructed using sectorial gross value added and hours of work. It is important to note that sectoral value added needs to be deflated by its price level to obtain real sectoral output (y_t^T and y_t^N). The log sectoral price can be computed by taking the weighted average of log industry prices, with the weights being the average of industry value added over the sample period. To investigate the impact of public capital on productivities, we incorporate information on public capital from the Investment and Capital Stock Dataset published by the IMF in 2019. Since the public capital measure is available at an annual frequency, we apply linear interpolation to transform the data to a quarterly frequency. The Hodrick-Prescott (HP) filter is then used to extract the cyclical components of the sectoral productivities and public capital. Using this data and by estimating the following models describing the sectoral productivity. In particular, we have

[16] Both data are obtained from the EUROSTAT database.

that

$$\begin{aligned}\log\left(\tilde{a}_t^i\right) &= \phi^i \log\left(k_t^G\right) + \log\left(a_t^i\right)\\ \log\left(a_t^i\right) &= \rho^i \log\left(a_{t-1}^i\right) + \varepsilon_t^i\end{aligned}$$

where $i \in \{T, N\}$. We find that $\phi_T = 0$ and $\phi_N = 0.08$.[17] It is worth noting that Leeper et al. (2010) emphasizes the challenging nature of determining the production elasticity of the public capital stock and uses the value of 0.05 and 0.1. Other papers acknowledge the difficulty of estimating this parameter and closely follow this estimate, e.g. Klein and Linnemann (2023). The value of 0.08 employed for the elasticity of the public capital stock for the non-tradable sector falls within the range commonly utilized in the literature.

The empirical analysis also provides information regarding the characteristics of the shocks in the tradable and the non-tradable sector. In particular, the persistence parameters ρ_T and ρ_N are estimated to be 0.80 and 0.75, respectively. The standard deviations of the shocks are $\sigma_T = 0.010$ for the tradable good sector and $\sigma_N = 0.005$ for the non-tradable good sector, reflecting their respective levels of variability. Furthermore, the correlation between the shocks in the two sectors ρ_{TN} is estimated to be 0.61, indicating a moderate level of correlation between the shocks affecting tradable and non-tradable activities. These findings provide insights into the dynamics and interdependencies of shocks in both sectors.

Other Parameters: Having specified the first set of parameters, we can now jointly calibrate the second set of parameters of our model. These parameters include the long-run values of foreign bond holdings ($\bar{d}$), the steady-state exogenous productivities (excluding the influence of public capital) in the tradable and non-tradable goods sectors (a^T and a^N), and the weight attached to public consumption in the utility function (κ). The calibration is conducted in two steps.

First, we employ five moments and eight steady state equilibrium conditions to jointly solve for three parameters $\left\{\bar{d}, \tilde{a}^T, \tilde{a}^N\right\}$ and nine steady state variables $\left\{c^T, c^N, y^T, y^N, h^T, h^N, p, g, w\right\}$. The data we use are: (i) the long-run average trade balance in France, which is approximately -0.016; (ii) we normalize the steady state TFP in the non-tradable good sector $\tilde{a}^N$ to one; (iii) the average share of the total tradable good sector output in the average share of the total tradable goods

[17]The regression shows a statistically insignificant coefficient on k_t^G in the tradable good sector. Hence we set ϕ_T to zero in the benchmark analysis. We relax this assumption by assuming a positive ϕ_T in a robustness check later on and our baseline results do not change much.

sector output in France's GDP, which is about 17.9 percent; and iv) the long-run average ratio of government expenditure to GDP, which equals 22.3 percent of GDP. The corresponding steady state equilibrium conditions are given by equations (8), (9), (10), (11), (12), (16), (21), and (22).

Second, we apply the values $\{c^T, c^N, y^T, y^N, h^T, h^N, p, g, w\}$ obtained in the first step in the first order conditions of the Ramsey problem (Appendix B), we can solve for parameter κ, the steady state public investment g^k, public consumption g^c, public capital stock k^G, as well as the steady state Lagrangian multipliers under the Ramsey problem. Once these are determined, we can solve for the steady state productivities a^T and a^N, which are given by $a^T = \tilde{a}^T \left(k^G\right)^{-\phi^T}$ and $a^N = \tilde{a}^N \left(k^G\right)^{-\phi^N}$.

Labor Market Frictions: Following the seminal papers by Schmitt-Grohé and Uribe (2012) and Schmitt-Grohé and Uribe (2016), we set the value of downward nominal wage rigidity to 0.99. This value of γ implies that nominal wages can decline by up to 4 percent annually. The total time endowment is normalized to one in our analysis.

Having described the calibration strategy used in this paper, Table 1 reports the values of the key parameters of our benchmark model.

Table 1: Model Parameters

Parameters	Model	Source
Preference and budget constraint		
CRRA coefficient	$\sigma = 1$	Assumed
Foreign interest rate	$r^* = 0.01$	Assumed
Discount factor	$\beta = 0.99$	Calibrate
Weight on government consumption	$\kappa = 1.05$	Calibrate
Share of tradable good consumption	$\psi = 0.41$	Calibrate
Foreign bond holding cost	$\chi = 0.01$	Assumed
Technology		
Productivity	$a^T = 1.27$ and $a^N = 0.96$	Calibrate
Share of labor in production	$\alpha^T = 0.56$ and $\alpha^N = 0.59$	Calibrate
Capital depreciation rate	$\delta = 0.025$	Literature
Contribution of public capital to productivity	$\phi^T = 0$ and $\phi^N = 0.08$	Estimate
Productivity shock in the tradable good sector	$\rho^T = 0.80$ and $\sigma^T = 0.010$	Estimate
Productivity shock in the non-tradable good sector	$\rho^N = 0.75$ and $\sigma^N = 0.005$	Estimate
Correlation between shocks	$\rho_{TN} = 0.61$	Estimate
Market Frictions		
Degree of DNWR	$\gamma = 0.99$	Literature
Total labor endowment	$h = 1$	Normalization

5 Quantitative Results

To have deeper insights of our framework, it is crucial that we outline the precise numerical strategy we are going to use to solve for the dynamic equilibrium, as well as elaborate on the specific experiments we plan to conduct.

In our quantitative analysis, to better isolate the short-run effects of productivity shocks we assume that three distinct spending policies result in the same optimal long-run allocation. As a result, when simulating the various policy options, we refrain from imposing their respective distorted steady states.

5.1 Numerical strategy

A key feature of our environment is that households and firms face a downward nominal rigidity while facing productivity shocks. This implies that our framework has occasionally binding constraints (OBC), which present some challenges when computing short-run dynamics. The literature has proposed several methods on how to compute such a model. One approach is to use global solution methods, such as the policy function iteration method, which can capture all of the model's non-linearities. However, a major drawback of such methods is that they do not scale well to larger models. This is the case we face in our framework. In particular, our model features multiple lagged variables, as we assume that public capital is formed after a period of 16 quarters following a public investment decision. This feature poses significant challenges when using the global method to solve our model with OBC.

An alternative to policy function iteration is perturbation methods. This is the approach we use to solve our model with OBC constraints. Specifically, we employ the DynareOBC toolkit developed by Holden (2016) and Holden (2021) to solve our model up to second order.[18] The key concept behind the DynareOBC method is to treat the OBC as an endogenous source of news, ensuring that where disturbances would cause bounds to be violated, anticipated news shocks return the bounded variable to the constraint. In other words, the DynareOBC method anticipates the effect of the OBC on the economic agents in the model and incorporates this anticipation into the solution. The method is described fully in Holden (2016) , with a companion paper, and Holden (2021), discussing the necessary and sufficient conditions for the existence and uniqueness of solutions at the bound. Swarbrick (2021) reviewed different solution methods and found that the DynareOBC performs well

[18] More details about the DynareOBC toolkit can be found at https://github.com/tholden/dynareOBC.

in solving DSGE models with OBCs compared to alternative methods.

Having calibrated our benchmark model and having specified a numerical strategy to compute the resulting short run dynamics after various shocks hit the small open economy, we can now further explore how the composition and size of government expenditures help mitigate the inefficiencies resulting from downward nominal wage rigidities and a fixed nominal exchange rate. To do so we perform a series of experiments.

5.2 A boom-bust episode

To illustrate how government spending affects the small open economy under a fixed rate regime, we simulate a boom-bust episode. Similar to Schmitt-Grohé and Uribe (2016), we define a boom-bust episode as a situation in which (i) productivity (a_t^T and/or a_t^N) is at or below trend in period 0, (ii) productivity (a_t^T or a_t^N) are more than one standard deviation above the trend in period 20, and (iii), the productivity, which increases by more than one standard deviation in period 20, is at least one standard deviation below the trend in period 30. We simulate the model for $200,000$ periods and select all sub-periods that satisfy our definition of a boom-bust episode. We then average across these periods, which we report in the different figures.

A key feature of our model is that there exists an asymmetry in the labor market's response when it comes to absorbing positive and negative shocks. Positive shocks trigger an efficient labor market response. This is the case as nominal wages increase, ensuring that firms align with their labor demand schedule and households meet their labor supply schedule. In other words, upward wage adjustments facilitate a flexible and efficient transmission of positive shocks. In contrast, the adjustment to negative shocks may deliver inefficiency. When nominal wages fail to decrease, beyond what a flexible labor market would predict, due to the downward nominal wage rigidity, households are not able to inelastically supply their labor. This indicates that downward wage adjustments are hindered, leading to labor market inefficiencies during negative shocks. As a result of this inefficiency, we observe involuntary unemployment.

Next, we examine the different macroeconomic responses and the propagation of shocks in the small open economy under five different fiscal policies: (i) optimal (g^k, g^c, s^c) policy; (ii) constant (g^k, g^c, s^c) policy; (iii) constant share and optimal s^c policy; (iv) constant g^k and optimal (g^c, s^c) policy; and (v) constant g^c and optimal (g^k, s^c) policy.

When characterizing the Ramsey policy, the government optimally implements adjustments in government investment, government consumption, and consumption subsidy to respond to economic shocks; i.e the government chooses (g^k, g^c, s^c). In contrast, in the constant (g^k, g^c, s^c) policy, the government does not adjust government spending or the consumption subsidy in response to economic shocks. In other words, there is no government short-run response to the exogenous shocks. In essence, this case mirrors a decentralized economy without any government intervention when confronted with exogenous shocks. In the constant share and optimal s^c policy, the government optimizes only the consumption subsidy but implements the same amount of total expenditure as in the optimal (g^k, g^c, s^c) policy, with the share of public investment being constant over the business cycle. In other words, the government spending in this policy is considered exogenous. To further explore the impact of different government spending components on macroeconomic variables and the transmission of shocks in the economy, we examine two policies that optimally adjust government consumption and investment separately. In one scenario we consider the constant g^k and optimal (g^c, s^c) policy, where the government fixes public investment at a constant level, equal to the steady-state public investment under the optimal (g^k, g^c, s^c) policy, while optimally choosing government consumption and the consumption subsidy simultaneously. In the other scenario we consider a constant g^c and optimal (g^k, s^c) policy, where the government fixes public consumption at a constant level, equal to the steady-state public consumption under the optimal (g^k, g^c, s^c) policy, while optimally choosing government investment and the consumption subsidy simultaneously.

In what follows we evaluate and compare all these different policies when the small open economy faces different shocks.

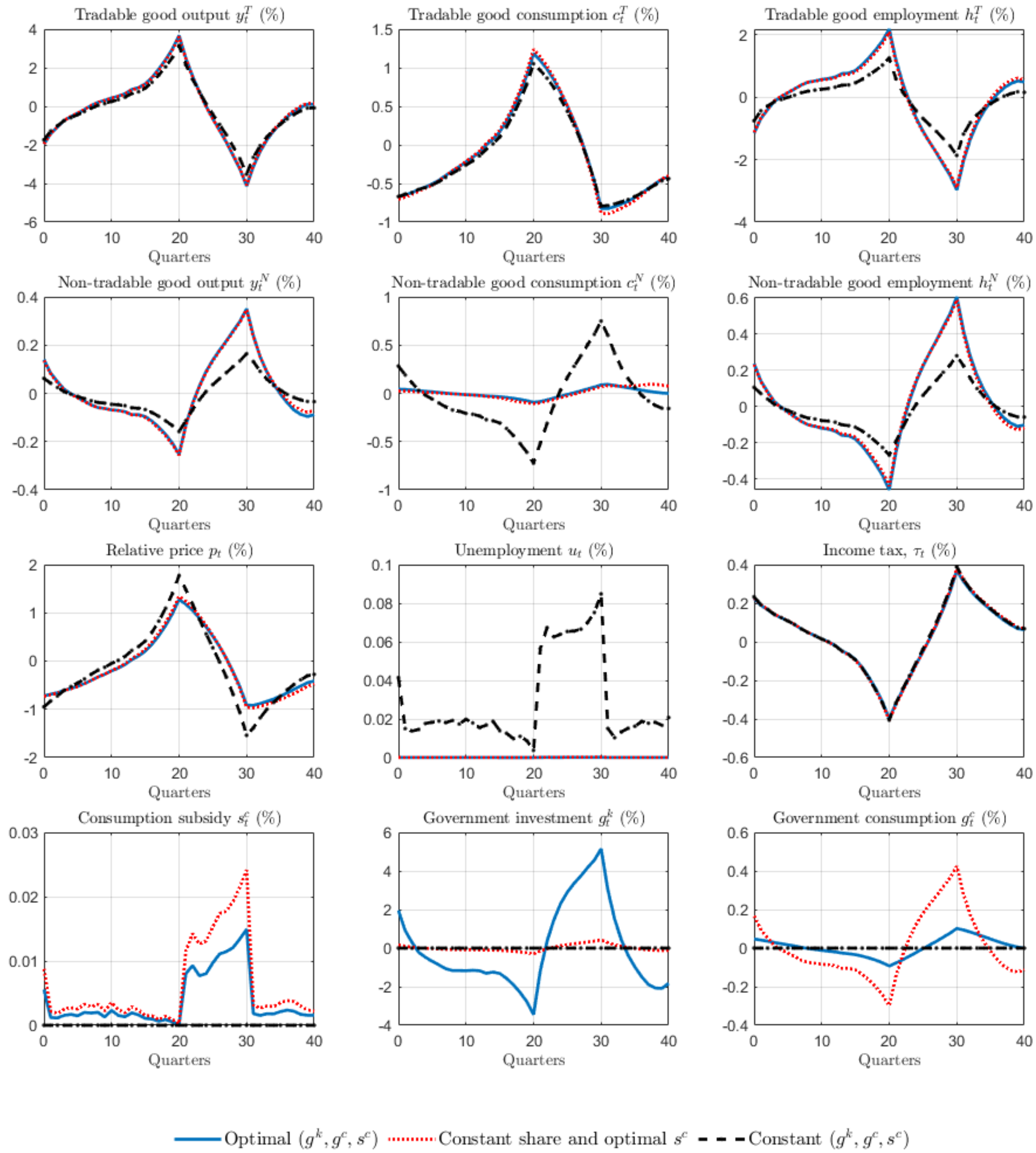
5.2.1 Productivity Shocks in the tradable goods sector

We begin by examining the boom-bust episode driven by shocks in a_t^T. Throughout this experiment, we maintain the productivity in the non-tradable good sector a_t^N at its steady state for all periods. Figures 1 and 2 depict the short run dynamics of the key macroeconomic aggregates.

As we can see in Figure 1, when the government can optimally choose the consumption subsidy, involuntary unemployment remains close to zero for all periods. However, if the government does not adjust consumption subsidy (case for constant (g^k, g^c, s^c) policy) there exists positive unemployment during the boom-bust episode.

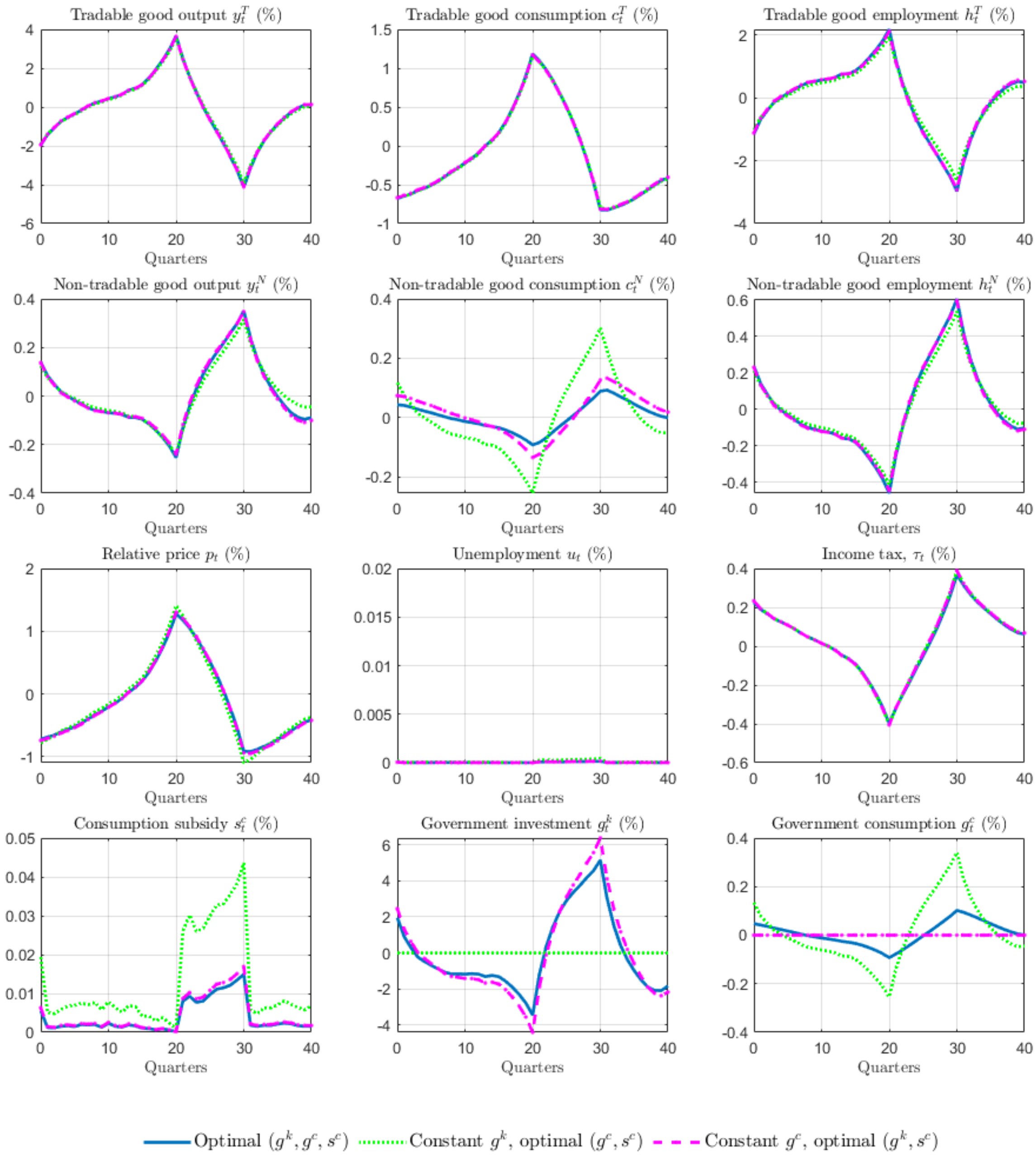
In terms of the tradable sector output, we observe that it follows a pattern similar to that of its productivity, a_t^T. Initially, tradable output increases, reaching its peak in period 20. Subsequently,

Figure 1: Boom-Bust episode with an a_t^T shock



it begins to decline, reaching its lowest level in period 30. There is little disparity in the tradable goods sector across the three different policies. In regard to the non-tradable sector, output moves in the opposite direction to that of the tradable sector throughout the entire boom-bust cycle. This is primarily due to the fact that the labor input in the non-tradable goods sector generally changes inversely relative to the tradable goods sector when involuntary unemployment is low. Additionally, we observe that the output of the non-tradable goods sector is slightly lower under the policies that

Figure 2: Boom-Bust episode with an a_t^T shock



rely on adjustments in either government spending or the consumption subsidy to respond to shocks, compared to the constant (g^k, g^c, s^c) policy when the economy is approaching the trough. However, this pattern reverses as the economy recovers from the recession. In our simulations, employment co-moves with output in each sector.

Consumption follows a trend similar to output in each sector. While there is no significant difference in tradable goods consumption among the policies, we note notable disparities in non-tradable goods consumption under the five policies. It is worth noting that if the government implements changes in spending or consumption subsidy when responding to shocks, the volatility of non-tradable goods consumption is considerably lower than under the constant (g^k, g^c, s^c) policy. Among the policies involving changes in spending or consumption subsidy during the boom-bust episode, the optimal (g^k, g^c, s^c) policy exhibits the lowest volatility in non-tradable good consumption.

Regarding the different government expenditures, Figure 2 shiows that both government investment and consumption spending co-move with output in the non-tradable goods sector if the government can rely on adjustments in spending when responding to economic shocks. In particular, the optimal Ramsey policy (g^k, g^c, s^c), public investment changes more aggressively than public consumption. This response indicates that the optimal response to shocks relies more heavily on public investment. As for the consumption subsidy, it increases when the economy approaches the trough and then decreases as the economy begins to recover. Figure 2 also illustrates that the rise in consumption subsidy during the downturn is more significant when there is a constant g^k and an optimal (g^c, s^c) policy. This leads to higher volatility in non-tradable goods consumption. In other words, the optimal response of the consumption subsidy is less volatile when government investment can optimally respond to economic shocks, resulting in lower volatility in non-tradable consumption.

It is important to highlight that in our environment, there are two primary *opposing forces* that affect the dynamics of the real exchange rate. The first one is the Balassa-Samuelson effect and the other one is the Froot-Rogoff effect. Note that when the tradable goods sector undergoes rapid expansion followed by contraction, the Balassa-Samuelson effect predicts an initial appreciation and subsequent depreciation of the real exchange rate. In contrast, the Froot-Rogoff effect, which is driven by the co-movement of total government expenditure with non-tradable goods output (which moves inversely to tradable goods output), produces opposing effects. Based on our simulation results, it is evident that the Balassa-Samuelson effect dominates the Froot-Rogoff effect. Therefore, the real exchange rate appreciates initially and then depreciates.

How do government spending and consumption subsidy policies help the economy achieve higher welfare? First, adjustments in the consumption subsidy help maintain low unemployment. The consumption subsidy decreases during a booming period, leading to relatively lower consumption of the non-tradable goods compared to scenarios where the consumption subsidy is not adjusted. Consequently, more labor is directed toward the tradable good sector, resulting in a lower wage rate. This in turn reduces the likelihood of encountering a binding downward nominal wage rigidity constraint in the future. When the economy enters a recession, the optimal consumption subsidy increases, which mitigates a rapid fall in non-tradable good consumption. This stimulates labor moving into the non-tradable goods sector and prevent wages from decreasing swiftly. As a result, involuntary unemployment is reduced.

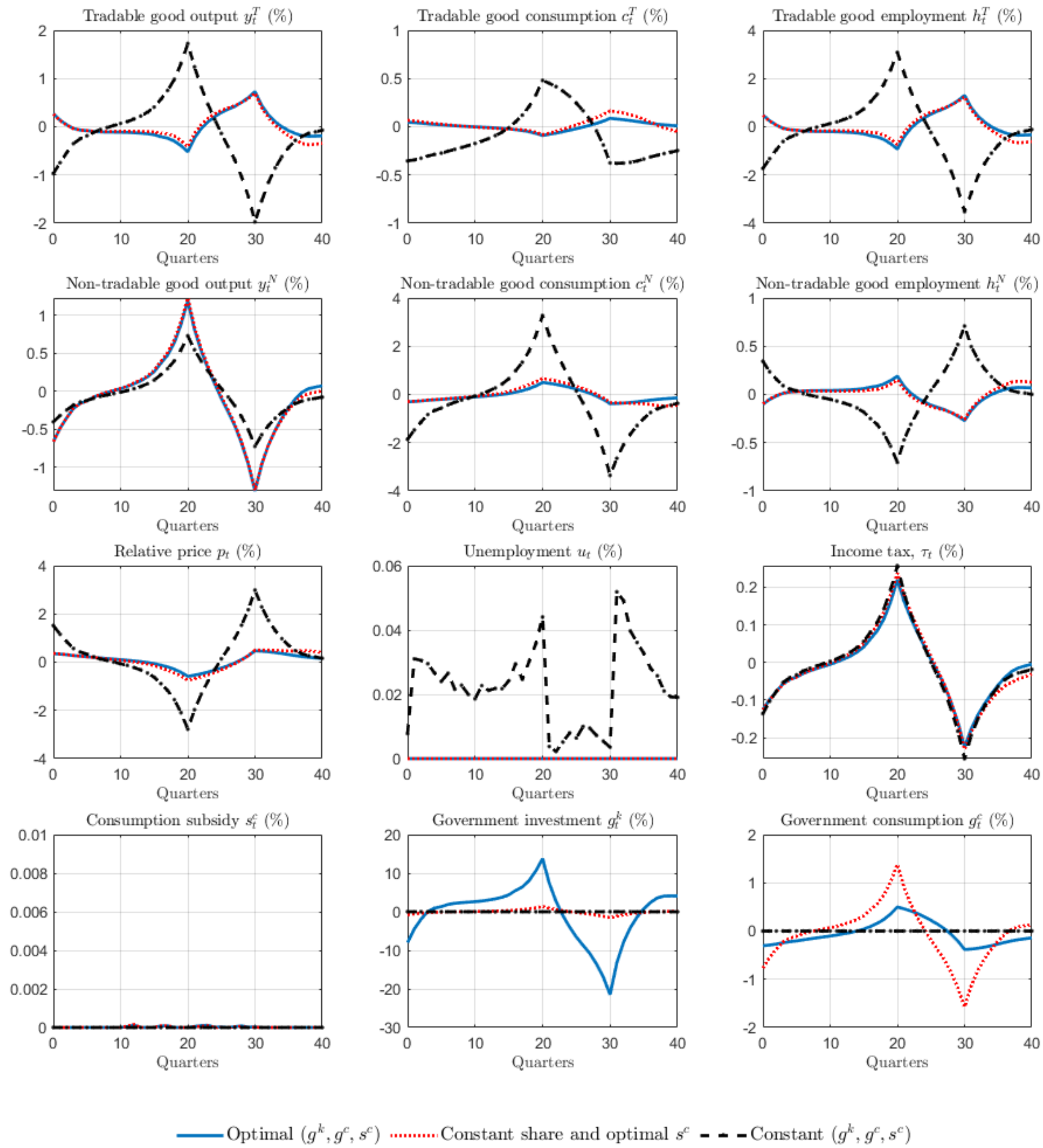
Another important results of our simulations are the role of adjustments in government spending in assisting households smooth their consumption. Consider two policies: (a) the optimal (g^k, g^c, s) policy; and (b) the constant share and optimal s policy. As illustrated in Figure 1, when the government adjusts both the composition and magnitude of spending (as in the optimal (g^k, g^c, s) policy), the consumption of the non-tradable good remains comparatively elevated relative to scenarios where the composition is held constant (as with the constant share and optimal s policy) until period 30. This trend reverses once the economy begins to recover after period 30. Our simulation results suggest that the volatility of consumption the non-tradable good is mitigated when the government has the flexibility to adjust its spending composition. Here is the rationale: when the output of non-tradable goods falls, the government reduces public investment more sharply than public consumption. This is the case as the decrease in public investment does not immediately affect output in the same period. Concurrently, this reduction in public investment crowds in private consumption by households. Therefore, adjustments in the composition of government spending can help smooth household consumption. Further illustration is provided in Figure 2, which shows that to effectively smooth consumption, both public investment and public consumption are necessary. As can be seen, when both public investment and public consumption are adjusted simultaneously, the volatility of non-tradable good consumption is lower compared to the cases when only one type of government spending is adjusted.

5.2.2 Productivity Shocks in the non-tradable good sector

We now analyze the boom-bust episode when they are driven by shocks in the productivity of the non-tradable sector, a_t^N. Similar to the previous analysis, we focus only on situations where there are no shocks to the tradable good sector productivity a_t^T. Figures 3 and 4 present the results.

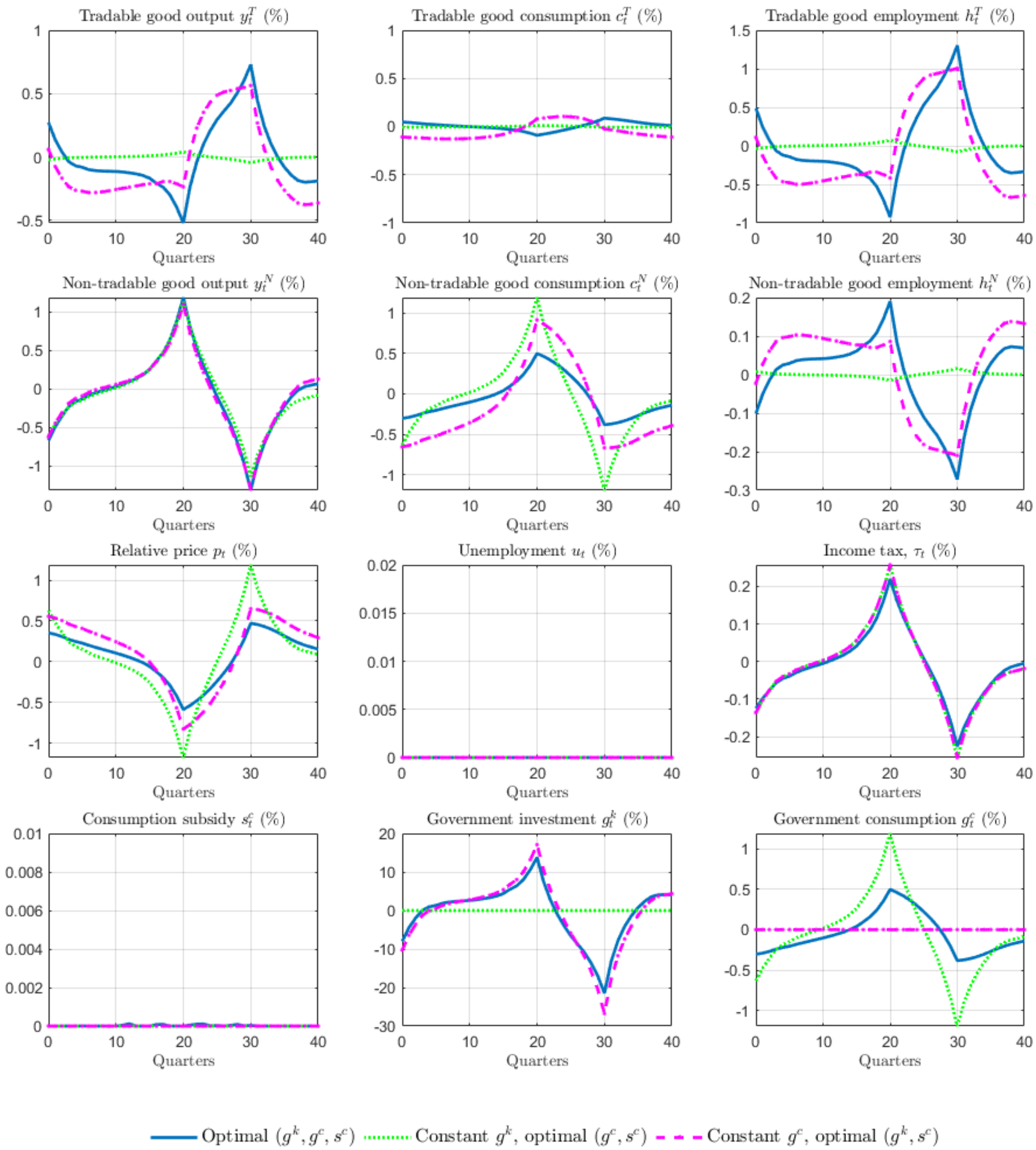
As we can see from the simulation in Figure 3, involuntary unemployment remains close to zero throughout all periods when a consumption subsidy is enacted. For output, the tradable good sector's output under the constant (g^k, g^c, s^c) policy behaves oppositely compared to that under the

Figure 3: Boom-Bust episode with an a_t^N shock



other two policies. More specifically, it initially rises and then falls after period 20, only to start increasing again from period 30. In contrast, the output of the non-tradable good sector co-moves in all policies. Sectoral employment and consumption also co-move with their respective sectoral outputs. Similar to the scenario with the a_t^T shock, consumption of the non-tradable good exhibits significantly lower volatility under both the optimal Ramsey policy (g^k, g^c, s^c) and the constant share plus optimal s policies when compared to the constant (g^k, g^c, s^c) policy.

Figure 4: Boom-Bust episode with an a_t^N shock



When analyzing the real exchange rate, we also have both the Froot-Rogoff effect and the Balassa-Samuelson effect at play. The pro-cyclicality of government spending (with the non-tradable good output) tends to lead to an initial appreciation and subsequent depreciation of the real exchange rate. However, the Balassa-Samuelson effect predicts opposite dynamics for the real exchange rate. Specifically, during the first 20 periods, as the productivity of the non-tradable goods sector increases, the real exchange rate is expected to depreciate in accordance with the Balassa-

Samuelson effect. It then tends to appreciate when the productivity of the non-tradable goods sector diminishes rapidly. Our simulation results indicate that, although the Balassa-Samuelson effect consistently dominates the Froot-Rogoff effect in the context of productivity shocks to the non-tradable sector, the Froot-Rogoff effect can substantially mitigate the Balassa-Samuelson effect when the government adjusts its spending. As depicted in Figure 3, the adjustments in government spending significantly can smooth the fluctuations in the relative price of the non-tradable good. Specifically, when the government increases its spending during an economic expansion, it results in a substantially stronger real exchange rate. This in turn encourages labor to shift toward the non-tradable good sector, thereby increasing the sector's output more rapidly.

The dynamics of the real exchange rate in this experiment can help understand why output and employment exhibit contrasting behavior under the constant (g^k, g^c, s^c) policy when compared to the optimal (g^k, g^c, s^c) and constant share plus optimal s^c policies. As the government refrains from adjusting government spending, the Froot-Rogoff effect becomes zero in this scenario. Consequently, the real exchange rate experiences an initial aggressive depreciation, which dominates the increase in productivity within the non-tradable good sector in our experiment. Hence, labor shifts from the non-tradable goods sector to the tradable one, leading to an increase in output in the tradable goods sector.

Note that when labor shifts into the tradable goods sector as a response to increased productivity in the non-tradable good sector, it paradoxically results in lower wages in the economy. In this scenario, the likelihood of a binding downward nominal wage rigidity increases. This results in higher levels of involuntary unemployment when the productivity in the non-tradable goods sector rises.

Intriguingly, in the a_t^N shock scenario, the changes in consumption subsidy in response to a_t^N shocks are relatively minor. This implies that government spending policies themselves are effective in mitigating involuntary unemployment. Additionally, we observe that adjustments in government investment exhibit greater volatility compared to adjustments in government consumption under the optimal (g^k, g^c, s^c) policy. The underlying rationale aligns with that of our prior analysis in the a_t^T shock case.

5.2.3 Combining a_t^T and a_t^N shocks

We now analyze the boom-bust episode when generated by two productivity shocks. Specifically, we consider periods that meet the following criteria: (i) both productivities are at or below the trend in the initial period; (ii) at least one productivities (a_t^T or a_t^N) is more than one standard deviation above trend at period 20; and (iii) the productivity that experiences an increase of more than one standard deviation at time 10 is at least one standard deviation below trend at period 20. We compute the average of macroeconomic aggregates over all the sub-periods. These results are depicted in Figures 5 and 6.

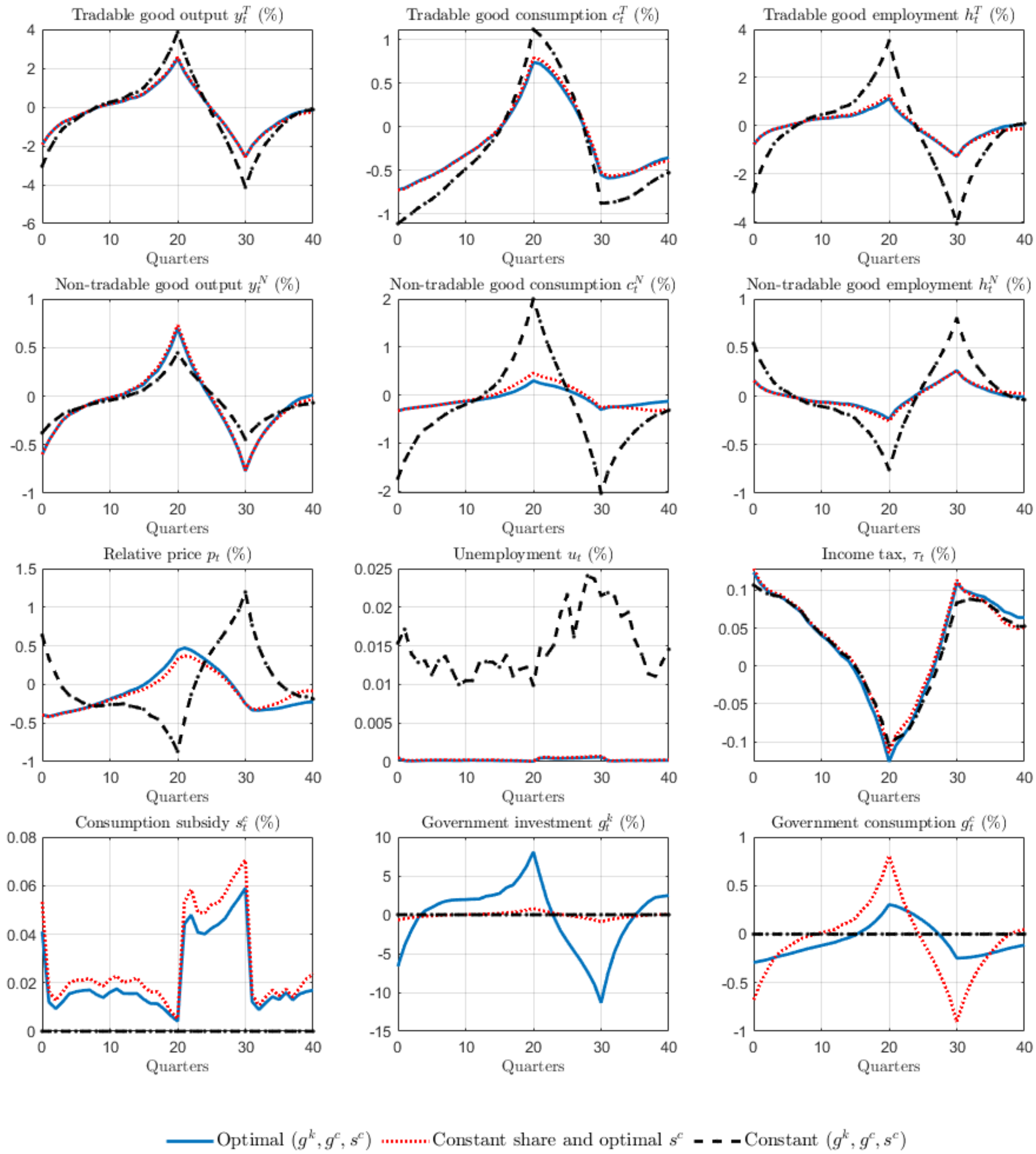
A few remarks are in order. First, all policies now produce a synchronous movement between the output in the tradable goods sector and the output in the non-tradable goods sector. This co-movement is largely attributable to the positive correlation between the productivities in these two sectors. Regarding employment, on average, employment in the tradable good sector increases during booming periods and declines during recessions. In contrast, employment in the non-tradable goods sector consistently moves in the opposite direction.

Second, consumption in both the tradable and non-tradable good sectors initially rises during economic boom periods, then falls when the economy enters a recession. Consumption of non-tradable goods under the optimal (g^k, g^c, s^c) policy and the constant share and optimal s^c policy remain similar during the first 30 periods. However, beyond that point, the optimal (g^k, g^c, s^c) policy generates higher consumption of non-tradable goods as the economy recovers from a recession.

Third, the real exchange rate, under policies that adjust government spending, co-moves with total government spending. Specifically, it appreciates during economic booms and depreciates during recessions. In contrast, when no government policies are implemented, the real exchange rate moves in the opposite direction compared to scenarios where government spending or consumption subsidy policies are in place. This suggests that the Froot-Rogoff effect has significant economic implications.

Fourth, government spending across all policies exhibits a pro-cyclical pattern. Moreover, the Ramsey policy reveals a preference for making adjustments in public investment rather than public consumption as it is the best way to respond to economic shocks.

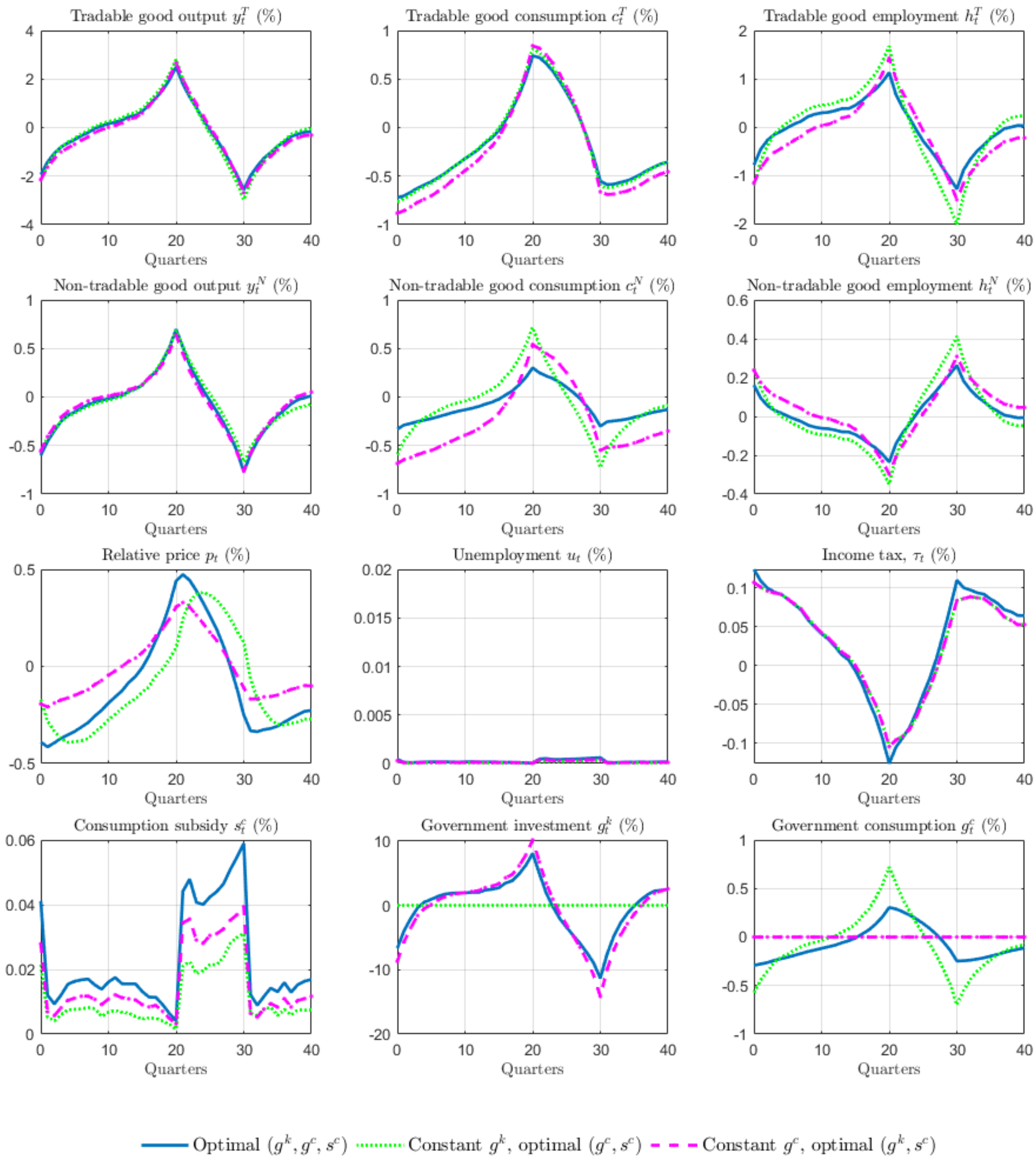
Figure 5: Boom-Bust episode with both a_t^T and a_t^N shocks



5.3 Welfare analysis

Having analyzed the transmission of shocks under different policies, we now evaluate welfare. To do so, we compute the welfare costs associated with switching from the Ramsey policy to the other policies we previously simulated. To compare these welfare costs, we define λ^j as the shares of consumption that a household would need to sacrifice if the government switches from the Ramsey

Figure 6: Boom-Bust episode with both a_t^T and a_t^N shocks



policy to the j policy. Specifically, λ^j satisfies the following condition:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{\left(\left(1-\lambda^j\right)c_t^{Ramsey}\right)^{1-\sigma}}{1-\sigma}+v\left(g_t^{c,Ramsey}\right)\right)\right]=\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma}+v\left(g_t^{c,j}\right)\right)\right]$$

where $j \in \{Opt(g^k, g^c, s^c)$, Const share, opt s, Const (g^k, g^c, s^c), Const g^k and Opt (g^c, s^c), Const g^c and Opt $(g^k, s^c)\}$. Table 2 reports the resulting welfare costs.

Table 2: Welfare cost as % consumption

Variable	Opt (g^k, g^c, s^c)	Const share, opt s^c	Const (g^k, g^c, s^c)	Opt (g^c, s^c)	Opt (g^k, s^c)
λ	0	0.033	0.017	0.005	0.001

As shown in Table 2, the constant share and opt s^c policy is associated with the lowest level of social welfare. This is primarily due to the non-optimal adjustments in government consumption and investment, despite the optimal choice of consumption subsidy. When adjustments in government spending (either public investment or public consumption) are possible, welfare improves compared to the constant (g^k, g^c, s^c) policy. This is particularly the case when adjustments are made in public investment spending. The welfare analysis indicates that while an appropriate consumption subsidy policy can effectively reduce involuntary unemployment, government spending plays a crucial role in enhancing social welfare. Our results particularly highlight the significance of government investment in absorbing exogenous shocks. Finally, it is noteworthy that the welfare differences between various policies are *small*. This is because we assume that equilibria under all policies converge to the same steady state. In other words, we are not considering the corresponding distorted steady states. Therefore, the observed differences are solely attributable to variations in short-run behavior.

5.4 Business Cycle Statistics

In this section, we analyze the business cycle properties of our model when productivity shocks affect both sectors. First, we report statistics, in Table 3, of involuntary unemployment under the four government policies and decentralized economy.[19]

As can be seen from Table 3 involuntary unemployment arises in approximately 6.6 percent of the total simulation periods under the Const (g^k, g^c, s^c) policy. We also compute the mean and maximum values of unemployment under the different policies. The average unemployment and maximum value of unemployment under this policy are 0.014 and 1.48 percent, respectively. No-

[19] Numerically, the unemployment computed from our code may take a very small positive value even when the DNWR is not binding. To address this, we interpret unemployment greater than 10^{-4} as indicative of positive involuntary unemployment.

Table 3: Unemployment

Variable	Opt (g^k, g^c, s^c)	Const share, opt s^c	Const (g^k, g^c, s^c)	Opt (g^c, s^c)	Opt (g^k, s^c)
Probability (%)	0	0	0.066	0	0
Mean (%)	0	0	0.014	0	0
Max (%)	0	0	1.48	0	0

tably, the absence of the probability of involuntary unemployment in the case of Const share, opt s^c underscores that a consumption subsidy is the most effective policy tool for reducing involuntary unemployment. It is important to note that the likelihood reported in Table 3 aligns with our previous boom-bust analysis, which shows that including the optimal consumption subsidy effectively maintains low unemployment levels during most periods. During an economic downturn, the optimal Ramsey policy involves an increase in the consumption subsidy. This adjustment helps alleviate a rapid decline in non-tradable goods consumption. It prompts labor to move towards the non-tradable goods sector, preventing a swift decrease in wages. Consequently, involuntary unemployment is minimized.

Next, it is important to investigate the volatility and the cyclicality of key macroeconomic aggregates under different policies. Table 4 presents the volatility and the cyclicality for the relevant equilibrium observable, which include output, hours of work and consumption in both sectors, the real exchange rate ($RER_t = p_t^{1-\psi}$), government investment, and government consumption.

Note that in our model, we define total output as $y_t = y_t^T + p_t y_t^N$. To calculate the volatility of the variables, we determine the standard deviation of the log deviations of the variables from their steady-state, relative to the standard deviation of the log deviation of total output. Cyclicality is measured by computing the correlation between the log deviations of variables from their steady state and the log deviation of the total output.

As reported in Table 4, the observed pattern in the magnitudes of the volatility of the macro aggregates for the Ramsey policy, i.e. Opt (g^k, g^c, s^c), and Const share, opt s^c, are consistent with the data. Specifically, in the data, the tradable sector exhibits volatility exceeding one for output, hours of work, and consumption, while the non-tradable sector's corresponding measures remain below one. For output in the tradable and non-tradable sectors and hours of work in the tradable sector, we see some volatility differences exist between these two policies. These, however, are moderate. For the consumption of tradable goods and nontradable goods, no significant volatility disparities between these two policies are apparent.

Table 4: Volatility and Cyclicality

Variable	Data	Opt (g^k, g^c, s^c)	Const share, opt s^c	Const (g^k, g^c, s^c)	Opt (g^c, s^c)	Opt (g^k, s^c)
			Volalitlity			
Tradable y^T (%)	2.00	2.43	2.37	2.87	2.85	2.95
Non-tradable y^N (%)	0.95	0.78	0.75	0.43	0.79	0.84
Tradable h^T (%)	1.07	1.52	1.49	2.66	1.79	1.94
Non-tradable h^N (%)	0.9	0.31	0.31	0.55	0.37	0.40
Tradable c^T (%)	1.01	1.06	1.06	1.29	1.24	1.46
Non-tradable c^N (%)	0.68	0.46	0.45	1.95	0.84	1.37
Real exchange rate (%)	1.96	0.45	0.45	0.72	0.56	0.54
Gov investment (%)	3.08	10.1	0.88	0	0	14.1
Gov consumption (%)	1.43	0.45	0.88	0	0.83	0
			Cyclicality			
Tradable y^T	0.83	0.68	0.67	-0.08	0.03	0.04
Non-tradable y^N	0.69	0.64	0.63	-0.57	0.39	0.45
Tradable h^T	0.63	0.24	0.20	-0.45	0.37	0.31
Non-tradable h^N	0.70	-0.24	-0.20	0.46	-0.37	-0.31
Tradable c^T	0.57	0.92	0.91	0.27	1	0.73
Non-tradable c^N	0.72	0.57	0.48	-0.57	0.39	0.16
Real exchange rate (%)	-0.27	0.77	0.79	0.97	0.78	0.75
Gov investment (%)	0.17	0.51	0.63	0	0	0.49
Gov consumption (%)	-0.42	0.59	0.63	0	0.39	0

One important feature that we observe in the data is the higher volatility in government investment compared to government consumption. The Ramsey policy, Opt (g^k, g^c, s^c), is the only policy capable of generating this observed pattern. However, it is important to note that the magnitude of volatility for government investments in our model is significantly higher than for consumption. Quantitatively, the volatility in optimal public investment under the Ramsey policy is more than 20 times that of the volatility in optimal public consumption (while the data counterpart is around two). As per our previous analysis, the government heavily relies on adjustments in government investment to respond to shocks, leading to high public investment volatility under the Ramsey policy, while government consumption shows lower volatility.

Regarding cyclicality, the sectoral outputs and consumptions all are pro-cyclical for the Ramsey policy, i.e. Opt (g^k, g^c, s^c), and Const share, opt s. These are consistent with the data. The correlations between sectoral outputs and total output, as well as the correlation between consumption of tradable goods and total output, exhibit negligible differences under the two policies. However, the correlation between the non-tradable consumption and total output in the Ramsey policy is notably higher at 0.57, compared to the Const share, opt s^c case, which is at 0.48. This implies that the cyclicality of non-tradable consumption in the Opt (g^k, g^c, s) model is closer to its counterpart in the data, which stands at 0.72. As for government expenditures, data indicates that government investment tends to move in line with the business cycle (procyclical), while government consumption shows a countercyclical behavior. The model replicates the procyclical trend for both policies. However, a discrepancy arises in the correlation between government consumption and total output under the two policies in the model, where it is positive, while in the data, it is negative.

Finally, it is not too surprising that we do not fully capture the dynamics real exchange rate (RER) when compared to the data counterpart. This is the case as our framework has that that the law of one price (LOP) holds. But as Engel (1993) argues that the dynamics of RER in a small open economy can largely be driven by the deviations of LOP.

5.5 Asymmetric Responses to shocks

Our model features a key asymmetry when it comes to responses to shocks. Specifically, while the nominal wage cannot fall below a fraction of the previous wage level during business cycles, it has no corresponding upper limit during booms. This asymmetry leads to the natural question of whether macroeconomic aggregates will exhibit asymmetric adjustments during economic booms and busts. In this section, we investigate how government spending policies are affected by such asymmetry introduced by the DNWR. Relative to Born et al. (2023), we can determine how much of the asymmetric response is driven by productive government investment.

In what follows, we define a period as an economic expansion (downturn) if the total output exceeds (falls below) the level observed in the previous period. Table 5 presents the means of different components of government spending and total government spending during economic upturns and downturns. To calculate the means of government spending, we compute the average log deviations of those variables from their steady-state levels.

In our analysis of government spending, the results indicate that under the Constant Share scenario (denoted as Const share, optimal s^c), there is no significant asymmetry in deviations between economic upturns and downturns for total government spending, as well as for government consumption and investment. However, in the Optimal Scenario (g^k, g^c, s^c), a pronounced asymmetry is observed in government investment between economic upturns and downturns. Specifically, the decline in government investment is more severe during economic downturns compared to the

Table 5: Asymmetric responses

Variable	Opt (g^k, g^c, s^c)	Const share, opt s^c	Const (g^k, g^c, s^c)	Opt (g^c, s^c)	Opt (g^k, s^c)
Economic upturn					
Total Gov spending (%)	0.27	0.27	0	0.18	0.26
Gov investment (%)	3.53	0.27	0	0	3.51
Gov consumption (%)	0.03	0.27	0	0.19	0
Consumption subsidy (%)	0	0	0	0	0
Economic downturn					
Total Gov spending (%)	-0.25	-0.25	0	-0.18	-0.25
Gov investment (%)	-4.32	-0.25	0	0	-4.68
Gov consumption (%)	-0.01	-0.25	0	-0.19	0
Consumption subsidy (%)	0.04	0.05	0	0.02	0.03

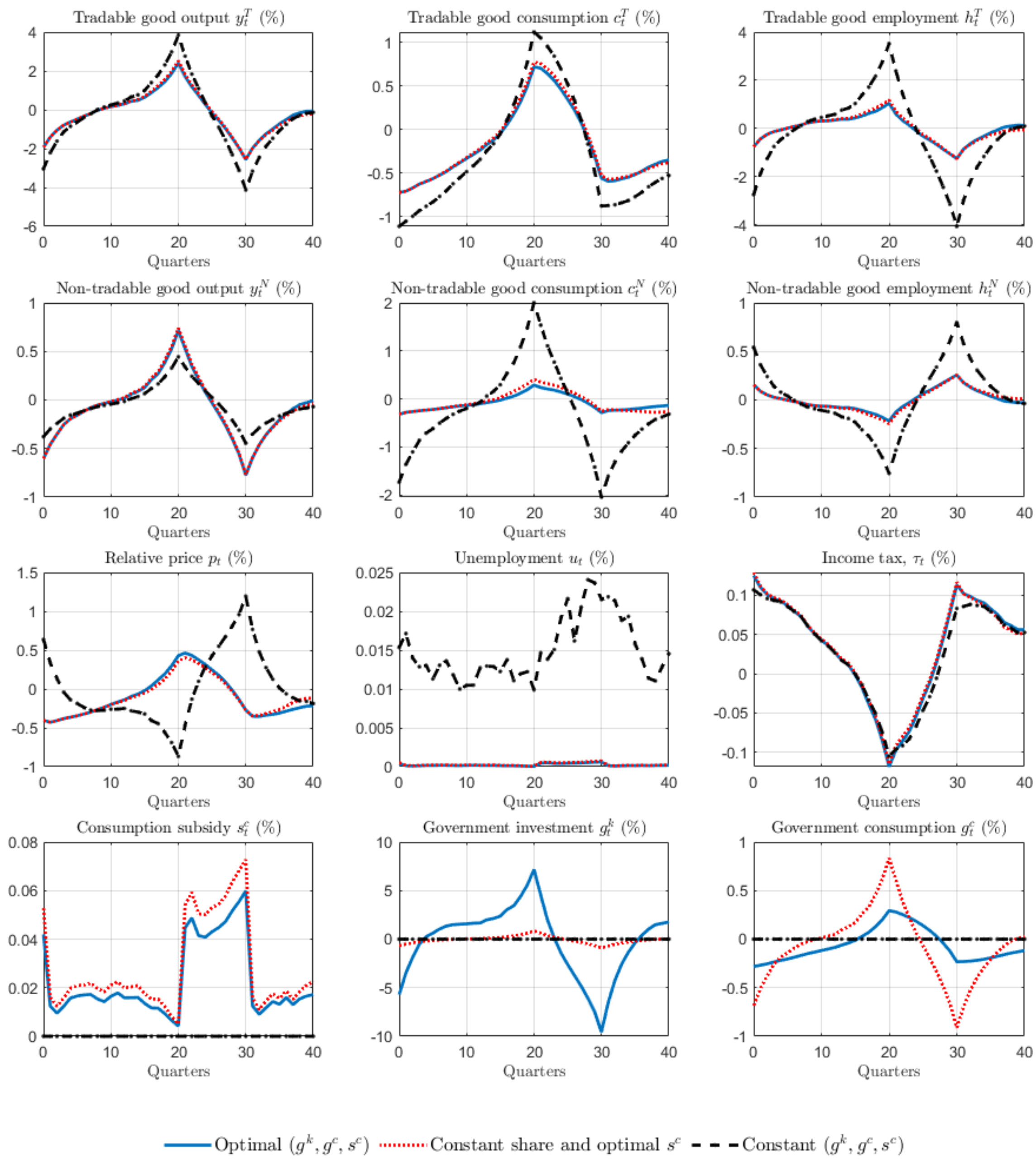
increase in government investment during upturns. This finding is not surprising, as the loss in welfare from reducing government investment during downturns is minimal compared to government consumption. This is because public capital takes time to become operational and productive in both the tradable and non-tradable sectors.

Regarding the consumption subsidy, it has been observed that the government consistently increases this subsidy during economic downturns while maintaining it at a minimal level during periods of economic boom. This asymmetric pattern in the allocation of consumption subsidies aligns with the rationale discussed in Schmitt-Grohé and Uribe (2012). That is, governments set lower consumption subsidies during economic booms to prevent wages from rising too rapidly. Conversely, during recessions, the government encourages household consumption to stimulate aggregate demand. This strategy helps increase labor demand and reduce involuntary unemployment (caused by the downward nominal wage rigidity) when the economy enters recession.

5.6 Robustness checks

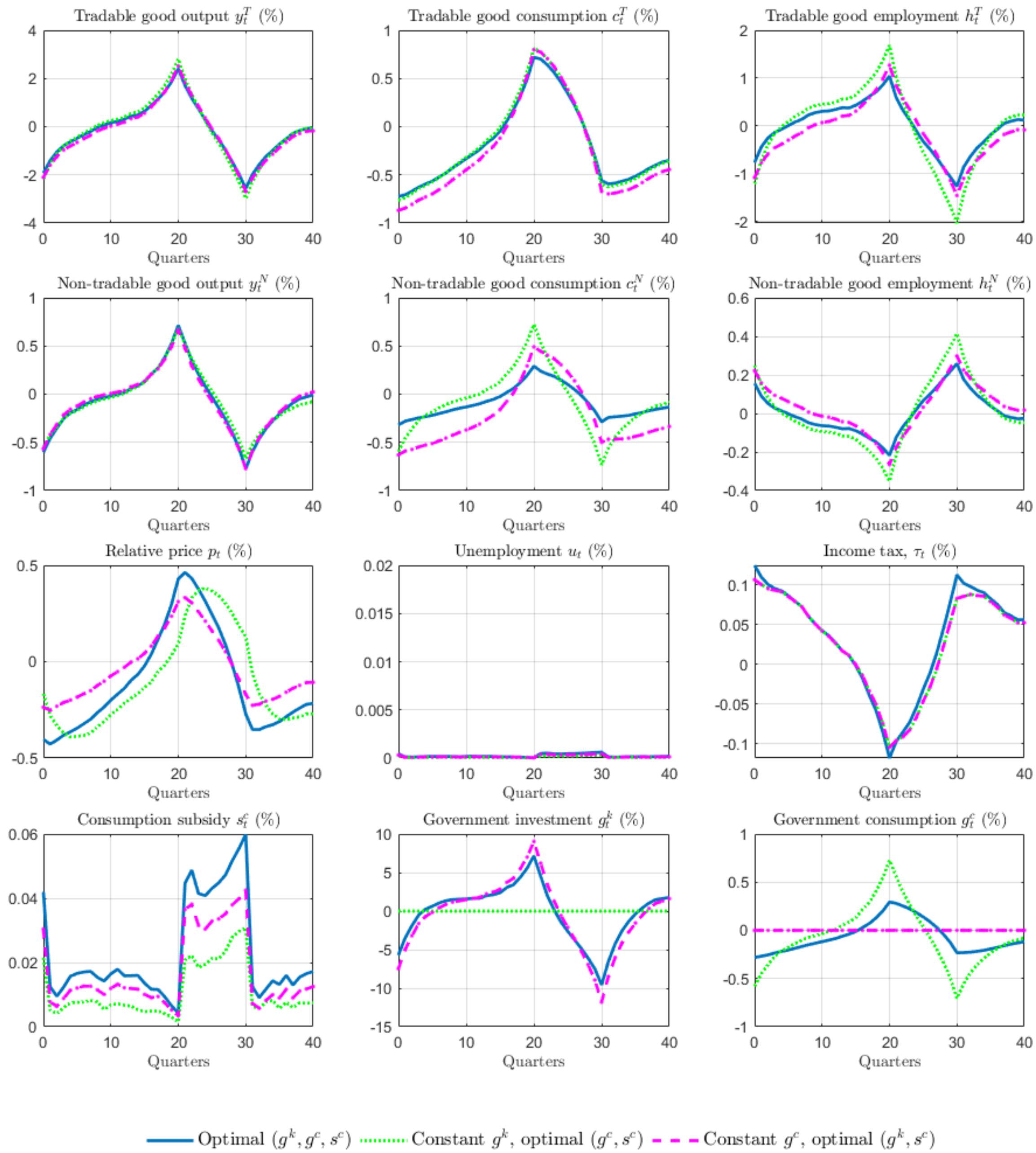
In this section, we consider two robustness checks. First, we consider a positive value of ϕ^T (instead of setting ϕ^T to zero as in the baseline calibration) and examine its impact on key macroeconomic aggregates. The responses of the macroeconomic aggregates during the boom-bust episode under this scenario are illustrated in Figures 7 and 8. In the second robustness check, we consider a shortened time period required for public investment to turn into productive public capital. Specifically, we assume that it takes 8 periods for public investment to become public capital. The responses of the macroeconomic aggregates during the boom-bust episode under this scenario are illustrated in Figures 9 and 10.

Figure 7: Boom-Bust episode when $\phi^T = \phi^N$



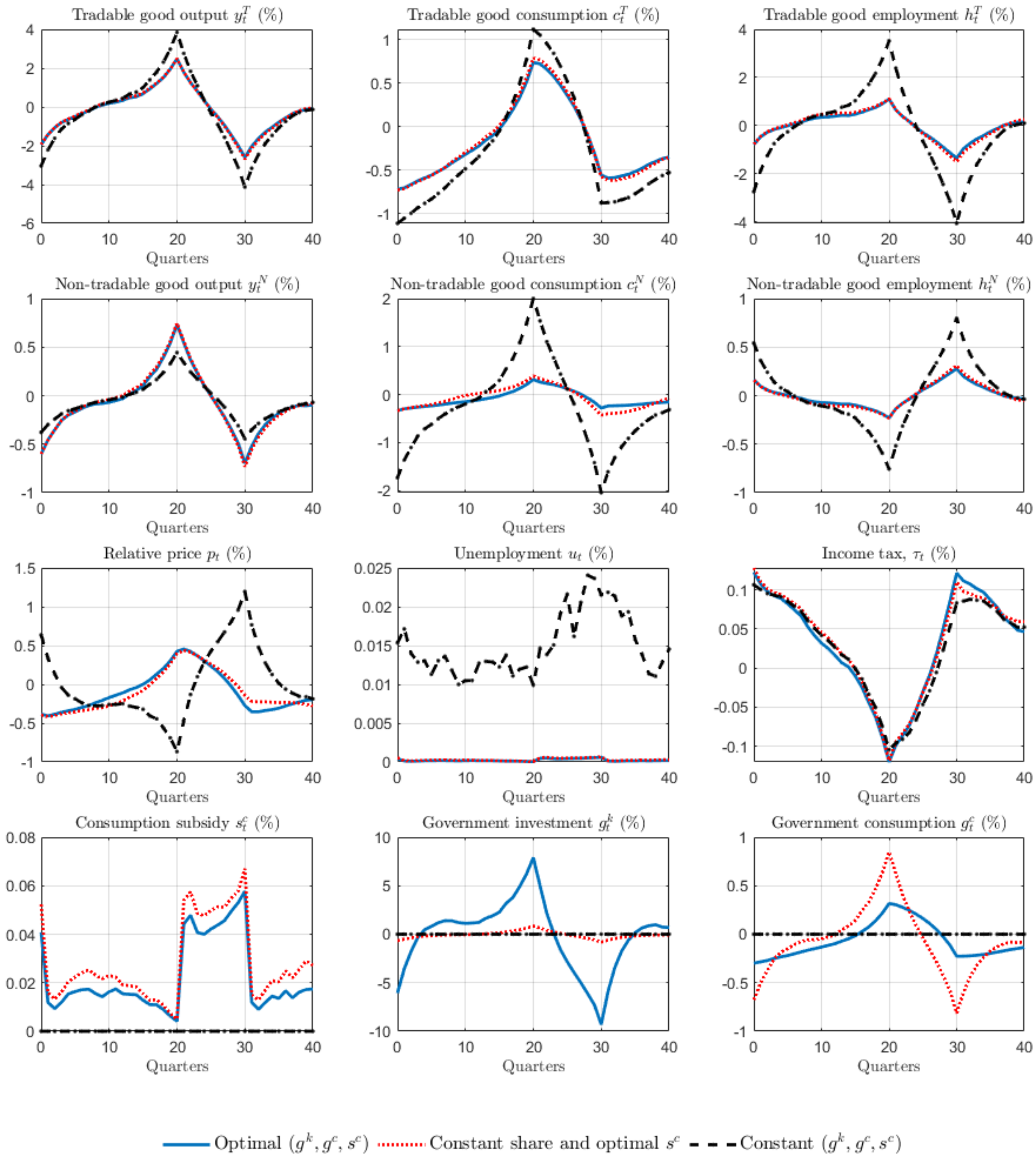
The results depicted in Figures 7-8 and 9-10 validate the robustness of the qualitative predictions of our baseline findings. Specifically, optimal adjustments in the consumption subsidy play a crucial role in sustaining low unemployment levels. Moreover, changes in government spending, under the

Figure 8: Boom-Bust episode when $\phi^T = \phi^N$



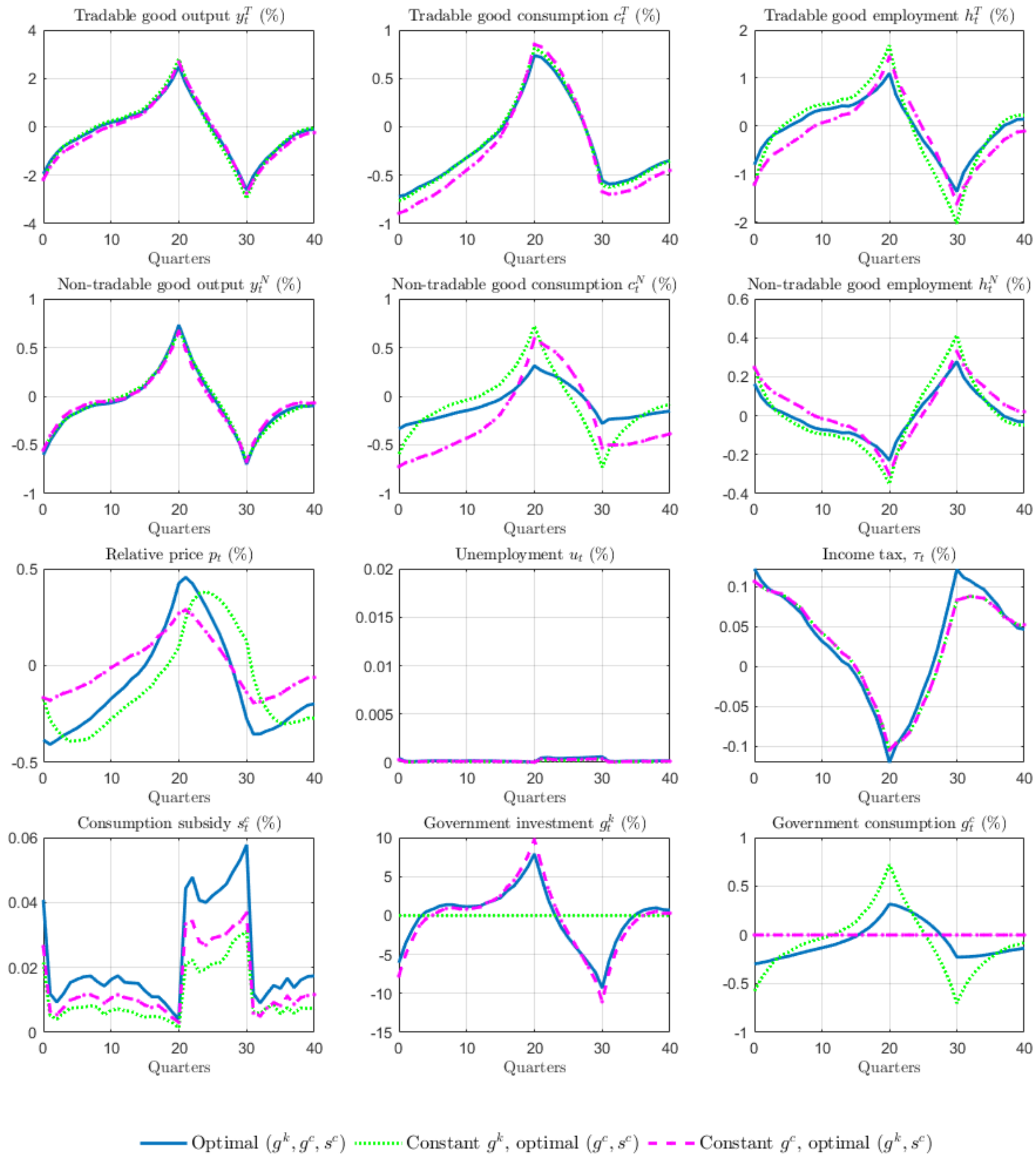
Opt (g^k, g^c, s^c) policy, contribute to smoothing household consumption, thereby further enhancing social welfare.

Figure 9: Boom-Bust episode with 8 periods delay



6 Conclusions

In this paper we have analyzed the mechanism through which government consumption and investment affect the transmission of shocks. We do so in the context of a two-sector small open economy that belongs to a monetary union and faces downward nominal wage rigidities. Most of the literature has previously focused on the effects of various tax policies, while government



consumption investment and financing policies have received far less attention. In contrast, we introduce government consumption, that is valued by households, as well as government investment in the form of a gradually accumulating stock of productivity-augmenting infrastructure capital. In addition, we consider a consumption subsidy. These various spending programs are financed by income taxes.

We characterize the Ramsey optimal policy by adjusting the composition between public investment and public consumption as well as sets a consumption subsidy. When calibrated to France, our simulation results show that total government spending and the composition between public investment and public consumption play different roles in enhancing social welfare. Specifically, optimal adjustments in the consumption subsidy play a crucial role in maintaining low unemployment. Moreover, variations in government spending not only amplify the impact of the consumption subsidy on unemployment, but also help households smooth private consumption. This type of intervention results in the highest welfare. Finally, in terms of the real exchange rate, our environment exhibits two opposing forces: the Balassa-Samuelson and Froot-Rogoff effects. Based on our simulation results, the Balassa-Samuelson effect dominates. Therefore, during a boom-bust cycle, the real exchange rate initially appreciates and then depreciates. All these findings are robust to various productivity shocks and the size of the capital share in the production of tradable and non-tradable goods

Summarizing, we have highlighted the importance of explicitly considering the composition of government expenditures when studying the transmission of shocks in a small open economy with fixed exchange rates. Adjusting government investment offers two key benefits. First, a gradual decline in government consumption has a minimal impact on reducing social welfare. This is the case as government consumption directly affects households' utility every period. Second, the direct effect of government investment on current output is relatively small as it takes time for public investment to deliver public capital that can be used in the production. Thus, by incorporating government investment in economic models to gain a more accurate understanding of the economy's resilience to shocks.

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Appendix

A Competitive equilibrium

Given $\left\{\tau_t,\, g_t^c,\, g_t^k,\, r^*\right\}$, we have 12 endogenous variables $\left\{c_t^T,\, c_t^N,\, \lambda_t,\, d_t,\, h_t^T,\, h_t^N,\, y_t^T,\, y_t^N,\, h_t,\, k_t^G,\, p_t,\, w_t\right\}$. The complete set of conditions describing the dynamics of the competitive equilibrium are as follows

1. c_t^T :
$$\psi c_t^{-\sigma}\left(\frac{c_t}{c_t^T}\right) = \lambda_t\left(1+\tau_t^c\right) \tag{23}$$

2. c_t^N :
$$\left(1-\psi\right) c_t^{-\sigma}\left(\frac{c_t}{c_t^N}\right) = \lambda_t p_t\left(1+\tau_t^c\right) \tag{24}$$

3. λ_t :
$$1+\chi\left(d_t-\bar{d}\right) = \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right] \tag{25}$$

4. d_t :
$$0 = \left(1+r^*\right) d_{t-1} + y_t^T - \left(c_t^T + d_t + \frac{\chi}{2}\left(d_t-\bar{d}\right)^2\right) \tag{26}$$

5. h_t^T :
$$0 = \alpha_T a_t^T\left(k_t^G\right)^{\phi_T}\left(h_t^T\right)^{\alpha_T-1} - w_t \tag{27}$$

6. h_t^N :
$$0 = \alpha_N p_t a_t^N\left(k_t^G\right)^{\phi_N}\left(h_t^N\right)^{\alpha_N-1} - w_t \tag{28}$$

7. y_t^T :
$$0 = a_t^T\left(k_t^G\right)^{\phi_T}\left(h_t^T\right)^{\alpha_T} - y_t^T \tag{29}$$

8. y_t^N :
$$0 = a_t^N\left(k_t^G\right)^{\phi_N}\left(h_t^N\right)^{\alpha_N} - y_t^N \tag{30}$$

9. h_t :
$$0 = h_t - \left(h_t^T + h_t^N\right) \tag{31}$$

10. k_t^G :
$$0 = g_t^k + \left(1-\delta\right) k_t^G - k_{t+1}^G \tag{32}$$

11. p_t :
$$0 = y_t^N - \left(c_t^N + g_t^c + g_t^k\right) \tag{33}$$

12. w_t :
$$\left(\bar{h}-h_t\right)\left(w_t-\gamma w_{t-1}\right) = 0 \tag{34}$$

    where
$$w_t \geq \gamma w_{t-1} \text{ and } h_t \leq \bar{h}$$

B Ramsey Policy

We consider the following functional form for the utility of government consumption $v\left(g_{t+s}^c\right) = \kappa \frac{(g_t^c)^{1-\sigma}}{1-\sigma}$.

1. c_t^T :
$$\psi c_t^{-\sigma}\left(\frac{c_t}{c_t^T}\right) + \mu_t^{h^N}\frac{w_t}{c_t^T} = \mu_t^d \tag{35}$$

2. d_t :
$$1 + \chi\left(d_t - \bar{d}\right) = \mathbb{E}_t\left[\frac{\mu_{t+1}^d}{\mu_t^d}\right] \tag{36}$$

3. h_t^T :
$$0 = \left(\alpha_T - 1\right)\mu_t^{h^T}\frac{w_t}{h_t^T} + \alpha_T\mu_t^{y^T}\frac{y_t^T}{h_t^T} - \mu_t^h \tag{37}$$

4. h_t^N :
$$0 = \left(\alpha_N - 1\right)\mu_t^{h^N}\frac{w_t}{h_t^N} + \alpha_N\mu_t^{y^N}\frac{y_t^N}{h_t^N} - \mu_t^h \tag{38}$$

5. y_t^T :
$$0 = \mu_t^d - \mu_t^{y^T} \tag{39}$$

6. y_t^N :
$$0 = \left(1-\psi\right)c_t^{-\sigma}\left(\frac{c_t}{c_t^N}\right) - \mu_t^{h^N}\frac{w_t}{c_t^N} - \mu_t^{y^N} \tag{40}$$
where
$$c_t^N = y_t^N - g_t^c - g_t^k$$

7. k_{t+1}^G :
$$0 = \beta\mathbb{E}_t\left[\phi_T\left(\mu_{t+1}^{y^T}\frac{y_{t+1}^T}{k_{t+1}^G} + \mu_{t+1}^{h^T}\frac{w_{t+1}}{k_{t+1}^G}\right) + \phi_N\left(\mu_{t+1}^{y^N}\frac{y_{t+1}^N}{k_{t+1}^G} + \mu_{t+1}^{h^N}\frac{w_{t+1}}{k_{t+1}^G}\right)\right] + \left(1-\delta\right)\beta\mathbb{E}_t\mu_{t+1}^{k^G} - \mu_t^{k^G} \tag{41}$$

8. h_t :
$$0 = \mu_t^h - \mu_t^w\left(w_t - \gamma w_{t-1}\right) \tag{42}$$

9. w_t :
$$0 = -\mu_t^{h^T} - \mu_t^{h^N} + \mu_t^w\left(1 - h_t\right) - \gamma\beta\mathbb{E}_t\left[\mu_{t+1}^w\left(1 - h_{t+1}\right)\right] \tag{43}$$

10. g_t^c :
$$0 = v'\left(g_t^c\right) - \left(1-\psi\right)c_t^{-\sigma}\left(\frac{c_t}{c_t^N}\right) + \mu_t^{h^N}\frac{w_t}{c_t^N} \tag{44}$$

11. g_t^k :
$$0 = \beta^T\mu_{t+T}^{k^G} - \left(1-\psi\right)c_t^{-\sigma}\left(\frac{c_t}{c_t^N}\right) + \mu_t^{h^N}\frac{w_t}{c_t^N} \tag{45}$$

12. μ_t^d :

$$0 = (1 + r^*) d_{t-1} + y_t^T - \left(c_t^T + d_t + \frac{\chi}{2} \left(d_t - \bar{d}\right)^2\right) \tag{46}$$

13. $\mu_t^{h^T}$:

$$w_t = \alpha_T a_t^T \left(k_t^G\right)^{\phi_T} \left(h_t^T\right)^{\alpha_T - 1} \tag{47}$$

14. $\mu_t^{h^N}$:

$$w_t = \alpha_N p_t a_t^N \left(k_t^G\right)^{\phi_N} \left(h_t^N\right)^{\alpha_N - 1} \tag{48}$$

15. $\mu_t^{y^T}$:

$$0 = a_t^T \left(k_t^G\right)^{\phi_T} \left(h_t^T\right)^{\alpha_T} - y_t^T \tag{49}$$

16. $\mu_t^{y^N}$:

$$0 = a_t^N \left(k_t^G\right)^{\phi_N} \left(h_t^N\right)^{\alpha_N} - y_t^N \tag{50}$$

17. μ_t^h :

$$0 = h_t - \left(h_t^T + h_t^N\right) \tag{51}$$

18. $\mu_t^{k^G}$:

$$0 = g_{t-T}^k + (1 - \delta) k_t^G - k_{t+1}^G \tag{52}$$

19. μ_t^w :

$$w_t = \max\left\{\frac{\alpha_T y_t^T + \alpha_N p_t y_t^N}{\bar{h}}, \gamma w_{t-1}\right\} \tag{53}$$

In what follows we also allow the composition of government spending to be subject to shocks. For example, we consider an experiment in which the fraction of public investment follows an AR(1) process given by:

$$\log\left(\frac{g_t^k}{g_{ss}^k}\right) = \rho_g \log\left(\frac{g_{t-1}^k}{g_{ss}^k}\right) + \sigma^{g^k} \varepsilon_t^{g^k}$$

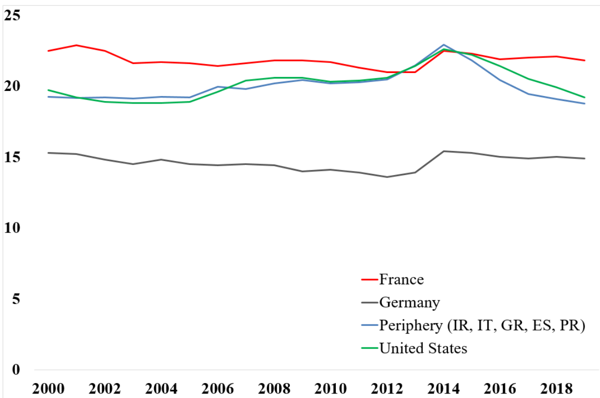
This allows us to compare the results of this experiment (the public investment follows an exogenous path) with the Ramsey planner allocation.

C Additional figures and tables

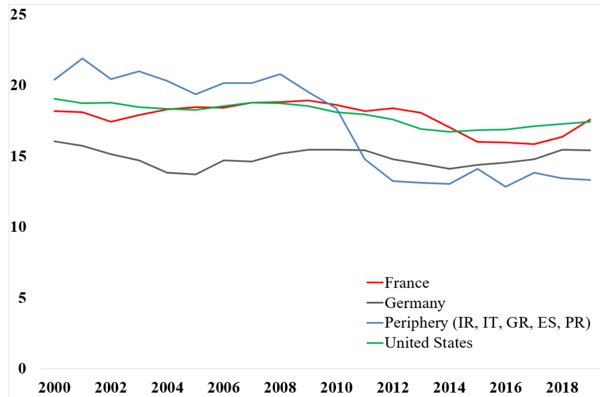
Figure C.1 depicts evolution of total government expenditures and its composition for a set of Euro countries and the United States.

Figure C.2 displays the average tax wedge for two household types in France, Germany, the US, and the average of periphery countries from 2000 to 2019. We have excluded the data after 2019 due to the impact of Covid. It is worth noting that despite the exclusion of post-2019 data, the overall pattern remains relatively stable. The average tax wedge, as defined by the OECD tax database measures the effective tax rate on labor costs. It is calculated by expressing the sum of personal income tax, employee and employer social security contributions, and any payroll taxes, minus any benefits received by the employee, as a percentage of labor costs. There is relatively little variation in the average tax wedge among the four types of households throughout the period under examination.

Figure C.1: Government Spending

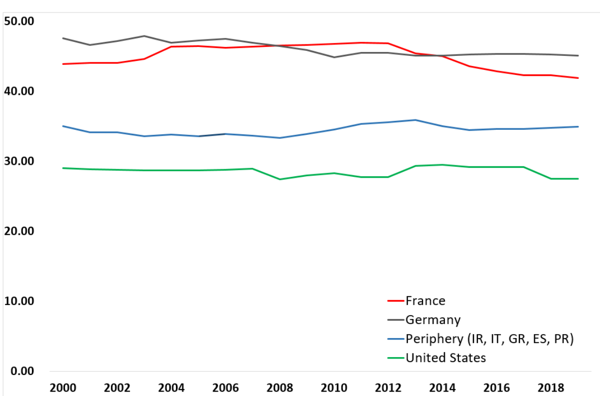


(a) Size of gov spending

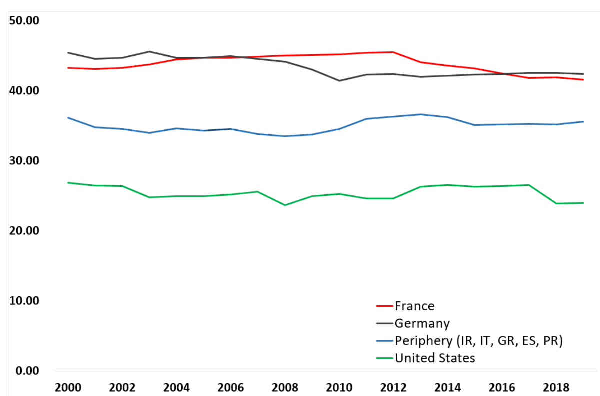


(b) Composition of gov spending

Figure C.2: Average Tax Wedge (% of Labour Costs)



(a) Single person at 67% of average earnings, without child



(b) Two-earner married couple, one at 100% of average earnings and the other at 67%, with two children