Measuring Global Interest Rate Comovements with Implications for Monetary Interdependence

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Reserve Bank of Australia Conference on Causes, Challenges and Consequences of the Low Interest Rate Environment)
June 27, 2022

Introduction

- Develop a new general measure to identify the strength of the comovements of global interest rates
- Interest rate comovement important for
  - ability to conduct independent monetary policy
  - policy co-ordination
  - business cycle synchronisation
  - financial integration
  - bond market contagion
  - portfolio diversification

Monetary interdependence indexes

- Monetary interdependence index (*MI*)
  - Aizenman, Chinn and Ito (2008)
  - based on the interest rate correlation

$$MI = 1 - (\rho + 1)/2$$

    - $\rho = 1$ - perfect monetary **interdependence** $MI = 0$
    - $\rho = -1$ - perfect monetary **independence** $MI = 1$
- Monetary conditions index
  - Freedman (1994) - Bank of Canada
  - weighted average of the interest rate and the exchange rate
- Extended Taylor rule
  - Belke amd Cui (2010), Beckmann et al. (2017), Gray (2013)
  - interest rate function of domestic inflation, output gap, international interest rates

Non-linearities in monetary interdependence

- Current approaches focus on second order moments between rates
- No focus on risk reflected in volatility
- No focus on risk aversion reflected in skewness
- Framework allows for non-linearities through higher-order comoments
  - coskewness - relationship between volatility and level
  - cokurtosis - relationship between skew and level
  - covolatility - relationship between volatility and volatility
  - and second-order comoments as a special case

Non-linearities in monetary interdependence

- Coskewness
  - interest rate volatility models are function of interest rate level
  - expansion to multivariate setting of local and global rates results in coskewness between rates
- Cokurtosis
  - skewness in interest rates in one market is related to the level of interest rates in another
- Covolatility
  - co-risk - interest rate volatility can spillover across local and global rates
- Other forms of nonlinearities linking rates
  - zero lower bound
  - unconventional monetary policy
  - non-linear Taylor rules

Our contribution

- Use entropy theory to measure the degree of uncertainty between interest rate comovements, (Fry-McKibbin, Hsiao and Martin, 2021)
- Account for higher order co-moments
  - bivariate measures of interest rate interdependence
  - aggregate measure of global interest rate interdependence
  - identify individual contributions of each channel to interdependence
- Sample
  - Australia, Canada, Europe, Japan, New Zealand, Switerzerland, UK with the U.S.
  - Krippner shadow short rates
  - Jan 1999 - May 2020
  - Compare pre-UMP (Jan 1999 - Aug 2008) and UMP periods (Sep 2008 - May 2020)

Outline

- Theoretical framework and implementation
- Statistical properties of global interest rates
- Simulation effects of the ZLB
- Empirical results
    - global monetary interdependence
    - country monetary interdependence
    - decompositions
    - using money market and long term rates
- Conclusion

Theoretical framework and implementation

Steps

- define the (negative) entropy of the joint distribution of interest rates
- specify the joint distribution using the generalized normal distribution
- construct the measure corresponding to the special case of independence
- re-scale the interdependence measure to assist interpretation

Measuring interdependence of global interest rates

- Joint probability distribution $f(r_1, r_2, \Theta)$
  - r_1 and r_2 represent interest rates
  - unknown parameter vector Θ
- Measure of interdependence between the two markets is

$$E\left[\log f\left(r_1, r_2; \Theta\right)\right] = \int\int \log\left(f\left(r_1, r_2; \Theta\right)\right) f\left(r_1, r_2; \Theta\right) dr_1 dr_2,$$

  - the negative of the entropy (uncertainty) of the distribution
  - interdependence measure has the properties
    - increases as interest rates become more dependent
    - decreases as interest rates become more independent
  - global interest rate independence
    - when $E\left[\log f\left(r_1, r_2; \Theta\right)\right]$ reaches a minimum (maximum entropy)

Measuring interdependence of global interest rates

- **Interdependence expressed relative to independence**
  - to capture relative changes in interest rate comovements over time

$$\psi_{1,2} = 1 - \frac{E\left[\log f\left(r_1, r_2; \Theta_1\right)\right]}{E\left[\log f\left(r_1, r_2; \Theta_0\right)\right]}.$$

  - $\psi_{1,2} = 0$ corresponds to independence
  - $\psi_{1,2} > 0$ increasing interdependence

- **Aggregate interest rate interdependence**
  - Average across the bivariate measures

$$\Psi = \frac{1}{n-1} \sum_{j=2}^{n} \psi_{1,j}.$$

Generalized exponential distribution

- Specify the joint distribution $f\left(r_1, r_2; \Theta\right)$
- Approach based on the generalized exponential family of Lye and Martin (1993) and Cobb et al. (1983)

$$f(r_1, r_2; \Theta) = \exp\left(h(\Theta) - \eta(\Theta)\right)$$

- The adopted parameterization of generalized normal distribution is

$$h_t\left(\Theta\right) = -\frac{1}{2}\left(\frac{z_{1t}^2 + z_{2t}^2 - 2\theta_0 z_{1t} z_{2t}}{1-\theta_0^2}\right) + \theta_1 z_{1t} z_{2t}^2 + \theta_2 z_{1t}^2 z_{2t} + \theta_3 z_{1t} z_{2t}^3 + \theta_4 z_{1t}^3 z_{2t} + \theta_5 z_{1t}^2 z_{2t}^2 - z_{1t}^4 - z_{2t}^4,$$

- $z_{it} = \left(r_{it} - E\left(\ r_{it}\right)\right) / \sqrt{var\left(r_{it}\right)},$
- Unknown parameters $\Theta = \left(\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5\right)$
- First term is the bivariate normal distribution
  - θ_0 interdependence through interaction of $z_{1t} z_{2t}$,
  - θ_1 to θ_5 higher order comoments

Implementation

- Maximize the log-likelihood function of the bivariate generalized exponential distribution
- Interdependence measure Ψ_t is the log-likelihood function evaluated at the maximum likelihood estimates
- Nonlinear function of the parameters - use an iterative gradient algorithm

Data

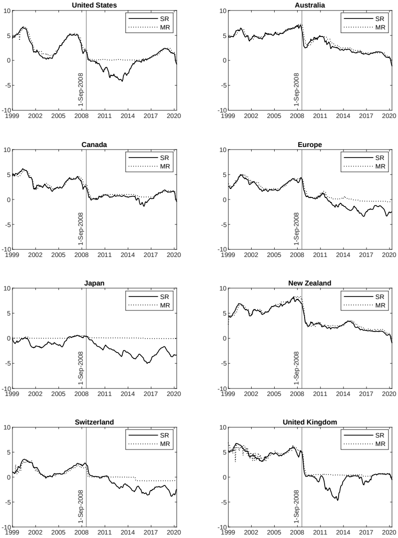
- Interest rates - monthly January 1999 - May 2020
  - Krippner shadow interest rates
  - overnight money market
  - long-term rates
- Australia, Canada, Euro Area, Japan, New Zealand, Switzerland, UK
- Measure interdependence relative to the U.S.
- Condition on macroeconomic variables of domestic and US real output growth, inflation rates, VIX using a VAR

Krippner shadow interest rates

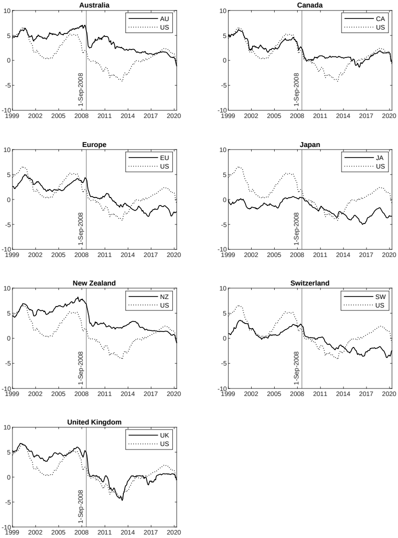
Use Krippner shadow interest rates (Krippner 2013, 2015)

- Alternative measure of the stance of monetary policy but not constrained by zero lower bound
- Yield curve which reflects unconventional and conventional monetary policy
- Shadow yield curve not constrained by zero lower bound
- Use data on daily government interest rates for a range of maturities up to 30 years spliced with overnight indexed swap rate
- Two factor Nelson-Siegal type factor model of the level and slope of the yield curve
- The estimate of the short rate from the yield curve is the SSR

Shadow rates versus money market rates



Country shadow rates versus the U.S. shadow rates



Descriptive statistics of monthly shadow short rates - Pre-UMP (January 1999 to August 2008)

Country	Skewness	Kurtosis	JB stat	p-value
United States	-0.022	1.611	9.335	0.009
Australia	0.319	2.153	5.432	0.066
Canada	0.416	1.917	9.012	0.011
Europe	0.136	1.812	7.176	0.028
Japan	0.059	1.594	9.626	0.008
New Zealand	0.003	2.080	4.094	0.129
Switzerland	0.106	1.767	7.569	0.023
United Kingdom	0.207	2.495	2.061	0.357

Descriptive statistics of monthly shadow short rates - UMP (September 2008 - May 2020)

Country	Skewness	Kurtosis	JB stat	p-value
United States	-0.251	2.027	7.042	0.030
Australia	0.402	2.736	4.209	0.122
Canada	-0.312	3.178	2.478	0.290
Europe	0.614	3.041	8.869	0.012
Japan	0.385	2.885	3.566	0.168
New Zealand	0.485	5.760	50.268	0.000
Switzerland	0.604	2.791	8.832	0.012
United Kingdom	-0.885	4.310	28.484	0.000

Define higher-order comoments

Define comoments (z_{it} standardized data)

- Correlation

$$\widehat{\rho} = \frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t},$$

- Coskewness

$$CS_{12} = \frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t}^{2}, \ CS_{21} = \frac{1}{T}\sum_{t=1}^{T} z_{1t}^{2}z_{2t},$$

- Cokurtosis

$$CK_{13} = \frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t}^{3}, \ CK_{31} = \frac{1}{T}\sum_{t=1}^{T} z_{1t}^{3}z_{2t}$$

- Covolatility

$$CV_{22} = \frac{1}{T}\sum_{t=1}^{T} z_{1t}^{2}z_{2t}^{2}$$

Tests of independence

Six tests of H_0 : independence from Fry-McKibbin, Hsiao and Martin (2021)

$$TSTAT_1 = \sqrt{T}\widehat{\rho} \quad (1)$$

$$TSTAT_2 = \frac{\sqrt{T}CS_{12}}{\sqrt{4\widehat{\rho}+2}} \quad (2)$$

$$TSTAT_3 = \frac{\sqrt{T}CS_{21}}{\sqrt{4\widehat{\rho}+2}} \quad (3)$$

$$TSTAT_4 = \frac{\sqrt{T}\,(CK_{13}-3\widehat{\rho})}{\sqrt{18\widehat{\rho}+6}} \quad (4)$$

$$TSTAT_5 = \frac{\sqrt{T}\,(CK_{31}-3\widehat{\rho})}{\sqrt{18\widehat{\rho}+6}} \quad (5)$$

$$TSTAT_6 = \frac{\sqrt{T}(CV_{22}-(1+2\widehat{\rho}^2))}{\sqrt{4\widehat{\rho}^4+16\widehat{\rho}^2+4}} \quad (6)$$

Lagrange Multiplier tests of independence of global shadow rates (r_i) with the U.S. (r_{us}) - p-values

Country	Joint	Corr.	Coskewness		Cokurtosis		Covol.
		$r_{us}r_i$	$r_{us}r_i^2$	$r_{us}^2r_i$	$r_{us}r_i^3$	$r_{us}^3r_i$	$r_{us}^2r_i^2$
Aust.	0.000	0.000	0.000	0.000	0.000	0.000	0.208
Can.	0.000	0.000	0.000	0.020	0.000	0.000	0.001
Euro.	0.000	0.000	0.193	0.264	0.000	0.000	0.011
Japan	0.015	0.744	0.278	0.149	0.488	0.545	0.888
NZ	0.000	0.000	0.000	0.028	0.000	0.000	0.672
Switz.	0.000	0.000	0.487	0.900	0.000	0.000	0.071
UK	0.000	0.000	0.806	0.063	0.000	0.000	0.012

- Tests of independence from FM, Martin and Hsiao, 2021
- Evidence of interdependence except for Japan

Simulation effects of the ZLB

- Show pitfalls of using correlation as a measure of interdependence if there are non-linearities
- Perform tests of independence on censored and uncensored simulated interest rate data
- DGP - r^*_{1t} and r^*_{2t}, evolve according to

$$\begin{aligned} r^*_{1t} &= 0.5882 + 0.7385\, z_{1t}, && (7)\\ r^*_{2t} &= 0.1594 + 0.7360\, z_{2t}, && (8)\\ z_{it} &\sim N(0,1), \qquad i = 1, 2, && (9)\\ E(z_{1t} z_{2t}) &= \rho, && (10) \end{aligned}$$

- ρ - measures strength of dependence

Simulation experiments of interest rate interdependence

- Impose the zero lower bound

$$r_{it} = \begin{cases} r_{it}^* & : \quad r_{it}^* > 0 \\ 0 & : \quad r_{it}^* \leq 0 \end{cases} \tag{11}$$

- Three cases
  - no censoring $r_{it} = r_{it}^*$, - negative interest rates are allowed
  - partial censoring - the second interest rate r_{2t} is censored
  - full Censoring - all interest rates are restricted to being non-negative

Simulation of zero-lower bound *rho* $= 0$

Prob of rejecting null of independence, $T = 1000$, replications = 50000.

Test	Statistic	Rejection Probability		
		No Censoring	Partial Censoring	Full Censoring
			$\rho = 0.0$	
Correlation	$TSTAT_1$	0.0516	0.0504	0.0512
Coskew(1,2)	$TSTAT_2$	0.0497	0.1273	0.1262
Coskew(2,1)	$TSTAT_3$	0.0509	0.0499	0.0450
Cokurt(1,3)	$TSTAT_4$	0.0506	0.3024	0.3040
Cokurt(3,1)	$TSTAT_5$	0.0492	0.0485	0.1007
Covol(2,2)	$TSTAT_6$	0.0467	0.1218	0.1100

- No censoring - no size distortions
- Full censoring - almost all oversized

Simulation of zero-lower bound *rho* $= 0.1$

Prob of rejecting null of independence, $T = 1000$, replications = 50000.

Test	Statistic	Rejection Probability		
		No Censoring	Partial Censoring	Full Censoring
			$\rho = 0.1$	
Correlation	$TSTAT_1$	0.8866	0.8076	0.7766
Coskew(1,2)	$TSTAT_2$	0.0487	0.5699	0.5522
Coskew(2,1)	$TSTAT_3$	0.0501	0.0526	0.2429
Cokurt(1,3)	$TSTAT_4$	0.0489	0.3445	0.3644
Cokurt(3,1)	$TSTAT_5$	0.0488	0.0502	0.1402
Covol(2,2)	$TSTAT_6$	0.0468	0.1236	0.2086

- Good power for correlation with no censoring
- Once fully censor, all higher order comoments show power when there should not be

Simulation of zero-lower bound *rho* $= -0.1$

Prob of rejecting null of independence, $T = 1000$, replications = 50000.

Test	Statistic	Rejection Probability		
		No Censoring	Partial Censoring	Full Censoring
			$\rho = -0.1$	
Correlation	$TSTAT_1$	0.8846	0.8061	0.7686
Coskew(1,2)	$TSTAT_2$	0.0503	0.5724	0.5318
Coskew(2,1)	$TSTAT_3$	0.0504	0.0525	0.1467
Cokurt(1,3)	$TSTAT_4$	0.0503	0.3409	0.3146
Cokurt(3,1)	$TSTAT_5$	0.0489	0.0491	0.0778
Covol(2,2)	$TSTAT_6$	0.0488	0.1240	0.1723

Conclusions from simulation experiments

- Correlation parameter - designed to model linear dependence
  - misleading
  - not a sufficient statistic if nonlinearities are present
- Higher-order comoments
  - capture the nonlinearities linking interest rates brought about by the effects of imposing a zero lower bound on interest rates

Interdependence indices (%) b/w country-U.S. shadow rates

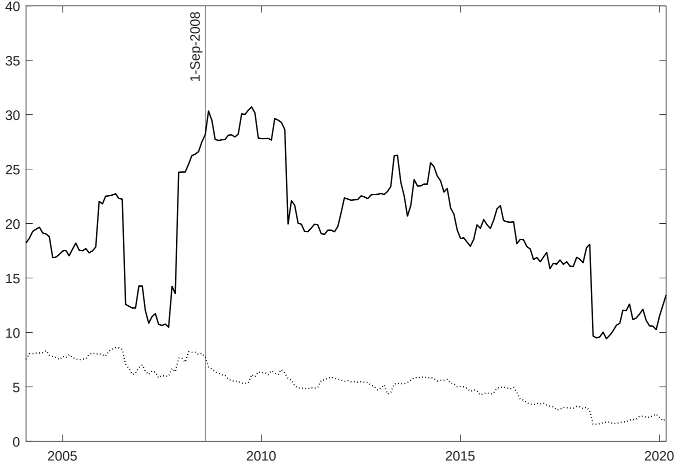
Country	Pre-UMP	UMP
Global	22.594	17.582
	(5.194)	(3.229)
Australia	21.035	13.030
	(11.159)	(4.093)
Canada	26.006	14.805
	(12.960)	(4.485)
Europe	37.860	19.648
	(22.733)	(11.788)
Japan	14.654	22.087
	(14.567)	(7.066)
New Zealand	19.945	9.412
	(15.334)	(6.439)
Switzerland	18.621	12.257
	(6.773)	(14.874)
United Kingdom	20.034	31.832
	(4.391)	(4.734)

Summary of results

- Interdependence index expressed as a percentage relative to 0 (independence)
- Global interdependence
  - pre-UMP 22.6%, significant
  - UMP 17.6%, significant
- Country specific interdependence - pre-UMP
  - all indices significant except Japan and New Zealand
  - highest interdependence - Europe then Canada
  - lowest interdependence - Japan
- Country specific interdependence - UMP
  - most countries reduction in interdependece
  - except Japan (now significant) and UK

Time varying global interdependence index

Figure 1: Interdependence index using a rolling 5 year window (solid line) against correlation (dashed lined) where $\theta_1 = \theta_2 ... 0$



Time varying interdependence index

- Large difference between correlation interdependence and higher order interdependence
- Pre-UMP
  - volatile with range between 10% and 30%
- UMP
  - interdependence increases with onset of GFC
  - maintained through the European debt crisis to 2011
  - reverts back to levels similar to before GFC
  - correlation interdependence declines through time

Decompositions

- Contribution of each comoment to monetary interdependence

$$
\begin{aligned}
C_{11} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\widehat{\theta}_0}{1-\widehat{\theta}_0^2} z_{1t}z_{2t} && \text{[Covariance(1,1)]}\\
S_{12} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_1 z_{1t}z_{2t}^2 && \text{[Coskewness(1,2)]}\\
S_{21} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_2 z_{1t}^2 z_{2t} && \text{[Coskewness(2,1)]}\\
K_{13} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_3 z_{1t}z_{2t}^3 && \text{[Cokurtosis(1,3)]}\\
K_{31} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_4 z_{1t}^3 z_{2t} && \text{[Cokurtosis(3,1)]}\\
V_{22} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_5 z_{1t}^2 z_{2t}^2, && \text{[Covolatility(2,2)]}
\end{aligned}
\tag{12}
$$

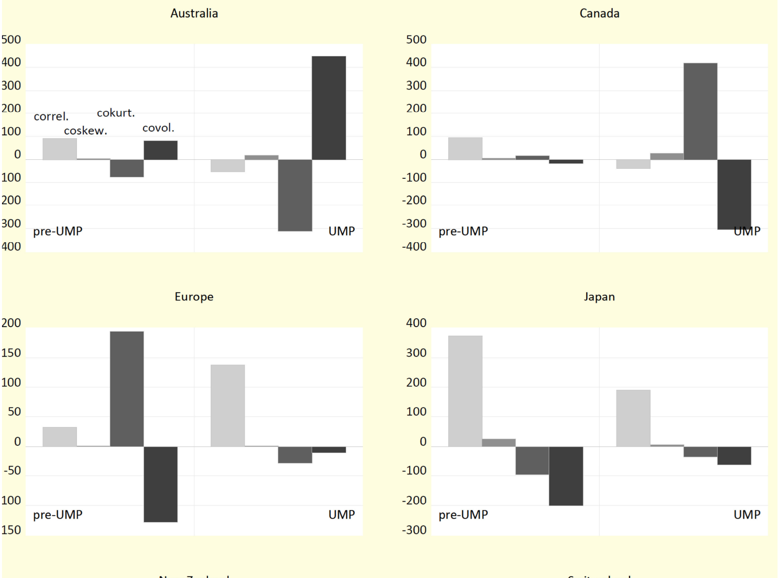
- Contribution of all comoments

$$
TOTAL = C_{11} + S_{12} + S_{21} + K_{13} + K_{31} + V_{22}, \tag{13}
$$

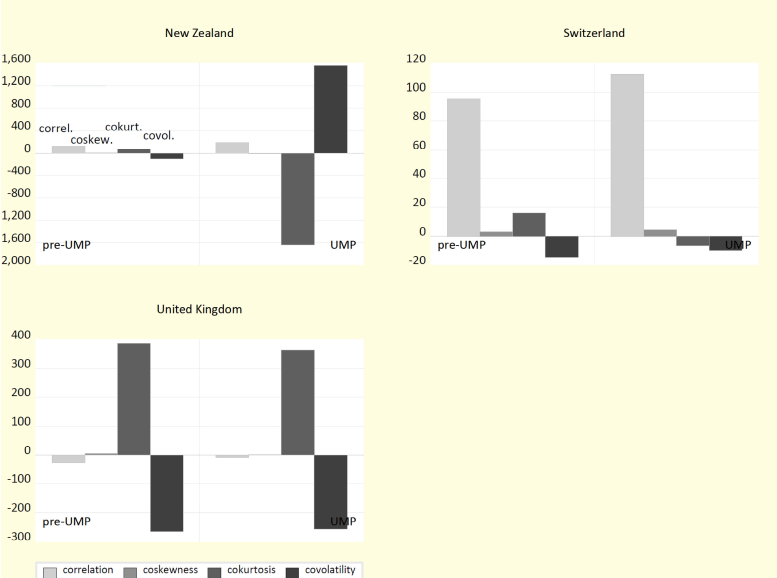
Decompositions

- For % contribution of each comoment divide (12) by (13)
- Positive value - comoment increases interdependence

Decomposition of interdependence indexes (%)



Decomposition of interdependence indexes (%)



Decomposition of interdependence indexes

- Cokurtosis and covolatility often largest in magnitude, offsetting
- Second order comoments dominate Japan and Switzerland
- Second order comoments dominate Europe in the UMP period

Interdependence index (%) based on money market rates

Country	Pre-UMP	UMP
Global	4.323	10.355
	(1.052)	(2.163)
Australia	1.032	17.538
	(2.343)	(8.893)
Canada	5.879	0.000
	(2.674)	(0.000)
Europe	4.915	1.019
	(1.662)	(2.363)
Japan	1.794	10.011
	(2.829)	(2.557)
New Zealand	4.461	16.446
	(1.726)	(6.257)
Switzerland	3.175	13.565
	(2.721)	(4.867)
United Kingdom	9.003	3.551
	(4.525)	(3.793)

Interdependence index (%) based on money market rates

- Results suggest monetary policy became more interdependent
- Opposite of shadow rates
- ZLB gives appearance rates more interdependent than actually are
- Canadian monetary policy becomes independent of US monetary policy

Interdependence index (%) based on long-term rates

Country	Pre-UMP	UMP
Global	17.217	12.931
	(6.117)	(1.111)
Australia	24.051	17.675
	(3.260)	(3.468)
Canada	28.116	22.733
	(2.786)	(3.538)
Europe	17.762	6.474
	(39.245)	(2.225)
Japan	5.139	2.469
	(2.465)	(3.085)
New Zealand	21.877	17.201
	(2.706)	(2.572)
Switzerland	7.435	6.454
	(2.512)	(2.576)
United Kingdom	16.142	17.512
	(3.623)	(2.905)

Interdependence index (%) based on long-term rates

- Consistent with results for shadow rates
- More considerable reductions in interdependence than money market rates

Conclusions

- Contribution: interdependence of international interest rates
  - new and more general approach
  - allow for non-linear channels through higher-order comoments, including coskewness, cokurtosis and covolatility
- Feature - use of shadow rates
  - does not suffer distortions present when at or near the zero lower bound
  - positive and significant interdependence with the US for most
  - international rates more independent post-UMP
  - higher-order comoment effects important
- Using money market rates after UMP distorts the degree of interdependence between rates
- Less of an issue for longer-term rates