Reserve Bank of Australia

RESEARCH DISCUSSION PAPER

Optimal Monetary Policy with Real-time Signal Extraction from the Bond Market

Kristoffer Nimark

OPTIMAL MONETARY POLICY WITH REAL-TIME SIGNAL EXTRACTION FROM THE BOND MARKET

Kristoffer Nimark

Abstract

Monetary policy is conducted in an environment of uncertainty. This paper sets up a model where the central bank uses real-time data from the bond market together with standard macroeconomic indicators to estimate the current state of the economy more efficiently, while taking into account that its own actions influence what it observes. The timeliness of bond market data allows for quicker responses of monetary policy to disturbances compared to the case when the central bank has to rely solely on collected aggregate data. The information content of the term structure creates a link between the bond market and the macroeconomy that is novel to the literature. To quantify the importance of the bond market as a source of information, the model is estimated on data for the United States and Australia using Bayesian methods. The empirical exercise suggests that there is some information in the US term structure that helps the Federal Reserve to identify shocks to the economy on a timely basis. Australian bond prices seem to be less informative than their US counterparts, perhaps because Australia is a relatively small and open economy.

Table of Contents

OPTIMAL MONETARY POLICY WITH REAL-TIME SIGNAL EXTRACTION FROM THE BOND MARKET

Kristoffer Nimark

1.   Introduction

Hayek (1945) famously argued that market economies are more efficient than planned economies because of markets' ability to efficiently use information dispersed among market participants. According to Hayek, prices in competitive markets reflect all information that is known to anyone and competitive markets can potentially allocate resources more efficiently. In most western economies there is now little planning and almost all prices are determined by market forces without interference from any central authority. However, there is one important exception: the market for short-term nominal debt where central banks borrow and lend at fixed interest rates. In the presence of nominal frictions in product or wage markets, this practise can improve welfare by reducing the volatility of inflation and output. Hayek's insight, though formulated in a more general setting of a planned economy, was that even a central bank that shares the objective of the representative agent may not be *able* to implement an optimal stabilising policy due to incomplete information. In this paper, the central bank would implement an optimal stabilising policy if it knew the state of the economy with certainty, and any deviation from optimal policy is due only to information imperfections. Under this assumption this paper demonstrates how the central bank can make use of Hayek's insight and use the market for debt of longer maturities as a source of information that makes a more efficient estimation of the state of the business cycle possible, and thus reduces deviations from optimal policy. That this is close to how some central banks think about and use the term structure is illustrated by a quote by the Chairman (then Governor) of the Federal Reserve Board Ben Bernanke:

> To the extent that financial markets serve to aggregate private-sector information about the likely future course of inflation, data on asset prices and yields might be used to validate and perhaps improve the Fed's forecasts. (Bernanke 2004)

The suggestion that the bond market can provide information that is valuable to policy-makers is thus not news to the policy-makers themselves. Rather, the contribution of the present paper is to provide a coherent framework for analysing and estimating the interaction between information contained in the term structure and the monetary policy-making process. In the model presented below, the central bank has to set interest rates in an uncertain environment, where the yield curve is informative about the state of the economy and thus also informative about the desired interest rate. This has the consequence that the macroeconomy is not independent of the term structure. The only direct effect of interest rates on the macroeconomy is from the short rate set by the central bank to aggregate demand, as is standard in the New-Keynesian literature. However, there is also an indirect feedback from rates on longer-maturity bonds to the macroeconomy through the informational content of the term structure. The mechanism is the following. Bonds are traded daily and the affine form of the bond pricing function makes the bond pricing equation with macro factors formally equivalent to a linear measurement of the state of the economy. The term structure can thus be viewed as a more timely measure of the state of the economy than collected aggregate information that is available only with delay and sometimes significant measurement error. A movement in the term structure can then signal a shift in the underlying macro factors that induces the central bank to re-evaluate what the optimal short-term interest rate should be. The shift in the term structure thus feeds into a change in demand through the change in the short-term interest rate.

In the present model the policy-makers exploit the fact that bond market participants' expectations about the future are revealed by the term structure. As pointed out by Bernanke and Woodford (1997), letting monetary policy react mechanically to expectations may lead to a situation where expectations become uninformative about the underlying state and no equilibrium exists. They further argue that 'targeting expectations' by policy-makers cannot be a substitute to structural modelling. In the proposed framework below, the information in the term structure is *complementary* to other information and firmly connected to an underlying structural model. Policy-makers then avoid the potential pitfalls of a pure 'expectations targeting' regime.

There is a large literature concerning the information content of the term structure. Mostly, it has focused on whether the term structure, often modelled as the spread between short and long rates, can help predict future outcomes of macro

variables.[1] More recent work by Ang, Piazzesi and Wei (2003) suggests that the best predictor of GDP is the short end of the yield curve. The negative correlation between short interest rates and future output is hardly surprising, given the evidence of the real effects of monetary policy. The conclusion of Ang *et al* highlights the need to distinguish between information in the term structure that tells us something about the transmission mechanism of monetary policy, and information that can be used by a central bank in the policy process when the transmission mechanism is assumed to be known. This paper is solely concerned with the latter.

There are two other potentially important types of information that could be revealed by the term structure that the present model is silent about. Goodfriend (1998) discusses the Federal Reserve's responses to 'inflation scares' in the 1980s, which he defines as increases in the long-term yields. He interprets these as doubts by market participants about the Federal Reserve's commitment to fighting inflation. The present paper does not address questions about central bank credibility, but takes a perfectly credible central bank with a publicly known inflation target as given. The model presented here is also not suited to analysing or interpreting market perceptions of the reasons for a change in the monetary policy stance, as done by Ellingsen and Söderström (2001). The policy-makers' relative preferences for stabilising inflation or the output gap are assumed to be known to bond market participants. In this paper, we restrict our attention to what the term structure can tell us about the state of the business cycle.

The practical relevance of any information contained in the term structure is ultimately an empirical question. When bond markets are noisy, observing the term structure is not very informative. In order to quantify the informational content of the term structure, the variances of the non-fundamental shocks in the term structure are estimated simultaneously with the structural parameters of the macroeconomy. The estimation methodology is similar to recent work by Hördahl, Tristani and Vestin (2006) who estimate the term structure dynamics jointly with a small empirical macro model where the central bank is assumed to be perfectly informed. Hördahl *et al* impose only a no-arbitrage condition on the pricing of bonds while in this paper the bond pricing function and the dynamics of the macroeconomy are derived from the same underlying utility function. This makes

---

[1] For example, Harvey (1988), Mishkin (1990) and Estrella and Mishkin (1998).

the analysis more stringent, but it comes at the cost of an empirically less flexible bond pricing function.

In the next section a model is presented where the central bank extracts information from the term structure about unobservable shocks while recognising that its own actions influence the term structure itself. In Section 3 the model is estimated to quantify the importance of the yield curve as a source of information. Section 4 concludes.

2. The Macroeconomy, the Term Structure and Monetary Policy Under Imperfect Information

This section presents a model where the central bank extracts information about the state of the economy from the term structure of interest rates. Movements in the term structure will then have a direct impact on the macroeconomy through its effect on the central bank's estimate of the state and therefore also on the setting of the policy instrument. This means that the macroeconomy, the term structure model and the filtering problem of the central bank have to be solved simultaneously. This makes the model different from other recent papers, for example Hördahl *et al* (2006) and Wu (2002), where the macroeconomic model can be solved separately from the term structure. Here, the macroeconomy is described first without specifying an explicit interest rate function, but merely noting that it is set by the central bank to minimise a loss function that in principle could be derived from micro foundations. The filtering problem of the central bank is then solved, taking the term structure model as given. Finally, in the last part the term structure model is derived and it is demonstrated how it influences the dynamics of the macroeconomy through an information channel.

2.1 The Macroeconomy

We use a standard business cycle model of the macroeconomy with monopolistically competitive firms that sell differentiated goods. Prices are set according to the Calvo mechanism, with a fraction of firms using a rule of thumb rather than optimising as in Galí and Gertler (1999). Households supply labour and consume goods. In addition to their own current consumption, they also care about the lagged aggregate consumption level.

2.1.1 Households and firms

Consider a representative household $j \in (0,1)$ that wishes to maximise the discounted sum of expected utility

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s U \left(C_{t+s}(j), N_{t+s}(j) \right) \right\} \tag{1}$$

where $\beta \in (0,1)$ is the household's subjective discount factor and the period utility function in consumption C_t and labour N_t is given by

$$U \left(C_t(j), N_t(j) \right) = \frac{\left(C_t(j) H_t^{-\eta} \right)^{1-\gamma}}{(1-\gamma)} - \frac{N_t(j)^{1+\varphi}}{1+\varphi}. \tag{2}$$

The variable H_t

$$H_t = \int C_{t-1}(j) \, dj \tag{3}$$

is a reference level of consumption that we interpret as external habits that makes marginal utility of consumption an increasing function of lagged aggregate consumption. The habit specification helps to explain the inertial movement of aggregate output as well as the procyclicality of asset prices.[2] Differentiated goods indexed by $i \in (0,1)$ are produced with a technology that is linear in labour and subject to a persistent productivity shock A_t

$$Y_t(i) = A_t N(i) \tag{4}$$

that follow an AR(1) process in logs

$$a_t = \rho a_{t-1} + \varepsilon_t^a \tag{5}$$

$$\varepsilon_t^a \sim N \left(0, \sigma_{a\varepsilon}^2 \right). \tag{6}$$

Firms set prices according to the Calvo (1983) mechanism where a fraction θ of firms reset their price in a given period. Of the firms resetting their price, a fraction $(1-\omega)$ optimise their price decision and take into account that their price may be effective for more than one period while a fraction ω of price-setters use a 'rule of thumb' as in Galí and Gertler (1999). The 'rule of thumb' price-setters set their price equal to the last period's average reset price plus the lagged inflation rate.

---

[2] See Campbell and Cochrane (1999) for the implications of habits for asset prices.

2.1.2 The linearised model

The linearised structural equations are given by Equations (7) and (8)

$$y_t = \mu_{yf} E_t y_{t+1} + \mu_{yb} y_{t-j} - \phi \left[i_t - E_t \pi_{t+1} \right] + \varepsilon_t^y \tag{7}$$

$$\pi_t = \mu_{\pi f} E_t \pi_{t+1} + \mu_{\pi b} \pi_{t-1} + \kappa m c_t + \varepsilon_t^\pi \tag{8}$$

where $\{y_t, \pi_t, i_t\}$ is real output, inflation and the short nominal interest rate in period t. The coefficients $\{\mu_{yf}, \mu_{yb}, \mu_{\pi f}, \mu_{\pi b}, \phi, \kappa\}$ are derived from the utility function (2) and the parameters in the price-setting equation are specified in the Appendix. Marginal cost in period t, mc_t, can be found by equating the marginal utility of consuming the real wage paid for an additional unit of labour with the household's disutility of providing the additional unit of labour. The real marginal cost then equals the market-clearing real wage divided by productivity

$$mc_t = (\varphi + \gamma) y_t + \eta (1 - \gamma) y_{t-1} - (1 + \varphi) a_t \tag{9}$$

where the relationship

$$n_t = y_t - a_t \tag{10}$$

was used to substitute out labour supply. Potential output, $\bar{y}_t$, defined as the level of output that is compatible with no acceleration in inflation, then is

$$\bar{y}_t \equiv \frac{\eta (1 - \gamma)}{(\varphi + \gamma)} y_{t-1} + \frac{1 + \varphi}{(\varphi + \gamma)} a_t. \tag{11}$$

The short-term interest rate is set by a monetary authority to minimise the expected value of the loss function

$$\mathbf{L}_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left[\lambda_y (y_{t+k} - \bar{y}_{t+k})^2 + \pi_{t+k}^2 + \lambda_i (i_{t+k} - i_{t+k-1})^2 \right] \right]. \tag{12}$$

The weights λ_y and λ_i can be chosen such that the loss function (12) is a second-order approximation of the utility function of the representative agent.[3] However, we do not necessarily want to impose this restriction when we estimate the model. Equations (5), (7), (8) and (9) can be written more compactly as

$$\left[\begin{array}{c} X_{1,t+1} \\ E_t X_{2,t+1} \end{array} \right] = A \left[\begin{array}{c} X_{1,t} \\ X_{2,t} \end{array} \right] + B i_t + C \varepsilon_t \tag{13}$$

---

[3] See Amato and Laubach (2004).

where

$$X_{1,t} = \left[a_t, y_{t-1}, \pi_{t-1}, \varepsilon_t^y, \varepsilon_t^\pi, i_{t-1}, \Delta i_t \right]' \tag{14}$$

$$X_{2,t} = \left[y_t, \pi_t \right]' \tag{15}$$

$$\varepsilon_t = \left[\varepsilon_t^a, \varepsilon_t^y, \varepsilon_t^\pi \right]'. \tag{16}$$

2.2 Monetary Policy and Real-time Signal Extraction

Monetary policy operates in an uncertain environment where some state variables are only observed with error and delay, and some variables, like productivity and thus potential output, are not observed at all. Variables that are not observable but relevant for monetary policy have to be inferred from the variables that are observable. In such a setting, Svensson and Woodford (2004) show that a form of certainty equivalence holds. That is, with a quadratic objective function and linear constraints, the optimal interest rate can be expressed as a linear function of the central bank's estimate of the state $X_{1,t|t}$

$$X_{1,t|t} = E_t \left[X_{1,t} \mid I_t \right]$$

where I_t is the information set of the central bank at time t. The coefficients in the policy function are then the same as they would be if the central bank could observe the pre-determined state perfectly. The coefficient vector F of the optimal interest function

$$i_t = F X_{1,t|t} \tag{17}$$

can thus be found by standard full-information methods, for instance by the algorithm in Söderlind (1999). Here we describe how the central bank can apply the Kalman filter to estimate the state $X_{1,t}$. The affine function that maps the pre-determined state into bond prices, characterised by the matrices Q_1 and Q_2, is taken as given and deriving the equilibrium dynamics of the model is then a straightforward application of the procedure in Svensson and Woodford (2004).

Partition the coefficient matrices in (13) conformably to the pre-determined and forward-looking variables and substitute in the interest rate function to get

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} F X_{1,t|t} + C \varepsilon_t. \tag{18}$$

The equilibrium dynamics of the model can then be described by Equations (19)–(23)

$$X_{1,t} = HX_{1,t-1} + JX_{1,t-1|t-1} + C\varepsilon_t \tag{19}$$

$$X_{2,t} = G^1 X_{1,t} + \left(G - G^1\right) X_{1,t|t} \tag{20}$$

$$X_{1,t|t} = X_{1,t|t-1} + K\left[Z_t - L_1 X_{1,t|t-1} - L_2 X_{1,t|t}\right] \tag{21}$$

$$Z_t = \mathbf{z} + L_1 X_{1,t} + L_2 X_{1,t|t} + \mathbf{v}_t \tag{22}$$

$$\mathscr{Y}_t = \mathbf{q} + Q_1 X_{1,t} + Q_2 X_{1,t|t} + \mathbf{v}_t^Y \tag{23}$$

where Z_t is the vector of variables that are observable to the central bank and $\mathscr{Y}_t$ is a vector of bond yields of different maturities. The system of Equations (19)–(23) can be written solely as functions of the actual state, the central bank's estimate of the state and the shock vectors ε_t and $\mathbf{v}_t$. The coefficient matrices G and G^1 are derived in Svensson and Woodford (2004) and satisfy Equations (24) and (25)

$$G = (A_{22} - GA_{12})^{-1}\left[-A_{21} + GA_{11} + (GB_1 - B_2)F\right] \tag{24}$$

$$G^1 = A_{22}^{-1}\{-A_{21} + [G^1 + (G - G^1)KL_1]H\} \tag{25}$$

where the following definitions were used

$$H \equiv A_{11} + A_{12}G^1 \tag{26}$$

$$J \equiv B_1 F + A_{12}\left(G - G^1\right). \tag{27}$$

The Kalman gain matrix K is given by

$$K = PL_1'(L_1 PL_1' + \Sigma_{vv})^{-1} \tag{28}$$

$$P = H(P - PL_1'(L_1 PL_1' + \Sigma_{vv})^{-1}L_1 P)H' + \Sigma_{\varepsilon\varepsilon} \tag{29}$$

$$\Sigma_{vv} = \begin{bmatrix} \Sigma_{vv}^{cb} & 0 \\ 0 & \Sigma_{vv}^{\mathscr{Y}\mathscr{Y}} \end{bmatrix} \tag{30}$$

where P is the one-period-ahead forecast error, $\Sigma_{\varepsilon\varepsilon}$ is the covariance matrix of the structural shocks $\{\varepsilon_t^a, \varepsilon_t^y, \varepsilon_t^\pi\}$ and Σ_{vv} is the covariance matrix of the errors in the measurement Equation (22). The coefficient matrices G, G^1, the Kalman gain K and the one-period forecast error P have to be determined jointly by finding a fixed point of the system described by the Equations (24), (25), (28) and (29). Before we can solve for the equilibrium dynamics we need to specify the selection matrices L_1 and L_2 in the observation Equation (22). We thus have to decide what the central bank can observe.

2.2.1 Variables observable by the central bank

The central bank observes bond yields contemporaneously, while output and inflation are only observable with a one-period lag. This is a compromise that is necessary due to the division of time into discrete periods that do not conform to the exact delays of data releases, though it does capture some essential features of data availability. Data on real GDP are released with a significant delay while bond prices are observed every day that bonds are traded. The compromise is the observation of the price level. CPI data are usually released the month after observation so the one-quarter lag is thus too long for most countries.[4] We can write the measurement Equation (22) as

$$Z_t = \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ \mathscr{Y}_t \end{bmatrix} + \begin{bmatrix} v_t^y \\ v_t^\pi \\ \mathbf{0} \end{bmatrix} \tag{31}$$

and the matrices L_1 and L_2 are then given by

$$L_1 = \begin{bmatrix} \mathbf{0}_{2\times 1} & I_2 & \mathbf{0}_{2\times 4} \\ & Q_1 & \end{bmatrix} \tag{32}$$

$$L_2 = \begin{bmatrix} \mathbf{0}_{2\times 7} \\ Q_2 \end{bmatrix}. \tag{33}$$

The information set of the central bank is given by

$$I_t^{cb} \equiv \left\{ A, B, C, Q_1, Q_2, \Sigma_{\varepsilon\varepsilon}, \Sigma_{vv}, Z_{t-s} \mid s \geq 0 \right\} \tag{34}$$

that is, in addition to observing the vector Z_t, the central bank also knows the structure of the economy.

2.3 The Law of Motion for the State of the Economy

In full-information models, the relevant state for the pricing of bonds is simply the same as the state of the economy. In the present model the central bank cannot observe the state of the economy with certainty and uses the Kalman filter to estimate it. The central bank's information set is a subset of the information set of the bond market participants. This assumption allows us to model bond market

---

[4] One exception is Australia, where data on the CPI are collected quarterly.

participants as if they know the central bank's estimate of the state. We define the extended state $\overline{X}_t$ as

$$\overline{X}_t \equiv \begin{bmatrix} X_{1,t} & X_{1,t|t} \end{bmatrix}' \tag{35}$$

and we want to find a system of the form

$$\overline{X}_t = M\overline{X}_{t-1} + N\begin{bmatrix} \varepsilon_t \\ \mathbf{v}_t \end{bmatrix} \tag{36}$$

$$\mathcal{Y}_t = \mathbf{q} + Q\overline{X}_t + \mathbf{v}_t^{\mathcal{Y}} \tag{37}$$

that is, we conjecture that yields are an affine function of the extended state plus a noise term $\mathbf{v}_t^{\mathcal{Y}}$. We start by substituting the observation Equation (22) into the central bank's, updating Equation (21) to get

$$X_{1,t|t} = X_{1,t|t-1} + K\begin{bmatrix} L_1 X_{1,t} + L_2 X_{1,t|t} + \mathbf{v}_t - L_1 X_{1,t|t-1} - L_2 X_{1,t|t} \end{bmatrix} \tag{38}$$

Using Equations (19)–(21), definitions (26) and (27) and rearranging them, we get

$$X_{1,t|t} = [(H+J) + KL_1 J - KL_1(H+J)]X_{1,t-1|t-1} \tag{39}$$
$$+ KL_1 H X_{1,t-1} + K\mathbf{v}_t + KL_1 \varepsilon_t$$

The matrices K, L_1 and L_2 depend on the coefficients in the conjectured term structure function (23) and the covariance matrix of measurement errors/non-macro factors $\mathbf{v}_t^{\mathcal{Y}}$ denoted by $\Sigma_{vv}^{\mathcal{Y}\mathcal{Y}}$. Combining Equations (19) and (39) we get the conjectured form from Equation (36)

$$\overline{X}_t = \begin{bmatrix} H & J \\ KL_1 H & [(H+J) + KL_1 J - KL_1(H+J)] \end{bmatrix} \overline{X}_{t-1}$$
$$+ \begin{bmatrix} C_1 & 0 \\ KL_1 C_1 & K \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \mathbf{v}_t \end{bmatrix}. \tag{40}$$

2.4   The Term Structure and the State of the Economy

In this section we derive the law of motion for the nominal stochastic discount factor that is used to price the bonds from the utility function of the representative household. However, the framework presented here is general enough to accommodate any affine asset-pricing function and it is thus not necessary to impose that the macro model and the bond pricing function are determined by

the same underlying micro foundations. Define the nominal stochastic discount factor M_{t+1} as

$$E_t M_{t+1} \equiv E_t \beta \frac{U_{ct+1} P_t}{U_{ct} P_{t+1}} \tag{41}$$

where U_{ct} is the marginal utility of consumption in period t. If we assume that the distribution of M_{t+1} is log normal, that is, if $m_{t+1} = \log M_{t+1}$ and

$$m_{t+1} \sim N(\overline{m}_{t+1}, \sigma_m^2) \tag{42}$$

then the expected value of m_{t+1} is

$$E_t m_{t+1} = \overline{m}_{t+1} - \frac{\sigma_m^2}{2}. \tag{43}$$

Plugging in the utility function (2) into (41) and (43) we get

$$E_t \widetilde{m}_{t+1} = -\gamma E_t c_{t+1} + (\gamma - \eta + \gamma \eta) c_t + \eta (1 - \gamma) c_{t-1} - E_t \pi_{t+1} \tag{44}$$

where $\widetilde{m}$ denotes the deviation of m from its mean. Using that in equilibrium

$$E_t \widetilde{m}_{t+1} + i_t = 0 \tag{45}$$

must hold and that the interest rate i_t is a function of the state $\overline{X}_t$

$$i_t = \overline{F} X_t \tag{46}$$
$$\overline{F} = \begin{bmatrix} \mathbf{0}_{1x7} & F \end{bmatrix} \tag{47}$$

we get the following simple expression for the the expected value of m_{t+1}

$$E_t m_{t+1} = \widetilde{\beta} - \overline{F} X_t - \frac{V' \Sigma V}{2} \tag{48}$$

where V' and Σ in the variance term is given by

$$V' = \begin{bmatrix} -\gamma & -1 \end{bmatrix} \begin{bmatrix} G^1 & G - G^1 \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ KL_1 C_1 & K \end{bmatrix} \tag{49}$$

$$\Sigma = \begin{bmatrix} \Sigma_{\varepsilon\varepsilon} & 0 \\ 0 & \Sigma_{vv} \end{bmatrix}. \tag{50}$$

The log of the price at time t of a nominal bond paying one dollar in period $t+n$ will then be

$$\log \mathcal{P}_t^n = \overline{A}_n + \overline{B}_n' X_t \tag{51}$$

where the constant $\bar{A}_n$ and the vector $\bar{B}_n$ are given by the recursive relations

$$
\begin{aligned}
\bar{A}_n &= -\bar{i} + \bar{A}_{n-1} - \bar{B}'_{n-1}V + \frac{1}{2}\bar{B}'_{n-1}N\Sigma N'\bar{B}_{n-1} & (52) \\
\bar{B}_n &= -\bar{F} + M'\bar{B}_{n-1} & (53)
\end{aligned}
$$

starting from

$$
\begin{aligned}
\bar{A}_1 &= -\bar{i} & (54) \\
\bar{B}_1 &= -\bar{F}' & (55)
\end{aligned}
$$

where $\bar{i}$ is the average short interest rate. To find the vector of yields of selected maturities $\mathscr{Y}_t$, collect the appropriate constants $\bar{A}_n$ and vectors $\bar{B}_n$ as

$$
\mathscr{Y}_t = \begin{bmatrix} -\bar{A}_1 \\ \vdots \\ -\frac{1}{n}\bar{A}_n \end{bmatrix} + \begin{bmatrix} -\bar{B}_1 \\ \vdots \\ -\frac{1}{n}\bar{B}_n \end{bmatrix} \bar{X}_t + \mathbf{v}_t^{\mathscr{Y}} \tag{56}
$$

where the yield of an n periods to maturity bond is found by dividing the price by n. Partitioning the stacked vectors $-\frac{1}{n}\bar{B}_n$ appropriately gives the desired form

$$
\mathscr{Y}_t = \mathbf{q} + \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \bar{X}_t + \mathbf{v}_t^{\mathscr{Y}}. \tag{57}
$$

Equation (57) has a dual interpretation. On one hand it can be used to express bond yields as a function of the state and the vector of shocks to the term structure, $\mathbf{v}_t^{\mathscr{Y}}$, are then residuals, i.e., the component of the yields that cannot be explained by the state. A small variance of $\mathbf{v}_t^{\mathscr{Y}}$ should then be interpreted as that the term structure model provides a good fit of the observed yields. Equation (57) can also be viewed as a measure equation of the state. The vector of shocks $\mathbf{v}_t^{\mathscr{Y}}$ are then measurement errors and when the variance of $\mathbf{v}_t^{\mathscr{Y}}$ is small, the signal-to-noise ratio is high and the term structure is very informative about the state of the economy. In the special case of the rank of Q_1 being equal to the dimension of the state and $\Sigma_{vv}^{\mathscr{Y}\mathscr{Y}} = \mathbf{0}$, the model replicates the full-information dynamics, since the state can then be backed out perfectly from the term structure. In the opposite case, when the variances of $\Sigma_{vv}^{\mathscr{Y}\mathscr{Y}}$ are very large, the model will replicate the dynamics when the central bank can only observe imperfect but direct measures of the lagged aggregate variables.

3. The Dynamics of the Estimated Model

The dynamic implications of allowing the central bank to extract information from the term structure depend on the magnitude of the noise in the bond market. There is little information in the term structure when bond prices are very noisy, and including it in the information set of the central bank then has little effect on the dynamics of inflation and output. It is therefore of interest to quantify the variances in the model. The parameters of the model are estimated by Bayesian methods using quarterly data for the US ranging from 1982:Q1 to 2005:Q4 and for Australia ranging from 1993:Q1 to 2005:Q3. The shorter sample period for Australia is motivated by the fact that Australia experienced a large downward shift in the level of inflation in the early 1990s. Preliminary estimations suggest that including this shift in the sample may bias the results. First, it makes the estimates more sensitive to the detrending method used. Second, including a non-typical period when inflation is brought down by contractionary monetary policy may bias the estimates of the preference parameters of the Reserve Bank of Australia (RBA) to make it appear more averse to inflation volatility than is actually the case.

The interest rates included are the Federal Funds rate and secondary market rates for 6- and 12-month Treasury bills for the US, and the cash rate and the 180-day bank bill rate and the 12-month Treasury bond rate for Australia. For both the US and Australia, non-farm real GDP and CPI (excluding food and energy) are used as a measure of output and to calculate quarterly inflation rates. The data were detrended using the Hodrick-Prescott filter and the first eight observations were used as a convergence sample for the Kalman filter. The prior modes, distributions and estimated posteriors are reported in Table 1 together with the prior $L(\widehat{\Theta})$ and the likelihood conditional on the data set Z, $\mathscr{L}(\widehat{\Theta} \mid Z)$. The Appendix contains more details on the estimation procedure.

The posterior estimates for the structural parameters governing the behaviour of households and firms are similar across the US and Australia. The higher estimated value of λ_i indicates that the RBA seems to be smoothing interest rates more than the Federal Reserve.

The measurement error in the short-term interest rate is bounded between zero and 1/100 of the variance of the short interest rate, reflecting that this is a precise measure of the policy instrument. The prior distribution of the variance of all other

Table 1: Parameter Estimates

Parameter	Prior mode	Prior s.e.	Distribution	Posterior US	s.e.	Posterior Australia	s.e.
γ	3	0.32	normal	3.43	0.24	3.03	0.22
φ	10	1.41	normal	8.49	0.08	9.65	0.18
η	1	0.32	normal	0.90	0.04	0.89	0.08
θ	0.75	0.11	beta	0.93	0.06	0.80	0.04
ω	0.3	0.03	beta	0.41	0.01	0.35	0.03
β	0.99	0.02	beta	1.00	0.012	0.99	0.10
ρ	0.8	0.28	beta	0.82	0.007	0.92	0.11
λ_y	0.5	0.32	normal	0.40	0.03	0.45	0.09
λ_i	0.5	0.32	normal	0.20	0.01	0.48	0.13
σ_a^2			flat	3.0×10^{-3}	0.02	1.3×10^{-5}	1.2×10^{-4}
σ_y^2			flat	6.9×10^{-5}	1.5×10^{-6}	2.5×10^{-5}	1.1×10^{-5}
σ_π^2			flat	2.5×10^{-6}	9.0×10^{-7}	1.3×10^{-4}	4.5×10^{-5}
σ_{vycb}^2	$\frac{\sigma_{Zy}^2}{5}$	$\left(\frac{\sigma_{Zy}^2}{10}\right)^{1/2}$	normal	1.1×10^{-4}	1.2×10^{-6}	7.7×10^{-12}	4.2×10^{-8}
$\sigma_{v\pi cb}^2$	$\frac{\sigma_{Z\pi}^2}{5}$	$\left(\frac{\sigma_{Z\pi}^2}{10}\right)^{1/2}$	normal	5.0×10^{-15}	1.9×10^{-6}	3.9×10^{-5}	2.2×10^{-5}
σ_{vy}^2	$\frac{\sigma_{Zy}^2}{5}$	$\left(\frac{\sigma_{Zy}^2}{10}\right)^{1/2}$	normal	4.2×10^{-6}	2.3×10^{-6}	9.5×10^{-6}	3.3×10^{-6}
$\sigma_{v\pi}^2$	$\frac{\sigma_{Z\pi}^2}{5}$	$\left(\frac{\sigma_{Z\pi}^2}{10}\right)^{1/2}$	normal	3.2×10^{-7}	1.2×10^{-7}	3.7×10^{-5}	1.4×10^{-5}
$\sigma_{\mathscr{Y}1}^2$	$\frac{\sigma_{Z\mathscr{Y}1}^2}{5}$	$\left(\frac{\sigma_{Z\mathscr{Y}1}^2}{10}\right)^{1/2}$	normal	1.5×10^{-6}	1.2×10^{-6}	3.9×10^{-7}	1.3×10^{-6}
$\sigma_{\mathscr{Y}2}^2$	$\frac{\sigma_{Z\mathscr{Y}2}^2}{5}$	$\left(\frac{\sigma_{Z\mathscr{Y}2}^2}{10}\right)^{1/2}$	normal	4.4×10^{-6}	6.7×10^{-7}	2.2×10^{-5}	5.6×10^{-6}
$\sigma_{\mathscr{Y}4}^2$	$\frac{\sigma_{Z\mathscr{Y}4}^2}{5}$	$\left(\frac{\sigma_{Z\mathscr{Y}4}^2}{10}\right)^{1/2}$	normal	5.1×10^{-5}	8.4×10^{-6}	4.8×10^{-5}	1.1×10^{-5}
$\mathscr{L}(\widehat{\Theta})$				14.3		8.9	
$\mathscr{L}(Z \mid \widehat{\Theta})$				1 915.5		848.2	
$\mathscr{L}(\widehat{\Theta} \mid Z)$				1 929.8		857.2	

measurement errors are normal, with means equal to 1/5 of the variance of the corresponding data series. We have two sets of measurement errors on output and inflation. The measurement errors in the theoretical model capture the variance of the central bank's misconception of the lagged aggregate variables and are denoted by σ^2_{vycb} and $\sigma^2_{v\pi cb}$. The second set, which are the econometric measurement errors, corresponds to the measurement errors in the data series that are used to estimate the model. These are denoted by σ^2_{vy} and $\sigma^2_{v\pi}$ and their estimated values are discussed in Section 3.2 in the context of the overall fit of the model. The posterior estimates of the model measurement errors on lagged output and inflation as well as the non-fundamental shocks in the term structure, are larger for Australia than for the US, and the consequences of this are analysed in the next section.

3.1 Impulse Responses and the Role of Term Structure Information

Figures 1 and 2 below illustrate the impulse responses of output, inflation and the short interest rate for the US and Australia to productivity, demand and cost-push shocks.

In the US, the short interest rate responds more to the shocks in the impact period than in Australia. Part of this difference can be explained by differences in the informational content of the term structure in the two economies. In Figure 3 the impulse responses to the same shocks as above are plotted for Australia, but with no noise in the term structure. The Australian central bank then has more accurate information and we can see that the responses to shocks are much larger in the impact period. Output now immediately increases in response to a positive productivity shock since short rates fall by more. The Australian central bank can then also counter a cost-push shock by lowering demand already in the impact period.

The analysis above emphasises the beneficial aspects of responding to the term structure, but since the term structure is noisy, this means that sometimes the central bank will inadvertently respond to non-fundamental shocks. The responses of output, inflation and the short interest rate in the US and Australia to a unit non-fundamental shock to the 1-year bond rate are plotted in Figure 4.

The short interest rate in both the US and Australia increases in response to the non-fundamental shock in the 1-year interest rate, leading to a fall in both output and inflation.The response is quite small for both countries, with a 100 basis point

Figure 1: **Impulse Responses of US Data to Technology, Demand
and Cost-push Shocks**
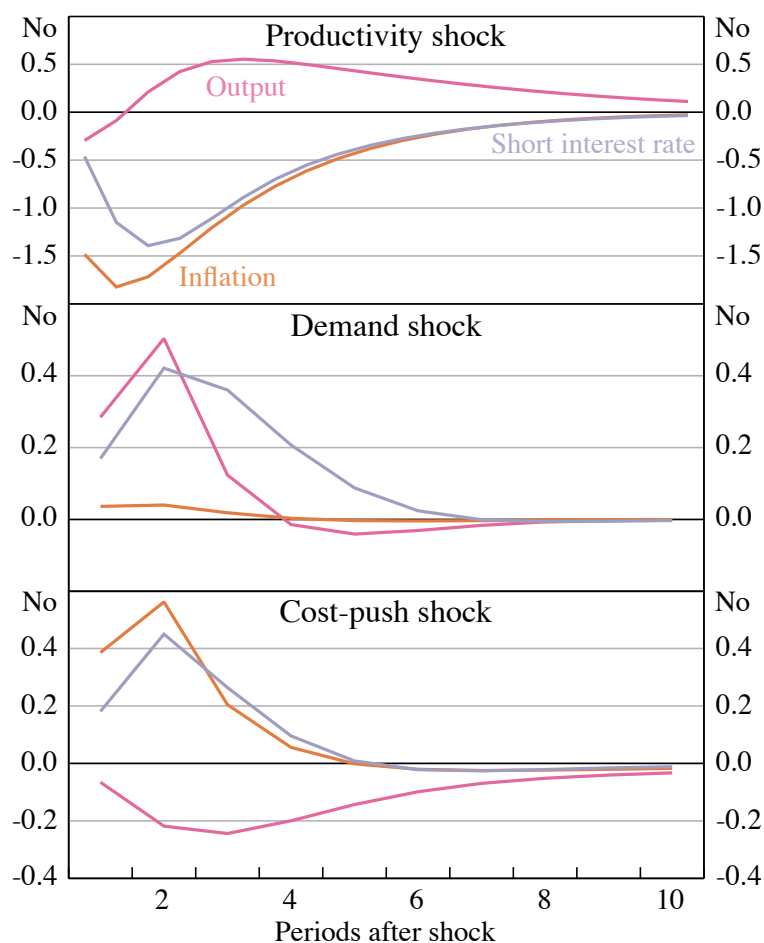
increase in the 1-year yield resulting in an increase of approximately 3 basis points in the short rate. The increase in the short rate leads to lower output and inflation.

3.2 The Fit of the Model

In order to assess the fit of the model, we can look at the estimated variances of the errors in the empirical measurement equation. Table 2 reports the ratio of the variance of the measurement errors in the observation over the variance of the corresponding variable for both the US and Australia. A large value indicates that the model is not very good at explaining the movements of the corresponding observed variable, since that means that a large portion of the variance of the variable does not conform to the model's dynamic and cross-equational implications. In general, the fit of the model is worse for Australia than for the US. There may be several explanations for this finding. First, Australia is a

Figure 2: **Impulse Responses of Australian Data to Technology, Demand and Cost-push Shocks**
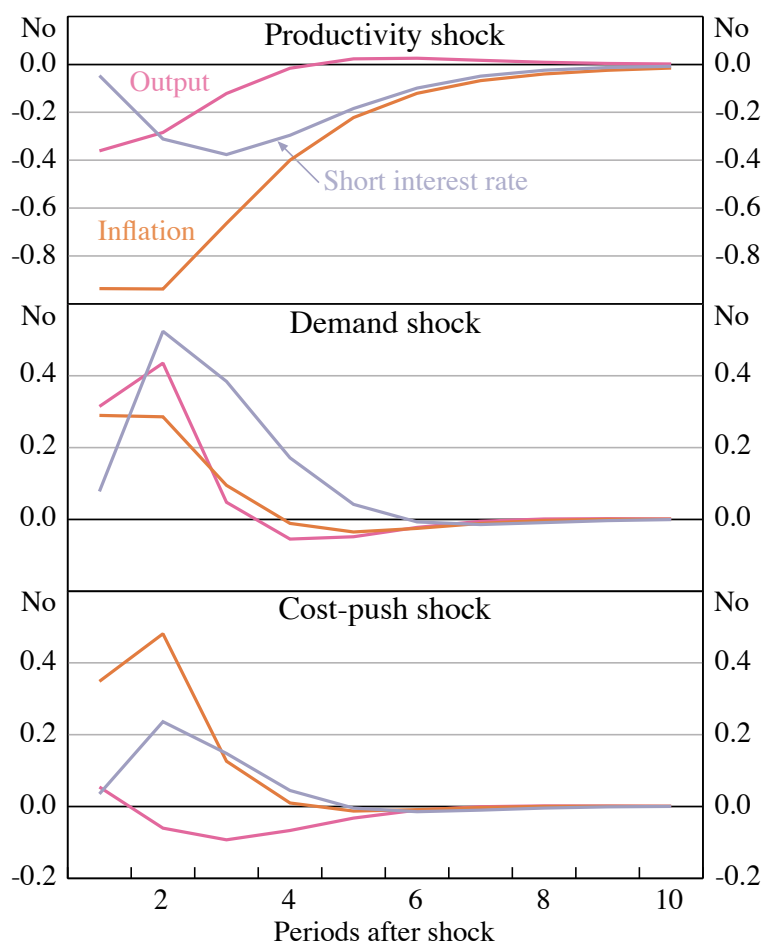
small, open economy while the US is a large and relatively more closed economy. This means that the baseline closed-economy New-Keynesian model employed here may be too simple to capture the dynamics of the Australian economy. The Australian economy is likely to be more affected by terms of trade shocks and shocks that affect capital flows and the exchange rate. As far as the responses of the endogenous variables to these types of disturbances are not nested in the demand, cost-push and productivity shocks of the model, this will reduce the fit. Another possible explanation, at least for the worse fit of long interest rates, is that the Australian bond market is not as deep as the US bond market. With fewer traders active, the market may be less efficient at aggregating dispersed information.

Figures 5 and 6 plot the actual data series and the model's fitted series of the observable endogenous variables. The model does a good job at tracking output, inflation, the short interest rate and the 6-month yield in the US, but has problems

Figure 3: **Impulse Response of Australian Estimates with No Noise in the Term Structure**
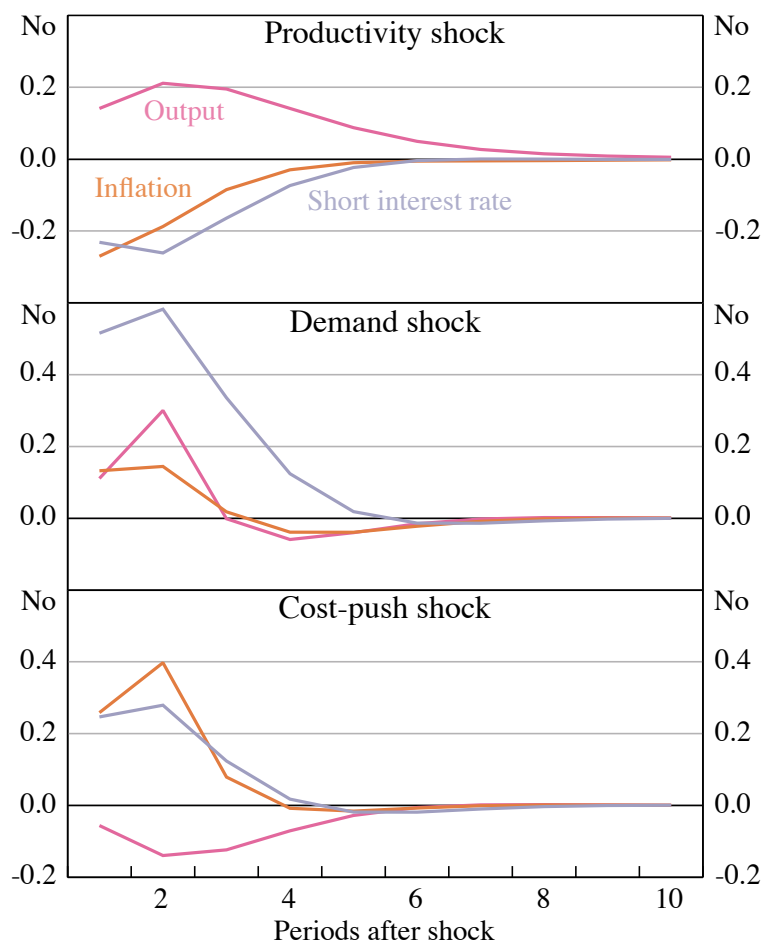
matching the volatility of the 1-year yield. The dual of the model's inability to match the long rate is that the long rate does not provide a lot of information about the underlying state of the model. The fit of the model is overall worse for Australia.

Table 2: Noise-over-variance Measure of Fit		
	US	Australia
σ_{vy}^2/σ_y^2	0.0099	0.24
$\sigma_{v\pi}^2/\sigma_\pi^2$	0.010	0.22
$\sigma_{v\mathscr{Y}1}^2/\sigma_{\mathscr{Y}1}^2$	0.010	0.0008
$\sigma_{v\mathscr{Y}2}^2/\sigma_{\mathscr{Y}2}^2$	0.011	0.31
$\sigma_{v\mathscr{Y}4}^2/\sigma_{\mathscr{Y}4}^2$	0.354	0.60

Figure 4: **Impulse Responses to Non-fundamental Shock to 1-year Bond Rate**
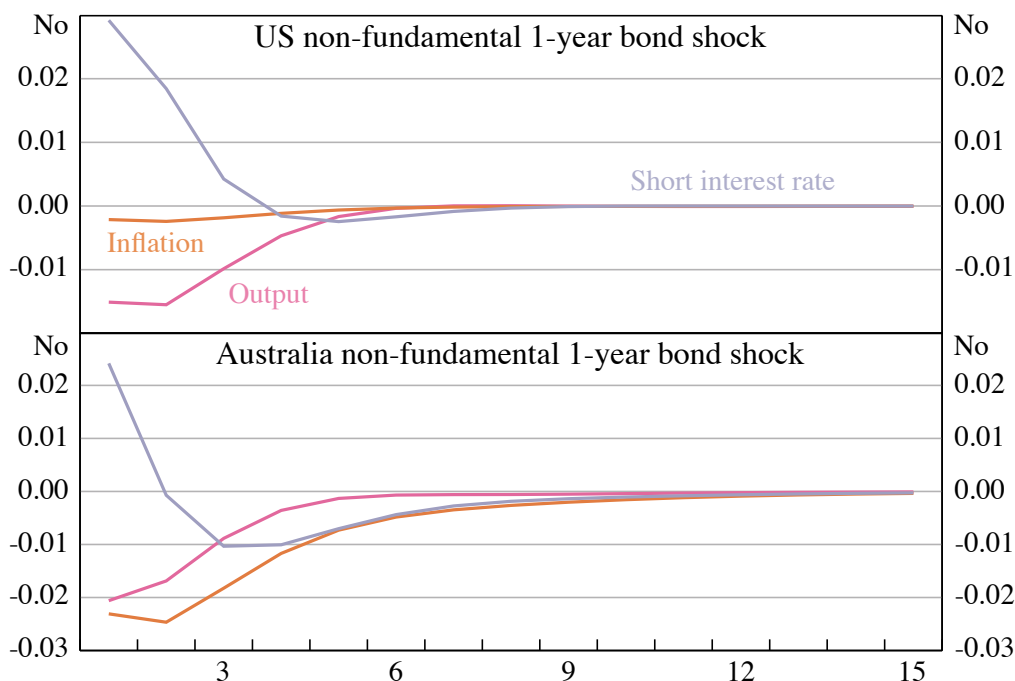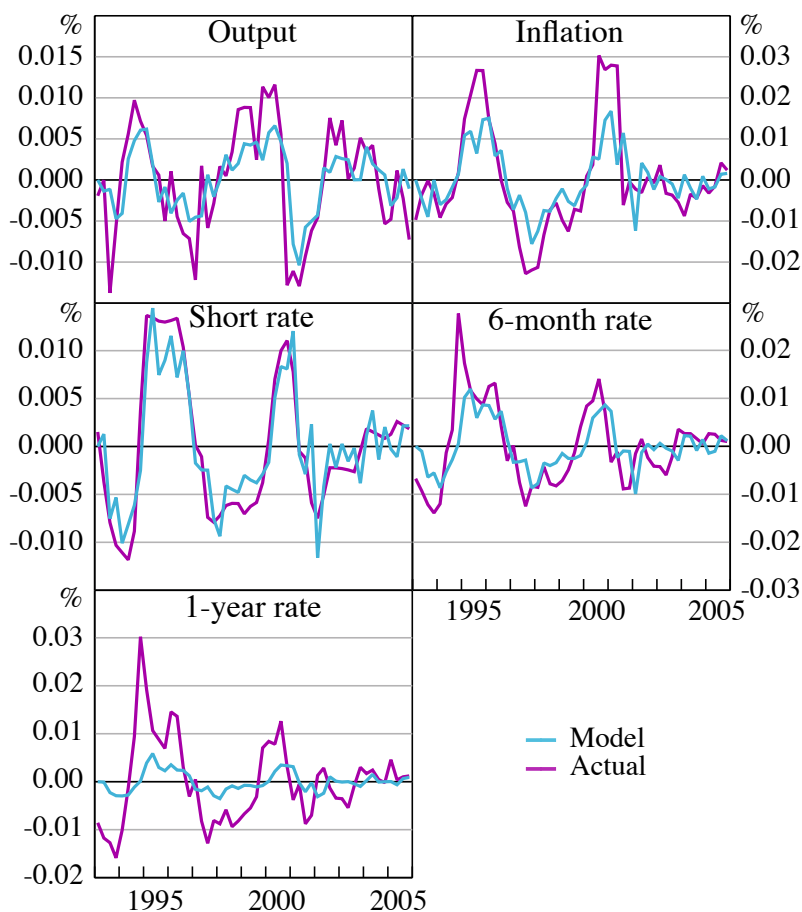

3.3 Variance Decomposition

The estimated model can be used to decompose historical variances of the endogenous variables into their exogenous sources. Table 3 reports the results of this exercise. In the US, inflation variance is driven almost entirely by cost-push shocks. The two main sources of fluctuations in output are productivity shocks and non-fundamental shocks to the 6-month bond rate, with respective shares of 77 per cent and 19 per cent. The proportion of variation attributed to non-fundamental sources may seem large. The present model attributes monetary policy shocks, that is, non-systematic movements in the interest rates to misconceptions about the state of the economy on behalf of the central bank, and thus gives a clear interpretation of the policy shock. From this perspective, 19 per cent is comparable to that found by other studies, where the central bank is assumed to be perfectly informed (for example Smets and Wouters 2004). That the 'policy shocks' appear to come from the bond market suggests that the misconceptions about the state of the economy may have been shared by the bond market participants. More than half of the variance of the US policy instrument is in response to productivity and demand shocks. That demand shocks explain only a small fraction of inflation and

Figure 5: US Data and Model's Fitted Series
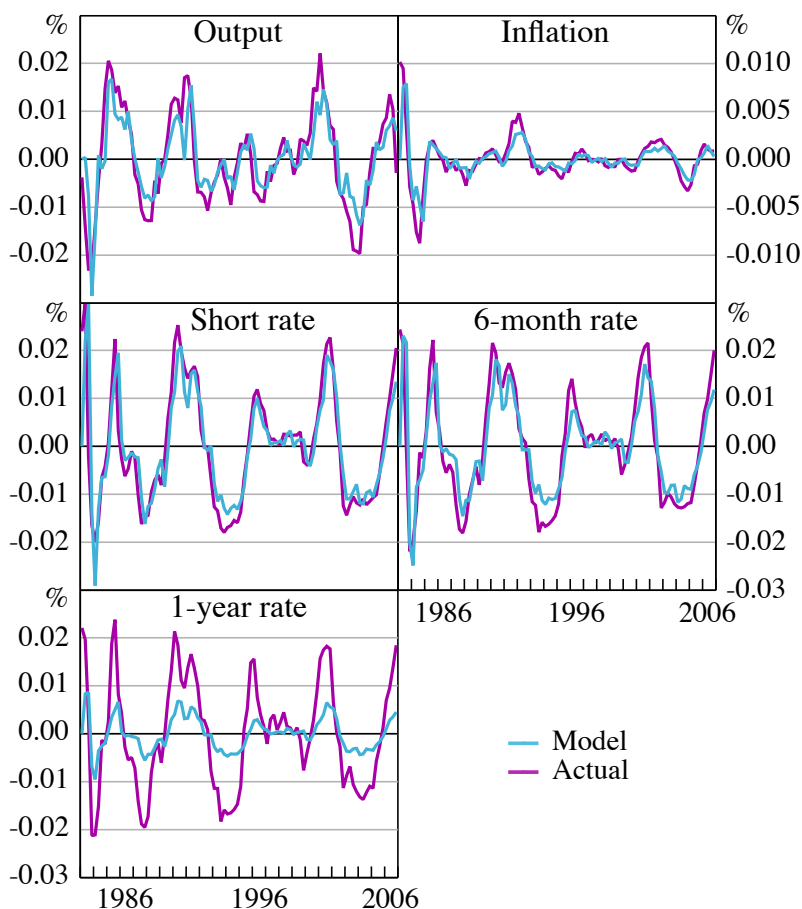
output variation suggests that these responses have been quite successful. About 20 per cent of the interest rate variance is explained by the non-fundamental shock to the 6-month interest rate. The instrument is the channel through which these shocks feed into the economy, and it may appear as if responding to these shocks is not optimal. However, since the central bank is assumed to respond with statistically optimal weights to movements in the term structure, the benefits of having more accurate estimates of the state, on average, outweighs the cost of occasionally responding to 'false alarms'.

The variance decomposition for Australia has a quite different pattern. Productivity shocks seem to play a very limited role for output, inflation and interest rate variances. A large portion of output-gap variance (75 per cent) is driven by demand shocks, and they also explain about a third of the variance of the short interest rate. Inflation is almost entirely explained by cost-push shocks. Central bank misconceptions of the inflation rate explains 10 per cent

Figure 6: Australian Data and Model's Fitted Series

of the short interest-rate variance while noise in the 6-month rate explains about 17 per cent. The responses of monetary policy to these non-fundamental shocks in turn explains 10 per cent of the variance of output.

Table 3: Variance Decomposition

	ε_t^a	ε_t^y	ε_t^π	v_t^{ycb}	$v_t^{\pi cb}$	$v_t^{\mathscr{Y}1}$	$v_t^{\mathscr{Y}2}$	$v_t^{\mathscr{Y}4}$
US								
y_t	0.77	0.04	0	0	0	0	0.19	0
π_t	0.10	0.03	0.84	0	0.02	0	0.01	0
i_t	0.35	0.42	0	0	0	0	0.20	0
Australia								
y_t	0.01	0.75	0.10	0.01	0.03	0	0.10	0
π_t	0.02	0.01	0.94	0	0.02	0	0.01	0
i_t	0.02	0.32	0.35	0.03	0.10	0	0.17	0.01

3.4 Robustness

In the baseline estimation above it was assumed that the central banks had an explicit interest rate smoothing objective. An alternative explanation of inertial interest rate changes is that central banks move slowly because they want to accumulate more information before they move. The model was re-estimated after imposing no interest rate smoothing, that is, by setting $\lambda_i = 0$. This leads to large reductions in the posterior probabilities for both the US and Australia, and the corresponding posterior odds ratios do not support the hypothesis that $\lambda_i = 0$.[5]

In the estimation, it was also imposed that the central banks of both the US and Australia used the information in the term structure to set policy. If this assumption is incorrect, it would bias the estimates of the noise in the term structure upwards. The log likelihood for both the US and Australia decreases slightly when we re-estimate the model without assuming that the central banks use the information in the term structure. The change for the US is slightly larger, probably because the assumption that the central bank observes the term structure should make less of a difference for Australia where the term structure is relatively more noisy.

4.   Conclusions

This paper has presented a general equilibrium model of monetary policy where the central bank operates in an uncertain environment and uses information contained in the term structure to estimate the underlying state of the economy more efficiently. This set-up creates a link between the term structure and the macroeconomy that is novel to the literature. A movement in the term structure signals that a change in the short-term interest rate set by the central bank may be desirable, which when implemented, in turn affects aggregate demand. Söderlind and Svensson (1997) warn that 'central banks should not react mechanically to [market expectations]' since this may lead to a situation of 'the central bank chasing the market, and the market simultaneously chasing the central bank'. This argument is formalised in Bernanke and Woodford (1997) where the authors show that if central banks react to market expectations, a situation with a multiplicity of equilibria or where no equilibrium exists may arise. In this

---

[5]   More details on posterior mode estimates and likelihoods for the alternative specifications are available from the author upon request.

paper we have argued that there may be benefits from systematic reactions to market expectations, but with some important qualifications. The non-existence of equilibria arises in the model of Bernanke and Woodford because the central bank can extract the underlying state perfectly from observing the expectations of the private sector. Inflation will thus always be on target. But if inflation is always on target and private agents only care about accurate inflation forecasts, there is no incentive for the private sector to pay a cost to be informed about the underlying shock, and observing expectations will not reveal any information. The model here differs because, to the extent that there is noise in the bond market, the central bank cannot extract the underlying shock perfectly. Thus there will always exist a cost of information-gathering that is small enough to make it profitable for the private sector to acquire information about the underlying shock, even if private agents only cared about having accurate inflation forecasts. Additionally, in this model the forecasting problem of private agents involves more than accurately forecasting inflation, since bond prices depend on real factors through the stochastic discount factor as well as the price level. Insofar as the real discount factor is affected by the underlying state, agents will have an incentive to collect information about it, regardless of the behaviour of inflation.

Ultimately, the informational content of the term structure is an empirical question. The model presented here provides a coherent framework within which any information about the state of the economy that is contained in the term structure can be quantified in a general equilibrium setting. The model explicitly takes into account that the central bank may use the information in the term structure to set policy and therefore influences what it observes. The model was estimated on US and Australian data using Bayesian methods. The empirical exercise suggests that there is some information in the US term structure that allows the Federal Reserve to respond to shocks in a timely manner, while the Australian term structure appears to be more noisy and less informative for the monetary policy process. This difference may be explained by the fact that Australia is a small and relatively open economy, and hence difficult to represent using a closed-economy model.

Appendix A: The Model

The parameters of the linearised model

$$a_t = \rho a_{t-1} + \varepsilon_t^a \tag{A1}$$

$$y_t = \mu_{yf} E_t y_{t+1} + \mu_{yb} y_{t-j} - \phi \left[i_t - E_t \pi_{t+1} \right] + \varepsilon_t^y \tag{A2}$$

$$\pi_t = \mu_{\pi f} E_t \pi_{t+1} + \mu_{\pi b} \pi_{t-1} + \kappa mc_t + \varepsilon_t^\pi \tag{A3}$$

are given by

$$\mu_{yf} \equiv \frac{\gamma}{\gamma - \eta + \gamma \eta}, \quad \mu_{yb} \equiv \frac{-\eta(1-\gamma)}{\gamma - \eta + \gamma \eta}$$

$$\phi \equiv \frac{1}{\gamma - \eta + \gamma \eta}$$

$$\mu_{\pi f} \equiv \frac{\beta \theta}{\theta + \omega(1 - \theta(1-\beta))}, \quad \mu_{\pi b} \equiv \frac{\omega}{\theta + \omega(1 - \theta(1-\beta))}$$

$$\kappa \equiv \frac{(1-\omega)(1-\theta)(1-\theta\beta)}{\theta + \omega(1 - \theta(1-\beta))}.$$

The model can be put in compact form

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = A \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + B i_t + C \varepsilon_t \tag{A4}$$

$$X_{1,t} = \left[\bar{y}_t, y_{t-1}, \pi_{t-1}, \varepsilon_t^y, \varepsilon_t^\pi, i_{t-1}, \Delta i_t \right]' \tag{A5}$$

$$X_{2,t} = [y_t, \pi_t]' \tag{A6}$$

where the coefficient matrices A, B and C are given by

$$A = A_0^{-1} A_1, \quad B = A_0^{-1} B_1, \quad C = A_0^{-1} \begin{bmatrix} C_1 \\ \mathbf{0}_{2 \times 1} \end{bmatrix}$$

$$
A_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_{yf} & \phi \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_{\pi f}
\end{bmatrix}
\tag{A7}
$$

$$
A_1 = \begin{bmatrix}
\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & -\mu_{yb} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\kappa\frac{1+\varphi}{(\varphi+\gamma)} & \kappa\frac{\eta(1-\gamma)}{(\varphi+\gamma)} & -\mu_{\pi b} & 0 & 0 & 0 & 0 & -\kappa(\varphi+\gamma) & 1
\end{bmatrix}
\tag{A8}
$$

$$
B_1 = \begin{bmatrix}
\mathbf{0}_{5x1} \\
1 \\
0 \\
\phi \\
0
\end{bmatrix}, \quad
C_1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\tag{A9}
$$

$$
\mathbf{L}_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left[\lambda_y (y_{t+k} - \bar{y}_{t+k})^2 + \pi_{t+k}^2 + \lambda_i (i_t - i_{t-1})^2 \right] \right].
\tag{A10}
$$

The likelihood function

To compute the likelihood of the model, we follow the method of Hansen and Sargent (2004). Form a state space system of the AR(1) process of the state $\overline{X}_t$

$$\overline{X}_t = M\overline{X}_{t-1} + N \begin{bmatrix} \varepsilon_t \\ \mathbf{v}_t \end{bmatrix} \tag{A11}$$

$$\widehat{Z}_t = \widehat{\mu} + D\overline{X}_t + \widehat{\mathbf{v}}_t \tag{A12}$$

$$E\mathbf{v}_t\mathbf{v}_t' = \begin{bmatrix} \Sigma_{vv}^{cb} & \mathbf{0} \\ \mathbf{0} & \Sigma_{vv}^{\mathcal{Y}\mathcal{Y}} \end{bmatrix}, \quad E\widehat{\mathbf{v}}_t\widehat{\mathbf{v}}_t' = \begin{bmatrix} \Sigma_{vv}^{ZZ} & \mathbf{0} \\ \mathbf{0} & \Sigma_{vv}^{\mathcal{Y}\mathcal{Y}} \end{bmatrix} \tag{A13}$$

where $\widehat{Z}_t$ is the vector of variables that are observable (to us as econometricians) and Σ_{vv}^{ZZ} is the covariance matrix of the econometric measurement errors on output and inflation. Construct the innovation series $\{u_t\}_{t=0}^{T}$ from the innovation representation

$$\widehat{X}_{t+1} = \widehat{\mu} + M\widehat{X}_t + Ku_t \tag{A14}$$
$$\widehat{Z}_t = D\widehat{X}_t + u_t \tag{A15}$$

by rearranging to

$$u_t = \widehat{Z}_t - D\widehat{X}_t \tag{A16}$$
$$\widehat{X}_t = \widehat{\mu} + M\widehat{X}_{t-1} + Ku_t \tag{A17}$$

where K is the Kalman gain matrix

$$K = PD'(DPD' + \widehat{\Sigma}_{vv})^{-1} \tag{A18}$$
$$P = M(P - PD'(DPD' + \widehat{\Sigma}_{vv})^{-1}DP)M' + N\Sigma_{\varepsilon\varepsilon}N'. \tag{A19}$$

The log likelihood $\mathscr{L}(\widehat{Z} \mid \Theta)$ of observing the data Z for a given set of parameters Θ can then be computed as

$$\mathscr{L}(Z|\Theta) = -.5\sum_{t=0}^{T}\left[p\ln(2\pi) + \ln|\Omega| + u_t'\Omega^{-1}u_t\right] \tag{A20}$$

where

$$\Omega = MPM' + \widehat{\Sigma}_{vv}. \tag{A21}$$

The posterior mode $\widehat{\Theta}$ is then given by

$$\widehat{\Theta} = \arg\max \mathscr{L}(\Theta) + \mathscr{L}(Z \mid \Theta) \tag{A22}$$

where $\mathscr{L}(\Theta)$ denotes the log of the prior likelihood of the parameters Θ. The posterior mode was found using Bill Goffe's simulated annealing minimiser (available at <http://cook.rfe.org/>). The posterior standard errors was calculated using Gary Koop's Random Walk Metropolis-Hastings distribution simulator (available at <http://www.wiley.co.uk/koopbayesian>).

References

Amato J and T Laubach (2004), 'Implications of habit formation for optimal monetary policy', *Journal of Monetary Economics*, 51(2), pp 305–325.

Ang A and M Piazzesi (2003), 'A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables', *Journal of Monetary Economics*, 50(4), pp 745–787.

Ang A, M Piazzesi and M Wei (2003), 'What does the yield curve tell us about GDP Growth?', Columbia University and University of California, Los Angeles, mimeo.

Bartolini L, G Bertola and A Prati (2002), 'Day-to-day monetary policy and the volatility of the Federal Funds interest rate', *Journal of Money, Credit and Banking*, 34(1), pp 137–159.

Bernanke B (2004), 'What policy makers can learn from asset prices', Remarks Before The Investment Analysts Society of Chicago, Illinois, 15 April, available at <http://www.federalreserve.gov/boarddocs/speeches/2004/20040415/default.htm>.

Bernanke B and M Woodford (1997), 'Inflation forecasts and monetary policy', *Journal of Money, Credit and Banking*, 29(4, Part 2), pp 653–684.

Campbell JY and JH Cochrane (1999), 'By force of habit: a consumption-based explanation of aggregate stock market behavior', *Journal of Political Economy*, 107(2), pp 205–251.

Ellingsen T and U Söderström (2001), 'Monetary policy and market interest rates', *American Economic Review*, 91(5), pp 1594–1607.

Estrella A and F Mishkin (1998), 'Predicting U.S. recessions: financial variables as leading indicators', *Review of Economics and Statistics*, 80(1), pp 45–61.

Galí J and M Gertler (1999), 'Inflation dynamics: a structural econometric analysis', *Journal of Monetary Economics*, 44(2), pp 195–222.

Goodfriend M (1998), 'Using the term structure of interest rates for monetary policy', Federal Reserve Bank of Richmond *Economic Quarterly*, 84(3), pp 13–30.

Hansen LP and TJ Sargent (2004), 'Recursive models of dynamic linear economies', New York University and University of Chicago, unpublished.

Harvey CR (1988), 'The real term structure and consumption growth', *Journal of Financial Economics*, 22(2), pp 305–333.

Hayek FA (1945), 'The use of knowledge in society', *American Economic Review*, 35(4), pp 519–530.

Hördahl P, O Tristani and D Vestin (2006), 'A joint econometric model of macroeconomic and term-structure dynamics', *Journal of Econometrics*, 131(1–2), pp 405–444.

Lippi F and S Neri (2003), 'Information variables for monetary policy in a small structural model of the euro area', Bank of Italy, mimeo.

Mishkin FS (1990), 'The information in the longer maturity term structure about future inflation', *Quarterly Journal of Economics*, 105(3), pp 815–828.

Smets F and R Wouters (2004), 'Comparing shocks and frictions in the U.S. and in the euro area business cycles: a Bayesian DSGE approach', European Central Bank Working Paper No 391.

Söderlind P (1999), 'Solution and estimation of RE macromodels with optimal policy', *European Economic Review*, 43(4–6), pp 813–823.

Söderlind P and LEO Svensson (1997), 'New techniques to extract market expectations from financial instruments', *Journal of Monetary Economics*, 40(2), pp 383–429.

Svensson LEO and M Woodford (2004), 'Indicator variables for optimal policy under asymmetric information', *Journal of Economic Dynamics and Control*, 28(4), pp 661–690.

Wu T (2002), 'Macro factors and the affine term structure of interest rates', Federal Reserve Bank of San Francisco Working Paper No 2002-06.