Firm Demographics and Sectoral Reallocation*

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Abstract

The US economy has experienced declining business dynamism alongside substantial sectoral reallocation, particularly the decline of manufacturing. We develop a multisector model of firm entry-exit and growth, with endogenous firm selection, to jointly investigate these phenomena. The model features feedback effects from firm demographics—age profiles of exit rates and average firm size—in response to aggregate and sectoral shocks. We establish equilibrium existence and uniqueness, and characterize transitional dynamics. Quantitatively, we evaluate the role of three forces driving manufacturing's decline: population growth, sectoral technology, and international trade. The model replicates key patterns of firm dynamics and sectoral reallocation in the data, including manufacturing's employment share declining from 26% to 9% and firm share from 7.4% to 4.3%. Transitional dynamics generated by changes in the firm-age distribution account for one-half of the firm share decline explained by all three forces combined. Population growth changes are essential to explain firm entry-exit patterns in both manufacturing and nonmanufacturing. Sectoral technology explains two-thirds of manufacturing employment decline, while trade drives the decline in the manufacturing firm share.

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1 Introduction

The US economy has experienced a widespread decline in business dynamism over the past four decades. The firm entry rate has declined from 14% to 8%, exit rates have declined, and average firm size has increased. Recent work shows that declining population growth is a key driver of these aggregate patterns (Hopenhayn et al., 2022; Karahan et al., 2024; Peters and Walsh, 2021). Lower population growth implies fewer entrants relative to the existing stock of incumbent firms, which shifts the firm-age distribution towards older firms. Firm demographics—the age profiles of exit rates and average firm size—–then amplify this initial shock: since older firms exhibit lower exit rates and larger average size, firm aging generates feedback effects that explain the decline in business dynamism.

The decline in US business dynamism has also coincided with substantial sectoral reallocation. Does the demographic shock from declining population growth apply uniformly across sectors? We divide the US economy into two sectors: manufacturing and nonmanufacturing. Consistent with the aggregate implications of declining population growth, both sectors exhibit a decline in entry rates which leads to firm aging. As in the aggregate, the exit rate decline in both sectors is mostly due to a composition effect: a declining profile of exit rates by firm age combined with firm aging.

These sectors, however, exhibit a striking divergence in other dimensions. While average firm size in nonmanufacturing has increased by 50% due to firm aging, it has declined by 30% in manufacturing *despite* firm aging. In addition, the US economy has reallocated both employment and firms away from manufacturing. The employment share of manufacturing has declined from 26% in 1978 to 9% in 2019, which represents a decline of about 8 million jobs. The share of firms in manufacturing has declined from 7.4% to 4.3% over the same time period. These sectoral facts raise important questions: can changes in population growth generate these diverging sectoral dynamics? What is the role of differences in sectoral firm demographics? What other forces are needed to explain these facts?

To investigate these questions, this paper generalizes the framework in Hopenhayn et al. (2022) to incorporate both firm demographics and sectoral dynamics. Firm entry, exit, growth, and endogenous firm selection generate firm demographics: age-profiles of average firm size and exit rates by firm age. Sectoral dynamics feature sectoral reallocation, input-output linkages, and rich firm dynamics within individual sectors. We establish existence and uniqueness of equilibrium in this richer environment and characterize the transitional dynamics following aggregate and sectoral shocks. A key feature that simplifies computation is that sectoral firm entry and sectoral output form a linear system, conditional on aggregate expenditure.

Firm entry in a sector depends on current entry in other sectors and the history of entry in the economy. Sectoral input-output linkages generate dependence on current entry in other sectors, as in Baqaee (2018) and Cavalcanti, Mendes and Pannella (2023).

History dependence of entry appears through two distinct channels: own-sector and cross-sector history dependence. Own-sector history dependence corresponds to the channel highlighted by Hopenhayn et al. (2022): past entry and survival determine the current stock of incumbent firms in a sector, which pins down the gap that needs to be filled by sectoral entry. Cross-sector history dependence is a novel channel through which current entry in a sector also depends on past entries in all other sectors. This dependence arises because all sectors are linked through a common resource constraint.

Beyond its direct effect on firm entry rates, population growth also generates sectoral reallocation due to differences in firm demographics across sectors. An increase in the rate of aggregate population growth raises entry rates in all sectors, but the magnitude varies by sector. Sectors that exhibit steeper firm-age profiles of exit rates and output-per-firm experience an increase in their firm share. The response of sectoral employment shares to population growth depends on how intensive sectoral production is in overhead labor. Sectors with steeper age profiles of exit and output-per-worker, which depends on overhead labor, experience an increase in employment shares.

We next investigate the importance of population growth and firm demographics for US sectoral dynamics. In particular, we study the decline of US manufacturing. In addition to changes in population growth, we incorporate two other driving forces that are important for the evolution of manufacturing: sectoral technology and international trade. A large literature on structural transformation has documented that improvements in sectoral technology reallocate resources away from manufacturing; Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). In recent decades, the net import share of manufacturing has steadily increased from 5% to 25%. Given that the rest of the economy has not experienced a similar increase in net imports, changes in international trade are an essential driving force to understand the decline in relative manufacturing output; Pierce and Schott (2016) and Fort, Pierce and Schott (2018).

We quantitatively evaluate the ability of these three driving forces—population growth, sectoral technology and international trade—to jointly replicate sectoral firm dynamics along with the observed manufacturing decline. Counterfactual exercises show that changes in population growth—the Baby Boom and its subsequent decline—played a significant role in generating firm entry-exit dynamics by sector. Even though a shock to population growth is an aggregate shock, it affects the two sectors differently because of the differences in firm demographics by sector. The decline in population growth since the 1970s lowers entry rates in nonmanufacturing more than in manufacturing. This effect reflects the stronger feedback effects due to firm-aging and steeper age-profiles of exit rates and average size in nonmanufacturing. The population growth force is primarily responsible for generating the observed 50% increase in average size in nonmanufacturing.

The three driving forces together successfully replicate the key patterns of US manufacturing decline, including the decline in manufacturing's employment share from 26% to 9% and its firm share from 7.4% to 4.3%. Transitional dynamics generated by changes in the firm-age distribution account for one-half of the firm share decline explained by all three forces combined. Decomposing these effects, we find that the observed changes in sectoral technology indicate a shift away from labor-intensive production in manufacturing, consistent with the decline in average size within firm-age bins in manufacturing. This change in sectoral technology explains two-thirds of the decline of manufacturing employment, with the remaining one-third attributed to changes in trade. The decline in the manufacturing firm share is largely driven by changes in trade. This decline would have been more severe in the absence of changes in population growth.

2 Theoretical Framework

There are I production sectors, indexed by $i = 1, \ldots, I$, each producing one of J commodities indexed by $j = 1, \ldots, J$, with $I = J$.

2.1 Consumers

There are L_t identical consumers who supply labor inelastically. Labor is the numeraire. Consumers have preferences over the J-vector of final-good quantities $\mathbf{d}_t$ represented by the utility function $C(\mathbf{d}_t)$. Equivalently, $C(\mathbf{d}_t)$ can be interpreted as an aggregator technology that combines the final-good commodities into a single composite consumption good. Let $\mathbf{p}_t$ denote the corresponding J-vector of commodity prices. Consumers maximize utility subject to the budget constraint $\mathbf{p}_t \cdot \mathbf{d}_t \leq E_t$, where aggregate expenditure E_t is taken as given by consumers but determined in equilibrium.[1]

2.2 Producers

2.2.1 Static Problem

A firm in sector i with productivity z has a production function

$$q_i(z, n, \mathbf{x}) = z^{1-\eta_i} f_i(n, \mathbf{x})^{\eta_i}, \quad 0 < \eta_i < 1,$$

where n is labor input and $\mathbf{x}$ is a J-vector of intermediate inputs. The aggregator f_i is homogeneous of degree one, differentiable, and increasing in all inputs. Firms also face a fixed cost c_i^f in units of labor, interpreted as an overhead or operating expense that induces endogenous exit.

For any commodity price vector $\mathbf{p}$ we can solve for profit functions $\pi_i(z, \mathbf{p})$, output functions $q_i(z, \mathbf{p})$, and input demand functions $n_i(z, \mathbf{p})$ and $x_{ji}(z, \mathbf{p})$. Because f_i is homothetic, all firms in a sector will choose inputs in the same proportion. The unit input requirements for sector i are defined as the solution to the

[1] The aggregator may be non-homothetic so final-good good demand $\mathbf{d}(\mathbf{p}_t, E_t)$ need not scale linearly with E_t.

cost-minimization problem:

$$c_i(\mathbf{p}) = \min_{n,\mathbf{x}} n + \mathbf{p} \cdot \mathbf{x} \qquad \text{subject to } f_i(n, \mathbf{x}) = 1.$$

where $c_i(\mathbf{p})$ is the (constant) marginal cost of the input bundle. Given $c_i(\mathbf{p})$, a firm with productivity z chooses scale y to solve

$$\max_y \; p_i z^{1-\eta_i} y^{\eta_i} - c_i(\mathbf{p}) y - c_i^f,$$

with first-order condition

$$\eta_i p_i z^{1-\eta_i} y^{\eta_i - 1} = c_i(\mathbf{p}).$$

The optimal scale is

$$y_i(\mathbf{p}, z) = \left(\frac{\eta_i p_i}{c_i(\mathbf{p})} \right)^{\frac{1}{1-\eta_i}} z.$$

Output is therefore

$$q_i(\mathbf{p}, z) = \left(\frac{\eta_i p_i}{c_i(\mathbf{p})} \right)^{\frac{\eta_i}{1-\eta_i}} z. \tag{1}$$

Profits are

$$\pi_i(\mathbf{p}, z) = p_i q_i(\mathbf{p}, z) - c_i(\mathbf{p}) y_i(\mathbf{p}, z) - c_i^f \tag{2}$$

$$= (1 - \eta_i) \left(\frac{\eta_i^{\eta_i} p_i}{c_i(\mathbf{p})^{\eta_i}} \right)^{\frac{1}{1-\eta_i}} z - c_i^f, \tag{3}$$

equal to $1 - \eta$ share of revenues, net of the fixed cost. Likewise, it can be shown that the demand of intermediate input j for a firm in sector i scales linearly in z:

$$x_{ji}(\mathbf{p}, z) = z \, x_{ji}(\mathbf{p}, 1). \tag{4}$$

Let $\zeta_i^w = \frac{\partial c_i}{\partial w} \frac{w}{c_i}$ be the elasticity of marginal cost with respect to the wage (with $w = 1$). Then variable labor per unit of the aggregator is $\zeta_i^w c_i(\mathbf{p})$, and total firm employment is

$$n_i(\mathbf{p}, z) = \zeta_i^w \left(\frac{\eta_i p_i}{c_i(\mathbf{p})^{\eta_i}} \right)^{\frac{1}{1-\eta_i}} z + c_i^f.$$

2.2.2 Dynamic Problem

A firm's idiosyncratic productivity z evolves according to a sector-specific, persistent, stationary Markov process $F_i(z'|z)$. The exogenous aggregate state $S_t = (\mathcal{T}_t, L_t)$ follows its own (potentially non-stationary) Markov process, where $\mathcal{T}_t \equiv \{\eta_{it}, c_{it}(\mathbf{p}_t)\}_{i \in I}$ summarizes *sectoral technology*. The value of a firm is

$$v_i(z; \mathbf{p}_t, S_t) = \max \Big\{ 0, \pi_i(z; \mathbf{p}_t, S_t) + \beta \mathbb{E}\left[v_i(z'; \mathbf{p}_{t+1}, S_{t+1}) | z, S_t \right] \Big\}, \qquad 0 < \beta < 1$$

for each sector i.[2] Each period, a firm compares its exit value, normalized to zero, to its continuation value.

2.2.3 Firm Entry

Entry costs differ across sectors. Entrants in sector i pay an entry cost c_e^i in units of labor. Upon paying this cost of entry, entrants draw their initial productivity according to distribution function G_i. Expected value for a potential entrant is

$$v_i^e(\mathbf{p}_t, S_t) = \int v_i(z; \mathbf{p}_t, S_t) dG_i(z) - c_i^e$$

2.3 Equilibrium

Definition 1. *An equilibrium consists of a price vector $\mathbf{p}_t$, firm measures vector $\boldsymbol{\mu}_t$, entry mass vector $\mathbf{m}_t$, exit thresholds vector $\mathbf{z}_t^*$, input demand vectors $\mathbf{n}_t$ and $(\mathbf{x}_{it})_{i=1}^I$, final demand vector $\mathbf{d}_t$ and aggregate expenditure E_t, such that firms maximize profits, consumers maximize utility, and:*

1. ***Entry:*** *$v_i^e(\mathbf{p}_t, S_t) \times m_{it} = 0$ and $v_i^e(\mathbf{p}_t, S_t) \leq 0, \quad \forall i$.*
2. ***Exit:*** *$v_i(z_{it}^*; \mathbf{p}_t, S_t) = 0, \quad \forall i$.*

[2] We assume that this is a small open economy with interest rate R such that $\beta R = 1$. The interest rate is invariant to fluctuations in $C(\mathbf{d}_t)$ in equilibrium if the small open economy borrows or lends in international credit markets in units of the consumption good. This assumption is only needed when not considering a steady state.

3. ***Law of Motion:*** *For all measurable $\mathcal{Z}$,*

$$\mu_{i,t+1}(\mathcal{Z}) = \int \int_{z' \geq z^*_{i,t+1}, z' \in \mathcal{Z}} dF_i(z'|z)\, d\mu_{it}(z) + m_{i,t+1}\left(\int_{z' \geq z^*_{i,t+1}, z' \in \mathcal{Z}} dG_i(z')\right), \quad \forall i \tag{5}$$

4. ***Good Markets Clearing:***

$$\int q_j(z, \mathbf{p}_t) d\mu_{jt}(z) = d_j(\mathbf{p}_t, E_t) + \sum_i \int x_{ji}(z, \mathbf{p}_t) d\mu_{it}(z), \quad \forall j \tag{6}$$

5. ***Labor Market Clearing:***

$$\sum_i \int n_i(z, \mathbf{p}_t) d\mu_{it}(z) + \sum_i M_{it} c_i^f + \sum_i m_{it} c_i^e = L_t \tag{7}$$

If in addition $\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t+1} = \boldsymbol{\mu}^*$, this is a stationary equilibrium.

2.4 Properties of the Equilibrium

We next develop a simple algorithm to construct the equilibrium. It relies on necessary conditions which have a unique solution. Prices can be obtained considering only the supply side of the model, assuming the free entry conditions of the equilibrium definition hold with equality every period.

Proposition 1 (Independence of Aggregate State)**.** *In the unique equilibrium, firm values are independent of the aggregate state $(\mathbf{p}_t, S_t)$ and depend only on the idiosyncratic productivity shock z. It follows that exit thresholds $\mathbf{z}^*_t$ are also independent of the aggregate state.*

Corollary 1. *Proposition 1 implies that exit rates by firm age are time-invariant.*

The proof relies on a guess and verify argument. Note that prices enter into the profit function just as an argument of the sector-specific variable profits $\tilde{\pi}_i(1; \mathbf{p}_t, S_t)$,

$$v_i(z; \mathbf{p}_t, S_t) = \max\left\{0, z\tilde{\pi}_i(1; \mathbf{p}, S) - c_i^f + \beta \mathbb{E}\left[v_i(z'; \mathbf{p}_{t+1}, S_{t+1})|z, S_t\right]\right\}$$

where

$$\tilde{\pi}_i(1; \mathbf{p}, S) = (1 - \eta_i) \left(\frac{p_i \eta_i^{\eta_i}}{c_i(\mathbf{p})^{\eta_i}} \right)^{\frac{1}{1-\eta_i}},$$

is the variable profit of a unit-idiosyncratic productivity firm. Based on this observation, we conjecture (and verify) that sector-specific variable profits are constant over time (relative to the numeraire):

$$\tilde{\pi}_i(1; \mathbf{p}_t, S_t) = \hat{\pi}_i.$$

If so, firm values satisfy

$$v_i(z; \hat{\pi}_i) = \max \left\{ 0,\ z\hat{\pi}_i - c_i^f + \beta \mathbb{E}[v_i(z'; \hat{\pi}_i) \mid z] \right\},$$

and so does the entrant value $v_i^e(\hat{\pi}_i)$. The latter is strictly increasing in $\hat{\pi}_i$, so there exists a unique vector $(\hat{\pi}_1, \ldots, \hat{\pi}_I)$ which satisfy the free entry conditions,

$$v_i^e(\hat{\pi}_i) = 0, \quad \forall i. \tag{8}$$

Note that each $\hat{\pi}_i$ is found using the value function of the corresponding sector separately.

Given $\hat{\pi}_i$'s for each sector, prices must satisfy

$$\hat{\pi}_i = (1 - \eta_{it}) \left(\frac{p_{it} \eta_{it}^{\eta_{it}}}{c_{it}(\mathbf{p}_t)^{\eta_{it}}} \right)^{\frac{1}{1-\eta_{it}}}, \quad \forall i, \tag{9}$$

every period. Taking logs,

$$\ln \hat{\pi}_i = \xi_{it} + \frac{1}{1 - \eta_{it}} \ln p_{it} - \frac{\eta_{it}}{1 - \eta_{it}} \ln c_{it}(\mathbf{p}_t), \quad \forall i,$$

with $\xi_{it} = \ln(1 - \eta_{it}) + \frac{\eta_{it}}{1-\eta_{it}} \ln \eta_{it}$. For any value of $c_{it}(\mathbf{p}_t)$ and η_{it} we can find the price p_i that equates the above to the target $\hat{\pi}_i$ and thus defines the mapping

$$T_i(\ln \mathbf{p}_t) = (1 - \eta_{it})(\ln \hat{\pi}_i - \xi_{it}) + \eta_{it} \ln c_{it}(\exp(\ln \mathbf{p}_t)), \quad \forall i,$$

This mapping takes the vector $\ln \mathbf{p}_t$ and maps it into a new vector $\ln \mathbf{p}_t'$. It follows immediately that a vector $\mathbf{p}_t^*$ is an equilibrium price vector if and only if it is a fixed point of this mapping. To show that the mapping T_i is a contraction, consider the partial derivative:

$$T_{ik}\left(\ln \mathbf{p}_t\right) = \frac{\partial T_i\left(\ln \mathbf{p}_t\right)}{\partial \ln p_{kt}} = \eta_{it}\left(\frac{p_{kt}x_{ki,t}}{c_{it}(\mathbf{p}_t)}\right),$$

where the last term in brackets follows from the fact that the elasticity of cost with respect to the price of an input is its share in total cost. Because cost shares and η_{it} are less than one, the mapping T_i is a contraction with modulus $\eta_{it}\left(\frac{p_{jt}x_{ki,t}}{c_{it}(\mathbf{p}_t)}\right)$.[3] Thus, iterations on this mapping converge to a unique fixed point $\mathbf{p}_t^*$, the unique equilibrium price vector.

Given prices $\mathbf{p}_t^*$, we now solve for entry and verify that the remaining equilibrium conditions hold.

Start with the good markets clearing conditions (6). These can be written compactly as

$$\mathbf{q}_t = \mathbf{\Omega}^\top\left(\mathbf{p}_t\right)\mathbf{q}_t + \mathbf{d}\left(\mathbf{p}_t, E_t\right)$$

where $\mathbf{q}_t = (Q_{1t}, \ldots, Q_{Jt})$ denote the vector of sectoral outputs and $\mathbf{\Omega}(\mathbf{p}_t) = (\Omega_{ji}(\mathbf{p}_t))$ is the input-output matrix of coefficients that summarize how much input j is used per unit of output in sector i. These coefficients reflect firms' optimal input choices at prices $\mathbf{p}_t$, rather than technological constraints. To see how they arise, let $q_i(\mathbf{p}_t, 1)$ denote the output of a firm with unit-idiosyncratic productivity in sector i. By (1) and (4), a firm with productivity z produces $z\, q_i(\mathbf{p}_t, 1)$,and optimally demands $z\, x_{ji}(\mathbf{p}_t, 1)$ units of input j. Thus input use per unit of output is $\Omega_{ji}(\mathbf{p}_t) = \frac{x_{ji}(\mathbf{p}_t,1)}{q_i(\mathbf{p}_t,1)}$.

Manipulating the expression further, the market-clearing condition can be conveniently written as

$$\mathbf{q}_t = \mathbf{\Psi}(\mathbf{p}_t)\,\mathbf{d}(\mathbf{p}_t, E_t), \tag{10}$$

where $\mathbf{\Psi}(\mathbf{p}_t) = \left[I - \mathbf{\Omega}^\top(\mathbf{p}_t)\right]^{-1} = (\Psi_{ji}(\mathbf{p}_t))$ is the Leontief inverse, a matrix that captures the direct and indirect input requirements needed to fulfill final good demands.

Equilibrium conditions (5), (7) and (10) yield a system of $J + 1$ equations that

[3] It is straightforward to show that the second Blackwell sufficient condition for a contraction, monotonicity, holds trivially. That is, for any $\ln(\mathbf{p}_t) \leq \ln(\mathbf{p}_t')$, $T_i\left(\ln \mathbf{p}_t\right) \leq T_i\left(\ln \mathbf{p}_t'\right)$.

uniquely determine $J+1$ unknowns, the entry mass vector $(m_{1t}, \ldots, m_{Jt})$ and the aggregate expenditure E_t, given equilibrium prices $\mathbf{p}_t^*$ determined by (9). In the remaining expressions we suppress the dependence on prices for ease of notation.

To solve for entry, write the output and labor market clearing conditions in terms of entrants and incumbents. Entrants are summarized by their average employment and output, n_{jt}^e and q_{jt}^e, while incumbents contribute total employment N_{jt}^{inc} and total output Q_{jt}^{inc}. Combining (10) and the J goods market clearing conditions (5), yields the expression

$$m_{jt} q_{jt}^e = \left(\sum_{i=1}^{I} \Psi_{ji,t} d_{it}(E_t) \right) - Q_{jt}^{\text{inc}}, \qquad \forall j \in J. \tag{11}$$

which states simply that output by entrants in each sector is equal to total sector output (the term in parentheses) minus the output already produced by incumbents in that sector. This expression pins down entry required by each sector $(m_{1t}, \ldots, m_{Jt})$ up to aggregate expenditure E_t, as all the other terms in this expression are known given prices. The Leontief inverse $(\Psi_{ji,t})$ captures the *static interdependence of entry* across sectors due to input-output linkages, as emphasized by paper such as Baqaee (2018) and Cavalcanti, Mendes and Pannella (2023). The incumbent term Q_{jt}^{inc} captures *own-sector history dependence of entry*, as emphasized by Hopenhayn et al. (2022): past entry and survival determine the stock of incumbent firms, which pins down the shortfall in output that must be filled by entrants.

The labor market clearing expressed in terms of entrants and incumbents is given by,

$$L_t = \sum_{j=1}^{J} n_{jt}^e m_{jt} + \sum_{j=1}^{J} N_{jt}^{\text{inc}}. \tag{12}$$

Substituting m_{jt} from (11) into (12) we obtain an expression that pins down expenditure E_t:

$$\sum_{j=1}^{J} \left(\frac{n_{jt}^e}{q_{jt}^e} \sum_{i=1}^{I} \Psi_{ji,t} d_{it}(E_t) \right) = L_t - \left(\sum_{j=1}^{J} N_{jt}^{\text{inc}} - \sum_{j=1}^{J} \frac{n_{jt}^e}{q_{jt}^e} Q_{jt}^{\text{inc}} \right). \tag{13}$$

This expression has an intuitive interpretation in terms of resources needed to sustain a given level of expenditure E_t. The expression inside the summation on the left-hand side computes, for each sector, the gross output requirement implied by final demand, $\sum_i \Psi_{ji,t} d_{it}(E_t)$, divided by average entrant output q^e_{jt}. This gives the number of entrants that would be needed to produce sectoral output if all production were carried out by entering firms. Multiplying by average entrant employment n^e_{jt} then yields the labor required in each sector. Summed across all sectors, the left-hand side pins down total resources needed, in units of labor, for a given level of E_t.

The right-hand side shows the amount of resources available in the economy. In addition to the labor not employed by incumbents, $L_t - \sum_{j=1}^{J} N^{inc}_{jt}$, these resources include incumbent firms. The adjustment term $\sum_{j=1}^{J} n^e_{jt}/q^e_{jt} \times Q^{\text{inc}}_{jt}$ converts incumbent output into units of labor. For sector j, Q^{inc}_{jt}/q^e_{jt} shows the number of entrants that incumbents embody, which multiplied by n^e_{jt} converts it to units of labor. When summed across sectors, this adjustment term shows the amount of resources represented by incumbent firms. Together, incumbent labor and the adjustment term, the term between parentheses on the right-hand-side, captures *cross-sector history dependence of entry*, a novel type of entry dependence across sectors: the history of entry in any given sector will affect current entry in all sectors through the mass of incumbents firms and their corresponding output and employment.

The system of $J+1$ equations and unknowns is solved sequentially: E_t is determined using (13), and then the entry vector $(m_{1t}, \ldots, m_{Jt})$ is solved using (11) as a linear system of J equations and unknowns.[4] These allocations and prices are, therefore, the unique equilibrium of the economy.

Having concluded the construction of the equilibrium, it is useful to compare our characterization of entry to that in Hopenhayn et al. (2022).

Proposition 2 (One-Sector Dynamic Entry Condition)**.** *When the economy consists of a single sector ($J = 1$), the system of entry equations collapses to the dynamic entry equation in Hopenhayn et al. (2022),*

$$m_t = \frac{L_t - N^{inc}_t}{n^e_t}.$$

[4]In the special case of homothetic utility, $d_{it}(E_t) = d_{it} \times E_t$, which makes (13) linear in E_t. As a consequence, the $J+1$ equations form a linear system.

Proof. Evaluating (11) and (13) at $J = 1$ yields

$$m_t q_t^e = \Psi_t d_t(E_t) - Q_t^{\text{inc}}, \tag{14}$$

$$\frac{n_t^e}{q_t^e}\Psi_t d_t(E_t) = L_t - N_t^{\text{inc}} + \frac{n_t^e}{q_t^e} Q_t^{\text{inc}}. \tag{15}$$

Solving (15) for $\Psi_t d_t(E_t)$ and substituting this expression into (14) yields

$$m_t q_t^e = \left(\frac{L_t - N_t^{\text{inc}} + \frac{n_t^e}{q_t^e} Q_t^{\text{inc}}}{n_t^e / q_t^e} \right) - Q_t^{\text{inc}}.$$

Dividing both sides by q_t^e and simplifying, the terms involving Q_t^{inc} cancel, and we obtain the stated expression: Current entry is determined as the gap between aggregate labor supply and the employment by incumbents, normalized by average size of entrants. ■

2.5 Population Dynamics and Reallocation

This section examines the implications of changes in population growth in the absence of any other shocks. Assume the representative consumer has homothetic demand, and let the economy be in a steady state with constant population growth g, so that $L_{t+1} = (1+g)L_t$. Begin by establishing characteristics of the lifecycle profile of firms.

Assumption 1. *Exit rate is strictly decreasing in firm age.*

Assumption 2. *Output-per-firm is strictly increasing in firm age.*

These assumptions arise naturally in the model as an outcome of i) selection on the fixed cost, and ii) if the distribution of entrant idiosyncratic productivity $G_i(z)$ has a lower mean than the stationary distribution of incumbent idiosyncratic productivity $F_i(z'|z)$. Assumption 1 is observable in the data. Assumption 2 is not directly observable but can be inferred by observed age profiles of employees-per-firm.

Proposition 3 (Population growth and firm share.)**.** *Suppose Assumptions* (1) *and* (2) *hold. Furthermore, suppose that age profiles of output-per-firm are identical across sectors. An increase in population growth raises the relative number of firms in sectors with steeper age-profiles of exit. If, in addition, sectors with steeper age-profiles of exit* *also have steeper age-profiles of output-per-firm, then the reallocation of firms toward these sectors is amplified.*

Proof. In steady state, total output and employment in each sector grow at the common rate g. Because prices are invariant to population growth, firm demographics within each sector are unaffected by changes in g.

In each sector, the steady-state law of motion implies that the entry rate λ_i must satisfy

$$\lambda_i = g + \xi_i(g).$$

where $\xi_i(g)$ is the sector's implied exit rate.[5] Thus, cross-sector differences arise solely from variation in exit-by-age profiles, which determine the multiplier $\xi_i(g)$.

An increase in population growth raises the required entry rate λ_i in every sector, thereby shifting each sector's age distribution toward younger firms. Sectors with steeper exit-by-age profiles exhibit larger multipliers $\xi_i(g)$, and therefore experience larger increases in entry and larger shifts towards young firms.

Consider the relative number of firms in sectors i and j:

$$\frac{M_i}{M_j} = \frac{Q_j/M_j}{Q_i/M_i} \times \frac{Q_i}{Q_j}.$$

Homothetic demand implies that relative outputs Q_i/Q_j do not change with changes in population growth. Changes in relative firm numbers therefore operate entirely through output per firm.

Output per firm reflects the firm-age distribution, since older cohorts produce more per firm than younger cohorts. When g rises, sectors with steeper exit profiles experience a larger reweighting toward young, low-output-per-firm cohorts. For a given level of sectoral output, these sectors require more firms; their mass of firms M_i therefore rises relative to sectors with flatter exit profiles.

If, in addition, these sectors have steeper output-per-firm age profiles, then the decline in output per firm induced by the shift toward younger cohorts is larger, further amplifying their increase in firm share. ■

[5] To derive the entry rate expression, note that the rate of growth in the number of firms in sector i is $\frac{M_{it}}{M_{i,t-1}} = \frac{N_{it}}{N_{i,t-1}} \frac{N_{i,t-1}/M_{i,t-1}}{N_{it}/M_{it}}$ and that the number of firms in a sector evolves according to $M_{it} = (1-\xi_{it})M_{i,t-1} + m_{it} \int_{z_i^*} dG_i(z)dz$. Combining these two equations yields the entry rate expression with entry rate λ_{it} defined as $m_{it} \int_{z_i^*} dG_i(z)dz / M_{i,t-1}$. See Hopenhayn et al. (2022) for more details. In steady state, employment and the number of firms in each sector grow at the rate of population growth.

We now turn to the effect of population growth on relative employment across sectors.

Assumption 3. *Output-per-worker is strictly increasing in firm age.*

Output per productive worker in the model is equalized across firms within a sector, so the age profile in output-per-worker is due exclusively to the fixed overhead cost, as smaller firms employ a larger share of their workforce as overhead and younger firms tend to be smaller than older firms, as explained earlier.

Proposition 4 (Population growth and employment share.)**.** *Suppose Assumptions* (1) *and* (3) *hold. Furthermore, suppose that age profiles of output-per-worker are identical across sectors. An increase in population growth raises the relative employment of sectors with steeper age-profiles of exit. If, in addition, sectors with steeper age-profiles of exit also have steeper age-profiles of output-per-worker, then the reallocation of firms toward these sectors is amplified.*

Proof. Using the decomposition

$$\frac{N_i}{N_j} = \frac{Q_j/N_j}{Q_i/N_i} \times \frac{Q_i}{Q_j},$$

homothetic demand again implies that relative outputs Q_i/Q_j are invariant to changes in g. Thus, changes in relative employment therefore operate entirely through differences in output per worker.

Output per worker in sector k can be written as

$$\frac{Q_k}{N_k} = \frac{Q_k}{N_k^{\text{prod}}} \times \frac{N_k^{\text{prod}}}{N_k},$$

where the first term is output per productive worker (identical across firms within a sector and independent of firm age), and the second term is the sector's productive-employment share. Because firms employ a fixed amount of non-productive (overhead) labor, the productive-employment share is increasing with firm age: younger firms allocate a larger fraction of their workforce to overhead.

Earlier results established that an increase in g shifts each sector's firm-age distribution toward younger firms, with larger shifts in sectors with steeper exit-by-age profiles. These sectors therefore experience larger declines in their productive-employment share when g rises. Since output per productive worker is age-invariant, output per worker Q_k/N_k falls more sharply in these sectors.

Sectors experiencing larger declines in output per worker require more workers to produce a given amount of sectoral output. Consequently, sectors with steeper exit profiles and steeper output-per-worker age profiles experience larger increases in relative employment when population growth rises. ■

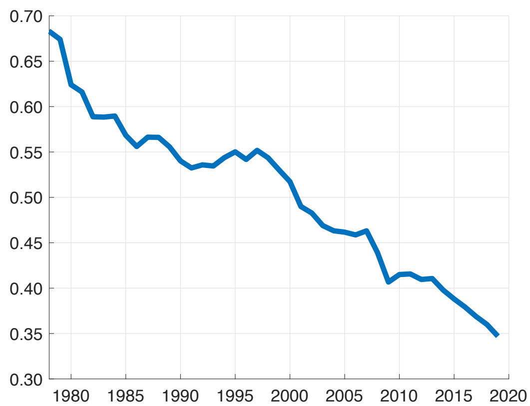
3 The Decline of US Manufacturing

In this section we use the theoretical framework to study the decline in US manufacturing. We divide the US economy into two sectors: Manufacturing and the rest of the economy, which we label Nonmanufacturing. In what follows, we first present facts about the evolution of US manufacturing since the 1970s. We then organize the facts using a simple accounting identity. Finally, we use the model to quantitatively investigate the sources of manufacturing decline.

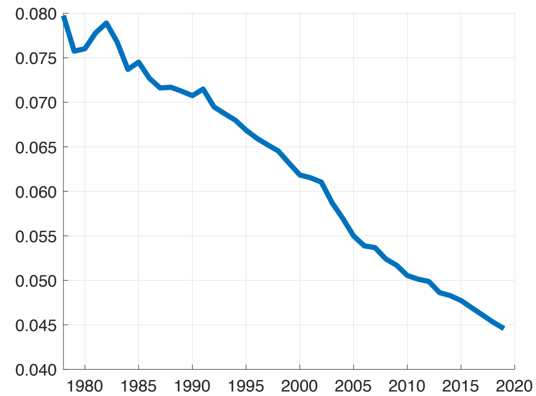
3.1 Motivating Facts

Figures 1a and 1b shows the evolution of the value of manufacturing gross output, relative to the rest of the economy, along with the relative number of firms. Since the 1970s, both series have exhibited a consistent decline. Relative output of manufacturing has declined from 0.68 to 0.35, while the relative number of firms has declined from 0.08 to 0.045. Figures 1c and 1d show the evolution of relative average size and output per worker in manufacturing over the same time period. Average size in manufacturing has declined steadily to about half its initial value. In contrast, relative output per worker has almost doubled. The facts about the evolution of US manufacturing presented in Figure 1 can be organized using a simple accounting identity.

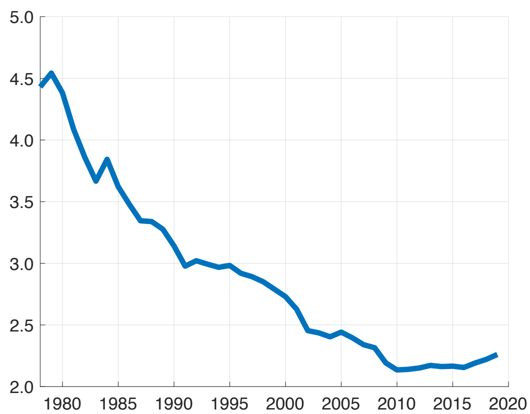
Consider the definition of output per worker, $OW \equiv Q/N$, where Q is the value of output and N is employment. Growth in output-per-worker can be decomposed into output growth minus employment growth, $\widehat{OW} = \widehat{Q} - \widehat{N}$. The definition of average size relates employment growth to firm growth, $\hat{e} = \widehat{N} - \widehat{M}$, where e is average size and M is the number of firms. Combining the two growth equations shows the relationship between output growth and firm growth, $\widehat{Q} = \widehat{M} + \hat{e} + \widehat{OW}$. In order to express it in relative terms, take the difference of this



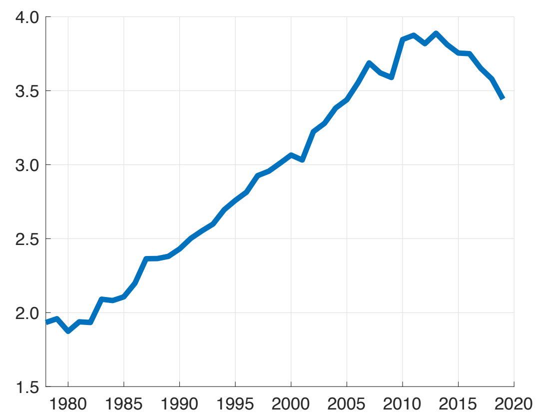
(a) Relative output



(b) Relative number of firms



(c) Relative average firm size



(d) Relative output per worker

Figure 1: Decline of Manufacturing

growth identity across sectors

$$\widehat{Q}_1 - \widehat{Q}_2 = \widehat{M}_1 - \widehat{M}_2 + \hat{e}_1 - \hat{e}_2 + \widehat{OW}_1 - \widehat{OW}_2 \tag{16}$$

The growth in relative output of a sector is equal to the growth in the relative number of firms, adjusted for the growth in relative output-per-worker at the firm level, which is equal to the growth in relative workers-per-firm plus growth in relative output-per-worker of the sector. Intuitively, the relative decline in firms in a sector is greater than the relative decline in output if average size or output-per-worker grow in relative terms, because fewer firms are needed to produce sectoral output.

Figure 2 shows how each component of the accounting identity matters for the decline in relative manufacturing output. By itself the observed change in relative output per worker would have nearly doubled relative manufacturing output. The fall in average size of manufacturing firms over the same time period, however, counteracts this effect. These two effects taken together generate only a slight decline, implying that relative manufacturing output would have declined little without the change in the relative number of firms. Therefore, the reallocation of firms away from manufacturing is crucial to understanding its relative decline.

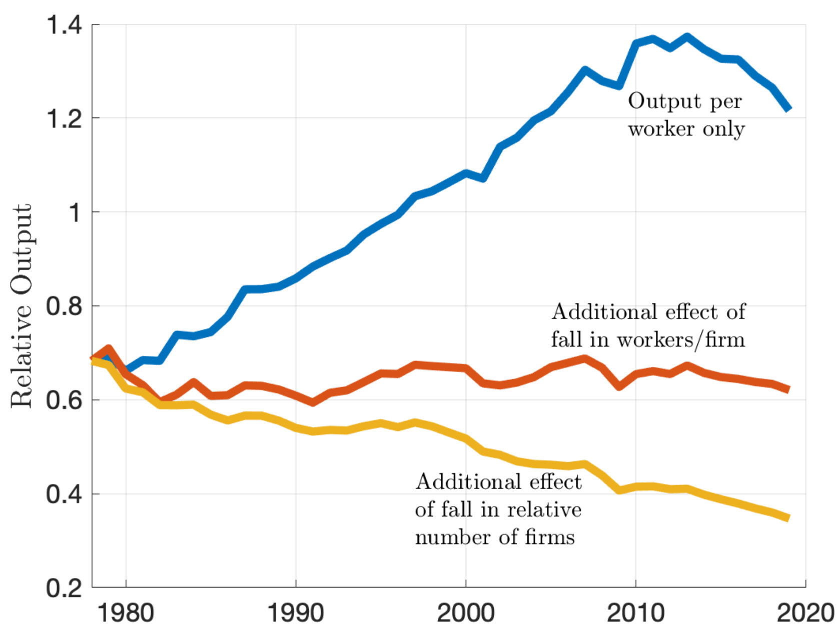


Figure 2: Accounting for Decline of Manufacturing

We evaluate the role of population growth, sectoral technology and international trade in driving the manufacturing decline. Any explanation of the entire decline in relative manufacturing output must satisfy the individual terms in the accounting decomposition above. Otherwise, that explanation will overload the other terms and be inconsistent with the facts. This observation guides our choice of driving forces. A large literature on structural transformation has documented that changes in output-per-worker in manufacturing reflect improvements in sectoral technology; Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). Any explanation of the evolution of manufacturing output-per-worker would be incomplete without allowing for changes in technology. We include population growth because it has been documented as a key driver of the trend in average firm size; Karahan et al. (2024) and Hopenhayn et al. (2022) and Peters and Walsh (2021). Manufacturing relative output includes domestic and imported goods. In recent decades, the net import share of manufacturing has steadily increased from 5% to 25%. Given that the rest of the economy has not experienced a similar increase in net imports, international trade is an essential driving force to understand the decline in relative manufacturing output; Pierce and Schott (2016) and Fort, Pierce and Schott (2018).

We quantitatively evaluate the ability of the decline in population growth, changes in sectoral technology and increase in net imports to generate the observed decline in manufacturing output. We can then ask a number of quantitative questions. What is the role of steady state vs transitional dynamics? What is the role of firm demographics and sectoral spillovers? How important is each driving force by itself? What is the new steady state? Given evolution of relative sectoral output, the quantitative success of the model is determined by how well the model matches the evolution of sectoral firm dynamics, including reallocation of firms. The entry equation (11) relates changes in relative sectoral output to sectoral entry, and therefore to the number of firms in each sector. While we calibrate the model to target an initial year, there is no apriori reason for the model to match subsequent changes in firm dynamics. After the initial year, the evolution of firm dynamics is determined by what the model implies for how the driving forces affect firm demographics along with the firm-age distribution, which is determined by entry and exit.

Methodology. The model time-period is set to one year. The input-output structure and sectoral firm dynamics in the model economy are calibrated to US manufacturing and nonmanufacturing sectors in 1978, the first year for which we have firm data. Given the quantitative importance of transitional dynamics, we assume that the economy was at a steady state in 1947. We then simulate the economy forward by feeding the post-1947 evolution of population growth, sectoral technology and net import shares from US data.

Given that population growth in the model refers to working-age population, we map it to data on US civilian labor force growth. We assume the functional forms of utility and sectoral production functions are Cobb-Douglas. This has the advantage that the corresponding parameters can be interpreted as shares that are observable in the data. The evolution of sectoral technology is inferred from data on final expenditure shares, input factor shares, and sectoral prices.[6]

The final-good demand aggregator over sectoral output is,

$$C(D_M, D_{NM}) = \left(\frac{D_M(\mathbf{p})}{\gamma_t}\right)^{\gamma_t} \left(\frac{D_{NM}(\mathbf{p})}{1-\gamma_t}\right)^{1-\gamma_t},$$

where the subscript i indexes the sector, D_i indicates final demand for the output of sector i, inclusive of imports, and γ_t indicates the expenditure share of manufacturing. The expenditure shares for each sector are allowed to be time-varying, as indicated by the subscript t, and set to match the expenditure shares each period in US data. Feeding the expenditure shares over time allows us to capture changes in sectoral output demand coming from changes in final usage.

On the production side, intermediate goods used by sector i are given by the intermediate good aggregator $X_{it}(\mathbf{x})$,

$$X_{it}(\mathbf{x}) = \left(\frac{X_{M,i}}{\nu_{it}}\right)^{\nu_{it}} \left(\frac{X_{NM,i}}{1-\nu_{it}}\right)^{1-\nu_{it}},$$

where $X_{M,i}$ indicates demand for manufacturing commodity by sector i, inclusive of imports, and $X_{NM,i}$ does the same for the nonmanufacturing commodity. ν_{it} is the time-varying intermediate usage share of the manufacturing good in sector i. With two sectors, the intermediate usage share of the manufacturing good by manufacturing is $\nu_{M,t}$ and by nonmanufacturing is $\nu_{NM,t}$. The intermediate good shares are observable in the data, and allow us to capture changes in sectoral output demand due to changes in the input-output structure.

The sectoral input aggregator is Cobb-Douglas in labor and the composite intermediate good

$$g_{it}(n, \mathbf{x}) = A_{it}^{\frac{1}{\eta_{it}}} \left(\frac{n_{it}}{\alpha_{it}}\right)^{\alpha_{it}} \left(\frac{X_{it}(\mathbf{x})}{1-\alpha_{it}}\right)^{1-\alpha_{it}}$$

where η_{it} and α_{it} map to the profit share and the labor share of sector i, which we feed from the data. In order to pin-down sectoral productivity A_{it}, we feed

[6] We include changes in final expenditure shares as part of sectoral technology because they affect the price of sectoral output.

sectoral prices from the data and find the value of A_{it} each period such that the profit of the unit-productivity firm is time-invariant.

International trade affects sectors in our framework because changes in imports and exports act as sectoral demand shocks. Imports of the manufactured good, either for final or intermediate usage, reduce demand for domestically produced manufactured goods and therefore represent a negative shock to demand. The goods market clearing condition for each sector needs to be augmented to include imports and exports,

$$(1-\chi_{jt})Q_{jt} = (1-\phi_{jt})\left(D_{jt} + \sum_i \int X_{ji,t}(\mathbf{p}_t, z)d\mu_i(z)\right)$$

where χ_j is the export share of gross output of commodity j, and ϕ_j is the import share of total usage. We feed import and export shares from the data.[7] Qualitatively, the effect of an increase in imports of a commodity are similar to the effects of a sector-specific decline in labor force growth. The age-profile of average size and exit rates are invariant to the increase in imports, but firm entry rates decline leading to firm aging in that sector.

Figures A-1, A-2, and A-3 summarize the evolution of all the driving forces that we feed through the model.

Firm dynamics. In order to be consistent with differences in sectoral firm dynamics, we allow the Markov process for the idiosyncratic state z to differ by

[7]The national income accounts do not split imports based on final consumption or intermediate usage, so we follow NIPA and assign the import share ϕ_j identically to final consumption and intermediate usage. This assignment rule implies domestic and imported goods in final consumption and intermediate usage satisfy:

$$d_j(\mathbf{p}) = (1-\phi_i)D_j(\mathbf{p}); \quad d_j^*(\mathbf{p}) = \phi_i D_j(\mathbf{p})$$
$$x_{ji}(\mathbf{p},1) = (1-\phi_i)X_{ji}; \quad x_{ji}^*(\mathbf{p},1) = \phi_i X_{ji}$$

The model equilibrium is consistent with any other split of imports into final and intermediate usage, provided markets clear. Note that in an open economy, the correct measure of income of the representative agent is Gross Domestic Absorption (GDA). GDA measures the total final demand for goods and services within a country's economy regardless of whether those goods and services are produced domestically or imported.

sector. The Markov process followed by the idiosyncratic state z is AR(1),

$$\log(z_{i,t+1}) = (1-\rho_i)\psi_{ij} + \rho_i \log(z_{it}) + \varepsilon_{i,t+1}; \qquad \varepsilon_{i,t+1} \sim \mathcal{N}(0, \sigma_i^{\varepsilon}), \tag{17}$$

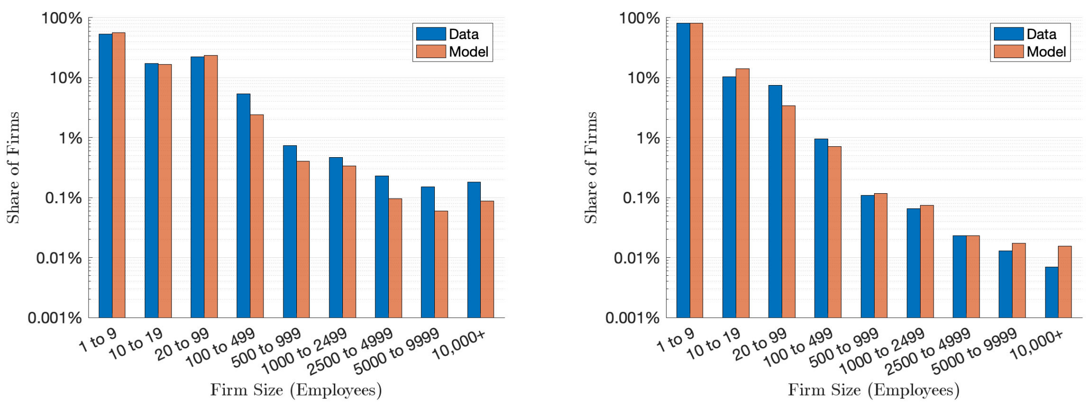
where i indexes the sector and j indexes whether the firm is a high-type or low-type.[8] The parameter ψ_{ij} is the sector-specific long-run mean that varies with type $j \in \{\ell, h\}$, ρ_i is the persistence, and σ_i^{ε} is the standard deviation of shocks. Firms draw their type after paying the sector-specific entry cost c_i^e. Once drawn, the type is fixed throughout the lifetime of the firm. The probability of drawing the high type is parameterized by ω_{ih}. In addition to drawing their type, entrants draw their idiosyncratic state from a lognormal distribution G_i with mean ψ_i^G and standard deviation σ_i^G. The state z then evolves according to the corresponding AR(1) process.

Together with the overhead parameters for high- and low-type firms in each sector, each sector has 11 parameters that need to be calibrated. The sectoral parameters are calibrated to match firm dynamics in each sector in 1978. The targeted variables for each sector include 1978-83 averages of average firm size, entry rate, average entrant size, 5-year growth and exit rates, share of firms with 1-9 employees, share of entrants with more than 250 employees, and the employment shares of firms with 100+, 1000+ and 10000+ employees.

We jointly calibrate 11 parameters for each sector, but each parameter exerts its primary influence on particular moments. The entry cost predominantly affects the entry rate. Because the entry equation requires matching average entrant size to identify the entry rate, we target both moments. Average entrant size responds mainly to ψ_i^G, the mean of the entrant productivity distribution. The dispersion parameter σ_i^G controls the right tail of this distribution, thereby targeting the

[8] We incorporate permanent differences across firms within each sector by introducing high and low types. This modeling choice addresses three empirical regularities. First, it can help generate the frequency with which young firms achieve large scale, as emphasized by (Luttmer, 2011). Second, it makes the model consistent with the finding in Haltiwanger, Jarmin and Miranda (2013) that firm-age retains explanatory power after controlling for firm size. Third, as documented by Holmes and Stevens (2014); Sterk ⓡ, Sedláček ⓡ and Pugsley (2021), it implies that firms of the same size may respond differently to changes in the economic environment. High-type firms are characterized by both a higher long-run mean in their productivity process and greater labor overhead, corresponding to "superstar" firms in the data. In the estimated model, these high-type firms represent approximately 5 percent of all firms but account for 30 percent of employment and are on average 7 times larger than their low-type counterparts.

concentration of entrants. Turning to the productivity process, the shock variance σ_i^{ε} determines the probability mass at productivity levels where firms exit, thus targeting the 5-year exit rate. The persistence parameter ρ_i governs convergence rates to long-run means and primarily influences the 5-year growth rate. High-type firms, which are substantially larger than low-type firms, contribute disproportionately to aggregate employment. We therefore calibrate three parameters characterizing these firms—the long-run mean ψ_{ih}, labor overhead c_{fh}, and share at birth ω_h—to match employment-weighted right-tail statistics: the shares employed at firms with 100+, 1000+, and 10,000+ workers. The low-type labor overhead $c_{f\ell}$ primarily pins down the share of firms in the smallest size bin (1 to 4 employees). Table 1 reports the calibrated parameters, targets, and model fit by sector.



(a) Manufacturing

(b) Nonmanufacturing

Figure 3: 1978 Firm Size Distribution

Figure 3 presents the firm size distribution for both sectors in the model and the data. Firm shares for firms with 10+ employees are nontargeted moments. Table A-1 in the Appendix shows firm demographics by sector in both the data and the calibrated model. In both sectors, the age profiles in model and data follow the expected pattern: average size and concentration are increasing in age, and exit rates are decreasing. Table A-2 presents the change in average size by age. In the data, average size in manufacturing has declined for all age groups, with older firms experiencing the strongest rate of decline. The model replicates the within-age group decline in manufacturing, because changes in manufacturing factor shares fed from the data capture the shift away from labor into intermedi-

Table 1: Calibration

Panel A: Parameter Values

	Manufacturing	Non-Manufacturing
Entry Cost, c_i^e	58.73	9.27
Mean of entrant dist, ψ_i^G	-2.14	-3.71
Std. dev. of entrant dist, σ_i^G	3.50	3.01
AR(1) Persistence, ρ_i	0.96	0.96
AR(1) std. dev., σ_i^ε	1.21	1.04
Long-run mean low-type, $\psi_{i\ell}$	-4.23	-10.64
Long-run mean high-type, ψ_{ih}	-4.23	-1.85
Operating cost low-type, $c_{i\ell}^f$	4.90	0.93
Operating cost high-type, c_{ih}^f	18.12	8.62
Share of high mean startups, ω_{ih}	0.75	0.09

Panel B: Model Fit

	Manufacturing	
	Data	Model
Average Firm Size	68.96	68.18
Entry Rate	9.29	10.97
Average Entrant Size	11.24	18.57
Concentration of entrants	50.08	49.07
5-year growth rate	84.93	108.25
5-year exit rate	51.83	46.21
Firm Share 1 to 9	52.64	56.60
Employment Share 100+	78.56	80.51
Employment Share 1000+	58.38	68.46
Employment Share 10,000+	38.98	48.44
	Non-Manufacturing	
	Data	Model
Average Firm Size	15.57	17.64
Entry Rate	12.14	14.44
Average Entrant Size	5.28	4.90
Concentration of entrants	25.57	14.17
5-year growth rate	77.55	67.58
5-year exit rate	56.39	55.87
Firm Share 1 to 9	80.12	81.60
Employment Share 100+	53.46	56.08
Employment Share 1000+	36.08	42.94
Employment Share 10,000+	20.62	25.34

Notes: Panel A reports calibrated parameter values. The remaining model parameters are the discount factor $\beta = 0.96$ and labor force growth rate in 1947 $g = 0.01$. Panel B reports data and model moments as time-averages from 1978 to 1983. Concentration of entrants is the share of employment in firms with more than 20 employees within the 0-year old category divided by total employment by 0-year old firms.

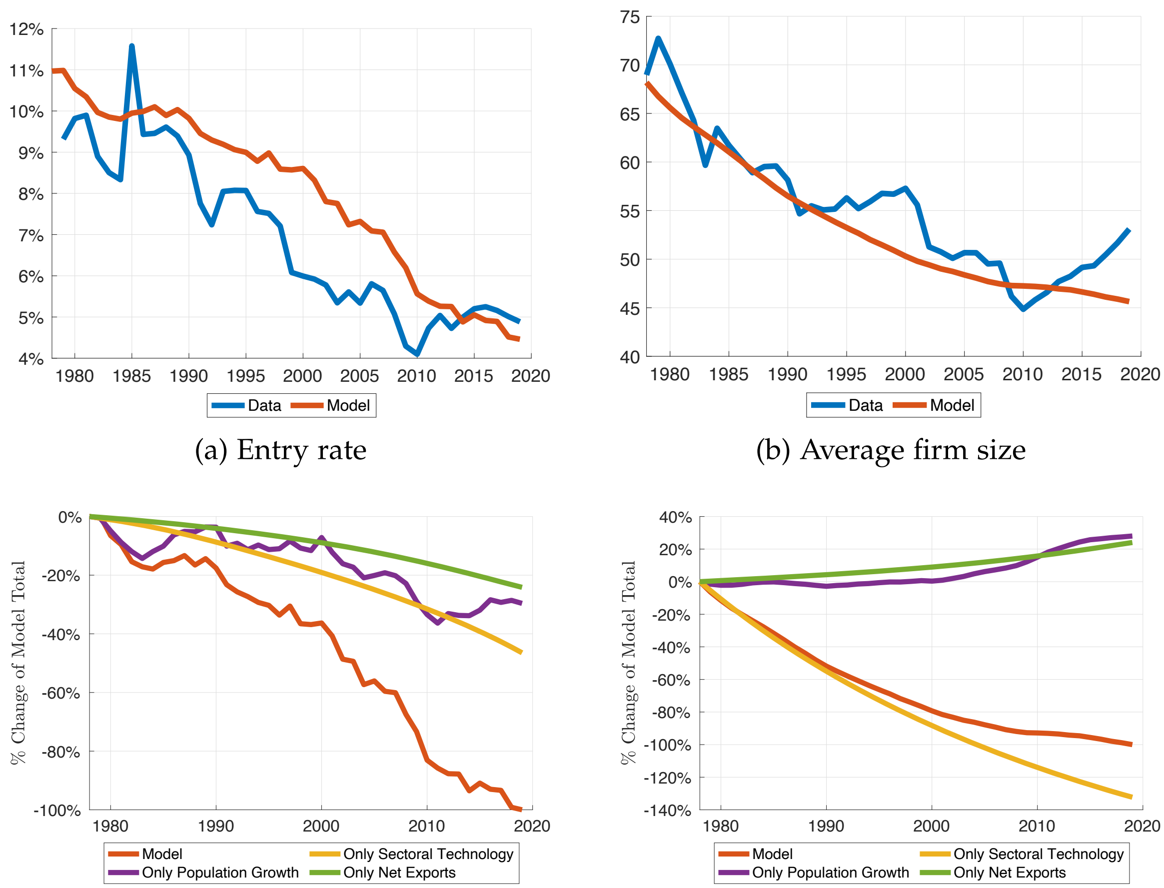
ates and profits. In nonmanufacturing, the changes in average size within each age group are not statistically different from zero.

3.2 Sectoral firm dynamics

Figure 4a shows the evolution of firm entry rates in manufacturing. Firm entry rates in manufacturing decline steadily in both model and data. In order to understand the role of the driving forces, Figure 4c presents the entry rate in counterfactual exercises in which only one of the three forces is active. All three forces generate a decline in the manufacturing entry rate. The largest decline in the manufacturing entry rate is generated by changes in sectoral technology alone, about half of the decline in the calibrated model. The decline in entry generated by population growth and net imports each is comparable in magnitude, about one-third of the total decline. Figure A-4 and A-5b in the Appendix show that a similar pattern holds for manufacturing firm exit.

The evolution of entry and exit rates determines the firm-age distribution, which then combined with the firm-age profiles implies the evolution of average size. The decline in entry implies that manufacturing experiences firm aging, which acts as a force to increase average firm size. Nevertheless, Figure 4b shows that average firm size in manufacturing has declined. Figure 4d shows why that is the case. The change in population growth and net imports, both forces that only affect firm aging, imply an increase in average size. However, changes in sectoral technology lowers average size within firm-age bins which more than counteracts these forces driving the overall decrease in manufacturing average size. In the absence of firm aging due to changes in population growth and net imports, manufacturing would have experienced a larger decline in average firm size.

Figure A-5 in the Appendix presents entry, exit and average size for nonmanufacturing, along with the counterfactual exercises. Firm entry and exit rates decline, while average size increases. These patterns are driven entirely by firm aging induced by changes in population growth. Given that nonmanufacturing produces more than 90% of gross output, these findings reiterate the result that firm aging induced by the decline in population growth drives the aggregate decline in firm entry-exit rates.



(a) Entry rate

(b) Average firm size

(c) Entry rate - Decomposition

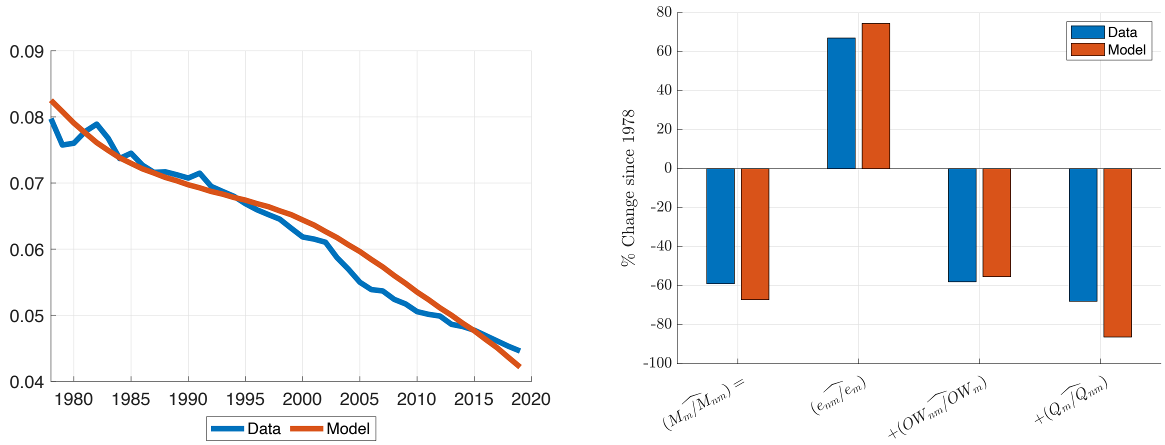
(d) Average firm size - Decomposition

Figure 4: Manufacturing firm dynamics

3.3 Sectoral Reallocation

Figure 5a shows the evolution of the relative number of firms in manufacturing in model and data, a statistic that is crucial to explain the observed decline in relative manufacturing output. In addition, Figure 5b decomposes the total change in the relative number of firms using the accounting identity (16) that organizes the manufacturing output decline. The figure highlights that the decline in the number of firms generated by the model is broadly consistent with the accounting identity (16). Alternative explanations that do not account for the decline in the relative number of firms will overload the sources of the output decline onto either output per worker or average firm size.

In order to better understand the decline in the relative number of manufacturing firms, we further decompose the sources of manufacturing decline into



(a) Evolution

(b) Decomposition

Figure 5: Relative Number of Manufacturing Firms

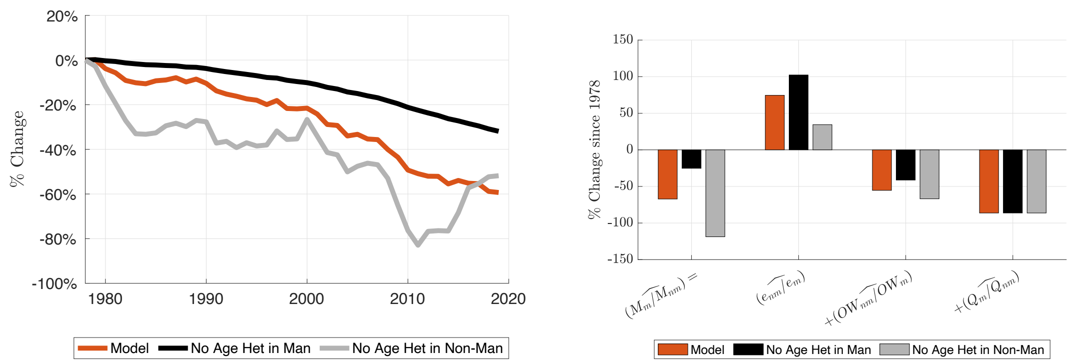
long-run and transitional effects. The *long-run effect* is the decline in manufacturing that can be attributed to the change between a 1978 steady state and a 2019 steady state. The steady states are computed by setting parameters related to population growth, sectoral technology and net exports equal to their 1978 or 2019 values, with all other parameters remaining unchanged. The *transitional effect* is the log-difference between the total effect and the long-run effect. Table 2 presents the results. The transitional effect is responsible for about 50% of the total decline in the relative number of manufacturing firms. The bulk of the transitional effect arises from the change in average size and output per worker implied by the 2019 steady state vs the total change. These effects arise mostly because the changes in manufacturing average size and output per worker along the transition are greater than the steady state, reflecting feedback effects due to changes in the firm-age distribution.

Firm demographics and sectoral spillovers. The multisector dynamic entry equation implies that entry rate in a sector is affected not only by its own firm demographics, but also by firm demographics in other sectors, via the resource constraint. Figure 6a shows how own-sector and cross-sector firm demographics affect the manufacturing entry rate. The entry-rate decline generated by the driving forces is insufficient when the feedback effects from own-sector firm demographics are missing. In contrast, the cross-sector effect generates a decline

Table 2: Decomposition: Steady State vs. Transitional Effects

		Total Change	=	Long-Run Effect	+	1978 Transition Effect	+	2019 Transition Effect
		$\log\left(\text{Tot}_{2019}/\text{Tot}_{1978}\right)$		$\log\left(\text{SS}_{2019}/\text{SS}_{1978}\right)$		$\log\left(\text{SS}_{1978}/\text{Tot}_{1978}\right)$		$\log\left(\text{Tot}_{2019}/\text{SS}_{2019}\right)$
+	Δ Qratio	-0.86		-0.86		0		0
$-$	Δ AFSratio	-0.74		-0.96		-0.05		0.27
$-$	Δ OWratio	0.55		0.45		0		0.11
=	Δ Mratio	-0.67		-0.34		0.05		-0.38

Notes.



(a) Manufacturing entry rate

(b) Decomposing Reallocation

Figure 6: Sectoral spillovers

in the entry rate that is too large compared to the baseline. The absence of age heterogeneity in nonmanufacturing implies that the fall in the entry rate in that sector is smaller. Consequently, the absorption of labor by nonmanufacturing is larger which leaves less labor available for entry in manufacturing.

Figure 6b shows the the change in the relative number of firms reflects the effect of firm demographics on the entry rate. The decomposition in (16) organizes the decline in the relative number of firms. Shutting down age heterogeneity does not change demand for sectoral output, so relative manufacturing output is the same in all cases. The change in relative output per worker is similar to the baseline, so the majority of the effect is due to changes in relative average size due to firm aging induced by the decline in entry. In the absence of age heterogeneity in manufacturing, average size growth due to firm aging occurs only in nonmanufacturing so the growth in relative average size, $\widehat{e_{nm}/e_m}$, is larger than the baseline. When there is no age heterogeneity in nonmanufacturing, the growth in average size in manufacturing due to firm aging is counteracted by the decline within firm-age bins. The net effect is such that relative average size in manufacturing declines, implying an increase in $\widehat{e_{nm}/e_m}$.

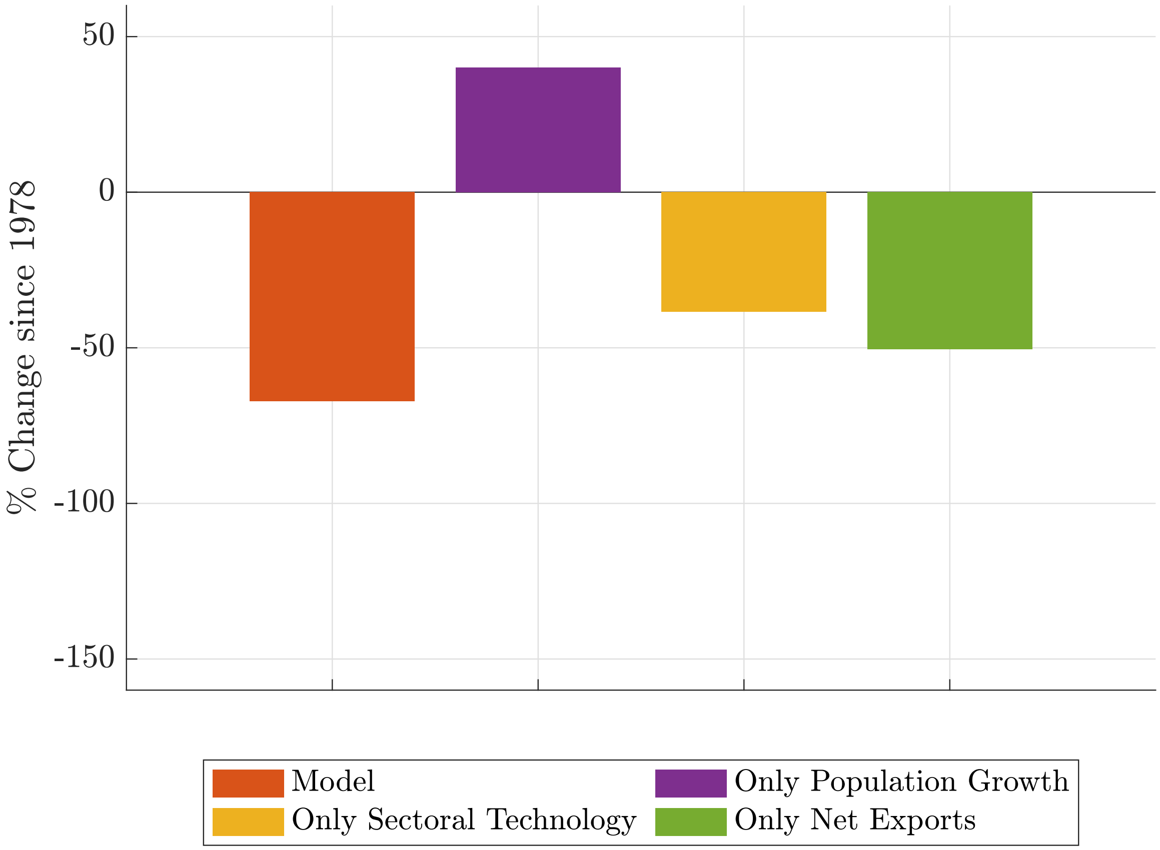


Figure 7: Driving forces and relative number of firms in manufacturing

The role of driving forces. Figure 7 isolates the role of the driving forces in explaining the decline in relative number of firms in manufacturing. Changes in international trade are the primary driver of the decline in the relative number of firms. Higher imports decrease relative demand for domestic manufacturing output and therefore reduces need for entry into manufacturing. Population growth by itself contributes to an increase in the relative number of firms in manufacturing because firm aging induced by population growth lowers relative average size in manufacturing. While firm aging increases average size in both the sectors increase, the relative increase in nonmanufacturing is greater because this sector has a steeper exit profile by firm-age, which implies that it experiences a steeper decline in entry and therefore more aging due to population growth. Changes in sectoral technology contribute to the decline in relative number of firms in manufacturing, but also have an additional affect because they capture the shift away from labor in manufacturing. These two effects together imply that changes in sectoral technology are particularly important for the decline in relative manufacturing employment.

4 Final Remarks

TBD

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Online appendices

Appendix A

A.1 Data Appendix

- Data Sources:

  1. Firm-level data is from the US Census Business Dynamics Statistics database
  2. All other data series are from the National Income and Product Accounts (NIPA-BEA) and Industry Economic Accounts (IEA-BEA).

     (a) Industry Economic Accounts: Make-Use Framework Tables, "The Use of Commodities by Industries, Before Redefinitions (Producers' Prices) - Sector", 1997-2021

     (b) Industry Economic Accounts: Input-Output Tables, Supplemental Estimate Tables, Historical Make-Use Tables, Use Tables/Before Redefinitions/Producer Value 1947-1996

     (c) GDP by Industry Accounts: Historical Industry Accounts Data, "GDP by Ind VA SIC": Value Added by Industry, Gross Output by Industry, Intermediate Inputs by Industry, the Components of Value Added by Industry, and Employment by Industry, 1947-1997

     (d) Sector Prices are retrieved from the BEA Chain-Type Price Indexes for Gross Output by 2-digit Industry.

       – Non-manufacturing prices are aggregated using Tornqvist index: the growth in the price index is a weighted average of price growth in each underlying sector with weights equal to expenditure shares.
       – Normalized by labor wage growth.
       – Smoothed by fitting an exponential curve with constant growth rate

- Notes: there are two publication categories that contain use tables by industry at the 2-digit level (referred to as "Sector" in the accounts). One is the *Input-Output* category (1 use table), and the other is the *Make-Use Framework* category (8 use tables). We use the table titled "The Use of Commodities by Industries, Before Redefinitions (Producers' Prices) - Sector" in the *Make-Use Framework* category for compatibility reasons with pre-1997 data. This table matches the values of the Use table in the *Input-Output* category with the exception that (i) the use of *Retail* and the *Transportation and Warehousing* output is *not* reassigned to other industries and (2) value added *is not* broken down into basic prices (before taxes on products and imports minus subsidies) and producer prices (after taxes). However, the

Value Added (producer prices) matches in both tables.[9] Reassigning Retail and Transport to other sectors yields much higher within-sector usage than historical estimates.

- We map commodities to industries, $\mathcal{I} = \mathcal{J}$.
- From the input-output tables, drop the following sectors: government, scrap, and other used incomparable imports
- Use the numbers for Manufacturing. Define Non-Manufacturing as Private Industries minus Manufacturing.
- Mapping to sectoral technology parameters:

$$\gamma_i = \frac{\text{Personal Consumption Expenditure in Sector } i \ (PCE_i)}{\sum_j PCE_j}$$

$$\nu_i = \frac{\text{Manufactured Usage in Sector } i}{\text{Total Intermediate Usage in Sector } i}$$

$$\alpha_i \approx \frac{\text{Compensation of Employees in Sector } i}{\text{Compensation of Employees in Sector } i + \text{Total Intermediate Usage in Sector } i}$$

$$1 - \eta_i \approx \frac{\text{Corporate Gross Operating Surplus in Sector } i}{\text{Comp of Emp in } i + \text{Corporate Gross Operating Surplus in } i \ + \text{Total Int Usage in Sector } i}$$

- Compensation of Employees is Wage and Salary accruals plus Supplements to wages and salaries
- Our measure of *Corporate Gross Operating Surplus* differs from the *Gross Operating Surplus* measured in NIPA in that it excludes the government sector, the noncorporate sector, and propietors' income. *Corporate Gross Operating Surplus* = Net Interest (corporate) + Business current transfer payments + Rental Income + Corporate profits before tax with IVA and CCadj + Capital consumption allowance (corporate) + Taxes on production and imports, less subsidies.
- Mapping to international trade parameters:

$$\chi_i = \frac{\text{Sector } i \text{ Exports}}{\text{Sector } i \text{ Gross Output}}$$

$$\phi_j = \frac{\text{Commodity } j \text{ Imports}}{\text{Commodity } j \text{ Total Intermediates} + \text{Commodity } j \text{ Consumption}}$$

A.2 Additional Figures and Tables

[9] V002 *Taxes on production and imports, less subsidies* on one table is equal to T00OTOP *Other taxes on production* minus T00OSUB *Less: Other subsidies on production* plus T00TOP *Taxes on products and imports* minus T00SUB *Less: Subsidies on products*

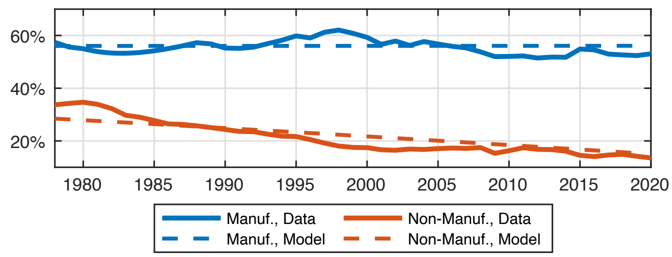
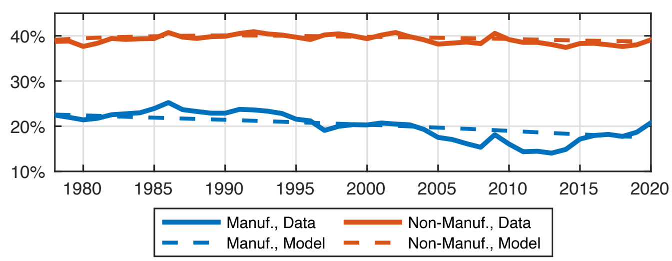
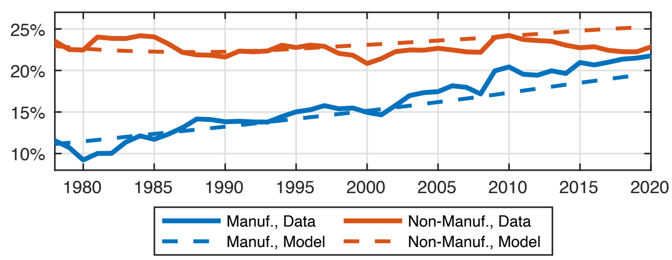
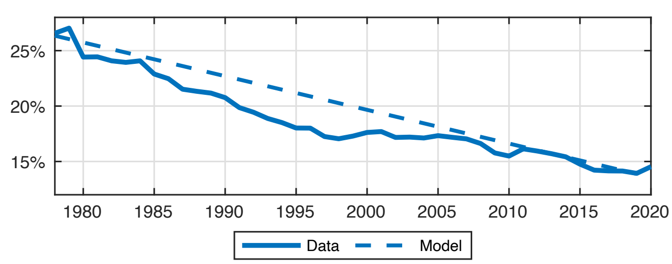
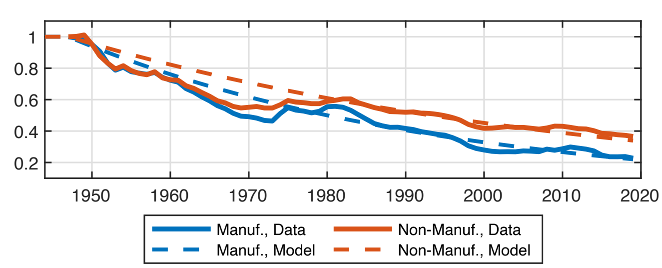
Variable	Data Mapping	Time Series
ν_{it}	Manufactured Usage Share	
α_{it}	Labor Share of Inputs	
$1 - \eta_{it}$	Profit Share of Gross Output	
γ_{mt}	Manufacturing Share of Final Good Expenditures	
$p_{it} \rightarrow A_{it}$	Sector Prices	

Figure A-1: Drivers: Sectoral Technology, $\mathcal{T}$

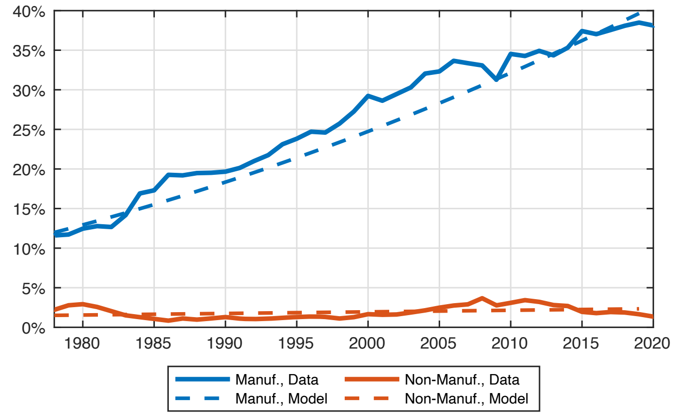
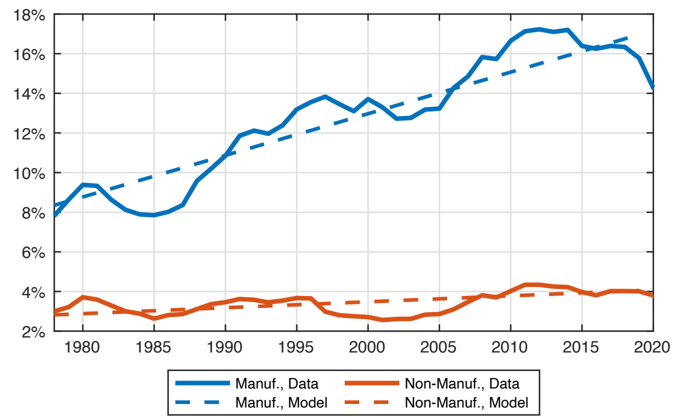
Variable	Data Mapping	Time Series
ϕ_{it}	Import Share of Total Usage	
χ_{it}	Export Share of Gross Output	

Figure A-2: Drivers: International Trade

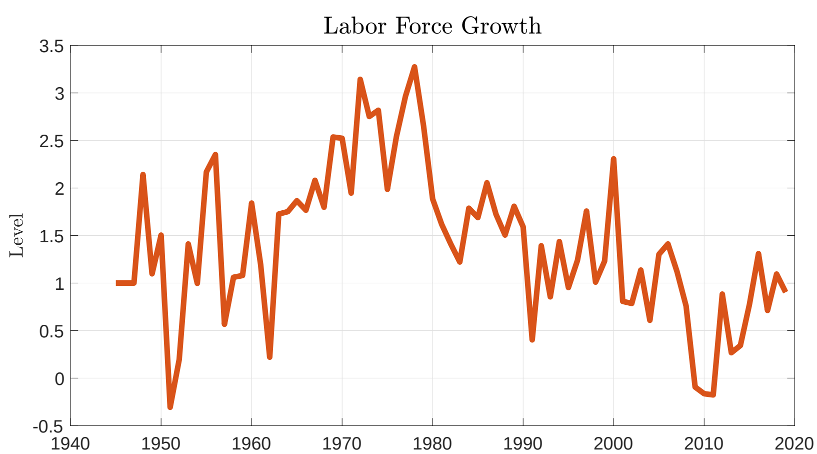
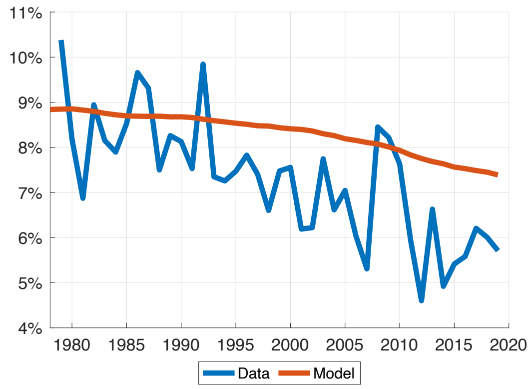
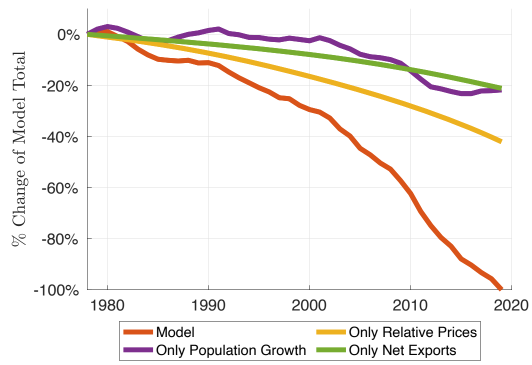


Figure A-3: Drivers: Population Growth

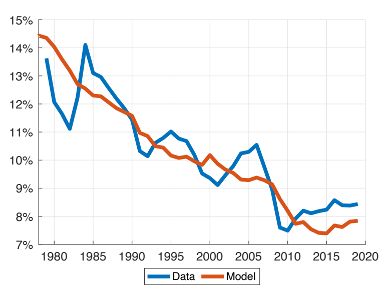


(a) Exit rate

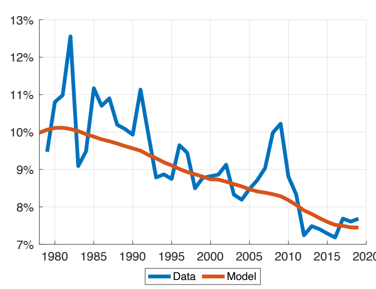


(b) Exit counterfactual

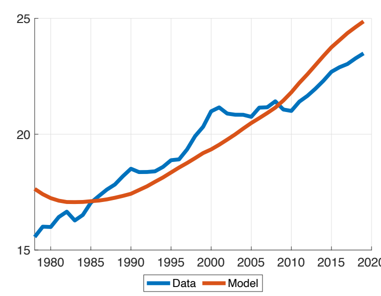
Figure A-4: Manufacturing exit rates



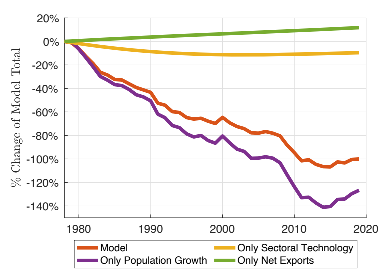
(a) Entry rate



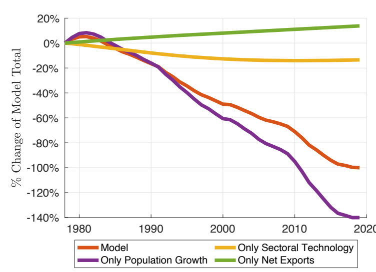
(b) Exit rate



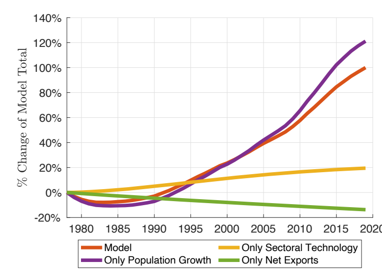
(c) Average firm size



(d) Entry counterfactual

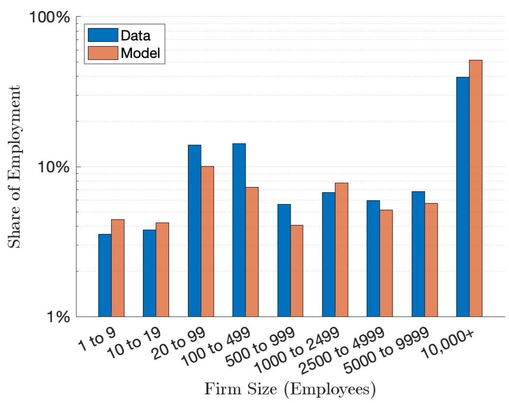


(e) Exit counterfactual

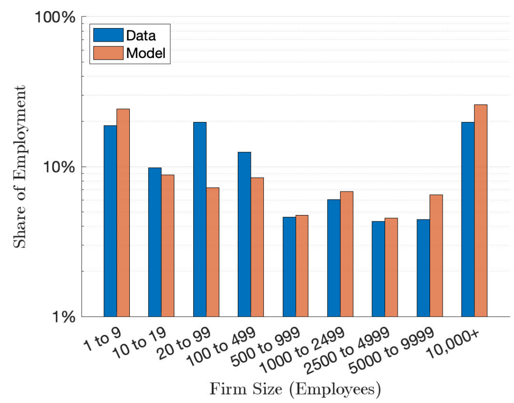


(f) Avg. size counterfactual

Figure A-5: Firm dynamics in Nonmanufacturing



(a) Manufacturing



(b) Nonmanufacturing

Figure A-6: 1978 Employment Size Distribution

Table A-1: Age Profiles by Sector

Panel A: Manufacturing						
	Exit Rate		Avg. Firm Size		Concentration	
Age	Data	Model	Data	Model	Data	Model
0	0.00	0.00	10.08	16.41	48.55	38.86
1	21.24	19.24	13.12	18.30	62.46	48.93
2	15.37	14.25	14.87	20.51	66.26	57.13
3	12.90	12.07	16.20	23.08	68.56	63.26
4	11.26	10.78	17.39	25.99	70.54	68.45
5	10.10	9.91	18.52	29.20	72.31	72.66
6 to 10	7.82	8.51	22.06	39.39	77.14	80.60
11 to 15	5.95	7.23	28.02	56.53	82.22	87.41
16 to 20	5.18	6.59	33.22	68.72	85.34	90.09
21 to 25	4.94	6.21	36.93	74.65	87.26	91.20
Above 25	4.18	5.66	111.99	78.72	96.14	92.15
Panel B: Non-Manufacturing						
	Exit Rate		Avg. Firm Size		Concentration	
Age	Data	Model	Data	Model	Data	Model
0	0.00	0.00	5.52	4.93	30.16	14.89
1	25.56	26.70	7.12	5.67	46.03	17.45
2	16.76	19.64	7.98	6.33	50.28	19.81
3	13.81	16.32	8.70	6.98	53.15	22.20
4	11.93	14.17	9.38	7.63	55.65	24.61
5	10.76	12.57	10.01	8.29	57.77	27.11
6 to 10	8.63	9.58	11.74	10.58	63.28	35.37
11 to 15	6.69	6.18	14.80	15.55	70.07	50.20
16 to 20	5.74	4.29	18.52	22.25	75.69	62.89
21 to 25	5.29	3.25	22.69	29.94	79.87	71.58
Above 25	4.58	2.22	67.22	51.02	93.00	83.09

Notes: Concentration is the share of employment in firms with more than 20 employees within the age category divided by total employment in the age category. Data moments are the average across all years in the sample. Model moments are the time average for the same years as the data moments.

Table A-2: Percentage Change in Average Firm Size by Age Group, 1978-2019

	Manufacturing		Non-Manufacturing	
Age	Data	Model	Data	Model
0	-1.07	-0.64	-0.12	0.04
1	-1.16	-0.83	-0.12	0.04
2	-1.24	-0.99	-0.09	0.04
3	-1.32	-1.13	-0.03	0.04
4	-1.30	-1.25	0.01	0.05
5	-1.34	-1.35	0.02	0.05
6 to 10	-1.82	-1.56	-0.16	0.05
11 to 15	-2.04	-1.73	-0.24	0.07
16 to 20	-2.05	-1.80	-0.09	0.08
21 to 25	-1.48	-1.82	0.19	0.09
Above 25	-1.74	-1.87	-0.60	0.10

Notes: Numbers are annual time trend of log-average firm size for comparable years.