Mind the gap when exiting low-for-long*

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Abstract

Keeping short-term interest rates low for long may induce agents to believe that interest rates will remain lower for longer than intended by central banks, widening a belief gap and stimulating the economy. When the interest rate is raised, however, agents perceive it as surprise monetary tightening, which causes economic contraction. This paper articulates this novel channel of low-for-long policy by building a New Keynesian model with learning and forward guidance. We find that a decrease in the nominal neutral rate in the perceived monetary policy rule drives the belief gap, and that the gap widens further if credibility about forward guidance is low. We estimate a perceived monetary policy rule using professional forecast data and provide empirical evidence that supports our model.

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1 Introduction

The conduct of monetary policy generates surprises when its action or the information released differs from that which agents had expected or believed. To promote the effectiveness of monetary policy and avoid unintended surprises, many central banks engage in communicating their views on the economies and how they lead to the adjustment of the stance of monetary policy, the latter being called a policy reaction function. Communicating a reaction function about short-term interest rates can be challenging, however. When interest rates stay constant at the effective lower bound (ELB) for a long time (low-for-long), agents would have difficulty in inferring the reaction function. A gap between the actual reaction function and what agents perceive can widen during low-for-long. And any correction of the gap can cause monetary policy surprises when the economy exits low-for-long.

After the global financial crisis of 2007–09 (GFC) many developed economies went through low-for-long, and most of them abruptly exited around 2022 in the face of a post-pandemic surge in inflation. During the rapid monetary tightening that ensued, there were some disruptions in the financial system including banking sector turmoil in March 2023. The taper tantrum of 2013 is another example in the preliminary phase of the exit from quantitative easing. Lagging behind these other developed economies, the Japanese economy is finally close to exiting its three-decade-long low interest rate environment as of Spring 2025.

This paper studies a mechanism of how a belief gap about a monetary policy reaction function can grow under low-for-long and looks at the implications of the belief gap during the exit. To that end, we build a New Keynesian (NK) model that incorporates agents' learning about a policy reaction function as well as the ELB and forward guidance with imperfect credibility. Using the model, we simulate a scenario of low-for-long and study how agents' belief evolves over time and what the consequences of a correction of a belief gap are. Moreover, we study them under various scenarios including an inflation surge when forward guidance keeps interest rates from rising. Furthermore, we conduct empirical analyses using data on professional forecasters to underpin our model's implications.

Our main findings are three-fold. First, low-for-long policy can induce boom and bust in the economy around the exit. Under such policy, a gap between agents' belief about the monetary policy stance implied by a policy reaction function and its actual stance widens downward nonlinearly. Agents learn from data and come to believe that the monetary policy stance is more accommodative, driven by a decrease in the perceived nominal neutral interest rate (i-star, for short) in the reaction function, and expect lower interest rates for longer. This expectation in turn stimulates the economy. When the interest rate is raised after low-for-long, however, the correction of the belief gap that has grown is perceived as surprise monetary tightening, causing economic contraction. Second, low credibility about forward guidance amplifies the boom and bust. Observing the interest rate at the ELB, if forward guidance is less credible, agents perceive that it pertains less to forward guidance and more to low notional interest rates implied by the reaction function, which leads to a further downward widening of the belief gap. Third, our estimation reports that the perceived i-star declined from 1990 especially during the ELB period. Importantly, consistent with our model's prediction about learning, we find that the perceived i-star we estimated responds positively to an identified monetary policy surprise.

Our model is based on a standard three-equation NK model with the ELB constraint. It features three additional ingredients: forward guidance with imperfect credibility, additional discounting in the linearized Euler equation, and agents' learning about a monetary policy reaction function. Forward guidance is added to generate low-for-long and modeled as keeping the policy rate at the ELB for extended periods. To capture the actual conduct of forward guidance that leaves room for discretion and to address the forward guidance puzzle (Del Negro et al., 2023), we assume that agents perceive that forward guidance will be actually implemented as announced with a certain probability in a spirit similar to Bodenstein et al. (2012). The subjective probability captures the degree of imperfect credibility about forward guidance. For simplicity we abstract from unconventional monetary policy except for forward guidance. Additional discounting in the Euler equation is introduced to help the model to generate low-for-long without causing a recession that is too severe in light of the experiences of the developed economies since the GFC. It is based on households' cognitive discounting studied by Gabaix (2020) or procyclical inequality among households in the tractable heterogeneous agent new Keynesian (HANK) model studied by Bilbiie (2024).

Regarding learning, we employ the anticipated utility approach of Kreps (1998). Under this approach, agents make decisions assuming that there will be no change in the monetary policy reaction function as is believed today with beliefs that have been inherited from the previous period. After observing the macroeconomic variables including the policy rate, agents update their belief and repeat this process in the next period. We use this approach because of its simplicity and applicability to a binding ELB constraint. We closely follow

Bodenstein et al. (2022), who study agents' learning under an ELB regime, and extend their learning algorithm to accommodate forward guidance with imperfect credibility.

We assume a relatively general form for a monetary policy reaction function, which is characterized by a constant term that captures i-star, a coefficient on the current inflation rate, and a coefficient on the output gap as well as a coefficient on the lagged policy rate. We allow for agents in the model to learn about the constant term and the two coefficients on inflation and output gap.

The model is then calibrated to the Japanese economy and simulated under a baseline scenario in which a severe negative demand shock hits and drives the economy into a prolonged period of the binding ELB constraint that spans seven years including two years of forward guidance. To focus on agents' learning, the scenario assumes no change in the actual monetary policy reaction function. In addition, to study learning under low-for-long, the scenario assumes a path of interest rates such that the central bank implements forward guidance as announced despite agents' imperfect credibility.

In the initial phase of the interest rate at the ELB, there is no significant change in agents' belief. This is because observing the interest rate at the ELB is too obvious and does not add meaningful information as inflation and output are far below their target or long-term levels. However, as the ELB regime is prolonged and the economy continues to recover, observing the interest rate at the ELB then starts to affect their beliefs. Agents see that their previous beliefs suggested that there was a significant probability of a positive interest rate; but the fact that the rate was set at the ELB implies that the interest rate implied by the reaction function may be lower than previously thought and so prompts agents to update their beliefs. The reasoning leads to a downward update of the interest rate implied by the reaction function, which pushes down i-star in the reaction function in particular. The same intuition applies to the period of forward guidance in which the interest rate is kept at the ELB. Hence, the agents' belief evolves non-linearly under the ELB regime: almost no change initially when the economic conditions clearly justify an ELB regime, but a sharp decline in the perceived i-star as the ELB period is prolonged while the economy continues to recover.

The baseline scenario simulation shows that the largest downward belief gap comes one period before the end of forward guidance. After low-for-long, the interest rate is set following the reaction function and the belief gap is corrected abruptly. The correction is perceived as monetary tightening and decreases output and inflation considerably. A sensitivity analysis suggests that the belief gap one period before the lift-off becomes larger, as forward guidance becomes less credible; the period of forward guidance becomes longer; the parameters of the reaction function are perceived to be more volatile; and the perceived law of motion for i-star becomes more persistent. The size of the belief gap and the effects of its correction during the exit depend on assumptions on these exogenous parameters, but the main result holds: as an ELB regime is prolonged, the belief gap widens downward, and its correction during the exit has contractionary effects on the economy.

The paper also examines alternative scenarios. An inflationary shock scenario during the period of forward guidance is considered in light of the recent inflation surge of 2021–2022. To reflect the almost three-decade long experience of low interest rates in Japan, a recurrent shock scenario is added, where the interest rate is kept at the ELB for twelve years – five years longer than the baseline scenario – as a severe negative shock hits recurrently. Under these scenarios the belief gap becomes wider than in the baseline scenario. A scenario of belief accommodations after the exit is also considered because a central bank may want to maintain an accommodative stance. In this scenario, a correction of the gap is postponed. Perceived monetary tightening when belief accommodations end can be less severe than in the case of the baseline scenario.

Finally, the paper examines market participants' perceptions of monetary policy empirically using survey data from professional forecasters on short-term interest rates, real GDP growth, and CPI inflation in Japan. The estimates of the perceived i-star gradually decline, especially from 2008 when the BOJ introduced various unconventional measures, including forward guidance. The gradual decline in the perceived i-star under the low interest environment with forward guidance is consistent with the simulation results of our model. We further examine the relationship between monetary policy shocks and the perceived i-star. A local projection analysis reveals that the perceived i-star falls a quarter later when a negative monetary policy surprise is observed. This gradual correction in the perceived i-star is consistent with our model where agents are learning policy rules from central bank actions.

Related literature This paper builds on the literature on agents' learning about parameters or variables pertaining to monetary policy, which includes learning about parameters that characterize a monetary policy reaction function (Tetlow and von zur Muehlen, 2001; Cogley et al., 2015; Bauer et al., 2024); monetary policy regimes (Gust et al., 2018; Mele et al., 2020; Bodenstein et al., 2022; Krane et al., 2023); shocks to the target rate of inflation (Erceg and Levin, 2003); trend inflation or long-run mean of inflation (Cogley and Sbordone, 2008; Carvalho et al., 2023; Hogen and Okuma, 2025); the real neutral interest rate (Rungcharoenkitkul and Winkler, 2023); the distribution of consumption and inflation (Christiano et al., 2024). Most closely related to our study is Bodenstein et al. (2022) which looks at agents' learning about a change in monetary policy strategies from an inflation-targeting rule to a price-level targeting rule under an ELB regime. We extend its learning algorithm to accommodate forward guidance with imperfect credibility.

This paper contributes to the literature on the potential side effects of low-for-long and unconventional monetary policy. Borio et al. (2017) and Claessens et al. (2018) empirically show that low-for-long interest rates adversely affect bank profitability. Abadi et al. (2023) theoretically demonstrate the existence of the reversal rate at which accommodative monetary policy becomes contractionary for lending in a NK model with banks. Acharya et al. (2022) and Acharya and Rajan (2024) argue that central bank balance sheet expansion and its subsequent shrinkage may increase both the probability and severity of episodes of liquidity stress. Orphanides (2024) points out that forward guidance can be a trap if surprises to inflation occur when forward guidance keeps the interest rate from rising. This paper adds to this literature another possible channel: low-for-long interest rates may induce agents to believe in lower-for-longer interest rates, and subsequent correction is contractionary.[1]

This paper is also related to the empirical studies on monetary policy rules perceived by economic agents. Most related is Bauer et al. (2024) who estimate a perceived monetary policy rule using professional forecast data. Their estimation results are consistent with a view that economic agents update their perceived monetary policy rule, especially the responsiveness to macroeconomic variables, by observing monetary policy actions. Motivated by our theoretical model's results, our estimation focuses on i-star instead of coefficients on inflation or output. Other papers that estimate perceived monetary policy rules include Hamilton et al. (2011) who use the United States federal funds futures; Bocola et al. (2024) who use daily bond yield data; Fendel et al. (2011) who use professional survey data for G7 countries, as we do for Japan in this paper; Schmidt and Nautz (2012) who use the Euro area financial market data; Carvalho and Nechio (2014) who use the Michigan Survey. We

[1] As a part of the European Central Bank's monetary policy review, Altavilla et al. (2021) argue that the side effects of the key monetary policy instruments employed since 2014 have been generally contained so far. In its monetary policy review, Bank of Japan (2024) concludes that although there have been certain side effects of large-scale monetary easing on financial markets and financial institutions' profits, its overall effect on the Japanese economy so far appears to have been positive.

contribute to this literature by shedding light on the dynamics of the perceived i-star in the low-for-long interest rate environment in Japan.

The rest of the paper is organized as follows. Section 2 presents the model including forward guidance and agents' learning. Section 3 simulates the model and presents the main results of the paper. Section 4 provides empirical support for the main results. Section 5 concludes.

2 The Model

We present a version of the NK model that incorporates forward guidance with imperfect credibility and discounting of future aggregate variables in the Euler equation. Imperfect credibility is introduced in order to capture uncertainty about whether forward guidance is implemented as originally announced. The discounting is introduced to address the so-called forward guidance puzzle and help the model generate a prolonged period of binding ELB.[2] The discounting can be interpreted as households' cognitive discounting as in Gabaix (2020) or procyclical inequality among households as in the tractable HANK model studied by Bilbiie (2024). In addition, we introduce private agents' learning about a monetary policy rule as in Bodenstein et al. (2022). To analyze the model, we consider a linearized model and focus on a targeted equilibrium where inflation in steady state is equal to the target rate of inflation set by the central bank.[3]

Section 2.1 presents a simple three-equation linearized model with rational expectations. Section 2.2 introduces forward guidance with imperfect credibility. Section 2.3 introduces learning and describes how private agents learn about a monetary policy rule.

2.1 The linearized model

First, a rational expectations model is presented. Let $\hat{y}_t$, $\hat{\pi}_t$, and $\hat{i}_t$ denote output, inflation, the nominal interest rate in deviation from their steady state value, respectively.[4] The

[2] For the forward guidance puzzle, see McKay et al. (2016) and Del Negro et al. (2023).

[3] In the context of the low-for-long experience in Japan, for the role of non-linearity, see Iiboshi et al. (2022); for the literature that focuses on a deflationary equilibrium as opposed to a targeted equilibrium, see Aruoba et al. (2017), Hirose (2020), and Coyle et al. (2025).

[4] For the interest rate, the deviation from steady state is expressed in terms of the gross interest rate, so that $\hat{i}_t$ is given by $\hat{i}_t = (i_t - i)/(1 + i)$, where i_t is the net nominal interest rate and i is its steady state. Similarly, the ELB in deviation from steady state is given by $\hat{\underline{i}} = (\underline{i} - i)/(1 + i)$, where $\underline{i}$ is the ELB of the net nominal interest rate.

linearized equations that describe the dynamics of inflation and output are given by

$$\hat{y}_t = M\mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) + z_t^d, \tag{1}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} + z_t^s, \tag{2}$$

where z_t^d and z_t^s are demand and supply shocks that follow an AR(1) process, respectively, $\mathbb{E}_t(\cdot)$ is an expectation operator conditional on information in period t, and $M \in [0,1]$ captures the discounting of future aggregate variables. The case of $M = 1$ corresponds to the standard NK model. The discounting of $M < 1$ can be derived from the model with households' behavioral cognitive discounting (Gabaix, 2020) or the tractable HANK model with cyclical inequality among households (Bilbiie, 2024). The other parameters are standard: $\sigma > 0$ captures relative risk aversion, β is a preference discount factor, and κ is the slope of the NK Phillips curve (2). Appendix A derives equations (1) and (2) from the original NK model.

Without forward guidance, a central bank sets the nominal interest rate following a monetary policy rule subject to the ELB constraint, given by

$$\hat{i}_t = \max\{\hat{i}_t^n, \underline{\hat{i}}\}, \tag{3}$$

$$\hat{i}_t^n = \rho_i\hat{i}_{t-1} + (1-\rho_i)\left(\hat{i}_t^* + \phi_{\pi t}\hat{\pi}_t + \phi_{yt}\hat{y}_t\right) + \epsilon_t^{mp}, \tag{4}$$

where $\hat{i}_t^n$ is the notional interest rate at which the central bank would set if there were no ELB, $\hat{i}_t^*$ is the nominal neutral interest rate (i-star), and ϵ_t^{mp} is an i.i.d. monetary policy shock that follows $N(0, \sigma_{mp}^2)$.[5] The coefficients on inflation and output, $\phi_{\pi t}$ and ϕ_{yt}, are allowed to be time-varying. Private agents learning about these variables will be introduced in 2.3.

To focus on private agents' learning about the i-star later in Section 2.3, we assume that it follows an exogenous AR(1) process:

$$\hat{i}_t^* = \rho^*\hat{i}_{t-1}^* + \epsilon_t^*, \tag{5}$$

where $0 \le \rho^* < 1$ and ϵ_t^* is an i.i.d. shock. The assumption of $\rho^* < 1$ ensures $\hat{i}_t^* = 0$ in steady state, that is, the i-star becomes equal to the steady state nominal interest rate that

[5] In steady state, the notional rate and the i-star coincide with the actual nominal interest rate: $i^n = i^* = i$. Similar to $\hat{i}_t$, the notional rate and the i-star in deviation from the steady state are given by $\hat{i}_t^n = (i_t^n - i)/(1+i)$ and $\hat{i}_t^* = (i_t^* - i)/(1+i)$, respectively.

is consistent with inflation on its target $\hat{\pi}_t = 0$. For the sake of exposition, we treat $\hat{i}^*_t$ as an exogenous time-varying policy parameter.

Without forward guidance and learning, the model is characterized by four equations (1)–(4) with four unknowns $\{\hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{i}^n_t\}$, for given $\hat{i}^*_t$, $\phi_{\pi t}$, and ϕ_{yt}.

2.2 Forward guidance with imperfect credibility

During the period specified by forward guidance the interest rate is set according to the forward guidance. Among a variety of forms of forward guidance, in light of recent developed economy experience of interest rates being kept at the ELB during the post Covid-19 pandemic surge in inflation, we focus on a lagged exit from the ELB.[6] Specifically, we consider a situation in which a severe shock hits and drives the economy into the ELB in period $t = 1$. A central bank announces that it will keep the interest rate at the ELB for an extended period until $t = T_1$, which is longer than the duration of the ELB without forward guidance. Under the forward guidance, the initial period $t = T_0$ in which the forward guidance starts binding so that the interest rate is set at the ELB is such that $i^n_{T_0} > \underline{i}$ for the first time since period $t = 1$.

In practice, the terminal period of forward guidance, T_1, may depend on the state of the economy when it is determined, and thus it can be expressed as T_{1t} with time subscript t. In this analysis, for simplicity, we assume that T_1 is unchanged at the original value when it was announced in $t = 1$. This specific forward guidance can be regarded as time-dependent or calendar-based, or it can also be regarded as state-dependent where there is no change in the state of the economy as projected in $t = 1$.

We assume that the terminal period of forward guidance is perfectly credible, but whether the central bank keeps the interest rate at the ELB as stipulated by the forward guidance is imperfectly credible. Specifically, agents believe that the ELB is continued with probability $q \in [0, 1]$, given that the promise made by the forward guidance has actually been implemented; with probability $1 - q$, the promise is reneged and the forward guidance is discarded thereafter. Consider a situation where the economy falls in the ELB in period $t = 1$ and

[6] From the modeling perspective, forward guidance can be modeled as monetary policy rules (Reifschneider and Williams, 2000; Debortoli et al., 2019), news shocks (Campbell et al., 2017), and a lagged exit (Bodenstein et al., 2012; Boneva et al., 2018). Forward guidance as news shocks can deliver the same outcomes as forward guidance as a lagged exit under some circumstances. For time-consistent forward guidance, see Batista et al. (2023). For empirical applications to the Japanese economy, see Ikeda et al. (2024) and Kubota and Shintani (2024) for rule-based forward guidance, and Hayashi and Koeda (2019) and Iiboshi et al. (2022) for threshold-based forward guidance as a lagged exit.

at the same time forward guidance to keep the interest rate at the ELB until $t = T_1$ is announced. In this situation, agents believe that the interest rate follows:

$$\hat{i}_t = \begin{cases} \underline{\hat{i}} & \text{if no renege before, for } t \in \{T_0, ..., T_1\}, \text{ with prob. } q \\ \max\{\hat{i}^n_t, \underline{\hat{i}}\} & \text{otherwise} \end{cases} \tag{6}$$

Forward guidance is perfectly credible if $q = 1$ and not credible if $q = 0$. We assume that the degree of credibility is exogenous. In practice, it would depend on economic agents' trust in a central bank, central bank communications, future uncertainty, a central bank's discretion, and time-inconsistency among others.[7]

2.3 Learning about a monetary policy rule

We now introduce private agents' learning about the monetary policy rule (4). Let $\mathbf{x}_t \stackrel{\text{def}}{=} [\hat{i}_{t-1}, \hat{\pi}_t, \hat{y}_t]'$ denote a column vector of the input variables in (4) and let $\boldsymbol{\theta}_t \stackrel{\text{def}}{=} [\hat{i}^*_t, \phi_{\pi t}, \phi_{yt}]'$ denote a column vector of time-varying parameters. A part of the notional rate that is systematically tied to inflation, output, and the lagged rate, $f(\mathbf{x}_t; \boldsymbol{\theta}_t) \stackrel{\text{def}}{=} \rho_i \hat{i}_{t-1} + (1 - \rho_i)(\hat{i}^*_t + \phi_{\pi t}\hat{\pi}_t + \phi_{yt}\hat{y}_t)$, is called the systematic part of the notional rate. With this notation, the notional rate can be written as $\hat{i}^n_t = f(\mathbf{x}_t; \boldsymbol{\theta}_t) + \epsilon^{mp}_t$.

Information structure The agents know the monetary policy rule that consists of (3) and (4), including its functional forms and parameter values, except for $\boldsymbol{\theta}_t$ and ϵ^{mp}_t. The agents believe that the i-star follows (5). The terminal date of forward guidance, T_1, is announced by the bank and is known to the agents, but the starting date T_0 is not known. With the starting date $T_0 = \tilde{T}_{0t}$ guessed in period t, the agents believe that the interest rate will follow equation (6) during the forward guidance period of $\{\tilde{T}_{0t}, ..., T_1\}$. They observe $\hat{\pi}_t$, $\hat{y}_t$, and $\hat{i}_t$ in period t, but do not observe $\boldsymbol{\theta}_t$, ϵ^{mp}_t, or ϵ^*_t.

Beliefs and decision making At the beginning of period t, the agents have a subjective belief about the mean of unobservable monetary policy parameters, $\tilde{\boldsymbol{\theta}}_{t|t-1}$, inherited from the previous period. In line with the learning literature such as Bodenstein et al. (2022), we employ the anticipated utility approach of Kreps (1998) and assume that agents make

[7] The assumption of constant q is made for simplicity. It can be made endogenous by assuming that agents learn q over time. The main results of the paper would continue to hold as long as there is an upper bound of q that is significantly less than unity.

decisions as if they expected no additional shock to $\boldsymbol{\theta}_t$ in the future.[8] Given $\tilde{\boldsymbol{\theta}}_{t|t-1}$, they form beliefs about the monetary policy shock, $\tilde{\epsilon}_t^{mp}$, and make individual decisions. Their beliefs are consistent with the macroeconomic variables, $\hat{\pi}_t$, $\hat{y}_t$, and $\hat{i}_t$, which are governed by equations (1)–(6), forward guidance, and the belief about the monetary policy shock that satisfies

$$\hat{i}_t^n = f(\mathbf{x}_t; \tilde{\boldsymbol{\theta}}_{t|t-1}) + \tilde{\epsilon}_t^{mp}, \tag{7}$$

with $\hat{i}_t^n$ given by equation (4).

How agents' belief about monetary policy shocks is formulated differs depending on whether the ELB constraint is binding. There are two cases: when the ELB constraint is slack or when it is binding. First, when the ELB constraint is slack and thus $\hat{i}_t = \hat{i}_t^n$, agents' belief about the monetary policy shock is given by

$$\tilde{\epsilon}_t^{mp} = \hat{i}_t - f(\mathbf{x}_t; \tilde{\boldsymbol{\theta}}_{t|t-1}) = \epsilon_t^{mp} + \left[f(\mathbf{x}_t; \boldsymbol{\theta}_t) - f(\mathbf{x}_t; \tilde{\boldsymbol{\theta}}_{t|t-1}) \right]. \tag{8}$$

The belief $\tilde{\epsilon}_t^{mp}$ consists of the true monetary policy shock plus a difference between the true systematic component of the rule and agents' belief about it. Next, in the case of the binding ELB constraint, what agents learn about $\hat{i}_t^*$ is the inequality related to the ELB constraint: $\hat{i}_t^n = f(\mathbf{x}_t; \tilde{\boldsymbol{\theta}}_{t|t-1}) + \tilde{\epsilon}_t^{mp} \leq \underline{\hat{i}}$. Given $\tilde{\boldsymbol{\theta}}_{t|t-1}$, there are many values of $\tilde{\epsilon}_t^{mp}$ that satisfy this inequality. The monetary policy rule (4) suggests that any value of $\tilde{\epsilon}_t^{mp}$ that satisfies the inequality leads to the same solution to equations (1)–(4).

In making decisions under the ELB in period t, the agents need to guess when the forward guidance will likely take effect by keeping the interest rate from rising or whether it has already taken effect. The terminal date T_1 is known, but the starting date T_0 is not. We assume that in period t the agents form expectations about T_0 such that the forward guidance will start to take effect at $\tilde{T}_{0t}$ when the expected notional rate becomes above the ELB for the first time from period t, i.e., $\tilde{T}_{0t} = \min\{t' | \mathbb{E}_t(\hat{i}_{t'}^n | \tilde{\boldsymbol{\theta}}_{t|t-1}) > \underline{\hat{i}}, t' \geq t\}$. Once the starting date coincides with the current period, $\tilde{T}_{0t} = t$, the agents believe that forward guidance has already taken effect and fix the starting date at $\tilde{T}_{0t'} = t$ for $t' \geq t$.[9]

[8]This assumption is consistent with how the model with the occasionally binding ELB constraint is solved by using the algorithm of Guerrieri and Iacoviello (2015), which we employ in this paper.

[9]The alternative simple assumption is that agents perceive $T_0 = 1$, that is, they perceive that the forward guidance starts taking effect as soon as it is announced in $t = 1$. This alternative assumption has no significant effect on the main results of the paper.

Learning and updating At the end of each period after the aggregate variables are realized, the agents update their belief about the policy parameters $\boldsymbol{\theta}_t$ in a Bayesian manner. Specifically, given the prior belief $\tilde{\boldsymbol{\theta}}_{t|t-1}$, they update it for a posterior belief $\tilde{\boldsymbol{\theta}}_{t|t}$ by solving the filtering problem that consists of the ELB constraint (3), the actual rule for the conceptual rate (4), and the law of motion for $\boldsymbol{\theta}_t$,

$$\boldsymbol{\theta}_t = F\boldsymbol{\theta}_{t-1} + \boldsymbol{\epsilon}_t^{\theta}, \tag{9}$$

where F is a diagonal matrix given by $F = \text{diag}(\rho^*, 1, 1)$ and $\boldsymbol{\epsilon}_t^{\theta} \sim \text{i.i.d.}N(0, \Sigma_\theta)$. Thus, agents perceive monetary policy coefficients $\phi_{\pi t}$ and ϕ_{yt} as a random walk.

To simplify the filtering problem that involves an occasionally binding constraint, we make two assumptions as in Bodenstein et al. (2022). First, the agents treat endogenous variables $\mathbf{x}_t$ as independent of shocks ϵ_t^{mp} and $\boldsymbol{\epsilon}_t^{\theta}$. Second, the posterior is approximated by a normal distribution. In addition, we assume that there is no new information relevant to the agents' learning when they believe that the interest rate is kept at the ELB due to forward guidance. This assumption is based on the nature of forward guidance such that the interest rate is set at the ELB irrespective of the current states of the economy if forward guidance is actually implemented as announced with probability q.

Under these assumptions, the prior belief about $\boldsymbol{\theta}_t$ is given by a normal distribution $N(\tilde{\boldsymbol{\theta}}_{t|t-1}, P_{t|t-1})$ and the posterior is given by $N(\tilde{\boldsymbol{\theta}}_{t|t}, P_{t|t})$, where $P_{t|\cdot}$ is an associated variance. From equation (9), the updated prior that is used for the agents to make decisions in $t+1$ is given by $\tilde{\boldsymbol{\theta}}_{t+1|t} = F\tilde{\boldsymbol{\theta}}_{t|t}$ and $P_{t+1|t} = FP_{t|t}F' + \Sigma_\theta$. Appendix B.1 describes how to obtain the posterior from the prior.[10]

3 Scenario Analyses

We simulate the model presented in Section 2 to study how agents' belief about the i-star changes during the prolonged period of the interest rate at the ELB and what implications those belief changes can have during the exit from low-for-long policy. This section is organized as follows. The model is parameterized in Section 3.1. A baseline scenario is presented in Section 3.2 under the assumption of no learning, where a severe demand shock drives the

[10] Appendix B.2 discusses a particle filter, which works without imposing the assumption that the posterior is approximated by a normal distribution. It shows that the results obtained by using the learning algorithm presented in the main text, which imposes the normality assumption, are quantitatively similar to those obtained by using a particle filter.

interest rate at the ELB for a prolonged period. Forward guidance and imperfect credibility are also briefly discussed. Learning is introduced in Section 3.3, which presents the evolution of agents' belief about the i-star during low-for-long policy and how its correction during the exit affects the economy. The underlying mechanisms behind the evolution of agents' belief are studied in Section 3.4. The sensitivity of the results to various assumptions is examined in Section 3.5. Additional scenarios are examined in Section 3.6.

3.1 Parameterization

We set parameter values to reflect the Japanese economy, although the model is highly stylized. One period of the model corresponds to a quarter of a year. The parameter values are set broadly in line with those reported in Iiboshi et al. (2022) and Hirose et al. (2024). Iiboshi et al. (2022) estimate the non-linear version of the model with the ELB, without additional discounting M, using Japanese quarterly data from 1983 to 2016. Hirose et al. (2024) estimate a similar model but with additional discounting, using U.S. quarterly data from 1983 to 2019. The values of parameters pertaining to equations (1)–(4) are set as $\sigma = 1.5$, $\kappa = 0.05$, $\beta = 0.998$, $\rho_i = 0.7$, $\phi_\pi = 2$, $\phi_y = 0.2$, and $M = 0.85$, where ϕ_π and ϕ_y denote benchmark monetary policy parameters. Given the target rate of inflation of 2 percent annually, the steady state nominal rate is set at $i = \pi/\beta - 1$.

The speed of learning is governed by the variance of monetary policy shocks σ^2_{mp} and the variance-covariance matrix Σ_θ that captures subjective uncertainty about the monetary policy parameters. Intuitively, according to equation (8) when the interest rate is above the ELB, the larger the ratio $\text{diag}(\Sigma_\theta)/\sigma^2_{mp}$, the more the agents attribute the perceived monetary policy shock $\tilde{\epsilon}^{mp}_t$ to changes in monetary policy parameter values. Since there is no empirical evidence regarding Σ_θ, we use the same values regarding σ_{mp} and variance attached to $\phi_{\pi t}$ as Bodenstein et al. (2022) who study how learning about a monetary policy rule occurs when there is a change from an inflation targeting rule to a price-level targeting rule in an ELB regime. For the variance pertaining to $\hat{i}^*_t$, we use the same value regarding $\phi_{\pi t}$. For the variance pertaining to $\hat{\phi}_{yt}$, we adjust the scale taking into account the benchmark values of $\phi_y = 0.2$ against $\phi_\pi = 2$. Specifically, we set $\sigma^2_{mp} = 0.01$ percent and $\Sigma_\theta/\sigma^2_{mp} = \text{diag}(0.4, 0.4, 0.4/100)$.

These parameter values imply the standard deviation of the monetary policy shock of

10 basis points, which is within the estimated values for the Japanese economy.[11] And the standard deviations of the innovations of $\phi_{\pi t}$ and ϕ_{yt}, are 0.06, and 0.006, respectively, which indicate that the standard deviations relative to a benchmark value are the same, $0.06/\phi_\pi = 0.006/\phi_y$, between the coefficients of inflation and output. The standard deviation of the innovation of $\hat{i}_t^*$ (in unit of percent) is 6 basis points, which is slightly smaller than the point estimates of equation (5) using the various estimates of the i-star for the Japanese economy.[12] Finally, the point estimates of equation (5) suggests the value of ρ^* between 0.9 to 0.96. We assume it is persistent and set at $\rho^* = 0.95$ within the range of the estimates.

3.2 Baseline scenario with forward guidance

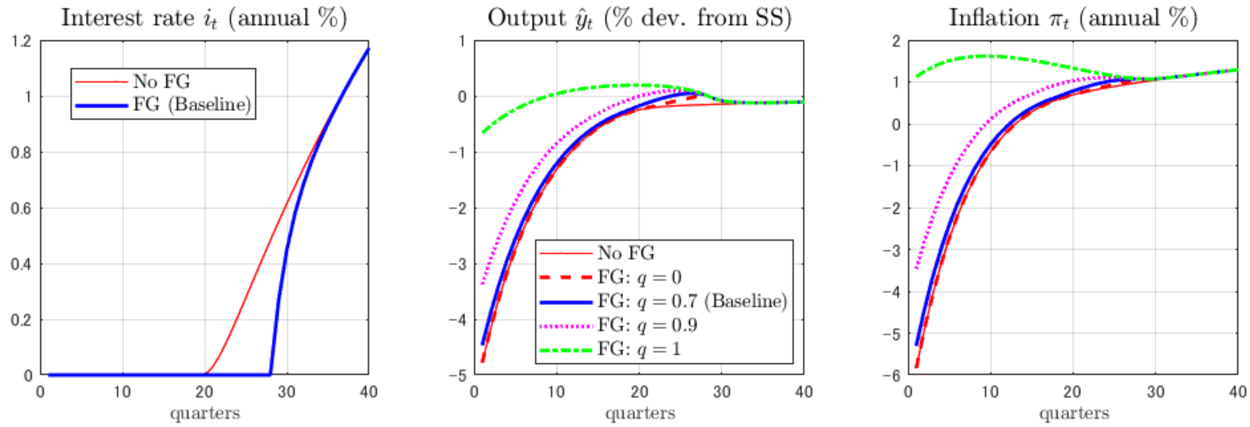
We construct a baseline scenario of low-for-long interest rates in view of the prolonged period of interest rates at the ELB in the Japanese economy from the late 1990s to early 2020s. We take a view that severe and persistent negative shocks hit the Japanese economy repeatedly during the period, such as deleveraging pressures from the bursting of asset and land prices in the early 1990s, the domestic financial crisis in the late 1990s, the global financial crisis of 2007–09, the great east Japan earthquake of 2011, and the COVID-19 pandemic. However, it is beyond the scope of the paper to explain low-for-long in the quarter century since the late 1990s.

To shed light on mechanisms of how agents' perception about the i-star in the monetary policy rule changes during low-for-long interest rates, and for clarity and simplicity, we focus on a scenario in which a single negative shock hits the economy and drives it into the prolonged period of low interest rates. Specifically, we consider a severe and persistent negative demand shock that hits in period $t = 1$ and drives the interest rates at the ELB for five years without forward guidance. To generate such a long period of binding ELB constraint with one time shock, the shock is made significantly persistent and severe: the AR(1) coefficient of z_t^d is set at 0.97 and the magnitude of the shock is set at $z_1^d = -0.55$

[11] According to the literature on estimating a dynamic stochastic general equilibrium model for the Japanese economy, the standard deviation of monetary policy shocks varies from about 6 basis points (Sugo and Ueda, 2008) to 10 basis points or greater (Kaihatsu and Kurozumi, 2014; Iiboshi et al., 2022).

[12] In general, the i-star is unobserved. Theoretically, it can be calculated as the sum of the natural real rate of interest and the long-term inflation expectations. For the estimate of the natural real rate of interest, we use six series estimated by Nakano et al. (2024). For the estimate of the long-term inflation expectations, we use those estimated by Hiraki and Hirata (2020). The point estimate of the standard deviation of the innovation of $\hat{i}_t^*$ lies around 7 basis points to 8 basis points. Thus, our choice of 6 basis points for the model's parameterization is conservative regarding the variability of the perceived i-star.

Figure 1: Baseline scenario and imperfect credibility about forward guidance



Note: 'FG' denotes forward guidance. The interest and inflation rates are shown in annual percentage level while the output is in terms of percentage deviation from steady state (SS).

percent.[13] In addition, we assume that the central bank responds to this situation in period $t = 1$ by announcing forward guidance of keeping the interest rate low for an extended period of two years. Thus, in terms of notations introduced in Section 2.2, the terminal date of the forward guidance is $T_1 = 28$ quarters under the baseline scenario. Importantly, to focus on agents' learning, we assume no change in the actual monetary policy rule (4) with $\hat{i}_t^* = 0$, $\phi_{\pi t} = \phi_\pi$, and $\phi_{yt} = \phi_y$. Finally, to clarify the role of learning and changes in agents' perceptions about monetary policy, we assume no learning in the baseline scenario and will introduce it subsequently. For the model's responses to shocks when the interest rate is positive, see Appendix A.3.

Figure 1 plots the baseline scenario along with the same scenario with a different degree of credibility about forward guidance and the case of no forward guidance. A severe shock hits the economy in $t = 1$ and drives the interest rate at the ELB for 20 periods without forward guidance, and in the baseline scenario with forward guidance, the interest rate is kept at the ELB until $t = T_1 = 28$, as shown in the left panel. Output and inflation drop sharply and the recoveries are slow, as shown in the middle and right panels. Admittedly, the decline of inflation is more severe than that experienced by the Japanese economy from the late 1990s to the early 2020s. Thus, the baseline scenario should be considered as a hypothetical scenario that captures low inflation and low output during the prolonged period of the ELB

[13]The high persistence of the shock in this simple model can be considered to reflect secular stagnation forces arising from a financial crisis and demographic aging, as studied by Ikeda and Kurozumi (2019) and Braun and Ikeda (2022) in rich quantitative models with nominal frictions, respectively.

in Japan. It is worth adding that the presence of additional discounting M in equation (1) helps the model generate deflation that is not unrealistically too severe (see Appendix A.3 for details).

A comparison between the case of no forward guidance ('No FG') and the cases of forward guidance ('FG') in Figure 1 shows forward guidance mitigates the stagnation pressure and the degree of its effect is increasing in the credibility parameter q. When it is perfectly credible, $q = 1$, the forward guidance may be too effective to believe it is true, which shares similarity with the forward guidance puzzle (Del Negro et al., 2023). In practice, no central bank would be able to commit credibly to the path of interest rates for future seven years and it is thus natural to assume $q < 1$. Although there is no empirical evidence about q, in light of some positive effects of monetary policy that can be interpreted as forward guidance for Japan (e.g., Kubota and Shintani, 2024), we set $q = 0.7$ as baseline.[14]

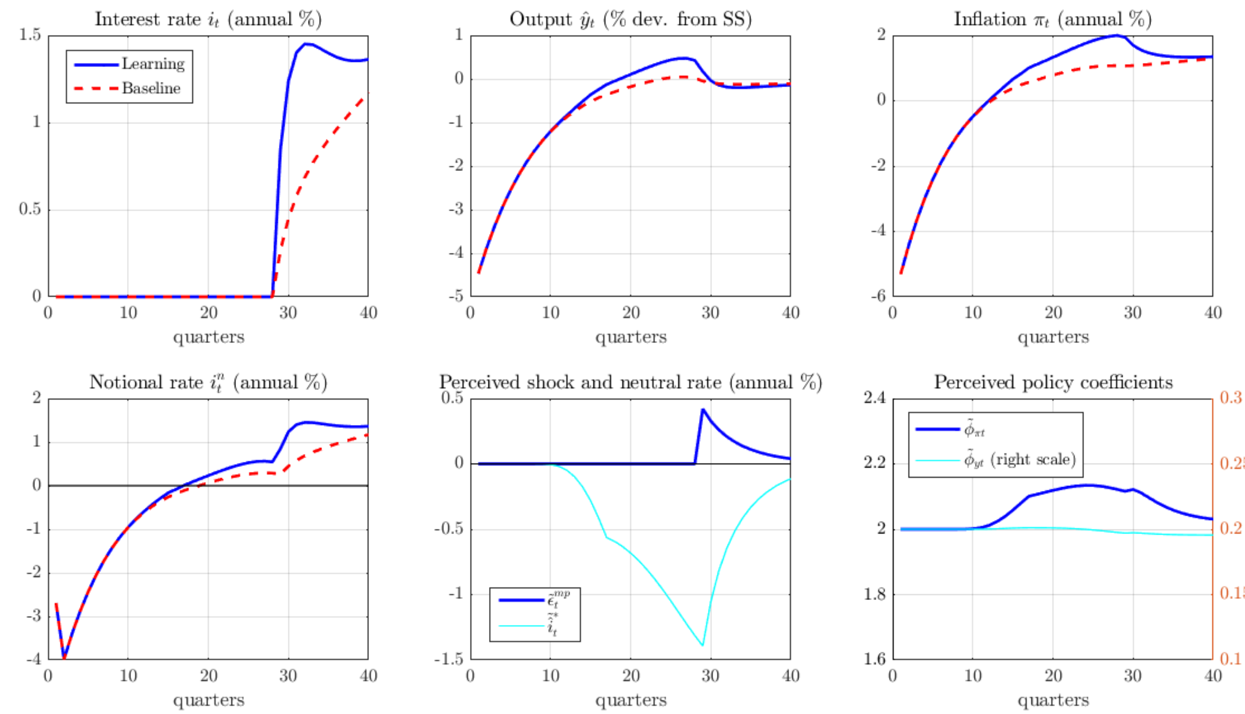
3.3 Baseline scenario when agents learn

Now we introduce agents' learning about a monetary policy rule into the model and simulate the baseline scenario. As described in Section 2.3, agents do not know key parameters in the monetary policy rule, $\boldsymbol{\theta}_t \overset{\text{def}}{=} [\hat{\imath}^*_t, \phi_{\pi t}, \phi_{yt}]'$. In every period, they have a belief $\tilde{\boldsymbol{\theta}}_t$ about the true parameter values $\boldsymbol{\theta}_t$, inherited from the previous period, and make a decision as if there will be no additional shock to $\tilde{\boldsymbol{\theta}}_t$. We focus on a path of interest rates such that the central bank keeps the interest rate at the ELB up to period $T_1 = 28$ as originally announced.

Figure 2 shows the baseline simulation results when agents learn. Compared to the baseline results that feature no learning, output and inflation in the model with learning show similar developments until around $t = 15$, but diverge thereafter, as shown in the upper middle and upper right panels. Specifically, output and inflation diverge upward until the lift-off from zero interest rate policy: output increases to a level greater than its steady state level, implying a positive output gap; inflation reaches to the level of the 2 percent target rate of inflation. However, when the economy exits the prolonged period of zero interest rates, output and inflation fall sharply. Hence, relative to the baseline results, in the

[14] Bodenstein et al. (2012) consider a central bank that faces the binding ELB constraint and commits to the optimal policy with probability q and reset the policy with the rest of the probability and argue that q is around 0.5 in the aftermath of the GFC in the U.S. and Sweden. Imperfect credibility is also supported by causal evidence provided by Coibion et al. (2023), who conduct a randomized control trial for U.S. households and find that the provision of information to households about current, past, and future policy rates has only limited effects on households' perceptions of both current and future rates.

Figure 2: Baseline scenario simulation when agents learn



Note: 'Learning' and 'Baseline' indicate the baseline scenario simulation using the model with and without learning, respectively. The notional rate in 'Learning' is the perceived notional rate during the period of the binding ELB constraint. The degree of credibility is set at $q = 0.7$.

model with learning there are boom and bust in output and inflation around the exit from low-for-long.

What drives the sharp decline in output and inflation during the exit is perceived monetary tightening, as shown in the lower middle panel of Figure 2. At the time of exit the perceived monetary policy shock $\tilde{\epsilon}_t^{mp}$ is more than 40 basis points. This perceived monetary tightening lowers output and inflation by 0.7 percentage points (pp) during the exit.

Since the structural monetary policy shock is assumed to be zero, $\epsilon_t^{mp} = 0$, equation (8) implies that the perceived monetary tightening $\tilde{\epsilon}_t^{mp}$ is caused by a difference in the actual and perceived systematic component of the monetary policy rule, $f(\mathbf{x}_t; \boldsymbol{\theta}_t) - f(\mathbf{x}_t; \tilde{\boldsymbol{\theta}}_{t|t-1})$, which in turn depends on differences in actual and perceived parameter values $\boldsymbol{\theta}_t - \tilde{\boldsymbol{\theta}}_{t|t-1}$. The decomposition of $\tilde{\epsilon}_t^{mp}$ into the three components in $\boldsymbol{\theta}_t - \tilde{\boldsymbol{\theta}}_{t|t-1}$ reveals that the perceived i-star, $\tilde{\hat{i}}_t^*$, accounts for almost all of the monetary tightening at the time of exit. Indeed, at the time of exit, the agents perceive that the i-star in the monetary policy rule is about -1.4 pp lower than its steady state level, and the deviations of their perceptions about monetary policy coefficients from the true values are relatively small, as shown in the lower middle and lower right panels of Figure 2. With these perceptions, the agents expect that the nominal interest rate will be low because of the low nominal rate in the monetary policy rule (4). However, this expectation is confounded at the time of exit, as the central bank follows the monetary policy rule (4) with $\hat{i}_t^* = 0$. The realized nominal interest rate that significantly exceeds the expectation is perceived as considerable monetary tightening.[15]

During the prolonged period of low interest rates, the agents' perception about the i-star in the monetary policy rule evolves in a nonlinear and cumulative manner. Three observations can be made from the lower middle panel of Figure 2. First, there is little change in the perception $\tilde{\hat{i}}_t^*$ during the first 3 years. Second, as the period of low interest rates is prolonged beyond 3 years, the perceived i-star starts decreasing and continues to do so until the agents perceive that the forward guidance starts taking effect in period $t = 17$. Third, as the period of forward guidance is prolonged, the perception continues to be revised downward significantly. From these observations, it appears that monetary policy that keeps interest rates at the ELB for a prolonged period contributes to building agents' perception that the i-star is lower than previously expected.

3.4 Understanding the gap

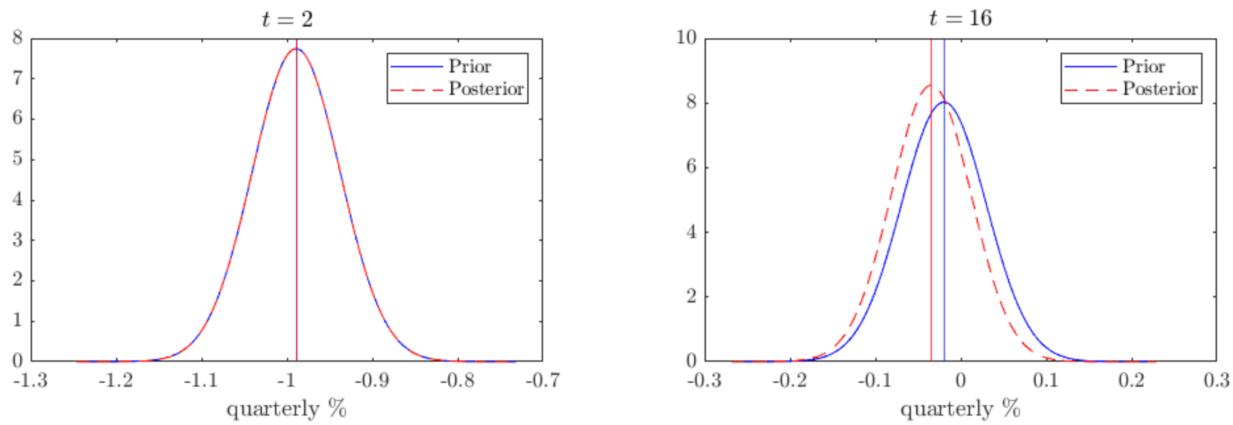
What drives the nonlinear and cumulative development in the perceived nominal rate in the model with learning and what is the underlying mechanism of perceiving low i-star during low-for-long? In this subsection, we address these questions with the aim of understanding the gap between the perceived i-star and its actual value in the monetary policy rule.

Let $f_{t|t-1}$ and $f_{t|t}$ denote the prior and posterior beliefs about the systematic part of the notional rate, $f_t = f(\mathbf{x}_t; \boldsymbol{\theta}_t)$. Since the agents have prior $\tilde{\boldsymbol{\theta}}_{t|t-1}$ inherited from the previous period, the prior about f_t is given by $f_{t|t-1} = f(\mathbf{x}_t; \tilde{\boldsymbol{\theta}}_{t|t-1})$. Under the assumptions about belief formation described in Section 2.3, when the interest rate is at the ELB because of low notional interest rates, as shown in Appendix B, the parameters of interest are updated as:

$$\tilde{\boldsymbol{\theta}}_{t|t} = \tilde{\boldsymbol{\theta}}_{t|t-1} + K_t \left(f_{t|t} - f_{t|t-1} \right), \tag{10}$$

[15]The model is agnostic about the source of a perceived change in the i-star in the monetary policy rule. It may be due to a change in the perceived trend inflation rate (Cogley and Sbordone, 2008; Carvalho et al., 2023), a change in the perceived target rate of inflation (Erceg and Levin, 2003), or a change in the perceived real neutral rate of interest (Rungcharoenkitkul and Winkler, 2023).

Figure 3: Beliefs about the systematic part of the notional rate



where K_t captures a gain from updated information about f_t. Since $\hat{i}_t^n$ is unobservable when the ELB constraint (3) is binding, agents form a posterior $f_{t|t}$ about the systematic part of $\hat{i}_t^n$ given the observation of $\hat{i}_t = \hat{\underline{i}}$, and update their belief about monetary policy parameters following equation (10).

During the initial phase of the low-for-long when the notional rate is in deep negative territory as shown in the lower left panel of Figure 2, the information gain of observing $\hat{i}_t = \hat{\underline{i}}$ is negligible, i.e., $f_{t|t} - f_{t|t-1}$ is close to zero. For example, in period $t = 2$ when the notional rate is actually at its lowest below zero, the decreases in output and inflation are so severe that it is obvious that the notional rate would be in deep negative territory, and observing the interest rate at the ELB does not add new information. Thus, there is no difference between the posterior and prior and no difference between their means as shown in the left panel of Figure 3. Therefore, during the downturn when output and inflation decrease substantially so that the notional rate is far below the ELB, there is essentially no new information from observing the binding ELB constraint, and there is essentially no change in the perceived i-star as shown in the lower middle panel of Figure 2.

As the notional rate recovers and becomes close to the ELB however, observing the interest rate at the ELB brings meaningful information about the notional rate. For example, in period $t = 16$, the agents' prior suggests that the perceived mean of the notional rate is still negative but close to the ELB of zero percent, as shown in the right panel of Figure 3. The prior distribution implies that the notional rate could have been positive. In this situation, on observing the interest rate at the ELB, the agents perceive that the notional rate would be below the ELB and use this information to shift their belief about the systematic part of the notional rate leftward, as shown in the right panel of Figure 3. The panel shows $f_{t|t} - f_{t|t-1} < 0$ and through equation (10) this leads to the downward update of the i-star, as shown in the lower middle panel of Figure 2.

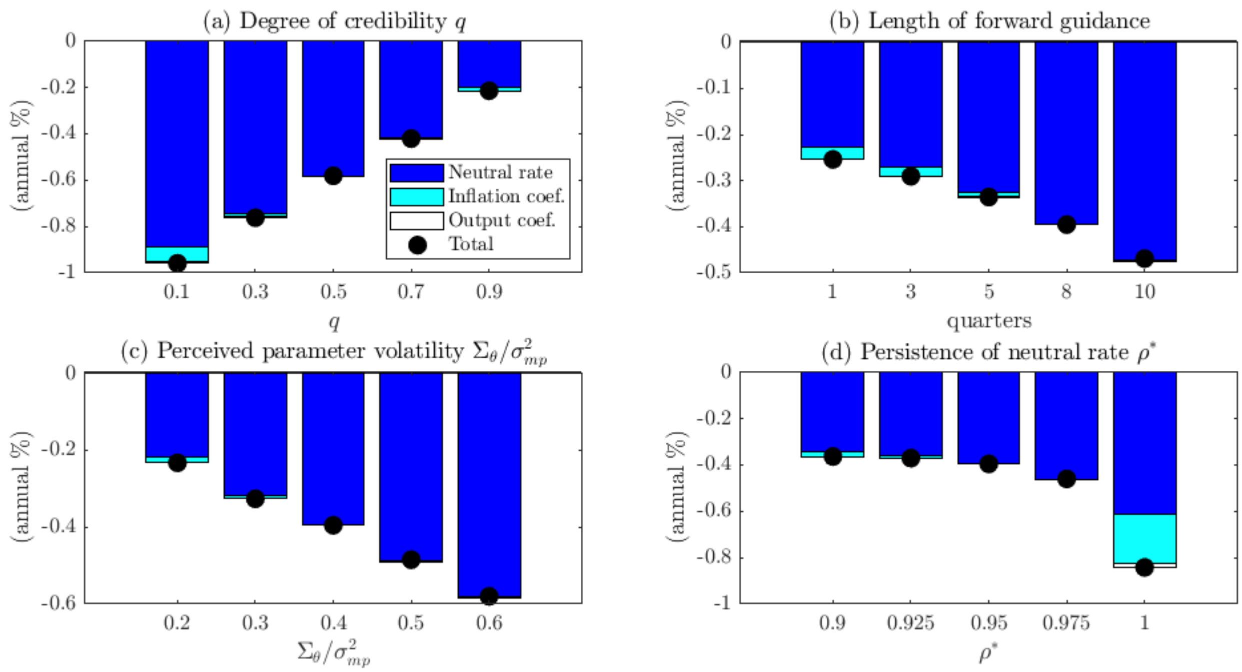
During the period of forward guidance, the agents understand that the announced forward guidance may be implemented but, due to imperfect credibility, they do not know whether it is actually implemented. With probability q, the agents perceive that the interest rate is kept at the ELB due to forward guidance, and in this case they receive no new information. But, with probability $1 - q$, the agents perceive that the interest rate is set at the ELB because of the below-ELB notional rate. Since their prior mean of the notional rate is in the positive area during the period of forward guidance, as shown in the lower left panel of Figure 2, observing the ELB shifts their posterior $f_{t|t}$ leftward, as in the right panel of Figure 3. And this leads to the stronger downward revision of their perceived i-star during the period of forward guidance, as shown in the lower middle panel of Figure 2.

3.5 Sensitivity

The simulation results shown above are based on the baseline scenario with agents' learning, which hinges on various assumptions. In this subsection, we study the sensitivity of the belief gaps – the deviations of the perceived systematic part of the notional nominal rate from its true value – to the degree of credibility, the length of forward guidance, the speed of learning, and the persistence of the perceived i-star. We focus on the belief gap in the initial period when interest rates begin rising since that is when the gap becomes largest and causes perceived monetary tightening. In addition to the sensitivity analysis, we also discuss the implications for the simulation results if low-for-long policy has an information effect.

Credibility about forward guidance Panel (a) of Figure 4 presents the belief gaps for various degrees of credibility about forward guidance, from $q = 0.1$ to 0.9, and the decomposition of the gaps into three components: the perceived i-star, inflation coefficient, and output coefficient, at the time of the lift-off. The case of $q = 0.7$ corresponds to the baseline scenario simulation shown in Figure 2. Panel (a) shows a clear pattern between the belief gap and credibility about forward guidance. The less credible forward guidance becomes, the more the deviation of the perceived i-star widens downward. Although the central bank keeps the interest rate at the ELB following its forward guidance, if agents believe that forward guidance is unlikely to be implemented due to low credibility, then the

Figure 4: Deviations of the perceived systematic part of the notional rate



Note: 'Neutral rate', 'Inflation coef.', and 'Output coef.' represent the contributions of the perceived i-star, inflation coefficient, and output coefficient, respectively, to the total deviation of the perceived systematic part of the notional rate from its true value one period before the end of forward guidance.

agents attribute the interest rate at the ELB to a low notional rate, resulting in a greater downward belief gap arising from a downward deviation of the perceived i-star. This result highlights the importance of the credibility of forward guidance: low credibility not only undermines the effectiveness of forward guidance but also exacerbates the potential side effects of forward guidance by widening the belief gap.

Length of forward guidance Panel (b) of Figure 4 shows the relationship between the length of forward guidance and the belief gap when interest rates are raised. The case of 8 quarters corresponds to the baseline scenario simulation shown in Figure 2. Panel (b) shows that the downward deviation of the belief gap, driven by the perceived i-star, becomes greater as the length of forward guidance becomes longer. This result can be expected from the lower middle panel of Figure 2, which shows that the downward deviation of the perceived i-star becomes greater non-linearly as the forward guidance is continued.

Speed of learning Panel (c) of Figure 4 shows the sensitivity of the belief gap to the perceived relative volatility of policy parameters, $\Sigma_\theta/\sigma^2_{mp}$, which govern the speed of learning.

The case of diagonal entries of $\Sigma_\theta/\sigma_{mp}^2$ of 0.4 for $\hat{i}_t^*$ and $\phi_{\pi t}$ and 0.4/100 for ϕ_{yt} corresponds to the baseline scenario result. With σ_{mp} fixed, an increase in the perceived relative volatility $\Sigma_\theta/\sigma_{mp}^2$ makes a change in perceived monetary policy parameter values more volatile and thus widens the belief gap downward, as shown in Panel (c). A change in the perceived i-star continues to be dominant and its downward deviation becomes greater as the relative volatility increases. The case of the relative volatility of 0.6 corresponds to the standard deviation of nearly 8 basis points for $\hat{i}_t^*$, which is within the estimates for the Japanese economy as mentioned in footnote 12. Hence, as shown in Panel (c), the belief gap of about minus 60 basis points, i.e., the perceived monetary tightening of about 60 basis points is not just a theoretical possibility. Moreover, if σ_{mp}^2 were perceived to be low during the ELB period, which can be the case in practice, the belief gap would widen downward further.

Persistence of the perceived i-star The persistence of the perceived i-star, ρ^*, in equation (5) also affects the belief gap. The higher the persistence, the less quickly does the perceived i-star return to the steady state. Panel (d) of Figure 4 shows that the downward belief gap widens, driven largely by the downward deviation of the perceived i-star, as the persistence parameter increases. In the case of $\rho^* = 1$, there is a significant downward contribution of the inflation coefficient. This is because the persistent i-star stimulates inflation and pushes it above its target around the last phase of the forward guidance. To reconcile it with the possibility of a notional rate below the ELB, the inflation coefficient needs to be updated downward, giving rise to the downward belief gap shown in the case of $\rho^* = 1$ in Panel (d).

Information effects of low-for-long We have focused on agents' learning about monetary policy under the implicit assumption that their perceptions about monetary policy do not affect those about the economy. However, after perceiving a change in the i-star in the monetary policy rule, the agents may perceive that the r-star in the economy might have changed as well. In Appendix C, we consider this 'information effect' of low-for-long and show that it attenuates the boom and bust in output and inflation observed in Figure 2. With the information effect put in place, a decrease in the perceived i-star in the monetary policy rule under low-for-long, which is expansionary, leads to a decrease in the perceived r-star in the economy, which is contractionary, and vice versa during the exit. If the information effect is strong enough, the economy stagnates relative to the model with no learning.

The literature is inconclusive about the information effects of monetary policy, and all the more so about the information effects of low-for-long.[16] Therefore, we keep the implicit assumption of no such effects in this paper.

3.6 Alternative scenarios

In addition to the baseline scenario, we consider three alternative scenarios: an inflationary shock scenario, a belief accommodation scenario, and a recurrent shock scenario, and assess what impact the scenarios have on the belief gap and its correction during the exit.

Inflationary shocks In light of the inflation surge of 2021–22, when the interest rate was still kept at the ELB in major jurisdictions, we consider an alternative scenario that adds to the baseline scenario inflationary supply and demand shocks. Specifically, these shocks are assumed to hit the economy unexpectedly during the period of forward guidance of keeping the interest rate at the ELB with innovations to z_t^d and z_t^s set at 0.04 percent for $t = 21, ..., 28$, where the AR(1) coefficients of the supply and demand shocks are set at 0.75. The positive demand shock increases output and inflation, while the positive supply shock decreases output but increases inflation. The combination of the two shocks is inflationary while having relatively muted effect on output.[17]

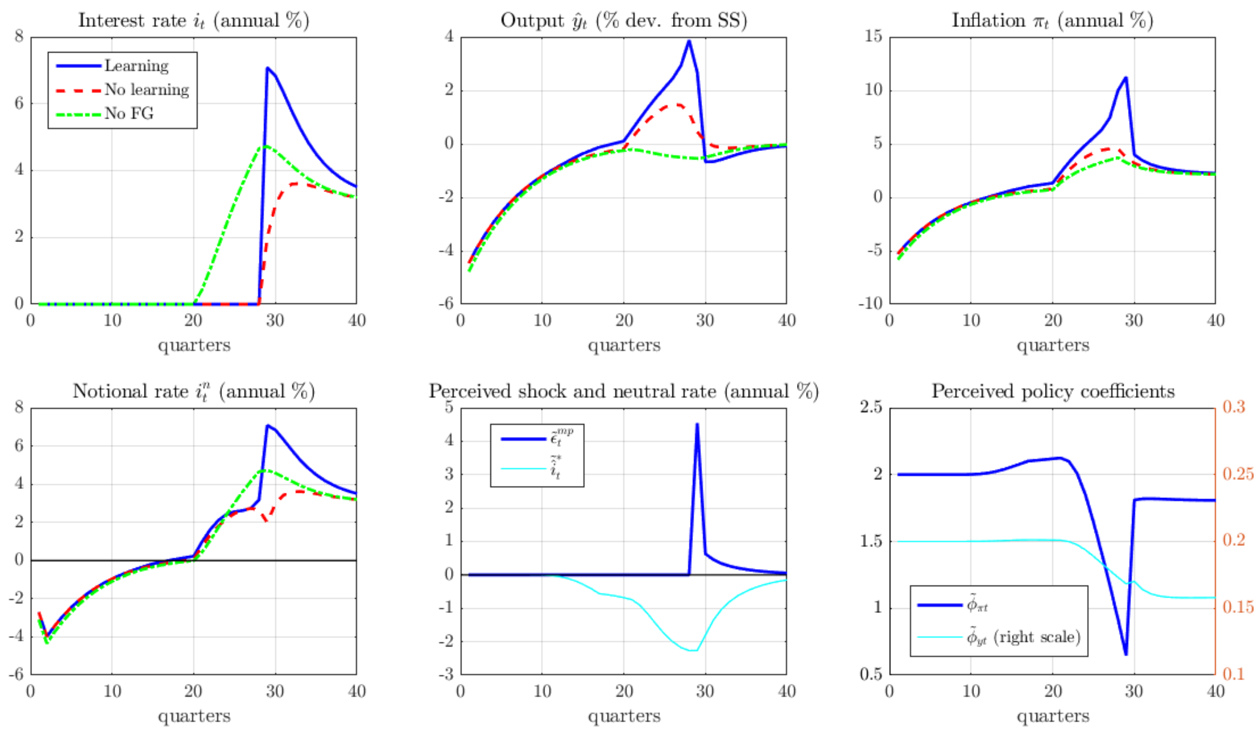
Figure 5 plots the results of the inflationary shock scenario for the model with learning ('Learning'), the model without learning ('No learning'), and the model without forward guidance and learning ('No FG'). In the case of no forward guidance, the interest rate begins to be raised in $t = 21$, and inflation surges nearly 4 percent in response to the series of the combination of supply and demand shocks, while the output hovers in the area just below its steady state level. With forward guidance, the interest rate is kept at the ELB until $T_1 = 28$. Keeping the interest rate at the ELB in spite of inflationary supply and demand shocks boosts the economy and generates a surge in inflation and output. In particular, inflation reaches nearly 5 percent as shown in the upper right panel of Figure 5.

With learning, the inflation surge is exacerbated considerably, rising above 10 percent. Although the shocks that hit the economy are the same, the inflation surge is more than

[16]For papers that support the information effects of monetary policy, see, e.g., Nakamura and Steinsson (2018) for the U.S. and Morita et al. (2025) for Japan. For those that provide an alternative view, see, e.g., Bauer and Swanson (2023) for the U.S.

[17]Our model is so stylized that it has to resort to repeated supply and demand shocks to reproduce a sharp and persistent increase in inflation and a moderate effect on output. See, Erceg et al. (2024) for an extended NK model that can explain such a phenomenon with supply shocks only.

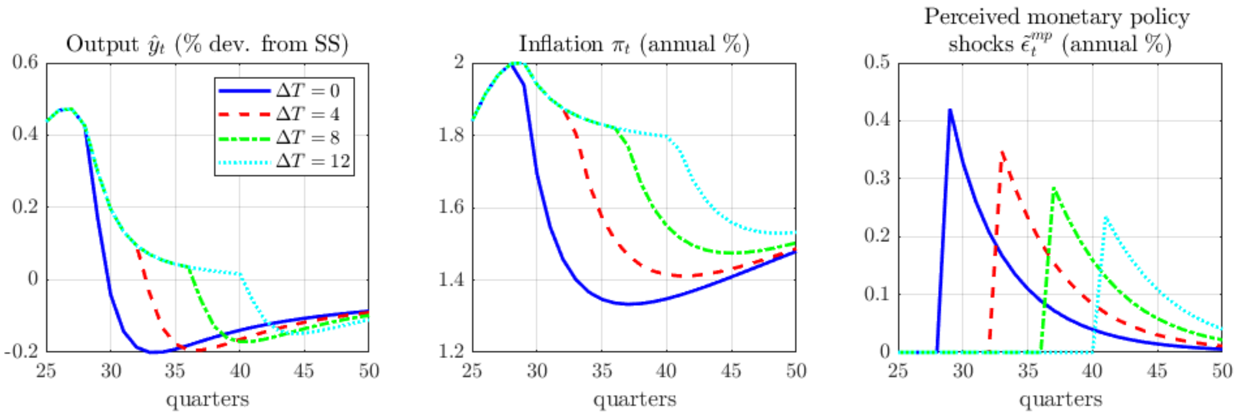
Figure 5: The inflationary shock scenario



Note: Unexpected supply and demand shocks are added from $t = 21$ to 28. 'Learning' and 'Baseline' use the model with and without learning, respectively, and 'No FG' uses the model without forward guidance and learning. The notional rate in 'Learning' is the perceived notional rate during the period of the binding ELB constraint.

doubled in the case of no learning. Observing the zero interest rate during the inflation surge, the agents update their belief in a way that lowers the perceived i-star as shown in the lower middle panel. In addition, the agents revise their belief in a way that lowers the inflation coefficient in the perceived monetary policy rule, since it is consistent with the interest rate at the ELB, which suggests a notional rate below the ELB with probability $1 - q$. With this belief, the agents expect a lower path of the nominal interest rate, which pushes inflation up further, a result that echos the forward guidance trap coined by Orphanides (2024). When exiting the low-for-long policy, the agents' belief about monetary policy is not realized and the gap between their belief and the actual monetary policy is perceived as considerable monetary tightening, as shown in the lower middle panel of Figure 5. In sum, a boom and bust arising from a correction of a gap between agents' belief and the actual policy is exacerbated if inflationary supply and demand shocks hit the economy during the period of forward guidance.

Figure 6: The belief accommodation scenario



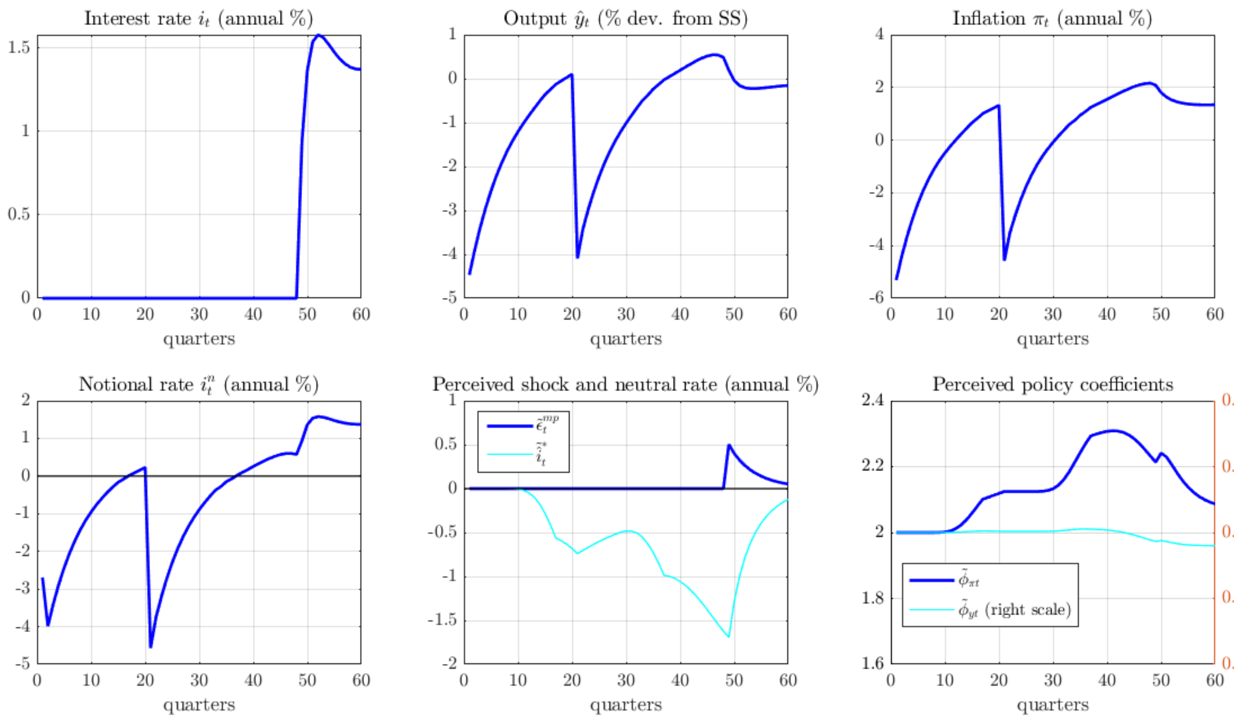
Note: ΔT^* denotes the number of periods in which agents' belief is accommodated by the central bank.

Accommodating beliefs In the baseline scenario with learning, agents' belief is corrected sharply after the end of forward guidance. In this situation, the central bank may want to consider accommodating the belief by offsetting the perceived monetary tightening with monetary easing $\epsilon_t^{mp} < 0$ to smooth the landing after interest rates are raised in $t = 29$. To articulate this scenario, suppose that the central bank sets an unexpected monetary policy shock ϵ_t^{mp} such that the perceived monetary policy shock $\tilde{\epsilon}_t^{mp}$ becomes zero in accordance with equation (8) for ΔT periods after interest rates are raised.

Figure 6 plots simulation results from $t = 25$ under the alternative scenario of accommodating beliefs for $\Delta T = 0, 4, 8, 12$. In the case of no accommodation of $\Delta T = 0$, output and inflation fall sharply as the belief gap that has widened under the low-for-long policy is corrected at the time of the lift-off. In the case of accommodations, a fall in output and inflation becomes more gradual and smoother as the length of belief accommodations gets longer from $\Delta T = 4$ to 12. Moreover, the perceived monetary tightening that comes after the end of belief accommodations becomes smaller as the central bank accommodates beliefs longer, as shown in the right panel of Figure 6. This is driven by the assumption of $\rho^* = 0.95$, which implies that agents expect the i-star in the monetary policy rule will converge to its steady state level. For example, with $\Delta T = 12$, the perceived i-star becomes about a half ($= 0.95^{12}$) of its original level at the beginning of the lift-off. Instead, if the agents believe that the i-star is more persistent, there will be less decline in the perceived monetary tightening and a less smooth landing after the lift-off.[18]

[18]To address the belief gap and better communicate policy, instead of accommodating beliefs, the central

Figure 7: The recurrent shock scenario



Note: A severe demand shock hits in $t = 21$ besides the shock in $t = 1$ assumed in the baseline scenario.

Recurrent shocks The Japanese economy went through a low-for-long environment from the late 1990s to the early 2020s due, possibly, to a series of severe negative shocks. In light of the experience, we consider a recurrent shock scenario in which a severe negative demand shock $\epsilon_t^d = -0.26$ percent hits in $t = 21$, which would induce an additional twenty periods of the binding ELB constraint without forward guidance. In response, as in the baseline scenario, we assume that the central bank introduces forward guidance of keeping the interest rate at the ELB for an additional eight periods. Except for the shock that hits in $t = 21$, the assumptions are the same as in the baseline scenario.

Figure 7 shows the simulated paths of the recurrent shock scenario. Output and inflation fall sharply and reach a second trough in $t = 21$ as the economy is hit by the second shock. This, combined with forward guidance, generates a prolonged period of the zero interest rate until $t = 48$ – twelve years after the initial shock hit. During this period, observing output, inflation, and the interest rate (at the ELB), the agents continue updating their belief about

bank may want to employ a new strategy such as a price-level targeting rule or a long-term interest rate targeting rule. However, under low-for-long, agents may be slow to learn a change in policy rules, which attenuates the intended effects of the new strategy. For papers that study learning a change in policy rules under low-for-long, see Bodenstein et al. (2022) and Krane et al. (2023).

monetary policy parameters. Specifically, after $t = 21$, the perceived i-star does not move much around $t = 35$ as the notional rate is in deep negative territory reflecting low levels of output and inflation. The perceived i-star starts to fall as the notional rate becomes close to zero. One period before the lift-off, the perceived i-star deviates downward by more than 1.5 pp, and this causes considerable monetary tightening perceived by the agents at the time of the lift-off. All in all, reflecting a correction of the belief gap accumulated during low-for-long, an additional five years of the zero interest rate period compared to the baseline scenario exacerbates a boom and bust around the lift-off of interest rates.

4 Empirical Analysis

In the previous section we have demonstrated within a NK framework that low-for-long policy induces agents to believe in lower-for-longer interest rates through learning by decreasing their perceived i-star in the monetary policy rule. In Section 4.1, we provide empirical evidence for a decline in the perceived i-star in Japan during the low interest rate environment. In Section 4.2, we examine whether the perceived i-star, estimated in Section 4.1, shows a property consistent with a learning mechanism highlighted in our theoretical model.

4.1 Estimating perceived i-star

Specifications Although it is difficult to measure perceptions directly, expectations can be extracted from market indicators or surveys. However, during the low interest rate environment of the late 1990s to the early 2020s in Japan, the short-term interest rate hovered around zero, and expected market rates, calculated from market indicators, exhibited minimal variations. To address this challenge, we utilize survey data to estimate a monetary policy rule perceived by professional forecasters over a long horizon as in Bauer et al. (2024).

To be consistent with the model presented in Section 2, consider the monetary policy rule (4):

$$i_t^n = \rho_i i_{t-1} + (1 - \rho_i)\left(i_t^* + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right) + \epsilon_t^{mp}, \tag{11}$$

where the interest rates including the i-star are expressed in terms of levels. We assume that professional forecasters form expectations about the short-term interest rate based on equation (11) and the ELB constraint (3). We further assume that if h is great enough, the the h-period ahead forecast of the interest rate is above the ELB. Under the assumption,

$\tilde{\mathbb{E}}_{jt}(i^n_{t+h}) = \tilde{\mathbb{E}}_{jt}(i_{t+h})$ holds, where $\tilde{\mathbb{E}}_{jt}(\cdot)$ is an expectation operator by professional forecaster j conditional on information in period t. Then, the expected h-period ahead interest rate can be expressed as follows:

$$\begin{aligned}\tilde{\mathbb{E}}_{jt}(i_{t+h}) =& \tilde{\mathbb{E}}_{jt}\left[\rho_i i_{t+h-1} + (1-\rho_i)(i^*_{t+h} + \phi_\pi \hat{\pi}_{t+h} + \phi_y \hat{y}_{t+h})\right] \\ &= \rho_i \tilde{\mathbb{E}}_{jt}(i_{t+h-1}) + (1-\rho_i)\tilde{\mathbb{E}}_{jt}(i^*_{t+h}) + \tilde{\mathbb{E}}_{jt}\left[(1-\rho_i)(\phi_\pi \hat{\pi}_{t+h} + \phi_y \hat{y}_{t+h})\right]. \end{aligned} \tag{12}$$

Although the expected perceived i-star $\tilde{\mathbb{E}}_{jt}(i^*_{t+h})$ is unobservable, it can be estimated if other variables—namely, $\tilde{\mathbb{E}}_{jt}(i_{t+h})$, $\tilde{\mathbb{E}}_{jt}(\hat{\pi}_{t+h})$, and $\tilde{\mathbb{E}}_{jt}(\hat{y}_{t+h})$—are observable at time t. However, the perceived i-star is inherently difficult to estimate due to its time-varying nature. Some empirical studies address this challenge by imposing statistical assumptions on the dynamics of i^*_t. In contrast, we estimate it by taking advantage of cross-sectional variations as well as time variations in survey data from professional forecasters, specifically those reported in Consensus Forecasts (CF).

From equation (12) our baseline specification for the expected short-term interest rate by forecaster j is given by:

$$\tilde{\mathbb{E}}_{jt}(i_{t+h}) = c + \rho_i \tilde{\mathbb{E}}_{jt}(i_{t+h-1}) + \Phi_\pi \tilde{\mathbb{E}}_{jt}(\hat{\pi}_{t+h}) + \Phi_y \tilde{\mathbb{E}}_{jt}(\hat{y}_{t+h}) + \epsilon_{jt}, \tag{13}$$

where ϵ_{jt} is an i.i.d. error term across j and over t. Then, we can infer the expected perceived i-star as $\widehat{\tilde{\mathbb{E}}_{jt}(i^*_{t+h})} = \widehat{c}/(1-\widehat{\rho_i})$, where a variable with $\widehat{\cdot}$ denotes its corresponding estimated value. We capture time variations in c using rolling regressions.

Data We use Consensus Forecasts (CF), which provides forecasts for GDP growth rates and CPI inflation rates on a calendar-year basis as well as 3-month interest rates for 3 and 12 months ahead. From these data we construct $\tilde{\mathbb{E}}_{jt}(i_{t+h})$, $\tilde{\mathbb{E}}_{jt}(i_{t+h-1})$, $\tilde{\mathbb{E}}_{jt}(\hat{\pi}_{t+h})$, and $\tilde{\mathbb{E}}_{jt}(\hat{y}_{t+h})$ to estimate equation (13). The number of leads for the interest rate is set at $h = 12$ months. For $\tilde{\mathbb{E}}_{jt}(i_{t+h-1})$, we use forecasts for the interest rate after three months. The forecasting horizon for inflation rates and GDP growth rates is set at the following year. For example, if forecasts are made in January 2024, then forecasts for 2025 calendar year are used.

To construct $\tilde{\mathbb{E}}_{jt}(\hat{\pi}_{t+h})$ and $\tilde{\mathbb{E}}_{jt}(\hat{y}_{t+h})$, which are in terms of deviations from steady state values in the theoretical model, we need to pay attention to what in the data corresponds to the steady state values. The steady state inflation rate is equal to the central bank's target rate of inflation, which is set at 1 percent before January 2013 and at 2 percent thereafter.[19] Interpreting $\hat{y}_t$ as an output gap, we construct forecasts made in t for the output gap in the target year T_t as follows.

$$\tilde{\mathbb{E}}_{jt}(\hat{y}_{T_t}) = (1 + \hat{y}_{T_t-1}) \cdot \frac{1 + \tilde{\mathbb{E}}_{jt}(g_{T_t})}{1 + g^*_{T_{t-1}}} - 1, \tag{14}$$

where $\tilde{\mathbb{E}}_{jt}(g_{T_t})$ denotes forecasts for the GDP growth rate for year T_t by forecaster j. For the lagged output gap $\hat{y}_{T_{t-1}}$ and the lagged potential growth rate $g^*_{T_{t-1}}$, we use those estimated by the Bank of Japan. The underlying assumption in (14) is that forecasters expect the potential growth rate to remain unchanged from year T_{t-1} to year T_t.

Since the frequency of the data is monthly and the forecast horizon is next calendar year for inflation and GDP, the timing of forecasts needs to be taken into account in estimating equation (13). Let m_t denote the month when the forecasts are made and let $\mathbb{I}(m_t = m)$ denote an indicator function that takes a value of one if the month is $m \in \{1, ..., 12\}$. With the constructed data on hand, we estimate a modified version of equation (13), given by

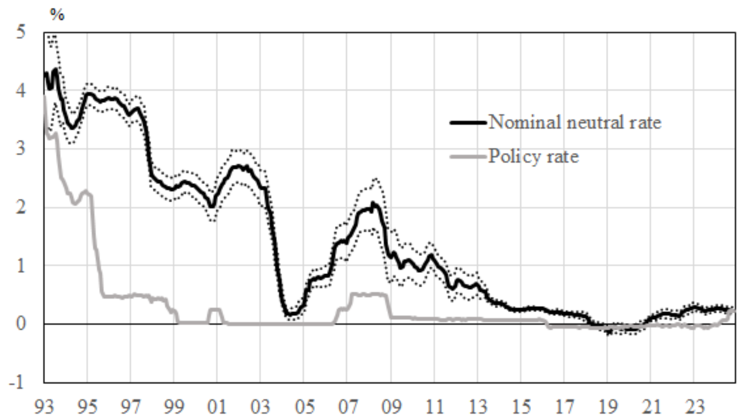
$$\begin{aligned} \tilde{\mathbb{E}}_{jt}(i_{t+12}) = c + \rho_i \tilde{\mathbb{E}}_{jt} i_{t+3} + \sum_{m=1}^{12} \mathbb{I}(m_t = m) \cdot \Phi_{\pi,m} \tilde{\mathbb{E}}_{jt}(\hat{\pi}_{T_t}) \\ + \sum_{m=1}^{12} \mathbb{I}(m_t = m) \cdot \Phi_{y,m} \tilde{\mathbb{E}}_{jt}(\hat{y}_{T_t}) + \epsilon_{jt}, \end{aligned} \tag{15}$$

This regression specification implies that if the survey is conducted in January 2024 (i.e., t = Jan. 2024), for example, then $m = 1$, and only $\Phi_{\pi,1}$ is active as the coefficient on the inflation rate in regressing $\tilde{\mathbb{E}}_{jt}(i_{t+12})$. The variation in the coefficients on the inflation rate and output gap across the months within a year helps account for differences in forecast horizons.

Estimation results We estimate the empirical specification (15) using a pooled ordinary least squares (OLS) regression with a rolling window of 36 months, spanning January 1990 to November 2024. The coefficient on the lagged short-term rate, ρ_i, is assumed to be 0.85

[19] The Bank of Japan set the price stability target at 2 percent in terms of the year-on-year rate of changes in the consumer price index in January 2013 as stated in "Joint Statement of the Government and the Bank of Japan on Overcoming Deflation and Achieving Sustainable Economic Growth" (`https://www.boj.or.jp/en/mopo/mpmdeci/mpr_2013/k130122c.pdf`). Prior to January 2013, the Bank set no explicit target, but aimed at 1 percent as of February 2012 as stated in "The Price Stability Goal in the Medium o Long Term" (`https://www.boj.or.jp/en/mopo/mpmdeci/mpr_2012/k120214b.pdf`).

Figure 8: Estimation results of the perceived nominal neutral rates



Note: The figure shows the estimated (expected) nominal neutral rate calculated as $\hat{c}/(1-\rho_i)$ by assuming $\rho_i = 0.85$, based on equation (15). The solid and dotted black lines indicate a point estimate and 90% confidence intervals, respectively. The gray line indicates the data on the short-term policy rate.

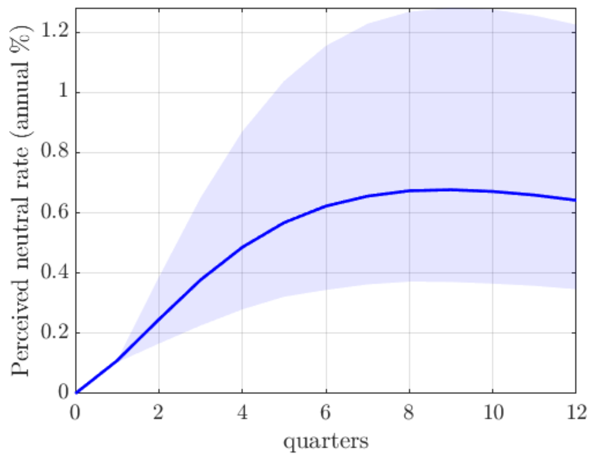
and constant over time.[20]

Figure 8 shows the estimated i-star in the perceived monetary policy rule. It shows a secular decline in the i-star during the sample period, albeit with some fluctuations. The estimated i-star starts declining gradually in the late 1990s and falls sharply in the early 2000s. The decline coincide with the BOJ's introduction of a zero-interest rate policy and forward guidance. The estimated i-star bottoms out in 2004 and reaches 2% in 2008. After the GFC, the estimated i-star continues to decline and reaches zero percent against the background of the prolonged period of low interest rates. This gradual and secular decline aligns with the scenario simulation results presented in Section 3: as a low interest rate environment is prolonged, agents update their beliefs to expect interest rates to stay lower for longer, driven by a decline in the perceived i-star in the monetary policy rule.

Figure 8 shows that the estimated i-star starts to rise gradually after 2021. Yet, in November 2024, it remains below 0.5 percent when the actual policy rate was 0.25 percent, whereas at the end of 2006, it is around 1.5 percent with the same policy rate of 0.25 percent. This contrast suggests that the perceived i-star remains low and there may be room for corrections going forward.

[20]The full-sample estimation yields an estimate of ρ_i equal to 0.88.

Figure 9: Impulse responses of the perceived neutral rate to perceived monetary policy shocks



Note: The solid line and the shaded area indicate the median and the 90 percent confidence intervals of the impulse responses of the perceived i-star to a perceived monetary policy shock that hits in $t = 0$ in the theoretical model with learning, where the shock is normalized to 1 percent annually. The impulse responses are drawn for 1,000 times by generating random shocks to $\epsilon_t^{mp} \sim N(0, \sigma_{mp}^2)$ and the prior mean of the i-star, where the distribution of the prior mean is set to the stationary distribution implied by the stochastic process (5).

4.2 Impulse responses of the perceived neutral rates

The model's prediction The empirical results in Section 4.1 show a secular decline in the perceived i-star in the monetary policy rule, but are silent about a mechanism that could underline the secular decline. We examine whether the perceived i-star, estimated in Section 4.1, is consistent with a learning mechanism, highlighted by the theoretical model in Section 2.

We focus on the impulse response of the perceived i-star to a monetary policy surprise, because the theoretical model has contrasting predictions about it depending on the presence of learning. With no learning, the model predicts no response. With learning, however, the model predicts a positive response. In the model, a monetary policy surprise corresponds to a perceived monetary policy shock $\tilde{\epsilon}_t^{mp}$, which consists of two components: a random deviation from the rule ϵ_t^{mp} and a gap between the perceived and actual systematic parts of the rules $\Delta f_t \equiv f(\mathbf{x}_t; \boldsymbol{\theta}_t) - f(\mathbf{x}_t; \tilde{\boldsymbol{\theta}}_{t|t-1})$. Be it $\epsilon_t^{mp} > 0$ or $\Delta f_t > 0$ that drives a positive perceived monetary policy shock, it induces agents to update their belief in a way that raises the perceived i-star, because agents attribute $\tilde{\epsilon}_t^{mp} > 0$ partly to their prior beliefs and think that they have perceived monetary tightening due partly to a low value of their prior mean of the i-star.

Figure 9 plots the impulse responses of the perceived i-star to perceived monetary policy shocks, which are computed from the model with learning by generating random shocks to ϵ_t^{mp} and the prior mean of the i-star. The model predicts that the responses are positive with 90 percent confidence intervals (the shaded area). In $t = 0$ when a shock hits, the perceived i-star is unchanged because it is pre-determined by assumption. From $t = 1$ on, it shows positive responses with some degree of variations, reflecting the different sources of the shocks, namely, ϵ_t^{mp} and Δf_t. Specifically, when Δf_t is dominant, the impulse responses show a high degree of persistence, as observed in Figure 9.

Specifications To validate the model's prediction, we investigate the relationship between monetary policy surprises and the perceived i-star by extending the empirical analysis of Bauer et al. (2024). For notional simplicity, let $\tilde{i}_t^*$ denote the perceived i-star in period t, estimated in Section 4.1.[21] We estimate a local projection for k-period ahead perceived i-star as follows:

$$\tilde{i}_{t+k}^* = \alpha_k + \beta_k \tilde{i}_{t-1}^* + \gamma_k MP_t + e_{t+k} \tag{16}$$

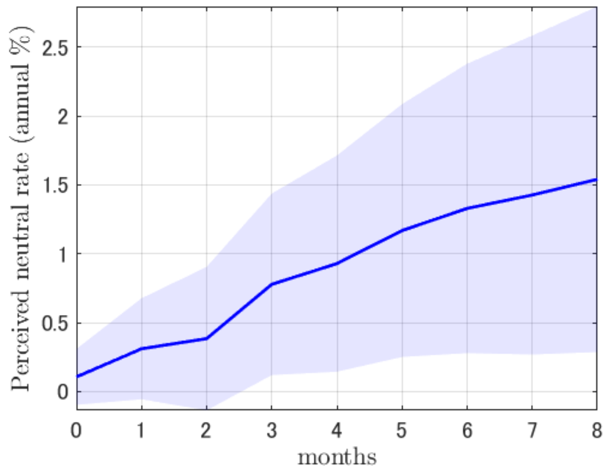
where MP_t indicates a monetary policy surprise. Following Swanson (2021, 2023), we use changes in the one-year Japanese government bond yield before and after days of monetary policy meetings and the governor's speeches and testimony in National Diet as a monetary policy surprise. To focus on the low-interest rate environment, we use the sample from January 2002 to November 2024.

Estimation results Figure 10 presents the estimation results. In response to monetary policy tightening (an unexpected increase in the JGB yield), the perceived i-star increases with some lags. The effect becomes statistically significant after three months, supporting the view that agents sort out monetary policy shocks and revise their perceived monetary policy rule through learning, as in our theoretical model.

To summarize, the empirical analysis supports the idea that the i-star in the perceived monetary policy rule might have declined during the prolonged period of low interest rates as agents have learned the central bank's stance on monetary policy from observed data. The estimated i-star is still at a low level at the end of the sample period of November 2024. Thus, there may be room for possible corrections of belief gaps. The local projection results shown in Figure 10 suggest that belief gaps can be a source of perceived monetary policy tightening

[21] If we follow the notations used in the specification (12), $\tilde{i}_t^*$ corresponds to $\tilde{\mathbb{E}}_{jt}(i_{t+12}^*)$, the expected i-star 12-month ahead conditional on information in period t.

Figure 10: Estimated impact of monetary policy shocks on the nominal neutral rate



Note: The figure shows the estimated impact of a one-percentage-point surprise increase in the one-year Japanese government bond yield. The x-axis represents the horizon in the local projection analysis. The solid line denotes the point estimate, while the shaded area indicates 90% confidence intervals.

as the perceived i-star is corrected in line with the theoretical model. Moreover, any such monetary policy tightening will have negative effects on the macroeconomic variables as well as on the financial variables, as empirically studied by e.g. Kubota and Shintani (2022, 2024), Ikeda et al. (2024), and Nakamura et al. (2024) for Japan.

5 Conclusion

This paper studies a new channel of low-for-long interest rates by building a NK model with learning, the ELB, and forward guidance with imperfect credibility. It highlights a possible challenge facing monetary policy in normalizing from low-for-long interest rates. During a low-inflation regime, low-for-long policy supports the economy by inducing agents to believe that interest rates will be kept lower and longer. When the time comes and the interest rate is raised, the correction of the belief has contractionary effects on the economy. While low-for-long policy is expansionary during a low-inflation regime, unwinding the policy toward a normal regime precipitates surprise monetary tightening through the correction of the belief that has grown under low-for-long policy. We study mechanisms behind this potential channel of monetary policy and how, under various assumptions and scenarios, the belief gap evolves during the ELB period and during the subsequent exit. We also provide some empirical evidence that supports the practical relevance of this channel.

References

Abadi, J., M. Brunnermeier, and Y. Koby (2023, August). The reversal interest rate. *American Economic Review 113*(8), 2084–2120.

Acharya, V. V., R. Chauhan, R. G. Rajan, and S. Steffen (2022). Liquidity dependence: Why shrinking central bank balance sheets is an uphill task. *Proceedings of the Jackson Hole Economic Symposium of the Federal Reserve Bank of Kansas City on "Reassessing Constraints on the Economy and Policy."*.

Acharya, V. V. and R. Rajan (2024). Liquidity, liquidity everywhere, not a drop to use: Why flooding banks with central bank reserves may not expand liquidity. *The Journal of Finance 79*(5), 2943–2991.

Altavilla, C., W. Lemke, T. Linzert, J. Tapking, and J. von Landesberger (2021). Assessing the efficacy, efficiency and potential side effects of the ECB's monetary policy instruments since 2014. *European Central Bank Occasional Paper Series*.

Aruoba, S. B., P. Cuba-Borda, and F. Schorfheide (2017, 04). Macroeconomic dynamics near the ZLB: A tale of two countries. *The Review of Economic Studies 85*(1), 87–118.

Bank of Japan (2024). Review of monetary policy from a broad perspective.

Batista, Q., T. Nakata, and T. Sunakawa (2023). Credible forward guidance. *Journal of Economic Dynamics and Control 153*, 104699.

Bauer, M. D., C. E. Pflueger, and A. Sunderam (2024, 06). Perceptions about monetary policy. *The Quarterly Journal of Economics 139*(4), 2227–2278.

Bauer, M. D. and E. T. Swanson (2023, March). An alternative explanation for the "fed information effect". *American Economic Review 113*(3), 664–700.

Bilbiie, F. O. (2024). Monetary policy and heterogeneity: An analytical framework. *Review of Economic Studies*. forthcoming.

Bocola, L., A. Dovis, K. Jørgensen, and R. Kirpalani (2024). Bond market views of the fed. *National Bureau of Economic Research Working Paper Series*.

Bodenstein, M., J. Hebden, and R. Nunes (2012). Imperfect credibility and the zero lower bound. *Journal of Monetary Economics 59*(2), 135–149.

Bodenstein, M., J. Hebden, and F. Winkler (2022). Learning and misperception of makeup strategies. *Journal of Economic Dynamics and Control 139*, 104417.

Boneva, L., R. Harrison, and M. Waldron (2018). Threshold-based forward guidance. *Journal of Economic Dynamics and Control 90*, 138–155.

Borio, C., L. Gambacorta, and B. Hofmann (2017). The influence of monetary policy on bank profitability. *International Finance 20*(1), 48–63.

Braun, R. A. and D. Ikeda (2022). Why aging induces deflation and secular stagnation. *mimeo*.

Campbell, J. R., J. D. M. Fisher, A. Justiniano, and L. Melosi (2017). Forward guidance and macroeconomic outcomes since the financial crisis. *NBER Macroeconomics Annual 31*, 283–357.

Carvalho, C., S. Eusepi, E. Moench, and B. Preston (2023, January). Anchored inflation expectations. *American Economic Journal: Macroeconomics 15*(1), 1–47.

Carvalho, C. and F. Nechio (2014). Do people understand monetary policy? *Journal of Monetary Economics 66*, 108–123.

Christiano, L. J., M. Eichenbaum, and B. K. Johannsen (2024). Slow learning. *mimeo*.

Claessens, S., N. Coleman, and M. Donnelly (2018). "low-for-long" interest rates and banks' interest margins and profitability: Cross-country evidence. *Journal of Financial Intermediation 35*, 1–16.

Cogley, T., C. Matthes, and A. M. Sbordone (2015). Optimized taylor rules for disinflation when agents are learning. *Journal of Monetary Economics 72*, 131–147.

Cogley, T. and A. M. Sbordone (2008, December). Trend inflation, indexation, and inflation persistence in the new Keynesian Phillips curve. *American Economic Review 98*(5), 2101–26.

Coibion, O., D. Georgarakos, Y. Gorodnichenko, and M. Weber (2023, 01). Forward guidance and household expectations. *Journal of the European Economic Association 21*(5), 2131–2171.

Coyle, P., N. Maezono, T. Nakata, and S. Schmidt (2025). Deflationary equilibrium with uncertainty. *mimeo*.

Debortoli, D., J. Galí, and L. Gambetti (2019). On the empirical (ir)relevance of the zero lower bound constraint. In *NBER Macroeconomics Annual 2019, volume 34*. University of Chicago Press.

Del Negro, M., M. P. Giannoni, and C. Patterson (2023). The forward guidance puzzle. *Journal of Political Economy Macroeconomics 1*(1), 43–79.

Erceg, C., J. Lindé, and M. Trabandt (2024). Monetary policy and inflation scares. *mimeo*.

Erceg, C. J. and A. T. Levin (2003). Imperfect credibility and inflation persistence. *Journal of Monetary Economics 50*(4), 915–944.

Fendel, R., M. Frenkel, and J.-C. Rülke (2011). 'ex-ante'taylor rules–newly discovered evidence from the g7 countries. *Journal of Macroeconomics 33*(2), 224–232.

Gabaix, X. (2020, August). A behavioral new Keynesian model. *American Economic Review 110*(8), 2271–2327.

Guerrieri, L. and M. Iacoviello (2015). Occbin: A toolkit for solving dynamic models with occasionally binding constraints easily. *Journal of Monetary Economics 70*, 22–38.

Gust, C., E. Herbst, and D. López-Salido (2018). Forward guidance with bayesian learning and estimation. Finance and Economics Discussion Series 2018-072, Washington: Board of Governors of the Federal Reserve System.

Hamilton, J. D., S. Pruitt, and S. Borger (2011). Estimating the market-perceived monetary policy rule. *American Economic Journal: Macroeconomics 3*(3), 1–28.

Hayashi, F. and J. Koeda (2019). Exiting from QE. *Quantitative Economics 10*, 1069–1107.

Hiraki, K. and W. Hirata (2020). Market-based long-term inflation expectations in Japan: A refinement on breakeven inflation rates. Working Paper Series 20-E-5, Bank of Japan.

Hirose, Y. (2020). An estimated DSGE model with a deflation steady state. *Macroeconomic Dynamics 24*(5), 1151–1185.

Hirose, Y., H. Iiboshi, M. Shintani, and K. Ueda (2024). Estimating a behavioral new Keynesian model with the zero lower bound. *Journal of Money, Credit and Banking 56*(8), 2185–2197.

Hogen, Y. and R. Okuma (2025). The anchoring of inflation expectations in Japan: A learning-approach perspective. *Japan and the World Economy 73*, 101293.

Iiboshi, H., M. Shintani, and K. Ueda (2022). Estimating a nonlinear new Keynesian model with the zero lower bound for Japan. *Journal of Money, Credit and Banking 54*(6), 1637–1671.

Ikeda, D. and T. Kurozumi (2019, October). Slow post-financial crisis recovery and monetary policy. *American Economic Journal: Macroeconomics 11*(4), 82–112.

Ikeda, D., S. Li, S. Mavroeidis, and F. Zanetti (2024, April). Testing the effectiveness of unconventional monetary policy in Japan and the United States. *American Economic Journal: Macroeconomics 16*(2), 250–86.

Kaihatsu, S. and T. Kurozumi (2014). Sources of business fluctuations: Financial or technology shocks? *Review of Economic Dynamics 17*(2), 224–242.

Krane, S., L. Melosi, and M. Rottner (2023). Learning monetary policy strategies at the effective lower bound with sudden surprises. *Federal Reserve Bank of Chicago Working Paper 2023-22*.

Kreps, D. M. (1998). Anticipated utility and dynamic choice. In D. P. Jacobs, E. Kalai, and M. I. Kamien (Eds.), *Frontiers of research in economic theory : the Nancy L. Schwartz Memorial Lectures, 1983-1997.* : Cambridge University Press.

Kubota, H. and M. Shintani (2022). High-frequency identification of monetary policy shocks in Japan. *The Japanese Economic Review 73*(3), 483–513.

Kubota, H. and M. Shintani (2024). Macroeconomic effects of monetary policy in Japan: an analysis using interest rate futures surprises. *Empirical Economics*.

McKay, A., E. Nakamura, and J. Steinsson (2016, October). The power of forward guidance revisited. *American Economic Review 106*(10), 3133–58.

Mele, A., K. Molnár, and S. Santoro (2020). On the perils of stabilizing prices when agents are learning. *Journal of Monetary Economics 115*, 339–353.

Morita, H., R. Matsumoto, and T. Ono (2025). Central bank information effects in Japan: the role of uncertainty channel. *Empirical Economics 68*, 855–877.

Nakamura, E. and J. Steinsson (2018, 01). High-frequency identification of monetary non-neutrality: The information effect*. *The Quarterly Journal of Economics 133*(3), 1283–1330.

Nakamura, F., N. Sudo, and Y. Sugisaki (2024). Assessing monetary policy surprises in japan by high frequency identification. *Journal of the Japanese and International Economies 71*, 101300.

Nakano, S., Y. Sugioka, and H. Yamamoto (2024). Recent developments in measuring natural rate of interest. Working Paper Series 24-E-12, Bank of Japan.

Orphanides, A. (2024). The Forward Guidance Trap. *Monetary and Economic Studies 42*.

Reifschneider, D. and J. C. Williams (2000). Three lessons for monetary policy in a low-inflation era. *Journal of Money, Credit and Banking 32*(4), 936–966.

Rungcharoenkitkul, P. and F. Winkler (2023). The natural rate of interest through a hall of mirrors. *mimeo*.

Schmidt, S. and D. Nautz (2012). Central bank communication and the perception of monetary policy by financial market experts. *Journal of Money, Credit and Banking 44*(2-3), 323–340.

Sugo, T. and K. Ueda (2008). Estimating a dynamic stochastic general equilibrium model for Japan. *Journal of the Japanese and International Economies 22*(4), 476–502.

Swanson, E. T. (2021). Measuring the effects of federal reserve forward guidance and asset purchases on financial markets. *Journal of Monetary Economics 118*, 32–53.

Swanson, E. T. (2023). The importance of fed chair speeches as a monetary policy tool. In *AEA papers and proceedings*, Volume 113, pp. 394–400. American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203.

Tetlow, R. J. and P. von zur Muehlen (2001). Simplicity versus optimality: The choice of monetary policy rules when agents must learn. *Journal of Economic Dynamics and Control 25*(1), 245–279. Computational Methods in Economic Dynamics and Finance.

Appendix

A Model

To make the paper self-contained we derive the log-linearized equations presented in the main text from the original equilibrium conditions. We first present the standard part of the model in Appendix A.1, discuss the source of additional discounting in Appendix A.2, and study the basic properties of the model in Appendix A.3.

A.1 The standard model

Households A representative household maximizes the utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t d_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} \right],$$

subject to the flow budget constraint

$$P_t c_t + B_t/(1+i_t) \leq W_t \ell_t + B_{t-1} + \Theta_t,$$

where c_t is consumption, ℓ_t is labor supply, P_t is the price level, B_t is the amount of nominal bonds in face value with the gross interest rate R_t, W_t is the nominal wage, and Θ_t consists of cash flows from firm profits. The variable d_t captures a composite shock to preferences, given by

$$d_t = \begin{cases} \exp(z_1^{*d} + z_2^{*d} + ... + z_t^{*d}) & \text{for } t = 1, 2, ... \\ 1 & \text{for } t = 0 \end{cases}$$

where z_t^{*d} is a preference (or demand) shock, which follows an AR(1) process

$$z_t^{*d} = \rho_d z_{t-1}^{*d} + \epsilon_t^{*d}.$$

Let Λ_t denote a Lagrange multiplier on the budget constraint in period t. The first-order conditions with respect to C_t, B_t, and ℓ_t are given, respectively, by

$$\begin{aligned} P_t \Lambda_t &= c_t^{-\sigma}, \\ d_t \Lambda_t &= \beta \mathbb{E}_t d_{t+1} \Lambda_{t+1} (1+i_t), \\ \varphi \ell_t^{\nu} &= \Lambda_t W_t. \end{aligned}$$

Define $\pi_t = P_t/P_{t-1}$ as the gross inflation rate. By substituting out Λ_t, the equations above can be written as the Euler equation and the labor supply curve, respectively:

$$c_t^{-\sigma} = \beta \mathbb{E}_t e^{z_{t+1}^d} c_{t+1}^{-\sigma} (1+i_t)/\pi_{t+1}, \quad \text{(A.1)}$$

$$w_t = \varphi c_t^{\sigma} \ell_t^{\nu}. \quad \text{(A.2)}$$

Final good firms A representative final good firm produces y_t by combining intermediate goods $\{y_t(i)\}_{i=0}^1$ according to

$$y_t = \left[\int_0^1 y_t(i)^{\frac{1}{\lambda_{p,t}}} di \right]^{\lambda_{p,t}}. \quad \text{(A.3)}$$

Let $P_t(i)$ denote the price of the i-th intermediate good. The firm maximizes profits $P_t y_t - \int_0^1 P_t(i) y_t(i) di$, taking prices as given. The solution yields demand for the i-th intermediate good, given by

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\lambda_{p,t}}{\lambda_{p,t}-1}} y_t. \tag{A.4}$$

The final good is used for consumption only, so that the good market clearing condition is given by $y_t = c_t$.

Intermediate goods firms There is a continuum of intermediate goods firms, each indexed by i. Each firm uses labor to produce an intermediate good according to

$$y_t(i) = l_t(i),$$

In every period, intermediate goods firms have an opportunity to reset their prices with probability $1 - \xi_p$. Consider the i-th intermediate good firm that has an opportunity to reset its price at $\tilde{P}_t$ in period t. It maximizes the discounted sum of profits,

$$\max_{\{\tilde{P}_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Lambda_{t,t+s} \left[P_{t+s}(i) y_{t+s}(i) - W_{t+s} y_{t+s}(i)\right],$$

subject to the demand curve (A.4), $P_{t+s}(i) = \pi P_{t+s-1}(i)$, and $P_t(i) = \tilde{P}_t$. Substituting equation (A.4) into the problem yields

$$\max_{\{\tilde{P}_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Lambda_{t,t+s} \left(\pi^s \tilde{P}_t - W_{t+s}\right) \left(\frac{\pi^s \tilde{P}_t}{P_{t+s}}\right)^{-\frac{\lambda_{p,t+s}}{\lambda_{p,t+s}-1}} y_t,$$

The first-order condition is

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \frac{P_{t+s} \Lambda_{t,t+s}}{\lambda_{p,t+s} - 1} \left[-\tilde{\Pi}^p_{t,t+s} \tilde{p}_t + \lambda_{p,t+s} w_{t+s}\right] \left(\tilde{\Pi}^p_{t,t+s} \tilde{p}_t\right)^{-\frac{\lambda_{p,t+s}}{\lambda_{p,t+s}-1}} y_{t+s}, \tag{A.5}$$

where $\tilde{p}_t = \tilde{P}_t / P_t$ and $\tilde{\Pi}^p_{t,t+s} = \pi^s / (P_{t+s}/P_t)$.

The price index is given by $P_t = \left[\int_0^1 P_t(i)^{1/(1-\lambda_{p,t})} di\right]^{1-\lambda_{p,t}}$ and it evolves following

$$P_t = \left\{\xi_p \left(\pi P_{t-1}\right)^{\frac{1}{\lambda_{p,t}-1}} + (1 - \xi_p) \tilde{P}_t^{\frac{1}{\lambda_{p,t}-1}}\right\}^{1-\lambda_{p,t}}$$

This equation can be written as

$$1 = \xi_p \left(\pi / \pi_t\right)^{\frac{1}{\lambda_{p,t}-1}} + (1 - \xi_p) \tilde{p}_t^{\frac{1}{\lambda_{p,t}-1}} \tag{A.6}$$

Government The government consists of a fiscal authority and a central bank. The fiscal authority issues nominal government bonds with zero net supply. The central bank sets the nominal interest rate i_t following a monetary policy rule with the effective lower bound (ELB) constraint:

$$i_t = \max\{i_t^n, \underline{i}\}, \tag{A.7}$$

$$\log\left(\frac{1 + i_t^n}{1 + i}\right) = \rho_r \log\left(\frac{1 + i_{t-1}}{1 + i}\right) + (1 - \rho_i) \left[i_t^* + \phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_y \log\left(\frac{y_t}{y}\right)\right] + \epsilon_t^{mp}, \tag{A.8}$$

where i_t^n is the notional net nominal interest rate and $\underline{i}$ is the ELB.

Log-linearization Since $c_t = y_t$, the Euler equation (C.1) can be log-linearized as

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) - \frac{\rho_d}{\sigma} z_t^{*d}.$$

This equation is the same as equation (1) with $M = 1$ in the main text if we set $z_t^d = -(\rho_d/\sigma) z_t^{*d}$. Log-linearizing equation (A.5) around steady state yields

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \left(-\hat{\Pi}_{t,t+s}^p - \hat{\tilde{p}}_t + \hat{w}_{t+s} + \hat{\lambda}_{p,t+s}\right)$$

Solving for $\hat{\tilde{p}}_t$ yields

$$\frac{1}{1-\beta\xi_p} \hat{\tilde{p}}_t = \hat{w}_t + \hat{\lambda}_{p,t} + \frac{\beta \xi_p}{1-\beta\xi_p} \mathbb{E}_t \left(\hat{\tilde{p}}_{t+1} + \hat{\pi}_{t+1}\right)$$

Log-linearizing equation (A.6) yields

$$0 = -\xi_p \hat{\pi}_t + (1-\xi_p)\hat{\tilde{p}}_t.$$

Combining the equations above, we obtain

$$\hat{\pi}_t = \kappa_p \hat{w}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_p \hat{\lambda}_{p,t}.$$

where $\kappa_p = (1-\xi_p)(1-\beta\xi_p)/\xi_p$. Note that $y_t = \ell_t = c_t$. Log-linearzing equation (A.2) yields

$$\hat{w}_t = (\sigma + \nu)\hat{y}_t.$$

Combining the two equations above yields equation (2) in the main text if we set $\kappa = \kappa_p(\sigma + \nu)$ and $z_t^s = \kappa_p \hat{\lambda}_{p,t}$. It is straightforward to see that monetary policy rules (3) and (4) in the main text can be derived from equations (A.7) and (A.8).

A.2 Additional discounting

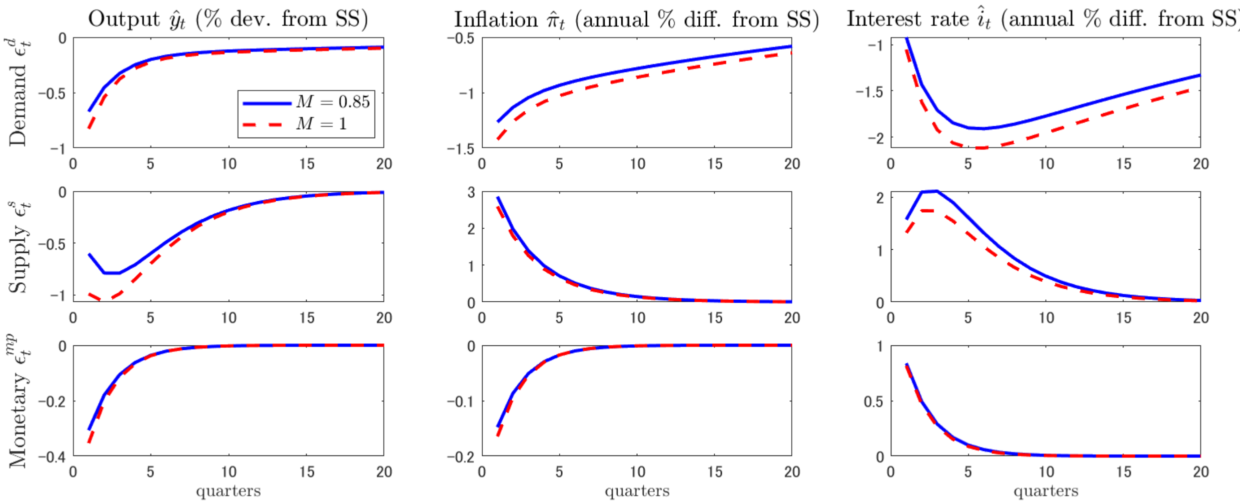
The additional discounting $M < 1$ attached to the log-linearized Euler equation (1) in the main text can be derived from two theories. First, Gabaix (2020) consider households' cognitive discounting of the future state variables and M measures attention to the future. Although $\mathbb{E}_t \hat{y}_{t+1}$ is discounted, as in Gabaix (2020) we assume that that the contemporaneous real interest rate, $\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}$, is correctly perceived, that is, the rational Fisher equation holds in the agents' perception of the world.

Second, Bilbiie (2024) studies a tractable HANK model with two types of households – savers and hand-to-mouth – whose type change stochastically and derives the aggregated Euler equation in the form of equation (1). He shows that $M < 1$ if income inequality between savers and hand-to-mouth is procylical, that is, if the income of savers relative to hand-to-mouth is procyclical. Intuitively, consider an exogenous increase in the output in the next period $\hat{y}_{t+1}$. Since income inequality is procyclical, it means lower relative income for hand-to-mouth in the next period. Households recognize the possibility of becoming hand-to-mouth in the next period and there arises demand for savings for self-insurance. But, in equilibrium there is zero saving since there is no supply of assets. The effect of an increase in $\hat{y}_{t+1}$ on $\hat{y}_t$ is muted and thus discounted to make saving zero in equilibrium.

A.3 Basic properties

Impulse responses in a non-ELB regime Figure A.1 plots impulse responses to the demand policy shock ($\epsilon_t^d = -0.25$ percent), the supply shock ($\epsilon_t^s = 0.25$ percent), and the monetary policy shock ($\epsilon_t^{mp} = 0.25$ percent), in the case of with and without additional discounting, respectively. In response to the negative demand shock, output decreases more persistently in the model with additional discounting than in the standard model without it. Inflation decreases and it is less responsive in the model with additional discounting than in the standard model. Reflecting the inflation responses, the policy interest rate drops less in the model with cognitive discounting.

Figure A.1: The impulse responses under a non-ELB regime



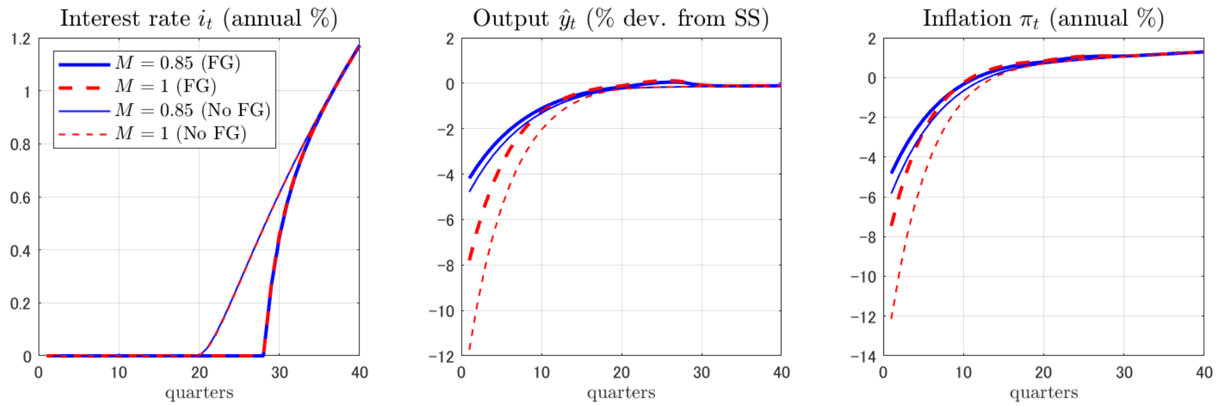
Note: '$M = 0.85$' corresponds to the model with $M = 0.85$ and '$M = 1$' corresponds to the model with $M = 1$.

In response to the inflationary supply shock, inflation rises similarly in the two models, but the decrease in output is less severe in the model with additional discounting than in the standard model. The inflationary supply shock leads to monetary tightening or a rise in the real interest rate, and equation (1) implies that the current output is given by minus the sum of discounted future real interest rates if the shock is ignored and that the future real interest rates are more heavily discounted in the model with additional discounting.

Finally, in response to the tightening shock to monetary policy, the responses of output, inflation, and the interest rate are more or less similar between the two models, although the responses of output and inflation are less responsive in the model with additional discounting.

Role of additional discounting in the ELB simulation Figure A.2 plots the baseline scenarios with and without forward guidance for the model with no additional discounting $M = 1$ as well as those for the model with $M = 0.85$. For the model with $M = 1$, the magnitude of the initial shock is set to generate interest rates at the ELB for 20 periods. The paths of interest rates are virtually indifferent between the two models as shown in the left panel. However, in the model with $M = 1$, output and inflation decrease much more than those in the model with $M = 0.85$. Specifically, a decrease in inflation to -12 annual percent would be too severe to be observed in practice. Moreover, in the model with $M = 1$, forward guidance becomes extremely powerful as shown in the middle and right panels. Hence, the presence of $M < 1$ helps the model generate practical responses of output and inflation as well as moderate positive effects of forward guidance.

Figure A.2: Role of additional discounting in the baseline scenario



Note: 'FG' denotes forward guidance. '$M = 0.85$' and '$M = 1$' indicate the model with $M = 0.85$ and $M = 1$, respectively. For the model with $M = 1$, the magnitude of the initial shock is set at $\epsilon_1^d = -0.5$ percent to generate interest rates at the ELB for 20 periods. In both models, $q = 0.7$ is assumed.

B Learning and updating algorithms

This appendix describes how agents update their belief about time-varying monetary policy parameters by learning from observable data. Appendix B.1 presents the main algorithm used in the main text, which closely follows that of Bodenstein et al. (2022). Appendix B.2 provides an alternative algorithm that employs a particle filter.

B.1 Main algorithm

Let $\boldsymbol{\theta}_t \equiv [\hat{i}_t^*, \phi_{\pi t}, \phi_{yt}]'$ denote a column vector of time-varying unobservable monetary policy parameters. The economy starts at $t = 1$ and the agents have a prior $N(\tilde{\boldsymbol{\theta}}_{1|0}, P_{1|0})$ over the true value of $\boldsymbol{\theta}_1$. The filtering problem facing the agents outside the period of forward guidance consists of the ELB constraint (3), the actual rule for the conceptual nominal interest rate (4), and the law of motion for time-varying parameters (9), which are shown here for convenience.

$$\hat{i}_t = \max\{\hat{i}_t^n, \hat{\underline{i}}\}, \tag{B.1}$$

$$\hat{i}_t^n - \rho_i \hat{i}_{t-1} = H_t \boldsymbol{\theta}_t + \epsilon_t^{mp}, \tag{B.2}$$

$$\boldsymbol{\theta}_t = F\boldsymbol{\theta}_{t-1} + \boldsymbol{\epsilon}_t^{\theta}, \tag{B.3}$$

with $\epsilon_t^{mp} \sim N(0, \sigma_{mp}^2)$, $\boldsymbol{\epsilon}_t^{\theta} \sim N(0, \Sigma_\theta)$, and $H_t \equiv [(1-\rho_i), (1-\rho_i)\hat{\pi}_t, (1-\rho_i)\hat{y}_t]$. The problem is to obtain the posterior $\tilde{\boldsymbol{\theta}}_{t|t}$ about the true parameters $\boldsymbol{\theta}_t$ given the prior $\tilde{\boldsymbol{\theta}}_{t|t-1}$.

When the ELB constraint (B.1) is slack, the problem is reduced to the standard Kalman filtering problem, and the posterior is given by

$$\begin{aligned}
\tilde{\boldsymbol{\theta}}_{t|t} &= \tilde{\boldsymbol{\theta}}_{t|t-1} + K_t(\hat{i}_t - \rho_i \hat{i}_{t-1} - H_t \tilde{\boldsymbol{\theta}}_{t|t-1}), \\
P_{t|t} &= (\boldsymbol{I} - K_t H_t) P_{t|t-1}, \\
K_t &= P_{t|t-1} H_t' (H_t P_{t|t-1} H_t' + \sigma_{mp}^2)^{-1},
\end{aligned}$$

where $\boldsymbol{I}$ is an identity matrix. From equation (B.3), the updated prior in the next period is given by $N(\tilde{\boldsymbol{\theta}}_{t+1|t}, P_{t+1|t})$, where $\tilde{\boldsymbol{\theta}}_{t+1|t} = F\tilde{\boldsymbol{\theta}}_{t|t}$ and $P_{t+1|t} = FP_{t|t}F' + \Sigma_\theta$.

Now consider the case of the binding ELB constraint (B.1). First, consider the case of no forward guidance. In this case, the conceptual interest rate $\hat{i}_t^n$ is unobservable and what is known is the inequality

$\hat{i}_t^n \leq \hat{\underline{i}}$. Following Bodenstein et al. (2022), we approximate the posterior distribution of the systematic part of the notional rate, $f_t \equiv \rho_i \hat{i}_{t-1} + H_t \boldsymbol{\theta}_t$, with a normal distribution. The prior of f_t is normal with mean $f_{t|t-1} \equiv \rho_i \hat{i}_{t-1} + H_t \tilde{\boldsymbol{\theta}}_{t|t-1}$ and variances $(\sigma^f_{t|t-1})^2 \equiv H_t P_{t|t-1} H_t'$. The posterior mean of f_t can be calculated as

$$
\begin{aligned}
f_{t|t} =& \mathbb{E}(f_t | \hat{i}_t^n \leq \hat{\underline{i}}, \mathbf{x}_t) = \mathbb{E}(f_t | f_t \leq \hat{\underline{i}} - \epsilon_t^{mp}, \mathbf{x}_t) = \frac{\mathbb{E}(f_t \mathbb{1}\{f_t \leq \hat{\underline{i}} - \epsilon_t^{mp}\} | \mathbf{x}_t)}{\mathbb{P}(f_t \leq \hat{\underline{i}} - \epsilon_t^{mp} | \mathbf{x}_t)} \\
=& \frac{\mathbb{E}[f_t \mathbb{E}(\mathbb{1}\{f_t \leq \hat{\underline{i}} - \epsilon_t^{mp}\} | f_t, \mathbf{x}_t) | \mathbf{x}_t]}{\mathbb{P}(f_t \leq \hat{\underline{i}} - \epsilon_t^{mp} | \mathbf{x}_t)} = \mathbb{E}\left[f_t \frac{\mathbb{P}(\epsilon_t^{mp} \leq \hat{\underline{i}} - f_t | f_t, \mathbf{x}_t)}{\mathbb{P}(f_t + \epsilon_t^{mp} \leq \hat{\underline{i}} | \mathbf{x}_t)} | \mathbf{x}_t \right] \\
=& \int_{-\infty}^{\infty} f \frac{\Phi\left(\frac{\hat{\underline{i}} - f}{\sigma_{mp}}\right)}{\Phi\left(\frac{\hat{\underline{i}} - f_{t|t-1}}{\sqrt{(\sigma^f_{t|t-1})^2 + \sigma^2_{mp}}}\right)} \frac{1}{\sqrt{2\pi}\sigma^f_{t|t-1}} \exp\left[-\frac{(f - f_{t|t-1})^2}{2(\sigma^f_{t|t-1})^2} \right] df.
\end{aligned}
$$

The posterior of the second moment of f_t can be similarly obtained. By using the first and second moments, the posterior of the variance, $\sigma^f_{t|t}$, can be calculated. We assume that the posterior of f_t is normally distributed: $f_t | \hat{i}_t^n \leq \hat{\underline{i}}, \mathbf{x}_t \sim N(f_{t|t}, (\sigma^f_{t|t})^2)$. The posterior of $\boldsymbol{\theta}_t$ can be updated by the following formula:

$$
\tilde{\boldsymbol{\theta}}_{t|t} = \tilde{\boldsymbol{\theta}}_{t|t-1} + K_t(f_{t|t} - \rho_i \hat{i}_{t-1} - H_t \tilde{\boldsymbol{\theta}}_{t|t-1}), \tag{B.4}
$$

$$
P_{t|t} = P_{t|t-1} - K_t[H_t P_{t|t-1} H_t' - (\sigma^f_{t|t})^2] K_t', \tag{B.5}
$$

$$
K_t = P_{t|t-1} H_t' (H_t P_{t|t-1} H_t')^{-1}. \tag{B.6}
$$

Next consider the period of forward guidance. If the agents observe the interest rate above the ELB, they learn that forward guidance has been lifted and update their belief as in the case of the non-binding ELB constraint. Instead, if the agents observe the interest rate at the ELB, with probability q, they believe that the interest rate is kept at the ELB due to the forward guidance. Since there is no new information relevant to their learning, they do not update the priors, so that $\tilde{\boldsymbol{\theta}}_{t|t} = \tilde{\boldsymbol{\theta}}_{t|t-1}$ and $P_{t|t} = P_{t|t-1}$ with probability q. With probability $1-q$, they believe that the interest rate is kept at the ELB due to the binding ELB constraint. In this case they update their priors following equations (B.4) and (B.5). To summarize, during the period of forward guidance, if the agents observe the interest rate at the ELB, their beliefs are updated as

$$
\begin{aligned}
\tilde{\boldsymbol{\theta}}_{t|t} &= \tilde{\boldsymbol{\theta}}_{t|t-1} + (1-q) K_t(f_{t|t} - \rho_i \hat{i}_{t-1} - H_t \tilde{\boldsymbol{\theta}}_{t|t-1}), \\
P_{t|t} &= P_{t|t-1} - (1-q) K_t[H_t P_{t|t-1} H_t' - (\sigma^f_{t|t})^2] K_t',
\end{aligned}
$$

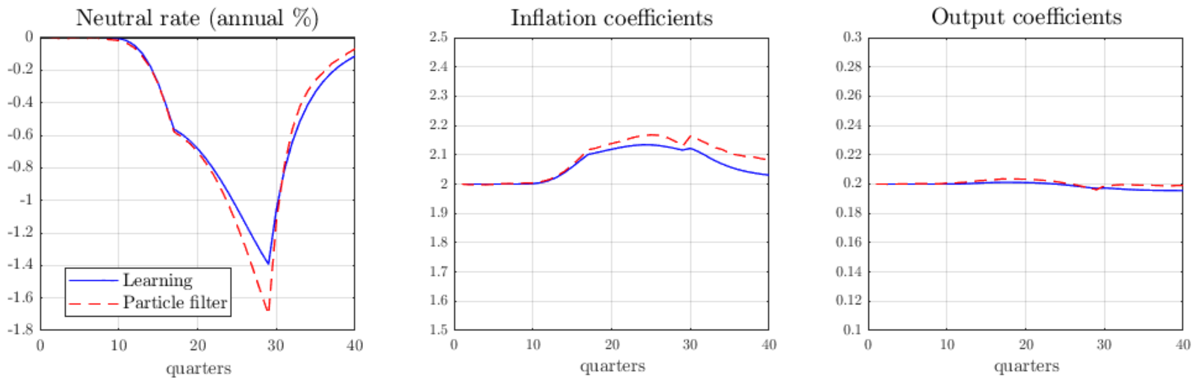
where K_t is given by equation (B.6).

B.2 Particle filter

As in the main algorithm, the economy starts at $t = 1$ and the agents have a prior $N(\tilde{\boldsymbol{\theta}}_{1|0}, P_{1|0})$ over the true value of $\boldsymbol{\theta}_1$. From the initial distribution a sample with size N is drawn: $\{\boldsymbol{\theta}^{(n)}_{t|t-1}\}_{n=1}^N$ where $t = 1$ and N is set at $N = 10,000$. By using equation (B.3) and randomly generated shocks $\{\boldsymbol{\epsilon}_t^{\theta(n)}\}$, candidates for the posterior are constructed as $\check{\boldsymbol{\theta}}^{(n)}_{t|t} = \boldsymbol{\theta}^{(n)}_{t|t-1} + \boldsymbol{\epsilon}_t^{\theta(n)}$. Note that F in equation (B.3) is already taken into account in the prior. For each n, the systematic part of the notional rate is obtained as $f^{(n)}_{t|t-1} = \rho_i \hat{i}_{t-1} + H_t \check{\boldsymbol{\theta}}^{(n)}_{t|t}$.

In the case of the non-binding ELB constraint, since $\epsilon_t^{mp} \sim N(0, \sigma^2_{mp})$, the likelihood of observing $\hat{i}_t = \hat{i}_t^n$ is given by $v_t^{(n)} = \phi((\hat{i}_t - f^{(n)}_{t|t-1})/\sigma_{mp})$, where $\phi(\cdot)$ denotes the standard normal probability density function. In the case of the binding ELB constraint without forward guidance, the likelihood of observing $\hat{i}_t = \hat{\underline{i}}$ is given by $v_t^{(n)} = \Phi((\hat{\underline{i}} - f^{(n)}_{t|t-1})/\sigma_{mp})$, where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function. In each case, the posterior $\{\boldsymbol{\theta}^{(n)}_{t|t}\}$ is obtained by re-sampling from $\{\check{\boldsymbol{\theta}}^{(n)}_{t|t}\}$ as $\boldsymbol{\theta}^{(n)}_{t|t} = \check{\boldsymbol{\theta}}^{(k)}_{t|t}$ with probability

Figure A.3: Comparison of learning algorithms



Note: 'Learning' denotes the main learning algorithm and 'Particle filter' denotes the algorithm that employs a particle filter.

$v_t^{(k)}/\sum_n v_t^{(n)}$. In the case of the binding ELB constraint with forward guidance, let $\{\boldsymbol{\theta}'^{(n)}_{t|t}\}$ denote the posterior obtained by assuming no forward guidance as explained previously. Then, the posterior is obtained by re-sampling from $\{\boldsymbol{\theta}'^{(n)}_{t|t}\}$ with probability $1-q$ and from $\{\check{\boldsymbol{\theta}}^{(n)}_{t|t-1}\}$ with probability q. This is because the agents believe that there is no forward guidance with probability q and that there is no change in the distribution except for the random shocks $\boldsymbol{\epsilon}_t^{\theta(n)}$ with probability $1-q$. In each case, the posterior mean $\tilde{\boldsymbol{\theta}}_{t|t}$ is given by the average over the posterior $\{\boldsymbol{\theta}^{(n)}_{t|t}\}$ and the prior is updated as $\boldsymbol{\theta}^{(n)}_{t+1|t} = F\boldsymbol{\theta}^{(n)}_{t|t}$.

Figure A.3 plots the evolution of the monetary policy parameter values perceived by the agents under the main algorithm and an alternative algorithm that employs a particle filter. The solid lines are the same as those presented in Figure 2. The perceived parameter values under the main algorithm (solid lines) are closely aligned with those under the alternative algorithm, although a decrease in the perceived i-star is slightly greater under the alternative algorithm. The result implies that the main algorithm, which makes a simplifying assumption of normality, works well in learning monetary policy parameter values under the effective lower bound constraint.

C Information effects of low-for-long

In this appendix we modify the model with learning to incorporate the information effects of low-for-long. Specifically, we consider a mechanism in which a change in the perceived i-star in the monetary policy leads to a change in the perceived r-star in the economy. Following Nakamura and Steinsson (2018), we incorporate the information effects of low-for-long into the model by modifying the Euler equation (1) as follows.

$$\hat{y}_t = M\mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{r}_t^*\right) + z_t^d, \tag{C.1}$$

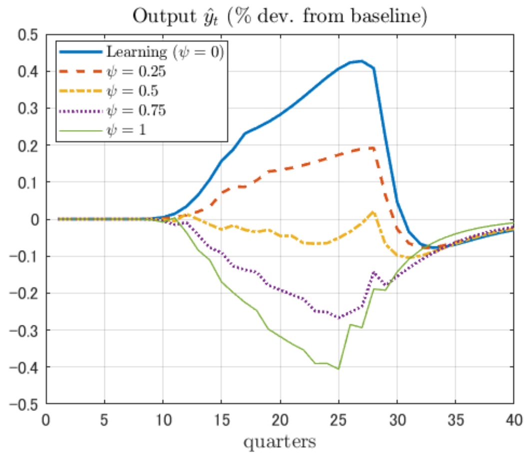
where the perceived r-star in the economy due to the information effects, $\hat{r}_t^*$, is given by

$$\hat{r}_t^* = \psi\hat{i}_t^*, \quad 0 \le \psi \le 1. \tag{C.2}$$

Equation (C.2) implies that a change in the perceived i-star, $\hat{i}_t^*$, affects the perceived r-star, $\hat{r}_t^*$, in the Euler equation. Parameter ψ governs the degree of the information effects. The case of $\psi = 0$ corresponds to no information effect, and the effect becomes stronger as ψ gets larger.

Figure A.4 plots the paths of output in the low-for-long simulation, relative to that of the baseline (the model with no learning), for various degrees of the information effects from $\psi = 0$ to $\psi = 1$. In the case of no information effects, i.e., $\psi = 0$, as explained in the main text, there is a boom in output in the latter half of the low-for-long policy, which lasts until $t = 28$, but it is collapsed during the exit. With the information

Figure A.4: The impact of the information effects of low-for-long



Note: The figure shows the paths of output under the low-for-long scenario, relative to that of the baseline (the model with no learning), for various degrees of the information effects.

effects put in place, the boom and bust in output is attenuated as ψ increases to 0.5. This is because a decrease in the perceived i-star during the latter half of low-for-long leads to a decrease in the perceived r-star, which is contractionary and thus partly offsets the expansionary effect of a decrease in the perceived i-star. As ψ becomes greater beyond 0.5, the contractionary effect becomes dominant, and output becomes lower than the case of no learning.