

# QUANTITATIVE EASING\*

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## Abstract

Is Quantitative Easing (QE) an effective substitute for conventional monetary policy? We study this question using a quantitative heterogeneous-agents model with nominal rigidities, as well as liquid and partially liquid wealth. The direct effect of QE on aggregate demand is determined by the difference in marginal propensities to consume out of the two types of wealth, which is large according to the model and empirical studies. A comparison of optimal QE and interest rate rules reveals that QE is indeed a very powerful instrument to anchor expectations and to stabilize output and inflation. However, QE interventions come with strong side effects on inequality, which can substantially lower social welfare. A very simple QE rule, which we refer to as *Real Reserve Targeting*, is approximately optimal from a welfare perspective when conventional policy is unavailable. We further estimate the model on U.S. data and find that QE interventions greatly mitigated the decline in output during the Great Recession.

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# 1 Introduction

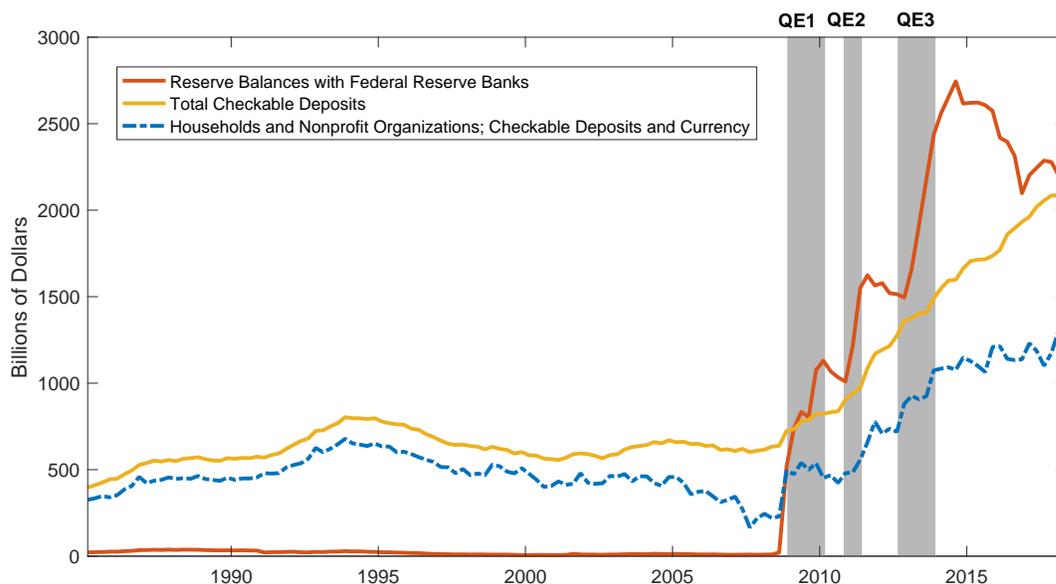
It has been over ten years since the U.S. Federal Reserve (Fed) initiated a colossal expansion of its balance sheet; the largest since the Great Depression. The 2008 financial crisis compelled the Fed to start providing loans to the banking sector, which was suffering from a freeze of interbank lending. However, as banks recovered from the crisis the Fed did not shrink its balance sheet but instead expanded it further, buying up assets such as long-term government debt in large quantities. This was done in a bid to stimulate aggregate demand, which slumped during the Great Recession. Known as *Quantitative Easing (QE)*, these interventions acted as a placeholder for conventional monetary policy, which had become powerless as the policy rate had hit the zero lower bound. Similar interventions took place in the UK and the Euro Area, as well as in Japan during the early 2000s.

While conducting QE, central banks received little guidance from economic theory, as this type of policy is completely ineffective in modern textbook models such as the standard New Keynesian (NK) model, see for instance [Woodford \(2012\)](#). Nevertheless, central bankers have carried on with QE, presumably believing that it is a useful instrument to manage aggregate demand. However, a decade into the balance sheet expansion it is still not well understood when to use QE, how aggressively to use it, and when to roll it back. This leaves central banks in a precarious position in the face of upcoming recessions, when the limits of conventional monetary policy might once again be reached.

This paper presents a quantitative NK model to provide policy makers with more guidance on how to use QE as a stabilization instrument. To this end, we extend the model to allow for household heterogeneity and assets with different degrees of liquidity, following a recent literature, see for instance [Kaplan et al. \(2017\)](#). In this setting, QE interventions can have powerful effects on aggregate demand, but they may also create strong side effects which exacerbate inequality and reduce social welfare.

In the model, QE stimulates aggregate spending by transforming the liquidity composition of households' asset portfolios, which consist of fully liquid deposits and partially liquid wealth stored in mutual funds. When conducting QE, the central bank buys assets from the mutual funds, which triggers the creation of additional deposits. These deposits end up being held by households, who use them to cushion the consumption effects of unemployment. Following a QE purchase, the households thus hold a larger fraction of their wealth in the form of fully liquid deposits and a smaller fraction within partially liquid mutual funds. This liquidity transformation enhances their ability to keep on spending during unemployment, and reduces their demand for precautionary saving while being employed. Both factors increase the aggregate demand for goods, which in the presence of nominal

Figure 1: Reserves and deposits in the United States.



Source: Federal Reserve Board, Flow of Funds accounts. Grey areas denote rounds of QE purchases by the Federal Reserve.

rigidities stimulates real activity. Similarly, the unwinding of QE tones down aggregate demand, depending on the speed of the exit strategy.

Before describing the model, we present a simple formula which captures the essence of the QE transmission mechanism and which can be used for back-of-the-envelope calculations. The key insight conveyed by this formula is that the direct effect of QE depends on the *difference* between the Marginal Propensities to Consume (MPCs) out of deposits and less liquid sources of wealth. Empirical estimates in the literature suggest that the gap between these two MPCs is large. An increase in deposit creation triggered by QE may therefore boost aggregate demand substantially.

Figure 1 shows the evolution of reserves at the Fed, the aggregate amount of checkable deposits, and the amount of deposits/currency held by households. As large-scale asset purchases by the Fed began, all three series increased sharply. This strongly suggests that QE triggered the creation of additional deposits, which in large part ended up being held by households.<sup>1</sup>

After calibrating the model to the U.S. economy, we investigate the efficacy of QE as a stabilization tool, relative to conventional interest rate policy. Following much of the

<sup>1</sup>An important reason for the close link between deposits and reserves is that the central bank funds its asset purchases by creating reserves, which are held as assets by banks. In turn, banks must fund these additional assets, which they achieve by creating deposits.

NK literature, we assume that monetary policy follows a policy rule.<sup>2</sup> In our case, this means that there is either a rule for interest rate policy or a rule for QE, each depending on output and inflation. QE is implemented by the central bank via purchases of long-term government debt, financed by the issuance of reserves.<sup>3</sup>

We evaluate the relative efficacy of QE along several dimensions. First, we consider the ability of QE to anchor expectations, i.e. to rule out fluctuations driven purely by changes in beliefs about the future. As is well known in the literature, conventional policy does so only when the interest rate rule satisfies the “Taylor principle”, meaning that the nominal interest rate responds strongly enough to changes in inflation (and output). We find that, likewise, the QE rule is not always successful in anchoring expectations. Nonetheless, it is successful under a very wide range of realistically achievable values of the policy coefficients. For example, expectations remain anchored under a special case of the QE rule in which the level of real reserves is held completely constant, a policy which we refer to as *Real Reserve Targeting (RRT)*.

Second, we consider the relative performance of QE in managing business cycles, i.e. in stabilizing output and inflation. In order to draw a fair comparison between conventional policy and QE, we give both policies the best possible chance in achieving the stabilization objective. This is done by evaluating both policies under the optimal policy coefficients. We consider different types of shocks, and also different weights on output versus inflation volatility.

A main finding is that, under a wide range of configurations, QE is actually more effective in stabilizing output and inflation than interest rate policy. This happens as QE tends to create a positive co-movement between output and inflation, due to aggregate demand effects. This co-movement in turn eases the trade-off between output and inflation volatility. We further find that in many instances, RRT performs better than interest rate policy, even though RRT is by itself a restricted and therefore suboptimal form of QE.

Third, we consider the ability of QE rules to improve welfare. We find that optimal QE rules tend to deliver lower welfare than optimal interest rate rules. Moreover, aggressive QE rules might be very detrimental to welfare. This might seem surprising, given that QE is relatively effective in stabilizing output and inflation. However, QE comes with strong *side effects* which adversely affect social welfare. In particular, when the central bank creates

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<sup>2</sup>We set aside the question of what are the optimal long-run levels of inflation, the nominal interest rates, and the optimal long-run size and composition of the central bank balance sheet. We thus focus purely on interest rate policy and QE as instruments for stabilization of the business cycle.

<sup>3</sup>Aside from long-term government debt, the Fed also purchased mortgage securities. We do not explicitly model such purchases, but they would create a very similar transmission mechanism in the model. [Fieldhouse et al. \(2018\)](#) provide empirical evidence that mortgage purchases by Government Sponsored Enterprises have expansionary macro effects.

movements in the amount of reserves and hence the supply of deposits, it varies the extent to which households can insure themselves against idiosyncratic income risk. The welfare costs of periods of low insurance are relatively large; they can dominate the total welfare gains from periods of high insurance and from reduced volatility of inflation and output. Thus, even though aggressive QE rules can be very powerful from a macro stabilization standpoint, such policies might not be advisable from a welfare perspective. In fact, we find RRT to be approximately optimal from a welfare perspective, when conventional policy is not available.

Having studied the efficacy of QE, we apply the model to the U.S. Great Recession. To this end, we estimate model parameters by Maximum Likelihood and match the model to the time series for household deposits as shown in Figure 1, as well as other macro time series. We then quantify the effects of QE using a counterfactual simulation. We find that QE had a very large, positive impact on U.S. output and inflation between 2009 and 2012, preventing a much deeper recession.

Finally, we explore two other unconventional policy options. The first is Forward Guidance, i.e. announcements about future interest rates. We show that once a QE rule is in place, the immediate effects of Forward Guidance are very small, which underscores the importance of interactions between different types of unconventional policy. Second, we consider the effects of a permanent expansion of the central bank balance sheet via QE. Such an expansion can move the economy to a new steady state with more liquidity and higher welfare. However, the transition path to such a new steady state is costly, due to the aforementioned side effects on welfare. We therefore find that there is only limited scope for improving welfare through permanent QE.

**Related literature.** The neutrality of central bank balance sheet policies in standard complete-markets models has been originally established by [Wallace \(1981\)](#), and was reiterated more recently by [Woodford \(2012\)](#). The underlying theoretical argument is a variation on the Modigliani-Miller and Ricardian Equivalence theorems. Perhaps in part because of this striking neutrality result, much of the recent NK literature on unconventional monetary policy has focused on Forward Guidance rather than on QE, see for instance [Del Negro et al. \(2012\)](#) and [McKay et al. \(2016\)](#).

That said, our model does have a number of precursors. [Chen et al. \(2012\)](#) analyze QE in a medium-scale DSGE model with segmented asset markets. They find that QE only has small effects. Large effects are found by [Del Negro et al. \(2017\)](#), who develop a quantitative model to evaluate the effects of liquidity provisions during the financial crisis. In their model, liquidity interventions ease financial constraints on the production side of

the economy. A similar channel operates in [Gertler and Karadi \(2012\)](#). By contrast, we focus on the role of QE as direct instrument to manage aggregate demand, which has been used well beyond the financial crisis.

The importance of household liquidity for optimal monetary policy is emphasized by [Bilbiie and Ragot \(2016\)](#). They show that liquidity frictions change the output-inflation trade-off, as inflation affects the extent to which households can self-insure using nominal assets. [Cui \(2016\)](#) studies the optimal monetary-fiscal policy mix in a model in which the liquidity of different asset classes differs endogenously, but without QE. [Harrison \(2017\)](#) studies optimal QE policy in a model with portfolio adjustment costs.

Unlike these studies, we use a quantitative model with incomplete markets in the Bewley-Huggett-Aiyagari tradition, combined with sticky prices in the NK tradition. A number of recent papers study the importance of household heterogeneity for the transmission of conventional monetary policy in this type of models, see for instance [Gornemann et al. \(2016\)](#), [Kaplan et al. \(2017\)](#), [Auclert \(2016\)](#), [Luetticke \(2015\)](#), [Ravn and Sterk \(2016\)](#), [Debortoli and Galí \(2017\)](#), [Challe \(2017\)](#), [Hagedorn \(2017\)](#), [Hagedorn et al. \(2017\)](#), and [Bhandari et al. \(2017\)](#). Our model fits into this category, but we instead study (optimal) QE. Heterogeneity also plays a role in [Sterk and Tenreyro \(2018\)](#), who study the distributional effects of open-market operations in a flexible-price model.

Finally, various authors have studied the empirical effects of large-scale asset purchases, generally finding evidence for expansionary macro effects. For example, [Weale and Wieladek \(2016\)](#) find that in the U.S., an asset purchase of one percent of GDP leads to an increase in real GDP of 0.58 percent and an increase in inflation of 0.62 percent. A survey of the broader literature on this topic can be found in [Bhattarai and Neely \(2016\)](#).

The remainder of this paper is organized as follows. Section 2 presents a simple formula for the effects of QE. The full model is presented in Section 3, whereas Section 4 discusses the calibration and the macro effects of QE. In Section 5 we evaluate the efficacy of (optimal) QE and conventional policy rules, while in Section 6 we estimate the model and study the macro effects of QE in the U.S. during the Great Recession. Section 7 discusses alternative unconventional policy options. Section 8 concludes.

## 2 A simple formula

Before we present the full model, we first provide a simple formula to gauge the effects of QE on aggregate demand. To this end, let us postulate an aggregate consumption demand function  $C(L, I, \Gamma)$ , where  $L$  denotes the (nominal) value of fully liquid assets held by households (e.g. deposits),  $I$  denotes the value of their illiquid, or partially liquid assets

(e.g. assets owned via mutual funds). The third argument,  $\Gamma$ , contains other relevant aspects of the economy, such as prices, and is denoted by a scalar for simplicity. The (average) marginal propensities to consume out of liquid and illiquid wealth are given by the respective derivatives of the aggregate demand function, and will be denoted by  $MPC^L \equiv C_L(L, I, \Gamma)$  and  $MPC^I \equiv C_I(L, I, \Gamma)$ .

When the central bank conducts QE, it purchases  $I$  in exchange for  $L$ .<sup>4</sup> Since this is a voluntary trade, QE does not directly change the total amount of wealth owned by households, i.e. any increase in  $L$  is matched by a decrease in  $I$  of the same magnitude. Denoting the value of assets purchased under QE by  $\Delta^{QE}$ , the consumption function becomes  $C(L + \Delta^{QE}, I - \Delta^{QE}, \Gamma(\Delta^{QE}))$ . By differentiating this function with respect to  $\Delta^{QE}$ , we obtain the following formula for the marginal effect of QE on aggregate demand:

$$\frac{\partial C}{\partial \Delta^{QE}} = \underbrace{MPC^L - MPC^I}_{\text{direct effect}} + \underbrace{GE}_{\text{indirect effect}},$$

where  $GE \equiv C_\Gamma(L, I, \Gamma) \frac{\partial \Gamma}{\partial \Delta^{QE}}$ . This formula splits the effects of QE into “direct” and “indirect” effects, in the spirit of a decomposition proposed by [Kaplan et al. \(2017\)](#) for conventional monetary policy.

The first term captures the direct effect. It is the difference between the MPCs out of liquid and illiquid wealth. Intuitively, QE directly triggers a liquidity transformation: it lowers households’ illiquid wealth holdings, while increasing their liquid wealth. The direct effect of this transformation on consumption depends on the difference in the marginal propensities to consume out of the two types of wealth. The second term captures the indirect general equilibrium effects triggered by QE.

Simple as it looks, the formula conveys a number of important insights. First, if the two types of wealth were equally liquid, as in many standard models, it would hold that  $MPC^L = MPC^I$ , other things equal. In this case, QE would have no direct effect on aggregate demand, echoing the neutrality result of [Wallace \(1981\)](#). Second, even in the extreme case in which  $MPC^I = 0$ , QE only has large effects to the extent that the marginal propensity to consume out of liquid wealth,  $MPC^L$ , is large. This point provides a way of understanding why for instance [Chen et al. \(2012\)](#) find that QE has small effects on the real economy, as it is well known that MPCs tend to be very small in representative-agent models. On the other hand, models with incomplete markets and borrowing constraints are well known to generate much higher MPCs out of liquid wealth. Moreover, when certain types of assets are subject to liquidity frictions, the MPCs out of these types of wealth tend

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<sup>4</sup>In the full model, the central bank issues reserves, held by banks, which then create deposits to fund those reserves.

to be small, even in incomplete-markets models.

Finally, the indirect GE effects depend crucially on the structure of the economy and in particular on price stickiness. With flexible prices, an increase in aggregate consumption demand is typically dampened by an increase in prices. With sticky prices, the increase in aggregate demand might be further amplified.

Are strong direct effects of QE in line with the data, i.e. is the difference between  $MPC^L$  and  $MPC^I$  large? A substantial body of empirical studies has found MPCs out of fully liquid wealth to be very sizable. For example, [Fagereng et al. \(2018\)](#) estimate an average MPC of 63 percent in the first year, based on high-quality administrative data on Norwegian lottery participants. The literature on MPCs out of less liquid sources of wealth is less extensive, but generally reports much smaller MPCs. [Di Maggio et al. \(2018\)](#) use Swedish data to estimate MPCs out of changes in stock market wealth, and estimate these to lie between 5 and 14 percent, much below typical estimates for the MPCs out of fully liquid wealth. Moreover, they report that –among the same individuals– MPCs out of fully liquid dividend payments are much higher. The empirical evidence is thus consistent with sizable direct effects.

Based on the above formula, we can obtain a back-of-the-envelope estimate for the direct effects of QE. This helps to get a sense of the quantitative importance of QE since the Great Recession. Between 2008 and 2017, checkable deposits increased from about one to six percent of annual GDP. [Figure 1](#) suggests that this increase was largely driven by QE. Assuming  $MPC^L = 0.63$  following [Fagereng et al. \(2018\)](#) and  $MPC^I = 0.095$ , the mid point of estimates provided by [Di Maggio et al. \(2018\)](#), this implies a direct effect of  $(6 - 1) \cdot (0.63 - 0.095) = 2.7$  percent of GDP.<sup>5</sup>

Thus, the data suggest that the direct effects of QE on GDP were substantial. However, the overall effect of QE depends also on the GE response to these direct effects. We will use the model to evaluate the overall effects of QE.

### 3 The model

This section presents a fully-fledged general equilibrium model. We use the model to contrast the effects of QE to conventional policy, allowing for a deeper understanding of when and how to apply QE. The key features of the model are that nominal prices are sticky, that agents face imperfectly insurable income risk, and that they hold both fully liquid and partially liquid assets.

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<sup>5</sup>A limitation of the back-of-the-envelope calculation is that one-year MPCs are used. The model presented in the next section accounts for cumulative effects over longer horizons.

The model economy is populated by households, firms, banks, mutual funds, a treasury and a central bank. The conceptual distinction between banks and mutual funds, and between the treasury and the central bank is not strictly necessary. One might consolidate them, but the distinction makes it easier to relate the model to reality.

**Households.** There is a continuum of infinitely-lived, ex-ante identical households, indexed by  $i \in [0, 1]$ . Household  $i$ 's preferences are represented by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t(i), N_t(i)), \quad (1)$$

where  $C_t(i)$  is a basket of goods consumed in period  $t$ ,  $N_t(i)$  denotes hours worked, supplied on a competitive labor market, and  $\beta \in (0, 1)$  is the subjective discount factor. Moreover,  $\mathbb{E}_t$  is the expectations operator conditional on information available in period  $t$ , and  $U(C, N)$  is a utility function which is increasing and concave in consumption and decreasing in hours worked. The consumption basket is given by  $C_t(i) \equiv \int_0^1 \left( C_t(i, j)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dj \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$ , where  $C_t(i, j)$  denotes the household's consumption of good  $j$  and  $\varepsilon_t > 1$  is the elasticity of substitution between goods, which is exogenous. Following the NK literature, variations in  $\varepsilon_t$  can be thought of as "cost push" shocks, since they affect mark-ups charged by firms. Household optimization implies that the price of the consumption basket is given by  $P_t = \int_0^1 (P_t(j)^{1 - \varepsilon_t} dj)^{\frac{1}{1 - \varepsilon_t}}$ , where  $P_t(j)$  is the price of good  $j$ .

Households are subject to idiosyncratic unemployment risk. When unemployed the household cannot supply labor, i.e.  $N_t(i) = 0$ , so it has no labor income. When employed, the household can freely choose the number of hours worked, earning a real wage rate  $w_t$  per hour. Unemployed households become employed with an exogenous probability  $p^{UE}$ , whereas employed households become unemployed with a probability  $p^{EU}$ . These transitions take place at the very end of each period.

Households can hold deposits, denoted by  $D_t(i)$  in real terms, which pay a nominal interest rate and are fully liquid, in the sense that there are no transaction costs involved. Deposits provide households with a means of self insurance against the idiosyncratic income risks associated with unemployment, helping them to cushion the decline in consumption when they lose their job. However, households must obey a borrowing constraint:

$$D_t(i) \geq \chi, \quad (2)$$

where  $-\chi$  is a borrowing limit.

Households further own wealth stored in mutual funds. Such wealth is only partially

liquid and is therefore only of limited use as a means of self insurance. Realistically, mutual funds often charge substantial transaction fees when a household buys into a mutual fund or sells out of a fund, and households often rely on costly financial advice when adjusting their financial portfolios.<sup>6</sup> Also, and perhaps more importantly, the aforementioned empirical evidence on MPCs out of different types of wealth strongly suggests that wealth stored in mutual funds is much less easily spent than deposit wealth.

To capture the partial liquidity of mutual fund wealth, we assume that a household cannot directly control the wealth stored in the fund. Instead, it receives a certain liquid payout from the fund, denoted  $X_t(i)$ . The fund potentially differentiates this amount across households, providing an amount  $X_t(i) = X_t^E$  to the employed and an amount  $X_t(i) = X_t^U \geq X_t^E$  to the unemployed. This differentiation captures the possibility that households may partially draw upon mutual fund wealth to cushion the impact of job loss on consumption. We will refer to the difference in payouts,  $\mu_t \equiv X_t^U - X_t^E \geq 0$ , as *liquidation* of mutual fund wealth during unemployment.<sup>7</sup>

When unemployed, a household further receives an unemployment benefit given by  $\Theta^U \geq 0$ . This benefit is provided by a government agency which runs a balanced budget. It therefore imposes a premium on the employed, given by  $\frac{u}{u-1}\Theta^U$ , where  $u = p^{EU}/(p^{EU} + p^{UE})$  is the unemployment rate.<sup>8</sup> The budget constraint of the household, in real terms, is given by:

$$C_t(i) + D_t(i) = w_t N_t(i) + \frac{R_{t-1}}{\Pi_t} D_{t-1}(i) + \Theta_t(i) + X_t(i) - T_t, \quad (3)$$

where  $\Theta_t(i) = \Theta^U$  if the household is unemployed and  $\Theta_t(i) = -\frac{u}{u-1}\Theta^U$  if the household is employed. Moreover,  $R_{t-1}$  is the gross nominal interest rate on deposits from period  $t-1$  to period  $t$ ,  $\Pi_t = P_t/P_{t-1}$  is the corresponding gross rate of inflation, and  $T_t$  is a lump-sum tax levied to finance government expenditures other than benefits. In each period, a household  $i$  chooses  $C_t(i)$ ,  $D_t(i)$  and  $N_t(i)$  to maximize (1) subject to the constraints (2) and (3), and the constraint that it can only supply labor when employed.

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<sup>6</sup>Moreover, many mutual funds impose a “back-loaded” fee structure, which means that selling fees are higher when a household has purchased the mutual fund shares relatively recently. This practice is geared to avoid rapid outflows from a fund. Such fee structures make mutual funds an even less attractive vehicle for self insurance.

<sup>7</sup>Strictly speaking, the fund provides some income insurance to the households by differentiating payouts. Kaplan et al. (2017) consider a richer setup in which partially liquid wealth is subject to adjustment costs. In their setting, households liquidate a limited amount of their partially liquid wealth following a large enough negative income shock. Our setup captures this outcome in a simple way, enabling us to drop the distribution of partially liquid wealth as a state variable, keeping track of only the distribution of fully liquid wealth (i.e. deposits).

<sup>8</sup>McKay and Reis (2016) provide an in-depth analysis of the stabilization role of social insurance in a NK model with heterogeneous agents.

**Firms.** Each consumption good is produced by a different firm. The structure of household preferences implies that firms are monopolistically competitive in the goods market. Firms operate a linear technology using labor only, i.e. their output is given by  $Y_t(j) = A_t N_t(j)$ . Here,  $A_t$  denotes Total Factor Productivity (TFP), which is exogenous and subject to stochastic shocks.

Firms also face a quadratic cost of price adjustment following [Rotemberg \(1982\)](#), given in real terms by  $Adj_t(j) = \phi \left( \frac{P_t(j) - P_{t-1}(j)}{P_{t-1}(j)} \right)^2 Y_t$ , where  $\phi \geq 0$  is a parameter which governs the cost of price adjustment, and  $Y_t = \int_0^1 Y_t(j) dj$  denotes aggregate output. The dividends paid by firm  $j$  are given, in real terms, by  $Div_t(j) = \frac{P_t(j)}{P_t} Y_t(j) - w_t N_t(j) - Adj_t(j)$  where in equilibrium it holds that  $P_t(j) = P_t$ . Therefore, aggregate dividends satisfy

$$Div_t = Y_t - w_t N_t - Adj_t, \quad (4)$$

where  $Adj_t = \int_0^1 Adj_t(j) dj = \phi (\Pi_t - 1)^2 Y_t$ . Firms maximize the present value of profits which leads to the following relation, commonly known as the New Keynesian Phillips Curve:

$$1 - \varepsilon_t + \varepsilon_t \frac{w_t}{A_t} = \phi (\Pi_t - 1) \Pi_t - \phi \mathbb{E}_t \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} (\Pi_{t+1} - 1) \Pi_{t+1}, \quad (5)$$

where  $\Lambda_{t,t+1}$  is the stochastic discount factor used by the firms and mutual funds. For simplicity, we assume  $\Lambda_{t,t+1} = \beta$ . We also assume that the distribution of initial prices is the same across firms, so they behave symmetrically. Accordingly, we drop the index  $j$  from now on.

**Mutual funds.** There is a representative and competitive mutual fund which owns the equity in the firms, as well as long-term treasury debt. We model the latter following [Woodford \(2001\)](#). A unit of long-term debt pays  $\rho^k$  dollars in any period  $t + k + 1$  going forward, where  $0 \leq \rho < \beta^{-1}$ . In the steady state, the duration of long-term government debt is given by  $\frac{1}{1 - \beta\rho}$ . The budget constraint of the mutual fund is given by:

$$u X_t^U + (1 - u) X_t^E = Div_t + (1 + \rho q_t) \frac{B_{t-1}^m}{\Pi_t} - q_t B_t^m, \quad (6)$$

where  $Div_t \equiv \int_0^1 Div_t(j) dj$  are aggregate dividends transferred from the firms to the fund,  $B_t^m$  is the amount of long-term treasury debt held by the mutual fund, and  $q_t$  is the price of government debt issued in period  $t$  which is determined according to:

$$q_t = \mathbb{E}_t \Lambda_{t,t+1} \frac{1 + \rho q_{t+1}}{\Pi_{t+1}}. \quad (7)$$

Note that mutual funds do not hold deposits. In equilibrium, the return on deposits is dominated by the return on long-term government debt. The reason is that households value deposits for precautionary savings reasons, which drives down the real interest rate on deposits. If the mutual funds were to hold on to deposits, they would depress their returns performance while at the same time deprive their owners, the households, from liquidity.

**Banks.** There is a perfectly competitive banking sector. Banks can hold reserves at the central bank, denoted by  $M_t$  in real terms, which pay a nominal interest rate  $R_t$ , controlled by the central bank. In order to fund these assets, banks must create liabilities, i.e. deposits. No-arbitrage implies that reserves and deposits carry the same nominal interest rate  $R_t$ . In equilibrium, banks therefore earn no profits. Consolidation of the balance sheet of the banking sector implies that:<sup>9</sup>

$$\int_0^1 D_t(i) di = M_t. \quad (8)$$

**Treasury.** Real government expenditures are exogenous and denoted by  $G_t$ . In our quantitative exercises, we will consider shocks to  $G_t$ . The treasury targets a constant real level of long-term debt, denoted  $B_t = B$ , during each period. The budget constraint of the treasury is given by:

$$G_t = q_t B - (1 + \rho q_t) \frac{B}{\Pi_t} + T_t^{cb} + T_t, \quad (9)$$

where  $T_t^{cb}$  is a seigniorage transfer received from the central bank.

**Central bank.** The central bank can issue reserves ( $M_t$ ) and can also purchase long-term government debt. The budget constraint of the central bank, in real terms, is given by:

$$T_t^{cb} + \frac{R_{t-1}}{\Pi_t} M_{t-1} + q_t B_t^{cb} = M_t + (1 + \rho q_t) \frac{B_{t-1}^{cb}}{\Pi_t}, \quad (10)$$

where  $B_t^{cb}$  denotes the central bank's holdings of long-term government debt. We further assume that if the central bank purchases government debt, it finances these purchases by

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<sup>9</sup>An alternative interpretation of the model is that households can directly hold central bank liabilities (money). It would also be straightforward to allow the banking sector to create additional deposits without holding reserves. However, this would not impact directly on our key mechanism, which requires QE to trigger the creation of *additional* deposits, as strongly suggested by Figure 1.

issuing reserves:<sup>10</sup>

$$q_t B_t^{cb} - (1 + \rho q_t) \frac{B_{t-1}^{cb}}{\Pi_t} = M_t - \frac{R_{t-1}}{\Pi_t} M_{t-1}. \quad (11)$$

We consider two versions of the model, each with a different conduct of monetary policy. In the first version, the central bank conducts conventional interest rate policy. In this case, the central bank sets the interest rate on reserves according to the following rule:

$$\widehat{R}_t = \widehat{\Pi}_t \xi_{\Pi}^R \widehat{Y}_t \xi_Y^R, \quad (12)$$

where hats denote variables relative to their steady-state values, i.e. relative to their state variables:  $\widehat{Y}_t \equiv Y_t/\bar{Y}$ ,  $\widehat{\Pi}_t \equiv \Pi_t/\bar{\Pi}$  and  $\widehat{R}_t \equiv R_t/\bar{R}$ , where  $\bar{R}$ ,  $\bar{\Pi}$  and  $\bar{Y}$  are the steady-state values of  $R$  and  $\Pi$ , and  $Y$ , respectively. In the above policy rule,  $\xi_{\Pi}^R$  and  $\xi_Y^R$  are stabilization coefficients which determine the response of monetary policy to fluctuations in output and inflation. We further assume that under conventional policy the central bank does not own any government debt ( $B_t^{cb} = 0$ ) and that the real amount of reserves (and hence aggregate deposits) is held at a constant level ( $M_t = \bar{M}$ ).<sup>11</sup>

In the second version of the model, the central bank conducts QE rather than interest rate policy. In this case, QE sets the total amount of reserves according to the following rule:<sup>12</sup>

$$\widehat{M}_t = \widehat{\Pi}_t^{\xi_{\Pi}^{QE}} \widehat{Y}_t^{\xi_Y^{QE}} z_t^{QE}, \quad (13)$$

where  $\widehat{M}_t = M_t/\bar{M}$  is amount of real reserves relative to the steady state and  $z_t^{QE}$  is an exogenous shock to the QE rule, akin to conventional monetary policy shocks often considered in the NK literature. We will study this shock to better understand the workings of QE. In the above rule,  $\xi_{\Pi}^{QE}$  and  $\xi_Y^{QE}$  are policy coefficients which are, respectively, the elasticities of real reserves with respect to inflation and output.

As mentioned above, the central bank implements the QE rule with purchases (or sales) of government bonds, as in Equation (11). We further assume that when QE is used, the nominal interest rate is pegged at  $R_t = \bar{R}$ , reflecting the reality that QE is typically used when the nominal interest rate cannot be moved.

An interesting special case of the QE rule sets both stabilization coefficients to zero, i.e.

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<sup>10</sup>Vice versa, we assume that changes in reserves are associated only with purchases/sales of government debt. That is, we do not consider ‘‘helicopter drops’’.

<sup>11</sup>We abstract from the zero lower bound on the net nominal interest rate ( $R_t - 1$ ). However, we will assume that the net nominal interest rate is pegged at zero in the model version with QE. Regarding QE policy, we similarly do not impose a lower bound on  $B_t^{cb}$ , i.e. the central bank itself could in principle issue long-term debt.

<sup>12</sup>This rule can be reformulated as one in nominal reserves, the current and lagged price level and nominal output.

$\xi_{\Pi}^{QE} = \xi_Y^{QE} = 0$ . In this case monetary policy directly targets a certain level of real reserves given by  $M_t = \bar{M}z_t^{QE}$ . We refer to this policy as *Real Reserve Targeting (RRT)*. This policy implies that, in the absence of QE shocks, the level of real reserves is constant and hence the nominal amount of reserves moves one for one with the price level.

**Equilibrium.** Given laws of motion for the exogenous states  $\{\varepsilon_t, A_t, G_t, z_t^{QE}\}$ , the equilibrium is defined as joint law of motion for household choices  $\{N_t(i), C_t(i), D_t(i)\}_{i \in [0,1]}$ , mutual fund choices  $\{X_t^E, X_t^U, B_t^m\}$ , lump-sum taxes and government debt  $\{T_t, B\}$ , central bank choices  $\{R_t, M_t, B_t^{cb}, T_t^{cb}\}$ , aggregate quantities  $\{Y_t, N_t\}$ , prices and dividends  $\{\Pi_t, w_t, q_t, Div_t\}$ , such that at any point in time (i) Each household  $i \in [0, 1]$  maximizes (1) subject to the constraints (2) and (3); (ii) Firms in total produce  $Y_t = A_t N_t$ , pay out dividends according to (4), and set nominal prices such that the New Keynesian Phillips Curve (5) holds; (iii) Mutual funds pay out  $X_t^E$  and  $X_t^U = X_t^E + \mu_t$ , satisfying (6); they also price the long-term government bonds according to (7); (iv) The banks create deposit such that (8) holds; (v) The treasury's and central bank's budget constraints, (9) and (10), hold; (vi) The central bank either conducts conventional policy, i.e. it satisfies Equation (12), and sets  $B_t^{cb} = 0$ , and  $M_t = \bar{M}$ , or it conducts QE policy, i.e. it satisfies Equations (11) and (13), and sets  $R_t = \bar{R}$ ; (vii) The markets for deposits/reserves clear, i.e. Equation (8) holds. Also the markets for long-term government debt, labor, and goods clear, i.e.

$$\begin{aligned} B &= B_t^{cb} + B_t^m, \\ N_t &= \int_0^1 N_t(i) di, \\ Y_t &= \int_0^1 C_t(i) di + G_t + \phi (\Pi_t - 1)^2 Y_t. \end{aligned}$$

Finally, we assume that each of the stochastic driving forces  $z \in \{\varepsilon, A, G, z^{QE}\}$  follows an independent process of the form  $\ln z_t = (1 - \lambda_z) \ln \bar{z} + \lambda_z \ln z_{t-1} + \nu_t$ . Here,  $\lambda_z \in [0, 1)$  is a persistence parameter and  $\nu_t$  is an i.i.d. innovation, drawn from a distribution with mean zero and a standard deviation given by  $\sigma_z \geq 0$ . We allow  $\lambda_z$  and  $\sigma_z$  to potentially differ across the four types of shocks, and we will discuss their calibration below. We further normalize  $\bar{z}^{QE} = \bar{A} = 1$  and will discuss the calibration of  $\bar{G}$  and  $\bar{\varepsilon}$  below.

## 4 Household heterogeneity and the effects of QE

We calibrate the model to the U.S. economy and set the length of a period to one quarter. Table 1 presents the parameter values. We assume the following utility function:

$$U(C, N) = \frac{C^{1-\sigma} - 1}{1 - \sigma} - \frac{\kappa_0}{1 + \kappa_1} N^{1+\kappa_1},$$

Here,  $\sigma > 0$  is the coefficient of relative risk aversion, which we set equal to one. Moreover,  $\kappa_1 > 0$  is the inverse Frisch elasticity of labor supply, which is also set to one. Finally,  $\kappa_0 > 0$  is a parameter scaling the disutility of labor, which we calibrate such that employed workers on average supply  $\bar{N} = 1/3$  unit of labor in the steady state. We further calibrate the steady-state elasticity of substitution between goods as  $\bar{\varepsilon} = 9$ , which implies a steady-state markup of 12.5 percent, and  $\beta = 0.99$ , which corresponds to an annual subjective discount rate of four percent. We target an unemployment rate of  $u = 0.045$  and an unemployment inflow rate of  $p^{EU} = 0.044$ , corresponding to a monthly inflow rate of about 1.5 percent, as measured in the Current Population Survey. The implied unemployment outflow rate is  $p^{UE} = 0.934$ . The unemployment benefit is targeted to be 25 percent of average wage income in the steady state, which implies that  $\Theta^U = 0.25 \frac{\bar{\varepsilon}-1}{\bar{\varepsilon}} \bar{N} = 0.071$ .<sup>13</sup>

We assume that the mutual fund's liquidation policy is constant over time, i.e.  $\mu_t = \mu$ , where we calibrate  $\mu$  such that the net real interest rate on deposits in the steady state is zero, i.e.  $\bar{R}/\bar{\Pi} = 1$ . This results in  $\mu = 0.0634$  (see Section 4.2 for more discussion on this). The price adjustment cost parameter is set to  $\phi = 47.1$ , which corresponds to an average price duration of three quarters in the Calvo equivalent of the model.

To facilitate comparison of the two policies, we calibrate the model such that the steady states of the model version with QE and the version with conventional policy coincide. Specifically, we assume that in both cases the central bank targets zero inflation in the steady state, i.e.  $\bar{\Pi} = 1$ . The implied nominal steady-state interest rate is  $\bar{R} = 1$ .<sup>14</sup> We further set  $\rho = 0.947$ , which implies a duration of government debt of four years. The borrowing limit,  $-\chi$ , is set to zero.<sup>15</sup>

<sup>13</sup>Statutory benefits are typically around 40 percent of labor income. However, the actual amount received by households is much lower due to limited eligibility and take-up. Chodorow-Reich and Karabarbounis (2016) argue that taking into account all these factors reduces the benefit to around 6 percent of income. Our calibration strikes a balance between their number and the statutory rate.

<sup>14</sup>Note that in the version with conventional policy, we abstract from the Zero Lower Bound on the nominal interest rate. In our comparison exercises, we thus ask whether effective QE is more or less effective than a hypothetical conventional policy that would not be subject to the ZLB. Alternatively, we could have calibrated the model version with conventional policy to be away from the ZLB, but this would make a clear comparison more difficult since the steady states of the two model versions would be different.

<sup>15</sup>We have solved a model with a positive borrowing limit. We obtained very similar results to our

Table 1: Parameter values and steady-state targets.

Parameter	Description	Value	Notes
$\beta$	subjective discount factor	0.99	subjective annual discount rate: 4%
$\sigma$	coefficient of relative risk aversion	1	convention
$\kappa_0$	labor disutility parameter	11.4296	average labor supply employed: 1/3
$\kappa_1$	inverse Frisch elasticity	1	convention
$p^{EU}$	unemployment inflow rate	0.044	monthly rate: 1.5% (CPS)
$p^{UE}$	unemployment outflow rate	0.934	steady-state unemployment rate: 4.5%
$\Theta^U$	unemployment benefit	0.0741	benefit 25% of avg. real wage
$\mu$	mutual fund liquidation coefficient	0.0634	real interest rate: 0 %
$\chi$	borrowing limit	0	see footnote 15
$\bar{\varepsilon}$	elasticity of substitution varieties	9	markup: 12.5%
$\phi$	price adjustment cost parameter	47.1	average price duration: 3 quarters
$\bar{G}$	real government expenditures	0.0732	expenditures-to-annual-output: 23%
$B$	government debt parameter	0.0398	median holdings liquid wealth (SCF), see text
$\rho$	decay government debt	0.9470	duration of government debt: 4 years
$\bar{D} = \bar{M}$	steady-state deposits (=reserves)	0.1009	deposits-to-annual-output (FoF): 7.5%
$\bar{\Pi}$	long-run inflation target	1	net inflation rate: 0%

The steady-state values of government expenditures, deposits and government debt, i.e.,  $\bar{G}$ ,  $\bar{D} = \bar{M}$ , and  $B$  are chosen to hit the following targets. We target a ratio of government expenditures to output of 23 percent, in line with national accounts data, and a deposit-to-annual-output ratio of 7.5 percent, in line with data from the Flow Of Funds (FoF) accounts. Moreover, we target data on liquid wealth from the Survey of Consumer Finances (SCF). In particular, we target the median amount of transaction accounts (deposits in the model) held by a household with median income, as a fraction of (pre-tax) median income. This ratio is about 26 percent in the SCF, averaged over the years 1989-2016. While not explicitly targeted, our model implies a ratio of the value of government debt to annual output, i.e.  $\frac{\bar{q}B}{4Y}$  of 58 percent.

## 4.1 Computation

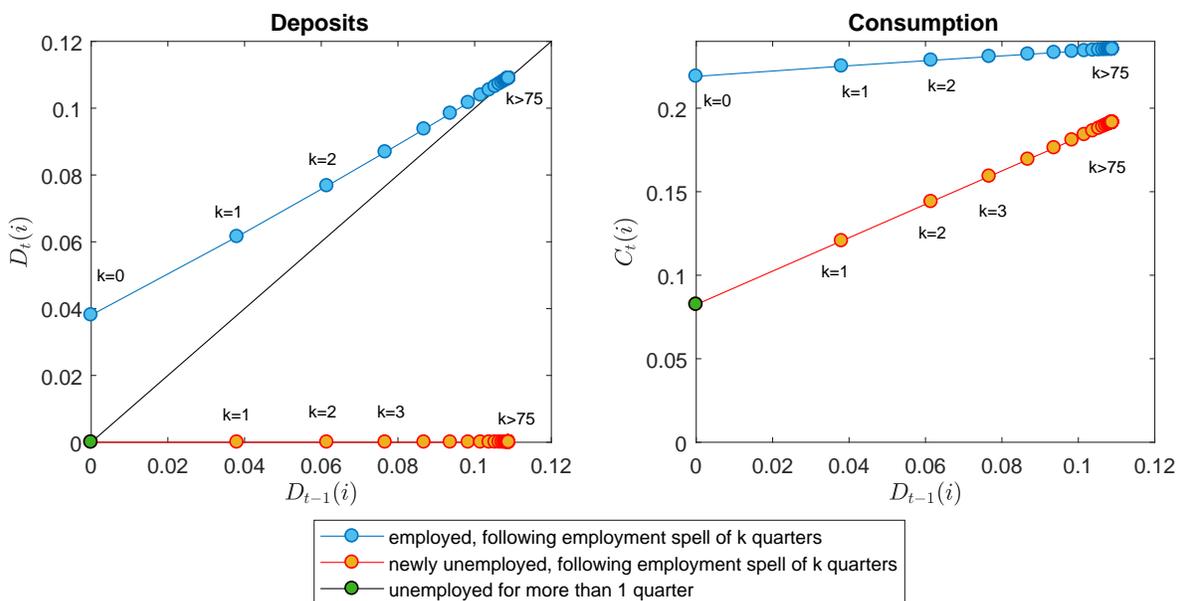
In the next section, we will analyze the dynamics of the model in response to aggregate shocks. As in typical heterogeneous-agents models, the wealth distribution then becomes a time-varying state, which generally makes it difficult to solve for the equilibrium (let alone search for optimal policy parameters or estimate the model), see for instance [Krusell and Smith \(1998\)](#).

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baseline, since we target the same steady-state real interest rate. Details of this version are available upon requests.

In our case, it turns out that the model can be solved much faster and more easily than is often the case. In particular, the wealth distribution can be represented as a finite-dimensional object, due to the fact that the amount of liquidity in the steady state is not too large. Under these circumstances, all those who become unemployed exhaust their deposits within the first quarter, hitting the borrowing constraint right away. This implies that all employed households with the same employment duration behave identically, as do the newly unemployed with the same employment duration before job loss, and those who have been unemployed for more than one quarter.

Figure 2: Decision rules (steady state).



Notes: markers denote mass points of the liquid wealth distribution observed in the steady-state equilibrium. The black line is the 45-degree line.

We exploit this outcome, to solve the model as easily as a typical medium-scale DSGE model with a representative agent. In particular, we group agents who were employed in quarter  $t - 1$  into cohorts, indexed by the length of the employment spell in the previous quarter, denoted by  $k \geq 0$ . The cohort with  $k = 0$  is entirely employed and all enter the period with zero deposits as they were previously unemployed. Hence they make identical decisions. Therefore, agents in cohort  $k = 1$  all start with the same level of deposits,  $D_{t-1}(i)$ . Hence, conditional on their employment status, all agents within cohort  $k = 1$  make the same decisions. Extending this logic, within any cohort  $k \geq 1$  a fraction  $p^{EU}$  of the agents has become unemployed in the current quarter. They all behave identically and move out

of the cohort in the next quarter. The remaining fraction of the cohort  $1 - p^{EU}$  remains employed. Again, they all behave identically and move on to become cohort  $k + 1$  in the next quarter. Turning to the households who were unemployed in quarter  $t - 1$ , we note that all behave identically as they have depleted their deposits.

Figure 2 illustrates the steady-state choices of deposits and consumption of the different cohorts. Note that for larger values of the employment spell  $k$ , cohorts converge to a certain level of deposits and consumption. We use a total of 75 cohorts, and group all cohorts with  $k \geq K$  into one bin. We thus need to keep track of  $K$  state variables characterizing the wealth distribution. In our quantitative exercises, we set  $K = 75$ . The precise value of the cutoff  $K$  is quantitatively irrelevant as long as it is not too small. To appreciate this point, note that from Figure 2 it can be seen the behavior of cohorts beyond  $k = 15$  is almost indistinguishable.

To solve the model, we apply a first-order perturbation method for dynamic analysis, using the popular `dynare` software package. Our method may be of independent interest and is described in more detail in the Appendix.<sup>16</sup>

## 4.2 Model implications for micro-level consumption

We now explore the implications of the calibration for micro-level consumption behavior, and in particular for Marginal Propensities to Consume. This is important since the simple formula presented in Section 2 makes clear that these MPCs are key determinants of the power of QE. We evaluate MPCs at different horizons, the importance of which has been recently emphasized by Auclert et al. (2018).

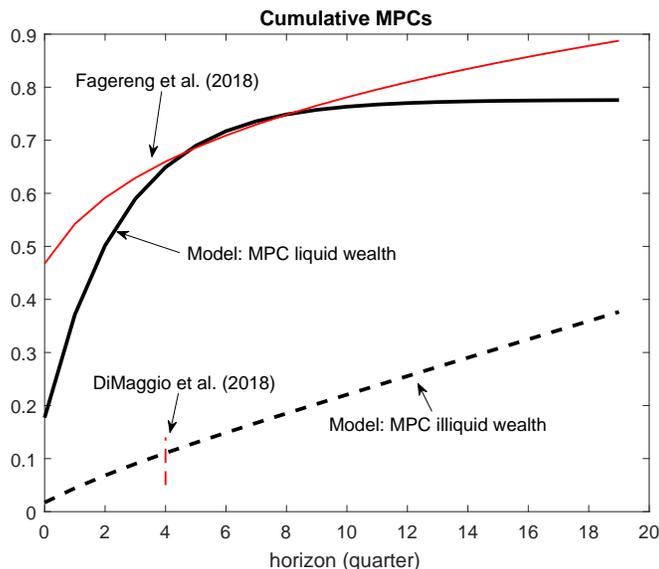
Figure 3 shows the average MPC out of *liquid* wealth across households, cumulated over time and evaluated at the steady state of the economy. In the initial quarter, household spend on average 17 percent of additional liquid wealth. Over the first year, the MPC out of liquid wealth is about 59 percent. The model also plots the average empirical MPCs out of liquid wealth as estimated by Fagereng et al. (2018) for Norwegian households, which turn out to be somewhat higher than in the model, depending on the horizon. Over longer horizons, the cumulative MPCs do not converge to one, since households use part of the additional wealth to reduce labor supply.

Analogously, the dashed line in panel A plots the average MPC out of *illiquid* (partially

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<sup>16</sup>We thus keep track of  $K = 75$  variables characterizing the wealth distribution. However, we obtained very similar results with as few as  $K = 20$  state variables. This is a much lower number than required by similar, perturbation-based solution methods. For example, the popular method of Reiter (2009) typically requires hundreds of state variables to obtain good accuracy. LeGrand and Ragot (2017) solve models by truncating idiosyncratic histories. In our application, even with a truncation cutoff lowered to  $K = 20$ , this would still imply  $2^{20}$  state variables, i.e. more than a million.

Figure 3: Average Marginal Propensities to Consume (steady state).



Notes: the figure shows average MPCs across households. The model MPC out of liquid wealth is computed as the response of a household to a surprise marginal amount of additional deposit wealth. The model MPC out of illiquid wealth is computed by giving the households a surprise unit of wealth stored in the mutual fund (see the main text for more details). Both types of MPCs are computed using a first-order perturbation method, keeping all aggregates (including prices) constant.

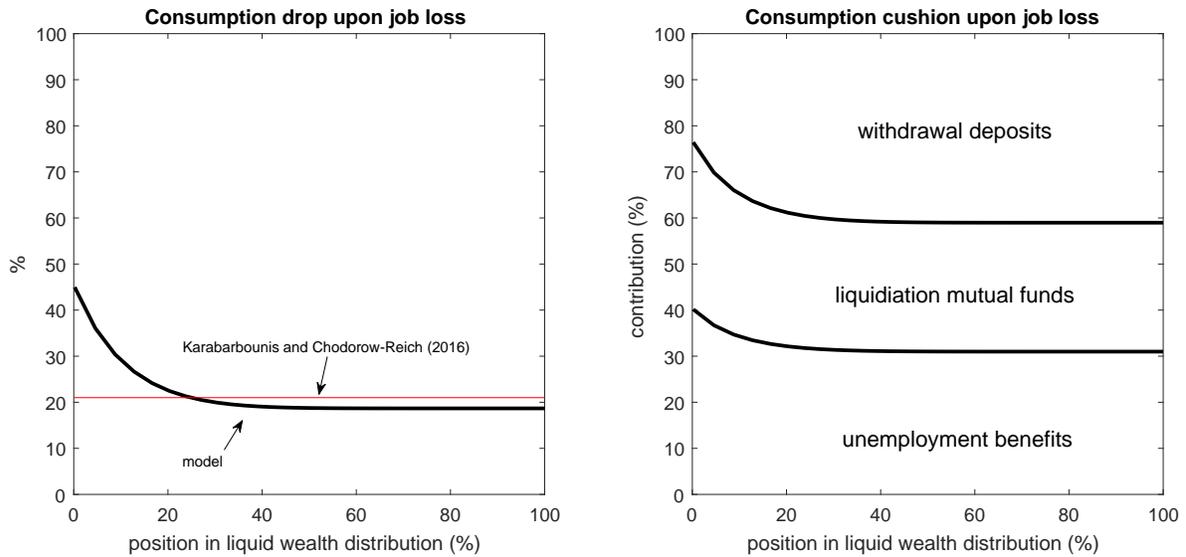
liquid) wealth. To compute this MPC, we surprise a household with an additional amount of firm equity wealth stored in the mutual fund, which in perpetuity pays off additional dividends. The amount of firm equity given to the household is chosen such that the present value of the additional dividends is equivalent to one unit of consumption. Here, the present value is computed assuming an 8 percent annual discount rate, in line with average equity returns observed in the data. Clearly, MPCs out of illiquid wealth are much lower than MPCs out of liquid wealth. Over the first year, the MPC is 9 percent, which is in the range of estimates provided by [Di Maggio et al. \(2018\)](#), based on Swedish data.

Thus, our low-liquidity calibration successfully matches recent empirical evidence on MPCs out of both liquid and illiquid wealth. As a direct result, the model also matches well the difference in MPCs out of liquid and illiquid wealth, which is crucial for the strength of the direct effects generated by QE, as argued previously. If anything, the model might slightly understate this difference.

One might also wonder about the ability of households to smooth consumption in the face of unemployment shocks. The left panel of [Figure 4](#) plots the model-implied drop in consumption upon job loss, as a function of the household's position in the distribution of liquid wealth (deposits). The line is downward-sloping, as households with more liquid

wealth are better able cushion the consumption effect of becoming unemployed. The average consumption drop is 22 percent, which is very close to the empirical estimate of Chodorow-Reich and Karabarbounis (2016), who report a 21 percent drop based on data from the Consumer Expenditure Survey.

Figure 4: Consumption behavior upon job loss.



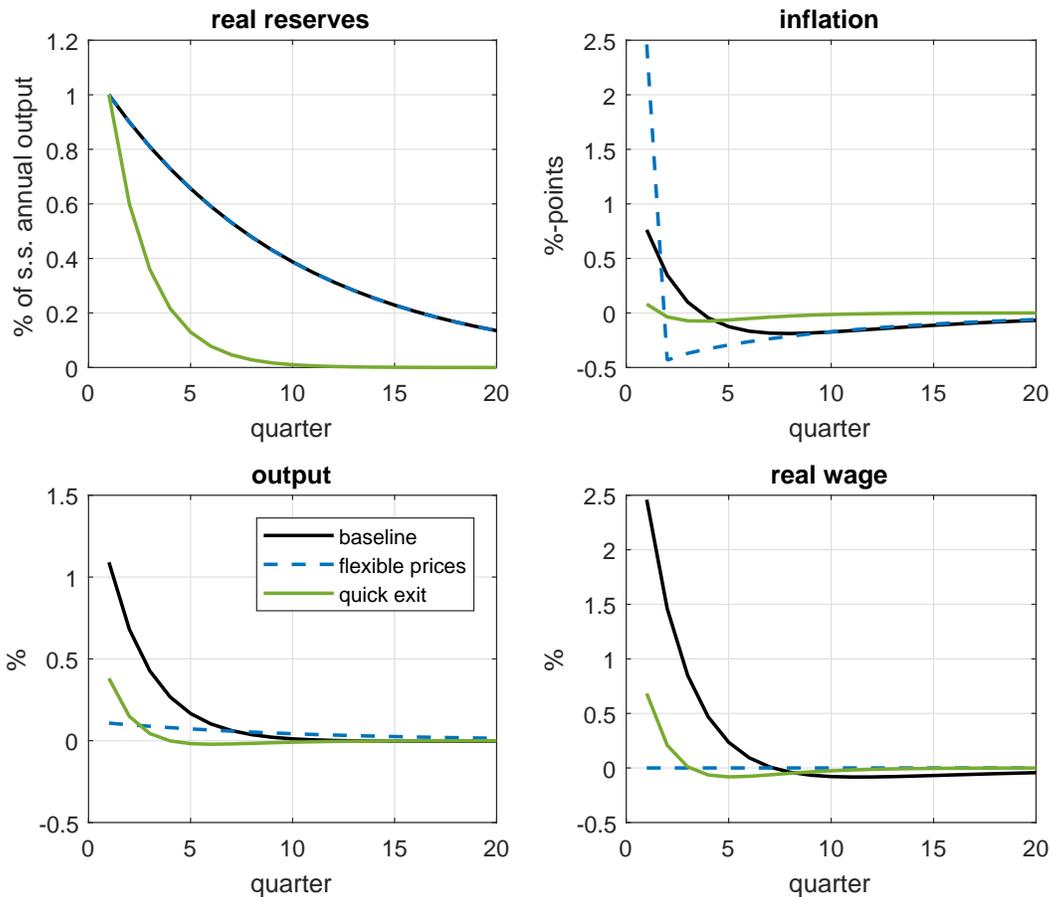
Notes: the black line in the left panel plots  $100 \cdot (1 - C_t(i)/C_{t-1}(i))$  for households who lost their jobs in the current quarter  $t$ . The right panel shows the contributions of the components of the “consumption cushion”, for households who lost their job in the current quarter  $t$ . The consumption cushion is defined as  $cush_t(i) \equiv w_{t-1}N_{t-1}(i) - (C_{t-1}(i) - C_t(i))$ . The contribution of unemployment benefit is computed as  $(\Theta^U - \Theta^E)/cush_t(i)$ , the contribution of liquidation of mutual funds as  $\mu/cush_t(i)$ , and the contribution of deposit withdrawal is computed as  $(D_t(i) - D_{t-1}(i))/cush_t(i)$ . Both panels show outcomes in the deterministic steady state.

The right panel of Figure 4 shows the composition of the “consumption cushion” upon job loss. The cushion is defined as the difference between the drop in labor income and the drop in consumption upon job loss. Between 30 and 40 percent of the consumption cushion is financed by unemployment benefits, depending on the amount of liquid assets owned by the households. These benefits directly help households alleviate the fall in consumption. Around 30 percent of the consumption cushion is due to the liquidation of mutual fund wealth. The remainder of the cushion is due to the withdrawal of deposits.

### 4.3 The impact of a QE shock

Before we compare the efficacy of QE to conventional policy, we conduct a simple experiment which helps to understand how QE affects inflation and the real economy. To this end, we consider an exogenous shock to QE, i.e., a positive innovation to  $z_t^{QE}$ . For transparency, we

Figure 5: Responses to an expansionary QE shock.



Notes: the shock is scaled such that real reserves increase by an amount equivalent to one percent of annual output on impact. The policy rule assumes  $\xi_{\Pi}^{QE} = \xi_Y^{QE} = 0$  (Real Reserve Targeting). The baseline and flexible price responses assume a persistence coefficient of  $\lambda_{z^{QE}} = 0.9$ , whereas the “quick exit” response assumes a persistence coefficient of  $\lambda_{z^{QE}} = 0.5$ .

consider a version with Real Reserve Targeting (RRT, i.e.  $\xi_{\Pi}^{QE} = \xi_Y^{QE} = 0$ ), so that there is no feedback from output and inflation to real reserves. The shock is scaled such that real reserves ( $M_t$ ) increase by 1 percent of annual output on impact. We further assume a persistence coefficient of  $\lambda_{z^{QE}} = 0.9$ , which implies that the QE expansion has a half life of about 1.7 years.

The black solid lines in Figure 5 plot the responses to the QE expansion in the baseline model. Immediately after the central bank starts purchasing government debt, output increases by 1.09 percent on impact and by 0.61 percent on average during the first year following the intervention. Inflation also responds strongly. One year after the intervention, the price level has increased by 1.16 percent. Real wages also increase substantially, reflect-

ing the increase in labor demand which ensues from the increase in goods demand. As QE is rolled back, this increase dies out.

Next, we consider a version of the model with flexible prices (i.e. setting  $\phi = 0$ ), illustrated by the blue dashed lines in Figure 5. In this case, the effect on output is much smaller, whereas there is a large spike in inflation on impact. Intuitively, the increase in prices strongly dampens the increase in goods demand following the QE intervention. That is, indirect effects mostly offset the direct effects. Real wages remain constant under flexible prices. The fact that the QE shock still creates a small increase in output under flexible prices is associated with labor supply effects and re-distributions of nominal wealth.

Finally, the green solid line in Figure 5 shows the effects in the baseline model when the QE expansion is less persistent, setting  $\lambda_{zQE} = 0.5$ , so that the exit is quicker. In that case, the initial expansion in output and inflation is much smaller. Intuitively, the contractionary effects associated with the quick unwinding of QE are immediately anticipated following the intervention, which dampens its effectiveness on impact. Thus, the overall power of a QE intervention depends crucially not only on the degree of price stickiness, but also on expectations regarding its persistence.

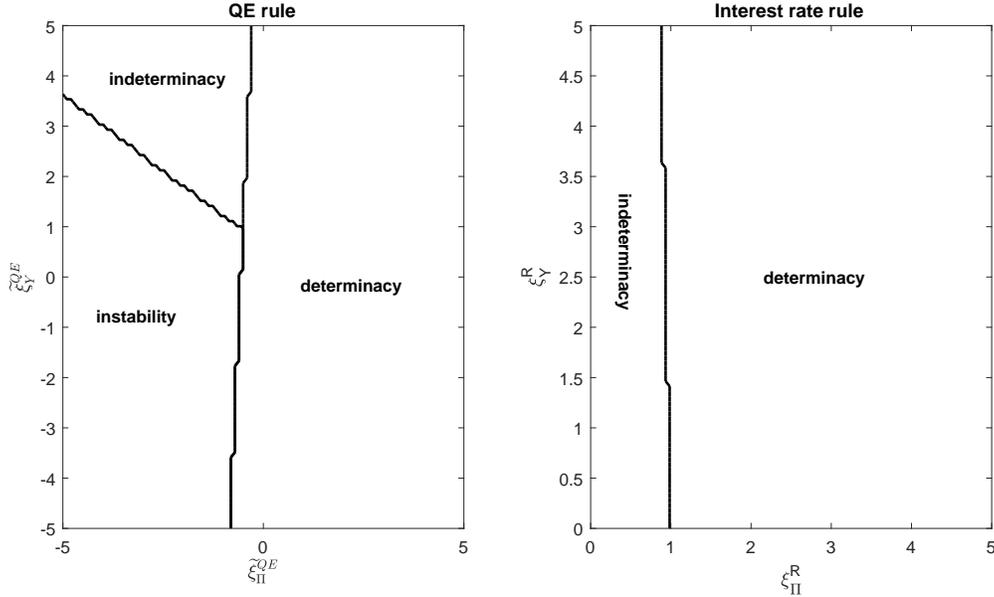
## 5 The efficacy of QE versus conventional policy

We now evaluate the efficacy of QE, drawing a comparison to conventional policy. We compare the two policies along three dimensions. First, we consider their ability to anchor expectations and thereby rule out expectations-driven fluctuations. Second, we evaluate their success in stabilizing output and inflation, traditionally a key objective of central banks. Finally, we consider their ability to mitigate the welfare costs of business cycles. The latter is affected not only by output and inflation volatility, but also by considerations regarding self-insurance and inequality.

### 5.1 Anchoring expectations

A widely appreciated objective of monetary policy is to anchor expectations. When expectations become disanchored, high inflation or deflation may arise purely due to changes in beliefs about the future. The “Taylor principle”, arguably the most celebrated policy recommendation of the NK model, concerns precisely this issue. The principle states that the central bank should let the nominal interest rate respond sufficiently aggressively to movements in inflation and/or output. When policy satisfies the Taylor principle, expectations remain anchored and belief-driven fluctuations are ruled out. [Clarida et al. \(2000\)](#)

Figure 6: Determinacy under an interest rate rule and a QE rule.



Notes: determinacy outcomes are obtained by analyzing the eigenvalues of the system of model equations, after a first-order perturbation around the deterministic steady state. “Determinacy” refers to an outcome in which the number of eigenvalues inside the unit circle coincides with the number of state variables in the system, whereas “Indeterminacy” (“Instability”) refers to an outcome in which there are more (fewer) eigenvalues inside the unit circle than there are state variables.

argue that a switch from passive to aggressive interest rate policy since Fed president Paul Volcker contributed to a dramatic decline in inflation and output volatility.

In December 2008, the Federal Funds rate was reduced to (almost) zero and stayed there until 2016. Potentially, this opened up the door to a disanchoring of expectations, as the ability of interest rate policy to respond to output and inflation had been curtailed. However, this does not happen if subsequent unconventional policy is able to successfully replace conventional policy and thus re-anchor expectations.

We now use the model to investigate the ability of QE and conventional policy to avoid belief-driven fluctuations. In more technical terms, we investigate whether the equilibrium is locally determinate around the steady state under each of the two policy rules. To this end, we consider a range of values for the stabilization coefficients of the interest rate rule and the QE rule. For ease of interpretation, we introduce re-scaled versions of the QE stabilization coefficients:  $\tilde{\xi}_Y^{QE} \equiv \frac{\bar{M}}{4Y} \xi_Y^{QE}$  and  $\tilde{\xi}_{\Pi}^{QE} \equiv \frac{\bar{M}}{16Y} \xi_{\Pi}^{QE}$ . Here,  $\tilde{\xi}_Y^{QE}$  is the response of reserves –in units of annual steady-state output– to a one percent increase in output. Moreover,  $\tilde{\xi}_{\Pi}^{QE}$  is the response of reserves –again in units of annual steady-state output– to a one percentage point increase in annualized inflation.

Figure 6 illustrates the outcomes regarding local determinacy under the various policy

configurations. The left panel shows outcomes under the QE rule, for a range of values of  $\tilde{\xi}_Y^{QE}$  and  $\tilde{\xi}_\Pi^{QE}$ . The figure shows that local determinacy does not arise under all combinations of the QE coefficients. In particular, it may fail to hold when the coefficient on inflation is sufficiently negative. The threshold for determinacy lies at around  $\tilde{\xi}_\Pi^{QE} = -0.5$ .<sup>17</sup> By contrast, the output coefficient  $\tilde{\xi}_Y^{QE}$  has relatively little impact on equilibrium determinacy.

Still, the equilibrium is locally determinate under a wide range of realistically achievable parameters of the QE rule. For example, determinacy is obtained under a Real Reserve Targeting (RRT) policy which sets  $\tilde{\xi}_\Pi^{QE} = \tilde{\xi}_Y^{QE} = 0$ . Intuitively, by targeting the amount of real reserves, the central bank creates a real anchor to the expectations of households and firms.

The right panel of Figure 6 shows outcomes under conventional policy, i.e. an interest rate rule. A substantial region of the parameter space implies local indeterminacy. Only if  $\xi_\Pi^R$  and  $\xi_Y^R$  are sufficiently high do we obtain determinacy. This reflects the Taylor Principle, which states that in a basic NK model  $\xi_\Pi^R > 1$  is typically a necessary and sufficient condition for determinacy (given  $\xi_Y^R = 0$ ). Figure 6 shows that this result applies approximately also to the incomplete-markets model considered here, although not precisely.

## 5.2 Managing aggregate fluctuations

Having established that QE can be an effective instrument to anchor expectations, we now study its power in mitigating fluctuations in aggregate output and inflation, relative to conventional policy. To this end, we set up a direct horse race between the two policy rules. Let us introduce the following loss function:

$$\mathbb{L}(\omega) = \omega Var(\hat{Y}_t) + (1 - \omega) Var(\hat{\Pi}_t).$$

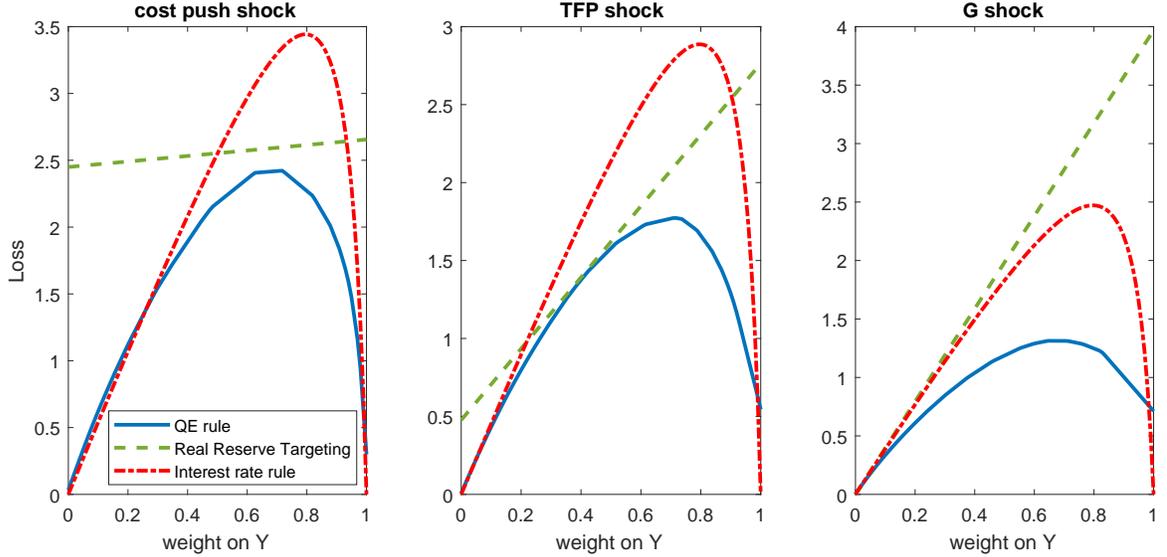
The loss function is a weighted average of the unconditional volatility of output and inflation, where the parameter  $\omega \in [0, 1]$  controls the relative weight given to output volatility.

Our purpose is to compare the ability of QE and conventional policy rules to minimize the loss  $\mathbb{L}$ . To compare the two types of policy on a fair basis, we evaluate the two rules at the *optimal* values of the policy rule coefficients, given  $\mathbb{L}(\omega)$ . In this way, the two policy rules are each given the best possible chance in achieving the objective. To implement this strategy,

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<sup>17</sup>Recall that this value of the policy coefficient means that in response to a 1 percentage point decline in annual inflation, the central bank buys government debt worth of 0.5 percent of annual GDP. To better understand why determinacy is not obtained under sufficiently negative values of  $\tilde{\xi}_\Pi^{QE}$ , note from Figure 5 that expansionary QE initially increases inflation, but reduces it over longer horizons. When  $\tilde{\xi}_\Pi^{QE} < 0$  a positive feedback between QE and inflation arises over longer horizons, which undermines a unique and stable equilibrium path.

Figure 7: Loss function under different shocks and policy configurations.



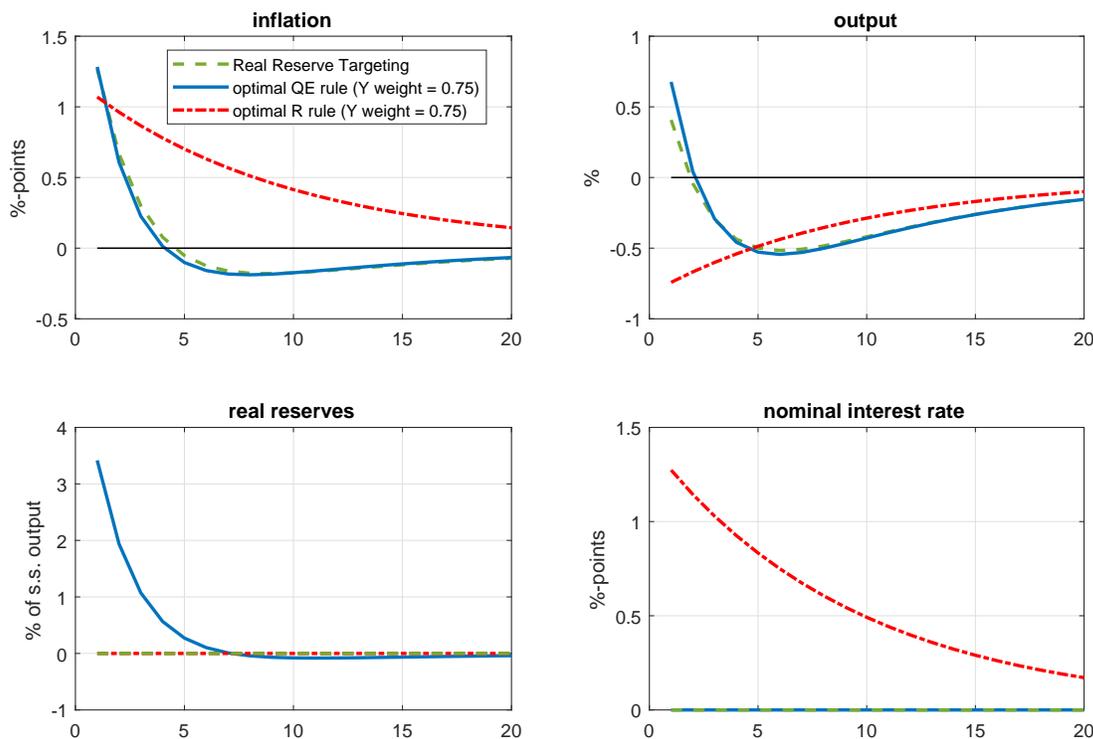
Notes: value of  $100 \cdot \mathbb{L}(\omega)$  as a function of the output weight  $\omega$ , under the optimal interest rate rule, the optimal QE rule, and the RRT rule. The optimal rules were found by searching for the values of the policy coefficients ( $\{\tilde{\xi}_{\Pi}^{QE}, \tilde{\xi}_Y^{QE}\}$  in case of QE,  $\{\xi_{\Pi}^R, \xi_Y^R\}$  in case of conventional policy) which minimize  $\mathbb{L}(\omega)$ , given  $\omega$ . To this end, we constructed a large grid for each of the two sets of policy coefficients, and solved the model at each of the grid points. Next, we constructed a grid for  $\omega$  and searched for the policy coefficients which minimize  $\mathbb{L}(\omega)$  for a given value of  $\omega$ . We did so individually for each of the three types of aggregate shocks, under our baseline calibration with sticky prices.

we first search over the values of, respectively,  $\{\tilde{\xi}_{\Pi}^{QE}, \tilde{\xi}_Y^{QE}\}$  and  $\{\xi_{\Pi}^R, \xi_Y^R\}$  which minimize  $\mathbb{L}(\omega)$ , and do so for a range of value of  $\omega$  between zero and one, each time computing the minimized objective. We also compute  $\mathbb{L}(\omega)$  under Real Reserve Targeting (RRT). In that case, we simply fix the QE rule coefficients to  $\tilde{\xi}_{\Pi}^{QE} = \tilde{\xi}_Y^{QE} = 0$  rather than searching for optimal parameters.

We consider, individually, three types of aggregate shocks: cost push shocks, TFP shocks, and government expenditure shocks, as defined previously. In each of the three cases, we assume a persistence parameter of  $\lambda_{z^{QE}} = 0.9$  and set the volatility of the shock innovations,  $\sigma_z$ , such that under an interest rate rule with  $\xi_{\Pi}^R = 1.5$  and  $\xi_Y^R = 0$ , the unconditional volatility of output is one percent.

Figure 7 plots the objective under the optimal interest rate rule, the optimal QE rule, and under RRT, for the three types of shock and the full range of output volatility weights. A striking outcome revealed by the figure is that, under a wide range of configurations, the QE policy rule is substantially more successful in stabilizing business cycles than the interest rate rule. This is particularly the case for intermediate values of  $\omega$ , i.e. when the objective is to stabilize both output and inflation. When the objective is mainly to stabilize inflation

Figure 8: Responses to a cost push shock.



Notes: responses to a cost push shock (i.e. a shock to  $\varepsilon_t$ ) under the optimal QE rule, the optimal interest rate rule, both given an output weight of  $\omega = 0.75$ , and the RRT rule. See the main text and the note of Figure 7 for details.

(i.e.  $\omega$  is close to zero) or output (i.e.  $\omega$  is close to one), the performance of the two rules is similar. We thus find that QE is not only an effective substitute for conventional policy to anchor expectations, but also to simultaneously stabilize aggregate output and inflation.

RRT is by construction less successful than the optimal QE rule, since RRT is nested in the QE rule. But interestingly, in a wide range of cases RRT actually performs better than the optimal interest rate rule, even though under RRT the policy rule coefficients have been fixed rather than optimized. Thus, despite being a very simple policy, RRT rivals conventional interest rate policy, both in terms of anchoring expectations and in terms of stabilizing output and inflation.

To understand why the QE rule and RRT are relatively successful in stabilizing both output and inflation, it is useful to consider the co-movement between the two variables. When the two variables co-move imperfectly, it is generally difficult to stabilize both variables with conventional monetary policy, given that a change in the interest rate tends to move both variables in the same direction. This is illustrated in Figure 8, which shows the

responses to a cost push shock. Under the (optimal) interest rate rule, output and inflation move in opposite directions following the shock. If the interest rate rule were more aggressive on inflation, the volatility of inflation would be dampened but the volatility of output would be increased, and vice versa.

Under QE and RRT, however, output and inflation co-move much more positively, which eases the policy trade-off. To understand what gives rise to the positive co-movement, note that the nominal interest rate is pegged under QE. Therefore, the expected real interest rate is given by  $\mathbb{E}_t \frac{\bar{R}}{\Pi_{t+1}}$ , where  $\bar{R}$  is fixed. Since a persistent cost push shock triggers a persistent increase in inflation, the real interest rate must fall. A decline in the real interest rate in turn stimulates aggregate consumption demand, pushing up output. Hence, output and inflation move in the same direction. By contrast, under conventional policy the real interest rate is given by  $\mathbb{E}_t \frac{R_t}{\Pi_{t+1}}$ , where  $R_t$  can move. Under the Taylor principle, the central bank increases the nominal interest rate more than one-for-one with inflation, which tends to create an increase rather than a decrease in the real interest rate. This in turn lowers consumption demand and pushes down output, exacerbating the negative co-movement.

### 5.3 Improving social welfare

So far, we have found that QE can be very effective in achieving two traditionally important central bank objectives: anchoring expectations and stabilizing the aggregate business cycle. We now consider the broader welfare implications. In representative-agent NK models, welfare is typically well approximated by a weighted combination of only the volatility of output and inflation. In that case, the sort of analysis we conducted in the previous subsection could also be used to evaluate the broader welfare effects of monetary policy rules.

In heterogeneous-agents economies like the one considered here, there is no direct mapping from aggregate output and inflation volatility to welfare. With incomplete markets and idiosyncratic risk, welfare also depends on factors concerning consumption insurance and inequality, which play no role in representative-agent models. Therefore, policies which are successful in stabilizing output and inflation might be undesirable from a broader welfare perspective. Moreover, optimal policy might sacrifice stability of output and inflation in order to avoid undesirable side effects on inequality.

To investigate these issues, we introduce the following utilitarian welfare objective, taken from a timeless perspective:

$$\mathbb{W} = \mathbb{E} \int_0^1 U(C(i), N(i)) di,$$

where  $\mathbb{E}$  is the unconditional expectations operator. Given this objective, we repeat the exercise of the previous subsection. That is, we again search for the policy parameters which optimize the objective, and then evaluate welfare under the optimal policy coefficients. As before we consider cost push shocks, TFP shocks and government expenditure shocks, and we also evaluate the objective under RRT.

Figure 5.3 is a contour plot of welfare outcomes for different values of the policy coefficients under QE and the interest rate rule. Red markers indicate the optimal policy coefficients, and the numbers next to the markers denote the associated welfare outcome. The latter is measured as a welfare cost of business cycles, expressed in percentages of average consumption in the steady state. Blue markers indicate Real Reserve Targeting (RRT).

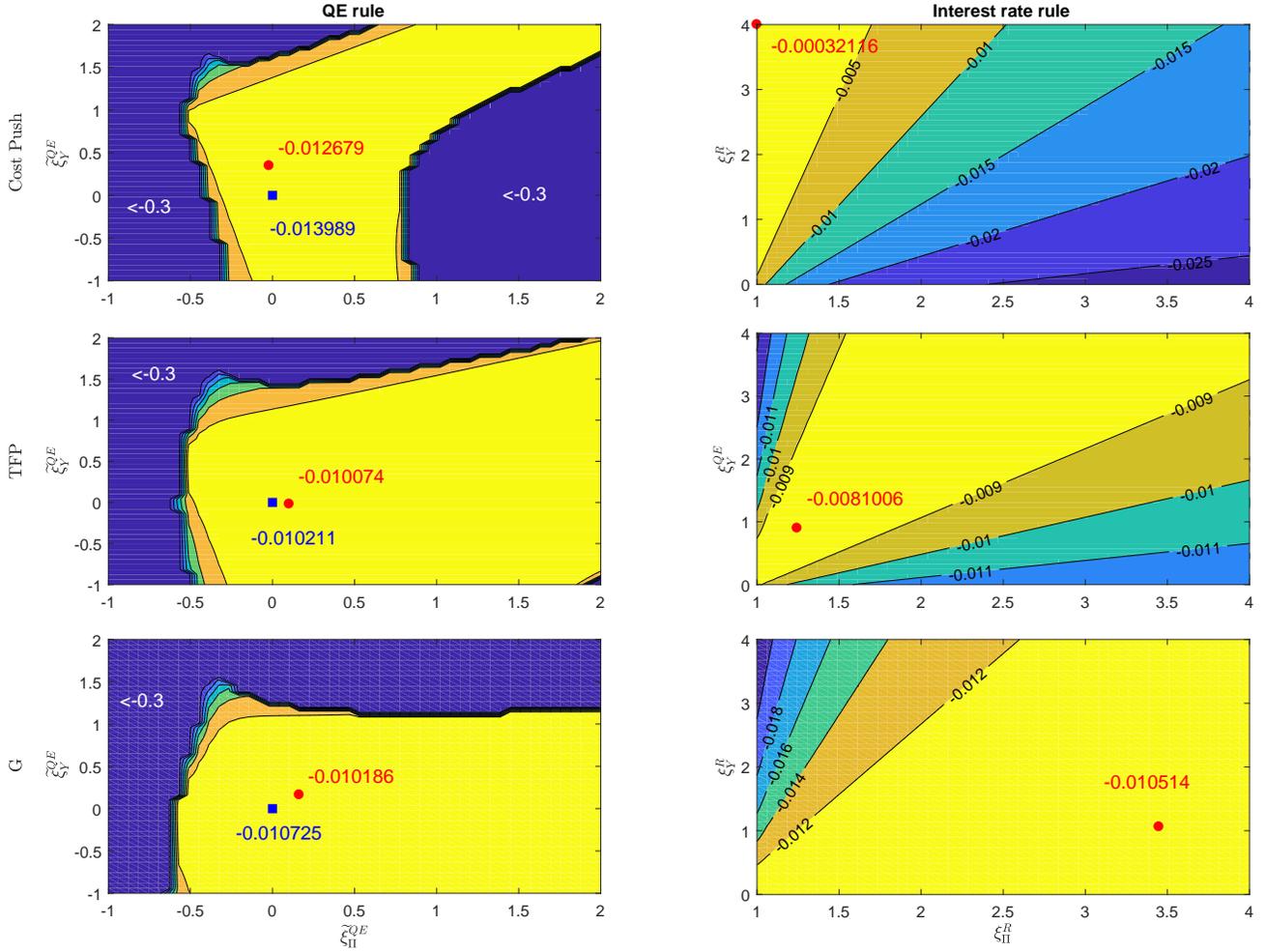
Three striking results follow from Figure 5.3. First, welfare under the optimal QE policy and RRT is generally lower than welfare under the optimal interest rate rule. The only exception is the government expenditure shock, under which the optimal QE policy performs marginally better. Under the cost push shock, however, the optimal QE policy performs substantially worse than the optimal interest rate rule. Second, under all three shocks the optimal QE rule is still quite successful in mitigating the welfare costs of business cycles, generating a loss equivalent to only 0.01 percent of consumption.<sup>18</sup> However, away from the optimal policy coefficients welfare can drop sharply. The dark blue areas in the left panels in Figure 5.3 denote configurations for which the welfare cost exceeds 0.3 percent of steady-state consumption; in some cases the cost is much more than that. Under the interest rule, by contrast, welfare is much less sensitive to the precise policy coefficients. Third, the optimal QE policy is similar to RRT, both in terms of the coefficients and in terms of the associated welfare outcome.

To understand these results, it is helpful to consider in more detail the side effects that QE can have on insurance and inequality. Note that when  $\tilde{\xi}_{\Pi}^{QE}$  and  $\tilde{\xi}_Y^{QE}$  are unequal to zero, the aggregate amount of reserves and hence the supply of deposits varies over the business cycle, as the central bank adjusts the amount of QE in response to changes in inflation and output. The variation in reserves in turn creates time variation in the supply of deposits, and hence in the extent to which households can insure themselves against idiosyncratic income risks. Via this channel, QE can contribute to time variation in consumption inequality. Periods of low insurance push unemployed households closer to zero consumption, which creates relatively large welfare costs given that households are risk averse. These adverse side effects on welfare may prompt the central bank to keep real reserves more or less constant

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<sup>18</sup>Recall that the shock volatility parameter  $\sigma_z$  was calibrated such that under an interest rate rule with  $\xi_{\Pi}^R = 1.5$  and  $\xi_Y^R = 0$ , the volatility of aggregate output is one percent.

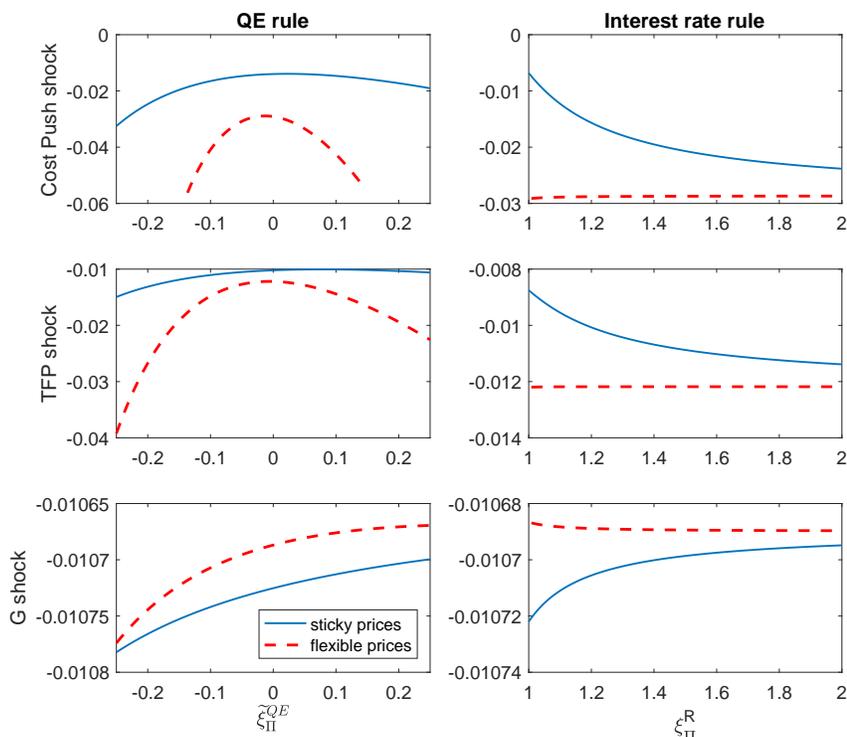
Figure 9: Welfare impact of business cycles (% s.s. consumption).



Notes: Welfare impact of business cycles as a function of the policy coefficients, expressed as percentage equivalents of average steady-state consumption. Yellow (dark blue) areas denote the highest (lowest) levels of welfare. Red round markers indicated the optimal policy, whereas blue square markers indicate RRT. We constructed a grid for the policy coefficients, solved the model at each grid point, and computed welfare ( $W$ ). We then considered a steady-version of the model with an additional lump-sum tax,  $\tau^c$ . We then solved for the level of  $\tau^c$  which renders welfare in this steady-state model exactly equal to the model with shocks, on each of the grid point. The figure plots  $-\tau^c$  as a percentage of average steady-state consumption. The top row show results for the model with cost push shocks, the middle row for a model with TFP shocks, and the bottom row for a model with government expenditure shocks. Shock volatility parameters were calibrated such that under an interest rate rule with  $\xi_{\Pi}^R = 1.5$  and  $\xi_Y^R = 0$ , the volatility of aggregate output is one percent.

even if this means that fluctuations in aggregate output and inflation are larger than they could be under more active QE policy. By contrast, conventional policy does not directly affect the amount of insurance, and hence conventional policy comes with less severe side effects on welfare.

Figure 10: Welfare impact of business cycles (% s.s. consumption) under sticky and flexible prices



Notes: welfare impact of business cycles, expressed in percentage consumption equivalents, in the baseline model and under flexible prices ( $\phi = 0$ ), setting  $\xi_Y^{QE} = \xi_Y^R = 0$ . See the note of Figure 9 for a description of the procedure.

These points are illustrated in Figure 10, which plots welfare in the baseline model as well as in a version with flexible prices, both under different policy configurations. For simplicity, we vary only the stabilization coefficients on inflation i.e. we set  $\xi_Y^{QE} = \xi_Y^R = 0$ . The right column of Figure 10 show that under conventional policy, the removal of price stickiness creates a much flatter welfare function. Intuitively, under flexible prices monetary policy is unable to affect output and inflation the Phillips Curve. As the traditional lever of monetary policy has been removed, the precise aggressiveness of the policy becomes close to irrelevant to welfare.

By contrast, removing price stickiness under the QE rule creates *more* curvature in the welfare function. That is, when prices are flexible the values of the stabilization coefficients

matter even more for welfare. This indicates that under the QE rule, much of the welfare effects of the stabilization policy operate not via the traditional monetary policy channel of the NK model, but rather via direct side effects on welfare and insurance. Therefore, the optimal QE rule is geared towards avoiding the side effects which come with time-variation in deposit supply, rather than towards stabilizing aggregate output and inflation.<sup>19</sup>

## 6 The macro effects of QE since the Great Recession

In this section we quantify the macro effects of QE on the U.S. economy since the Great Recession, when the nominal interest rate was at the zero lower bound, starting from 2008Q3. To this end, we structurally estimate the model, using data on the deviation of real output from its potential, the government-to-output ratio, the deposits-to-output ratio, and CPI inflation. To measure potential output, we use estimates from the Congressional Budget Office. The data are normalized around 2008Q3 and are shown in Figure 11.

We estimate the version of the model with QE, and four shocks: cost push shocks, TFP shocks, government expenditure shocks, and QE shocks. We assume that all four shocks follow first-order autoregressive processes, as before, and we estimate the associated parameters. One might think of QE shocks as discretionary policy interventions. At the same time we also allow for systematic responses via the QE rule, and we estimate the stabilization coefficients on output and inflation. The remaining parameters are calibrated as described above. The model is estimated by Maximum Likelihood.

Table 2 displays the estimated parameter values. The implied magnitude of QE shocks is substantial. At the same time, we also find a systematic component to the QE rule. In particular, we estimate the coefficient on inflation  $\tilde{\xi}_{\Pi}^{QE}$  to be significantly negative. The point estimate for  $\tilde{\xi}_Y^{QE}$  is positive, though not very significantly so. Thus, QE since the Great Recession appears best described as a mix of systematic and discretionary interventions.

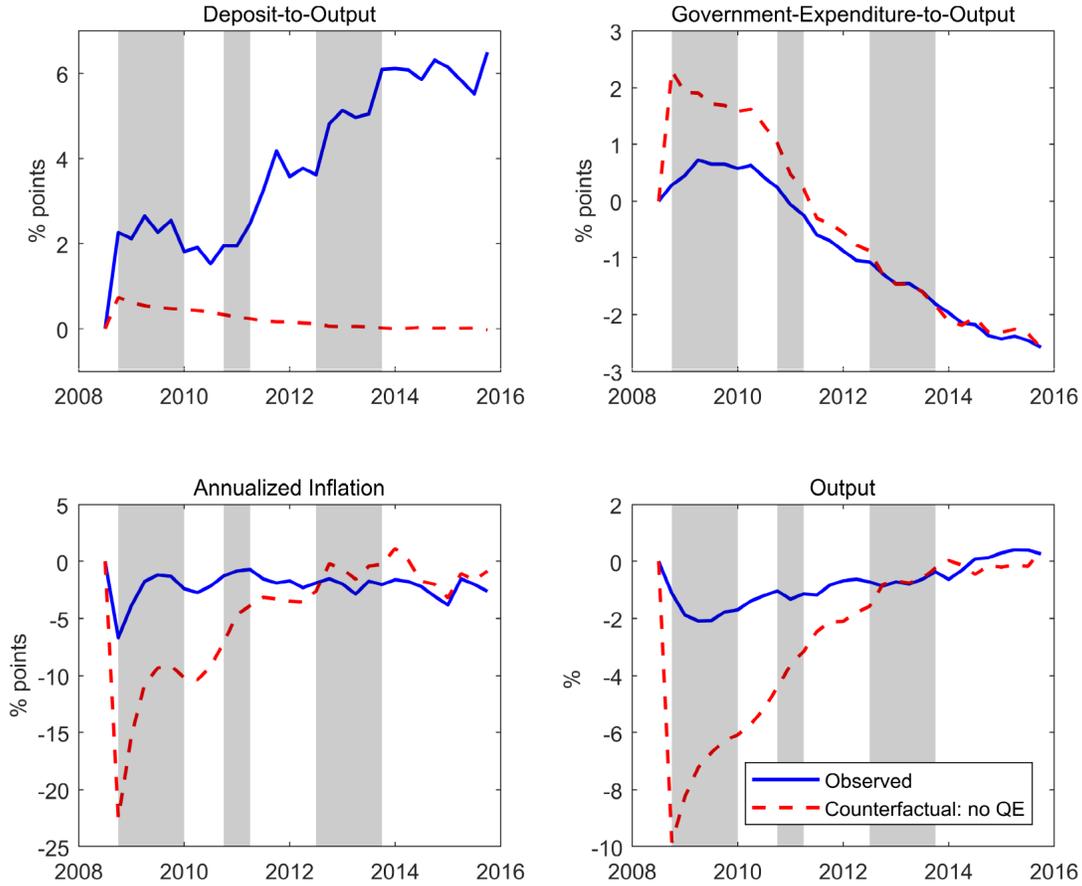
With the estimated model at hand, we quantify the effects of active QE on the macro economy. We do so by simulating a counterfactual in which we both set  $\tilde{\xi}_Y^{QE} = \tilde{\xi}_{\Pi}^{QE} = 0$  and shut down the QE shocks. In this case, real reserves and deposits remain fixed at their steady-state level throughout the sample period.

Figure 11 shows the results of this counterfactual. The difference between the two lines in the upper left panel captures the Fed’s asset purchases, which resulted in large-scale deposit

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<sup>19</sup>Note that under flexible prices, fluctuations in inflation are typically larger than under sticky prices. Given a QE rule with a certain non-zero coefficient on inflation, this generates larger fluctuations in the amount of reserves, and hence deposits. Therefore, welfare becomes more sensitive to the QE policy coefficients.

Figure 11: The impact of QE in the U.S. since the Great Recession



Notes: data series and a counterfactual simulation without QE. For a description of the data series, see the main text. In the counterfactual, we set  $\tilde{\xi}_Y^{QE} = \tilde{\xi}_\Pi^{QE} = 0$  and shut down the QE shocks. Grey areas denote rounds of QE purchases by the Federal Reserve. Time series have been normalized around 2008Q3.

Table 2: Estimated parameter values.

Parameter	Description	value	std. error	t-statistic
$\lambda_\varepsilon$	persistence cost push shock	0.014	0.317	0.044
$\lambda_A$	persistence TFP shock	0.965	0.031	31.479
$\lambda_G$	persistence G shock	0.995	0.019	51.288
$\lambda_{z^{QE}}$	persistence QE shock	0.738	0.052	11.837
$\sigma_\varepsilon$	st.dev. cost push innovation	0.118	0.026	4.574
$\sigma_A$	st.dev. TFP innovation	0.005	0.001	4.602
$\sigma_G$	st.dev. G innovation	0.007	0.001	6.516
$\sigma_{z^{QE}}$	st.dev. QE innovation	0.158	0.032	4.890
$\tilde{\xi}_Y^{QE}$	QE coef. output	0.389	0.196	1.982
$\tilde{\xi}_\Pi^{QE}$	QE coef. inflation	-0.396	0.119	3.335

Notes: parameters have been estimated using Maximum Likelihood. See the main text and Appendix for a description of the data series and the sample period.

creation. The lower right panel shows that QE had a large positive impact on aggregate output. Without active QE interventions, the recession would have been much deeper. For example, the fall in output relative to potential would have been about 10%, compared to about 1.7% at the beginning of QE in 2008Q3. Note that this effect is much larger than the direct effect computed in Section 2. Thus, in the initial years the direct effects of QE were amplified by general equilibrium effects. However, after the recession the effects of QE gradually fade out, even though the policy itself had not been rolled back. Over longer horizons, general equilibrium effects turn from an amplifying into a dampening factor, as prices have had more time to adjust. Note also that the third round of QE did not create large additional effects, according to the model, as it had been anticipated by the private sector.

Figure 11 also shows large positive effect of QE on inflation, although this effect was relatively short-lived and switched sign during 2013. The latter result reflects the overshooting of inflation also visible in Figure 5. In these responses, the effect on inflation dies out much faster than the effect on output.

## 7 Other unconventional policy options

In this section, we discuss two alternative unconventional policies. We will first consider Forward Guidance, and then analyze the potential benefits of “permanent” QE.

### 7.1 Forward Guidance

Among unconventional policies, the main alternative to QE is Forward Guidance: an announcement about monetary policy in the future. In the standard NK model without QE, Forward Guidance is an extremely effective policy once the zero lower bound on the nominal interest rate binds. In fact, macroeconomic responses to Forward Guidance turn out to be so enormous that they might call into question the basic tenets of the NK model, see [Del Negro et al. \(2012\)](#).

To address this puzzle, [McKay et al. \(2016\)](#) revisit the effects of Forward Guidance in an incomplete-markets NK model (without QE). They show that the output response to a five-year-ahead announcement is dampened substantially, relative to a representative-agent version of the model. Nonetheless, the effects of Forward Guidance remain large in comparison to empirical evidence, as presented for instance in [Del Negro et al. \(2012\)](#). [Hagedorn et al. \(2017\)](#) consider an incomplete-markets NK model with a target for nominal expenditure growth and show that in this setting the effects of Forward Guidance are much

smaller.

We explore the effects of Forward Guidance in our model, with a QE policy on the part of the central bank. For transparency, we assume that a QE rule with Real Reserve Targeting is in place, i.e. we set  $\xi_{\Pi}^{QE} = \xi_Y^{QE} = 0$ . We then consider a pre-announced decline in the nominal interest rate of 50 basis points (corresponding to about 2 percentage points on an annualized basis) which lasts for one quarter. During all other periods, the net nominal interest rate remains fixed at zero. We consider a Forward Guidance announcement two years ahead, and another one five years ahead.

Figure 12 shows the effects of the two Forward Guidance shocks. The figure shows that once the nominal interest rate is actually reduced, there is a strong decline in the real interest rate. During the quarters leading up to the implementation there is a small expansion in output, followed by a minor contraction after the implementation. Importantly, the output increase in the initial period of the announcement is almost negligible.<sup>20</sup> The impact response of the real interest rate (and hence inflation) is also extremely small. Moreover, the initial responses are declining in the announcement horizon. Finally, the lower right panel of Figure 12 shows that the Forward Guidance shock does have a substantial initial effect on the price of long-run treasury debt.

We thus conclude that once we account for incomplete markets *and* QE policy, the effects of forward guidance on output and inflation are no longer puzzlingly large. Rather, they are very small. An implication of this finding is that, in comparison, QE stands out as the more effective stabilization policy, at least when the nominal interest rate is immutable in the short run.

## 7.2 Permanent QE

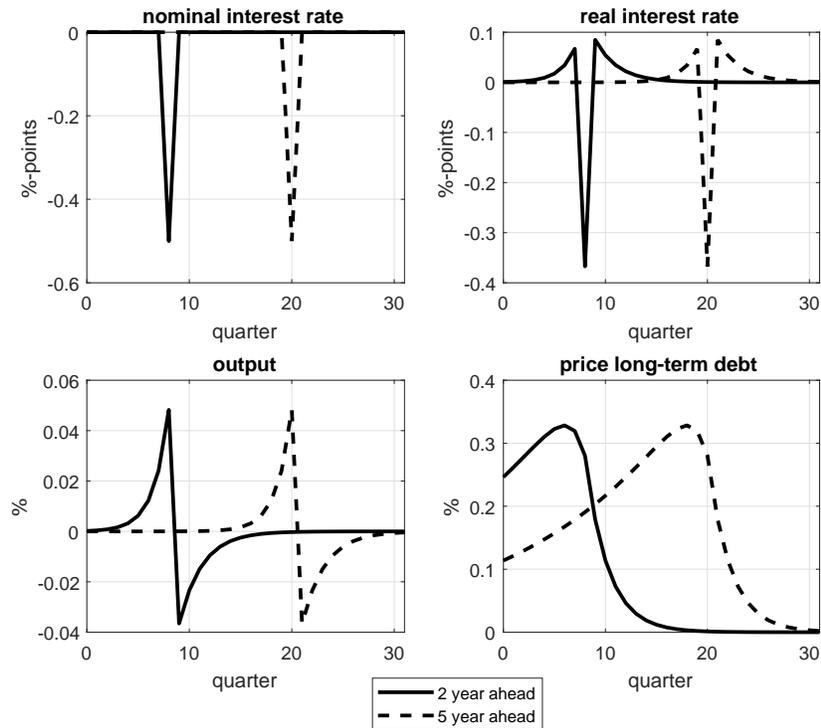
Finally, we explore the possibility of “permanent QE”, i.e. a central bank purchase of government debt which is never reversed. Potentially, such a policy could improve welfare as it increases the amount of liquidity in the hands of households, enabling them to better self-insure against idiosyncratic income risk. One may therefore wonder if it is optimal in our model for the central bank to conduct large permanent QE operations, flooding the market with liquidity.

We explore this possibility in the model, by simulating the effects of a (semi-) permanent increase (or decrease) in real reserves implemented via QE. We do so for a range of magnitudes of the QE intervention and compute the impact on welfare in each case. Figure

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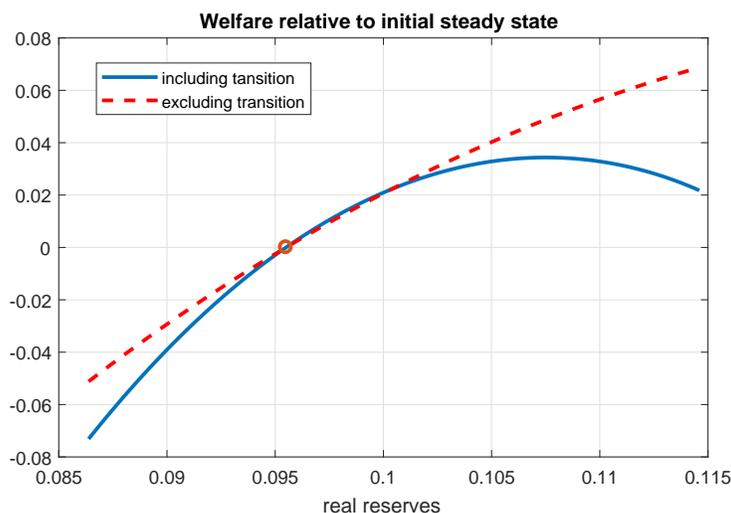
<sup>20</sup>The output response is less than 0.0002 percent. Putting this number in perspective, McKay et al. (2016) report an initial output increase of 0.25 (0.1) percent under complete (incomplete) markets, in response to a forward guidance shock to the real interest rate of 50 basis points, 20 quarters ahead.

Figure 12: Responses to a Forward Guidance shock.



Notes: responses to a forward guidance shock, reducing the quarterly nominal interest rate by 50 basis points for one quarter, and announced 2 or 5 years ahead. Responses were computed in the model version with QE, setting  $\xi_{\Pi}^{QE} = \xi_Y^{QE} = 0$ , and letting the nominal interest rate  $R_t$  vary with the forward guidance shock, starting from  $R_t = \bar{R}$ .

Figure 13: Welfare impact of permanent QE.



Notes: The figure plots the effect of permanent QE on the present value of welfare, starting from the initial steady state (red round marker), including and excluding the transition path. The effect of permanent QE on the economy was computed by simulating a one-time permanent increase in  $z_t^{QE}$ .

13 shows the results of this exercise. The red dashed line shows the level of welfare in the new steady state as a function of the level of reserves in the new steady state, excluding the transition path. As expected, welfare is increasing in the amount of steady-state liquidity (reserves).

The blue solid line again shows the effect of permanent QE on welfare, but this time including welfare along the transition path towards the new steady state. Interestingly, it is no longer the case that welfare is monotonically increasing in the size of the QE intervention. This happens as the transition to the new steady state is costly from a welfare perspective, due to the side effects created by QE. For instance, the announcement of permanent QE drives up aggregate demand, and hence the wage rate which in turn reduces firms' markups. The latter two effects create a redistribution from the employed to the unemployed, which is unfavorable from a social welfare perspective. We find that welfare is maximized at a level of reserves that is around 13 percent higher than in the original steady state (indicated by a red marker), which corresponds to a 3.8 percent increase in reserves as a fraction of annual output. Thus, the scope for improving welfare by permanently flooding the market with liquidity is rather limited, once the transition path to the new steady state is taken into account.

## 8 Concluding remarks

We have used a quantitative New Keynesian model with incomplete markets and fully and partially liquid assets to study the efficacy of Quantitative Easing (QE) as a cyclical stabilization instrument, relative to conventional interest rate policy. The direct impact of QE is determined by the difference in marginal propensities to consume out of the two types of wealth, which is large according to recent empirical evidence.

Our main finding is that a QE rule can be a very effective tool to stabilize the macro economy and that it has greatly dampened the decline in output during the Great Recession. We also find that, despite its effectiveness, it is generally not desirable to replace conventional interest rate rule with a QE rule. The reason is that such a rule can come with strong side effects on welfare, which arise from time-variation in the amount of available deposits, which exacerbate time-varying consumption inequality. The latter can be very costly from a welfare perspective as some households already consume little even without time-variation in self-insurance.

Because of the side effects, optimal QE rules avoid large fluctuations in reserves, even if this means that output and inflation volatility are higher than they could be under aggressive QE policy. In fact, a simple policy which keeps real reserves, and hence deposits, completely constant, Real Reserve Targeting (RRT), emerges as approximately welfare optimal in the model, provided that conventional interest rate policy cannot be used. In future work, it would be interesting to study the optimal simultaneous use of QE and interest rate policy.

We conclude with a note on conventional monetary policy. We have assumed that the central bank directly controls the short-term interest rate, following the NK literature. In practice, most central banks implement interest rate policy through open market operations. That is, they lower short-term interest rates by purchasing T-bills (short-term government debt) and issuing reserves, which in turn triggers deposit creation. To the extent that, from the private sector's perspective, deposits are more liquid than T-bills, the stimulating effects of QE emphasized in this paper may apply to conventional policy as well. We leave a quantitative exploration of this possibility for future research.

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# Appendix

## A. Data used in the calibration and estimation

For households' deposits data, we use "Households and Nonprofit Organizations; Checkable Deposits and Currency" from the U.S. Flow-of-Funds accounts. For consumption data, we use "Personal Consumption Expenditures" from U.S. BEA (Bureau of Economic Analysis) minus "Personal Consumption Expenditures: Durable Goods". For government expenditures data, we use "Government Consumption Expenditures and Gross Investment" from BEA. Output is defined as the sum of consumption and government expenditures. For inflation, we use the growth rate of "Consumer Price Index for All Urban Consumers". The above four series are obtained from 1985Q1 to 2018Q2, and we use the sample averages of each series to calibrate the model in the steady state.

For the estimation exercise, we only use the sub-sample period 2008Q3-2015Q4 because the nominal interest rate (i.e., the Fed Funds rate) is at (almost) zero during this period. We use the deposits-to-output ratio, the government-expenditures-to-output ratio, and inflation. For the output deviation, we do not use detrended output because the sample is too short. Instead, we first obtain the real potential GDP estimated by the U.S. Congressional Budget Office (CBO); the output deviation is then the difference between observed real GDP in natural log terms and the CBO estimated potential real GDP in natural log terms. To simplify the estimation exercise, we normalize all variables in 2008Q3 to zero.

For Survey of Consumer Finance (SCF), we use 2016 SCF Chartbook<sup>21</sup>. Specifically, we use "Median value of before-tax family income for families with holdings" Table on Page 7 and "Median value of transaction accounts for families with holdings" Table on Page 151.

## B. Computation

In the presence of aggregate shocks, the distribution of liquid wealth (deposits) fluctuates over time, which is relevant to the state of the economy. When solving for equilibrium dynamics, we therefore need to keep track of this distribution. In the calibrated model, it turns out that the liquid wealth distribution consists of only mass points. This happens as households who become unemployed spend all their liquid wealth in the initial quarter of unemployment, hitting the no-borrowing constraint within the first quarter of unemployment. Thus, all the unemployed choose  $D_t(i) = 0$ .

It follows that any household which transitions from unemployment to employment holds exactly zero deposits. As a result, all employed households with the same employment duration behave identically (see also the discussion in Section 4.1). Moreover, all households which have been unemployed for more than one quarter consume simply their current net income, whereas the newly unemployed households consume their current income plus their liquid wealth (which in turn depends on their previous employment duration).

Let us introduce some notation indicating various "cohorts" of employed and unemployed households. Let a superscript  $E$  denote the employed,  $EU$  the newly unemployed, and

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<sup>21</sup><https://www.federalreserve.gov/econres/files/BulletinCharts.pdf>.

*UU* those who have been unemployed for at least one quarter. Further, let  $k$  denote the employment duration of a household up until the current period (i.e. excluding the current period). For example  $C_t^E(k)$  with  $k = 0$  denotes the consumption level of a currently employed household which was unemployed in the previous quarter and  $C_t^{EU}(k)$  with  $k = 3$  denotes a newly unemployed household, who had completed an employment spell of 3 quarters upon job loss.

We can now characterize the household's choices with the following system of equations. For employed households we have the following equations:

$$C_t^E(k) + D_t^E(k) = w_t N_t^E(k) + \Theta_t^E + X_t^E - T_t, \quad k = 0, \quad (14)$$

$$C_t^E(k) + D_t^E(k) = w_t N_t^E(k) + \frac{R_{t-1}}{\Pi_t} D_{t-1}^E(k-1) + \Theta_t^E + X_t^E - T_t, \quad \forall k \geq 1, \quad (15)$$

$$[C_t^E(k)]^{-\sigma} = \beta \mathbb{E}_t \left[ \frac{R_t}{\Pi_{t+1}} [(1 - p^{EU}) (C_{t+1}^E(k+1))^{-\sigma} + p^{EU} (C_{t+1}^{EU}(k+1))^{-\sigma}] \right], \quad \forall k \geq 0, \quad (16)$$

$$w_t [C_t^E(k)]^{-\sigma} = \kappa_0 [N^E(k)]^{\kappa_1}, \quad \forall k \geq 0. \quad (17)$$

For the newly unemployed households (*EU*) and the remaining unemployed households (*UU*) we have:

$$C_t^{EU}(k) + D_t^{EU}(k) = \frac{R_{t-1}}{\Pi_t} D_{t-1}^E(k-1) + \Theta_t^U + X_t^U - T_t, \quad \forall k \geq 1, \quad (18)$$

$$D_t^{EU}(k) = 0, \quad \forall k \geq 1, \quad (19)$$

$$C_t^{UU} + D_t^{UU} = \Theta_t^U + X_t^U - T_t, \quad (20)$$

$$D_t^{UU} = 0. \quad (21)$$

The above system contains three blocks of equations. Equations (14), (15), (16), and (17) are budget constraints. Moreover, (16), (19), and (21) characterize the optimal choices for deposits (using the fact that the employed are at the no-borrowing constraint, whereas the unemployed are on the Euler equation for deposits), and (17) is the first-order optimality condition for labor supply of the employed households.

In practice, we truncate the above system at a certain employment duration, i.e. we let  $k = 0, 1, 2, 3, \dots, K$ , which renders the state-space finite dimensional. As can be seen from Figure 2, under our calibration, households converge fairly quickly to a maximum amount of assets. In our application, we set  $K = 75$  and verify that results are insensitive to the truncation threshold.<sup>22</sup> We close the system by setting for the final cohort of employed

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<sup>22</sup>Setting the threshold as low as  $K = 20$  delivers very similar results.

households:

$$\begin{aligned}
C_t^E(K) + D_t^E(K) &= w_t N_t^E(K) + \frac{R_{t-1}}{\Pi_t} D_{t-1}^E(K) + \Theta_t^E + X_t^E - T_t, \\
(C_t^E(K))^{-\sigma} &= \beta \mathbb{E}_t \left[ \frac{R_t}{\Pi_{t+1}} \left[ (1 - p^{EU}) (C_{t+1}^E(K))^{-\sigma} + p^{EU} (C_{t+1}^{EU}(K))^{-\sigma} \right] \right], \\
w_t [C_t^E(K)]^{-\sigma} &= \kappa_0 [N^E(K)]^{\kappa_1}.
\end{aligned}$$

These equations impose that beyond an employment duration of  $k = 75$  quarters (i.e., more than 18 years), all households behave identically. This is not a very restrictive cutoff, since households already behave practically identically beyond an employment duration of 10 to 20 quarters, see Figure 2. We solve the above system jointly with the remaining model equations, which are given by:

$$\begin{aligned}
1 - \varepsilon_t + \varepsilon_t \frac{w_t}{A_t} &= \phi (\Pi_t - 1) \Pi_t - \phi \mathbb{E}_t \left[ \beta \frac{Y_{t+1}}{Y_t} (\Pi_{t+1} - 1) \Pi_{t+1} \right], \\
u X_t^U + (1 - u) X_t^E &= Div_t + (1 + \rho q_t) \frac{B_{t-1}^m}{\Pi_t} - q_t B_t^m, \\
X_t^U &= X_t^E + \mu, \\
q_t &= \mathbb{E}_t \left[ \beta \frac{1 + \rho q_{t+1}}{\Pi_{t+1}} \right], \\
G_t &= q_t B - (1 + \rho q_t) \frac{B}{\Pi_t} + T_t^{cb} + T_t, \\
T_t^{cb} + \frac{R_{t-1}}{\Pi_t} M_{t-1} + q_t B_t^{cb} &= M_t + (1 + \rho q_t) \frac{B_{t-1}^{cb}}{\Pi_t}, \\
B_t^{cb} + B_t^m &= B, \\
\sum_{k=0}^K \psi^E(k) D_t^E(k) &= M_t, \\
\sum_{k=0}^K \psi^E(k) N_t^E(k) &= N_t, \\
Y_t - G_t + \phi Y_t (\Pi_t - 1)^2 &= \sum_{k=0}^K \psi^E(k) C_t^E(k) + \sum_{k=0}^K \psi^{EU}(k) D_t^{EU}(k) + \psi^{UU} C_t^{UU}, \\
Y_t &= A N_t,
\end{aligned}$$

and in addition the policy equations for either QE or conventional policy and the exogenous evolution of  $(\varepsilon_t, A_t, G_t, z_t^{QE})$ , as stated in the main text. In the above equations,  $\psi^E(k)$ ,  $\psi^{EU}(k)$ , and  $\psi^{UU}$  are population share parameters, which satisfy  $\psi^{UU} = u - p^{EU}(1 - u)$ ,  $\psi^E(k) = p^{UE}(1 - p^{UE})^k u$  for  $k < K$ ,  $\psi^E(K) = 1 - \sum_{k=0}^{K-1} \psi^E(k) - u$ ,  $\psi^{EU}(k) = p^{UE}(1 - p^{UE})^k p^{EU} u$  for  $k < K$ , and  $\psi^{EU}(K) = p^{EU} \psi^E(K)$ . For equilibrium dynamics, we use a first-order perturbation method to solve for the joint system.

Finally, we discuss how to verify easily that in equilibrium the unemployed hit the no-

borrowing constraint in deposits. This is the case if:

$$[C_t^{EU}(K)]^{-\sigma} > \beta \mathbb{E}_t \left[ \frac{R_t}{\Pi_{t+1}} [p^{UE}(C_{t+1}^E(k=0))^{-\sigma} + (1-p^{UE})(C_{t+1}^{UU})^{-\sigma}] \right].$$

This equation implies that the newly unemployed with the longest previous employment spell do not want to save, i.e., they are at the constraint. If this condition holds, then the same is true for all the other unemployed households, since these are less wealthy, which implies that  $C_t^{UU} \leq C_t^{EU}(K)$  and  $C_t^{EU}(k) \leq C_t^{EU}(K)$ . See also Figure 2 for an illustration of this point. We verify that the above equation holds in the steady state.<sup>23</sup>

### C. Steady state and calibration

This appendix shows how to solve for the steady-state economy in a systematic way, which is useful for the calibration exercise. The calibration strategy is shown after we discuss how to solve the steady state efficiently. A more “black-box” alternative is to solve the entire system of steady-state equations all at once using a numerical solution routine. The procedure below, however, makes it easier to hit certain calibration targets.

#### Solving for the steady-state equilibrium

We first show how to solve for the steady-state equilibrium, given all parameters of the model. To save notation, we let all variables refer to their steady-state levels from now on. In the steady state,  $q = 1/(\beta^{-1} - \rho)$  and the wage rate is  $w = (\varepsilon - 1)/\varepsilon$ . The steady-state inflation is  $\Pi = 1$ . The mutual fund has an exogenous extra amount of liquidation  $\mu$  for unemployed agents. The government targets exogenous levels of  $G$ ,  $B$ ,  $M$ ,  $\Theta^U$ , and thus  $\Theta^E$  (because of the budget-neutral insurance). Next, we solve for the resulting tax policy  $T$  and interest rate policy  $r$ , together with dividend payout to employed agents  $X^E$ .

Suppose we have an initial guess of  $(T, r, X^E)$ . There are  $K$  cohorts of employed agents. The labor supply decision of the  $k$ th cohort satisfies

$$\frac{w}{C^E(k)} = \kappa_0 N^E(k),$$

which means that the labor income is  $wN^E(k) = \frac{w^2}{C^E(k)\kappa_0}$ . To this end, we first solve the consumption and saving choice. For the  $K$ th cohort, the Euler equation for deposits is given by:

$$\frac{1}{C^E(K)} = \beta r \left[ p^{EU} \frac{1}{C^{UU} + D^E(K)r} + (1-p^{EU}) \frac{1}{C^E(K)} \right],$$

and the budget constraint is

$$C^E(K) = \frac{w^2}{C^E(K)\kappa_0} + D^E(K)(r-1) + \tilde{\Theta}^E$$

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<sup>23</sup>Under a local perturbation, the constraint then also holds outside the steady state.

where  $\tilde{\Theta}^E = \Theta^E + X^E - T$ . The above two equations pin down  $C^E(K)$  and  $D^E(K)$ . There is even an analytical solution as  $C^E(K)$  can be solved from a quadratic equation. To see this, rearrange the Euler equation

$$\frac{1 - \beta r (1 - p^{EU})}{C^E(K)} = \frac{\beta r p^{EU}}{C^{UU} + r D^E(K)} \rightarrow D^E(K) = \frac{\beta p^{EU}}{1 - \beta r (1 - p^{EU})} C^E(K) - \frac{C^{UU}}{r},$$

which can be used to express the budget constraint as a quadratic equation of  $C^E(K)$ :

$$\left[ 1 - \frac{(r-1)\beta p^{EU}}{1 - \beta r (1 - p^{EU})} \right] [C^E(K)]^2 + \left( \frac{r-1}{r} C^{UU} - \tilde{\Theta}^E \right) C^E(K) - \frac{w^2}{\kappa_0} = 0.$$

Since  $\left[ 1 - \frac{(r-1)\beta p^{EU}}{1 - \beta r (1 - p^{EU})} \right] > 0$  and  $-\frac{w^2}{\kappa_0} < 0$ , the only positive root is

$$C^E(K) = \frac{\tilde{\Theta}^E - \frac{r-1}{r} C^{UU} + \sqrt{\left( \tilde{\Theta}^E - \frac{r-1}{r} C^{UU} \right)^2 + 4 \left[ 1 - \frac{(r-1)\beta p^{EU}}{1 - \beta r (1 - p^{EU})} \right] \frac{w^2}{\kappa_0}}}{2 \left[ 1 - \frac{(r-1)\beta p^{EU}}{1 - \beta r (1 - p^{EU})} \right]}.$$

For cohort  $k = K - 1, K - 2, \dots, 1$ , the Euler equation for deposits is given by

$$\frac{1}{C^E(k)} = \beta r \left[ p^{EU} \frac{1}{C^{UU} + D^E(k)r} + (1 - p^{EU}) \frac{1}{C^E(k+1)} \right],$$

and the budget constraint is

$$C^E(k) = \frac{w^2}{C^E(k)\kappa_0} + D^E(k-1)r - D^E(k) + \tilde{\Theta}^E.$$

This means that, given a value  $D^E(K-1)$  which is arbitrarily close to but smaller than  $D^E(K)$ , we can solve backwards for consumption and deposits of the employed households as:

$$C^E(k) = \beta^{-1} r^{-1} \left[ p^{EU} \frac{1}{C^{UU} + D^E(k)r} + (1 - p^{EU}) \frac{1}{C^E(k+1)} \right]^{-1},$$

$$D^E(k-1) = \frac{1}{r} \left[ D^E(k) + C^E(k) - \frac{w^2}{C^E(k)\kappa_0} - \tilde{\Theta}^e \right],$$

for  $i = K - 1, K - 2, \dots, 1$ .

For the unemployed agents without any savings, the budget constraint implies:

$$C^{UU} = \Theta^{UU} + X^U - T$$

Let us now guess  $C^E(1)$ . For the  $i = 0$ th cohort, the Euler equation and the budget constraint are

$$C^E(0) = \beta^{-1} r^{-1} \left[ p^{EU} \frac{1}{C^{UU} + D^E(0)r} + (1 - p^{EU}) \frac{1}{C^E(1)} \right]^{-1}$$

$$C^E(0) = \frac{w^2}{C^E(0)\kappa_0} - D^E(0) + \tilde{\Theta}^E.$$

Since we know  $C^E(1)$ , the two equations solve the two unknowns  $C^E(0)$  and  $D^E(0)$ .

For any  $k = 1, 2, \dots, K - 1$  cohort we then obtain

$$C^E(k) = \frac{w^2}{C^E(k)\kappa_0} + rD^E(k-1) - D^E(k) + \tilde{\Theta}^E,$$

and also  $D^E(k) = \frac{w^2}{C^E(k)\kappa_0} + rD^E(k-1) + \tilde{\Theta}^E - C^E(k)$ . From the Euler equation

$$\frac{1}{C^E(k)} = \beta r \left[ p^{EU} \frac{1}{C^{UU} + D^E(k)r} + (1 - p^{EU}) \frac{1}{C^E(k+1)} \right],$$

we obtain  $C^E(k+1) = \left[ \frac{1}{\beta r(1-p^{EU})C^E(k)} - \frac{p^{EU}}{(1-p^{EU})(C^{UU} + D^E(k)r)} \right]^{-1}$ . Therefore, given  $C^E(1)$ , we can calculate  $D^E(1)$ ,  $C^E(2)$ ,  $D^E(2)$ , ...  $C^E(K-1)$  and  $D^E(K-1)$ . We look for  $C^E(1)$  such that  $D^E(K-1) = D^E(K) - \epsilon$  where  $\epsilon$  is an arbitrary small and positive number (i.e., the amount of savings converge to the fixed point). This is effectively a shooting algorithm.

Aggregate labor supply is then given by

$$N = \sum_{k=0}^K \psi^E(k) N^E(k) = \sum_{k=0}^K \psi^E(k) \frac{w}{\kappa_0 C^E(k)}. \quad (22)$$

Now, we turn to the government side. Notice that the debt held by the central bank is

$$B^{cb} = \frac{M(1-r)}{q - (1 + \rho q)},$$

and the total debt held by the mutual fund is thus

$$B^m = B - B^{cb}.$$

Finally, after obtaining all equilibrium objects with a given  $(T, r, X^E)$ , we check the following three equations. Since the total payout is  $(1 - p^U)X^E + p^U X^U = X^E + p^U \mu$ , we know that

$$X^E + p^U \mu = N(1 - p^U) - wN + (1 + \rho q - q)B^m. \quad (23)$$

The market clearing for reserves is given by:

$$M = \sum_{k=0}^K \psi^E(k) D_t^E(k). \quad (24)$$

The goods market clearing is given by:

$$\sum_{k=0}^K \psi^E(k) C^E(k) + \sum_{k=0}^K \psi^{EU}(k) D^{EU}(k) + \psi^{UU} C^{UU} + G = AN. \quad (25)$$

These three equations above solve the three unknowns  $(T, r, X^E)$ . That is, if these three equations do not hold, we change our initial guess of  $(T, r, X^E)$  and iterate the computation.

## Calibration

The above strategy for calculating the steady-state equilibrium objects will be used in the following calibration exercise.

The average labor supply is targeted (1/3 in our calibration), so  $N$  is known. Recall that the wage rate is  $w = (\varepsilon - 1)/\varepsilon$ , and we thus know the average labor income in the model. The unemployment benefit is calibrated to be a fraction (0.25 in our calibration) of average labor income, so  $\Theta^U$  is known. Because of the budget-neutral unemployment insurance,  $\Theta^E$  is also known. Without loss of generality, we normalize the steady-state TFP to  $A = 1$ , so that  $Y = N$ . We target the reserves-to-output ratio  $M/4Y$  (or  $M/Y$ ), the government-expenditure-to-output ratio  $G/Y$ , the real interest rate  $r$ , and the median deposits-to-income ratio. With these targets, we directly obtain  $G$  and  $M$ . The following discussion shows how we calibrate  $B$ ,  $\kappa_0$ , and  $\mu$ .

First, we have an initial guess of the disutility of labor supply parameter  $\kappa_0$  and the extra liquidation parameter  $\mu$ ; these will be used when we solve for the households' decision rules.

Second, given  $(\kappa_0, \mu)$ , the lump-sum tax  $T$ , or equivalently the level of government debt  $B$ , is set such that the model hits the median deposits-to-income ratio. To see this, notice that the consolidated fiscal and monetary budget constraint implies that

$$G + (r - 1)M + (1 + \rho q - q)B^m = T. \quad (26)$$

For any given  $T$ , we obtain  $B^m = [T - (r - 1)M - G]/(1 + \rho q - q)$ ; using (23) gives  $X^E$ . Now, we have  $(T, r, X^E)$ , and we can follow the strategy specified before to calculate steady-state equilibrium objects. The lump-sum tax  $T$  is adjusted so that the model generates the median deposits-to-income ratio as in the data. In addition, since the level of total debt satisfies  $B = B^m + B^{cb}$ , we can also view that  $B$  is calibrated to hit the median deposits-to-income ratio, where  $B$  is given by

$$B = B^m + B^{cb} = B^m + M \frac{(1 - r)}{q - (1 + \rho q)}.$$

Finally, to hit the steady-state labor supply  $N$  and the real interest rate  $r$ , the guessed values of  $\kappa_0$  and  $\mu$  are adjusted such that the labor market clearing condition (22) and the reserve market clearing condition (24) are satisfied.<sup>24</sup> Notice that for a different pair of  $(\kappa_0, \mu)$  one needs to re-calibrate  $B$ .

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<sup>24</sup>Equation (25) is not used here as we have used the consolidated fiscal and monetary budget constraint in calibration. The households' budget constraints and the consolidated government budget constraint imply the goods market clearing condition (25).