Abstract

The development of Asia exposed commodity-exporting economies to unprecedented changes in their terms of trade. Using a small open economy model we estimate changes in the long-run level and variance of Australia’s terms of trade and study the quantitative implications of these changes. We find that long-run commodity prices increased by 40 per cent in mid-2003 and that the volatility of shocks to commodity prices doubled soon after. The increase in the level of commodity prices is smaller than single-equation estimates suggest. But our inferences rely on many observables that in general equilibrium also respond to shifts in the long-run level of the terms of trade.
1 Introduction

Over the past two decades, the increasing development of Asia has exposed many economies to a surge in global commodity prices. There have been commodity price booms before, but this one, fuelled by an unprecedented era of high growth in China, has been by far the largest and most persistent.

A recurring question for commodity exporting economies is the extent to which the recent increases in commodity prices are permanent.\(^1\) One view is that these fluctuations are the result of an unprecedented, and permanent, shift in the demand for commodities associated with the economic development of Asia. According to this view, the level of commodity prices has reached a permanently higher ‘new normal’.\(^2\) Others argue that these fluctuations are unrelated to long-run fundamentals but are instead the result of increased speculation in commodity markets.\(^3\) This view also implies a ‘new normal’ for commodity prices, but one associated with greater volatility rather than a permanently higher level. Understanding whether a ‘new normal’ exists, what it entails, and what its implications are has become of first order importance for these economies.

In this paper, we take these competing hypotheses to the data. To this end, we set up a model with multiple productivity trends that can capture the drifts in relative prices that we see in the data. We do so because if a permanent change in the terms of trade influences other relative prices, as theory asserts, then it is important to account for existing trends in relative prices to identify shifts in the long-run level of the terms of trade. We estimate the model allowing – but not requiring – the long-run level of commodity prices and the volatility of shocks to commodity prices to change. A permanent change in the long-run level of commodity prices gives rise to a transition towards a new balanced growth path. It is a matter of econometric identification that we tackle in this paper to distinguish trends that belong to the balanced growth path, cycles around those trends, and fluctuations that originate from a transition towards a new balanced growth path.

We estimate the model on Australian data because Australia is a commodity-exporting economy that has been exposed to the rise of Asia. Over the past two decades, the foreign currency price of Australia’s commodity exports increased by more than 200 per cent. As Figure 1 shows, Australia’s commodity export prices have experienced similar fluctuations to those of world commodity prices. This suggests that our analysis may be of interest to other commodity exporting economies as well.

\(^1\)Because for commodity-exporting economies the bulk of recent terms of trade fluctuations comes from commodity prices movements, we use these terms interchangeably.

\(^2\)See Bernanke (2008), Bloxham et al. (2012), Stevens (2011), and Yellen (2011).

\(^3\)See Tasker (2013).
We find support in the data for both hypotheses. In particular, we find that the long-run level of Australia’s commodity prices increased by 40 per cent in mid 2003 and that the volatility of shocks to commodity prices more than doubled soon after that. An increase of 40 per cent in the long-run level of commodity prices is probably less than one would infer from visual inspection of Figure 1; it is also less than the 100 per cent increase suggested by single equation reduced-form specifications. Our inferences, however, rely on more observables than single equation estimates. An increase of 100 per cent in our model is unlikely, not because commodity prices would disagree, but because other observable series would disagree. In general equilibrium, the estimated long-run level of commodity prices depends on the cross-equations restrictions that rational expectations imposes on every observable series.

The economic implications of the 40 per cent increase in the long-run level of commodity prices that we estimate are, however, significant. To name a few: the commodity sector’s share of exports increases from 35 to 52 per cent; consumer price inflation falls temporarily by 2 percentage points with tradable inflation and non-tradable inflation strongly offsetting each other; the trade deficit widens by around 2 percentage points of GDP for a decade and the real exchange appreciates permanently by 30 per cent.

Our work builds on that of Rabanal (2009) and Siena (2014), who set up models with different productivity trends in the tradable and non-tradable sectors. We add capital
accumulation with a differential trend in the relative price of investment goods as well
as a commodity-exporting sector that takes the relative price of its output as given. Our work also relates to a large literature on the role of terms of trade shocks in open
economies to which we cannot do justice in the space we have here. 4 Mendoza (1995)
studies the contribution of terms of trade shocks to the business cycle in a calibrated
open economy RBC model and finds that terms of trade shock account for 50 per cent of
output fluctuations. Other papers, like Dib (2008) and Medina and Soto (2007), study
the impact of terms of trade shocks in DSGE models with nominal frictions. Our work
differs from this literature because we distinguish temporary shocks to the terms of trade – which has been the focus in the small open economy literature – from permanent shifts
in the long-run level of the terms of trade.

The rest of the paper is structured as follows. Section 2 discusses the model. Many
details can be found in the online appendix. 5 Section 3 discusses our empirical approach
which involves calibration and estimation of date breaks and parameters. Section 4
describes the main results. Section 5 analyses the responses of the estimated model to
temporary and permanent changes in the terms of trade. Section 6 concludes suggesting
avenues for further research.

2 Model

We extend the standard small open economy model with nominal rigidities in the fol-
lowing ways: we include capital accumulation, non-tradable and commodity-exporting
sectors; and, following Rabanal (2009), we include trends in sector-specific productivity
technology processes that give rise to inflation differentials in steady state. 6 Since the
model is large, we present the basic ingredients in the main text and leave a comprehen-
sive presentation to the appendix.

4 Instead, we point the reader to Ostry and Reinhart (1992), Bidarkota and Crucini (2000), Bleaney
and Greenaway (2001), Broda (2004), Blattman et al. (2007), Jäskelä and Smith (2013), Charnavoki
and Dolado (2014) and the references therein.
5 Our online appendix can be found here http://sites.google.com/site/marianokulish/
6 For a version of the small open model with nominal rigidities see, for example, Gali and Monacelli
(2005).
2.1 Households

The preferences of a typical household in the small open economy are given by:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \zeta_t \left[ \log(C_t - hC_{t-1}) - \varepsilon_{L,t} \frac{L_t^{1+\nu}}{1+\nu} \right] \right\} \]

where \( E_0 \) denotes the time 0 conditional expectation, \( \beta \) is the household’s discount factor, \( C_t \) is consumption, \( L_t \) is labour supply and \( h \in [0,1] \) governs the degree of external habit formation. The variable \( \zeta_t \) is an intertemporal preference shock that follows the stochastic process:

\[ \log \zeta_t = \rho \zeta \log \zeta_{t-1} + u_{\zeta,t} \]

with \( u_{\zeta,t} \) independently and identically distributed \( N(0,\sigma^2_\zeta) \). The variable \( \varepsilon_{L,t} \) is a labour supply shock that follows the process:

\[ \log \varepsilon_{L,t} = \rho \varepsilon \log \varepsilon_{L,t-1} + u_{L,t} \]

with \( u_{L,t} \) independently and identically distributed \( N(0,\sigma^2_L) \).

Aggregate labour supply consists of labour supplied to the home-tradable goods producing sector, \( L_{H,t} \), the non-tradable goods producing sector, \( L_{N,t} \), and the commodity-exporting sector, \( L_{X,t} \), according to the Constant Elasticity of Substitution (CES) bundle:

\[ L_t = \left[ \xi_H L_{H,t}^{1+\omega} + \xi_N L_{N,t}^{1+\omega} + \xi_X L_{X,t}^{1+\omega} \right]^{\frac{1}{1+\omega}} \]

Workers view employment in different sectors as imperfect substitutes. The parameter \( \omega \) controls the willingness of workers to move between sectors in response to wage differentials, while the parameters \( \xi_H, \xi_N \) and \( \xi_X \) govern the relative desirability of supplying labour to each sector.

Households enter the period with \( K_{j,t} \) units of capital from sector \( j \in \{H,N,X\} \), \( B_t \) units of one-period risk-free bonds denominated in domestic currency and \( B^*_t \) units of one-period risk-free bonds denominated in foreign currency. During the period, the household receives wages, returns on capital and profits and pays lump sum transfers to the government. The household uses its income to purchase new bonds, to invest in new
capital and to purchase consumption goods. The resulting flow budget constraint is:

\[
P_t C_t + \sum_{j \in \{H,N,X\}} P_{t,j} I_{j,t} + B_{t+1} + S_t B_t^* + R_t^r S_t B_t^* + \sum_{j \in \{H,N,X\}} (W_{j,t} L_{j,t} + R_{j,t} K_{j,t} + \Gamma_{j,t}) - T_t \leq R_t - 1 B_t + R_{t,F} - 1 S_t B_t^* + \sum_{j \in \{H,N,X\}} (W_{j,t} L_{j,t} + R_{j,t} K_{j,t} + \Gamma_{j,t}) - T_t \]

where \(P_t\) is the consumer price index, \(P_{t,j}\) is the price of the aggregate investment good, \(I_{j,t}\) is investment in sector \(j\), \(W_{j,t}\), \(R_{j,t}\) and \(\Gamma_{j,t}\) are the wage rate, the rate of return on capital and profits in sector \(j\), \(T_t\) are lump-sum transfers, \(R_t\) and \(R_{t,F}\) are the gross interest rates on risk-free bonds in domestic and foreign currency and \(S_t\) is the nominal exchange rate, defined as the domestic price of foreign currency.

The capital stock of each sector evolves according to the law of motion:

\[
K_{j,t+1} = (1 - \delta) K_{j,t} + \bar{V}_t \left[ 1 - \Upsilon \left( \frac{I_{j,t}}{I_{j,t-1}} \right) \right] I_{j,t}
\]

for \(j \in \{H,N,X\}\) where \(\delta\) is the capital depreciation rate and \(\Upsilon\) is an investment adjustment cost with the standard restrictions that in steady state \(\Upsilon(\bullet) = \Upsilon'(\bullet) = 0\) and \(\Upsilon''(\bullet) > 0\). \(\bar{V}_t\) governs the efficiency with which investment adds to the capital stock. It follows the process:

\[
\bar{V}_t = v \left( \frac{1}{z_t} \right)^t V_t
\]

where \(z_t\) is the differential between the growth rate of real investment and the growth rate of labour-augmenting technology, \(z\). \(V_t\) is a stationary autoregressive process that affects the marginal efficiency of investment of the form:

\[
\log V_t = \rho_V \log V_{t-1} + u_{V,t}
\]

where \(u_{V,t}\) is identically and independently distributed \(N(0, \sigma_V^2)\). On the balanced growth path \(I_{j,t}\) grows at \(z \times z_t\). The term on the right hand side of Equation (4), \(\bar{V}_t I_{j,t}\), grows at \(z\). Thus, the trend in \(\bar{V}_t\) enables a balanced growth path in which real investment grows faster than real consumption.

As explained by Schmitt-Grohe and Uribe (2003), to ensure stationarity we link the interest rate that domestic residents pay for foreign borrowing to the economy’s net-
foreign asset position. The interest rate on foreign bonds is given by

\[ R^F_t = R^*_t \exp \left[ -\psi_b \left( \frac{S_t B^*_t}{NGDP_t} - b^* \right) + \tilde{\psi}_{b,t} \right] \]  

(7)

where \( R^*_t \) is the foreign interest rate, \( b^* \) is the steady-state net foreign asset-to-GDP ratio and \( NGDP_t \) is nominal GDP. \( \tilde{\psi}_{b,t} \) is a risk-premium shock that follows the process:

\[ \tilde{\psi}_{b,t} = \rho_{\psi} \tilde{\psi}_{b,t-1} + u_{\psi,t} \]  

(8)

where \( u_{\psi,t} \) is independently and identically distributed \( N(0, \sigma^2_{\psi}) \).

2.2 Final goods producing firms

The economy features two final goods: a composite consumption good and a composite investment good. We describe each in turn.

2.2.1 Final consumption goods

Final consumption goods are produced by a representative competitive firm that combines non-tradable and tradable consumption goods according to the technology:

\[ C_t = \left[ \gamma_{T,t} C_{T,t}^{\eta} + \gamma_{N,t} C_{N,t}^{\eta} \right]^{\frac{1}{\eta-1}} \]

where \( C_{N,t} \) is the output of the non-traded sector that is directed towards consumption and has price \( P_{N,t} \) while \( C_{T,t} \) is the output of the traded sector that is directed towards consumption and has price \( P_{T,t} \). The deterministic processes \( \gamma_{T,t} \) and \( \gamma_{N,t} \) ensure, as in Rabanal (2009), that expenditure shares remain stationary along the balanced growth path.\(^7\) \( C_{T,t} \) is a composite of domestically-produced and imported tradable goods assembled according to the technology:

\[ C_{T,t} = \frac{(C_{H,t})^{\gamma_H} (C_{F,t})^{\gamma_F}}{\gamma_H^{\gamma_H} (\gamma_F)^{\gamma_F}} \]

The Cobb-Douglas specification guarantees that the expenditure shares in the tradable consumption basket remain constant. This assumption is convenient to find the normalisations to make the system stationary. Otherwise, \( \gamma_H \) and \( \gamma_F \) would have to trend to keep nominal expenditure shares constant in steady state. The trends in \( \gamma_H \) and \( \gamma_F \)

\(^7\)See the online appendix for details about the normalisations.
together with the differential growth rate of the home-tradable producing goods, $z_H$, and the differential growth rate of the foreign goods producing sector, $z^*$, would determine the differential growth rate of the tradable basket, that is, $z_T$. But to find the trends in $\gamma_H$ and $\gamma_F$ one must know $z_T$.

The non-traded, domestically-produced traded and imported consumption goods are all bundles of a continuum of imperfectly substitutable goods:

$$C_{j,t} \equiv \left( \int_0^1 C_{j,t}(i)^{\theta_j} di \right)^{\frac{\theta_j}{\theta_j-1}}$$

for $j \in \{H, N, F\}$. Profit maximisation and the zero-profit condition imply that the price of the final consumption good is a CES aggregate of the prices of the non-tradable and tradable consumption goods:

$$P_t = \left[ \gamma_{T,t} P_{T,t}^{1-\eta} + \gamma_{N,t} P_{N,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (9)$$

and the price of the tradable consumption good is a Cobb-Douglas aggregate of the home-produced and imported goods:

$$P_{T,t} = (P_{H,t})^{\gamma_H} (P_{F,t})^{\gamma_F} \quad (10)$$

### 2.2.2 Final Investment Goods

Final investment goods are produced by a representative competitive firm according to the technology:

$$I_t = z_t \left( I_{T,t} \right)^{\gamma_H} \left( I_{N,t} \right)^{\gamma_F} \left( \gamma_H \right)^{\gamma_H} \left( \gamma_F \right)^{\gamma_F}$$

where $I_{N,t}$ is the output of the non-traded sector directed towards the production of investment, $I_{T,t}$ is the output of the traded sector that is directed towards investment and $z_t$ is a productivity trend that jointly with the growth rates of $I_{T,t}$ and $I_{N,t}$ determines the steady state growth rate of final investment, $z_I$.

$I_{T,t}$ is a composite of domestically-and foreign produced tradable goods that is assembled according to the technology:

$$I_{T,t} = \left( I_{H,t} \right)^{\gamma_H} \left( I_{F,t} \right)^{\gamma_F} \left( \gamma_H \right)^{\gamma_H} \left( \gamma_F \right)^{\gamma_F}$$

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8This is also the case for investment, $I_{j,t}$ for $j \in \{H, N, F\}$.

9Ireland and Schuh (2008) and Justiniano et al. (2011) are examples of closed economy models with a trend in the price of investment goods and a wedge between the growth rates of real investment and real output.
The corresponding price indices are:

\[ P^I_t = (P^I_{T,t})^{\gamma^I_t} (P^I_{N,t})^{\gamma^I_N} \]  

(11)

and

\[ P^I_{T,t} = (P^I_{H,t})^{\gamma^I_H} (P^I_{F,t})^{\gamma^I_F} \]  

(12)

As the shares of non-tradable, domestically-produced tradable and imported goods in the investment and consumption composites differ, the price of final consumption goods, \( P_t \), will, in general, differ from the price of investment goods, \( P^I_{t,t} \). Similarly, the price of tradable consumption goods, \( P^I_{T,t} \), will differ from the price of tradable investment goods, \( P^I_{T,t} \).

2.3 Intermediate goods producing firms

The economy features four intermediate good producers: commodity firms, non-tradable firms, domestic tradable firms and importing firms. We describe each in turn.

2.3.1 Commodity-exporting firms

Commodity firms produce a homogeneous good in a perfectly competitive market using the Cobb-Douglas production function:

\[ Y_{X,t} = A_t \tilde{Z}_{X,t} (K_{X,t})^{\alpha_X} (Z_tL_{X,t})^{1-\alpha_X} \]  

(13)

where \( Z_t \) is a labour-augmenting technology shock, common to all producing sectors, whose growth rate, \( z_t = Z_t/Z_{t-1} \), follows the process:

\[ \log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + u_{z,t} \]  

(14)

where \( z > 1 \) determines the trend growth rate of real GDP and \( u_{z,t} \) is independently and identically distributed \( N(0, \sigma_z^2) \). The sector-specific productivity process, \( \tilde{Z}_{X,t} \), follows

\[ \tilde{Z}_{X,t} = z_X Z_{X,t} \]  

(15)

where \( z_X > 0 \) determines the differential growth rate, along the balanced growth path, between the output of the commodity-exporting sector and real GDP. The stationary
process $Z_{X,t}$ gives rise to temporary departures from the differential trend by:

$$\log Z_{X,t} = \rho_X \log Z_{X,t-1} + u_{X,t}$$

(16)

where $u_{X,t}$ is independently and identically distributed $N(0, \sigma^2_X)$. In Equation (13) $A_t$ is a stationary technology shock, also common to all sectors, that follows the process:

$$\log A_t = \rho_A \log A_{t-1} + u_{A,t}$$

(17)

where $u_{A,t}$ is independently and identically distributed $N(0, \sigma^2_A)$.

Commodity producers take prices as given. These prices are set in world markets and are unaffected by domestic economic developments. Specifically, we assume that the price of commodities, in foreign currency terms, is equal to:

$$P^*_X,t = \tilde{\kappa}_t P^*_t$$

(18)

where $P^*_t$ is the foreign price level and $\tilde{\kappa}_t$, which governs the relative price of commodities, follows the exogenous process:

$$\tilde{\kappa}_t = \kappa_t \left(\frac{z^*_X}{z^*_t}\right)^t$$

(19)

where $z^*_t$ is the differential growth rate of foreign output and $z^*_X$ is the differential growth rate of foreign production of commodities. The drift in the relative price of commodities reflects the relative productivity growth of the commodity sector and the foreign economy. Along the balanced growth path, relative commodity prices experience transitory shocks according to the process:

$$\log \kappa_t = (1 - \rho_\kappa) \log \kappa + \rho_\kappa \log \kappa_{t-1} + u_{\kappa,t}$$

(20)

where $u_{\kappa,t}$ is independently and identically distributed $N(0, \sigma^2_\kappa)$. For the stochastically detrended variables, $\kappa$ determines the unconditional mean of the terms of trade and, in turn, is one of the determinants of the economy’s steady state. In estimation, we allow for breaks in $\kappa$ and in $\sigma_\kappa$, possibly occurring at different dates in the sample.

The law of one price holds for commodities. This means that their price in domestic currency terms is:

$$P_{X,t} = S_t P^*_{X,t}$$

(21)
2.3.2 Non-tradeable goods producing firms

Non-tradeable firms sell differentiated products, which they produce using the Cobb-Douglas production function:

\[
Y_{N,t}(i) = A_t \tilde{Z}_{N,t} \left( K_{N,t}(i) \right)^{\alpha_N} \left( Z_t L_{N,t}(i) \right)^{1-\alpha_N}
\]  

(22)

\( \tilde{Z}_{N,t} \) is sector-specific productivity process that follows:

\[
\tilde{Z}_{N,t} = z_N^t Z_{N,t}
\]

where \( z_N > 0 \) and \( Z_{N,t} \) are transitory deviations from the sector-specific trend that follow the process:

\[
\log Z_{N,t} = \rho_N \log Z_{N,t-1} + u_{N,t}
\]  

(23)

where \( u_{N,t} \) is independently and identically distributed \( N(0, \sigma_N^2) \). We introduce price stickiness into this sector by assuming that firms can only change prices at some cost, following a Rotemberg (1982) pricing mechanism:10

\[
\frac{\psi_N}{2} \left( \frac{P_{N,t}(i)}{\Pi^N P_{N,t-1}(i)} - 1 \right)^2 P_{N,t} Y_{N,t}
\]

where \( \psi_N \) governs the size of the price adjustment cost and \( \Pi^N \) is the steady state inflation rate of non-tradable goods prices.

Aggregate non-tradeable output is defined by a CES aggregator:

\[
Y_{N,t} \equiv \left( \int_0^1 Y_{N,t}(i)^{\frac{\theta_N-1}{\theta_N}} di \right)^{\frac{\theta_N}{\theta_N-1}}
\]

2.3.3 Domestic tradeable goods producing firms

Domestic tradable firms produce differentiated products using the Cobb-Douglas production function:

\[
Y_{H,t}(i) = A_t \tilde{Z}_{H,t} \left( K_{H,t}(i) \right)^{\alpha_H} \left( Z_t L_{H,t}(i) \right)^{1-\alpha_H}
\]  

(24)

10We assume that these price adjustment costs do not affect the cash flow of firms, but only affect their objective function (see De Paoli et al. (2010) for a discussion of this approach). Therefore, they do not appear in the resource constraint or net export equations. Assuming instead that these adjustment costs are real costs would yield equivalent results as quadratic terms do not appear in the linearized system.
\( \tilde{Z}_{H,t} \) is a stationary sector-specific TFP shock that follows:

\[
\tilde{Z}_{H,t} = z_H^t Z_{H,t}
\]

where \( z_H > 0 \) and \( Z_{H,t} \) are temporary deviations from that trend according to the process:

\[
\log Z_{H,t} = \rho_H \log Z_{H,t-1} + u_{H,t}
\]  

(25)

where \( u_{H,t} \) is independently and identically distributed \( N(0, \sigma_H^2) \). Like their non-tradable counterparts, tradable firms can only change prices at some cost, following a Rotemberg (1982) pricing mechanism:

\[
\frac{\psi_H}{2} \left( \frac{P_{H,t}(i)}{\Pi^H P_{H,t-1}(i)} - 1 \right)^2 P_{H,t} Y_{H,t}
\]

where \( \psi_H \) governs the size of the price adjustment cost and \( \Pi^H \) is the steady state inflation rate of domestic-tradable goods prices. Domestic tradable output, \( Y_{H,t} \) is an aggregate of the output of each of the domestic tradable firms:

\[
Y_{H,t} \equiv \left( \int_0^1 Y_{H,t}(i) \frac{\theta_H^{-1}}{\theta_H} \, di \right)^{\theta_H^{-1}}
\]

2.3.4 Importing Firms

Importing firms purchase foreign good varieties at the price \( \varsigma S_t P_t^* \) and sell them in the domestic market at price \( P_{F,t}(i) \). The parameter \( \varsigma \) represents a subsidy to importing firms, funded by lump-sum taxation. We set the subsidy equal to \( \varsigma = (\theta_F - 1)/\theta_F \), thereby ensuring that markups in this sector are zero in equilibrium.

Importing firms can only change prices at some cost, following a Rotemberg (1982) pricing mechanism:

\[
\frac{\psi_F}{2} \left( \frac{P_{F,t}(i)}{\Pi^F P_{F,t-1}(i)} - 1 \right)^2 P_{F,t} Y_{F,t}
\]

2.4 Foreign sector, net exports and the current account

Following Gertler et al. (2007), we postulate a foreign demand function for domestically produced tradable goods, \( C_{*,t}^* \), of the form:

\[
C_{H,t}^* = \gamma_{H,t}^* \left( \frac{P_{H,t}}{S_t P_t^*} \right)^{-\eta^*} \tilde{Y}_t^*
\]  

(26)
Foreign output, $\tilde{Y}_t^*$, follows the non-stationary process

$$\tilde{Y}_t^* = Z_t(z^*)'Y_t^*$$

Transitory deviations from foreign trend growth are captured by $Y_t^*$ which follows:

$$\log Y_t^* = \rho_Y^* \log Y_t^* + u_{Y,t}^*$$

(27)

where $u_{Y,t}^*$ is independently and identically distributed $N(0, \sigma_Y^2)$. Foreign inflation is assumed to follow:

$$\log \Pi_t^* = (1 - \rho_{\Pi}^*) \log \Pi^* + \rho_{\Pi}^* \log \Pi_{t-1}^* + u_{\Pi,t}^*$$

(28)

and the foreign interest rate follows:

$$\log R_t^* = (1 - \rho_R^*) \log R^* + \rho_R^* \log R_{t-1}^* + u_{R,t}^*$$

(29)

where the independently and identically distributed shocks $u_{\Pi,t}^*$ and $u_{R,t}^*$ are distributed $N(0, \sigma_{\Pi}^2)$ and $N(0, \sigma_R^2)$.

Net exports are given by:

$$N_X_t = P_{H,t} C_{H,t}^* + P_{X,t} Y_{X,t} - P_{F,t} (C_{F,t} + I_{F,t})$$

(30)

and so the current account equation is given by:

$$S_t B_{t+1}^* = R_t^F S_t B_t^* + N_X_t$$

(31)

### 2.5 Monetary Policy

The domestic central bank follows a Taylor rule that responds to deviations of output growth and inflation from their steady-state levels

$$\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_R) \left[ \phi_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \phi_y \log \left( \frac{Y_t}{z_{Y_{t-1}}} \right) \right] + u_{R,t}$$

(32)

where $\Pi_t = P_t / P_{t-1}$ is the inflation rate in terms of final consumption goods prices and $\Pi$ is the central bank’s inflation target.
2.6 Market Clearing

Market clearing for investment goods requires that production of these goods equals to quantity demanded by the three domestic production sectors

\[ I_t = \mathcal{I}_{H,t} + \mathcal{I}_{N,t} + \mathcal{I}_{X,t} \]  

(33)

For the non-tradable, domestic tradable and import sectors, market clearing requires that the quantity produced equals the quantity demanded:

\[ Y_{N,t} = C_{N,t} + I_{N,t} \]  

(34)

\[ Y_{H,t} = C_{H,t} + C^*_H + I_{H,t} \]  

(35)

\[ Y_{F,t} = C_{F,t} + I_{F,t} \]  

(36)

Nominal GDP is defined as:

\[ NGDP_t = P_{N,t} Y_{N,t} + P_{H,t} Y_{H,t} + P_{X,t} Y_{X,t} \]  

(37)

and real GDP is defined as:

\[ Y_t = \frac{P_N}{P} Y_{N,t} + \frac{P_H}{P} Y_{H,t} + \frac{P_X}{P} Y_{X,t} \]  

(38)

2.7 Balanced Growth

A novel feature of this model as that of Rabanal (2009) is the existence of trends in aggregate as well as sector-specific productivity. Next, we describe how the variables behave along a balanced growth path and the normalisations that induce stationarity.

Along a balanced growth path aggregate variables, including GDP, consumption and the capital stock, grow at the rate of aggregate productivity, \( z \). Sectoral variables, such as output of non-tradable goods, \( Y_{N,t} \), and the quantity of these goods that enter consumption and investment baskets, that is \( C_{N,t} \) and \( I_{N,t} \), grow at aggregate productivity adjusted by the sector specific trend. For example, the steady state growth rate of non-tradable output is \( z \times z_{N} \).

Balanced growth requires that the shares of each sector in nominal GDP remain constant. For this to hold, the relative prices of each sector must offset the sector-specific productivity growth rate. For example, the relative price of non-tradable goods to the price of consumption goods, \( P_{N,t}/P_t \), must grow at \( z^{-1}_N \) along a balanced growth path.

When bundles are Cobb-Douglas, expenditure shares are constant and balanced
growth is satisfied regardless of trends in relative prices because income and substitution effects are offsetting. However, in the more general CES specification for the bundles, balanced growth requires that the weights satisfy the following processes: 

$$\gamma_{N,t} = \frac{\gamma_N}{(z_N)^{\eta-1}}$$

and

$$\gamma_{T,t} = \frac{\gamma_T}{(z_T)^{\eta-1}}.$$ 

### 3 Empirical Analysis

The model’s structural parameters can be thought of as belonging to two categories: those that can only determine dynamics – persistence parameters, adjustment cost parameters, policy rule parameters and standard deviations – and those that, in addition to influencing the dynamics, pin down the steady state. We estimate the first category of parameters and, like Adolfson et al. (2007), calibrate the parameters that determine the steady state, with the exception of \(h\), the parameter that determines the degree of habit persistence, and \(1 + \Delta \kappa\), the long-run level of commodity prices in the final steady state.

#### 3.1 Calibration

We calibrate for two reasons. First, not all parameters are well identified given the usual choice of observable variables. Second, estimation could imply a steady state at odds with long-run features of the data. Accordingly, we set the parameters governing the steady state so that the model’s balanced growth path is in line with the first moments of the data.

The traditional approach of matching sample means seems inappropriate in our case because we postulate a possible break in the long-run level of the terms of trade, which in turn leads to changes in unconditional means. We instead focus on matching the features of the data over the first part of the sample, before commodity prices started to rise rapidly in the early 2000s. To be precise, we calibrate the model’s parameters to match means over the period 1993 to 2002, which was a time of relative stability in the terms of trade. In the initial steady state, we first set \(\kappa\) to 1. Before other parameters are calibrated, this choice is a normalisation. After that, of course, a change in \(\kappa\) alters the steady state. For the data to be consistent with the model in estimation we normalise the index of real commodity prices so that it averages 1 over the sub-sample.

Our calibration strategy is as follows. We calibrate the model at a quarterly frequency. We assume that the steady-state rate of labour augmenting TFP growth, \(z\), is 1.0049,
which matches the average growth rate of per capita GDP over our sample.\footnote{We calibrate this parameter using the average growth over our full sample of data rather than the shorter 1993 to 2002 sample because the shorter sample featured an unusually rapid period of economic growth in Australia associated with a steep recovery from a deep recession in the early 1990s and a period of rapid productivity growth due, in part, to a series of microeconomic reforms in the 1980s. Consequently, the full-sample average is likely to better reflect the average long-run TFP growth rate.} We set the central bank’s target inflation rate, \( \Pi \), to deliver an average annual inflation rate of 2.5 per cent, which is the middle of the Reserve Bank of Australia’s stated inflation target. We set the household’s discount rate, \( \beta \), equal to 0.99625. Together, these three parameters imply a steady state nominal interest rate of 6 per cent.

We set the foreign productivity growth differential, \( z^* \), to match the average growth rate of Australia’s major trading partners. We set the sector-specific productivity growth differentials, \( z_N \) and \( z_H \), so that the inflation rates of tradable and non-tradable goods match their rates in the data. This means that, in steady state, non-tradable prices increase faster than consumer prices, while tradable prices increase more slowly.

We set the capital shares in each sector, \( \alpha_N \), \( \alpha_H \) and \( \alpha_X \), according to their average values in national accounts data.\footnote{The data appendix provides additional detail regarding our classification of industries into tradable, non-tradable and commodity-exporting.} The markup parameters, \( \theta_N \), \( \theta_H \) and \( \theta_F \), are set so that each sector has an average markup of 10 per cent. The price adjustment cost parameters that determine the slope of Phillips curves, however, are estimated.

We set the parameter governing the elasticity of labour supply, \( \eta \), to 2, which is a standard value in the literature. In line with Horvath (2000), we set the parameter governing the willingness of workers to move between sectors in response to wage differentials, \( \omega \), to 1. We set the parameters \( \xi_N \), \( \xi_H \) and \( \xi_X \) so that the shares of hours worked in each sector approximates those in the data.

The shares in the Cobb-Douglas bundles are set to match averages in the data. The parameters, \( \gamma_H^* \), \( \gamma_N \) and \( \gamma_T \), are set to approximate the share of exports in GDP and the share of non-tradable and tradable goods in the domestic consumption basket in the data.

Table 2 compares the moments implied by the model’s steady state to their empirical counterparts. At the calibrated parameters values of Table 1 the model’s steady state does quite well. However, investment growth is somewhat lower than in our sample. Because investment is volatile and can contract significantly in recessions, it is likely that the growth rate of investment in our sample (which does not include a recession) is overstated. If we extend the sample to begin in 1990:Q1 – which includes a recession – the average growth rate of investment is 3.5 per cent. We set \( z_v \) so as to match the rate of inflation of investment goods prices. It turns out that this implies a growth rate of...
## Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household discount factor</td>
<td>0.99625</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.005</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Labour supply parameter</td>
<td>2</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Intersectoral labour supply elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\xi_N$</td>
<td>Constant on non-tradable labor supply</td>
<td>100</td>
</tr>
<tr>
<td>$\xi_H$</td>
<td>Constant on tradable labour supply</td>
<td>209</td>
</tr>
<tr>
<td>$\xi_X$</td>
<td>Constant on commodities labour supply</td>
<td>4167</td>
</tr>
<tr>
<td>$\psi_b$</td>
<td>Risk premium</td>
<td>0.001</td>
</tr>
<tr>
<td>$\gamma_N$</td>
<td>Non-tradables consumption weight</td>
<td>0.48</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>Home-produced tradables weight</td>
<td>0.643</td>
</tr>
<tr>
<td>$\gamma_I^N$</td>
<td>Non-tradables investment weight</td>
<td>0.664</td>
</tr>
<tr>
<td>$\gamma_I^H$</td>
<td>Home-produced tradables investment weight</td>
<td>0.172</td>
</tr>
<tr>
<td>$\gamma^*_H$</td>
<td>Determinant of foreign demand</td>
<td>0.877</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution</td>
<td>0.8</td>
</tr>
<tr>
<td>$\eta^*$</td>
<td>Elasticity of substitution</td>
<td>0.8</td>
</tr>
<tr>
<td>$z$</td>
<td>Steady-state TFP growth</td>
<td>1.0049</td>
</tr>
<tr>
<td>$z_v$</td>
<td>Investment growth rate differential</td>
<td>1.0035</td>
</tr>
<tr>
<td>$z_N$</td>
<td>Non-tradable growth differential</td>
<td>0.999</td>
</tr>
<tr>
<td>$z_H$</td>
<td>Home tradable growth differential</td>
<td>1.002</td>
</tr>
<tr>
<td>$z_X$</td>
<td>Commodity growth differential</td>
<td>1.0</td>
</tr>
<tr>
<td>$z^*$</td>
<td>Foreign growth differential</td>
<td>1.00033</td>
</tr>
<tr>
<td>$\alpha_N$</td>
<td>Capital share in non-tradables</td>
<td>0.358</td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>Capital share in tradables</td>
<td>0.438</td>
</tr>
<tr>
<td>$\alpha_X$</td>
<td>Capital share in commodities</td>
<td>0.70</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Domestic inflation target</td>
<td>1.0062</td>
</tr>
<tr>
<td>$\Pi^*$</td>
<td>Foreign inflation target</td>
<td>1.0055</td>
</tr>
<tr>
<td>$\theta_N$</td>
<td>Markup in non-tradables</td>
<td>11</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>Markup in home tradables</td>
<td>11</td>
</tr>
<tr>
<td>$\theta_F$</td>
<td>Markup in imports</td>
<td>11</td>
</tr>
<tr>
<td>$b^*$</td>
<td>Steady state net foreign assets</td>
<td>0</td>
</tr>
</tbody>
</table>

**Notes:**
investment of 3.4 per cent.

3.2 Estimation

We use Bayesian methods, as is common in the estimated DSGE literature.\textsuperscript{13} Our case, however, is non-standard because we allow for structural change and jointly estimate two sets of distinct parameters: the structural parameters of the model, $\vartheta$, that have continuous support and the dates of structural changes, $T = (T_\kappa, T_\sigma)$ that have discrete support; $T_\kappa$ is the date break in the mean and $T_\sigma$ is the date break in the variance of commodity prices. We set the trimming parameter for both date breaks to 25 per cent of the sample. This implies that the minimum length of a segment is 20 observations.

Next, we describe how we construct the joint posterior density of $\vartheta$ and $T$:

$$p(\vartheta, T|Y) \propto L(Y|\vartheta, T)p(\vartheta, T),$$

where $Y \equiv \{y_{t}^{\text{obs}}\}_{t=1}^{T}$ is the data and $y_{t}^{\text{obs}}$ is a $n_{\text{obs}} \times 1$ vector of observable variables.

The likelihood is given by $L(Y|\vartheta, T)$. The priors for the structural parameters and the date breaks are taken to be independent, so that $p(\vartheta, T) = p(\vartheta)p(T)$. We use a flat prior for $T$ so $p(T) \propto 1$, which is proper given its discrete support. Kulish and Pagan (2012) discuss how to construct $L(Y|\vartheta, T)$ in models with forward-looking expectations and structural changes. Appendix C describes the posterior sampler.

We estimate the model using quarterly Australian macroeconomic data for the period 1993Q1 to 2013Q4. The starting date coincides with the start of inflation targeting in Australia and represents a period over which the macroeconomic policy environment has been broadly stable.

Our data series includes aggregate and sectoral Australian variables and foreign variables. The aggregate data include real GDP, investment, consumption, net exports, hours worked, the cash rate, trimmed mean inflation and the percentage change in the nominal exchange rate. The national accounts variables and hours are all expressed in per capita terms and seasonally adjusted. Output, investment, consumption and hours all enter in percentage changes, while net exports enter as a share of nominal GDP. We also include two sectoral variables in the model: the inflation rate of non-tradable goods and the ratio of nominal non-tradable consumption to aggregate nominal consumption. The foreign variables that we include in the model are output growth, interest rates and inflation. We take the growth rate of the Australian major trading partner GDP series constructed by the Reserve Bank of Australia as the measure of foreign output growth.

\textsuperscript{13}See An and Schorfheide (2007) for a description of these techniques.
Table 2: Steady State Properties of the Model

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>1993 - 2013</th>
<th>1993 - 2002</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Macro Aggregates</strong> (annual per cent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Per capita output growth</td>
<td>1.96</td>
<td>2.64</td>
<td>1.96</td>
<td></td>
</tr>
<tr>
<td>Per capita investment growth</td>
<td>4.31</td>
<td>4.39</td>
<td>3.43</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>2.63</td>
<td>2.50</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
<td>Tradable inflation</td>
<td>1.65</td>
<td>2.09</td>
<td>2.09</td>
<td></td>
</tr>
<tr>
<td>Non-tradable inflation</td>
<td>3.41</td>
<td>2.86</td>
<td>2.86</td>
<td></td>
</tr>
<tr>
<td>Investment deflator inflation</td>
<td>1.29</td>
<td>1.09</td>
<td>1.09</td>
<td></td>
</tr>
</tbody>
</table>

| **Expenditure** (per cent of GDP) |         |             |             |       |
| Consumption             | 74.6    | 75.8        | 72.7        |       |
| Investment              | 26.3    | 24.7        | 27.3        |       |
| Exports                 | 19.5    | 19.3        | 19.7        |       |

| **Consumption basket** (per cent of consumption) |         |             |             |       |
| Non-tradable consumption | 55.2    | 53.4        | 53.4        |       |
| Home tradable consumption | 27.3    | 30.0        | 30.0        |       |
| Imported tradable consumption | 17.5    | 16.6        | 16.6        |       |

| **Investment basket** (per cent of investment) |         |             |             |       |
| Non-tradable investment | 67.4    | 66.4        | 66.4        |       |
| Home tradable investment | 2.6     | 5.7         | 5.8         |       |
| Imported investment     | 30.0    | 27.9        | 27.8        |       |

| **Exports** (per cent of exports) |         |             |             |       |
| Resource exports         | 42.1    | 34.6        | 34.6        |       |
| Other exports             | 57.9    | 65.4        | 65.4        |       |

| **Employment** (per cent of hours worked) |         |             |             |       |
| Non-tradable             | 67.6    | 65.0        | 64.3        |       |
| Home tradable            | 31.1    | 34.0        | 33.2        |       |
| Mining                   | 1.3     | 1.0         | 2.5         |       |

Notes: Model ratios calculated at initial regime with $\bar{\kappa} = 1.0$ and $h = 0.5$. 

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For interest rates we use the average of the policy rates in the US, the Euro area and Japan.\textsuperscript{14} For the foreign inflation rate, we use the trade-weighted average inflation rate of Australia’s major trading partners. The 14 series we use in estimation are shown in Figure 2. Appendix A contains a complete description of the data sources, calculations and transformations.

Figure 2: Observable variables

Macroeconomic data are measured with noise and economic concepts in the model do not always match the measures in the data. As is standard in the literature, we add measurement error in estimation. We set the variance of each measurement error to 5 per cent of the variance of its corresponding observable series.

\textsuperscript{14}Prior to the introduction of the euro, we construct this series using the German policy rate.
3.3 Priors

We choose a uniform prior with support −0.25 to 3.5 for \( \Delta \kappa \), which is the parameter of most interest in our analysis. This means that the estimation is free to choose a steady state in which the long-run level of commodity prices would have declined by 25 per cent, would have increased by 350 per cent or would have taken any value in between. We choose the same inverse gamma priors for the standard deviation of the shocks to commodity prices in the low volatility regime, \( \sigma_\kappa \), as we do for the standard deviation of the shocks to commodity prices in the high volatility regime, \( \sigma'_\kappa \).

Our other choices are in line with the literature. We impose loose beta distributions on the autoregressive parameters and inverse gamma distributions on the standard deviations of the shocks. For the parameters of the monetary policy rule we set a prior mean of 1.5 for the response of the Cash Rate to inflation and of 0.3 for the response to real output growth.

4 Results

4.1 Structural parameters and date breaks

Table 3 summarises the estimates of key structural parameters. We consign the estimates of the parameters of the exogenous process to Table 7 in the appendix below.

In estimation we allow for breaks in \( \kappa \) and \( \sigma_\kappa \) because we want to allow the model to fit the data without necessarily having to resort to a change in \( \kappa \). As it turns out, the data strongly prefer a specification in which \( \kappa \) and \( \sigma_\kappa \) both increase. The long-run level of commodity prices, \( 1 + \Delta \kappa \), is estimated to have increased by around 40 per cent, with a distribution that ranges between 30 and 50 per cent. The left panel of Figure 3 shows the posterior distribution of \( \Delta \kappa \). The posterior distribution significantly shrinks the uncertainty relative to our uninformative prior on \( \Delta \kappa \) and the density is bounded well away from zero.

At the same time, the estimates point to a significant increase in the volatility of shocks to commodity prices. At the mode, the standard deviation of shocks to commodity prices has more than doubled. The right panel of Figure 3 shows the posterior distribution of the ratio of the standard deviations of shocks to commodity prices in the two regimes, that is \( \sigma'_\kappa / \sigma_\kappa \). The distribution has no mass at unity or values below. Thus, there is no likelihood that the standard deviation of commodity price shocks has fallen or stayed the same.

The monetary policy rule, habits and investment adjust costs parameters are in line
Table 3: Prior and Posterior Distribution of Structural Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shape</td>
<td>Mean</td>
</tr>
<tr>
<td><strong>Commodity Prices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \kappa$</td>
<td>Uniform [-0.25,3.00]</td>
<td>0.42</td>
</tr>
<tr>
<td>$\sigma_{\kappa}$</td>
<td>Inv. Gamma 0.1 2</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma'_{\kappa}$</td>
<td>Inv. Gamma 0.1 2</td>
<td>0.11</td>
</tr>
<tr>
<td>$\rho_{\kappa}$</td>
<td>Beta 0.5 0.5</td>
<td>0.90</td>
</tr>
<tr>
<td><strong>Monetary Policy Rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Beta 0.5 0.2</td>
<td>0.88</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>Normal 1.5 0.5</td>
<td>2.24</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Normal 0.3 0.2</td>
<td>0.34</td>
</tr>
<tr>
<td><strong>Frictions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>Beta 0.5 0.2</td>
<td>0.66</td>
</tr>
<tr>
<td>$\gamma''$</td>
<td>Normal 2 1</td>
<td>2.89</td>
</tr>
<tr>
<td>$Slope_N$</td>
<td>Gamma 5 3</td>
<td>2.68</td>
</tr>
<tr>
<td>$Slope_H$</td>
<td>Gamma 5 3</td>
<td>13.41</td>
</tr>
<tr>
<td>$Slope_F$</td>
<td>Gamma 5 3</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Notes: $Slope_N = 100(\theta_N - 1)/\psi_N$, $Slope_H = 100(\theta_H - 1)/\psi_H$,
        $Slope_F = 100(\theta_F - 1)/\psi_F$

with findings in the literature (Jääskelä and Nimark (2011) and Adolfson et al. (2013)).
Like Rabanal (2009), we find heterogeneity in the degree of price stickiness across sectors.
At the mode the slope of the Phillips curve in the tradable sector is five times steeper
than in the non-tradable sector which is in turn steeper than in the importing goods
producing sector.

Figure 4 shows the cumulative posterior densities of $T_\kappa$, the date break in the uncondi-
tional mean of commodity prices, and $T_{\sigma_\kappa}$, the date break of the variance. The data
strongly prefer 2003:Q2 for the date break in the unconditional mean. The probability
that the break occurred in this quarter is around 95 per cent. This estimate is close
to that of Gruen (2011) who dates the start of boom to 2002:Q2. Single equation Bai
and Perron (1998) tests place the date break in $\kappa$ a quarter later, that is 2003:Q3. The
date break in volatility, $T_{\sigma_\kappa}$, shown in the bottom panel, is estimated to have occurred
after the increase in the unconditional mean. The posterior density for $T_{\sigma_\kappa}$, however, is
bi-modal. It peaks in 2005:Q2 and then again in the second quarter of 2008:Q2.
4.2 Estimated transitional dynamics

To get a sense of the magnitudes involved, we compute the transitional dynamics of commodity prices and other selected variables implied by the posterior distribution of $\Delta \kappa$. To construct these, we sample from the joint posterior distribution of date breaks and structural parameters. For each sample, we compute the non-stochastic transition path; this is the path the economy would have followed in the absence of shocks but in the presence of $\Delta \kappa$ occurring at $T_{\kappa}$.

Figure 5 plots observed commodity prices and their estimated long-run level. Actual commodity prices were close to their estimated long-run level until 2005. After that, increases in commodity prices were mostly driven by larger temporary shocks. The estimation was free to capture all of the increase in commodity prices after 2003 with an increase in the volatility of shocks to commodity prices. But it chose to explain some of the increase with a permanent change of the long-run level of commodity prices.

The increase in the long-run level of commodity prices has implications for other variables. Figure 6 shows the estimated transitional dynamics of the ratio of the price of non-tradable goods to consumer goods, net exports to GDP, investment growth and consumption growth.

The top left panel of Figure 6 plots the observed relative price of non-tradables goods and the posterior distribution implied by the transition induced by $\Delta \kappa$. Because there is wedge between non-tradable inflation and consumer price inflation, this relative price
trends at a rate that is the same on both balanced growth paths. According to this figure, the increase in the relative price of non-tradables goods over recent years is largely explained by the shift in the long-run level of the terms of trade.

The top right panel of Figure 6 shows that the increase in the long-run commodity prices leads initially to a persistent increase in the trade deficit. This is explained in part by an increase in consumption, due to higher permanent income, and in part by an increase in investment as the economy expands its productive capacity in the commodity sector. In Section 5 we flesh out the mechanism determining the economy’s response to changes in long-run commodity prices.

4.3 Single-equation estimates

Next, we compare our inferences about long-run commodity prices with those obtained using single equation estimates. Using the same priors, we estimate Equation (5) on the commodity price series alone. At the mode, we find a 114 per cent increase in the long-run level of commodity prices and a tripling in the volatility of shocks to commodity prices,
Figure 5: Observed and long-run commodity prices
Figure 6: Data and estimated transitional dynamics

\[ \frac{P_N}{P} \]

Net Exports

Consumption Growth

Investment Growth
although both variances are estimated to be lower than in our multivariate estimation.\textsuperscript{15} We find a mode of 0.02 for $\sigma_\kappa$ and a mode of 0.07 for $\sigma_\kappa'$. 

Figure 7 shows the posterior distribution of the implied transitional dynamics for long-run commodity prices from this single equation exercise. The commodity price series can be fitted better with lower variances and a larger break of the long-run mean. However, we consider our exercise above to provide more plausible estimates of the long-run level of commodity prices because it relies on more observables. There is a trade off in general equilibrium between fitting the commodity price series and fitting the other observables. We show this in Figure 8, which plots the transitional dynamics of the same observable variables shown in Figure 6 but calculated using the posterior distribution of long-run commodity prices from Figure 7.

A 114 per cent increase in the long-run level of commodity prices would require larger shocks to explain the deviations between these non-stochastic transitional dynamics and the data. For example, the net exports to GDP ratio should have decreased by 15 percentage points, quarterly consumption growth should have increased to 5 per cent

\textsuperscript{15}We have also used the tests of Bai and Perron (1998) on the commodity price series without placing priors and allowed up to a maximum of 5 breaks. The tests prefer one break in 2003:Q3 and the estimates suggest a 90 per cent increase of the long-run level of commodity prices.
and quarterly investment growth by close to 20 per cent. These are numbers which are far from the empirical regularities seen in the data.

This single equation exercise is informative in two ways. First, because with theory in hand it allows us to assess the plausibility of reduced form estimates. Second, it sheds light on how $\Delta \kappa$ is identified in estimation. In principle, given our loose prior on $\Delta \kappa$, the estimation procedure was free to fit commodity prices as seen in Figure 7. The fact that it did not choose that parameterisation is because it comes at the expense of a significant loss of fit for the other observables.

Relative to the single equation estimates, our general equilibrium estimation reduces the size of the estimated increase in long-run commodity prices from 114 per cent to 42 per cent, and increases the precision of the estimates. It is fair to wonder if our estimate of $\Delta \kappa$ would be equal zero if we were to remove commodity prices from the set of observable variables. In fact, we have run this estimation and find statistically significant evidence of a 20 per cent increase in $\Delta \kappa$.\textsuperscript{16} The evidence for an increase in the long-run level of the terms of trade comes from all observables and not just from commodity prices.

\textsuperscript{16}These additional results can be found in Online Appendix.
Table 4: Change in Economic Structure

<table>
<thead>
<tr>
<th>Concept</th>
<th>Initial Structure</th>
<th>Final Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expenditure</strong> (per cent of GDP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>72.7</td>
<td>72.0</td>
</tr>
<tr>
<td>Investment</td>
<td>27.3</td>
<td>28.0</td>
</tr>
<tr>
<td>Exports</td>
<td>19.7</td>
<td>22.0</td>
</tr>
<tr>
<td><strong>Production</strong> (per cent of GDP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-tradable</td>
<td>57.0</td>
<td>57.6</td>
</tr>
<tr>
<td>Home tradable</td>
<td>36.2</td>
<td>32.3</td>
</tr>
<tr>
<td>Commodities</td>
<td>6.8</td>
<td>10.2</td>
</tr>
<tr>
<td><strong>Exports</strong> (per cent of exports)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resource exports</td>
<td>34.6</td>
<td>51.8</td>
</tr>
<tr>
<td>Other exports</td>
<td>65.4</td>
<td>48.2</td>
</tr>
<tr>
<td><strong>Relative Prices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>100</td>
<td>70</td>
</tr>
</tbody>
</table>

4.4 Steady State Ratios and Variance Decompositions

The estimated changes in the unconditional mean and volatility of commodity prices alters some characteristics of the steady state.

Table 4 illustrates how the structure of the economy changes as a result of the estimated 42 per cent increase in the long-run level of commodity prices. Other characteristics of the balanced growth path, the growth rate of real quantities and rates of inflation are invariant to the long-run by the level of commodity prices.

The increase in the long-run level of commodity prices causes a shift in resources from the non-commodity tradeable sector to the commodities sector. The commodity sector’s share of exports increases from 35 per cent to 52 per cent. And its share of value added increases from 6.8 per cent of GDP to 10.2 per cent of GDP. There is also a significant impact on the real exchange rate, which appreciates permanently by 30 per cent.17

The change in the steady state and the change in the volatility of shocks to commodity prices alter the relative contribution that shocks have to the observable variables. To measure the implications of these changes, we compute unconditional variance decompositions at the mode for variables of interest in two regimes: a low regime (κ = 1 and σ_κ = .05) and a high regime (κ = 1.42 and σ_κ = .11).18 The contribution of most shocks is similar in the two regimes. For example, shocks to non-tradeable technology explain

---

17We define this as S_tP^*_t / P.  
18We do not report variance decompositions for the high-κ low-σ_κ-regime because the joint posterior implies a low probability for this regime.
Table 5: Variance Decomposition: Low-regime (%)

<table>
<thead>
<tr>
<th></th>
<th>$\pi_t$</th>
<th>$\pi_{N,t}$</th>
<th>$\Delta y_t$</th>
<th>$\Delta I_t$</th>
<th>$\Delta c_t$</th>
<th>$\Delta s_t$</th>
<th>$\tau_{F,t}$</th>
<th>$n_{x_t}$</th>
<th>$r_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_R$</td>
<td>13.2</td>
<td>15.4</td>
<td>1.4</td>
<td>2.0</td>
<td>1.4</td>
<td>1.7</td>
<td>0.5</td>
<td>1.1</td>
<td>10.0</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>5.7</td>
<td>13.7</td>
<td>12.1</td>
<td>73.9</td>
<td>1.9</td>
<td>0.1</td>
<td>0.3</td>
<td>3.1</td>
<td>14.2</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>1.5</td>
<td>2.2</td>
<td>9.3</td>
<td>1.8</td>
<td>20.1</td>
<td>0.2</td>
<td>0.5</td>
<td>1.5</td>
<td>2.2</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>12.0</td>
<td>9.0</td>
<td>1.4</td>
<td>2.7</td>
<td>45.4</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>16.1</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>21.6</td>
<td>15.6</td>
<td>11.5</td>
<td>6.3</td>
<td>6.2</td>
<td>0.3</td>
<td>1.1</td>
<td>1.0</td>
<td>13.2</td>
</tr>
<tr>
<td>$\sigma_\psi$</td>
<td>6.3</td>
<td>0.2</td>
<td>10.6</td>
<td>0.3</td>
<td>1.0</td>
<td>71.3</td>
<td>0.2</td>
<td>0.3</td>
<td>6.1</td>
</tr>
<tr>
<td>$\sigma_N$</td>
<td>4.7</td>
<td>34.6</td>
<td>8.8</td>
<td>8.4</td>
<td>8.5</td>
<td>0.9</td>
<td>67.9</td>
<td>10.5</td>
<td>7.7</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>32.4</td>
<td>3.9</td>
<td>15.1</td>
<td>2.4</td>
<td>12.4</td>
<td>0.7</td>
<td>2.4</td>
<td>0.7</td>
<td>24.1</td>
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<tr>
<td>$\sigma_X$</td>
<td>0.1</td>
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<td>0.0</td>
<td>0.4</td>
<td>1.4</td>
<td>17.6</td>
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</tr>
<tr>
<td>$\sigma_{y*}$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_{y*}$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>1.6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_{r*}$</td>
<td>1.9</td>
<td>4.3</td>
<td>2.4</td>
<td>2.1</td>
<td>3.0</td>
<td>22.1</td>
<td>21.9</td>
<td>49.2</td>
<td>4.7</td>
</tr>
<tr>
<td>$\sigma_\kappa$</td>
<td>0.0</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>3.5</td>
<td>14.8</td>
<td>0.2</td>
</tr>
</tbody>
</table>

a much of the variation in non-tradeable inflation and the real exchange rate before and after the changes. Investment growth, as one would expect, is largely driven by shocks to the marginal efficiency of investment. Aggregate demand shocks ($\zeta$) explain almost half of consumption growth and shocks to labour-augmenting TFP explain another 20 per cent of consumption growth. As is typically the case in the literature, we find that risk premium shocks account for the bulk of the fluctuations in the nominal exchange rate.

Shocks to commodity prices make a modest contribution to economic fluctuations in the low regime, with the exception perhaps of net exports, where shocks to commodity prices account for 15 per cent of the variance. In the high regime, shocks to commodity prices now make a more significant contribution to fluctuations of the real exchange rate and a much more significant contribution to the fluctuations in net exports. And, they also make a modest, but visible, contribution to the nominal exchange rate.
Table 6: Variance Decomposition: High-regime (%)  

<table>
<thead>
<tr>
<th></th>
<th>(\pi_t)</th>
<th>(\pi_{N,t})</th>
<th>(\Delta y_t)</th>
<th>(\Delta I_t)</th>
<th>(\Delta c_t)</th>
<th>(\Delta s_t)</th>
<th>(\tau_{F,t})</th>
<th>(nx_t)</th>
<th>(r_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_R)</td>
<td>13.4</td>
<td>15.1</td>
<td>1.0</td>
<td>2.0</td>
<td>1.4</td>
<td>1.6</td>
<td>0.5</td>
<td>0.4</td>
<td>9.9</td>
</tr>
<tr>
<td>(\sigma_v)</td>
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<td>13.9</td>
<td>9.7</td>
<td>74.1</td>
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<td>0.1</td>
<td>0.3</td>
<td>1.2</td>
<td>14.7</td>
</tr>
<tr>
<td>(\sigma_z)</td>
<td>1.6</td>
<td>2.2</td>
<td>7.1</td>
<td>1.8</td>
<td>20.1</td>
<td>0.1</td>
<td>0.8</td>
<td>0.7</td>
<td>2.0</td>
</tr>
<tr>
<td>(\sigma_A)</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>(\sigma_\zeta)</td>
<td>12.9</td>
<td>8.6</td>
<td>1.0</td>
<td>2.7</td>
<td>45.2</td>
<td>0.2</td>
<td>0.5</td>
<td>0.1</td>
<td>16.2</td>
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<tr>
<td>(\sigma_L)</td>
<td>22.1</td>
<td>15.1</td>
<td>8.7</td>
<td>6.2</td>
<td>6.2</td>
<td>0.2</td>
<td>0.9</td>
<td>0.3</td>
<td>12.7</td>
</tr>
<tr>
<td>(\sigma_\psi)</td>
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<td>0.1</td>
<td>5.4</td>
<td>0.3</td>
<td>0.7</td>
<td>68.5</td>
<td>0.2</td>
<td>0.1</td>
<td>4.1</td>
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<tr>
<td>(\sigma_N)</td>
<td>5.3</td>
<td>33.2</td>
<td>6.7</td>
<td>8.4</td>
<td>9.3</td>
<td>0.7</td>
<td>44.9</td>
<td>4.5</td>
<td>8.0</td>
</tr>
<tr>
<td>(\sigma_H)</td>
<td>31.6</td>
<td>3.9</td>
<td>9.2</td>
<td>2.4</td>
<td>12.1</td>
<td>0.6</td>
<td>2.0</td>
<td>0.2</td>
<td>22.9</td>
</tr>
<tr>
<td>(\sigma_X)</td>
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<td>0.7</td>
<td>48.6</td>
<td>0.0</td>
<td>0.1</td>
<td>0.7</td>
<td>2.4</td>
<td>14.3</td>
<td>3.4</td>
</tr>
<tr>
<td>(\sigma_\gamma)</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(\sigma_\kappa)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(\sigma_\tau)</td>
<td>1.4</td>
<td>3.3</td>
<td>1.3</td>
<td>1.9</td>
<td>2.4</td>
<td>21.1</td>
<td>20.0</td>
<td>20.6</td>
<td>3.6</td>
</tr>
<tr>
<td>(\sigma_\kappa)</td>
<td>0.3</td>
<td>3.7</td>
<td>1.2</td>
<td>0.3</td>
<td>0.5</td>
<td>4.6</td>
<td>27.6</td>
<td>57.8</td>
<td>2.4</td>
</tr>
</tbody>
</table>

5 The Implications of a Change in the Long-run Level of Commodity Prices

The responses of commodity prices to temporary and permanent shocks are different. We reproduce Equation (20) here to explain why:

\[
\log \kappa_t = (1 - \rho_\kappa) \log \kappa + \rho_\kappa \log \kappa_{t-1} + u_{\kappa,t}
\]

A temporary shock, \(u_{\kappa,t}\), raises commodity prices on impact, but implies an expected path of falling commodity prices towards their original steady state. In contrast, a permanent change in \(\kappa\) has a contemporaneous impact which is dampened by the persistence of the process, \((1 - \rho_\kappa) \log(1 + \Delta \kappa)\), but implies an expected path of commodity prices to a permanently higher level.

To illustrate the economic implications of these differences, we compare similar sized temporary and permanent shocks to commodity prices. In line with our estimates above, we consider a 42 per cent increase.

As the bottom left panel of Figure 9 shows, the paths of commodity prices are different on impact and are expected to settle at different long-run levels as well. Permanent income increases significantly more with a change in the long-run level of commodity prices than it does with a transitory shock. Consequently, aggregate demand expands by
more, as is evident in the responses of consumption and investment growth.

A fundamental difference between permanent and transitory shocks is the response of the nominal exchange rate. In response to a change in the long-run level of commodity prices, the initial appreciation of the exchange rate is half as large as the long-run change in $\kappa$. With a temporary shock, the appreciation of the exchange rate is only five percent of the change in $\kappa_t$. Because the determination of the exchange rate depends on expectations, it appreciates on impact by more when the change in commodity prices is permanent in anticipation of larger future inflows of foreign currency. Expected inflows of foreign currency are larger not only because commodity prices are expected to be permanently higher but also because commodity sector output expands permanently in response to permanent shocks. Thus, the price effect is magnified further by a quantity effect when the change is permanent.

For a permanent shock, the initial change in the nominal exchange rate leads to a gradual, persistent and permanent appreciation of the real exchange rate. This result is in line with Chen and Rogoff (2003), who find a strong connection between the real exchange rate and commodity prices for Australia and Cashin et al. (2004), who report evidence of a long-run relationship between the real exchange rate and real commodity prices for 20 commodity-exporting countries.

The larger appreciation of the exchange rate in the permanent case leads to cheaper imported goods which, in turn, leads to a larger fall in tradable inflation. Higher permanent income increases the demand for non-tradable output, which puts upward pressure on non-tradable inflation. Once again, this effect is larger when the change in commodity prices is permanent. Aggregate inflation, however, falls by 2 percentage points in the permanent case because the appreciation of the exchange rate is sufficiently large that the deflationary impact from import prices dominates the increase in non-tradable prices that comes from rising incomes.

Domestic demand for domestically produced tradable goods rises when commodity prices increase permanently. But the exchange rate appreciation leads to a significant fall in non-commodity exports. This is large enough to ensure that the production of domestic tradable goods initially falls. Output of domestically-produced tradeable goods experiences a brief recovery, led by the expansion of domestic demand. However, the long-run increase in commodity prices ultimately leads to a reallocation of inputs across sectors and gradually the commodity sector expands to take advantage of these higher prices, as does the non-tradable sector.

When the change in commodity prices is permanent, the commodity sector initially contracts. This is because the large nominal exchange rate appreciation occurs immedi-
ately, while the impact that a change in long-run commodity prices on commodity prices today, \( \kappa_t \), is small if the process is persistent. As a consequence, the domestic-currency price of commodities initially falls, which delays the expansion of the commodity sector.

When the shock is temporary, consumption growth, investment growth and the net exports to GDP ratio increase. When the change in commodity prices is permanent, consumption increases by more as permanent income has increased and investment increases by more as the economy builds up its productive capacity to supply commodities and non-tradeable goods. Net exports initially fall because the increase in domestic demand increases imports and because the exchange rate appreciation leads to a contraction in the value and volume of exports. GDP ultimately expands, although there is an initial contraction due to the fall in exports caused by the appreciation of the exchange rate.

Monetary policy initially lowers the nominal interest rate in response to the disinflation and the initial contraction in GDP. Eventually, nominal interest rates increase as the disinflationary impact of the exchange rate appreciation passes and output growth remains higher in the transition to the new steady state.
6 Conclusion

The recent development of Asia exposed many economies to unprecedented increases in global commodity prices. A recurrent question for these economies has been the degree to which these changes are permanent. Our objective has been to estimate the permanent change in the level and volatility of export prices and to measure the consequences with the aid of a structural model.

Using Australian data, we detect a change in the long-run level and volatility of its terms of trade. We find that the long-run level of commodity-export prices increased by 42 per cent around 2003:Q2. This estimate is less than what casual observation might suggest and less than what single equation structural break tests suggest. In forward-looking general equilibrium a change in the long-run mean of the terms of trade manifests itself in other observable series we use in estimation. Single equation inferences that are not grounded in theory can yield implausible predictions. Using more observables in structural estimation grounds estimates of the long-run level of commodity prices. Although general equilibrium estimates are less than what may be inferred otherwise, the estimated change in the long-run mean of 40 per cent has a significant impact on the economy.

A contribution of our paper is that we treat trends and cycles in a model-consistent way. Economic theory asserts that permanent changes in the terms of trade must influence other relative prices. In the data of open economies non-tradable prices, consumer prices, investment prices, foreign output, foreign prices, real investment and real output all trend at different rates. Because our model’s balanced growth path has these trends, we can identify the extent to which trends, cycles and breaks drive the fluctuations in the data.

There are questions we leave for future research. We have not studied the design of optimal policy. Our model has a monetary authority, but abstracts from a fiscal one. In recent years, commodity exporting economies have designed fiscal rules (sovereign wealth funds) that respond to commodity price fluctuations (Céspedes and Velasco (2011)). Our model can be extended to study the design of fiscal policy in the presence of a shifting long-run mean in commodity prices.

One relevant characteristic of the Australian economy is that the commodities exported are not consumed domestically. Other economies that have also experienced similar unprecedented increases in their terms of trade, food-producing economies, consume a significant fraction of the goods they also export. The implications of commodity price shocks can be expected to be different for these economies. Thus, extending the model
along these lines and taking it to the data of other commodity-exporting economies is worthwhile. For other countries still, commodities also enter as intermediate goods in production and the implications of commodity price booms in this case are also likely to be different. Studying and estimating the impact of long-run shifts in commodity prices for these alternative specifications is an avenue that we leave for further research.
A Data Sources

A.1 Data used in estimation

The model is estimated using 14 macroeconomic time series. Real GDP, consumption, investment, net exports and hours worked are taken from the Australian Bureau of Statistics’ National Accounts series (ABS cat. 5206.0). All data are seasonally adjusted and measured in chain volume terms, except for the net exports-to-GDP ratio which is seasonally adjusted and measured in current prices and hours worked, which is seasonally adjusted. We convert all of the real activity series into per capita series by dividing by the Australian population, which we derive from the ABS’ GDP per capita series. Our final domestic activity series is the ratio of nominal non-traded consumption to aggregate consumption. We discuss the construction of the non-traded consumption series below.

Our measures of Australian inflation are the Consumer Price Index and Non-Tradeables Price Index, both excluding interest and tax. These series are published by the Australian Bureau of Statistics (ABS Cat. 6401.0). Our measure of the nominal exchange rate is the Australian trade weighted index and our policy interest rate is the cash rate. We source these series from the Reserve Bank of Australia (RBA) statistical tables F1 and F11.

For foreign GDP we use the index of Australia’s major trading partners’ GDP, calculated at purchasing power parity exchange rates, produced by the RBA. For foreign interest rates, we use the average policy rate in the United States, Japan and the euro area (we use German interest rates before the introduction of the euro). Finally, we calculate foreign inflation implicitly using Australian inflation and the real exchange rate index, published by the Reserve Bank of Australia.

A.2 Data used in calibration

This section describes the calculation of the sectoral data used to calibrate the model.

- ** Tradable and Non-tradable consumption:** We allocated private consumption categories in the national accounts to either tradable or non-tradable consumption to match the components of the published tradable and non-tradable consumption price indices (see ABS Cat. 6461 Appendix 2 for these categories). Tradable consumption is the sum of private Food, Cigarettes and Tobacco, Alcohol, Clothing and Footwear, Purchase of Vehicles, Communications and Recreation and Culture consumption. It also includes 24 per cent of healthcare consumption, reflecting the share of pharmaceutical products, which are tradable, in total healthcare consumption and 50 per cent of other household services. Non-tradable consumption
includes Rent, Electricity, Gas & Water, Operation of Vehicles, Transport Services, Education, Hotels, Cafes & Restaurants, Insurance & Financial Services as well as Healthcare and Other Households Services not allocated to tradable consumption. We assume that the allocation of private consumption between tradable and non-tradable goods is the same as for private consumption. We measure imported consumption as the sum of consumption goods imports, 75 per cent of services imports and 25 per cent of intermediate imports.

- ** Tradable and Non-Tradable Investment:** We define tradable investment as the sum of Machinery & Equipment Investment and 36 per cent of both Non-Residential Construction and Public Investment. We base the tradable shares of construction on Table 1 of Burstein et al. (2004). Non-tradable investment is total investment less tradable investment. We measure imported investment as the sum of capital goods imports plus 25 per cent of services imports and 75 per cent of intermediate imports.

- **Resource and Non-Resource Exports:** We use the measure of Resource Exports published in ABS Cat. 5302.0. Non-resource exports equal total exports less resource exports.

- **Hours Worked:** As hours worked data is unavailable at an industry level in Australia, we construct our measure using employment data. This will result in some inaccuracy if average hours worked varies across industries. We define tradable employment as the sum of Agriculture, Mining, Wholesale Trade, Accomodation & Food and Transport, Postal & Warehousing employment. Our measure of employment in the resources sector is Mining employment. As discussed in Plumb et al. (2013), this understates total employment in the resources sector as it excludes workers in industries that service the operations of the mining sector. Our model calibration takes account of this feature of the data. Non-tradable employment is all employment not categorized as Tradable or Mining.

**B The Posterior Sampler**

To simulate from the joint posterior of the structural parameters and the date breaks, $p(\vartheta, T|Y)$, we use the Metropolis-Hastings algorithm following a strategy similar to Kulish et al. (2014). As we have continuous and discrete parameters we modify the standard setup for Bayesian estimation of DSGE models. We separate the parameters into two
blocks: date breaks and structural parameters. To be clear, though, the sampler delivers draws from the joint posterior of both sets of parameters.

The first block of the sampler is for the date breaks, $T$. As is common in the literature on structural breaks (Bai and Perron (1998)), we set the trimming parameter to 25 percent of the sample size so that the minimum length of a segment has 20 observations. Within the feasible range we draw from a uniform proposal density and randomize which particular date break in $T$ to update. This approach is motivated by the randomized blocking scheme developed for DSGE models in Chib and Ramamurthy (2010).

The algorithm for drawing for the date breaks block is as follows: Initial values of the date breaks, $T_0$, and the structural parameters, $\vartheta_0$, are set. Then, for the $j^{th}$ iteration, we proceed as follows:

1. randomly sample which date break to update from a discrete uniform distribution with support ranging from one to the total number of breaks, in our case two.

2. randomly sample the corresponding elements of the proposed date breaks, $T'_j$, from a discrete uniform distribution $[T_{\text{min}}, T_{\text{max}}]$ and set the remaining elements to their values in $T_{j-1}$

3. calculate the acceptance ratio $\alpha^T_j \equiv \frac{p(\vartheta_{j-1}, T'_j|Y)}{p(\vartheta_{j-1}, T_{j-1}|Y)}$

4. accept the proposal with probability $\min\{\alpha^T_j, 1\}$, setting $T_j = T'_j$, or $T_{j-1}$ otherwise.

The second block of the sampler is for the $n_\vartheta$ structural parameters.\footnote{In our application $n_\vartheta = 37$.} It follows a similar strategy to the date-breaks-block described above - we randomize over the number and which parameters to possibly update at each iteration. The proposal density is a multivariate Student’s $t-$ distribution.\footnote{The hessian of the proposal density is computed at the mode of the structural parameters.} Once again, for the $j^{th}$ iteration we proceed as follows:

1. randomly sample the number of parameters to update from a discrete uniform distribution $[1, n_\vartheta]$

2. randomly sample without replacement which parameters to update from a discrete uniform distribution $[1, n_\vartheta]$

3. construct the proposed $\vartheta'_j$ by drawing the parameters to update from a multivariate Student’s $t-$ distribution with 10 degrees of freedom and with location set at the
corresponding elements of \( \vartheta_{j-1} \), scale matrix based on the corresponding elements of the negative inverse Hessian at the posterior mode multiplied by a tuning parameter \( \nu = 0.15 \).

4. calculate the acceptance ratio \( \alpha_{j}^{\vartheta} \equiv \frac{p(\vartheta_{j}', T_{j}|Y)}{p(\vartheta_{j-1}, T_{j}|Y)} \) or set \( \alpha_{j}^{\vartheta} = 0 \) if the proposed \( \vartheta_{j}' \) includes inadmissible values (e.g. a proposed negative value for the standard deviation of a shock or autoregressive parameters above unity) preventing calculation of \( p(\vartheta_{j}', T_{j}|Y) \)

5. accept the proposal with probability \( \min\{\alpha_{j}^{\vartheta}, 1\} \), setting \( \vartheta_{j} = \vartheta_{j}' \), or \( \vartheta_{j-1} \) otherwise.

We use this multi-block algorithm to construct a chain of 575,000 draws from the joint posterior, \( p(\vartheta, T|Y) \), throwing out the first 25 per cent as burn-in. Trace plots show that the sampler mixes well.

C Additional Results
Table 7: Prior and Posterior Distribution of Shock Processes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shape</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Mode</th>
<th>Mean</th>
<th>5 %</th>
<th>95 %</th>
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<tbody>
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<td>$\rho_H$</td>
<td>Beta</td>
<td>0.5</td>
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<td>0.91</td>
<td>0.90</td>
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<td>0.99</td>
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<td>0.81</td>
<td>0.70</td>
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<td>Beta</td>
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<td>0.2</td>
<td>0.70</td>
<td>0.61</td>
<td>0.34</td>
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</tr>
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<td>0.31</td>
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<td>0.87</td>
<td>0.82</td>
<td>0.66</td>
<td>0.93</td>
</tr>
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<td>0.68</td>
<td>0.54</td>
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</tr>
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<td>0.2</td>
<td>0.60</td>
<td>0.59</td>
<td>0.35</td>
<td>0.81</td>
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<td>0.97</td>
<td>0.96</td>
<td>0.98</td>
</tr>
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<td>0.020</td>
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<td>Inv. Gamma</td>
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Notes:
References


