Financial Markets and Unemployment*

Tommaso Monacelli  Vincenzo Quadrini
Università Bocconi  University of Southern California
Antonella Trigari
Università Bocconi
October 25, 2011

Abstract

We study the importance of financial markets for (un)employment fluctuations in a model with searching and matching frictions where firms issue debt under limited enforcement. Higher debt allows employers to bargain lower wages which in turn increases the incentive to create jobs. The transmission mechanism of ‘credit shocks’ is fundamentally different from the typical credit channel and the model can explain why firms cut hiring after a credit contraction even if they do not have shortage of funds for hiring workers. The empirical relevance of these shocks is validated by the structural estimation of the model. The theoretical predictions are also consistent with the estimation of a structural VAR whose identifying restrictions are derived from the theoretical model.

Keywords: Limited enforcement, wage bargaining, unemployment, credit shocks.

JEL classification: E24, E32, E44.

*We thank Wouter Den Haan and John Haltiwanger for insightful comments and seminar participants at Atlanta Fed, Ente Luigi Einaudi, European Summer Symposium in International Macroeconomics, European University Institute, NBER Summer Institute, New York Fed, NYU Abu Dhabi, Ohio State University, Philadelphia Fed, Princeton University, St. Louis Fed, Stanford University, University of Bonn, University of Lausanne, University of Porto, University of Southern California, University of Wisconsin.
1 Introduction

The recent financial turmoil has been associated with a severe increase in unemployment. In the United States the number of unemployed workers jumped from 5.5 percent of the labor force to about 10 percent and continues to stay close to 9 percent despite four years have passed since the beginning of the recession. Because the financial sector has been at the center stage of the recent crisis and the growth rate in the volume of credit has dropped significantly from its pick (see top panel of Figure 1), it is natural to ask whether the contraction of credit is an important driving force of the unemployment hike and sluggish recovery.

One possible channel through which de-leveraging could affect the real economy is by forcing employers to cut investment and hiring because of financing difficulties. This is the typical ‘credit channel’ formalized in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Although there is compelling evidence that the credit channel did play an important role during the crisis when the volume of credit contracted sharply and the liquidity dried up together with widening interest rate spreads, this channel appears less important for explaining the sluggish recovery of the labor market after the initial drop in employment. As shown in the bottom panel of Figure 1, the liquidity held by US businesses contracted during the crisis, consistent with the view of a credit crunch. However, after the initial drop, the liquidity of nonfinancial businesses quickly rebounded and shortly after the crisis firms seem to hold more liquidity than prior to the crisis. Therefore, despite de-leveraging, in more recent periods firms seem to have enough resources to finance investment and hiring.

The fact that firms have rebuilt their liquidity poses some doubts that the standard credit channel is the primary explanation for the sluggish recovery of the labor market (although the credit channel did play an important role in the initial stage of the crisis). Should we then conclude that de-leveraging is not important for understanding the sluggish recovery of the labor market? In this paper we argue that, even if firms have enough funds to sustain hiring, de-leveraging can still induce a decline in employment that is very persistent. This is not because lower debt impairs the hiring ability of firms but because, keeping everything else constant, it places workers in a more favorable position in the negotiation of wages. Therefore, the availability of credit could affect the ‘willingness’, not (necessarily) the ‘ability’ to hire.

To illustrate the mechanism we use a theoretical framework that shares
Figure 1: Liquidity and debt in the US nonfinancial business sector. *Liquidity* is the sum of foreign deposits, checkable deposits and currency, time and savings deposits. *Debt* is defined as credit markets instruments. Data is from the Flows of Funds Accounts.
the basic ingredients of the models studied in Pissarides (1987) and Mortensen and Pissarides (1994) where firms are created through the random matching of job vacancies and workers. We extend the basic structure of these models in two directions. First, we allow firms to issue debt under limited enforcement. Second, we introduce an additional source of business cycle fluctuation which affects directly the enforcement constraint of borrowers and the availability of credit.

Because of the matching frictions and the wage determination process based on bargaining, firms prefer to issue debt even if there is no fixed or working capital that needs to be financed. The preference for debt derives exclusively from the wage determination process, that is, bargaining, whose empirical relevance is shown in Hall and Krueger (2010). When wages are determined through bargaining, higher debt reduces the net bargaining surplus which in turn reduces the wages paid to workers. This creates an incentive for the employer to borrow until the borrowing limit binds. The goal is to study how exogenous or endogenous changes in this limit affect the dynamics of the labor market.

Central to our mechanism is the firm’s capital structure as a bargaining tool in the wage determination process. Both anecdotal and statistical evidence point to this channel. Consider the anecdotal evidence first. An illustrative example, also suggested in Matsa (2010), is provided by the case of the New York Metro Transit Authority. In 2004 the company realized an unexpected 1 billion dollars surplus, largely from a real estate boom. The Union, however, claimed rights to the surplus demanding a 24 percent pay raise over three years.\footnote{From The New York Times, Transit Strike Deadline: How extra Money Complicates Transit Pay Negotiations, 12/15/2005: “The unexpected windfall was supposed to be a boom[.] but has instead become a liability.[.] How, union leaders have asked, can the authority boast of such a surplus and not offer raises of more than 3 percent a year? Why aren’t wages going up more?” In a similar vein: “The magnitude of the surplus […] has set this year’s negotiations apart from prior ones, said John E. Zuccotti, a former first deputy mayor. It’s a much weaker position than the position the M.T.A. is normally in: We’re broke and we haven’t gotten any money […]. The playing field is somewhat different. They haven’t got that defense”.} Another example comes from Delta Airlines. The company weathered the 9/11 airline crisis but its excess of liquidity allegedly reduced the need to cut costs. This hurt the firm’s bargaining position with workers and three years after 9/11 it faced severe financial challenges.\footnote{From The Wall Street Journal, Cross Winds: How Delta’s Cash Cushion Pushed It Onto Wrong Course, 10/29/2004: “In hindsight, it is clear now that Delta’s pile of}
The idea that debt allows employers to improve their bargaining position is supported by several empirical studies in corporate finance. Bronars and Deere (1991) document a positive correlation between leverage and labor bargaining power, proxied by the degree of unionization. Matsa (2010) finds that firms with greater exposure to (union) bargaining power have a capital structure more skewed towards debt. Atanassov and Kim (2009) find that strong union laws are less effective in preventing large-scale layoffs when firms have higher financial leverage. Gorton and Schmid (2004) study the impact of German co-determination laws on firms’ labor decisions and find that firms that are subject to these laws exhibit greater leverage ratios. Chen, Chen and Liao (2011) show that labor union strength relates positively to bond yield spreads.

All the aforementioned studies suggest that firms may use financial leverage strategically in order to contrast the bargaining power of workers. Although there are theoretical studies in the micro-corporate literature that investigates this mechanism (see Perotti and Spier (1993)), the implications for employment dynamics at the macroeconomic level have not been fully explored. The goal of this paper is to investigate these implications. In particular, we study how the labor market responds to a shock that affects directly the availability of credit for employers. This shock resembles the ‘financial shock’ studied in Jermann and Quadrini (2009) but the transmission mechanism is different. While in Jermann and Quadrini the financial shock is transmitted through the standard credit channel (higher cost of financing employment), in the current paper the financing cost remains constant over time. Instead, the reduction in borrowing places firms in a less favorable bargaining position with workers and, as a result, they create fewer jobs.

Credit shocks can generate sizable employment fluctuations in our model. Furthermore, as long as the credit contraction is persistent—a robust feature of the data—the impact on the labor market is long-lasting. In this vein, the properties of the model are consistent with recent findings that recessions associated with financial crisis are more persistent than recessions associated with systemic financial difficulties. See IMF (2009), Claessens, Kose, and Terrones (2008), Reinhart and Rogoff (2009). Models with the standard credit channel can generate severe drops in employment in response to

---

cash and position as the strongest carrier after 9/11 lured the company’s pilots and top managers onto a dire course. Delta’s focus on boosting liquidity turned out to be its greatest blessing and curse, helping the company survive 9/11 relatively unscathed but also putting off badly needed overhauls to cut costs”.
a credit contraction but the drop is usually not very persistent. Typically, an unexpected credit contraction could generate a large macroeconomic response in these models because it is costly to replace debt with equity in the short-run. However, once the adjustment has taken place, which happens relatively quickly, the lower debt is no longer very important for the hiring and investment decisions of firms. Our model, instead, can generate the persistence because workers maintain a favorable bargaining position as long as the debt remains low.

There are other papers in the macro-labor literature that have embedded credit market frictions in search and matching models. Chugh (2009) and Petrosky-Nadeau (2009) are two recent contributions. However, the transmission mechanism proposed by these papers is still based on the typical credit channel. More specifically, since firms could be financially constrained, the cost of financing new vacancies plays a central role in the transmission of shocks. Also related is Wasmer and Weil (2004). They consider an environment in which bargaining is not between workers and firms but between entrepreneurs and financiers. In this model financiers are needed to finance the cost of posting a vacancy and the higher surplus extracted by financiers is similar to a higher cost of financing investments. Thus, the central mechanism is still of the credit channel type.\(^3\)

In order to assess the empirical relevance of credit shocks for employed fluctuations, we conduct a structural estimation of the model using Bayesian methods. The estimation shows that credit shocks contribute significantly to employment fluctuations in general and to the sluggish labor market recovery experienced in the aftermath of the recent financial crisis. We also estimate a structural VAR where the shocks are identified using short-term restrictions derived from the theoretical model. We find that the response of employment (and unemployment) to credit shocks is statistically significant and economically sizable. Although the VAR analysis does not allow us to separate the standard credit channel from the channel emphasized in this paper, the empirical results are consistent with the predictions of the model.

The paper is organized as follows. Section 2 presents the theoretical model. Section 3 provides first some analytical intuitions for the response of the economy to shocks and then shows additional properties numerically. After extending the model in Section 4 in ways that improve the dynamics

\(^3\)Wasmer and Weil (2004) also discuss the possibility of extending the model with wage bargaining. However, the analysis with wage bargaining is not fully explored in the paper.
of wages, Sections 5 and 6 bring the model to the data using two approaches: structural estimation and estimation of a three dimensional VAR. The final Section 7 concludes.

2 Model

There is a continuum of agents of total mass 1 with lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t c_t$. At any point in time agents can be employed or unemployed. They save in two types of assets: shares of firms and bonds. Risk neutrality implies that the expected return from both assets is equal to $1/\beta - 1$. Therefore, the net interest rate is constant and equal to $r = 1/\beta - 1$.

Firms: Firms are created through the matching of a posted vacancy and a worker. Starting in the next period, a new firm produces output $z_t$ until the match is separated. Separation arises with probability $\lambda$. An unemployed worker cannot be self-employed but needs to search (costlessly) for a job. The number of matches is determined by the function $m(v_t, u_t)$, where $v_t$ is the number of vacancies posted during the period and $u_t$ is the number of unemployed workers. The probability that a vacancy is filled is $q_t = m(v_t, u_t)/v_t$ and the probability that an unemployed worker finds a job is $p_t = m(v_t, u_t)/u_t$.

At any point in time firms are characterized by three states: a productivity $z_t$, an indicator of the financial conditions $\phi_t$ that will be described below, and a stock of debt $b_t$. The productivity $z_t$ and the financial state $\phi_t$ are exogenous stochastic variables, common to all firms (aggregate shocks). The stock of debt $b_t$ is chosen endogenously. Although firms could choose different levels of debt, in equilibrium they all choose the same $b_t$.

The dividend paid to the owners of the firm (shareholders) is defined by the budget constraint

$$d_t = z_t - w_t - b_t + \frac{b_{t+1}}{R},$$

where $R$ is the gross interest rate charged on the debt. As we will see, $R$ is different from $1 + r$ because of the possibility of default when the match is separated.

Timing: If a vacancy is filled, a new firm is created. The new firm starts producing in the next period, and therefore, there is no wage bargaining
in the current period. However, before entering the next period, the newly created firm chooses the debt $b_{t+1}$ and pays the dividend $d_t = b_{t+1}/R_t$ (the initial debt $b_t$ is zero). There is not separation until the next period. Once the new firm enters the next period, it becomes an incumbent firm.

An incumbent firm starts with a stock of debt $b_t$ inherited from the previous period. In addition, it knows the current productivity $z_t$ and the financial variable $\phi_t$. Given the states, the firm bargains the wage $w_t$ with the worker and output $z_t$ is produced. The choice of the new debt $b_{t+1}$ and the payment of dividends arise after wage bargaining. After the payments of dividends and wages and after contracting the new debt, the firm observes whether the match is separated. It is at this point that the firm chooses whether to default. Therefore, each period can be divided in three sequential steps: (i) wage bargaining, (ii) financial decision, (iii) default. These sequential steps are illustrated in figure 2.

Figure 2: Timing for an incumbent firm

Remarks on timing: We would like to clarify the importance of the timing assumptions. Although this will become clear later, it will be helpful to stress the relevance of our assumptions here. First, the sequential timing of decisions for an incumbent firm is irrelevant for the dynamic properties of equilibrium employment. For example, the alternative assumption that incumbent firms choose the new debt before or jointly with the bargaining of wages will not affect the dynamics of employment. For new firms, instead,
the assumption that the debt is chosen in the current period while wage bargaining does not take place until the next period is crucial for the results. As an alternative, we could assume that bargaining takes place in the same period in which a vacancy is filled as long as the choice of debt is made before going to the bargaining table with the new worker. For presentation purposes, we assumed that the debt is raised after matching with a worker (but before bargaining the wage). Alternatively, we could assume that the debt is raised before posting a vacancy but this would not affect the results. What is crucial is that the debt of a new firm is raised before bargaining for the first time with the new worker.

The second point we would like to stress is that the assumption that wages are bargained in every period is not important. We adopted this assumption in order to stay as close as possible to the standard matching model (Pissarides (1987)). In Section 4 we show that the employment dynamics do not change if we make different assumptions about the frequency of bargaining. All we need is that there is bargaining when a new worker is hired.

**Financial contract and borrowing limit:** We assume that lending is done by competitive intermediaries who pool a large number of loans. We refer to these intermediaries as lenders. The amount of borrowing is constrained by limited enforcement. After the payments of dividends and wages, and after contracting the new debt, the firm observes whether the match is separated. It is at this point that the firm chooses whether to default. In the event of default the lender will be able to recover only a fraction $\chi_t$ of the firm’s value.

Denote by $J_t(b_t)$ the equity value of the firm at the beginning of the period, which is equal to the discounted expected value of dividends for shareholders. This function depends on the individual stock of debt $b_t$. Obviously, higher is the debt and lower is the equity value. It also depends on the aggregate states $s_t = (z_t, \phi_t, B_t, N_t)$, where $z_t$ and $\phi_t$ are exogenous aggregate states (shocks), $B_t$ is the aggregate stock of debt and $N_t = 1 - u_t$ is employment. We distinguish aggregate debt from individual debt since, to derive the equilibrium, we have to allow for individual deviations. We use the time subscript $t$ to capture the dependence of the value function from the aggregate states, that is, we write $J_t(b_t)$ instead of $J(z_t, \phi_t, B_t, N_t; b_t)$. We will use this convention throughout the paper.

We begin by considering the possibility of default when the match is
separated. In this case the value of the firm is zero. The lender anticipates that the recovery value is zero in the event of separation and the debt will not be repaid. Therefore, in order to break-even, the lender imposes a borrowing limit insuring that the firm does not default when the match is not separated and charges an interest rate premium to cover the losses realized when the match is separated.

If the match is not separated, the value of the firm’s equity is $\beta E_{t+1}(b_{t+1})$, that is, the next period expected value of equity discounted to the current period. Adding the present value of debt, $b_t/(1+r)$, we obtain the total value of the firm. If the firm defaults, the lender recovers only a fraction $\chi_t$ of the total value of the firm. Therefore, the lender is willing to lend as long as the following constraint is satisfied

$$\chi_t \left[ \frac{b_{t+1}}{1 + r} + \beta E_{t+1}(b_{t+1}) \right] \geq \frac{b_{t+1}}{1 + r}.$$  

The variable $\chi_t$ is stochastic and affects the borrowing capacity of the firm. Henceforth, we will refer to unexpected changes in $\chi_t$ as ‘credit shocks’.

By collecting the term $b_{t+1}/(1+r)$ and using the fact that $\beta(1+r) = 1$, we can rewrite the enforcement constraint more compactly as

$$\phi_t E_{t+1}(b_{t+1}) \geq b_{t+1},$$  

where $\phi_t \equiv \chi_t/(1 - \chi_t)$. We can then think of credit shocks as unexpected innovations to the variable $\phi_t$. This is the exogenous state variable included in the set of aggregate states $s_t$.

We now have all the elements to determine the actual interest rate that lenders charge to firms. Since the loan is made before knowing whether the match is separated, the interest rate charged by the lender takes into account that the repayment arises only with probability $1 - \lambda$. Assuming that financial markets are competitive, the zero-profit condition requires that the gross interest rate $R$ satisfies

$$R(1 - \lambda) = 1 + r.$$  

The left-hand side of (2) is the lender’s expected income per unit of debt. The right-hand side is the lender’s opportunity cost of funds (per unit of debt). Therefore, the firm receives $b_{t+1}/R$ at time $t$ and, if the match is not separated, it repays $b_{t+1}$ at time $t+1$. Because of risk neutrality, the interest rate is always constant, and therefore, $r$ and $R$ bear no time subscript.
**Firm’s value:** Central to the characterization of the properties of the model is the wage determination process which is based on bargaining. Before describing the bargaining problem, we define the value of the firm recursively taking as given the wage bargaining outcome. This is denoted by \( w_t = g_t(b_t) \). The recursive structure of the problem implies that the wage is fully determined by the states at the beginning of the period.

The equity value of the firm can be written recursively as

\[
J_t(b_t) = \max_{b_{t+1}} \left\{ z_t - g_t(b_t) - b_t + \frac{b_{t+1}}{R} + \beta(1 - \lambda)E_t J_{t+1}(b_{t+1}) \right\}
\]

subject to

\[
\phi_t E_t J_{t+1}(b_{t+1}) \geq b_{t+1}.
\]

Notice that the only choice variable in this problem is the debt \( b_{t+1} \). Also notice that the firm takes the current wage as given but it fully internalizes that the choice of debt \( b_{t+1} \) affects future wages. This is captured implicitly by the next period value \( J_{t+1}(b_{t+1}) \).

Because of the additive structure of the objective function, the optimal choice of \( b_{t+1} \) does not depend neither on the current wage \( w_t = g_t(b_t) \) nor on the current liabilities \( b_t \).

**Lemma 1** The new debt \( b_{t+1} \) chosen by the firm depends neither on the current wage \( w_t = g_t(b_t) \) nor on the current debt \( b_t \).

**Proof 1** Since \( w_t \) and \( b_t \) enter the objective function additively and they do not affect neither the next period value of the firm’s equity nor the enforcement constraint, the choice of \( b_{t+1} \) is independent of \( w_t \) and \( b_t \).

As we will see, this property greatly simplifies the wage bargaining problem we will describe below.

**Worker’s values:** In order to set up the bargaining problem, we define the worker’s values ignoring the capital incomes earned from the ownership of bonds and firms (interests and dividends). Since agents are risk neutral and
the change in the dividend of an individual firm is negligible for an individual worker, we can ignore these incomes in the derivation of wages.

When employed, the worker’s value is

$$W_t(b_t) = g_t(b_t) + \beta \mathbb{E}_t \left[ (1 - \lambda)W_{t+1}(b_{t+1}) + \lambda U_{t+1} \right],$$

(4)

which is defined once we know the wage function $w_t = g_t(b_t)$. The function $U_{t+1}$ is the value of being unemployed and is defined recursively as

$$U_t = a + \beta \mathbb{E}_t \left[ p_t W_{t+1}(B_{t+1}) + (1 - p_t)U_{t+1} \right],$$

where $p_t$ is the probability that an unemployed worker finds a job and $a$ is the flow utility for an unemployed worker.

While the value of an employed worker depends on the aggregate states and the individual debt $b_t$, the value of being unemployed depends only on the aggregate states since all firms choose the same level of debt in equilibrium. Thus, if an unemployed worker finds a job in the next period, the value of being employed is $W_{t+1}(B_{t+1})$.

**Bargaining problem:** Let’s first define the following functions

$$\hat{J}_t(b_t, w_t) = \max_{b_{t+1}} \left\{ z_t - w_t - b_t + \frac{b_{t+1}}{R} + \beta (1 - \lambda) \mathbb{E}_t J_{t+1}(b_{t+1}) \right\}$$

(5)

$$\hat{W}_t(b_t, w_t) = w_t + \beta \mathbb{E}_t \left[ (1 - \lambda)W_{t+1}(b_{t+1}) + \lambda U_{t+1} \right].$$

(6)

These are the values of a firm and an employed worker, respectively, given an arbitrary wage $w_t$ paid in the current period and future wages determined by the function $g_{t+1}(b_{t+1})$. The functions $J_t(b_t)$ and $W_t(b_t)$ were defined in (3) and (4) for a particular wage equation $g_t(b_t)$.

Given the relative bargaining power of workers $\eta \in (0, 1)$, the current wage is the solution to the problem

$$\max_{w_t} \hat{J}_t(b_t, w_t)^{1-\eta} \left[ \hat{W}_t(b_t, w_t) - U_t \right]^\eta.$$

(7)

Let $w_t = \psi_t(g; b_t)$ be the solution, which makes explicit the dependence on the function $g$ determining future wages. The rational expectation solution to
the bargaining problem is the fixed-point to the functional equation $g_t(b_t) = \psi_t(g; b_t)$.

We can now see the importance of Lemma 1. Since the optimal debt chosen by the firm after the wage bargaining does not depend on the wage, in solving the optimization problem (7) we do not have to consider how the choice of $w_t$ affects $b_{t+1}$. Therefore, we can derive the first order condition taking $b_{t+1}$ as given. After some re-arrangement this can be written as

$$J_t(b_t) = (1 - \eta)S_t(b_t), \quad (8)$$

$$W_t(b_t) - U_t = \eta S_t(b_t), \quad (9)$$

where $S_t(b_t) = J_t(b_t) + W_t(b_t) - U_t$ is the bargaining surplus. As it is typical in search models with Nash bargaining, the surplus is split between the contractual parties proportionally to their relative bargaining power.

**Choice of debt:** Let’s first rewrite the bargaining surplus as

$$S_t(b_t) = z_t - a - b_t + \frac{b_{t+1}}{R} + (1 - \lambda)\beta E_t S_{t+1}(b_{t+1}) - \eta \beta p_t E_t S_{t+1}(B_{t+1}). \quad (10)$$

Notice that the next period surplus enters twice but with different state variables. In the first term the state variable is the individual debt $b_{t+1}$ while in the second is the aggregate debt $B_{t+1}$. The reason is because the value of being unemployed today depends on the value of being employed in the next period in a firm with the aggregate value of debt $B_{t+1}$. Instead, the value of being employed today also depends on the value of being employed next period in the same firm. Since the current employer is allowed to choose a level of debt that differs from the debt chosen by other firms, the individual state next period, $b_{t+1}$, could be different from $B_{t+1}$. In equilibrium, of course, $b_{t+1} = B_{t+1}$. However, to derive the optimal policy we have to allow the firm to deviate from the aggregate policy.

Because the choice of $b_{t+1}$ does not depend on the existing debt $b_t$ (see Lemma 1), we have

$$\frac{\partial S_t(b_t)}{\partial b_t} = -1. \quad (11)$$
Before using this property, we rewrite the firm’s problem (3) as
\[
J_t = \max_{b_{t+1}} \left\{ z_t - g_t(b_t) - b_t + \frac{b_{t+1}}{R} + \beta(1 - \lambda)(1 - \eta)E_tS_{t+1}(b_{t+1}) \right\} \tag{12}
\]
subject to
\[
(1 - \eta)\phi_tE_tS_{t+1}(b_{t+1}) \geq b_{t+1},
\]
where we used \(W_{t+1}(b_{t+1}) - U_{t+1} = \eta S_{t+1}(b_{t+1})\) from (8) and the surplus is defined in (10).

Denoting by \(\mu_t\) the Lagrange multiplier associated with the enforcement constraint, the first order condition is
\[
\eta - \left[ 1 + (1 - \eta)\phi_t \right] \mu_t = 0. \tag{13}
\]
In deriving this expression we used (11) and \(\beta R(1 - \lambda) = \beta(1 + r) = 1\). We can then establish the following result.

**Lemma 2**. The enforcement constraint is binding \((\mu_t > 0)\) if \(\eta \in (0,1)\).

**Proof 2**. It follows directly from the first order condition (13).

A key implication of Lemma 2 is that, provided that workers have some bargaining power, the firm always chooses to maximum debt and the borrowing limit binds. To gather some intuition about the economic interpretation of the multiplier \(\mu_t\), it will be convenient to re-arrange the first order condition as
\[
\mu_t = \left(\frac{1}{1 + (1 - \eta)\phi_t}\right) \times \left(\frac{1}{\frac{1 - \eta}{R}}\right). \tag{13}
\]
The multiplier results from the product of two terms. The first term is the change in next period liabilities \(b_{t+1}\) allowed by a marginal relaxation of the enforcement constraint, that is, \(b_{t+1} = \phi_t(1 - \eta)E_tS(z_{t+1}, B_{t+1}, b_{t+1}) + \bar{a}\), where \(\bar{a} = 0\) is a constant. This is obtained by marginally changing \(\bar{a}\). In
fact, using the implicit function theorem, we obtain $\frac{\partial b_{t+1}}{\partial a} = \frac{1}{1+(1-\eta)\phi_t}$, which is the first term.

The second term is the net gain, actualized, from increasing the next period liabilities $b_{t+1}$ by one unit (marginal change). If the firm increases $b_{t+1}$ by one unit, it receives $1/R$ units of consumption today, which can be paid as dividends. This unit has to be repaid next period. However, the effective cost for the firm is lower than 1 since the higher debt allows the firm to reduce the next period wage by $\eta$, that is, the part of the surplus going to the worker. Thus, the effective repayment incurred by the firm is $1 - \eta$. This cost is discounted by $R = (1 + r)/(1 - \lambda)$ because the debt is repaid only if the matched is not separated, which happens with probability $1 - \lambda$. Therefore, the multiplier $\mu_t$ is equal to the total change in debt (first term) multiplied by the gain from a marginal increase in borrowing (second term).

2.1 Firm entry and general equilibrium

So far we have defined the problem solved by incumbent firms. We now consider more explicitly the problem solved by new firms. In this setup new firms are created when a posted vacancy is filled by a searching worker. Because of the matching frictions, a posted vacancy will be filled only with probability $q_t = m(v_t, u_t)/v_t$. Since posting a vacancy requires a fixed cost $\kappa$, vacancies will be posted only if the value is not smaller than the cost.

We start with the definition of the value of a filled vacancy. When a vacancy is filled, the newly created firm starts producing and pays wages in the next period. The only decision made in the current period is the debt $b_{t+1}$. The funds raised by borrowing are distributed to shareholders. Therefore, the value of a vacancy filled with a worker is

$$Q_t = \max_{b_{t+1}} \left\{ \frac{b_{t+1}}{1 + r} + \beta(1 - \eta)E_t S_{t+1}(b_{t+1}) \right\}$$

subject to

$$\phi_t(1 - \eta)E_t S_{t+1}(b_{t+1}) \geq b_{t+1}.$$  

Since the new firm becomes an incumbent starting in the next period, $S_{t+1}(b_{t+1})$ is the surplus of an incumbent firm defined in (10).
As far as the choice of $b_{t+1}$ is concerned, a new firm faces a similar problem as incumbent firms (see problem (12)). Even if the new firm has no initial debt and it does not pay wages, it will choose the same stock of debt $b_{t+1}$ as incumbent firms. This is because the new firm faces the same enforcement constraint and the choice of $b_{t+1}$ is not affected by $b_t$ and $w_t$ as established in Lemma 1. This allows us to work with a ‘representative’ firm.

We are now ready to define the value of posting a vacancy. This is equal to $V_t = q_t Q_t - \kappa$. As long as the value of a vacancy is positive, more vacancies will be posted. Thus, in equilibrium we must have $V_t = 0$ and the free entry condition can be written as

$$q_t Q_t = \kappa. \tag{15}$$

In a general equilibrium all firms choose the same level of debt. Therefore, $b_t = B_t$. Furthermore, assuming that the bargaining power of workers is positive, firms always borrow up to the limit, that is, $B_{t+1} = \phi_t (1 - \eta) \mathbb{E}_t S_{t+1}(B_{t+1})$. Using the free entry condition (15) Appendix A derives the wage equation

$$w_t = (1 - \eta) a + \eta (z_t - b_t) + \frac{\eta [p_t + (1 - \lambda) \phi_t] \kappa}{q_t (1 + \phi_t)}. \tag{16}$$

The wage equation makes clear that the initial debt $b_t$ acts like a reduction in output in the determination of wages. Instead of getting a fraction $\eta$ of the output, the worker gets a fraction $\eta$ of the output ‘net’ of debt. Thus, for a given bargaining power $\eta$, the larger is the debt and the lower is the wage received by the worker.

3 Response to shocks

The goal of this section is to show how employment responds to shocks (credit and productivity). We first provide some analytical intuition and we simulate the model numerically.

3.1 Analytical intuition

The key equation that defines job creation is the free entry condition $q_t Q_t = \kappa$. Once we understand how the value of a filled vacancy $Q_t$ is affected by shocks, we can then infer the impact of the shocks on job creation. More specifically, if the value of a filled vacancy $Q_t$ increases, the probability of
filling a vacancy \( q_t = m(v_t, u_t) / v_t \) must decline. Since the number of searching workers \( u_t \) is given in the current period, this must be associated with an increase in the number of posted vacancies. Thus, more jobs are created.

Because of the general equilibrium effects of a shock, it is not possible to derive closed form solutions for the impulse responses. However, we can derive analytical results if we assume that the shock affects only a single (atomistic) firm. In this way we can abstract from general equilibrium effects and provide simple analytical intuitions. This is the approach we take in this section. The full general equilibrium responses will be shown numerically in the next subsection.

**Credit shocks:** Starting from a steady state equilibrium, suppose that there is one firm with a newly filled vacancy for which the value of \( \phi_t \) increases. The increase is purely temporary and it reverts back to the steady state value starting in the next period. We stress that the change involves only one firm so that we can ignore the general equilibrium consequences of the change.

The derivative of \( Q_t \) with respect to \( \phi_t \) is

\[
\frac{\partial Q_t}{\partial \phi_t} = \left[ \frac{1}{1 + r} + \beta (1 - \eta) \frac{\partial E_t S_{t+1}(b_{t+1})}{\partial b_{t+1}} \right] \frac{\partial b_{t+1}}{\partial \phi_t}.
\]

Applying the implicit function theorem to the enforcement constraint holding with equality, that is, \( b_{t+1} = \phi_t (1 - \eta) E S_{t+1}(b_{t+1}) \), we can rewrite the derivative as

\[
\frac{\partial b_{t+1}}{\partial \phi_t} = \frac{(1 - \eta) E_t S_{t+1}(b_{t+1})}{1 - (1 - \eta) \phi_t E_t \frac{\partial S_{t+1}(b_{t+1})}{\partial b_{t+1}}},
\]

Substituting \( \partial E_t S_{t+1}(b_{t+1}) / \partial b_{t+1} = -1 \) (see equation (11)) we obtain

\[
\frac{\partial Q_t}{\partial \phi_t} = \frac{\eta (1 - \eta) \beta E_t S_{t+1}(b_{t+1})}{1 + (1 - \eta) \phi_t},
\]

where we have used \( \beta = 1/(1 + r) \).

From this equation we can see that an increase in \( \phi_t \) raises the value of a newly filled vacancy \( Q_t \), provided that the worker has some bargaining power, that is, \( \eta > 0 \). The intuition is straightforward. If the new firm can increase its debt in the current period, the firm can pay more dividends now and less
dividends in the future. However, the reduction in future dividends needed to repay the debt is smaller than the increase in the current dividends because the higher debt allows the firm to reduce the next period wages. Effectively, part of the debt will be repaid by the worker, increasing the firm’s value today.

In deriving this result we assumed that the change in $\phi_t$ was only for one firm so that we could ignore the general equilibrium effects induced by this change. However, since $\phi_t$ is an aggregate variable, this change increases the value of a vacancy for all firms and more vacancies will be posted. The higher job creation will have some general equilibrium effects that cannot be characterized analytically. The full general equilibrium response will be shown numerically.

**Productivity shocks:** Although the main focus of the paper is on credit shocks, it will be helpful to investigate how the ability to borrow affects the propagation of productivity shocks since this has been the main focus of a large body of literature.

In general, productivity shocks generate an employment expansion because the value of a filled vacancy increases. This would arise even if the level of debt is constant, which is the case in the standard matching model. In the case in which the constant debt is zero we revert exactly to the standard matching model. However, if the debt is not constrained to be constant but changes endogenously, then the impact of productivity shocks on employment could be amplified.

As for the case of credit shocks, we consider a productivity shock that affects only one newly created firm and abstract from general equilibrium effects. We further assume that the productivity shock is persistent. The persistence implies that the new firm will be more productive in the next period when it starts producing. If the increase in $z_t$ is purely temporary, the change will not have any effect on the value of a new match.

The derivative of $Q_t$ with respect to $z_t$ is

$$\frac{\partial Q_t}{\partial z_t} = \beta (1 - \eta) \frac{\partial \mathbb{E}_t S_{t+1}(b_{t+1})}{\partial z_t} + \left[ \frac{1}{1 + r} + \beta (1 - \eta) \frac{\partial \mathbb{E}_t S_{t+1}(b_{t+1})}{\partial b_{t+1}} \right] \frac{\partial b_{t+1}}{\partial z_t}.$$

Applying the implicit function theorem to the enforcement constraint
\[ b_{t+1} = (1 - \eta) \phi_t E_t S_{t+1}(b_{t+1}) \]

we obtain

\[
\frac{\partial b_{t+1}}{\partial z_t} = \frac{(1 - \eta) \phi_t E_t \frac{\partial S_{t+1}(b_{t+1})}{\partial z_t}}{1 - (1 - \eta) \phi_t E_t \frac{\partial S_{t+1}(b_{t+1})}{\partial b_{t+1}}}. 
\]

Since \( \partial E_t S_{t+1}(b_{t+1}) / \partial b_{t+1} = -1 \) (see equation (11)), substituting in the derivative of the firm’s value \( Q_t \) and using \( \beta = 1/(1 + r) \) we obtain

\[
\frac{\partial Q_t}{\partial z_t} = \beta (1 - \eta) \phi_t E_t S_{t+1}(b_{t+1}) \frac{\partial z_t}{\partial z_t} + \eta \left( \frac{(1 - \eta) \phi_t E_t S_{t+1}(b_{t+1})}{1 + (1 - \eta) \phi_t E_t} \right). \quad (18)
\]

We can now compare this expression to the equivalent expression we would obtain if the borrowing constraint was exogenous. More specifically, we replace the enforcement constraint (1) with the borrowing limit \( b_{t+1} \leq \bar{b} \) where \( \bar{b} \) is constant. Under this constraint we have that \( \partial b_{t+1} / \partial z_t = 0 \). Therefore,

\[
\frac{\partial Q_t}{\partial z_t} = \beta (1 - \eta) \phi_t E_t S_{t+1}(b_{t+1}) \frac{\partial z_t}{\partial z_t}. \quad (19)
\]

Comparing (18) to (19) we can see that, when the borrowing limit is endogenous, there is an extra term in the derivative of \( Q_t \) with respect to \( z_t \). This term is positive if \( \eta > 0 \). Therefore, the change in the value of a filled vacancy in response to a productivity improvement is bigger when the borrowing limit is endogenous. Intuitively, the increase in productivity raises the value of the firm. This allows for more debt which in turn increases the value of a filled vacancy \( Q_t \).

Of course, this does not tell us whether the amplification effect is large or small. However, we can derive some intuition of what is required for the amplification effect to be large. In particular, as we can see from equation (18), we need that the value of a match is highly sensitive to the productivity shock, that is, we need \( \partial E_t S_{t+1}(b_{t+1}) / \partial z_t \) to be large. This essentially requires large asset price responses to productivity shocks. In this sense the model shares the same features of the models proposed by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) where the amplification of productivity shocks depends on the response of asset prices.

### 3.2 Numerical simulation

We now show the responses to shocks in the general equilibrium through the numerical simulation of the baseline model. Since the goal of the numerical
simulation is only to illustrate the qualitative properties of the model, we avoid a lengthy discussion of the parameter values which are reported in Table 1. A full quantitative analysis will be conducted in Section 5. As we will see, the parameters used here are those estimated in Section 5.

Table 1: List of parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor for entrepreneurs, $\beta$</td>
<td>0.990</td>
</tr>
<tr>
<td>Matching parameter, $\xi$</td>
<td>0.773</td>
</tr>
<tr>
<td>Matching parameter, $\alpha$</td>
<td>0.649</td>
</tr>
<tr>
<td>Relative bargaining power, $\eta$</td>
<td>0.672</td>
</tr>
<tr>
<td>Probability of separation, $\lambda$</td>
<td>0.049</td>
</tr>
<tr>
<td>Cost of posting vacancy, $\kappa$</td>
<td>0.711</td>
</tr>
<tr>
<td>Utility flow unemployed, $a$</td>
<td>0.468</td>
</tr>
<tr>
<td>Enforcement parameter, $\tilde{\phi}$</td>
<td>3.637</td>
</tr>
</tbody>
</table>

**Responses to credit shocks:** Figure 3 plots the responses of debt, employment, output and wages to a negative credit shock. The credit variable $\phi_t$ is assumed to follow a first order autoregressive process with parameters $\rho_\phi = 0.965$ and $\sigma_\phi = 0.143$. Since the model is solved by linearizing the dynamic system around the steady state, the response to a positive credit shock will have the same shape but with inverted sign.

The response of employment is quite persistent, reflecting the persistence of the shock. The mechanism that generates this dynamics should be clear by now. Since firms are forced to cut their debt, workers are able to negotiate higher future wages starting from the next period. The response of wages is plotted in last panel of Figure 3. At impact the wage falls below the steady state but then, starting from the next period, it raises above the steady state. Since new firms start paying wages in the next period, what matters for job creation is the response starting in period 1, that is, one period after the shock. Thus, the anticipated cost of labor for new matches increase in response to a negative credit shock and this discourages job creation.

The initial drop in the wage of incumbent workers can be explained as follow. All bargaining parties understand that, starting from the next period
Figure 3: Impulse responses to a negative credit shock.

wages are going to increase. Since the wage paid when the shock hits is bargained before changing the debt, the total net surplus has not changed yet (besides the changes induced by some general equilibrium feedbacks). This means that the lifetime values received by both parties remain the same. But then, if the value received by workers does not change at impact but there is the anticipation of higher future wages, the current wage has to decline.

The credit shock does not affect the value received by ‘incumbent’ workers and firms (besides, again, the impact coming from general equilibrium effects). So it may appear counterintuitive why an incumbent firm choose to borrow up to the limit if, effectively, the surplus and the division of the surplus do not change. This is due to the lack of commitment from the firm. Since the new debt is chosen unilaterally by the firm after bargaining the wage, the firm prefers higher debt to reduce future wages. This is anticipated by workers who demand higher wages today to compensate for the lower wages expected in the future. If the firm could credibly commit before bargaining the wage, it would agree not to raise the debt.\(^4\) We will come

\(^4\)This mechanism has some similarities with the model studied by Barro and Gordon
back to the dynamics of wage in the next section where we consider possible extensions of the model.

Responses to productivity shocks: Figure 4 plots the impulse responses to a negative productivity shock. The variable $z_t$ is assumed to follow a first order autoregressive process with parameters $\rho_z = 0.944$ and $\sigma_z = 0.005$. We also report the response when the debt limit is exogenously fixed to the steady state value. In this case we impose the borrowing constraint $b_{t+1} \leq \bar{\phi} J$, where $\bar{\phi}$ and $\bar{J}$ are the steady state values of the financial variable $\phi_t$ and of the firm’s value $J_t(b_t)$.

![Figure 4: Impulse responses to a negative productivity shock.](image)

Productivity shocks are amplified somewhat when the borrowing limit is endogenous. However, the magnitude of the amplification is not large. This (1983): since workers anticipate that the central bank inflates ex-post, they demand higher nominal wages today. Differently from that model, however, there are not real costs from deviating, at least from the point of view of an individual firm. As long as new firms can choose the debt before bargaining with new workers, what happens after the firm becomes incumbent is irrelevant for the dynamics of employment.

21
is because productivity shocks do not generate large changes in the value of the firm. As observed in Section 3.1, large amplification effects require sizable movements in $E_t S_{t+1}(b_{t+1})$, that is, in asset prices. Standard business cycle models, however, have difficulties generating large fluctuations in asset prices and this is even harder when preferences are linear. In general, the response of the economy to productivity shocks is similar to the standard matching model. This is not surprising since the version of our model with exogenous borrowing is the standard matching model.

4 Model extension

In this section we propose two extensions of the model that could improve the dynamics of wages. First we assume that each firm is a monopolistic producer, that is, it produces a differentiated good used as an input in the production of final goods. The second assumption is that, after the initial wage bargaining for a new worker, wages are renegotiated infrequently. As we will see, the new features will have very minor implications for the dynamics of employment but will affect the dynamics of wages.

4.1 Monopolistic competition

In this section we assume that each firm/match is a monopolistic producer of a differentiated good. The differentiated goods produced by each firm, denoted by $y_i$, contribute to aggregate output according to $Y = (\int_0^\infty y_i^\varepsilon d\varepsilon)^{1/\varepsilon}$, where $N$ is the total number of differentiated goods which is equal to employment. Furthermore, to make monopolistic competition relevant, we assume that there is also an intensive margin for the production of good/firm $i$. The production technology takes the form $y_i = zl_i$ where $l_i$ is effort/hours supplied by the worker with dis-utility $\mathcal{A} l_i^{1+\phi} / (1 + \phi)$. The intensive margin gives us additional flexibility in separating employment from output.

A well known feature of models with monopolistic competition is that the demand for the differentiated good and the profits of each producer are increasing functions of aggregate production. In our model with equilibrium unemployment, aggregate production depends on how many matches are active which is also equal to the number of employed workers. Therefore, higher is the employment rate and higher is the demand for each intermediate good. Because of this, Appendix B shows that the revenues of an individual
firm can be written in reduced form as

$$\pi_t = \tilde{z}_t N_t^{\nu}.$$  \hfill (20)

The variable $\tilde{z}_t$ is a monotone transformation of productivity $z_t$ and $N_t$ is aggregate employment taken as given by an individual firm. We call this term net surplus flow instead of output for reasons that will become clear below. Therefore, the introduction of monopolistic competition only requires the replacement of firm level production $z_t$ with the net surplus flows $\pi_t = \tilde{z}_t N_t^{\nu}$.

We can now easily describe how a credit shock affects wages. Thanks to the dependence of the surplus flow from aggregate employment, a positive credit shock has two effects on the wages paid to newly hired workers. On the one hand, taking as given aggregate employment, the higher leverage allows firms to pay lower wages, which increases the incentive to hire more workers. On the other hand, the increase in aggregate employment, $N_t$, raises the surplus flow $\pi_t$ which, through the bargaining of the surplus, increases wages. Therefore, whether a credit shock is associated with an increase or decrease in the wages paid to new hired workers depends on the relative importance of these two effects.

**Numerical simulation:** There are only two new parameters, $\varepsilon$ and $\varphi$. The first determines the price mark-up and the second the elasticity of effort or labor utilization. We set $\varepsilon = 0.75$ which implies a price mark-up of $1/\varepsilon - 1 = 0.33$. Then we choose the value of $\varphi$ so that the elasticity of workers’ effort is equal to 1, that is, $1/\varphi = 1$.

Figure 5 plots the impulse responses to a credit shock. We first notice that the responses of debt and employment are not very different from the baseline model. The dynamics of wages, however, is different. In particular, the wage falls at impact and, contrary to the baseline model, it does not raise above the steady state for several periods. What this means is that the wages of new hires are almost unaffected by the credit shock.

**4.2 Optimal labor contracts and infrequent negotiation**

Although it is common in the searching and matching literature to assume that wages are renegotiated every period, in general there is not a theoretical or empirical justification for adopting this assumption. An alternative approach is to characterize the optimal contract first and then design possible mechanisms for implementing the optimal contract.
Figure 5: Impulse responses to a negative credit shock - Extended model with monopolistic competition and endogenous effort/hours.

Suppose that, when the worker is first hired, the parties bargain an optimal long-term contract. The optimal contract chooses the sequence of wages paid to the worker at any point in time, contingent on all possible contingencies directly related to the firm. The state-contingent sequence of wages maximizes the total surplus which is shared according to the relative bargaining weight $\eta$. The sequence of wages must satisfy the participation constraints for the firm and the worker. What this means is that, at any point in time, the value of the firm cannot be negative and the value for the worker cannot be smaller than the value of being unemployed.

It turns out that the sequence of wages that characterizes the optimal contract is not unique. The multiplicity has a simple intuition. Since production does not depend on wages, the choice of a different sequence does not affect the surplus of the match. For example, the firm could pay slightly lower wages at the beginning and slightly higher wages in later periods. This is also an optimal contract as long as the initial worker's value is the same and the participation constraints are not violated. The second condition is
typically satisfied if \( \eta \) is not too close to 0 or 1 and shocks are bounded. The assumption of risk neutrality plays a crucial role for this result. With concave utility of at least one of the parties, like in Michelacci and Quadrini (2009), the optimal sequence of wages would be unique.

Given the multiplicity, we have different ways of implementing the optimal contract. One possibility is to choose a sequence of wages that is equal to the sequence obtained when the wage is re-bargained with some probability \( \psi \). As long as this sequence does not violate the participation constraints, it also implements the optimal contract. Another way of thinking is that, when the firm and the worker meet, they decide not only the division of the surplus (through bargaining) but also the frequency with which they renegotiate the contract. Since the parties are indifferent on the frequency, we could choose a frequency that seems more empirically relevant. Although the choice of a particular frequency is arbitrary, it cannot be dismissed on the ground that it is suboptimal.

Appendix C derives the key equations under the assumption that wage contracts are renegotiated by each firm with probability \( \psi \) and wages stay constant until they are renegotiated. The net surplus generated by a match \( S_t(b_t) \) is still given by (10) while the net value of an employed worker when the contract is renegotiated is

\[
W_t(b_t) - U_t = \frac{w_t - a}{1 - \beta(1 - \lambda)(1 - \psi)} + \Omega_t(b_t),
\]

with the function \( \Omega_t(b_t) \) defined recursively as

\[
\Omega_t(b_t) = \eta \beta [(1 - \lambda) - \psi] \mathbb{E}_t S_{t+1}(b_{t+1}) + \beta(1 - \lambda)(1 - \psi) \mathbb{E}_t \Omega_{t+1}(b_{t+1}).
\]

See Appendix C for the detailed derivation.

We can see from equation (21) that the worker’s value has two components. The first component derives from contingencies in which the contract is not renegotiated. The second component derives from contingencies in which the contract is renegotiated.

**Numerical simulation:** There is only one additional parameter to be calibrated, \( \psi \), which we set to 0.25. If we think of a period as a quarter, this implies that wages are renegotiated, on average, every year. This extension does not affect the other parameters which we set to the same values used in the previous subsection for the model with monopolistic competition.
Figure 6: Impulse responses to a negative credit shock - Extended model with monopolistic competition and infrequent negotiation.

Figure 6 plots the impulse responses to a credit shock generated by the model with monopolistic competition and infrequent negotiation. The responses of debt and employment are not very different from the baseline model. Average wages, however, fall below the steady state for few quarters (thanks to monopolistic competition) and their volatility is significantly smaller (thanks to infrequent negotiation). Therefore, the consideration of monopolistic competition and infrequent bargaining could allow the model to generate a more plausible dynamics of wages.

5 Empirical application: Structural estimation

The analysis conducted so far has illustrated the dynamic responses of the model to credit shocks numerically. The goal was primarily to show the qualitative properties of the model. In this and next sections we try to bring the model to the data more systematically using two approaches. In this section we conduct a structural estimation using Bayesian methods and
in the next section we estimate a three-dimensional VAR whose identifying restrictions are derived from the structural model.

5.1 Estimation with three shocks and three variables

Before proceeding with the structural estimation we add a third shock to the model. In particular, we consider a shock to the efficiency of the matching function. The matching function takes the form \( m_t = \xi v_t^\alpha u_t^{1-\alpha} \), where \( m_t, v_t, u_t \) are, respectively, the number of new matches, the number of posted vacancies and the number of searching workers. In the previous analysis \( \xi \) was a normalizing constant. Now we assume that it is stochastic and follows the first order autoregressive process \( \ln \xi_t = \bar{\xi} + \rho \ln \xi_{t-1} + \epsilon_{\xi,t} \).

We would like to clarify that our interest is not in capturing the importance of this particular shock. Rather we interpret this shock as capturing the potential impact of other shocks that are not explicitly modeled. In this way we do not force the model to replicate the empirical series only with productivity and credit shocks. Also, this allows us to estimate the model with an additional empirical series. As we will show below, innovations to the variable \( \xi_t \) may also capture the impact of shocks that are transmitted through the standard credit channel.

With the addition of the matching shock, we have three exogenous states: productivity, \( z_t \), credit, \( \phi_t \), and matching, \( \xi_t \). They all follow independent autoregressive processes characterized by the parameters \( \rho_j \) and \( \sigma_j \) (where \( j = z, \phi, \xi \)).

Since we have three shocks, we use three empirical time series: (i) real log-GDP; (ii) log-employment in the non-farm sector; (iii) net issuance of debt in the business sector divided by business GDP. The estimation is performed using first differences over the sample period 1984.I-2011.II.

Five of the model parameters are pre-set. They are the discount factor \( \beta \), the frequency of negotiation \( \psi \), the probability of separation \( \lambda \), the average efficiency of the matching function \( \bar{\xi} \) and the cost of posting a vacancy \( \kappa \). The discount factor is set to the standard quarterly value of \( \beta = 0.99 \) and the frequency of negotiation to \( 1/\psi = 4 \) (annually on average). The reason these two parameters are pre-set is because they do not affect the properties of the three variables included in the estimation set. The remaining three parameters are pinned down using the following steady state targets: (i) 5 percent unemployment rate; (ii) 70 percent probability of filling a vacancy; (ii) 93 percent probability of finding a job for an unemployed worker. Only
one of the three targets—the steady state unemployment rate—has some relevance of the properties of the variables included in the estimation set. However, since this number is quite standard in the literature we decided to impose it in advance.

All remaining parameters are estimated with Bayesian methods. Table 2 reports the prior distributions, the mode and the thresholds for the bottom and top percentiles of the posterior distributions.

Table 2: Bayesian estimation with three variables.

<table>
<thead>
<tr>
<th>Estimated parameter</th>
<th>Prior[mean,std]</th>
<th>Mode</th>
<th>Posterior thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching share parameter, $\alpha$</td>
<td>Beta[0.5,0.1]</td>
<td>0.649</td>
<td>0.621 0.662</td>
</tr>
<tr>
<td>Bargaining power workers, $\eta$</td>
<td>Beta[0.5,0.1]</td>
<td>0.672</td>
<td>0.665 0.693</td>
</tr>
<tr>
<td>Utility flow unemployed, $a$</td>
<td>Beta[0.4,0.1]</td>
<td>0.468</td>
<td>0.442 0.472</td>
</tr>
<tr>
<td>Mean enforcement parameter, $\phi$</td>
<td>IGamma[8,5]</td>
<td>3.637</td>
<td>3.589 3.634</td>
</tr>
<tr>
<td>Mark-up parameter, $\varepsilon$</td>
<td>Beta[0.8,0.05]</td>
<td>0.938</td>
<td>0.925 0.949</td>
</tr>
<tr>
<td>Elasticity of effort/hours $\varphi$</td>
<td>Beta[1,0.1]</td>
<td>0.999</td>
<td>0.948 0.985</td>
</tr>
<tr>
<td>Productivity shock persistence, $\rho$</td>
<td>Beta[0.5,0.20]</td>
<td>0.944</td>
<td>0.922 0.962</td>
</tr>
<tr>
<td>Productivity shock volatility, $\sigma$</td>
<td>IGamma[0.001,0.05]</td>
<td>0.095</td>
<td>0.094 0.096</td>
</tr>
<tr>
<td>Credit shock persistence, $\rho_\varphi$</td>
<td>Beta[0.5,0.20]</td>
<td>0.965</td>
<td>0.945 0.977</td>
</tr>
<tr>
<td>Credit shock volatility, $\sigma_\varphi$</td>
<td>IGamma[0.001,0.05]</td>
<td>0.143</td>
<td>0.130 0.157</td>
</tr>
<tr>
<td>Matching shock persistence, $\rho_\xi$</td>
<td>Beta[0.5,0.20]</td>
<td>0.983</td>
<td>0.977 0.986</td>
</tr>
<tr>
<td>Matching shock volatility, $\sigma_\xi$</td>
<td>IGamma[0.001,0.05]</td>
<td>0.056</td>
<td>0.053 0.065</td>
</tr>
</tbody>
</table>

The prior distributions are chosen as follows. Direct estimate of the matching parameter $\alpha$ range from very low values of 0.3 to high values of 0.7 (see Petrongolo and Pissarides (2001)). Thus, we assume that the mean of the prior distribution is the intermediate value of 0.5 with a large standard deviation of 0.1. For the bargaining power $\eta$ there is not direct evidence and it is common to use a value of 0.5. This justifies our choice of a mean value of 0.5 and a large standard deviation of 0.1. A similar approach is used for the utility flow from unemployment. Since there is no agreement on the right value of $a$, we use a mean value of 0.4 and a large standard deviation of 0.1. When evaluated at the mean value of all other parameters, $a = 0.4$ implies that the period utility from being unemployed is 50 percent the period utility from being employed.

The parameter $\phi$ determines the ratio of debt over output. The value of 8 implies a debt-to-output ratio of 1 when evaluated at the mean values of all other parameters. At the same time, however, we allow for a large standard
The variance decomposition of several variables of interest are reported in Table 3. As can be seen, credit shocks contribute significantly to the volatility of employment. The contribution of productivity shocks is quite limited which was to be expected given the result of Shimer (2005). If we impose a higher flow utility from unemployment along the lines of Hagedorn and Manovskii (2008), productivity could contribute more to employment fluctuation. However, productivity shocks are important determinants of output fluctuation. This happens directly through the change in the productivity of each worker and through the change in effort/hours induced by the productivity improvement. Finally, matching shocks are important contributors to the variance of employment. As stressed above, this should not be assigned a deep structural meaning but should be interpreted a residual capturing many sources of fluctuations that have not been explicitly modeled in the paper.

Figure 4 plots the time series contribution of productivity and credit
shocks to the empirical series of employment growth. The difference between the actual data and the sum of the contributions of productivity and credit shocks is the contribution of matching shocks. The series are constructed using the mode values of the parameters. As can be seen from the figure, credit shocks contribute significantly to the fluctuation of employment. In particular we observe that credit shocks contribute with some lag to the actual dynamics of employment. For example, during the last recession, credit shocks seem to have contributed to the decline in employment (negative growth) but not at the pick of the recession. This is fully consistent with the view that our channel contributes to the sluggish recovery after the crisis hit. The initial drop was caused by something else including liquidity shocks propagated through the standard credit channel. As we will see next, the impact of liquidity shocks could be captured in the model by the matching shock $\xi_L$.

5.2 Estimation with four shocks and four variables

Since wages play a central role in the model, it would be desirable to include empirical series of wages in the estimation. However, once we add wages, we have four empirical variables but only three shocks. Therefore, in order to perform the estimation we need to add a fourth shock. We choose to add iid measurement errors in the empirical series of wages. This seems the most natural choice since the general view is that wage series are imperfectly measured. With this addition, we re-estimate the model using four empirical time series in first differences: (i) real log-GDP; (ii) log-employment in the non-farm sector; (iii) net issuance of debt in the business sector divided by business GDP; (iv) hourly real wages in non-farm business.

Table 5 reports the prior distributions, the mode and the thresholds for the bottom and top percentiles of the posterior distributions. Compared to the previous estimation we have two additional parameters: the standard deviation of the measurement error, \( \sigma_w \) and the parameter that determines the frequency of negotiation \( \psi \). Since we include the wage series in the estimation, this parameter becomes relevant. The estimated mode is slightly below 2. This implies that renegotiation takes place on average every five quarters. The average debt-to-output ratio is about 1 and the period utility from unemployed is about 89 percent the period utility from being employed.

### Table 5: Bayesian estimation with four variables.

<table>
<thead>
<tr>
<th>Estimated parameter</th>
<th>Prior [mean, std]</th>
<th>Mode</th>
<th>Posterior thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matching share parameter, ( \alpha )</td>
<td>Beta [0.5, 0.1]</td>
<td>0.762</td>
<td>0.749 0.793</td>
</tr>
<tr>
<td>Bargaining power workers, ( \eta )</td>
<td>Beta [0.5, 0.1]</td>
<td>0.272</td>
<td>0.252 0.268</td>
</tr>
<tr>
<td>Utility flow unemployed, ( \alpha )</td>
<td>Beta [0.4, 0.1]</td>
<td>0.768</td>
<td>0.765 0.794</td>
</tr>
<tr>
<td>Mean enforcement parameter, ( \phi )</td>
<td>IGamma [8, 5]</td>
<td>8.009</td>
<td>7.987 8.002</td>
</tr>
<tr>
<td>Negotiation frequency, ( \psi )</td>
<td>Beta [0.25, 0.05]</td>
<td>0.188</td>
<td>0.174 0.195</td>
</tr>
<tr>
<td>Std measurement error wages, ( \sigma_w )</td>
<td>IGamma [0.001, 0.05]</td>
<td>0.009</td>
<td>0.008 0.010</td>
</tr>
<tr>
<td>Mark-up parameter, ( \varepsilon )</td>
<td>Beta [0.8, 0.05]</td>
<td>0.958</td>
<td>0.952 0.973</td>
</tr>
<tr>
<td>Elasticity of effort, ( \varphi )</td>
<td>Beta [1, 0.1]</td>
<td>0.907</td>
<td>0.906 0.934</td>
</tr>
<tr>
<td>Productivity shock persistence, ( \rho_z )</td>
<td>Beta [0.5, 0.20]</td>
<td>0.923</td>
<td>0.919 0.934</td>
</tr>
<tr>
<td>Productivity shock volatility, ( \sigma_z )</td>
<td>IGamma [0.001, 0.05]</td>
<td>0.005</td>
<td>0.004 0.006</td>
</tr>
<tr>
<td>Credit shock persistence, ( \rho_\phi )</td>
<td>Beta [0.5, 0.20]</td>
<td>0.967</td>
<td>0.959 0.975</td>
</tr>
<tr>
<td>Credit shock volatility, ( \sigma_\phi )</td>
<td>IGamma [0.001, 0.05]</td>
<td>0.136</td>
<td>0.135 0.152</td>
</tr>
<tr>
<td>Matching shock persistence, ( \rho_\xi )</td>
<td>Beta [0.5, 0.20]</td>
<td>0.982</td>
<td>0.976 0.986</td>
</tr>
<tr>
<td>Matching shock volatility, ( \sigma_\xi )</td>
<td>IGamma [0.001, 0.05]</td>
<td>0.032</td>
<td>0.029 0.037</td>
</tr>
</tbody>
</table>

Table 6 reports the variance decomposition for the variable of interest and
Table 6: Variance decomposition. Estimation with four variables

<table>
<thead>
<tr>
<th></th>
<th>TFP shock</th>
<th>Credit shock</th>
<th>Matching shock</th>
<th>Measure error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>50.8</td>
<td>22.0</td>
<td>27.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Employment</td>
<td>6.3</td>
<td>42.0</td>
<td>51.7</td>
<td>0.0</td>
</tr>
<tr>
<td>New debt/output</td>
<td>3.2</td>
<td>73.8</td>
<td>23.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Hourly wage</td>
<td>0.1</td>
<td>1.6</td>
<td>1.5</td>
<td>96.8</td>
</tr>
</tbody>
</table>

Figure 7 plots the time series contribution of productivity and credit shocks to employment growth. Although the estimation with the wage series reduces somewhat the contribution of credit shocks to the volatility of employment, its contribution remains substantial. The volatility of wages, however, is now mostly explained by measurement errors. The contribution of credit shocks to quarter by quarter employment shown in Figure 7 is very similar to the contribution resulting from the previous estimation without wages.

5.3 Costly financing and the credit channel

So far we have abstracted from frictions in financial markets that could affect more directly job creation. The only channel through which financial markets affect employment is through the negotiation of wages. By doing so we have excluded the standard credit channel which, according to recent contributions, could also be important in understanding the dynamics of the labor market. We now extend the model to allow for a more direct role of financial markets and we show that the credit channel cannot be easily separated from the efficiency of the matching function $\xi_t$. Thus, fluctuations in $\xi_t$ could also result from shocks that affect the cost of external finance.

A typical feature of the matching model is that jobs are created through the posting of vacancies and this requires a cost $\kappa$. This cost has the typical features of an investment whose return takes place over several periods. We can then think of the debt raised by new firms as contributing to the financing of this investment. By further assuming that there are agency problems associated with the investment $\kappa$, we are able to introduce a more direct mechanism through which financing affects job creation. This additional mechanism has the feature of the standard credit channel.

Suppose that new firms raise debt before posting vacancy. In the baseline model we assumed that the debt was raised after knowing the outcome of the match. However, this different timing for initial borrowing is inconsequential. If the firm raises debt before posting a vacancy, the lender knows that the debt will not be repaid if the vacancy is not filled and this happens with probability $1 - q_t$. Anticipating this, the lender charges a higher interest but the expected cost of the debt for the firm is still $r = 1/\beta - 1$. The timing in subsequent periods remain the same.

As long as the lender can control the investment $\kappa$, that is, it can insure that the firm does post the vacancy and search for a worker, the different timing is irrelevant. The frictions arise if the lender has not full control over the use of the raised funds. In particular, we now assume that after receiving the loan, a new firm has the ability to default before incurring the cost $\kappa$. If the firm defaults it retains the funds raised from the bank, that is, $\frac{b_{t+1} + R^m}{R^m}$, where $R^m$ is the gross interest rate charged by the bank. The firm raises funds to finance $\kappa$ but after receiving the funds it can default instead of investing. The value of default is the funds received from the lender, that is, $\frac{b_{t+1} + R^m}{R^m}$. 

33
Alternatively, the firm could post the vacancy whose expected value is

\[ V_t = - \left( \kappa - \frac{b_{t+1}}{R^n} \right) + q_t \beta (1 - \eta) \mathbb{E}_{t} S_{t+1}(b_{t+1}). \]  

(23)

The first term in parenthesis is the equity financing of \( \kappa \). Since the firm raises \( b_{t+1}/R^n \) with debt, the difference must be financed with equity.

Because of free entry, the value of posting a vacancy is zero in equilibrium, that is, \( V_t = 0 \). Therefore, the value of defaulting, \( \frac{b_{t+1}}{R^n} \), is bigger than the value of not defaulting, \( V_t \). This implies that a new firm is unable to raise debt and the investment \( \kappa \) must be fully financed with equity.

Now suppose that the lender could enforce the investment \( \kappa \) with costly monitoring. Specifically, the lender can insure that the firm makes the investment by incurring the cost \( \zeta_t \kappa \), where \( \zeta_t \) is stochastic and captures time-varying frictions in financial markets. This would make the cost of borrowing higher but allows the firm to borrow. Since the cost of debt is now higher than the cost of equity, would the new firm continue to borrow? As long as the monitoring cost \( \zeta_t \) is not too large and the bargaining power of workers \( \eta \) sizable, the firm prefers debt over equity because this allows to bargain lower wages in the event of a successful match.

Consider now the break-even condition for the lender which is equal to

\[ \frac{b_{t+1}}{R^n} + \zeta_t \kappa = \frac{q_t b_{t+1}}{1 + r}. \]

Using this condition to eliminate \( R^n \) in equation (23), the choice of debt for a new firm can be written as

\[ V_t = \max_{b_{t+1}} \left\{ \frac{q_t b_{t+1}}{1 + r} - (1 + \zeta_t) \kappa + \beta (1 - \eta) q_t \mathbb{E}_{t} S_{t+1}(b_{t+1}) \right\} \]  

(24)

subject to

\[ \phi_t (1 - \eta) \mathbb{E}_{t} S_{t+1}(b_{t+1}) \geq b_{t+1}, \]

Competition for entry then implies that \( V_t = 0 \) and the free entry condition can be written as

\[ (1 + \zeta_t) \kappa = q_t \left\{ \frac{b_{t+1}}{1 + r} + \beta (1 - \eta) \mathbb{E}_{t} S_{t+1}(b_{t+1}) \right\}, \]
which is very similar to the free entry condition in the baseline model. The only difference is the monitoring cost $\zeta_t$ which was zero in the baseline model. Shocks to $\zeta_t$ are equivalent to an increase in the cost of posting a vacancy.

The new free entry condition makes clear that the monitoring cost cannot be easily separated from the matching shock. Since the probability of filling a vacancy is equal to $q_t = \xi_t(v_t/u_t)^{1-\alpha}$, what matters for job creation is the ratio $(1 + \zeta_t)/\xi_t$. It is in this sense that the matching shock $\xi_t$ can be interpreted as also capturing the impact of shocks that affect employment through the standard credit channel.

6 Empirical application: VAR estimation

In this section we further investigate the empirical relevance of credit shocks using a structural VAR where the shocks are identified through short-term restrictions that are derived from the theoretical model. Let’s start with the theoretical model. The solution of the linearized system around the steady state has the following representation:

$$
\begin{pmatrix}
  z_t \\
  \phi_t \\
  b_t \\
  e_t
\end{pmatrix}
= 
\begin{pmatrix}
  \rho_z & 0 & 0 & 0 \\
  0 & \rho\phi & 0 & 0 \\
  a_{bz} & a_{b\phi} & a_{bb} & a_{be} \\
  a_{ez} & a_{e\phi} & a_{eb} & a_{ee}
\end{pmatrix}
\begin{pmatrix}
  z_{t-1} \\
  \phi_{t-1} \\
  b_{t-1} \\
  e_{t-1}
\end{pmatrix}
+ 
\begin{pmatrix}
  \epsilon_{z,t} \\
  \epsilon_{\phi,t} \\
  0 \\
  \epsilon_{\xi,t}
\end{pmatrix}
$$

(25)

where $z_t$ is the aggregate productivity, $\phi_t$ the variable that captures the aggregate financial conditions, $e_t = 1 - u_t$ the employment rate, $b_t$ the stock of debt at the beginning of the period. These four variables are the states of the dynamic (linearized) system and $\epsilon_{z,t}, \epsilon_{\phi,t}, \epsilon_{\xi,t}$ are innovations to productivity, credit and matching efficiency. For simplicity we have assumed that the shock to the matching technology is $iid$. The analysis that follows will also go through with persistent matching shocks but with more complex innovations to the reduced form system.

Ideally, we would like to estimate directly the above dynamic system which is a first order VAR. This would be possible if we have data for each of the four variables. The problem is that we have data for $z_t, b_t, e_t$, but not for $\phi_t$. Therefore, we have to use an alternative approach. The approach we use is to substitute out $\phi_t$ and $\phi_{t-1}$ using the third equation in system (25)
evaluated at $t$ and $t+1$, that is,

$$b_t = a_{bz}z_{t-1} + a_{bb}\phi_{t-1} + a_{bb}b_{t-1} + a_{be}e_{t-1},$$

$$b_{t+1} = a_{bz}z_t + a_{bb}\phi_t + a_{bb}b_t + a_{be}e_t.$$

Using these two equations to eliminate the variables $\phi_t$ and $\phi_{t-1}$ in the second and fourth equations of system (25) we obtain

$$
\begin{pmatrix}
  z_t \\
  b_{t+1} \\
  e_t
\end{pmatrix}
= 
\begin{bmatrix}
  \rho & 0 & 0 \\
  \psi_{bz} & \psi_{bb} & \psi_{be} \\
  \psi_{e2} & \psi_{eb} & \psi_{ee}
\end{bmatrix}
\begin{pmatrix}
  z_{t-1} \\
  b_{t} \\
  e_{t-1}
\end{pmatrix}
+ 
\begin{bmatrix}
  0 & 0 & 0 \\
  0 & \gamma_{bb} & 0 \\
  0 & \gamma_{eb} & 0
\end{bmatrix}
\begin{pmatrix}
  z_{t-2} \\
  b_{t-1} \\
  e_{t-2}
\end{pmatrix}
+ 
\begin{bmatrix}
  \pi_{zz} & 0 & 0 \\
  \pi_{bz} & \pi_{bb} & \pi_{be} \\
  0 & 0 & \pi_{ee}
\end{bmatrix}
\begin{pmatrix}
  \epsilon_{z,t} \\
  \epsilon_{\phi,t} \\
  \epsilon_{\xi,t}
\end{pmatrix}
$$

This is a second order dynamic system in three observable variables. The zeros in the matrix that multiplies the innovations $\epsilon_{z,t}$, $\epsilon_{\phi,t}$, $\epsilon_{\xi,t}$ provide the restrictions we use to identify the shocks. The first two restrictions impose that $z_t$ is only affected by productivity innovations $\epsilon_{z,t}$, given the exogenous nature of this variable. The additional two restrictions impose that shocks to productivity and credit cannot affect employment at impact. This follows from the property of the model for which employment responds with one period lag.

There are additional restrictions that the structural model imposes to the VAR. However we only impose the short term restrictions just described which are sufficient for the identification of the three shocks.

Data: The three dimensional VAR is estimated using quarterly growth rates of TFP, Credit to the Private Sector and Employment over the period 1984.1-2009.3. The TFP growth is constructed using the utilization-adjusted TFP series constructed by John Fernald (2009). The growth in private credit is constructed using data from the Flow of Funds. Specifically, we use new borrowing (financial market instruments) for households and nonfinancial businesses dividend by the stock of debt (again, financial market instruments). For employment we have three series. The first series includes all civilian employment from the BLS. The second series includes all employees in private industries, also from the BLS. The third series includes all employees in the nonfarm sector, from the Current Employment Statistics survey.
Impulse responses: We first estimate the VAR system with $e_t$ measured by ‘employment in the private sector’ and five lags. Results using the other two definitions of employment (not reported) are similar.

The impulse response functions of Private Credit and Employment to credit and TFP shocks are plotted in Figure 7. As far as the credit shock is concerned, we see that this generates an expansion in the growth rate of private credit that lasts for many quarters. Therefore, these shocks tend to generate long credit cycles. Credit shocks generate an expansion in the growth rate of employment that is statistically significant for four quarters.

TFP shocks also generate an expansion in the growth rate of private credit but the impact is much less persistent. The growth rate of employment goes up but the overall impact is smaller than the impact of credit shocks.

Overall, the results presented in Figure 7 are consistent with the properties of the theoretical model. In particular, we see that credit shocks have a statistical significant impact on employment and TFP shocks lead to a credit expansion. As long as a credit expansion allows for more job creation, the financial mechanism allows for some amplification of productivity shocks.
Separating the transmission through the credit channel: The VAR analysis described above does not allow us to separate the transmission mechanism of credit shocks emphasized in this paper from the typical credit channel. Separating our mechanism from a standard credit channel is a difficult task. Here we try a possible implementation which, however, is not strictly derived from the theoretical model.

The idea is as follows. If the importance of the credit channel is reflected in interest rate spreads, then a credit shock that changes the stock of debt but does not affect the interest rate spread can be identified as a shock that is transmitted through our channel, not the credit channel. To pursue this idea we extend the VAR by adding interest spreads. The short-term restrictions are then given by the zeros in the following matrix that multiplies the innovations \((\epsilon_{z,t}, \epsilon_{\phi,t}, \epsilon_{\xi,t}, \epsilon_{r,t})\):

\[
\begin{bmatrix}
\pi_{zz} & 0 & 0 & 0 \\
\pi_{bz} & \pi_{bb} & \pi_{be} & \pi_{br} \\
0 & 0 & \pi_{ee} & 0 \\
\pi_{rz} & 0 & \pi_{re} & \pi_{rr}
\end{bmatrix}
\]

The variables in the system are now \((z_t, b_{t+1}, e_t, r_t)\) where \(r_t\) is the interest rate spread on corporate bonds. A credit shock is a shock that affects the stock of debt but it does change, at impact, neither employment nor the credit spread. As common in the literature we use the difference in yields for Aaa and Baa rated bonds. As can be seen from Figure 8, the response of employment to credit shocks does not change significantly with the inclusion of interest rate spreads. The estimation also suggests that credit spread impact on employment but this is in addition to our channel.

7 Conclusion

In this paper we have studied the importance of financial flows for employment (and unemployment) fluctuations. We have extended the basic matching model by allowing firms to issue debt under limited enforcement of financial contracts. Our approach goes beyond a mere cumulation of frictions, respectively in financial and labor markets. Firms have an incentive to borrow in order to affect wage bargaining as emphasized in the corporate finance literature. Our paper embeds this mechanism in a general equilibrium environment and investigates its role for the dynamics of aggregate employment.
Figure 8: Four variables VAR: TFP, private credit, unemployment and corporate interest rate spreads.

In our model the ability to borrow can change exogenously in response to credit shocks or endogenously in response to productivity shocks. Independently of the sources of credit expansion, higher debt allows firms to bargain more favorable wages. Through this mechanism, credit shocks can generate large and persistent employment fluctuations. The determination of wages based on bargaining is central to these results.

The paper has also investigated the empirical relevance of credit shocks through the structural estimation of the model. The estimation shows that the contribution of credit shocks to the fluctuation of employment is quantitatively important. This findings are also supported by the estimation of a structural VAR whose identifying restrictions are derived from the structural model.
Appendix

A Wage equation

Consider the value of a filled vacancy defined in (14). Using the binding enforcement constraint $b_{t+1} = \phi_t(1 - \eta)\mathbb{E}_t S_{t+1}(B_{t+1})$ to eliminate $b_{t+1}$, the value of a filled vacancy becomes

$$Q_t = (1 + \phi_t)\beta(1 - \eta)\mathbb{E}_t S_{t+1}(B_{t+1}).$$

Next we use the free entry condition $V_t = q_t Q_t - \kappa = 0$. Eliminating $Q_t$ using the above expression and solving for the expected value of the surplus we obtain

$$\mathbb{E}_t S_{t+1}(B_{t+1}) = \frac{\kappa}{q_t(1 + \phi_t)\beta(1 - \eta)}. \quad (26)$$

Substituting into the definition of the surplus—equation (10)—and taking into account that $b_{t+1} = \phi_t(1 - \eta)\mathbb{E}_t S_{t+1}(B_{t+1})$, we get

$$S_t(B_t) = z_t - a - b_t + \frac{[1 - \lambda - p_t \eta + \phi_t(1 - \lambda)(1 - \eta)]\kappa}{q_t(1 + \phi_t)(1 - \eta)}. \quad (27)$$

Now consider the net value for a worker,

$$W_t(B_t) - U_t = w_t - a + \eta(1 - \lambda - p_t)\beta\mathbb{E}_t S_{t+1}(B_{t+1})$$

Substituting $W_t(B_t) - U_t = \eta S_t(B_t)$ in the left-hand-side and eliminating $\mathbb{E}_t S_{t+1}(B_{t+1})$ in the right-hand-side using equation (26) we obtain

$$\eta S_t(B_t) = w_t - a + \frac{\eta(1 - \lambda - p_t)\kappa}{q_t(1 + \phi_t)(1 - \eta)} \quad (28)$$

Finally, combining (27) and (28) and solving for the wage we get

$$w_t = (1 - \eta)a + \eta(z_t - b_t) + \frac{\eta[p_t + (1 - \lambda)\phi_t]\kappa}{q_t(1 + \phi_t)}$$

which is the expression reported in (16).

B Monopolistic competition

Each firm, indexed by $i$, produces an intermediate good used in the production of final goods. The production function for final goods is

$$Y = \left(\int_0^N y_i^\epsilon dt\right)^{\frac{1}{\epsilon}}. \quad (29)$$
Notice that the integral is over the interval \([0, N]\) since there are \(N\) producers equivalent to the number of employed workers. The inverse demand function is

\[ P_i = Y^{1-\varepsilon} y_i^{\varepsilon-1}, \quad (30) \]

where \(P_i\) is the unit price for intermediate good \(i\) in terms of final goods and \(1/(1 - \varepsilon)\) is the elasticity of demand.

To make the monopolist structure relevant, we need to introduce some margin along which the firm can change the quantity of intermediate goods produced. One way to do this is to assume that there is also an intensive margin in the use of labor. Suppose that the production function for good \(i\) takes the form

\[ y_i = z l_i, \quad (31) \]

where \(l_i\) is efforts or hours supplied by the worker with disutility cost \(A l_i^{1+\varphi}/(1+\varphi)\). An alternative interpretation is that \(l_i\) represents costly utilization of labor.

The monopoly revenue is \(P_i y_i\), that is, the unit price multiplied by output. Substituting the demand function (30) and the production function (31), the revenue can be written as \(Y^{1-\varepsilon} (z l_i)^{\varepsilon}\). The optimal input \(l_i\) solves the problem

\[ \max_{l_i} \left\{ Y^{1-\varepsilon} (z l_i)^{\varepsilon} - \frac{A l_i^{1+\varphi}}{1+\varphi} \right\}, \quad (32) \]

with first order condition \(\varepsilon Y^{1-\varepsilon} z \varepsilon l_i^{\varepsilon-1} = A l_i^{1+\varphi}\).

We can now impose the equilibrium condition \(l_i = L\) and individual production becomes \(y_i = z L\). Aggregate production is equal to \(Y = z L N^{1+\varepsilon}\) and the unit price of intermediate goods is \(P_i = P = N^{1+\varepsilon}\). Finally, the individual revenue is equal to \(z L N^{1+\varepsilon}\).

Using these results in the first order condition for the intensive margin, we can solve for the input \(L = \left(\frac{\varepsilon}{A}\right)^{\frac{1}{2}} N^{\frac{1}{2+\varepsilon}}\). Then substituting in (32) and re-arranging, the revenue net of the disutility from working (net surplus flow) can be written as

\[ \pi = \left[ \left(\frac{\varepsilon}{A}\right)^{\frac{1}{2}} \left(1 - \frac{\varepsilon}{1+\varphi}\right) \right] z^{1+\varphi} N^{\frac{(1-\varepsilon)(1+\varphi)}{\varphi \varepsilon}}. \quad (33) \]

It is now easy to see the equivalence between this function and the net surplus flow reported in (20). If we define \(\tilde{z} = \left[ \left(\frac{\varepsilon}{A}\right)^{\frac{1}{2}} \left(1 - \frac{\varepsilon}{1+\varphi}\right) \right] z^{1+\varphi}\), which is a monotone function of \(z\), and we define \(\nu = \frac{(1-\varepsilon)(1+\varphi)}{\varphi \varepsilon}\), the surplus flow defined in (33) is exactly equal to (20).
C Model with infrequent negotiation

Suppose that wages are negotiated (bargained) when a new match is formed and then they are renegotiated in future periods with some probability $\psi$. In the interim periods wages are kept constant.

To avoid some unnecessary complications, we make the following assumption:

**Assumption 1** The enforcement constraint takes the form $\phi_t \mathbb{E}_t J_{t+1}(b_{t+1}) \geq b_{t+1}$, where $J_{t+1}(b_{t+1})$ is the next period equity value of the firm when the next period wage is renegotiated with certainty.

This assumption insures that the borrowing limit is independent of the current wage, which is different across firms depending on the renegotiation history. In this way all firms continue to choose the same debt even if they pay different wages. The assumption that the collateral value depends on the equity value of the firm when the next period wage is renegotiated with certainty can be justified with the assumption that, in case of default, wages are always renegotiated. Since the lender gets a fraction of the firm’s value, this assumption implies that the collateral is a fraction $\phi_t$ of the equity value of the firm when the next period wage is renegotiated with certainty (since wages are renegotiated in case of default). See Section 2 for the derivation of the enforcement constraint.

The value for a newly hired worker who bargains the first wage at time $t$ is

$$W_t(b_t) = w_t + \beta \mathbb{E}_t \left\{ (1 - \lambda) \left[ \psi W_{t+1}(b_{t+1}) + (1 - \psi) W_{t+1}(b_{t+1}) \right] + \lambda U_{t+1} \right\}, \quad (34)$$

where $W_{t+1}(b_{t+1})$ is the value at time $t+1$ if there in not renegotiation and the worker receives the wage negotiated at time $t$. Therefore, the first subscript denotes the last period in which the wage was negotiated and the second subscript denotes the period in which the wage is paid.

The value of being unemployed is

$$U_t = a + \beta \mathbb{E}_t \left[ p_t W_{t+1}(B_{t+1}) + (1 - p_t) U_{t+1} \right]. \quad (35)$$

Subtracting (35) to (34) and re-arranging we get

$$W_t(b_t) - U_t = w_t - a + \beta \mathbb{E}_t \left\{ (1 - \lambda) \left[ \psi \left( W_{t+1}(b_{t+1}) - U_{t+1} \right) + (1 - \psi) \left( W_{t+1}(b_{t+1}) - U_{t+1} \right) \right] - p_t \left( W_{t+1}(B_{t+1}) - U_{t+1} \right) \right\} \quad (36)$$
Since in equilibrium $b_{t+1} = B_{t+1}$, we can rewrite the equation as

$$W_t(b_t) - U_t = w_t - a + \beta[(1 - \lambda)\psi - p_t]E_t\left(W_{t+1}(b_{t+1}) - U_{t+1}\right) + \beta(1 - \lambda)(1 - \psi)E_t\left(W_{t,t+1}(b_{t+1}) - U_{t+1}\right)$$

(37)

To simplify notations, define

$$\rho = \beta(1 - \lambda)(1 - \psi)$$

$$\delta_t = \beta[(1 - \lambda)\psi - p_t]$$

$$\hat{W}_t(b_t) = W_t(b_t) - U_t$$

$$\hat{W}_{\tau,t}(b_t) = \hat{W}_{\tau,t}(b_t) - U_t,$$

where $\tau \leq t$ is the time subscript for the last period in which the wage was renegotiated. If $\tau = t$ we have $\hat{W}_{\tau,t}(b_t) = \hat{W}_t(b_t)$.

Using this notation, the net value of the worker can be written as

$$\hat{W}_t(b_t) = w_t - a + \delta_t E_t\hat{W}_{t+1}(b_{t+1}) + \rho E_t\hat{W}_{t,t+1}(b_{t+1})$$

(38)

The next period value without bargaining is

$$\hat{W}_{t,t+1}(b_{t+1}) = w_t - a + \delta_{t+1} E_{t+1}\hat{W}_{t+2}(b_{t+2}) + \rho E_{t+1}\hat{W}_{t,t+2}(b_{t+2})$$

(39)

Substituting in (38) at $t + 1, t + 2, t + 3, \ldots$, the net value for the worker can be written as

$$\hat{W}_t(b_t) = \frac{w_t - a}{1 - \rho} + \Omega_t(b_t),$$

(40)

where the function $\Omega_t(b_t)$ is defined as

$$\Omega_t(b_t) = E_t\delta_t\hat{W}_{t+1}(b_{t+1}) + \rho E_t\delta_{t+1}\hat{W}_{t+2}(b_{t+2}) + \rho^2 E_t\delta_{t+2}\hat{W}_{t+3}(b_{t+3}) + \ldots$$

The function $\Omega_t(b_t)$ has a recursive structure and can be written recursively as

$$\Omega_t(b_t) = \delta_t E_t\hat{W}_{t+1}(b_{t+1}) + \rho E_t\Omega_{t+1}(b_{t+1}).$$

(41)

Using the bargaining outcome $\hat{W}_t(b_t) = \eta S_t(b_t)$ in (40) and (41), we obtain

$$\eta S_t(b_t) = \frac{w_t - a}{1 - \rho} + \Omega_t(b_t),$$

(42)
\[ \Omega_t(b_t) = \eta \delta_t E_t S_{t+1}(b_{t+1}) + \rho E_t \Omega_{t+1}(b_{t+1}). \]  
\[ \text{(43)} \]

Finally, the surplus is the same as in the baseline model, that is,
\[ S_t(b_t) = z_t - a_t - b_t + \frac{b_{t+1}}{R} + (1 - \lambda - \eta p_t) \beta E_t S_{t+1}(b_{t+1}). \]  
\[ \text{(44)} \]

### C.1 Evolution of aggregate wages

Denote by \( \bar{w}_{t-1} \) the average wage in period \( t-1 \). Then the average wage in period \( t \) is equal to
\[ \bar{w}_t = \left( \frac{(1 - \lambda) N_{t-1}}{N_t} \right) \left[ (1 - \psi) \bar{w}_{t-1} + \psi w_t \right] + \left( \frac{m(v_{t-1}, u_{t-1})}{N_t} \right) w_t, \]  
\[ \text{(45)} \]

where \( m(v_{t-1}, u_{t-1}) \) is the number of new matches.

To determine the average wage at time \( t \), we need to know the average wage in the previous period and the share of employment that bargains a new wage at time \( t \). This share is equal to
\[ s_t = \psi(1 - \lambda) N_{t-1} + m(v_{t-1}, u_{t-1}) \]  
\[ \text{(49)} \]

Using \( s_t \), the average wage equation can be written as
\[ \bar{w}_t = (1 - s_t) \bar{w}_{t-1} + s_t w_t. \]

### C.2 Summary

The consideration of infrequent negotiation is captured by the following equations
\[ \eta S_t(b_t) = \frac{w_t - a}{1 - \rho} + \Omega_t(b_t) \]  
\[ \text{(46)} \]
\[ \Omega_t(b_t) = \eta \delta_t E_t S_{t+1}(b_{t+1}) + \rho E_t \Omega_{t+1}(b_{t+1}) \]  
\[ \text{(47)} \]
\[ \bar{w}_t = (1 - s_t) \bar{w}_{t-1} + s_t w_t \]  
\[ \text{(48)} \]
\[ s_{t+1} = \frac{\psi(1 - \lambda) N_t + m(v_t, u_t)}{N_{t+1}} \]  
\[ \text{(49)} \]

Notice that equation (46) replaces the equation for the worker’s value in the baseline model with period-by-period bargaining. Equations (47)-(49) are additional. The set of state variables is expanded with the new states \( s_t \) and \( \bar{w}_{t-1} \).
References


International Monetary Fund (2009). World Economic Outlook, Chapter 3, April.


