

# Seemingly Undervalued Currencies

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December 8, 2010

(preliminary version)

## Abstract

The paper demonstrates that two common criteria used to assess exchange rate undervaluations - deviations from the purchasing power parity and the existence of a large current account surplus - are not reliable. A rise in the sex ratio (increasing relative surplus of men in the marriage market), which is a recent major social development in China, Singapore, Vietnam, and several other economies, could simultaneously generate both phenomena, even without currency manipulations. Empirically, those countries with a high sex ratio tend to have a low real exchange rate, beyond what can be explained by the Balassa-Samuelson effect, financial underdevelopment, and exchange rate regime classifications. Once these factors are accounted for, the Chinese real exchange rate is estimated to be undervalued by only a relatively trivial amount.

**Key words:** surplus men, equilibrium real exchange rate, currency manipulation

**JEL code:** F3, F4, J1 and J7

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# 1 Introduction

Real exchange rate undervaluation due to currency manipulation is a frequent topic in international economic policy discussions. Two commonly used criteria by researchers and international financial institutions for judging undervaluations are deviations from the purchasing power parity (PPP) and large and persistent current account surpluses. The goal of this paper is to demonstrate that neither is a reliable metric for assessing currency undervaluation. Specifically, structural factors, unrelated to currency manipulation for the purpose of gaining export competitiveness, can generate both phenomena. We argue that one shock that is both realistic and powerful enough in those economies facing accusation of currency manipulation is a rise in the sex ratio, or the increasing relative surplus of men relative to women in the marriage market. (Given a current account surplus, foreign exchange reserve accumulation could be a passive outcome of a country's capital account controls, rather than exchange rate interventions. In other words, if a country has no capital controls, e.g., Japan, a current account surplus shows up as an addition to its private sector's holding of foreign assets. With capital controls, which typically require compulsory surrender of foreign exchange earnings by firms or households, a current account surplus has to be converted into additional holding of foreign exchange reserves by the official sector.)

We highlight two channels through which structural factors, rather than currency manipulation, could lead to an appearance of currency undervaluation. The first is a savings channel. If an economy experiences a shock that raises its savings rate, then the real exchange rate often falls. To see this, we recognize that a rise in the savings rate implies a reduction in the demand for both tradable and nontradable goods. Since the price of the tradable good is tied down by the world market, this translates into a reduction in the relative price of the nontradable good, and hence a decline in the value of the real exchange rate (a departure from the PPP). The effect can be persistent if there are frictions that impede the reallocation of factors between the tradable and nontradable sectors.

What is the sex ratio imbalance? How would a rise in this imbalance trigger a significant increase in the savings rate? In the case of China, the sex ratio at birth rose from being approximately balanced in the early 1980s to about 120 boys/100 girls by 2007. As the competition for brides intensifies, young men and their parents raise their savings rate in order to improve their relative standing in the marriage market. If the biological desire to have a female partner is strong, the response of the savings rate to a rise in the sex ratio can also be quantitatively large. Of course, our theory has to investigate why the behavior by women or their parents does not undo the competitive savings story.

The empirical motivation for the savings channel comes from Wei and Zhang (2009). They provide evidence from China at both the household level and regional level. First, across rural households with a son, they document that the savings rate tends to be higher in regions with a higher sex ratio imbalance (holding constant family income, age, gender, and educational level of the household head and other household characteristics). In comparison, for rural households with a daughter, their savings rate appears to be uncorrelated with the local sex ratio. Across cities, both households with a son

and households with a daughter tend to have a higher savings rate in regions with a more skewed sex ratio, although the elasticity of the savings rate with respect to the sex ratio tends to be bigger for son families. Second, across Chinese provinces, they find a strong positive correlation between the local savings rate and the local sex ratio, after controlling for the age structure of the local population, income level, inequality, recent growth rate, local enrollment rate in the social safety net, and other factors. Third, to go from correlation to causality, they explore regional variations in the enforcement of the family planning policy as instruments for the local sex ratio, and confirm the findings in the OLS regressions. The sex ratio effect is both economically and statistically significant. While the Chinese household savings rate approximately doubled from 16% (of disposable income) in 1990 to 31% in 2007, Wei and Zhang (2009) estimate that the rise in the sex ratio could explain about half of the increase in the household savings rate.

The second theoretical channel works through effective labor supply. A rise in the sex ratio can also motivate men to cut down leisure and increase labor supply. This leads to an increase in the economy-wide effective labor supply. If the nontradable sector is more labor intensive than the tradable sector, this generates a Rybzinsky-like effect, leading to an expansion of the nontradable sector at the expense of the tradable sector. The increase in the supply of nontradable good leads to an additional decline in the relative price of nontradable and a further decline in the value of the RER. There is evidence from China that the effective labor supply is indeed larger in regions with a higher sex ratio (Wei and Zhang, 2010).

Putting the two channels together, a rise in the sex ratio generates a real exchange rate that appears too low relative to the purchasing power parity. Of course, if there are structural factors, other than a rise in the sex ratio, that have also triggered an increase in the aggregate savings rate (e.g., an increase in the government savings rate) or an increase in the effective labor supply (e.g., peculiar patterns of the rural-urban migration within a country), they would reinforce the mechanisms discussed in this paper, causing the real exchange rate to fall further.

A desire to enhance one's prospect in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission. A sex ratio imbalance at birth and in the marriage age cohort is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. In many economies, parents have a preference for a son over a daughter. This used to lead to large families, not necessarily an unbalanced sex ratio. However, in the last three decades, as the technology to detect the gender of a fetus (Ultrasound B) has become less expensive and more widely available, many more parents engage in selective abortions in favor of a son, resulting in an increasing relative surplus of men. The strict family planning policy in China, introduced in the early

1980s, has induced Chinese parents to engage in sex-selective abortions more aggressively than their counterparts in other countries. The sex ratio at birth in China rose from 106 boys per hundred girls in 1980 to 122 boys per hundred girls in 1997 (see Wei and Zhang, 2009, for more detail). It may not be a coincidence that the Chinese real exchange rate started to garner international attention around 2003 just when the first cohort born after the implementation of the strict family planning policy were entering the marriage market.

Throughout the model, we assume an exogenous sex ratio. While the sex ratio is endogenous in the long-run as parental preference should evolve, the assumption of an exogenous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio affects only the relatively young cohort's savings decisions, but not those who have passed half of their working careers. Second, in terms of cross country experience, most countries with a skewed sex ratio have not shown a sign of reversal. Korea is the only economy whose sex ratio appeared to have started to revert back from a very skewed level. This suggests that, if the sex ratio follows a mean reversion process, the speed of reversion is very low. A systematic reversal of the sex ratio is unlikely to happen in most economies in the short run.

There are four bodies of work that are related to the current paper. First, the theoretical and empirical literature on the real exchange rate is too voluminous to summarize comprehensively here. In the context of using theories to estimate equilibrium real exchange rate, xxx provides recent surveys. Second, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men (or parents with sons), it is a favorable shock to women (or parents with daughters). Could the latter group strategically reduce their savings so as to completely offset whatever increments in savings men or parents with sons may have? In other words, the impact on aggregate savings from a rise in the sex ratio appears ambiguous. Our model will address this question. In any case, the literature on status goods has no discernible impact in policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by US officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A third related literature is the economics of family, which is also too vast to be summarized here comprehensively. One interesting insight of this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make

use of this feature in our model as well. None of the papers in this literature explores the general equilibrium implications for aggregate savings from a change in the sex ratio. The fourth literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause responsible for a majority of the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for the parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily choose to have fewer children than the earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2009) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows. In Section 2, we construct a simple overlapping generations (OLG) model with only one gender, and show that structural shocks, by raising the savings rate, can simultaneously produce a real exchange rate depreciation and a current account surplus. In Section 3, we present an OLG model with two genders, and demonstrate that a rise in the sex ratio could lead to a rise in both the aggregate savings rate and the current account, and a fall in the value of the real exchange rate. In Section 4, we calibrate the model to see if the sex ratio imbalance can produce changes in the real exchange rate and current account whose magnitudes are economically significant. In section 5, we provide some empirical evidence on the connection between the sex ratio and the real exchange rate. Section 6 offers concluding remarks and discusses possible future research.

## 2 A benchmark model with one gender

We start with a simple benchmark model with one gender. This allows us to see the savings channel in a transparent way. The setup is standard, and the discussion will pave the way for a model in the next section that features two genders and an unbalanced sex ratio.

There are two types of agents: consumers and producers. Consumers consume and make the saving decisions to maximize their intertemporal utilities. Producers choose capital and labor input to maximize the profits.

### 2.1 Consumers

Consumers live for two periods: young and old. They receive labor income in the first period and nothing in the second period after retiring. In the first period, consumers consume a part of the labor income in the first period and save the rest for the second period.

The final good  $C_t$  consumed by consumers consists of two parts: a tradable good  $C_{Tt}$  and a nontradable good  $C_{Nt}$ .

$$C_t = \frac{C_{Nt}^\gamma C_{Tt}^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$$

We normalized the price of the tradable good to be one and let  $P_{Nt}$  denote the relative price of the nontradable good. The consumer price index is

$$P_t = P_{Nt}^\gamma$$

Consumers earn labor income when they are young and retire when they are old. The optimization problem for a representative consumer is

$$\max u(C_{1t}) + \beta u(C_{2,t+1})$$

with the intertemporal budget constraint

$$\begin{aligned} P_t C_{1t} &= (1 - s_t) y_t \\ P_{t+1} C_{2,t+1} &= R s_t y_t \end{aligned}$$

where  $y_t$  is the disposable income and  $s_t$  is the savings rate of the young cohort.  $R$  is the gross interest rate in terms of the tradable good.

The optimal condition for the representative consumer's problem is

$$\frac{u'_{1t}}{P_t} = \beta R \frac{u'_{2,t+1}}{P_{t+1}} \tag{2.1}$$

We start with the case of a small open economy, and assume that the law of one price for the tradable good holds. The price of the tradable good is determined by the world market, and is set to be one in each period. The interest rate  $R$  in units of the tradable good is also a constant.

## 2.2 Producers

There are two sectors in the economy: a tradable good sector and a non-tradable good sector. Both markets are perfectly competitive. For simplicity, we make the same assumption as in Obstfeld and Rogoff (1996) that only the tradable good can be transformed into capital used in production.<sup>1</sup>

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<sup>1</sup>Relaxing this assumption will not change any of our results qualitatively.

### 2.2.1 Tradable good producers

For simplicity, we assume a complete depreciation of capital in every period. Tradable producers will maximize

$$\max E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} [Q_{T,t+\tau} - w_{t+\tau} L_{T,t+\tau} - K_{T,t+\tau+1}]$$

where the production function is

$$Q_{Tt} = \frac{A_{Tt} K_{Tt}^{\alpha_T} L_{Tt}^{1-\alpha_T}}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}}$$

Without any unanticipated shocks, the factor demand functions are, respectively,

$$R = \frac{1}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} \alpha_T A_{Tt} \left( \frac{L_{Tt}}{K_{Tt}} \right)^{1-\alpha_T} \quad (2.2)$$

$$w_t = \frac{1}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} (1-\alpha_T) A_{Tt} \left( \frac{K_{Tt}}{L_{Tt}} \right)^{\alpha_T} \quad (2.3)$$

It is useful to note that when there is an unanticipated shock in period  $t$ , (2.2) does not hold since  $K_{Tt}$  is a predetermined variable.

### 2.2.2 Nontradable good producers

Nontradable good producers will maximize the following objective function:

$$\max E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} [P_{N,t+\tau} Q_{N,t+\tau} - w_{t+\tau} L_{N,t+\tau} - K_{N,t+\tau+1}]$$

with the production function given by

$$Q_{Nt} = \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}}$$

Without unanticipated shocks, we have

$$R = \frac{1}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} P_{Nt} \alpha_N A_{Nt} \left( \frac{L_{Nt}}{K_{Nt}} \right)^{1-\alpha_N} \quad (2.4)$$

$$w_t = \frac{1}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} P_{Nt} (1-\alpha_N) A_{Nt} \left( \frac{K_{Nt}}{L_{Nt}} \right)^{\alpha_N} \quad (2.5)$$

If there is an unanticipated shock in period  $t$ , (2.4) does not hold.

In equilibrium, the market clearing condition for the nontradable good pins down the price of the

nontradable good,

$$Q_{Nt} = \frac{\gamma P_t (C_{2t} + C_{1t})}{P_{Nt}} \quad (2.6)$$

The labor market clearing condition is given by

$$L_{Tt} + L_{Nt} = 1 \quad (2.7)$$

Assuming no labor income tax (for simplicity),  $y_t = w_t$ .

**Definition 1** *An equilibrium in the small open economy is a set  $\{s_t, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  that satisfies the following conditions:*

(i) *The households' savings rates,  $s_t = \{s_{it}, s_{-i,t}\}$ , maximize the household's welfare*

$$s_t = \arg \max \{V_t | s_{-i,t}, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$$

(ii) *The allocation of capital stock and labor, and the output of the non-tradable good clear the factor and the output markets, and maximize the firms' profit. In other words,  $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  solves (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).*

### 2.3 A shock to the savings rate and the effect on the exchange rate

To illustrate the idea that a shock that raises the savings rate could lower the value of the real exchange rate, we now consider an unanticipated increase in the discount factor  $\beta$  that makes the young cohort more patient. In period  $t$ , (2.3) and (2.5) hold, but (2.2) and (2.4) fail.

The market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt} A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} = \gamma (R s_{t-1} w_{t-1} + (1-s_t) w_t)$$

We can solve (2.1), (2.6), (2.3) and (2.5) to obtain the equilibrium in period  $t$ . Let  $R = \frac{RP_t}{P_{t+1}}$  denote the real interest rate. We assume that the utility function is of the CRRA form, i.e.,  $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ , and following Obstfeld and Rogoff (1996), that the nontradable good sector is relatively more labor-intensive, i.e.,  $\alpha_N < \alpha_T$ . We can obtain the following proposition.

**Proposition 1** *With an increase in the discount factor  $\beta$  of the young cohort, the aggregate savings rate rises, and the price of the nontradable good falls. As a result, the real exchange rate depreciates and the current account increases.*

**Proof.** See Appendix A. ■



In the period in which the shock occurs, as a representative consumer becomes more patient, he would save more and consume less. The reduction in aggregate consumption leads to a decrease in the relative price of nontradable good (and a depreciation of the real exchange rate). As the rise in savings is not accompanied by a corresponding rise in investment, the country's current account increases. In summary, without currency manipulations, real factors that lead to a rise in a country's savings rate can simultaneously produce a fall in the real exchange rate and a rise in the current account. The low value of the real exchange rate is not the cause of the current account surplus.

Note that the effect on the RER and the current account last for one period. In period  $t+1$ , since the shock has been observed and taken into account by consumers and firms, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1 - \alpha_T}} \quad \text{and} \quad P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1 - \alpha_T}}$$

In other words, the price of the nontradable good and the consumer price index go back to their initial levels. Later in the paper, we will demonstrate how frictions in the factor market can produce longer-lasting effects on the real exchange rate and the current account.

### 3 Unbalanced sex ratios and real exchange rates

In this section, we extend our benchmark model to a two-sex overlapping generations model. Within each cohort, there are women and men. A marriage can take place at the beginning of a cohort's second period, but only between a man and a woman in the same cohort. Once married, the husband and the wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and the wife can each consume more than half of their combined second period income. Everyone is endowed with an ability to give his/her spouse some additional emotional utility (or "love"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when an individual enters the marriage market. There are no divorces.

Each generation is characterized by an exogenous ratio of men to women  $\phi(\geq 1)$ . All men are identical *ex ante*, and all women are identical *ex ante*. Men and women are symmetric in all aspects - in particular, men do not have an intrinsic tendency to save more - except that the sex ratio may be unbalanced.

#### 3.1 A small open economy

We again start from a small open economy. As in the benchmark model, the price of the tradable good is always one and the interest rate in units of the tradable good is a constant  $R$ . As in Obstfeld

and Rogoff (1995), we assume for simplicity that only tradable goods can be transferred into capital used in production.<sup>2</sup>

### 3.1.1 Consumers

#### A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking into account the choices by men and all other women, and the likelihood that she will be married. If she fails in getting married, then her second-period consumption is

$$P_{t+1}C_{2,t+1}^{w,n} = Rs_t^w y_t^w$$

where  $R$ ,  $y_t^w$  and  $s_t^w$  are the gross interest rate of an international bond, her endowment, and her savings rate, respectively, all in units of the tradable good.

If she is married at the beginning of the second period, her second-period consumption is

$$P_{t+1}C_{2,t+1}^w = \kappa (Rs_t^w y_t^w + Rs_t^m y_t^m)$$

where  $y_t^m$  and  $s_t^m$  are her husband's first period endowment and savings rate, respectively.  $\kappa$  ( $\frac{1}{2} = \kappa < 1$ ) represents the notion that consumption within a marriage is a public good with congestion. As an example, if two spouses buy a car, both can use it. In contrast, if both are single, they may need to buy two cars. When  $\kappa = \frac{1}{2}$ , the husband and the wife only consume private goods. When  $\kappa = 1$ , then all the consumption is a public good with no congestion<sup>3</sup>.

The optimal savings rate is chosen to maximize the following objective function:

$$V_t^w = \max_{s_t^w} u(C_{1t}^w) + \beta E_t [u(C_{2,t+1}^w) + \eta^m]$$

subject to the budget constraints that

$$P_t C_{1t}^w = (1 - s_t^w) y_t^w \tag{3.1}$$

$$P_{t+1} C_{2,t+1}^w = \begin{cases} \kappa (Rs_t^w y_t^w + Rs_t^m y_t^m) & \text{if married} \\ Rs_t^w y_t^w & \text{otherwise} \end{cases} \tag{3.2}$$

where  $E_t$  is the conditional expectation operator.  $\eta^m$  is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function  $F^m$ . Bhaskar (2009) also introduces a similar "love" variable.

<sup>2</sup>This assumption does not change any qualitative results but greatly simplifies the derivations.

<sup>3</sup>By assuming the same  $\kappa$  for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow  $\kappa$  to be gender specific, and to be a function of the sex ratio and the relative wealth levels of the two spouses, along the lines of Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

### A Representative Man's Optimization Problem

A man's problem is symmetric to a women's problem. In particular, if he fails in getting married, his second period consumption is

$$P_{t+1}C_{2,t+1}^{m,n} = Rs_t^m y_t^m$$

If he is married, his second period consumption is

$$P_{t+1}C_{2,t+1}^m = \kappa (Rs_t^w y_t^w + Rs_t^m y_t^m)$$

A representative man will choose his savings rate to maximize the following value function

$$V_t^m = \max_{s_t^m} u(C_{1t}^m) + \beta E_t [u(C_{2,t+1}^m) + \eta^w]$$

subject to the budget constraints that

$$P_t C_{1t}^m = (1 - s_t^m) y_t^m \tag{3.3}$$

$$P_{t+1} C_{2,t+1}^m = \begin{cases} \kappa (Rs_t^w y_t^w + Rs_t^m y_t^m) & \text{if married} \\ Rs_t^m y_t^m & \text{otherwise} \end{cases} \tag{3.4}$$

where  $V^m$  is his value function when he chooses to enter the marriage market.  $\eta^w$  is the emotional utility he obtains from his wife, which is drawn from a distribution function  $F^w$ .

### The Marriage Market<sup>4</sup>

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" he/she can obtain from his/her spouse. The weights on the two criteria are implied by the utility functions specified earlier. More precisely, woman  $i$  prefers a higher ranked man to a lower ranked one, where the rank on man  $j$  is given by  $u(c_{2w,i,j}) + \eta_j^m$ . Symmetrically, man  $j$  assigns a rank to woman  $i$  based on the utility he can obtain from her  $u(c_{2m,j,i}) + \eta_i^w$ . (To ensure that the preference is strict for men and women, when there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers  $j$  if  $j < j'$  and a man does the same.) Note that "love" is not in the eyes of a beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round  $k-1$  makes a new proposal in round  $k$  to his most preferred woman among those who have not have rejected him. Each available women in round  $k$  holds the proposal

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<sup>4</sup>We use the word "market" informally here. The pairing of husbands and wives is not done through prices.

from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made, and the women accept the most attractive proposals.<sup>5</sup>

With many women and men in the marriage market, all women (and all men) approximately form a continuum and each individual has a measure close to zero. Let  $I^w$  and  $I^m$  denote the continuum formed by women and men respectively. We normalize  $I^w$  and let  $I^w = (0, 1)$ . Since the sex ratio is  $\phi$ , the set of men  $I^m = (0, \phi)$ . Men and women are ordered in such a way that a higher value in the set means a higher ranking by members of the opposite sex.

In equilibrium, there exists a unique mapping ( $\pi^w$ ) for women in the marriage market.

$$\pi^w : I^w \rightarrow I^m$$

That is, woman  $i$  ( $i \in I^w$ ) is mapped to man  $j$  ( $j \in I^m$ ), given all the initial wealth and emotional utility draws. This implies a mapping from a combination  $(s_i^w, \eta_i^w)$  to another combination  $(s_j^m, \eta_j^m)$ . In other words, for woman  $i$ , given all her rivals'  $(s_{-i}^w, \eta_{-i}^w)$  and all men's  $(s^m, \eta^m)$ , the type of husband  $j$  she can marry depends on her  $(s_i^w, \eta_i^w)$ . Before she enters the marriage market, she knows only the distribution of her own type but not the exact value. As a result, the type of her future husband  $(s_j^m, \eta_j^m)$  is also a random variable. Woman  $i$ 's second period expected utility is

$$\begin{aligned} & \int \max \left[ u(c_{2w,i,j}) + \eta_{\pi^w(i|s_i^w, \eta_i^w, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m)}^m, \quad u(Rs_i^w y_i^w) \right] dF^w(\eta_i^w) \\ &= \int_{\bar{\pi}_i^w} \left[ u(c_{2w,i,j}) + \eta_{\pi^w(i|s_i^w, \eta_i^w, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m)}^m \right] dF^w(\eta_i^w) + \int^{\bar{\pi}_i^w} u(Rs_i^w y_i^w) dF^w(\eta_i^w) \end{aligned}$$

where  $\bar{\pi}_i^w$  is her threshold ranking on men such that she is indifferent between marriage or not. Any lower-ranked man, or any man with  $\pi_i^w < \bar{\pi}_i^w$ , won't be chosen by her.

Since we assume there are (weakly) fewer women than men, we expand the set  $I^w$  to  $\tilde{I}^w$  so that  $\tilde{I}^w = (0, \phi)$ . In the expanded set, women in the marriage market start from value  $\phi - 1$  to  $\phi$ . The measure for women in the marriage market remains one. In equilibrium, there exists a unique mapping for men in the marriage market:

$$\pi^m : I^m \rightarrow \tilde{I}^w$$

where  $\pi^m$  maps man  $j$  ( $j \in I^m$ ) to woman  $i$  ( $i \in I^w$ ). Those men who are matched with a low value  $i < \phi - 1$  in set  $\tilde{I}^w$  will not be married. In that case,  $\eta_{\pi^m(j)}^w = 0$  and  $c_{2m,j,i} = Rs_j^m y_j^m$ . In general,

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<sup>5</sup>If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge. What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

man  $j$ 's second period expected utility is

$$\begin{aligned} & \int \max \left[ u(c_{2m,j,i}) + \eta_{\pi_j^m}^w(j|s_j^m, \eta_j^m, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w), \quad u(Rs_j^m y_j^m) \right] dF^m(\eta_j^m) \\ &= \int_{\bar{\pi}_j^m} \left[ u(c_{2m,j,i}) + \eta_{\pi_j^m}^w(j|s_j^m, \eta_j^m, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w) \right] dF^m(\eta_j^m) + \int^{\bar{\pi}_j^m} u(Rs_j^m y_j^m) dF^m(\eta_j^m) \end{aligned}$$

where  $\bar{\pi}_j^m$  is his threshold ranking on all women. Any woman with a poorer rank,  $\pi_j^m < \bar{\pi}_j^m$ , will not be chosen by him.

We assume that the density functions of  $\eta^m$  and  $\eta^w$  are continuously differentiable. Since all men (women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a positive *assortative* matching emerges for those men and women who are matched. In other words, there exists a mapping  $M$  from  $\eta^w$  to  $\eta^m$  such that

$$\begin{aligned} 1 - F^w(\eta^w) &= \phi(1 - F^m(M(\eta^w))) \\ &\Leftrightarrow \\ M(\eta^w) &= (F^m)^{-1} \left( \frac{F^w(\eta^w)}{\phi} + \frac{\phi - 1}{\phi} \right) \end{aligned}$$

For simplicity, we assume that  $\eta^w$  and  $\eta^m$  are drawn from the same distribution,  $F^w = F^m = F$ . Furthermore, the lowest possible value of the emotional utility  $\eta^{\min}$  is sufficiently large that everyone desires to be married. We also assume that there exists a small and exogenous possibility  $p$  that a woman may not find a marriage partner due to frictions in the marriage market. The last assumption plays no role in the analytical part of the model but will simplify the quantitative calibrations later. In equilibrium, given all her rivals' saving decisions and  $\eta^w$ , woman  $i$ 's second period utility is

$$(1-p) \left[ u \left( \frac{\kappa(Rs_t^w(i)y_t^w + Rs_t^m y_t^m)}{P_{t+1}} \right) + \int_{\eta^{\min}}^{\eta^{\max}} M(\tilde{\eta}_i^w) d\tilde{F}(\tilde{\eta}_i^w) \right] + pu \left( \frac{Rs_t^w y_t^w}{P_t} \right)$$

where  $\tilde{\eta}_i^w = u \left( \frac{\kappa(Rs_t^w(i)y_t^w + Rs_t^m y_t^m)}{P_{t+1}} \right) - u \left( \frac{\kappa(Rs_t^w y_t^w + Rs_t^m y_t^m)}{P_{t+1}} \right) + \eta^w$ .

Due to symmetry, we drop the sub-index  $i$ . A representative woman's first order condition, given men's savings decisions, is

$$-u'_{1w} \frac{y_t^w}{P_t} + \beta(1-p) \left[ u'_{2w} \frac{\partial C_{2,t+1}^w}{\partial s_t^w} + \frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}(\tilde{\eta}^w)}{\partial s_t^w} \right] + pu'_{2w,n} \frac{y_t^w}{P_{t+1}} = 0 \quad (3.5)$$

where

$$\frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}(\tilde{\eta}^w)}{\partial s_t^w} = \kappa u'_{2w} R \frac{y_t^w}{P_{t+1}} \left[ \int M'(\eta^w) dF(\eta^w) + M(\eta^{\min}) f(\eta^{\min}) \right]$$

Similarly, a representative man's second-period utility is

$$(1-p) \left[ \tilde{\delta}_j^m u \left( \frac{\kappa (Rs_t^w y_t^w + Rs_t^m(j) y_t^m)}{P_{t+1}} \right) + \int_{M(\eta^{\min})}^{\eta^{\max}} M^{-1}(\tilde{\eta}_j^m) d\tilde{F}(\tilde{\eta}_j^m) \right] + [(1-p)(1-\tilde{\delta}_j^m) + p] u \left( \frac{Rs_t^m(j) y_t^m}{P_{t+1}} \right)$$

where  $\tilde{\eta}_j^m = u \left( \frac{\kappa (Rs_t^w y_t^w + Rs_t^m(j) y_t^m)}{P_{t+1}} \right) - u \left( \frac{\kappa (Rs_t^w y_t^w + Rs_t^m y_t^m)}{P_{t+1}} \right) + \eta_j^m$  and  $\tilde{\delta}_j^m$  is the probability he gets married

$$\begin{aligned} \tilde{\delta}_j^m &= \Pr \left( u(C_{2,t+1}^w(j)) - u(C_{2,t+1}^w) + \eta_j^m \geq M(\eta^{\min}) \mid Rs^w y^w, Rs^m y^m \right) \\ &= 1 - F \left( M(\eta^{\min}) - u(C_{2,t+1}^w(j)) + u(C_{2,t+1}^w) \right) \end{aligned} \quad (3.6)$$

His first order condition is

$$-u'_{1m} \frac{y_t^m}{P_t} + \beta(1-p) \left[ \begin{aligned} &\delta^m u'_{2m} \frac{\partial C_{2,t+1}^m}{\partial s_t^m} + \int_{M(\eta^{\min})} \frac{\partial M^{-1}(\tilde{\eta}^m)}{\partial s_t^m} d\tilde{F}^m(\tilde{\eta}^m) \\ &+ f \left( M(\eta^{\min}) \right) u'_{2w} \frac{\partial C_{2,t+1}^m}{\partial s_t^m} (u_{2m} - u_{2m,n} + \eta^{\min}) \end{aligned} \right] + [(1-\delta^m)(1-p) + p] u'_{2m,n} \frac{y_t^m}{P_{t+1}} = 0 \quad (3.7)$$

where

$$\int_{M(\eta^{\min})} \frac{\partial M^{-1}(\tilde{\eta}^m)}{\partial s^m} d\tilde{F}^m(\tilde{\eta}^m) = \kappa u'_{2w} R \frac{y_t^m}{P_{t+1}} \int_{M(\eta^{\min})} (M^{-1})'(\eta^m) dF(\eta^m)$$

For simplicity, we assume that women and men will earn the same first period labor income and that there is no tax, i.e.,  $y_t^w = y_t^m = w_t$ . We now define an equilibrium in this economy.

**Definition 2** An equilibrium is a set of savings rates, capital and labor allocation by sector, and the relative price of nontradable good  $\{s_t^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  that satisfies the following conditions:

(i) The savings rates by the representative woman and the representative man, conditional on other women and men's savings rates,  $s_t^w = \{s_{it}^w, s_{-i,t}^w\}$  and  $s_t^m = \{s_{jt}^m, s_{-j,t}^m\}$ , maximize their respective utilities

$$\begin{aligned} s_{it}^w &= \arg \max \{ V_t^w \mid s_{-i,t}^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt} \} \\ s_{jt}^m &= \arg \max \{ V_t^m \mid s_t^w, s_{-j,t}^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt} \} \end{aligned}$$

(ii) The markets for capital, labor, and tradable and nontradable goods clear, and firms maximize their profits. In other words,  $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  solves (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).

### 3.1.2 Shocks on the sex ratio

We now consider an unanticipated shock on the young cohort's sex ratio, i.e., the sex ratio rises from one to  $\phi(> 1)$  from period  $t$  onwards. Since the shock is unanticipated, (2.2) and (2.4) do not hold in period  $t$ .

As in the benchmark model, the market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} = \gamma(Rs_{t-1}w_{t-1} + (1-s_t)w_t) \quad (3.8)$$

where

$$s_t = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$$

is the aggregate savings rate by the young cohort in period  $t$ .

By (2.3) and (2.5), we have

$$w_t = \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T)A_{Tt} \left( \frac{K_{Tt}}{1-L_{Nt}} \right)^{\alpha_T} = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N)A_{Nt} \left( \frac{K_{Nt}}{L_{Nt}} \right)^{\alpha_N} \quad (3.9)$$

We can solve (3.5), (3.7), (3.8) and (3.9) to obtain the equilibrium in period  $t$ . If utility function is of log form,  $u(C) = \ln C$  and  $\eta$  is drawn from a uniform distribution, we have the following proposition.

**Proposition 2** *As the sex ratio in the young cohort rises, a representative man weakly increases his savings rate while a representative woman weakly reduces her savings rate. However, the economy-wide savings rate increases unambiguously. The real exchange rate depreciates and the current account rises.*

**Proof.** See Appendix B. ■

A few remarks are in order. First, it is perhaps not surprising that the representative man raises his savings rate in response to a rise in the sex ratio because the need to compete in the marriage market becomes greater. Why does the representative woman reduce her savings rate? Because she anticipates a higher savings rate from her future husband, she does not need to sacrifice her first-period consumption as much as she otherwise would have to.

Second, why does the aggregate savings rate rise in response to a rise in the sex ratio? In other words, why is the increment in men's savings greater than the decline in women's savings? Intuitively, a representative man raises his savings rate for two reasons: in addition to improving his relative standing in the marriage market, he raises his savings rate to make up for the lower savings rate by his future wife. The more his future wife is expected to cut down her savings, the more he would have to raise his own savings to compensate. This ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. In addition, since men save more, the rising share of

men in the population would also raise the aggregate savings rate. While both channels contribute to a rise in the aggregate savings rate, it is easy to verify that the first channel (the incremental competitive savings by any given man) is more important than the second effect (a change in the composition of the population with different saving propensities).

Third, once we obtain an increase in the aggregate savings rate, the logic from the previous one-gender benchmark model applies. In particular, the relative price of the non-tradable good declines (and hence the real exchange rate depreciates), and the current account rises.

Similar to the benchmark model with a single gender, once the shock is observed and taken into account in period  $t + 1$ , (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1 - \alpha_T}} \quad \text{and} \quad P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1 - \alpha_T}}$$

This means that the real exchange rate and the current account will return to the previous values after one period.

### 3.2 Capital adjustment costs

With additional frictions, a shock to the sex ratio can only affect the real exchange rate for one period. If there are capital adjustment costs in each sector, the effect on the real exchange rate can be prolonged. We assume that the capital accumulation in each sector is as following:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{b}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t$$

where  $\delta$  is the depreciation rate and  $I_t$  is investment.  $\frac{b}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t$  represents the adjustment cost as in Chari, Kehoe and McGrattan (2002).

Then (2.2) and (2.4) become, respectively,

$$\begin{aligned} R &= 1 - \delta + \frac{1}{\alpha_T^{\alpha_T} (1 - \alpha_T)^{1 - \alpha_T}} \alpha_T A_{Tt} \left( \frac{L_{Tt}}{K_{Tt}} \right)^{1 - \alpha_T} - bR \left( \frac{I_{Tt}}{K_{Tt}} - \delta \right) - \frac{b}{2} \left( \left( \frac{I_{Tt}}{K_{Tt}} \right)^2 - \delta^2 \right) \quad (3.10) \\ R &= 1 - \delta + \frac{1}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1 - \alpha_N}} P_{Nt} \alpha_N A_{Nt} \left( \frac{L_{Nt}}{K_{Nt}} \right)^{1 - \alpha_N} - bR \left( \frac{I_{Nt}}{K_{Nt}} - \delta \right) - \frac{b}{2} \left( \left( \frac{I_{Nt}}{K_{Nt}} \right)^2 - \delta^2 \right) \quad (3.11) \end{aligned}$$

Without capital adjustment cost, i.e.,  $b = 0$ , the price of the nontradable good will go back to its equilibrium level in period  $t + 1$ . If  $b > 0$ , then

$$P_{Nt} = \frac{\frac{1}{\alpha_T^{\alpha_T} (1 - \alpha_T)^{1 - \alpha_T}} \alpha_T A_{Tt+1} \left( \frac{L_{Tt+1}}{K_{Tt+1}} \right)^{1 - \alpha_T} - bR \left( \frac{I_{Tt+1}}{K_{Tt+1}} - \frac{I_{Nt+1}}{K_{Nt+1}} \right) - \frac{b}{2} \left( \left( \frac{I_{Tt+1}}{K_{Tt+1}} \right)^2 - \left( \frac{I_{Nt+1}}{K_{Nt+1}} \right)^2 \right)}{\frac{1}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1 - \alpha_N}} \alpha_N A_{Nt+1} \left( \frac{L_{Nt+1}}{K_{Nt+1}} \right)^{1 - \alpha_N}}$$



$P_{Nt}$  is now a function of  $\frac{I_{Tt+1}}{K_{Tt+1}}$  and  $\frac{I_{Nt+1}}{K_{Nt+1}}$ . If  $\frac{I_{Tt+1}}{K_{Tt+1}} \neq \frac{I_{Nt+1}}{K_{Nt+1}}$ ,  $P_{Nt}$  is not a constant. This means that, with capital adjustment costs, the price of the nontradable good does not return immediately to its long-run equilibrium level. As a result, a rise in the sex ratio can have a long-lasting and depressing effect on the real exchange rate.

### 3.3 Two large countries

We now turn to a world with two large countries: Home and Foreign. Assume that they are identical in every respect except for their sex ratios. Specifically, in period  $t$ , the sex ratio of the young cohort in Home rises from one to  $\phi$  ( $\phi > 1$ ), while Foreign always has a balanced sex ratio. Households in each country consume a tradable good and a nontradable good.

$$C_t = \frac{C_{Nt}^\gamma C_{Tt}^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \text{ and } C_t^* = \frac{(C_{Nt}^*)^\gamma (C_{Tt}^*)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$$

where  $C_t$  and  $C_t^*$  represent home and foreign consumption indexes, respectively. We choose the tradable good as the numeraire. As a result, the consumer price index is

$$P_t = P_{Nt}$$

where  $P_{Nt}$  is the price of the home produced nontradable good. Similarly, the consumer price index in Foreign is:

$$P_t^* = (P_{Nt})^\gamma$$

The rise in Home's sex ratio in period  $t$  is assumed to be an unanticipated shock. As a result, (2.2) and (2.4) fail in both the home and foreign country. By the same reasoning, Home experiences a real exchange rate depreciation in period  $t$ , but a real appreciation in period  $t + 1$ .

We can write the current account in Home and Foreign as follows:

$$CA_t = s_t w_t - s_{t-1} w_{t-1} + K_t - K_{t+1} \text{ and } CA_t^* = s_t^* w_t^* - s_{t-1}^* w_{t-1}^* + K_t^* - K_{t+1}^*$$

Before the shock, we had

$$s_{t-1} = s_{t-1}^*, w_{t-1} = w_{t-1}^* \text{ and } K_t = K_t^*$$

In period  $t + 1$ , we have

$$P_{Nt} = P_{Nt}, w_{t+1} = w_{t+1}^*, \text{ and } P_{t+1} = P_{t+1}^*$$

and the demand for the nontradable good is

$$Q_{N,t+1} = \frac{\gamma w_{t+1} ((R-1)s_t + 1)}{P_{Nt}} \text{ and } Q_{N,t+1}^* = \frac{\gamma w_{t+1}^* ((R-1)s_t^* + 1)}{P_{Nt}}$$

Since Home now has a higher sex ratio than Foreign, we have  $s_t > s_t^*$ , and therefore

$$Q_{N,t+1} > Q_{N,t+1}^*$$

We assume that the nontradable sector is more labor-intensive, i.e.,  $\alpha_N < \alpha_T$ . Given the same technologies and the same labor endowments in the two countries, we have

$$K_{t+1} < K_{t+1}^*$$

In period  $t$ , nothing changes in the foreign country, then  $s_t^* w_t^* = s_{t-1} w_{t-1}$ . Following the same steps as in the case of a small open economy, we can show that  $s_t w_t > s_{t-1} w_{t-1} = s_t^* w_t^*$ . Then it is easy to show that  $CA_t > 0 > CA_t^*$ . In other words, Home exhibits a current account surplus while Foreign experiences a current account deficit.

### 3.4 Endogenous labor supply

We turn to the case of endogenous labor supply. Just as a male raises his savings rate to gain a competitive advantage in the marriage market, he may choose to increase his supply of labor for the same reason in response to a rise in the sex ratio. This can translate into an increase in the effective aggregate labor supply if women do not decrease their labor supply too much. If the production of the nontradable good is more labor-intensive, the increase in the effective labor supply can reduce the relative price of the non-tradable good (and the value of the real exchange rate). Therefore, endogenous labor supply could reinforce the savings channel from the sex ratio shock, leading to an additional reduction in the real exchange rate.

We allow each person to endogenously choose the first period labor supply and the utility function of the first period is  $u(C) + v(1 - L)$ , where  $L$  is the labor supply and  $v(1 - L)$  is the utility function of leisure. As in the standard literature, we assume that  $v' > 0$  and  $v'' < 0$ . Again, for simplicity, we assume no tax on the labor income. The utility function governing the leisure-labor choice is the same for men and women. In other words, by assumption, men and women are intrinsically symmetric except for their ratio in the society.

We can rewrite the optimization problem for a representative woman as following:

$$\max u(C_{1t}^w) + v(1 - L_t^w) + \beta E_t [u(C_{2,t+1}^w) + \eta^m]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^w &= (1 - s_t^w) w_t L_t^w \\ P_{t+1} C_{2,t+1}^w &= \begin{cases} \kappa (R s_t^w L_t^w + R s_t^m L_t^m) w_t & \text{if married} \\ R s_t^w w_t L_t^w & \text{otherwise} \end{cases} \end{aligned}$$

The first order condition with respect to her labor supply is

$$u'_{1w} \frac{(1 - s_t^w) w_t}{P_t} + \left[ u'_{2w} \frac{\partial C_{2,t+1}^w}{\partial L_t^w} + \frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial L_t^w} \right] + \beta p u'_{2w} \frac{\kappa R s_t^w w_t}{P_{t+1}} - v'_w = 0$$

Notice that  $\frac{\partial C_{2,t+1}^w}{\partial L_t^w} = \frac{\partial C_{2,t+1}^w}{\partial s_t^w} \frac{s_t^w}{L_t^w}$  and  $\frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial L_t^w} = \frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial s_t^w} \frac{s_t^w}{L_t^w}$ . Combining the equation above with (3.5), we have

$$\frac{w_t}{P_t} = \frac{v'_w}{u'_{1w}} \quad (3.12)$$

The optimization problem for a representative man is similar. If he decides to enter the marriage market

$$\max u(C_{1t}^m) + v(1 - L_t^m) + \beta E_t [u(C_{2,t+1}^m) + \eta^w]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^m &= (1 - s_t^m) w_t L_t^m \\ P_{t+1} C_{2,t+1}^m &= \begin{cases} \kappa (R s_t^m L_t^m + R s_t^w L_t^w) w_t & \text{if married} \\ R s_t^m w_t L_t^m & \text{otherwise} \end{cases} \end{aligned}$$

The optimization condition for the representative man's labor supply is

$$\frac{w_t}{P_t} = \frac{v'_m}{u'_{1m}} \quad (3.13)$$

On the supply side, all equilibrium conditions remain the same except for the labor market clearing condition, which now becomes

$$L_{Tt} + L_{Nt} = \frac{1}{1 + \phi} L_t^w + \frac{\phi}{1 + \phi} L_t^m \quad (3.14)$$

We now define an equilibrium in such an economy.

**Definition 3** An equilibrium is a set  $\{(s_t^w, L_t^w), (s_t^m, L_t^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  that satisfies the following conditions:

(i) The savings and labor supply decisions by women and men,  $(s_t^w, L_t^w) = \{s_{it}^w, s_{-i,t}^w, L_{it}^w, L_{-i,t}^w\}$

and  $(s_t^m, L_t^m) = \{s_{it}^m, s_{-i,t}^m, L_{it}^m, L_{-i,t}^m\}$ , maximize their utilities, respectively,

$$\begin{aligned} (s_{it}^w, L_{it}^w) &= \arg \max \{V_t^w | (s_{-i,t}^w, L_{-i,t}^w), (s_t^m, L_t^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\} \\ (s_{jt}^m, L_{jt}^m) &= \arg \max \{V_t^m | (s_t^w, L_t^w), (s_{-j,t}^m, L_{-j,t}^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\} \end{aligned}$$

(ii) The markets for both goods and factors clear, and firms' profits are maximized. In other words,  $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  solves (2.2), (2.3), (2.4), (2.5), (2.6) and (3.14).

As before, we assume that  $u(C) = \ln C$ . We let  $L_t$  denote the aggregate labor supply in period  $t$ , and assume that  $\frac{v''L}{v'}$  is non-decreasing in  $L$ .

**Proposition 3** *As the sex ratio (in the young cohort) rises, a representative man weakly increases both his labor supply and his savings rate, while a representative woman weakly reduces both her labor supply and her savings rate. However, the economy-wide labor supply and savings rate both increase unambiguously. The real exchange rate depreciates, and the current account rises.*

**Proof.** See Appendix C. ■

In response to a rise in the sex ratio, for the same reason that men may cut their consumption and increase their savings rate, they may cut down their leisure and increase their labor supply. Similarly, for women, for the same reason that induce them to reduce their savings, they may reduce their labor supply (and increase leisure). In the aggregate, for the same reason that the increase in savings by men is more than enough to offset the decrease in savings by women, the increase in labor supply by men is also larger than the decrease in labor supply by women. Therefore, the aggregate labor supply rises in response to a rise in the sex ratio.

With a fixed labor supply, it is worth remembering that the nontradable sector shrinks after a rise in the sex ratio. The reason is that a decline in the relative price of the nontradable goods (due to the savings channel) makes it less attractive for labor and capital to stay in the nontradable sector. Now, with an endogenous labor supply, the total effective labor supply increases after a rise in the sex ratio according to Proposition 3. By a logic similar to the Rybzinsky theorem, this by itself has a tendency to induce an expansion of the nontradable sector if the production of the nontradable good is more labor intensive.

Relative to the case of a fixed labor supply (and only the savings channel), adding the effect of endogenous labor supply leads to either an expansion of the nontradable sector, or at least a smaller reduction in the size of the nontradable sector. The exact scenario depends on parameter values. However, regardless of what happens to the size of the nontradable sector, the price of the nontradable good (and the value of the real exchange rate) must fall by a greater amount when the endogenous labor supply effect is added to the savings effect.

### 3.5 Parental savings

As documented in Wei and Zhang (2009), parental savings for children are likely to be an important part of household savings. To incorporate this feature, we consider an OLG model in which every cohort lives three periods (young, middle-aged, and old). Everyone works and earns labor income in the first two periods. If one gets married, the marriage takes place at the beginning of the second period, and the couple produces a single child right away. The child is a boy with a probability of  $\frac{\phi}{1+\phi}$ , and a girl with a probability of  $\frac{1}{1+\phi}$ . If the child ever gets married, the marriage takes place at the beginning of the child's second period, which is also the beginning of the parents' third period. They derive direct emotional utility from having a child, although the value of this emotional utility could depend on the gender of the child. Parents are also altruistic toward their child and can save for their children (in addition to saving for themselves). For simplicity, we assume that children do not make reverse financial transfers to their parents.

With this setup, the optimization problem for a representative woman who enters the marriage market now becomes

$$V_t^w = \max_{s_t^w} u(c_{1t}^w) + \beta E [u(c_{2,t+1}^w) + \eta^m] + \beta^2 E [u(c_{3,t+2}^w) + \eta^m] + \beta \theta E \left[ \frac{\phi (V_{t+1}^m + \eta^s)}{1 + \phi} + \frac{V_{t+1}^w + \eta^d}{1 + \phi} \right]$$

where  $V_{t+1}^m$  ( $V_{t+1}^w$ ) is the life utility of the woman's son (daughter) if she gets married.  $\eta^s$  and  $\eta^d$  are the emotional utility parents obtain from having a son and a daughter, respectively.  $\theta$  is the parameter shows the degree of altruism.

If her first-period savings rate is  $s_t^w$ , her first-period consumption is

$$P_t c_{1t}^w = (1 - s_t^w) y_t^w$$

In the second period, if she fails to get married, she would consume the same amount in the second and the third periods:

$$P_{t+1} c_{2,t+1}^{w,n} = P_{t+2} c_{3,t+2}^{w,n} = \frac{R}{1 + R} (R s_t^w y_t^w + y_t^w)$$

Parents can save for their child, and that savings rate potentially depends on the gender of their child. Let  $T_{t+1}^i$  be the amount of parental savings for their child, where  $i = w$  (a daughter) or  $m$  (a son). Parental savings augments a young person's first-period income. We represent this by assuming that a person's first-period income is a positive and concave function of his/her parents' real savings:

$$\frac{y_t^i}{P_t} = Y \left( \frac{T_t^i}{P_t} \right)$$

and  $Y' > 0$ ,  $Y'' < 0$ ,  $Y' \rightarrow \infty$  as  $T^i \rightarrow 0$  and  $Y' \rightarrow 0$  as  $T^i \rightarrow \infty$ .<sup>6</sup>

<sup>6</sup>We use this assumption to avoid the inequality constraint  $T^i \geq 0$ , which greatly simplifies the calculation.

The representative woman's second and third period consumptions are

$$P_{t+1}c_{2,t+1}^{w,i} = P_{t+2}c_{3,t+2}^{w,i} = \frac{\kappa R}{1+R} (Rs_t^w y_t^w + Rs_t^m y_t^m + y_t^w + y_t^m - T_{t+1}^i)$$

where  $i$  stands for the child's gender.

As in the benchmark model, we assume a uniform distribution for  $\eta^i$ . The optimization condition for the representative woman is

$$-u'_{1w} + \left( \beta \frac{P_t}{P_{t+1}} + \beta^2 \frac{P_t}{P_{t+2}} \right) R \left[ (1-p) \frac{\kappa R}{1+R} \left( 1 + \frac{1}{\phi} + M(\eta^{\min}) f(\eta^{\min}) \right) \cdot \left( \frac{\phi}{1+\phi} u'_{2w,m} + \frac{1}{1+\phi} u'_{2w,w} \right) + p u'_{2w,n} \right] = 0 \quad (3.15)$$

and by the Benveniste and Scheinkman formula (1979),

$$\frac{\partial V_t^w}{\partial T_t^w} = u'_{1t,w} Y' \left( \frac{T_t^w}{P_t} \right)$$

Similarly, for a representative man, the optimal condition is

$$-u'_{1m} + \left( \beta \frac{P_t}{P_{t+1}} + \beta^2 \frac{P_t}{P_{t+2}} \right) \left[ (1-p) \frac{\kappa R}{1+R} \left( \delta^m + \left( \frac{u_{2m} + \eta^{\max} - u_{2m,n}}{\eta^{\max} - \eta^{\min}} \right) \right) \left( \frac{\phi}{1+\phi} u'_{2w,m} + \frac{1}{1+\phi} u'_{2w,w} \right) + ((1-\delta^m)(1-p) + p) u'_{2m,n} \right] = 0 \quad (3.16)$$

and,

$$\frac{\partial V_t^m}{\partial T_t^m} = u'_{1t,m} Y' \left( \frac{T_t^m}{P_t} \right)$$

Parents' optimal savings for their child in period  $t$  satisfies the following conditions:

$$-(1+\beta) \frac{\kappa R}{1+R} u'_{2t,w} + \theta \frac{\partial V_t^w}{\partial T_t^w} = 0 \quad (3.17)$$

$$-(1+\beta) \frac{\kappa R}{1+R} u'_{2t,m} + \theta \frac{\partial V_t^m}{\partial T_t^m} = 0 \quad (3.18)$$

We define the equilibrium in this case as following.

**Definition 4** An equilibrium is a set  $\{s_t^w, s_t^m, T_t^m, T_t^w, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  that satisfies the following conditions:

(i) The women and men's savings rates,  $s_t^w = \{s_{it}^w, s_{-i,t}^w\}$  and  $s_t^m = \{s_{jt}^m, s_{-j,t}^m\}$ , solve their maximization problems, respectively,

$$s_{it}^w = \arg \max \{V_t^w | s_{-i,t}^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$$

$$s_{jt}^m = \arg \max \{V_t^m | s_t^w, s_{-j,t}^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$$

(ii) The parental savings  $\{T_t^m, T_t^w\}$  solve the equations (3.17) and (3.18).

(iii)  $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$  solves (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).

To simplify the derivations, we follow Du and Wei (2010) and assume that  $Y = \left(\frac{T}{P}\right)^\xi$  where  $\frac{1}{2} \leq \xi < 1$ . We totally differentiate (3.15), (3.16), (3.17) and (3.18). We assume that the utility function is of the logarithmic form, and that  $E\eta^m$  and  $E\eta^w$  are sufficiently large, so that marriage is strongly attractive. With these assumptions, we have the following proposition:

**Proposition 4** *As the sex ratio in the young cohort rises, not only does a representative man weakly increase his first-period savings rate, but parents with a son also increase their savings for their son. The savings responses by young women and by parents with a daughter are ambiguous. However, the economy-wide savings rate increases unambiguously. The real exchange rate depreciates and the current account rises.*

**Proof.** See Appendix D. ■

## 4 Calibrations

We start from a simple OLG model in which every cohort lives two periods and there are no capital adjustment costs. We then add some more realisms by (1) assuming a 50-period life and (2) introducing capital adjustment costs. In the latter case, when every person lives 50 periods, he/she works in the first 30 periods and retires in the last 20 periods.

### 4.1 Parameters

We assume log-utility function,  $u(C) = \ln(C)$ . We take the annual interest rate  $R = 1.04$  and  $\beta = R^{-1}$ . We assume the tradable sector has a larger capital intensity,  $\alpha_T = 0.6$  and the nontradable sector has a lower capital intensity  $\alpha_N = 0.3$ . The share of the nontradable good consumption in the aggregate consumption is set to be 0.7,  $\gamma = 0.7$ . Within a family, the congestion for family consumption,  $\kappa = 0.8$ .

The emotional utility  $\eta$  needs to follow a continuously differential distribution. We assume a truncated normal distribution which might be more realistic than the uniform distribution used in the analytical model. We choose a standard deviation that is relatively tight,  $\sigma = 0.01$ . This limits the extent of heterogeneity among women (or men) in the eye of the opposite sex. We truncate the distribution at 1% in the left tail and at 99% in the right tail.

We choose the mean value of the emotional utility/love in the following way: holding all other factors constant, we compute the income compensation to a life-time bachelor that can makes him

indifferent between being married and being single.

$$u\left(\frac{1}{1+\beta}(1+x)y\right) = u\left(\frac{1}{1+\beta}y\right) + E(\eta)$$

where  $xy$  is the compensation paid to a life-time bachelor for being single and  $\frac{1}{1+\beta}(1+x)y$  is his second period consumption. We calculate the value of  $x$  based on Blanchflower and Oswald (2004). Regressing self-reported well-being scores on income, marriage status, and other determinants, they estimate that a lasting marriage is, on average, worth \$100,000<sup>7</sup> per year in the United States (compared to being widowed or separated) during 1972-1998. During the same period, the GDP per person employed in the U.S. is about \$48,000. The marriage is worth more than twice the average income for employed people in the U.S. We take the ratio  $x = 2$  as the benchmark and then the mean value of the emotional utility/love is:

$$E(\eta) = u\left(\frac{3y}{1+\beta}\right) - u\left(\frac{y}{1+\beta}\right)$$

#### Choice of Parameter Values

Parameters	Benchmark	Source and robustness checks
Discount factor	$\beta = 0.45$	Prescott (1986), discount factor takes value 0.96 based on annual frequency. We take 20 years as one period, then $\beta = 0.96^{20} \simeq 0.45$
Share of nontradable good in the consumption basket	$\gamma = 0.7$	Burstein et al (2001)
Nontradable sector capital-intensity	$\alpha_N = 0.3$	Burstein et al (2001)
Tradable sector capital-intensity	$\alpha_T = 0.6$	Burstein et al (2001)
Share of capital input	$\alpha = 0.35$	Bernanke, Gertler and Gilchrist (1999)
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ in the robustness checks.
Marriage market friction <sup>8</sup>	$p = 0.02$	$p = 0.05$ in the robustness checks
Love, standard deviation	$\sigma = 0.01$	$\sigma = 0.05$ in the robustness checks
Love, mean	$x = 2$	$x = 0.5$ in the robustness checks

## 4.2 Results for the 2-period OLG model

In Figure 1, we set parameter  $\kappa$  equal to 0.8. The benchmark case sets  $x = 2$ ,  $\sigma = 0.01$ , and  $p = 0.02$ . With an unbalanced sex ratio ( $\phi > 1$ ), the real exchange rate depreciates. As the sex ratio rises from 1 to 1.5, the extent of real exchange rate depreciation increases from 0% to about 8%. At the same time, the economy-wide savings rate rises from 12% to 20%, and the current account

<sup>7</sup>In 1990 dollars.

<sup>8</sup> $p$  is the exogenous possibility that any individual (a women or a man) entering the marriage market is bumped off the market independent of the sex ratio.



surplus rises from 0% to 9% of GDP. As the first set of robustness checks, we experiment with different combinations of  $m=0.5$  or 2,  $\sigma = 0.05$  or 0.01, and  $p = 0.02$  or 0.05. The results are also reported in Figure 1, and generally do not deviate from the benchmark very much.

We also set  $\kappa$  to be 0.7 or 0.9, respectively, and experiment with different combinations of other parameters. The results are reported in Figures 2 and 3. Generally speaking, the real exchange rate always depreciates more with a higher sex ratio. Both the savings rate and the current account (as a share of GDP) rise in response to a rise in the sex ratio.

We now consider endogenous labor supply in Figure 4. With  $\kappa = 0.8$ ,  $x = 2$ ,  $\sigma = 0.01$ , and  $p = 0.02$ , we obtain a much stronger exchange rate depreciation. As the sex ratio rises from 1 to 1.5, the extent of the real exchange rate depreciation also rises from 0% to about 35%. The aggregate savings rate rises from 12% to 24%, while the current account surplus rises from 0% first to close to 6% of GDP and reverse slightly to 4% of GDP. Robustness checks with other combinations of the parameters are reported in Figures 5 and 6. The results are broadly in line with the benchmark calibration. In particular, with an endogenous labor supply, a given rise in the sex ratio leads to a greater response in the real exchange rate.

While the aggregate savings rate always rises with the sex ratio, the modest non-monotonic picture of the current account response deserves some comments. With a fixed labor supply, a rise in the sex ratio leads to an expansion of tradable good production but a contraction of nontradable good production. This leads to very little change in the aggregate (domestic) investment rate. As a result, a higher sex ratio leads to a higher savings rate, which produces an increase in the current account balance. In contrast, with an endogenous labor supply, a higher sex ratio leads to an increase in the effective labor supply. Both the tradable good and the nontradable good sectors could expand (or at least the non-tradable sector shrinks by a smaller amount than in the case of a fixed labor supply), which leads to an increase in aggregate domestic investment. As Figures 4-6 show, for the initial rise in the sex ratio (from 1 to 1.15), the current account surplus increases monotonically, indicating that the increase in the aggregate savings rate outpaces the increase in the aggregate investment rate. After that point, any additional increase in the sex ratio leads to a smaller current account surplus, indicating that the incremental savings rate is smaller than the incremental investment rate. Since virtually all economies in the real world have sex ratios (for the pre-marital age cohort) less than 1.15, we do not expect to see the turning point in the current account in the data.

### 4.3 The OLG model in which a cohort lives 50 periods

We now extend our benchmark model by assuming that every cohort lives 50 periods. Everyone works in the first 30 periods, and retires in the remaining 20 periods. If one gets married, the marriage take place in the  $\tau$ th period. We have not been able to solve the problem that allows for parental savings for their child in the 50-period setup. Instead, we study a case in which men and women save for themselves. However, as we recognize the quantitative importance of parental savings in the data,

we choose  $\tau = 20$  as our benchmark case so the timing of the marriage is somewhere between the typical number of working years by parents when their child gets married. Generally speaking, the greater the value of  $\tau$ , the stronger is the aggregate savings response to a given rise in the sex ratio.

For a representative woman who decides to enter the marriage market, the optimization problem is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} u(c_t^w) + E_1 \left[ \sum_{t=\tau}^{50} \beta^{t-1} (u(c_t^w) + \eta^m) \right]$$

For  $t < \tau$ , when the woman is still single, the intertemporal budget constraint is

$$A_{t+1} = R(A_t + y_t^w - P_t c_t^w)$$

where  $A_t$  is the wealth held by the woman in the beginning of period  $t$ .  $y_t^w = w_t L_t^w$  is the labor income of a representative woman with age  $t$ .

After marriage ( $t \geq \tau$ ), then her family budget constraint becomes

$$A_{t+1}^H = \begin{cases} R(A_t^H + w_t L_t^w - P_t c_t) & \text{if } t \leq 30 \\ R(A_t^H - c_t^w) & \text{if } t > 30 \end{cases}$$

where  $A_t^H$  is the level of family wealth (held by wife and husband) at the beginning of period  $t$ .  $c_t$  is the public good consumption by wife and husband, which takes the same form as in the two period OLG model. The optimization problem for a representative man is similar. To simplify the calculation and generate interesting results, we assume that there is a lower bound of labor supply  $\bar{L}$ ,  $L_t^i \geq \bar{L}$  ( $i = w, m$ ).<sup>9</sup>

As in the standard literature, we will take  $R = 1.04$  as the annual gross interest rate in the calibration. The subjective discount factor now takes the value of  $\beta = 1/R$ . We assume capital is accumulated as following:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{b}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t$$

where  $\frac{b}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t$  represents the quadratic capital adjustment cost. We assume  $\delta = 0.1$  and  $b = 2.72$  in the benchmark calibration.<sup>10</sup>

To be concrete, we use demographic changes in China over the last two decades as a guide. As the Chinese data exhibits a steady increase in the sex ratio in the pre-marital age cohort (15-20 years old) since 2000, we let the sex ratio at birth in the model rise continuously and smoothly until it reaches 1.2 in period 20. The sex ratio at birth then stays at that level in all subsequent periods. We

<sup>9</sup>This assumption will not affect any qualitative results.

<sup>10</sup>We take the same value of the adjustment cost parameter ( $b$ ) as Chari, Kehoe and McGrattan (2002).

assume  $\tau = 20$ . The last assumption is meant to capture the notion that typical parents have worked for approximately 20 years by the time their children consider marriage.

The calibration result is shown in Figures 7. In the benchmark calibration, as the sex ratio rises from 1 in period 0 to 1.2 in period 20, the real exchange rate depreciates by more than 15 percent. The economy-wide savings rate and the current account rise by more than 9 percent of GDP. As a robustness check, if capital adjusts more slowly, i.e., with a higher cost of capital adjustment, the real exchange rate depreciates by about 18 percent. The converse is true when the adjustment cost is lower.

## 5 Empirics

In this section, we provide some suggestive cross-country evidence of how the sex ratio imbalance may affect the real exchange rate and the current account. We first run regressions based on the following specification:

$$\ln RER_i = \alpha + \beta \cdot \text{sex ratio} + \gamma \cdot Z + \varepsilon_i$$

where  $RER_i$  is the real exchange rate for country  $i$ .  $Z$  is the set for control variables other than sex ratio, which includes log GDP per capita, financial development index, and *de facto* exchange rate regime classifications.

The data of the real exchange rate and real GDP per capita is obtained from Penn World Tables 6.3. The variable “ $p$ ” (called “price level of GDP”) in the Penn World Tables is equivalent to the inverse of the real exchange rate defined in the standard literature: A higher value of  $p$  means a depreciation in the real exchange rate. The sex ratio data is obtained from the World Factbook. As we are not able to find the sex ratio for the cohort 15-25 for a large number of countries, we use age group 0-15 instead to maximize the country coverage.

We use two proxies for financial development. The first is the ratio of private sector credit to GDP, from the World Bank’s dataset. This is perhaps the most commonly used proxy in the standard literature. There is a clear outlier with this proxy: China has a very high level of credit to GDP ratio. However, 80% of the bank loans go to state-owned firms, which are potentially less efficient than private firms (see Allen, Qian, and Qian, 2004). To deal with this problem, we modify the index by multiplying the private credit to GDP ratio for China by 0.2. Because the first measure is far from being perfect, we also use a second measure, which is the level of financial system sophistication, from the Global Competitiveness Report (GCR).

For exchange rate regimes, we use two *de facto* classifications. The first comes from Reinhart and Rogoff (2004), who classify all regimes into four groups: peg, crawling peg, managed floating or free floating. The second classification comes from Levy-Yeyati and Sturzenegger (2005), who use three groups: fix, intermediate or free float. The year 2006 is chosen for current account information because it is relatively recent (when the global current account imbalances had become a global policy issue), and the data are available for a large section of countries.

Table 1 provides summary statistics for the key variables. Table 2a and 2b show the regression results that use private credit (% of GDP) and financial system sophistication index respectively. We find that results in both tables are very similar. In regression (1), we only consider the Balassa-Samuelson effect, and both tables show that this effect is very significant. In regression (2), we consider both the Balassa-Samuelson effect and financial development. We find that the real exchange rate tends to depreciate if the country has a relatively weak financial system and the effect is statistically significant. In regression (3), as well as the Balassa-Samuelson effect and financial development, we consider the impact from the sex ratio. We find that in countries with higher sex ratios, real exchange rates tend to be lower, and the effect is statistically significant. In regression (4) and (5), we control for *de facto* exchange rate regimes and find very similar results as in regression (3).

We also conduct a robustness check whereby the dependent variable is the average of log real exchange rate over a five-year period (2004-2008). Tables 3a and 3b show the regression results that use private credit (% of GDP) and the financial system sophistication index, respectively. Those results are very similar to Tables 2a and 2b.

We now turn to the relationship between the sex ratio and the current account. Tables 4a, 4b, 5a and 5b report the regression results whereby the dependent variable is the average ratio of non-governmental current account to GDP in year 2006. We find that the sex ratio has a significantly positive effect on the current account except for the last column in Table 4b.<sup>11</sup> To conduct a robustness check, we use the average ratio of non-governmental current account to GDP over the period from 2004 to 2008 as the dependent variable. We report the results in Tables 5a and 5b, and find the results similar to Tables 4a and 4b.

In sum, we find that the sex ratio has a significant impact on the real exchange rate and current account: as the sex ratio rises, a country tends to have a real exchange rate depreciation and current account surplus, which is consistent with our theoretical predictions. (An important caveat is that we do not have a clever idea to instrument for the sex ratio; future research will have to investigate the causality more thoroughly.)

To illustrate the quantitative significance of the empirical relations, we compute the extent of the Chinese real exchange rate undervaluation (or the value of the RER relative to what can be predicted based on fundamentals) by taking the point estimates in Columns 1-3 of Tables 2b, 3b, 4b, and 5b, respectively, at their face value. The results are tabulated in Table 6. Based on the data in the Penn World Table, the Chinese RER undervaluation, averaged over 2004-2008, is about 45%. Once we adjust for the Balassa-Samuelson effect, the extent of the undervaluation becomes 54% (Column 1 of Table 6) - apparently the Chinese RER is even lower than other countries at the comparable income level. If we additionally consider financial underdevelopment (proxied by the ratio of private sector loans to GDP), the Chinese RER undervaluation is reduced to 43%, which is still economically significant (Column 2, row 1 of Table 6). If we further take into account the sex ratio effect, the extent of the

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<sup>11</sup>Part of the reason is that the smaller sample size has raised the standard error and reduced the power of the test.

RER undervaluation becomes less than 4% (Column 3, row 1 of Table 6). The last number represents a relatively trivial amount of undervaluation since major exchange rates (e.g., the euro/dollar rate or the yen/dollar rate) could easily fluctuate by more than 4% in a month. If we proxy financial development by the rating of financial system sophistication from the GCR, and also take into account the sex ratio effect, the extent of the Chinese RER undervaluation becomes 0.8% (Column 3, row 2), an even smaller amount.

We can do similar calculations for the Chinese current account (as a share of GDP) in excess of the fundamentals. If we only take into the regularity that poorer countries tend to have a lower current account balance, the Chinese excess CA is on the order of 14%. If we take into account the sex ratio effect as well as financial underdevelopment, the excess amount of current account becomes somewhere between 0.5% and 2.9%, depending on which proxy for financial development is used. These numbers suggest that, if the sex ratio effect is not taken into account when it is a key fundamental, one might mistakenly exaggerate the role of currency manipulation.

## 6 Conclusion

A low value of the real exchange rate (i.e., deviations from the PPP from below), a large current account surplus, and accumulation of foreign exchange reserve are the commonly used criteria for judging currency undervaluation or manipulation. We argue that none of them is a logically sound criterion. Instead, a dramatic rise in the sex ratio for the premarital age cohort in China since 2002, unrelated to currency manipulation, could generate both a depreciation of the real exchange rate and a rise in the current account surplus. With capital controls (including mandatory surrender of foreign exchange earnings), a persistent current account surplus can mechanically be converted into a rise in a country's foreign exchange reserve.

The usual narrative about the Chinese external economy connects the three variables in the following way: The authorities intervene aggressively in the currency market in order to generate an artificial undervaluation of its currency. This generates a rise in the foreign exchange reserve holdings and a fall in the real exchange rate. As a result of the currency undervaluation, the country manages to produce a current account surplus. The model and the evidence in this paper encourage the reader to consider an alternative, logically equally plausible, way to connect the three variables: structural factors, such as a rise in the sex ratio, simultaneously generate a rise in the current account (through a rise in the savings rate) and a fall in the real value of the exchange rate. The low real exchange rate is not the cause of the current account surplus. With mandatory surrender of foreign exchange earnings required of by the country's capital control regime, the current account surplus is converted passively into an increase in the central bank's foreign exchange reserve holdings.

If other factors, in addition to a rise in the sex ratio, have also contributed to a rise in the Chinese savings rate, such as a rise in the corporate and government savings rates, they can complement

the sex ratio effect and reinforce an appearance of an undervalued currency even when there is no manipulation.

Empirically, countries with a high sex ratio do appear to be more likely have a low value of the real exchange rate and a current account surplus. If we take these econometric point estimates at face value, it appears that the Chinese real exchange rate has only a relatively trivial amount of undervaluation (0.8-3.9%) once we take into account the sex ratio effect in addition to the Balassa-Samuelson effect and financial underdevelopment. To be clear, this is not meant to be a proof of no currency undervaluation in any particular currency. Instead, it illustrates potential pitfalls in assessing the equilibrium exchange rate when important structural factors are not accounted for.

An extension of the model that allows for endogenous adjustment of the sex ratio will allow us to assess the speed of the reversal of the sex ratio and the unwinding of the current account surplus and currency "undervaluation." We leave this for future research.

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## A Proof of Proposition 1

**Proof.** We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

where

$$\begin{aligned} \Omega_{11} &= (u_1'' + \beta R u_2'') \frac{w_t}{P_t} \\ \Omega_{12} &= 0 \\ \Omega_{14} &= 0 \\ \Omega_{15} &= 0 \\ \Omega_{21} &= \gamma w_t \\ \Omega_{22} &= -\gamma(1 - s_t) \\ \Omega_{23} &= \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1-\alpha_N}} \\ \Omega_{24} &= \frac{P_{Nt} (1 - \alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1-\alpha_N}} \\ \Omega_{31} &= 0 \\ \Omega_{32} &= -1 \\ \Omega_{33} &= 0 \\ \Omega_{34} &= \left( \frac{\alpha_T}{1 - \alpha_T} \right)^{1-\alpha_T} (1 - \alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1 - L_{Nt})^{-\alpha_T-1} \\ \Omega_{41} &= 0 \\ \Omega_{42} &= -1 \\ \Omega_{43} &= \frac{w_t}{P_{Nt}} \\ \Omega_{44} &= - \left( \frac{\alpha_T}{1 - \alpha_T} \right)^{1-\alpha_T} (1 - \alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1 - L_{Nt})^{-\alpha_T-1} \end{aligned}$$

and

$$\begin{aligned}
z_1 &= -Ru'_2 \\
z_2 &= 0 \\
z_3 &= 0 \\
z_4 &= 0
\end{aligned}$$

The determinant of matrix  $\Omega$  is

$$\det(\Omega) = \Omega_{11} \cdot \det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}$$

and

$$\begin{aligned}
\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} &= \text{negative.terms} + \gamma(1-s_t) \left( \frac{w_t}{P_{Nt}} \right)^2 \frac{1-\alpha_T}{L_T} \\
&\quad - \left( \frac{w_t}{P_{Nt}} \right) \left( \frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right) C_{Nt}
\end{aligned}$$

Since the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, it follows that  $\gamma(1-s_t)w_t < P_{Nt}C_{Nt}$ . Therefore,

$$\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} < 0$$

and

$$\det(\Omega) > 0$$

Then it is easy to show that

$$\begin{aligned}
\frac{ds_t}{d\beta} &= \frac{\det \begin{pmatrix} z_1 & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ z_2 & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ z_3 & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ z_4 & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}}{\det(\Omega)} \\
&= \frac{z_1}{\Omega_{11}} > 0
\end{aligned}$$

and the price of the nontradable good

$$\begin{aligned} \frac{dP_{Nt}}{d\beta} &= \frac{\det \begin{pmatrix} \Omega_{11} & \Omega_{12} & z_1 & \Omega_{14} \\ \Omega_{21} & \Omega_{22} & z_2 & \Omega_{24} \\ \Omega_{31} & \Omega_{32} & z_3 & \Omega_{34} \\ \Omega_{41} & \Omega_{42} & z_4 & \Omega_{44} \end{pmatrix}}{\det(\Omega)} \\ &= \frac{z_1 \Omega_{21} \Omega_{32} (\Omega_{44} - \Omega_{34})}{\det(\Omega)} < 0 \end{aligned}$$

The labor input in the nontradable sector

$$\begin{aligned} \frac{dL_{Nt}}{d\beta} &= \frac{\det \begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & z_1 \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & z_2 \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & z_3 \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & z_4 \end{pmatrix}}{\det(\Omega)} \\ &= -\frac{z_1 \Omega_{21} \Omega_{32} \Omega_{34}}{\det(\Omega)} < 0 \end{aligned}$$

In period  $t + 1$ , the shock has been observed, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1 - \alpha_T}} \quad \text{and} \quad P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1 - \alpha_T}}$$

which means that after one period the shock occurs, the price of the nontradable good and the consumer price index will go back to their initial levels. As for the current account,

$$CA_t = P_{Nt} Q_{Nt} + Q_{Tt} + (R - 1) \cdot NFA_{t-1} - P_t C_t - K_{t+1}$$

where  $NFA_{t-1}$  is the net foreign asset holdings in period  $t - 1$  and  $K_{t+1}$  is the sum of capital input in both the nontradable sector and the tradable sector in period  $t + 1$ . Since

$$s_{t-1} w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_t w_t - s_{t-1} w_{t-1} - \Delta K_{t+1}$$

where  $\Delta K_{t+1} = K_{t+1} - K_t$ . The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w ((R - 1) s_t + 1)}{P_N}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since  $s_t > s_{t-1}$ ,  $Q_{N,t+1} > Q_{N,t-1}$ .

As  $\alpha_N < \alpha_T$ , the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period  $t + 1$ ,  $K_{t+1} < K_{t-1}$ .

In period  $t + 1$ ,

$$A_{Nt}K_{N,t+1}^{\alpha_N}L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w((R-1)s_t + 1)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N)[(R-1)s_t + 1]}{(1 - \alpha_T)R} w$$

and then

$$CA_t = s_t w_t - s_{t-1} w + \frac{(\alpha_T - \alpha_N)(R-1)(s_t - s_{t-1})}{(1 - \alpha_T)R} w$$

To show  $\frac{dCA_t}{d\beta} > 0$ , we only need to show  $\frac{d(s_t w_t - s_{t-1} w_{t-1})}{d\beta} > 0$ . One sufficient condition for the inequality is

$$s_t P_{Nt} > s_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$s_t \frac{dP_{Nt}}{d\beta} + P_{Nt} \frac{ds_t}{d\beta} > 0$$

which means

$$\frac{dP_{Nt}/d\beta}{ds_t/d\beta} + \frac{P_{Nt}}{s_t} > 0$$

Plugging the expressions of  $\frac{dP_{Nt}}{d\beta}$  and  $\frac{ds_t}{d\beta}$ , we have

$$\begin{aligned} \frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t)w_t C_{Nt} \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) + P_{Nt} C_{Nt} \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive terms}} + \text{positive term} \\ &= \frac{(P_{Nt} C_{Nt} - \gamma(1-s_t)w_t) \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive terms}} + \text{positive term} \end{aligned}$$

As shown above,  $P_{Nt} C_{Nt} - \gamma(1-s_t)w_t > 0$ , then  $\frac{dCA_t}{d\beta} > 0$ , in period  $t$ , the country will experience a current account surplus. ■

## B Proof of Proposition 2

**Proof.** If emotional utilities are large enough, when  $\phi = 1$ , or  $\phi$  is close to one, we have  $V^i > V_n^i$  ( $i = w, m$ ). Since  $\frac{1}{2} \leq \kappa \leq 1$ ,

$$\kappa(Rs^m w_t + Rs^w w_t) > \max(Rs^w w_t, Rs^m w_t)$$

which means that within the neighborhood of  $\phi = 1$ , we have  $\kappa u'_{2m} < u'_{2m,n}$ .

We proceed in two steps. In the first step, we assume that inequality  $\kappa u'_{2m} < u'_{2m,n}$  holds for all values of  $\phi$ , and prove that a higher sex ratio leads to a higher savings rate. In the second step, we prove by contradiction that the inequality indeed holds for all values of  $\phi$ .

We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where

$$\begin{aligned} \Omega_{11} &= \left[ u''_{1w} + \beta(1-p)\kappa^2 R u''_{2w} \left( 1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min}) \right) + p R u''_{2w,n} \right] \frac{w_t}{P_t} \\ \Omega_{12} &= \beta(1-p)\kappa^2 R u''_{2w} \left[ 1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min}) \right] \frac{w_t}{P_t} \\ \Omega_{13} &= 0 \\ \Omega_{14} &= (1-\sigma)\beta \left( (1-p)\kappa u'_{2w} \left[ 1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min}) \right] + p u'_{2w,n} \right) \frac{R\gamma P_{Nt}}{P_{t+1}} \\ \Omega_{15} &= 0 \end{aligned}$$

$$\begin{aligned}
\Omega_{21} &= \beta(1-p)\kappa^2 \left[ (\delta^m + 1) + f(M(\eta^{\min})) (u_{2m} + \eta^{\min} - u_{2m,n}) \right] u_{2m}'' R + R \frac{u_{2w}' u_{2m}'}{\eta^{\max} - \eta^{\min}} \left] \frac{w_t}{P_t} \\
\Omega_{22} &= \frac{w_t}{P_t} \left[ \begin{aligned} &\beta\kappa^2 R \delta^m u_{2m}'' + \beta\kappa^2 u_{2m}'' R + \beta R \kappa u_{2w}' f(M(\eta^{\min})) (\kappa u_{2m}' - u_{2m,n}') \\ &+ \beta\kappa^2 R u_{2w}' f(M(\eta^{\min})) (u_{2m} + \eta^{\min} - u_{2m,n}) + u_{1m}' \end{aligned} \right] \\
&\quad + \beta [p + (1-p)(1-\delta^m)] R u_{2m,n}'' \frac{w_t}{P_t} \\
\Omega_{23} &= \beta R \kappa u_{2w}' f(M(\eta^{\min})) (u_{2m} - u_{2m,n}) \frac{1-\sigma}{w_t} \\
\Omega_{24} &= (1-\sigma) \left( \begin{aligned} &(1-p) [\kappa \delta^m u_{2m}' + \kappa u_{2w}' f(M(\eta^{\min})) (u_{2m} + \eta^{\min} - u_{2m,n})] \\ &+ [(1-\delta^m)(1-p) + p] u_{2m,n}' \end{aligned} \right) \frac{R \gamma P_{Nt}}{P_{t+1}} \\
\Omega_{25} &= 0
\end{aligned}$$

$$\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1+\phi} \\
\Omega_{32} &= \frac{\gamma \phi w_t}{1+\phi} \\
\Omega_{33} &= -\gamma(1-s_t) \\
\Omega_{34} &= \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} \\
\Omega_{35} &= \frac{P_{Nt} (1-\alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}}
\end{aligned}$$

$$\begin{aligned}
\Omega_{41} &= 0 \\
\Omega_{42} &= 0 \\
\Omega_{43} &= -1 \\
\Omega_{44} &= 0 \\
\Omega_{45} &= \left( \frac{\alpha_T}{1-\alpha_T} \right)^{1-\alpha_T} (1-\alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1-L_{Nt})^{-\alpha_T-1}
\end{aligned}$$

$$\begin{aligned}
\Omega_{51} &= 0 \\
\Omega_{52} &= 0 \\
\Omega_{53} &= -1 \\
\Omega_{54} &= \frac{w_t}{P_{Nt}} \\
\Omega_{55} &= - \left( \frac{\alpha_N}{1-\alpha_N} \right)^{1-\alpha_T} (1-\alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N-1}
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= 0 \\
z_2 &= (1-p) \frac{\partial \delta^m}{\partial \phi} (u'_{2m}(n) - \kappa u'_{2m}) \\
z_3 &= -\frac{\gamma w_t (s_t^m - s_t^w)}{1 + \phi} \\
z_4 &= 0 \\
z_5 &= 0
\end{aligned}$$

If we assume the utility function is log,  $u(C) = \ln(C)$ , then

$$\Omega_{14} = \Omega_{23} = \Omega_{24} = 0$$

and

$$\det(\Omega) = \det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}$$

It is easy to show that

$$\det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0$$

and

$$\begin{aligned}
\det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} &= \text{negative terms} + \gamma(1-s_t) \left( \frac{w_t}{P_{Nt}} \right)^2 \frac{1-\alpha_T}{L_T} \\
&\quad - \left( \frac{w_t}{P_{Nt}} \right) \left( \frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right) C_{Nt}
\end{aligned}$$

Notice that the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, then  $\gamma(1-s_t)w_t < P_{Nt}C_{Nt}$ . Therefore,

$$\det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} < 0$$

and

$$\det(\Omega) < 0$$



Then

$$\frac{ds_t^m}{d\phi} = \frac{z_2 \Omega_{11}}{\det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} > 0$$

$$\frac{ds_t^w}{d\phi} = \frac{z_2 \Omega_{12}}{\det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} < 0$$

and the aggregate savings rate by the young cohort  $s_t = \frac{\phi}{1+\phi} s_t^m + \frac{1}{1+\phi} s_t^w$ ,

$$\frac{ds_t}{d\phi} = \frac{\phi}{1+\phi} \frac{ds_t^m}{d\phi} + \frac{1}{1+\phi} \frac{ds_t^w}{d\phi} + \frac{s_t^m - s_t^w}{(1+\phi)^2} > 0$$

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = - \frac{z_3 \frac{w_t^2}{P_{Nt}} \frac{\alpha_T}{L_{Tt}}}{\det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}} + \frac{z_2 (\Omega_{11} \Omega_{32} - \Omega_{12} \Omega_{31}) \frac{w_t}{P_{Nt}} \frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that  $\Omega_{11} \Omega_{32} - \Omega_{12} \Omega_{31} < 0$ , and since  $z_2 < 0$ ,  $\frac{dP_{Nt}}{d\phi} < 0$ , which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period  $t$ .

As for the current account,

$$CA_t = P_{Nt} Q_{Nt} + Q_{Tt} + (R-1) \cdot NFA_{t-1} - P_t C_t - K_{t+1}$$

where  $NFA_{t-1}$  is the net foreign asset holdings in period  $t-1$  and  $K_{t+1}$  is the sum of capital input in both the nontradable sector and the tradable sector in period  $t+1$ .

Notice that

$$s_{t-1} w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_t w_t - s_{t-1} w_{t-1} - \Delta K_{t+1}$$

where  $\Delta K_{t+1} = K_{t+1} - K_t$ . By Obstfeld and Rogoff (1995), if the sex ratio remains constant  $\phi$  after period  $t$ , the price of the nontradable good will go back to its initial level, which means that real exchange rate will appreciate in period  $t+1$ . In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we take the log utility function, the aggregate savings rate by the young cohort will remain same after period  $t$ .

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w ((R-1)s_t + 1)}{P_{N,t+1}}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since  $s_t > s_{t-1}$ ,  $Q_{N,t+1} > Q_{N,t-1}$ .

As in Obstfeld and Rogoff (1995), we assume that  $\alpha_N < \alpha_T$ , the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period  $t+1$ ,  $K_{t+1} < K_{t-1}$ .

In period  $t+1$ ,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w ((R-1)s_t + 1)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1} = \left[ \frac{\alpha_T}{(1-\alpha_T)} - \left( \frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) \gamma ((R-1)s + 1) \right] \frac{w}{R}$$

and then

$$\begin{aligned} \Delta K_{t+1} &= \gamma \left( \frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) (R-1) (s_t - s_{t-1}) \frac{w}{R} \\ CA_t &= s_t w_t - s_{t-1} w + \gamma \left( \frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) (R-1) (s_t - s_{t-1}) \frac{w}{R} \end{aligned}$$

To show  $\frac{dCA_t}{d\phi} > 0$ , we only need to show  $\frac{d(s_t w_t - s_{t-1} w_{t-1})}{d\phi} > 0$ . By (3.9), one sufficient condition is for the inequality is

$$s_t P_{Nt} > s_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$s_t \frac{dP_{Nt}}{d\phi} + P_{Nt} \frac{ds_t}{d\phi} > 0$$

which means

$$\frac{dP_{Nt}/d\phi}{ds_t/d\phi} + \frac{P_{Nt}}{s_t} > 0$$

Plugging the expressions of  $\frac{dP_{Nt}}{d\phi}$  and  $\frac{ds_t}{d\phi}$ , we have

$$\begin{aligned} \frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t)w_t C_{Nt} \left( \frac{w_t}{P_{Nt}} \right) \left( \frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right) + P_{Nt} C_{Nt} \left( \frac{w_t}{P_{Nt}} \right) \left( \frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right)}{s_t \cdot \text{positive .terms}} + \text{positive .term} \\ &= \frac{(P_{Nt} C_{Nt} - \gamma(1-s_t)w_t) \left( \frac{w_t}{P_{Nt}} \right) \left( \frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right)}{s_t \cdot \text{positive .terms}} + \text{positive .term} \end{aligned}$$

As shown above,  $P_{Nt} C_{Nt} - \gamma(1-s_t)w_t > 0$ , then  $\frac{dCA_t}{d\phi} > 0$ , in period  $t$ , the country will experience a

current account surplus.

The impact of a rise in the sex ratio on the social welfare is

$$\begin{aligned}
\frac{\partial U^w}{\partial \phi} &= w_t \left( -u'_{1w} + (1-p)\kappa\beta R \frac{P_{t+1}}{P_t} u'_{2w} + p\beta R \frac{P_{t+1}}{P_t} u'_{2w,n} \right) \frac{ds^w}{d\phi} + (1-p) \left( w_t\beta R \frac{P_{t+1}}{P_t} \kappa u'_{2w} \frac{ds^m}{d\phi} + \frac{\beta}{\phi^2} E[\eta] \right) \quad (\text{B.1}) \\
&> (1-p) \left( w_t\kappa\beta R \frac{P_{t+1}}{P_t} u'_{2w} \left( \frac{ds^m}{d\phi} + \frac{ds^w}{d\phi} \right) + \frac{\beta}{\phi^2} E[\eta] \right) > 0 \\
\frac{\partial U^m}{\partial \phi} &= w_t \left( -u'_{1m} + \beta R \frac{P_{t+1}}{P_t} \kappa\delta(1-p)u'_{2m} + \beta R \frac{P_{t+1}}{P_t} [(1-p)\delta + (1-\delta)] u'_{2m,n} \right) \frac{ds^m}{d\phi} \\
&\quad + (1-p)w_t\beta R \frac{P_{t+1}}{P_t} \kappa\delta u'_{2w} \frac{ds^w}{d\phi} + (1-p) \left[ \beta \frac{\partial \delta}{\partial \phi} (u_{2m} - u_{2m,n} + \eta^{\min}) + \beta \frac{\partial \left( \int_{M(\eta^{\min})} M^{-1}(\eta^m) dF(\eta^m) \right)}{\partial \phi} \right] \quad (\text{B.2}) \\
&< (1-p) \left( w_t\beta R \frac{P_{t+1}}{P_t} \kappa\delta u'_{2w} \frac{ds^w}{d\phi} - \frac{\beta}{\phi^2} (u_{2m} - u_{2m,n} + \eta^{\min}) - \frac{\beta}{\phi^2} E[\eta] \right) < 0
\end{aligned}$$

where the first inequality in (B.1) holds because

$$-u'_{1w} + (1-p)\beta R \frac{P_{t+1}}{P_t} \kappa u'_{2w} + p\beta R \frac{P_{t+1}}{P_t} u'_{2w,n} = -(1-p)\beta R \frac{P_{t+1}}{P_t} \kappa u'_{2m} \left[ \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min}) \right] < 0$$

and the first inequality in (B.2) holds because

$$\begin{aligned}
&-u'_{1m} + \beta R \frac{P_{t+1}}{P_t} \kappa(1-p)(1+\delta)u'_{2m} + \beta R \frac{P_{t+1}}{P_t} p(1-\delta)u'_{2m,n} \\
&= -(1-p)\beta R \frac{P_{t+1}}{P_t} \kappa u'_{2w} \left( \frac{u_{2m} + \eta^{\min} - u_{2m,n}}{\eta^{\max} - \eta^{\min}} \right) < 0
\end{aligned}$$

We now show by contradiction that  $\kappa u'_{2m} < u'_{2m,n}$  must hold for all  $\phi$ s. Suppose not, then  $\kappa u'_{2m} < u'_{2m,n}$  may fail sometime. Due to continuity of  $z_2$ , there exists a level of sex ratio  $\phi_0$  at which  $\kappa u'_{2m} = u'_{2m,n}$ , which implies that  $z_2 = 0$ .

As in Du and Wei (2010), we can show that

$$z_2|_{\phi=\phi_0} = 0 \text{ and } \left. \frac{d^k z_2}{d\phi^k} \right|_{\phi=\phi_0} = 0 \text{ for any } k > 0$$

which means that  $z_2 = 0$  for all  $\phi$ s. This contradicts the assumption that  $z_2 < 0$  when  $\phi = 1$ . Therefore, the inequality  $\kappa u'_{2m} < u'_{2m,n}$  holds for all  $\phi$ s. ■

## C Proof of Proposition 3

**Proof.** If  $u(C) = \ln C$ , for  $\phi < \phi_1$ , by the optimal labor supply condition, we have

$$0 < \frac{dL_t^i}{ds_t^i} = \frac{1}{1 - s_t^i} \frac{v_i' L_t^i}{v_i'' L_t^i} \quad (\text{C.1})$$

where  $i = w, m$ .

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where

$$\begin{aligned} \Omega_{11} &= \left[ u_{1w}'' + \beta(1-p)\kappa^2 R u_{2w}'' \left( 1 + \frac{1}{\phi} + M(\eta^{\min}) f(\eta^{\min}) \right) + \beta p R u_{2w,n}'' \right] \frac{w_t L_t^w}{P_t} \\ &\quad + \left[ -(1-s_t) u_{1w}'' + \beta(1-p)\kappa^2 R s_t^w u_{2w}'' \left( 1 + \frac{1}{\phi} + M(\eta^{\min}) f(\eta^{\min}) \right) + \beta p R s_t^w u_{2w,n}'' \right] \frac{w_t}{P_t} \frac{dL_t^w}{ds_t^w} \end{aligned}$$

$$\Omega_{12} = \beta(1-p)\kappa^2 R u_{2w}'' \left( 1 + \frac{1}{\phi} + M(\eta^{\min}) f(\eta^{\min}) \right) \left( L_t^m + s_t^m \frac{dL_t^m}{ds_t^m} \right) \frac{w_t}{P_t}$$

$$\Omega_{13} = 0$$

$$\Omega_{14} = 0$$

$$\Omega_{15} = 0$$

$$\Omega_{21} = \beta(1-p)\kappa^2 \left[ ((\delta^m + 1) + f(M(\eta^{\min}))(u_{2m} + \eta^{\min} - u_{2m,n})) u_{2m}'' R + R \frac{u_{2w}' u_{2m}'}{\eta^{\max} - \eta^{\min}} \right] \left( L_t^w + s_t^w \frac{dL_t^w}{ds_t^w} \right) \frac{w_t}{P_t}$$

$$\begin{aligned} \Omega_{22} &= \frac{w_t}{P_t} \left[ \beta \kappa^2 u_{2m}'' R + \beta R \kappa u_{2w}' f(M(\eta^{\min})) (\kappa u_{2m}' - u_{2m,n}') \right] \left( L_t^m + s_t^m \frac{dL_t^m}{ds_t^m} \right) - (1-s_t^m) u_{1m}'' \frac{w_t}{P_t} \frac{dL_t^m}{ds_t^m} \\ &\quad + \beta [p + (1-p)(1-\delta^m)] R u_{2m,n}'' \left( L_t^m + s_t^m \frac{dL_t^m}{ds_t^m} \right) \frac{w_t}{P_t} \end{aligned}$$

$$\Omega_{23} = 0$$

$$\Omega_{24} = 0$$

$$\Omega_{25} = 0$$

$$\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1+\phi} \left( L_t^w + s_t^w \frac{dL_t^w}{ds_t^w} \right) \\
\Omega_{32} &= \frac{\gamma \phi w_t}{1+\phi} \left( L_t^m + s_t^m \frac{dL_t^m}{ds_t^m} \right) \\
\Omega_{33} &= -\gamma \left[ \frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi} \right] \\
\Omega_{34} &= \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} \\
\Omega_{35} &= \frac{P_{Nt}(1-\alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}}
\end{aligned}$$

$$\begin{aligned}
\Omega_{41} &= -\frac{\alpha_T w_t}{\frac{1}{1+\phi} L_t^w + \frac{\phi}{1+\phi} L_t^m - L_{Nt}} \frac{1}{1+\phi} \frac{dL_t^w}{ds_t^w} \\
\Omega_{42} &= -\frac{\alpha_T w_t}{\frac{1}{1+\phi} L_t^w + \frac{\phi}{1+\phi} L_t^m - L_{Nt}} \frac{\phi}{1+\phi} \frac{dL_t^m}{ds_t^m} \\
\Omega_{43} &= -1 \\
\Omega_{44} &= 0 \\
\Omega_{45} &= \left( \frac{\alpha_T}{1-\alpha_T} \right)^{1-\alpha_T} (1-\alpha_T) A_{Tt} K_{Tt}^{\alpha_T} \left( \frac{1}{1+\phi} L_t^w + \frac{\phi}{1+\phi} L_t^m - L_{Nt} \right)^{-\alpha_T-1}
\end{aligned}$$

$$\begin{aligned}
\Omega_{51} &= 0 \\
\Omega_{52} &= 0 \\
\Omega_{53} &= -1 \\
\Omega_{54} &= \frac{w_t}{P_{Nt}} \\
\Omega_{55} &= -\left( \frac{\alpha_N}{1-\alpha_N} \right)^{1-\alpha_N} (1-\alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N-1}
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= 0 \\
z_2 &= (1-p) \frac{\partial \delta^m}{\partial \phi} (u'_{2m}(n) - \kappa u'_{2m}) \\
z_3 &= -\frac{\gamma w_t (s_t^m L_t^m - s_t^w L_t^w)}{1+\phi} \\
z_4 &= \frac{\alpha_T w_t}{\frac{1}{1+\phi} L_t^w + \frac{\phi}{1+\phi} L_t^m - L_{Nt}} \frac{L_t^m - L_t^w}{(1+\phi)^2} \\
z_5 &= 0
\end{aligned}$$

The determinant of matrix  $\Omega$  is

$$\det(\Omega) = \det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}$$

It is easy to show that

$$\det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0$$

and

$$\begin{aligned} \det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} &= \text{negative terms} + \gamma \left[ \frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi} \right] \left( \frac{w_t}{P_{Nt}} \right)^2 \frac{1-\alpha_T}{L_T} \\ &\quad - \left( \frac{w_t}{P_{Nt}} \right) \left( \frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right) C_{Nt} \end{aligned}$$

Notice that the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, then  $\gamma \left[ \frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi} \right] w_t < P_{Nt}C_{Nt}$ . Therefore,

$$\det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} < 0$$

and

$$\det(\Omega) < 0$$

Then

$$\begin{aligned} \frac{ds_t^m}{d\phi} &= -\frac{z_2\Omega_{11}}{\det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} > 0 \\ \frac{ds_t^w}{d\phi} &= \frac{z_2\Omega_{12}}{\det \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} < 0 \end{aligned}$$

By (C.1), we have

$$\begin{aligned} \frac{dL_t^m}{d\phi} &> 0 \\ \frac{dL_t^w}{d\phi} &< 0 \end{aligned}$$

The aggregate savings rate by the young cohort  $s_t = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$ ,

$$\frac{ds_t}{d\phi} = \frac{\phi}{1+\phi} \frac{ds_t^m}{d\phi} + \frac{1}{1+\phi} \frac{ds_t^w}{d\phi} + \frac{s_t^m - s_t^w}{(1+\phi)^2} > 0$$

The aggregate labor supply in period  $t$

$$\begin{aligned} \frac{dL_t}{d\phi} &= \frac{\phi}{1+\phi} \frac{dL_t^m}{d\phi} + \frac{1}{1+\phi} \frac{dL_t^w}{d\phi} + \frac{L_t^m - L_t^w}{(1+\phi)^2} \\ &= \frac{\phi}{1+\phi} \frac{dL_t^m}{ds_t^m} \frac{ds_t^m}{d\phi} + \frac{1}{1+\phi} \frac{dL_t^w}{ds_t^w} \frac{ds_t^w}{d\phi} + \frac{L_t^m - L_t^w}{(1+\phi)^2} \end{aligned}$$

Under the assumption  $\frac{v''L}{v'}$  is non-decreasing in  $L$ , by (C.1),  $\frac{dL_t^m}{ds_t^m} > \frac{dL_t^w}{ds_t^w}$ , then we have

$$\frac{dL_t}{d\phi} > 0$$

which means the aggregate labor supply is increasing in the sex ratio.

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = -\frac{z_3 \left( \frac{w_t^2}{P_{Nt}} \right) \frac{\alpha_T}{L_{Tt}} + z_4 (\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54})}{\det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}} + \frac{z_2 (\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31}) \frac{w_t^2}{P_{Nt}} \frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that  $\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54} < 0$ , then

$$\frac{dP_{Nt}}{d\phi} < 0$$

which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period  $t$ .

As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1) \cdot NFA_{t-1} - P_t C_t - K_{t+1}$$

where  $NFA_{t-1}$  is the net foreign asset holdings in period  $t-1$  and  $K_{t+1}$  is the sum of capital input in both the nontradable sector and the tradable sector in period  $t+1$ .

Notice that

$$s_{t-1}w_{t-1}L_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = \left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) w_t - s_{t-1} w_{t-1} L_{t-1} - \Delta K_{t+1}$$

where  $\Delta K_{t+1} = K_{t+1} - K_t$ . Following Obstfeld and Rogoff (1995), if the sex ratio remains constant at  $\phi$  after period  $t$ , the price of the nontradable good will go back to its initial level, which means that the real exchange rate will appreciate in period  $t+1$ . In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we take the log utility function, the aggregate savings rate by the young cohort will remain the same after period  $t$ .

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left( (R-1) \left( \frac{s_{t+1}^w}{1+\phi} + \frac{\phi s_{t+1}^m}{1+\phi} \right) + 1 \right)}{P_{N,t+1}}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since  $s_t > s_{t-1}$ ,  $Q_{N,t+1} > Q_{N,t-1}$ .

In period  $t+1$ ,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w \left( (R-1) \left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) + 1 \right)}{P_{N,t+1}}$$

In equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N) \left[ (R-1) \left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) + 1 \right]}{(1 - \alpha_T)R} w$$

and then

$$CA_t = \left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) w_t - s_{t-1} w_{t-1} L_{t-1} + \frac{(\alpha_T - \alpha_N)(R-1) \left( \left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) - s_{t-1} L_{t-1} \right)}{(1 - \alpha_T)R} w$$

To show  $\frac{dCA_t}{d\phi} > 0$ , we only need to show  $\frac{d \left( \left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) w_t - s_{t-1} w_{t-1} L_{t-1} \right)}{d\phi} > 0$ . By (3.9), one sufficient condition is for the inequality is

$$\left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) P_{Nt} > s_{t-1} L_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$\left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right) \frac{dP_{Nt}}{d\phi} + P_{Nt} \frac{d \left( \frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi} \right)}{d\phi} > 0$$



which means

$$\frac{dP_{Nt}/d\phi}{d\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)/d\phi} + \frac{P_{Nt}}{s_t} > 0$$

Plug the expressions of  $\frac{dP_{Nt}}{d\phi}$  and  $\frac{ds_t}{d\phi}$ , we have

$$\begin{aligned} \frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t)w_t C_{Nt} \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) + P_{Nt} C_{Nt} \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive .terms}} + \text{positive .term} \\ &= \frac{(P_{Nt} C_{Nt} - \gamma(1-s_t)w_t) \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive .terms}} + \text{positive .term} \end{aligned}$$

As shown above,  $P_{Nt} C_{Nt} - \gamma(1-s_t)w_t > 0$ , then  $\frac{dCA_t}{d\phi} > 0$ , in period  $t$ , the country will experience a current account surplus. ■

## D Proof of Proposition 4

**Proof.** As in the proof of Proposition 2, we showed that only the aggregate savings rate will influence the wage and the nontradable good price. As the aggregate savings rate rises, the nominal wage decreases and so does the nontradable good price. This in turn will lead to a depreciation in the real exchange rate and a rise in the current account.

Du and Wei (2010) showed that, in a one-good model, with parental savings, as the sex ratio rises, not only does a representative man weakly increase his first-period savings rate, but parents with a son also increase their savings for their son. The savings responses by young women and by parents with a daughter are ambiguous. However, the aggregate savings rate rises. Therefore, all the results in Proposition 2 still hold. ■

## E Welfare analysis and discussions of policy interventions

We conduct a simple welfare analysis and use it as a basis for evaluating policy interventions aimed at reducing current account imbalances. Consider a benevolent central planner who cares about the overall welfare of men and women when utility is transferable. The central planner can do anything, including cutting down the sex ratio. We first compute the welfare loss of a rise in the sex ratio. Then we compare the welfare consequences of two different ways to reduce the current account surplus: (i) taxing the tradable good and (ii), reducing the sex ratio.

There are two sources of market failures that the central planner would avoid: (a) men save competitively to improve their relative standing in the marriage market; and (b) both men and women may under-save as they do not take into account the benefits of their own savings for the well-being of

their future spouses. The central planner assigns the marriage market matching outcome and optimally chooses women's and men's savings rates to maximize the social welfare function,

$$\max U = \frac{1}{1+\phi}U^w + \frac{\phi}{1+\phi}U^m$$

The first order conditions are

$$-u'_{1w} + 2(1-p)\kappa u'_{2w} \frac{P_t}{P_{t+1}} + pu'_{2w,n} \frac{P_t}{P_{t+1}} = 0 \quad (\text{E.1})$$

$$-u'_{1m} + \frac{2(1-p)}{\phi} \kappa u'_{2w} \frac{P_t}{P_{t+1}} + \left(1 - \frac{1}{\phi}(1-p)\right) u'_{2w,n} \frac{P_t}{P_{t+1}} = 0 \quad (\text{E.2})$$

Comparing (E.1), (E.2) to (3.5) and (3.7), in general, it is not obvious whether women or men will save at a higher rate in a decentralized equilibrium than that under central planning. However, when  $\phi = 1$ , since women and men have the same optimization problem, if  $f(\eta^{\min})M(\eta^{\min}) > 0$ , then women and men will save more in the competitive equilibrium. If  $f(\eta^{\min})M(\eta^{\min})$  is sufficiently small, the competitive equilibrium is very close to the central planner's economy.

There are two opposing effects. On one hand, a part of the savings in the competitive equilibrium is motivated by a desire to out-save one's competitors in the marriage market. The increment in the savings, while individually rational, is not useful in the aggregate, since when everyone raises the savings rate by the same amount, the ultimate marriage market outcome is not affected by the increase in the savings. In this sense, the competitive equilibrium produces too much savings. On the other hand, because the savings contribute to a public good in a marriage (an individual's savings raises the utility of his/her partner), but an individual in the first period does not take this into account, he/she may under-save relative to the social optimum. These two effects offset each other. Therefore, when  $\phi = 1$ , the final savings rate in the decentralized equilibrium could be close to the social optimum.

In calibrations with a log utility function, we show that men's welfare under a decentralized equilibrium relative to the central planner's economy declines as the sex ratio increases. In comparison, women's relative welfare increases as the sex ratio goes up. The social welfare (a weighted average of men's and women's welfare) goes down as the sex ratio rises.

As a thought experiment, one may also consider what the central planner would do if she can choose the sex ratio (in addition to the savings rates) to maximize the social welfare. The new first order condition with respect to  $\phi$  is

$$U^m - U^w = 0 \quad (\text{E.3})$$

The only sex ratio that satisfies (E.3) is  $\phi = 1$ . In other words, the central planner would have chosen a balanced sex ratio. Deviations from a balanced sex ratio represent welfare losses.

We now consider the welfare effect of two policy interventions aimed at reducing the current account imbalance: i) taxing the tradable good and ii), reducing the sex ratio.

We first consider the case of taxing the tradable good. Suppose the home country will impose a tax  $\tau$  on the tradable good in period  $t$  and fully rebate this tax revenue to consumers, then the price taken by the tradable good producers will be  $1 - \tau$ . In period  $t + 1$ , when the current account goes back to zero, home will reduce the tax to zero. During the period in which the shock occurs, (3.9) becomes

$$w_t^\tau = \frac{1 - \tau}{\alpha_T^{\alpha_T} (1 - \alpha_T)^{1 - \alpha_T}} (1 - \alpha_T) A_{Tt} \left( \frac{K_{Tt}^\tau}{1 - L_{Nt}^\tau} \right)^{\alpha_T} = \frac{1}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1 - \alpha_N}} P_{Nt} (1 - \alpha_N) A_{Nt} \left( \frac{K_{Nt}^\tau}{L_{Nt}^\tau} \right)^{\alpha_N}$$

where variable  $Z^\tau$  denotes the variable when there is a tax on the tradable good.

As we have shown in the proof of Proposition 2,

$$CA_t = s_t y_t - s_{t-1} w + \gamma \left( \frac{\alpha_T}{1 - \alpha_T} - \frac{\alpha_N}{1 - \alpha_N} \right) (R - 1) (s_t - s_{t-1}) \frac{w}{R} \quad (\text{E.4})$$

where  $y_t$  is the first period income of the young cohort. We assume that a fraction  $a$  ( $0 \leq a \leq 1$ ) of the tax revenue will be distributed to the young cohort in period  $t$  while the rest will be refunded to the old cohort. Then the nontradable good market clearing condition can be re-written as

$$\frac{P_{Nt} A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1 - \alpha_N}}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1 - \alpha_N}} = \gamma (R s_{t-1} w_{t-1} + (1 - a) \tau Q_{Tt} + (1 - s_t) (w_t + a Q_{Tt})) \quad (\text{E.5})$$

and the wage parity is

$$w_t = \frac{1 - \tau}{\alpha_T^{\alpha_T} (1 - \alpha_T)^{1 - \alpha_T}} (1 - \alpha_T) A_{Tt} \left( \frac{K_{Tt}}{1 - L_{Nt}} \right)^{\alpha_T} = \frac{1}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1 - \alpha_N}} P_{Nt} (1 - \alpha_N) A_{Nt} \left( \frac{K_{Nt}}{L_{Nt}} \right)^{\alpha_N} \quad (\text{E.6})$$

Given  $K_{Tt}$  and  $K_{Nt}$  are predetermined, we can show the following proposition:

**Proposition 5** *If the tax revenue from the tradable good will only be refunded to the working people,*

(i) *If*

$$\alpha_T \left( R s_{t-1} \frac{w_{t-1}}{w_t} + 1 - s_t \right) + (1 - s_t) (w_t - P_{Nt}) (\alpha_T + L_{Nt} - 1) \geq 0$$

*taxing the tradable good cannot reduce the current account surplus.*

(ii) *If*

$$\alpha_T \left( R s_{t-1} \frac{w_{t-1}}{w_t} + 1 - s_t \right) + (1 - s_t) (w_t - P_{Nt}) (\alpha_T + L_{Nt} - 1) < 0$$

*taxing the tradable good can reduce the current account surplus. However, everyone in Home will experience a welfare loss (on top of the welfare loss associated with an unbalanced sex ratio).*

**Proof.** As we have shown in Proposition 2, if the utility function is of log form, then savings rates will not depend on the first period income. We then can take the savings rates as given. We totally

differentiate the system which consists of (E.5) and (E.6) and obtain

$$\Omega \cdot \begin{pmatrix} dP_{Nt} \\ dw_t \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} d\tau$$

where

$$\begin{aligned} \Omega_{11} &= C_{Nt} \\ \Omega_{12} &= -\gamma(1-s_t) \\ \Omega_{13} &= \gamma(1-a+a(1-s_t)) \frac{(1-\alpha_T)Q_{Tt}}{1-L_{Nt}} \\ \Omega_{21} &= 0 \\ \Omega_{22} &= 1 \\ \Omega_{23} &= -\alpha_T \frac{w_t}{1-L_{Nt}} \\ \Omega_{31} &= \frac{w_t}{P_{Nt}} \\ \Omega_{32} &= 1 \\ \Omega_{33} &= \alpha_N \frac{w_t}{L_{Nt}} \end{aligned}$$

and

$$\begin{aligned} z_1 &= \gamma(1-a+a(1-s_t))Q_{Tt} \\ z_2 &= -\frac{w_t}{1-\tau} \\ z_3 &= 0 \end{aligned}$$

The determinant of matrix  $\Omega$  is

$$\begin{aligned} \det(\Omega) &= wC_N \left( \frac{\alpha_N}{L_N} + \frac{\alpha_T}{1-L_N} \right) + \frac{w}{P_N} \left( \frac{\gamma(1-s_t)\alpha_T w}{1-L_N} - \frac{\gamma(1-a+a(1-s_t))(1-\alpha_T)Q_{Tt}}{1-L_N} \right) \\ &= \text{positive terms} + \frac{w}{P_N} \left( P_N C_N \left( \frac{\alpha_N}{L_N} + \frac{\alpha_T}{1-L_N} \right) - \gamma(1-a+a(1-s_t))w \right) \\ &> \frac{w}{P_N} (P_N C_N - \gamma(1-a+a(1-s_t))w) \end{aligned}$$

The last inequality holds because we use the fact that

$$\frac{\alpha_N}{L_N} + \frac{\alpha_T}{1-L_N} \geq (\sqrt{\alpha_N} + \sqrt{\alpha_T})^2$$

where the equality holds when  $L_N = \left(1 + \sqrt{\frac{\alpha_T}{\alpha_N}}\right)^{-1}$ . In the standard literature, both  $\alpha_N$  and  $\alpha_T$  take value greater than 0.25, then  $\frac{\alpha_N}{L_N} + \frac{\alpha_T}{1-L_N} > 1$ .

Notice that  $\gamma(1 - a + a(1 - s_t))w$  is only part of the demand for the nontradable good, which must be smaller than  $P_N C_N$ , therefore,  $\det(\Omega) > 0$ .

Then we can calculate

$$\begin{aligned} \left. \frac{dy_t}{d\tau} \right|_{\tau=0} &= \frac{dw_t}{d\tau} + Q_{Tt} \\ &= \frac{w_t}{\det(\Omega)} \left[ \frac{\alpha_T C_N}{1 - \alpha_T} + \gamma(1 - s_t) \left( \frac{\alpha_T}{1 - L_N} - 1 \right) \left( \frac{w_t}{P_{Nt}} - 1 \right) Q_{Tt} \right] \\ &= \frac{\gamma w_t^2}{P_{N,t+1}(1 - \alpha_T) \det(\Omega)} \left[ \alpha_T \left( R s_{t-1} \frac{w_{t-1}}{w_t} + 1 - s_t \right) + (1 - s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) \right] \end{aligned}$$

In period  $t - 1$ ,  $w_{t-1} = R^{-\frac{\alpha_T}{1 - \alpha_T}} < R^{\frac{\alpha_N - \alpha_T}{\alpha_T}} = P_{Nt}$ . In period  $t$ , when shock occurs, as we have shown in Proposition 2,  $\frac{w_t}{P_{Nt}}$  increases. However, it is unclear whether it exceeds one. Therefore, the sign of  $\left. \frac{dy_t}{d\tau} \right|_{\tau=0}$  is ambiguous.

If  $\alpha_T \left( R s_{t-1} \frac{w_{t-1}}{w_t} + 1 - s_t \right) + (1 - s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) \geq 0$ , then  $\left. \frac{dy_t}{d\tau} \right|_{\tau=0} \geq 0$ . By (E.4), taxing the tradable good cannot reduce the current account surplus caused by the unbalanced sex ratio. On the other hand, if  $\alpha_T \left( R s_{t-1} \frac{w_{t-1}}{w_t} + 1 - s_t \right) + (1 - s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) < 0$ , then  $\left. \frac{dy_t}{d\tau} \right|_{\tau=0} < 0$ . Taxing the tradable good can achieve the goal of cutting down the current account surplus. However, this also reduces the first period income by the young cohort. The welfare of young women and young men will be worse off.

And

$$\begin{aligned} \left. \frac{dC_{2t}}{d\tau} \right|_{\tau=0} &= -\frac{R s_{t-1} w}{P_{Nt}} \frac{dP_{Nt}}{d\tau} \\ &= -\frac{\gamma R s_{t-1} w}{P_{Nt}} (1 - s_t) \left( \frac{\alpha_N}{L_N} + \frac{2\alpha_T}{1 - L_N} - 1 \right) < 0 \end{aligned}$$

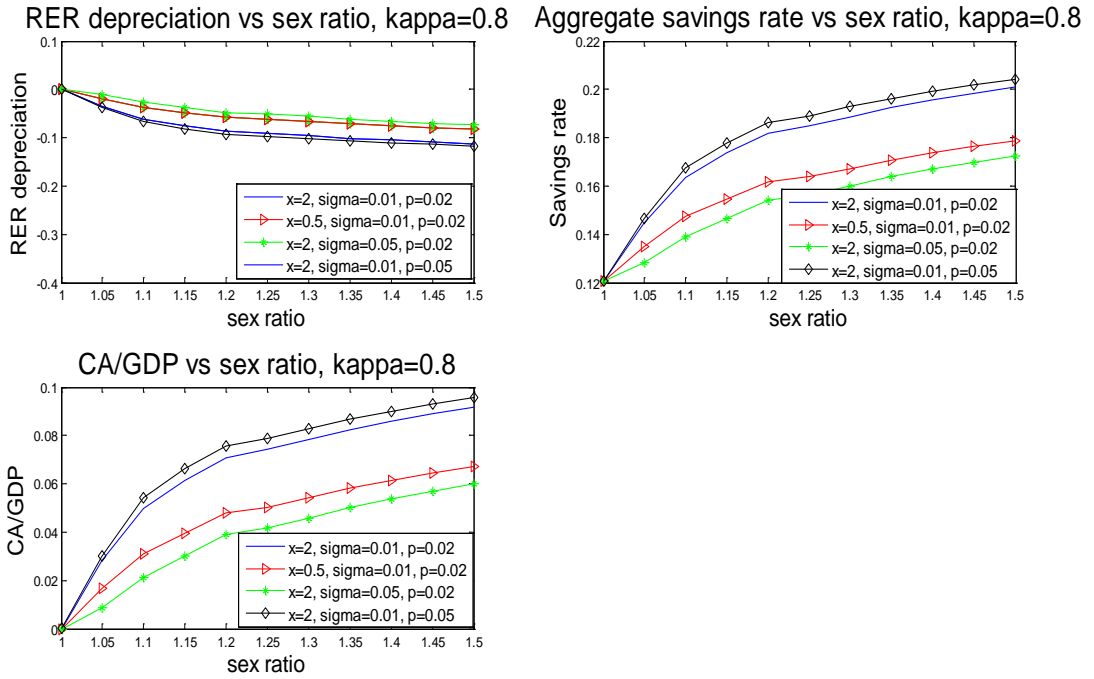
then the old cohort in period  $t$  also suffers from the tax on the tradable good sector.

Therefore, if  $\frac{\alpha_T \left( R s_{t-1} \frac{w_{t-1}}{w_t} + 1 - s_t \right)}{1 - \alpha_T} + (1 - s_t)(w_t - P_{Nt}) \frac{\alpha_T + L_{Nt} - 1}{1 - \alpha_T} < 0$ , taxing the tradable good will cut down the current account surplus, however, at the same time, it will reduce the economy-wide welfare. ■

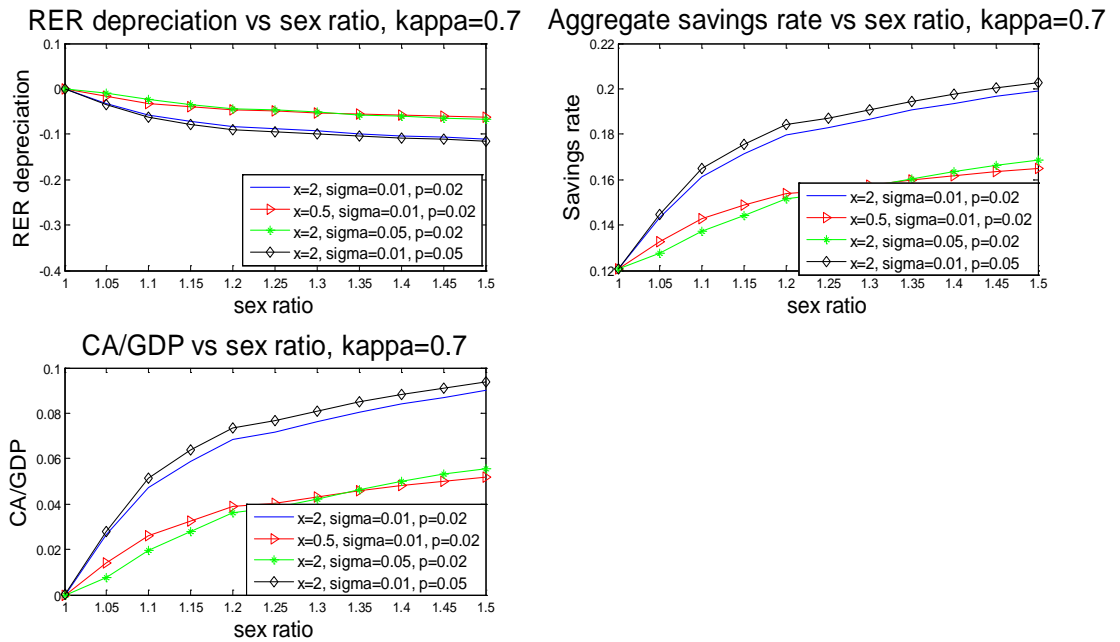
When Home taxes the tradable good sector, the wage rate in that sector decreases immediately, which induces a migration of labor from the tradable sector to the nontradable good sector. The tradable good sector shrinks. Since the young people also get all the tax refund, whether this tax refund can offset the decrease in wage rate is ambiguous. Since the total tax refund equals the tax on per unit tradable good multiplied by the quantity of tradable output, a shrinkage of the tradable good sector implies less tax revenue from the tradable sector and a smaller transfer to consumers. However, consumers only bear a part of the tax burden through a lower wage. Firms bear the other part of tax burden by receiving a lower return to capital. Since the entire tax revenue is transferred to consumers,

there is an indirect transfer from firms in the tradable sector to consumers. The net effect on the first period income of the young cohort is ambiguous.

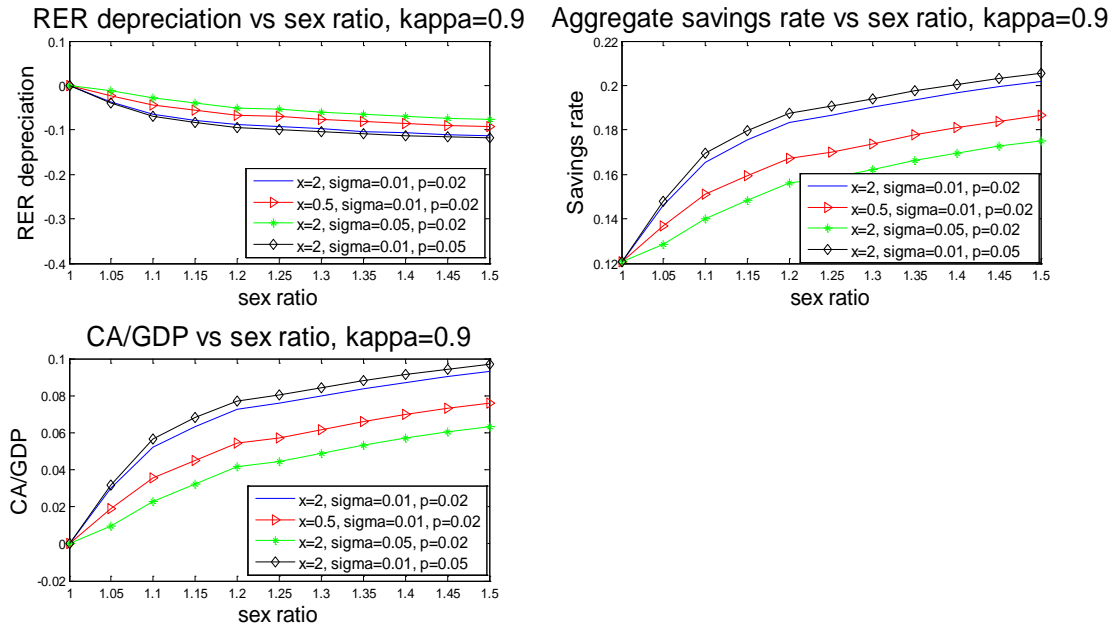
If the central planner can reduce the sex ratio, then as shown Proposition 2, a reduction in the sex ratio will yield a fall in the current account. Correspondingly, there will be a welfare gain for young men but a welfare loss for young women. The aggregate social welfare will improve.



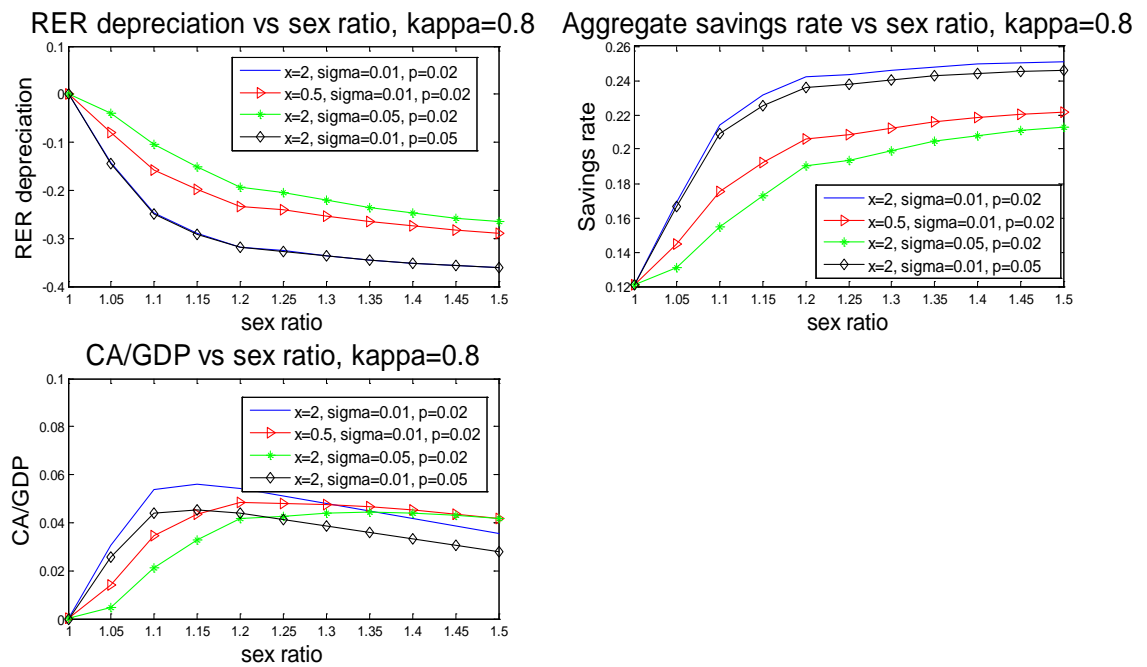
**Figure 1: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.8**



**Figure 2: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.7**

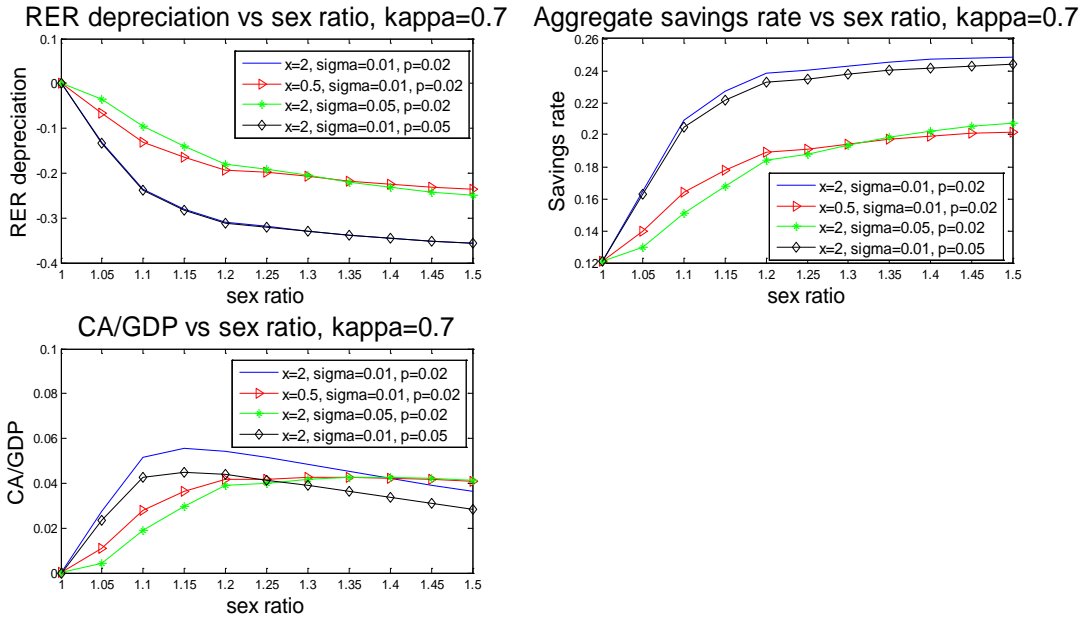


**Figure 3: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.9**

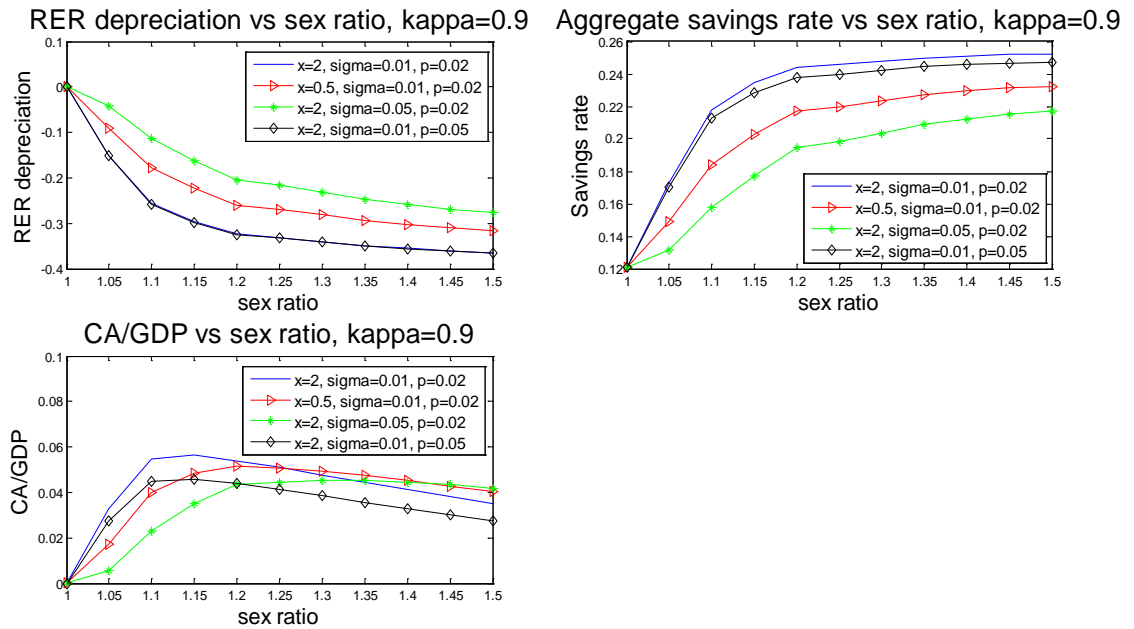


**Figure 4: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.8**





**Figure 5: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.7**



**Figure 6: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.9**

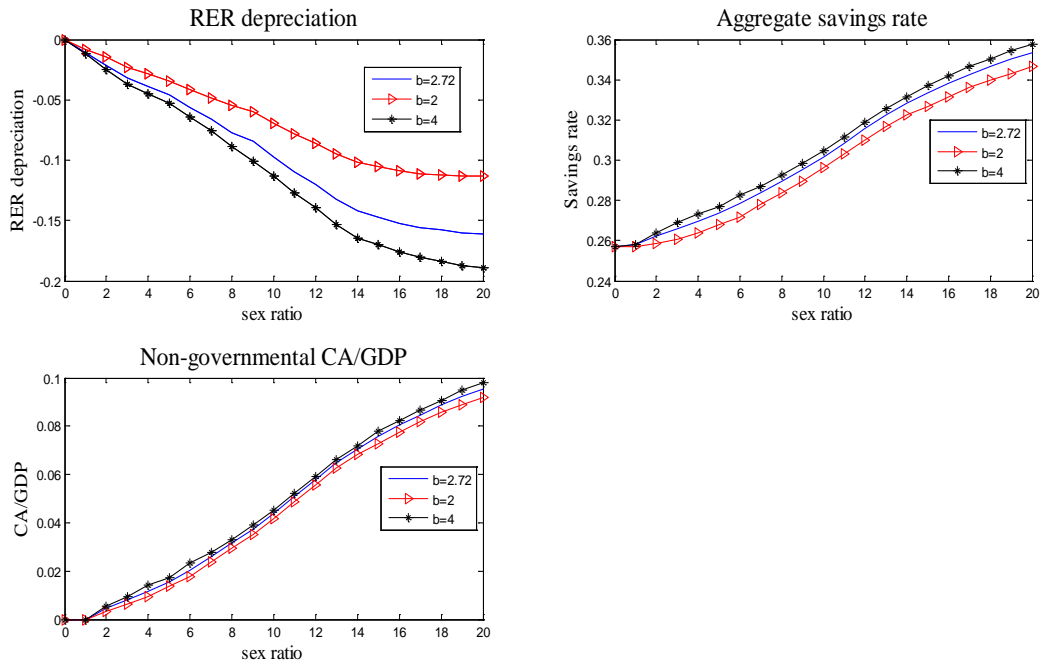


Figure 7: Impulse responses of RER, aggregate savings rate and CA/GDP,  $\tau=20$

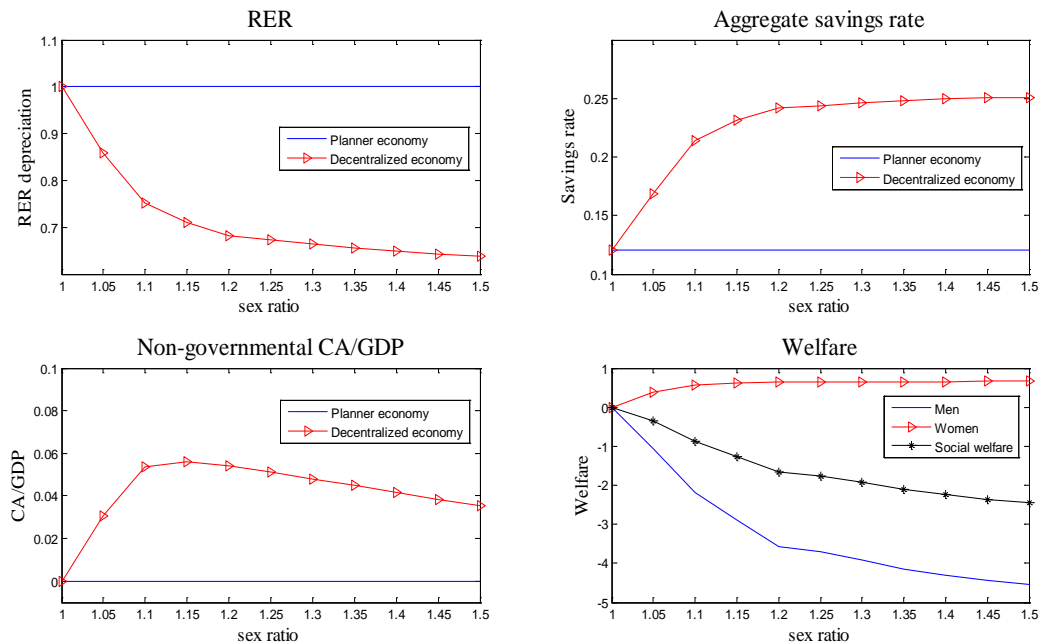


Figure 8: Planner's economy vs Decentralized economy, benchmark

**Table 1: Summary statistics, 2004-2008 average**

Variable	Mean	Median	Standard deviation	Min value	Max value
RER	-2.45	-2.21	1.52	-9.23	-0.63
Current account	-3.63	-2.93	9.32	-31.51	26.91
Real GDP per capita	12985.89	7747.42	13732.60	367.38	77057.20
Private credit (% of GDP)	56.63	38.70	52.26	2.08	319.72
Financial system sophistication	3.78	3.66	0.79	2.52	5.28
Sex ratio	1.04	1.04	0.02	1.00	1.13

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable “ $p$ ” (called “price level of GDP”) in the Penn World Tables is equivalent to the inverse of the real exchange rate defined in the standard literature: A lower value of  $p$  means a depreciation in real exchange rate.
- Current account in the two tables above is the non-governmental current account, i.e., we use the current account to GDP ratio minus the government savings to GDP ratio.
- We use the private credit (% of GDP) as one measure for financial development. For China, we modify the financial development measure by multiplying 0.2 to the private credit to GDP ratio.
- Financial system sophistication from the Global Competitiveness Report is another measure for the financial development.

**Table 2a: Real exchange rate vs sex ratio, year 2006, using private credit to GDP ratio as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)
Sex Ratio			-4.300***	-4.369***	-4.175**
			(1.635)	(1.588)	(1.624)
Ln(GDP per capita)	-0.292***	-0.179***	-0.235***	-0.291***	-0.240***
	(0.028)	(0.034)	(0.041)	(0.040)	(0.040)
Private credit (% of GDP)		-0.00366***	-0.00350***	-0.00207***	-0.00344***
		(0.001)	(0.001)	(0.001)	(0.001)
Crawling peg (RR)				0.359***	
				(0.072)	
Managed floating (RR)				0.0315	
				(0.083)	
Free floating (RR)				0.0647	
				(0.137)	
Intermediate (LYS)					0.0394
					(0.089)
Floating (LYS)					0.167**
					(0.078)
Observations	170	156	132	121	132
R-squared	0.402	0.485	0.552	0.671	0.568

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable “ $p$ ” (called “price level of GDP”) in the Penn World Tables is equivalent to the inverse of the real exchange rate defined in the standard literature: A lower value of  $p$  means a depreciation in real exchange rate.
- Similar results hold when we exclude China out of the sample.
- RR exchange rate regime index is obtained from Reinhart and Rogoff (2004). RR takes value 1, 2, 3 or 4 respectively when the *de facto* exchange rate regime is peg, crawling peg, managed floating or free floating.
- LYS exchange rate regime index is obtained from Levy-Yeyati and Sturzenegger (2005). LYS takes value 1, 2 or 3 when the *de facto* exchange rate regime is fix, intermediate or free float.
- We use the private credit (% of GDP) as the measure for financial development. For China, we modify the financial development measure by multiplying 0.2 to the private credit to GDP ratio.

**Table 2b: Real exchange rate vs sex ratio, year 2006, using financial system sophistication as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)
Sex Ratio			-6.161*** (1.953)	-6.662*** (2.094)	-7.207*** (2.117)
Ln(GDP per capita)	-0.292*** (0.028)	-0.474*** (0.082)	-0.437*** (0.076)	-0.415*** (0.079)	-0.437*** (0.077)
Financial system sophistication		-0.152* (0.089)	-0.235*** (0.086)	-0.218** (0.098)	-0.247*** (0.086)
Crawling peg (RR)				0.203* (0.118)	
Managed floating (RR)				-0.0763 (0.096)	
Free floating (RR)				0.0877 (0.154)	
Intermediate (LYS)					-0.103 (0.117)
Floating (LYS)					0.0868 (0.094)
Observations	170	53	53	51	53
R-squared	0.402	0.738	0.782	0.805	0.793

Standard errors in parentheses, \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable “ $p$ ” (called “price level of GDP”) in the Penn World Tables is equivalent to the inverse of the real exchange rate defined in the standard literature: A lower value of  $p$  means a depreciation in real exchange rate.
- Similar results hold when we exclude China out of the sample.
- RR exchange rate regime index is obtained from Reinhart and Rogoff (2004). RR takes value 1, 2, 3 or 4 respectively when the *de facto* exchange rate regime is peg, crawling peg, managed floating or free floating.
- LYS exchange rate regime index is obtained from Levy-Yeyati and Sturzenegger (2005). LYS takes value 1, 2 or 3 when the *de facto* exchange rate regime is fix, intermediate or free float.
- We use the financial system sophistication index from the Global Competitiveness Report as the measure for financial development in Table 2b.

**Table 3a: Real exchange rate vs sex ratio, 2004-2008, using private credit to GDP ratio as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)
Sex Ratio			-4.357*** (1.637)	-4.525*** (1.586)	-4.218** (1.622)
Ln(GDP per capita)	-0.297*** (0.028)	-0.176*** (0.034)	-0.233*** (0.041)	-0.292*** (0.040)	-0.239*** (0.041)
Private credit (% of GDP)		-0.00390*** (0.001)	-0.00371*** (0.001)	-0.00228*** (0.001)	-0.00365*** (0.001)
Crawling peg (RR)				0.359*** (0.073)	
Managed floating (RR)				0.037 (0.084)	
Free floating (RR)				0.0656 (0.137)	
Intermediate (LYS)					0.0471 (0.089)
Floating (LYS)					0.176** (0.078)
Observations	170	156	132	121	132
R-squared	0.406	0.495	0.562	0.679	0.579

Standard errors in parentheses, \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable “ $p$ ” (called “price level of GDP”) in the Penn World Tables is equivalent to the inverse of the real exchange rate defined in the standard literature: A lower value of  $p$  means a depreciation in real exchange rate.
- Similar results hold when we exclude China out of the sample.
- RR exchange rate regime index is obtained from Reinhart and Rogoff (2004). RR takes value 1, 2, 3 or 4 respectively when the *de facto* exchange rate regime is peg, crawling peg, managed floating or free floating.
- LYS exchange rate regime index is obtained from Levy-Yeyati and Sturzenegger (2005). LYS takes value 1, 2 or 3 when the *de facto* exchange rate regime is fix, intermediate or free float.
- We use the private credit (% of GDP) as the measure for financial development. For China, we modify the financial development measure by multiplying 0.2 to the private credit to GDP ratio.

**Table 3b: Real exchange rate vs sex ratio, year 2006, using financial system sophistication as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)
Sex Ratio			-6.082*** (1.982)	-6.501*** (2.190)	-7.185*** (2.140)
Ln(GDP per capita)	-0.297*** (0.028)	-0.482*** (0.083)	-0.445*** (0.077)	-0.423*** (0.082)	-0.446*** (0.078)
Financial system sophistication		-0.162* (0.090)	-0.244*** (0.087)	-0.239** (0.103)	-0.258*** (0.087)
Crawling peg (RR)				0.151 (0.128)	
Managed floating (RR)				-0.0703 (0.100)	
Free floating (RR)				0.0642 (0.160)	
Intermediate (LYS)					-0.119 (0.118)
Floating (LYS)					0.0887 (0.095)
Observations	170	53	53	51	53
R-squared	0.406	0.746	0.787	0.797	0.799

Standard errors in parentheses, \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable “ $p$ ” (called “price level of GDP”) in the Penn World Tables is equivalent to the inverse of the real exchange rate defined in the standard literature: A lower value of  $p$  means a depreciation in real exchange rate.
- Similar results hold when we exclude China out of the sample.
- RR exchange rate regime index is obtained from Reinhart and Rogoff (2004). RR takes value 1, 2, 3 or 4 respectively when the *de facto* exchange rate regime is peg, crawling peg, managed floating or free floating.
- LYS exchange rate regime index is obtained from Levy-Yeyati and Sturzenegger (2005). LYS takes value 1, 2 or 3 when the *de facto* exchange rate regime is fix, intermediate or free float.
- We use the financial system sophistication index from the Global Competitiveness Report as the measure for financial development in Table 3b.

**Table 4a: CA/GDP vs sex ratio, year 2006, using private credit to GDP as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)	(6)
Sex ratio				106.5***	106.8**	105.2*
				(46.69)	(47.5)	(54.2)
Ln(GDP per capita)	1.777*		3.754***	2.427*	2.19	5.97**
	(0.904)		(1.227)	(1.332)	(1.61)	(2.45)
Private credit (% of GDP)			-0.051**	-0.041*	-0.041**	-0.071***
			(0.022)	(0.022)	(0.021)	(0.023)
Share of working age people						-0.521*
						(0.311)
Social security expenditure (% of GDP)						0.437
						(0.219)**
Ln(RER)		-1.53			-1.22	
		(1.69)			(2.87)	
Africa						19.84***
						(6.35)
Asia						17.15***
						(5.21)
Europe						5.66
						(5.05)
North America						10.89*
						(5.62)
South America						17.42***
						(5.92)
Observations	93	92	92	92	91	60
R-squared	0.041	0.010	0.096	0.147	0.154	0.428

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

- In Table 4a, we use the average non-governmental current account to GDP ratio in 2006 as the dependent variable.
- We use the private credit (% of GDP) as the measure for financial development. For China, we modify the financial development measure by multiplying 0.2 to the private credit to GDP ratio.



**Table 4b: CA/GDP vs sex ratio, year 2006, using financial system sophistication as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)	(6)
Sex ratio				148.9***	131.8**	91.83
				(50.35)	(51.4)	(71.09)
Ln(GDP per capita)	1.78**		2.37	3.51**	5.55*	-0.456
	(0.90)		(2.24)	(2.09)	(3.12)	(3.35)
Financial system sophistication			-1.17	-3.00	-2.15**	1.78
			(2.42)	(2.09)	(2.50)	(3.07)
Share of working age people						0.093
						(0.467)
Social security expenditure (% of GDP)						-0.035
						(0.266)
Ln(RER)		1.53			-4.03	
		(1.69)			(4.56)	
Africa						17.33*
						(9.79)
Asia						8.54
						(7.97)
Europe						9.50
						(7.36)
North America						5.83
						(7.75)
South America						9.51
						(8.37)
Observations	93	92	43	43	43	36
R-squared	0.041	0.010	0.035	0.212	0.228	0.247

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

- In Table 4b, we use the average non-governmental current account to GDP ratio in 2006 as the dependent variable.
- We use the financial system sophistication index from the Global Competitiveness Report as the measure for financial development.

**Table 5a: CA/GDP vs sex ratio, 2004-2008, using private credit to GDP as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)	(6)
Sex ratio				105.4***	97.6**	90.37**
				(39.550)	(39.7)	(44.470)
Ln(GDP per capita)	2.181***		3.873***	2.673**	3.42***	5.136**
	(0.827)		(1.126)	(1.112)	(1.32)	(2.002)
Private credit (% of GDP)			-0.0454**	-0.0361*	-0.030	-0.0483**
			(0.021)	(0.019)	(0.019)	(0.020)
Share of working age people						-0.566**
						(0.262)
Social security expenditure (% of GDP)						0.271
						(0.185)
Ln(RER)		1.48			-1.76	
		(1.46)			(2.24)	
Africa						15.04***
						(5.207)
Asia						14.75***
						(4.351)
Europe						5.457
						(4.240)
North America						8.142*
						(4.721)
South America						13.91**
						(5.311)
Observations	104	102	103	99	102	61
R-squared	0.064	0.010	0.106	0.191	0.207	0.416

Standard errors in parentheses, \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

- In Table 5a, we use the average non-governmental current account to GDP ratio (2004-2008) as the dependent variable.
- We use the private credit (% of GDP) as the measure for financial development. For China, we modify the financial development measure by multiplying 0.2 to the private credit to GDP ratio.

**Table 5b: CA/GDP vs sex ratio, 2004-2008, using financial system sophistication as the measure of financial development**

	(1)	(2)	(3)	(4)	(5)	(6)
Sex ratio				139.3***	123.4**	98.49
				(44.630)	(46.3)	(61.090)
Ln(GDP per capita)	2.181***		2.63	3.710**	5.12**	0.354
	(0.827)		(1.926)	(1.782)	(2.44)	(2.891)
Financial system sophistication			-1.937	-3.801*	-2.64	1.297
			(1.991)	(1.904)	(1.98)	(2.642)
Share of working age people						-0.0619
						(0.394)
Social security expenditure (% of GDP)						-0.111
						(0.229)
		1.48			-3.11	
		(1.46)			(3.34)	
Africa						15.48*
						(8.318)
Asia						8.234
						(6.776)
Europe						9.175
						(6.251)
North America						6.304
						(6.577)
South America						4.62
						(8.106)
Observations	104	102	44	44	48	30
R-squared	0.064	0.010	0.044	0.231	0.230	0.314

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

- In Table 5b, we use the average non-governmental current account to GDP ratio (2004-2008) as the dependent variable.
- We use the financial system sophistication index from the Global Competitiveness Report as the measure for financial development.

**Table 6: Real exchange rate undervaluation and excess current account, China**

Financial development index	% of RER undervaluation			Excess current account		
	(1)	(2)	(3)	(4)	(5)	(6)
Private credit to GDP	54.44	42.94	3.93	13.97	12.46	2.90
financial system sophistication index	54.44	46.38	0.77	13.97	10.41	0.46

- Current account in Table 6 is the non-governmental current account (% of GDP), i.e., we exclude the government savings from the current account.
- We use two measures of financial development. One is the private credit (% of GDP) and the other is the financial system sophistication index from the Global Competitiveness Report. We modify the private credit index by dividing the ratio of private credit to GDP by two for China because approximately half of the bank credit goes to non-state owned firms.
- RER undervaluation is defined as the difference between the actual real exchange rate and the predicted value. In column (1), we consider the Balassa-Samuelson effect. In column (2), we consider the financial development and the Balassa-Samuelson effect. In column (3), we consider the sex ratio, financial development and the Balassa-Samuelson effect.
- Excess current account is computed by using the actual non-governmental current account (% of GDP) minus the predicted value from regressions. In column (4), the predicted value of non-governmental current account is obtained by regressing non-governmental CA/GDP on log per capita GDP. In column (5), the predicted value of non-governmental current account is obtained by regressing the actual non-governmental CA/GDP on log per capita GDP and financial development index. In column (6), the predicted value of non-governmental current account is obtained by regressing the actual non-governmental CA/GDP on log per capita GDP, financial development index and the sex ratio.