Financial Integration and Liquidity Crises*

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Abstract

This paper analyzes the effects of financial integration on the banking system. Financial integration allows banks in different countries to smooth local liquidity shocks by borrowing on the international interbank market. We show that, under realistic conditions, financial integration induces banks to reduce their liquidity holdings and to shift their portfolios towards more profitable but less liquid long-term investments. Integration helps to reallocate liquidity when different countries are hit by uncorrelated shocks. However, when an aggregate liquidity shock hits, the aggregate liquid resources in the banking system are lower than in autarky. Therefore, financial integration leads to larger spikes in interest rates on the interbank market in crisis episodes. For some parameter values, financial integration can lead to higher overall volatility of interbank rates. The model is consistent with recent trends in external positions and liquidity holdings of banks in the US and in Europe.

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1 Introduction

International financial integration allows banks located in different countries to smooth local liquidity shocks by borrowing and lending on international interbank markets. Everything else equal, this should have a stabilizing effect on financial markets, by allowing banks in need of funds to borrow from banks located abroad and not only from other local banks. That is, financial integration should help dampen the effects of local liquidity shocks. However, the availability of additional sources of funding changes the ex ante incentives of banks when they make their lending and portfolio decisions. In particular, banks who have access to an international pool of liquidity may choose to hold lower reserves of safe, liquid assets and choose less liquid and/or more risky investment strategies. Once this endogenous response is taken into account, the equilibrium effects of international integration on financial stability are less clear. If a correlated shock hits all countries (a "systemic" event) the lower holdings of liquid reserves in the system lead to a larger increase in interbank rates. In this paper, we explore these effects showing that financial integration can lead to lower holdings of liquid assets, to more severe crises and, in some cases, to higher volatility in interbank markets. Moreover, we show that all these effects can take place in an environment where integration is always welfare improving from an ex ante point of view.

The focus of our analysis is to understand the effects of integration on the banks' investment decisions ex ante and, in particular, on the response of equilibrium liquidity holdings. The direction of this response is non obvious because two forces are at work. On the one hand, internationally integrated banks have more opportunities to borrow if they are hit by a liquidity shock. This lowers their incentives to hold reserves of liquid assets. On the other hand, they also have more opportunities to lend their excess liquidity when they do not need it. This increases their incentives to hold liquid reserves. In our model we capture these two forces and show that under reasonable parameter restrictions the first force dominates and financial integration leads to lower holdings of liquid assets.

We consider a world with two ex ante identical regions. In each region banks offer state contingent deposit contracts to consumers, they invest the consumers' savings and allocate funds to them when they are hit by liquidity shocks à la Diamond and Dybvig [11]. Banks can invest in two assets: a liquid short-term asset and an illiquid long-term asset. When the two regions are hit by different average liquidity shocks there are gains from trade from sharing liquid resources through the international interbank market. However, when the two regions are both hit by a high liquidity shock, there is a worldwide liquidity shortage and the presence of the international interbank market is of little help. We consider different configurations of regional liquidity shocks allowing for various degrees of correlation between regional shocks. When the correlation between regional shocks is higher there is more aggregate uncertainty and the gains from integration are lower.

We analyze the optimal investment decision of banks under autarky and under financial integration and show that, under some conditions, banks invest a smaller fraction of their portfolio in liquid assets under integration. Moreover, we show that this effect is stronger when there is less aggregate uncertainty. In this case, it is more likely that banks are hit by different shocks, there is better scope for coinsurance, and the ex ante incentive to hold liquid assets is lower.

We then look at the implications for the equilibrium distribution of interest rates in the interbank market, comparing the equilibrium under integration and in autarky. When the two regions are hit by different shocks financial integration tends to reduce interest rates in the region hit by the high liquidity shock. This is the stabilizing effect of financial integration. However, financial integration makes things worse (ex post) in the state of the world where both regions are hit by a high liquidity shock. In this case, since banks are holding overall lower liquid reserves, the worldwide liquidity shortage is more severe and there is a spike in interest rates. Therefore, financial integration tends to make the distribution of interest rate more skewed, with low interest rates in "normal times," in which regional shocks offset each other, and occasional spikes when a worldwide shock hits. We identify the worldwide liquidity shock in the model with a "systemic" event and thus argue that financial integration can make systemic crises more severe. If the probability of a worldwide shock is small enough the overall effect of integration is to reduce interest rate volatility. However, if there is a sufficient amount of aggregate uncertainty interest rate volatility can increase as a consequence of financial integration.

We also look at the model implications for the equilibrium distribution of consumption, showing the real implications of financial integration are similar to the implications for the interest rate: the distribution of consumption tends to become more skewed after integration and can display higher volatility.

We conduct our exercise in the context of a model with minimal frictions, where banks allocate liquidity efficiently by offering fully state contingent deposit contracts. In this setup equilibria are Pareto efficient and the increased volatility that can follow from financial integration is not a symptom of inefficiency. In fact, in our model financial integration is always welfare improving. Although this result clearly follows from the absence of frictions in the model, it points to a more general observation: the effects of integration on volatility should not be taken as unequivocal evidence that integration is undesirable ex ante.

There is a large literature on the role of interbank markets as a channel for sharing liquidity risk among banks. In particular, our paper is related to Bhattacharya and Gale [7], Allen and Gale [4], and Freixas, Parigi and Rochet [15], who analyze the functioning of the interbank market in models where banks act as liquidity providers à la Diamond and Dybvig [11]. Allen and Gale [4] and Freixas, Parigi and Rochet [15] are concerned with the fact that interbank linkages can act as a source of contagion, generating chains of bank liquidations. In this paper, we focus on how different degrees of interbank market integration affect ex ante investment decisions. For this reason, we simplify the analysis and rule out bank runs and liquidations by allowing for fully state contingent deposit contracts. Our paper is also related to Holmstrom and Tirole [18], who emphasize the different role of aggregate and idiosyncratic uncertainty in the optimal allocation of liquidity.

A recent paper which also emphasizes the potentially destabilizing effects of integration is Freixas and Holthausen [14], who point out that integration may magnify the asymmetry of information, as banks start trading with a pool of foreign banks on which they have less precise information. Here we abstract from informational frictions in interbank markets, either in the form of asymmetric information (as, e.g., in Rochet and Tirole [22]) or moral hazard (as, e.g., in Brusco and Castiglionesi [8]).

The recent literature has emphasized a number of potential inefficiencies generating excessive illiquidity during systemic crises. In Wagner [24] interbank lending may break down due to moral hazard. In Acharya, Gromb and Yorulmazer [1] inefficiencies in interbank lending arise due to monopoly power. In Allen, Carletti and Gale [3] inefficiencies in the interbank market arise because interest rates fluctuate too much in response to shocks, precluding efficient risk sharing. In Castiglionesi and Wagner [10], inefficient liquidity provision is due to non exclusive contracts. This paper emphasizes the fact that the instability associated to integration can also be the product of an efficient response of banks' investment decisions.

Caballero and Krishnamurthy [9] have recently emphasized that the higher demand for liquid stores of values by emerging economies is an endemic source of financial instability. This demand pressure stretches the ability of the developed countries' financial system to transform illiquid assets into liquid liabilities, pushing them to hold larger holdings of risky assets. Here we emphasize a different but complementary channel by which financial globalization changes the balance sheet of financial intermediaries, emphasizing the endogenous illiquidity generated by increased access to international interbank markets.

The remainder of the paper is organized as follows. Section 2 lays out the empirical motivation for our theoretical work. Section 3 presents the model. Sections 4 and 5 characterize the equilibrium, respectively, in autarky and under financial integration. In Section 6 contains our main results on the effects of integration on liquid asset holdings.

Section 7 analyzes the consequences of integration on the depth of systemic crises, both in term of interbank market interest rates and in term of consumption. Section 8 concludes. All the proofs are in the Appendix.

2 Some motivating facts

To motivate our analysis, we begin by briefly documenting the recent increase in financial market integration and the contemporaneous reduction in the holding of liquid assets in the banking system.

The last fifteen years have witnessed a dramatic increase in the international integration of the banking system. Panel A in Figure 1 documents the increase in the cross-border activities of US banks between 1993 and 2007. To take into account the overall growth of the banking activities, we look at the ratio of the external positions of US banks towards all foreign financial institutions to total domestic credit.[1] This ratio goes from 11% to 21% between 1993 and 2007. If we restrict attention to the external position of the US banks towards foreign banks, the same ratio goes from 8% to 16%. In panel B of Figure 1, we look at the holdings of liquid assets by US banks over the same time period. In particular, we look at the ratio of liquid assets to total deposits.[2] This ratio decreased from 13% in 1993 to 3.5% in 2007. Therefore, the US banking system clearly displays a combination of increased integration and increased illiquidity.

A similar pattern arises if we look at banks in the Euro area. In panel A of Figure 2 we plot the ratio of the total external position of Euro-area banks to domestic credit and in panel B we plot their liquidity ratio.[3] The external position goes from 40% of domestic credit to 66% between 1999 and 2008. Restricting attention to the external position towards Euro-area borrowers (i.e., banks located in a Euro-area country different than the originating bank), we observe an increase from 25% until 43% in the same period. Finally, if we restrict attention to external loans to foreign banks, we see an increase from 19% to 30%. At the same time, also in the Euro area we see a reduction in liquidity holdings, with a liquidity ratio going from 26% in 1999 to 15% in 2008.

[1] The total external position of US and Euro area banks are from the BIS locational data. The data on domestic credit are from the International Financial Statistics of the IMF.

[2] Following Freedman and Click [13] we look at the "liquidity ratio" given by liquid assets over total deposits. The numerator is given by the sum of reserves and claims on central government from the IMF International Financial Statistics. The denominator is the sum of demand deposits, time and savings deposits, money market instruments and central government deposits, also from the IMF International Financial Statistics.

[3] The construction of the ratios and the data sources are the same as for Figure 1.

Figure 3 shows similar results for Germany alone. Notice that the level of financial integration and the liquidity ratio of the German banking system were relatively stable until 1998. The increase in financial integration and the concomitant decrease in the liquidity ratio only start in 1999.

As a final piece of motivating evidence, we look at the recent behavior of liquidity premia in interbank markets. Figure 4 reports the spread between LIBOR rate and Overnight Indexed Swap (OIS) with the same maturity, and the spread between the LIBOR and the secured interbank government (REPOs) of the same maturity. These spreads are good measures of the cost of liquidity since they factor out, respectively, expectations and default risk. Figure 4 illustrates both the stabilizing and the destabilizing effects of integration on interbank markets which we will emphasize in our model. In the period preceding the crisis, we observe unusually stable and low spreads. At the same time, since the onset of the mortgage crisis in the summer of 2007 we see a dramatic spike in spreads, reflecting a protracted illiquidity problem in interbank markets.

It is also useful to mention some cross sectional evidence showing that banks in emerging economies, which are typically less integrated with the rest of the world, tend to hold larger liquid reserves. Freedman and Click [13] show that banks in developing countries keep a very large fraction of their deposits in liquid assets. The average liquidity ratio for developing countries is 45% vs. an average ratio of 19% for developed countries. Clearly, there are other reasons behind this differences, besides international integration—Acharya, Shin and Yorulmazer [2] emphasize the role of poor legal and regulatory environments. However, this evidence is at least consistent with the view that financial integration may affect banks' liquidity ratios.

Another type of cross sectional evidence which is interesting for our exercise is in Ranciere, Tornell and Westerman [20], who show that countries affected by large financial crises display both higher variance and larger negative skewness in credit growth than countries characterized by a more stable financial system. Since the former group of countries are also more open to international capital flows, this evidence provides some support to our view that important implications of financial integration are to be observed in both the variance *and* the skewness of real aggregates.

3 The model

In this section we describe a simple model of risk sharing among banks located in different regions. The model is a multi-region version of Diamond and Dybvig [11] and is similar to

Allen and Gale [4], except that we allow for fully state contingent deposit contracts.

Consider an economy with three dates, $t = 0, 1, 2$, and a single consumption good that serves as the numeraire. There are two assets, both in perfectly elastic supply. The first asset, called the short asset or the liquid asset, yields one unit of consumption at date $t+1$ for each unit of consumption invested at date t, for $t = 0, 1$. The second asset, called the long asset or the illiquid asset, yields $R > 1$ units of consumption at date 2 for each unit of consumption invested at date 0.

There are two ex ante identical regions, A and B. Each region contains a continuum of ex ante identical consumers with an endowment of one unit of consumption good at date 0. In period 1, agents are hit by a preference shock $\theta \in \{0, 1\}$ which determines whether they like to consume in period 1 or in period 2. Their preferences are represented by the expected utility function

$$\mathrm{E}\left[\theta u\left(c_1\right) + \left(1-\theta\right) u\left(c_2\right)\right],$$

where $u(.)$ is continuously differentiable, increasing and strictly concave and satisfies the Inada condition $\lim_{c \to 0} u'\left(c\right) = \infty$. We will call early and late consumers, respectively, the consumers hit by the shocks $\theta = 1$ and $\theta = 0$.

The uncertainty about preference shocks is resolved in period 1 as follows. First, a regional liquidity shock is realized, which determines the fraction ω^i of early consumers in each region $i = A, B$. Then, preference shocks are randomly assigned to the consumers in each region so that ω^i consumers receive $\theta = 1$. The preference shock is privately observed by the consumer, while the regional shocks ω^i are publicly observed.

The regional shock ω^i takes the two values ω_H and ω_L, with $\omega_H > \omega_L$, with equal probability 1/2. Therefore, the expected value of the regional shock is

$$\omega_M \equiv \mathrm{E}\left[\omega^i\right] = \frac{\omega_H + \omega_L}{2}.$$

To allow for various degrees of correlation between regional shocks, we assume that the probability that the two regions are hit by different shocks is $p \in (0, 1]$. We then have four possible states of the world $S \in \mathcal{S} = \{HH, LH, HL, LL\}$ with the probabilities given in Table 1. In states HH and LL the two regions are hit by identical shocks while in states LH and HL they are hit by different shocks. A higher value of the parameter p implies a lower correlation between regional shocks and more scope for interregional risk sharing. A simple baseline case is when the regional shocks are independent and $p = 1/2$.

In each region there is a competitive banking sector. Banks offer fully state contingent deposit contracts: in period 0, a consumer transfers his initial endowment to the bank, which invests a fraction y in the short asset and a fraction $1 - y$ in the long asset; then, in period 1, after the aggregate shocks S is publicly observed, the consumer reveals his

State S	A	B	Probability
HH	ω_H	ω_H	$(1-p)/2$
LH	ω_L	ω_H	$p/2$
HL	ω_H	ω_L	$p/2$
LL	ω_L	ω_L	$(1-p)/2$

Table 1: Regional liquidity shocks

preference shock to the bank and receives the consumption vector $\left(c_1^S, 0\right)$ if he is an early consumer and the consumption vector $\left(0, c_2^S\right)$ if he is a late consumer. Therefore, a deposit contract is fully described by the array

$$\left\{y, \left\{c_t^S\right\}_{S\in\mathcal{S};t=1,2}\right\}.$$

The assumption of perfectly state contingent deposit contracts is not particularly realistic, but it helps to conduct our analysis in a setup where the only friction is a possible barrier to risk sharing across regions.

4 Autarky

We start the analysis with the autarky case, in which a bank located in a given region can only serve the consumers located in that region and cannot enter into financial arrangements with banks located in other regions.

Consider a representative bank in region A. Given that the liquidity shock in region B is irrelevant for the consumers in region A, the bank will find it optimal to offer deposit contracts that are only contingent on the local liquidity shock, denoted by $s \in \{H, L\}$, that is deposit contracts of the form

$$\left\{y, \left\{c_t^s\right\}_{s\in\{H,L\};t=1,2}\right\},$$

where c_t^s is the amount that a depositor can withdraw at time t if the local liquidity shock is ω_s.

Given that we have a competitive banking sector, the representative bank will maximize the expected utility of the consumers in the region. Therefore, the equilibrium allocation under autarky is given by the solution to the problem:

$$\max_{y,\{c_t^s\}} \frac{1}{2}\left[\omega_H u\left(c_1^H\right) + (1-\omega_H)\, u\left(c_2^H\right)\right] + \frac{1}{2}\left[\omega_L u\left(c_1^L\right) + (1-\omega_L)\, u\left(c_2^L\right)\right] \tag{1}$$

subject to

$$\begin{aligned} \omega_s c_1^s &\leq y \qquad s = L, H, \\ (1-\omega_s)\, c_2^s &\leq R\,(1-y) + y - \omega_s c_1^s \qquad\qquad s = L, H. \end{aligned}$$

The first constraint is a liquidity constraint stating that in every state s total payments to early consumers have to be covered by the returns of the short investment made in period 0. If this constraint is slack, the residual funds $y - \omega_s c_1^s$ are reinvested in the short asset in period 1. The second constraints states that total payments to late consumers are covered by the returns of the long investment plus the returns of the short investment made in period 1.[4] When the liquidity constraint is slack and $y - \omega_s c_1^s > 0$ we say that there is *positive rollover.*

The next proposition characterizes the autarky allocation.

Proposition 1 *The optimal allocation under autarky satisfies*

$$c_1^H < c_1^L \leq c_2^L < c_2^H.$$

No funds are rolled over between periods 1 *and* 2 *in state* H. *If positive rollover occurs in state* L *then* $c_1^L = c_2^L$.

The fact that it is never optimal to have positive rollover in state H is intuitive. If there is positive rollover after a high liquidity shock, then there must also be positive rollover after the low liquidity shock. But then some of the funds invested in the short asset at date 0 will be rolled over with certainty, yielding a return of 1 in period 2, while it would be more profitable to invest them in the long asset which yields $R > 1$. On the other hand, if the liquidity shock ω_L is sufficiently low it may be optimal not to exhaust all liquid resources to pay early consumers. In this case, the optimal allocation of funds between periods 1 and 2 requires that the marginal utility of early and late consumers is equalized, which implies $c_1^L = c_2^L$.

This proposition establishes that in autarky there is uncertainty about the level of consumption at time t and, in particular, we have $u'(c_1^H) > u'(c_1^L)$ and $u'(c_2^H) < u'(c_2^L)$. This means that in period 1, it would be welfare improving to reallocate resources from state L to state H, if resources could be transferred one for one between the two states.

[4] Since the type of each consumer is private information, the problem should include an incentive compatibility constraint of the form $c_1^s \leq c_2^s$ for $s = H, L$ (assuming the late consumers have the option to withdraw c_1^s and invest it in the short asset). However, as we will see in Proposition 1, this constraint is automatically satisfied by the solution to problem (1). The same is true under integration (see Proposition 3), so in both cases we can safely leave aside incentive compatibility.

Similarly, in period 2 it would be welfare improving to reallocate resources from state H to state L. Clearly, these transfers are not feasible in autarky. Financial integration opens the door to an efficient reallocation of liquidity across regions.

5 Financial integration

We now turn to the case of financial integration, in which banks located in one region can insure against regional liquidity shocks by trading contingent credit lines with banks located in the other region. Notice that this mechanism does not eliminate aggregate uncertainty. It is possible to coinsure in states LH and HL, but in states HH and LL this coinsurance is not possible. Since the probability of the first two states is p, the probability $(1-p)$ is a measure of residual aggregate uncertainty.

When the two regions are integrated, we considered a decentralized banking system where:

1. each regional bank offers deposit contracts to the consumers in its own region;

2. regional banks offer each other *contingent credit lines* of the following form: if the two regions are hit by different shocks, the bank in the region hit by the high liquidity shock H can borrow the amount $m_1 \geq 0$ from the other bank at time 1 and has to repay $m_2 \geq 0$ at time 2.

Suppose that a bank can choose any credit line $(m_1, m_2) \in R_+^2$. Competition implies that the representative bank in region A will choose a deposit contract $\{y, \{c_t^S\}\}$ and a contingent credit line $(m_1, m_2) \in R_+^2$ that maximize the expected utility of the representative consumer in that region, solving:

$$\max_{y,\{c_t^S\},(m_1,m_2)} \quad p\left\{\frac{1}{2}\left[\omega_H u\left(c_1^{HL}\right)+(1-\omega_H)\,u\left(c_2^{HL}\right)\right]+\frac{1}{2}\left[\omega_L u\left(c_1^{LH}\right)+(1-\omega_L)\,u\left(c_2^{LH}\right)\right]\right\} \qquad (2)$$

$$+(1-p)\left\{\frac{1}{2}\left[\omega_H u\left(c_1^{HH}\right)+(1-\omega_H)\,u\left(c_2^{HH}\right)\right]+\frac{1}{2}\left[\omega_L u\left(c_1^{LL}\right)+(1-\omega_L)\,u\left(c_2^{LL}\right)\right]\right\}$$

subject to

$$\begin{aligned}
\omega_H c_1^{HL} &\leq y+m_1, \qquad (1-\omega_H)\,c_2^{HL} \leq R\,(1-y)+\left(y+m_1-\omega_H c_1^{HL}\right)-m_2,\\
\omega_L c_1^{LH} &\leq y-m_1, \qquad (1-\omega_L)\,c_2^{LH} \leq R\,(1-y)+\left(y-m_1-\omega_L c_1^{LH}\right)+m_2\\
\omega_s c_1^{ss} &\leq y, \qquad (1-\omega_s)\,c_2^{ss} \leq R\,(1-y)+\left(y-\omega_s c_1^{ss}\right), \qquad s=H,L,
\end{aligned}$$

The first four constraints reflect the bank's budget constraints in the states in which the two regions are hit by asymmetric shocks. In state $S = HL$, the bank in region A has additional resources available to pay early consumers in period 1, given by m_1. Accessing the credit line, though, reduces the resources for late consumers by m_2. In state $S = LH$, the opposite happens, as the bank's correspondent draws on its credit line in period 1 and repays in period 2. The last two constraints represent the budget constraints in the states of the world in which the two regions are hit by identical shocks. Since in these states the contingent credit line is inactive, these constraints are analogous to the autarky case.

By symmetry, a bank in region B will solve the same problem, except that the roles of states HL and LH are inverted. This implies that finding a solution to problem (2) immediately gives us an equilibrium where the banks in region A and B choose symmetric credit lines and the market for credit lines clears.[5] Therefore, in the rest of this section we focus on characterizing solutions to (2).

Proposition 2 *Under financial integration, equilibrium consumption satisfies* $c_t^{HL} = c_t^{LH}$ *for* $t = 1, 2$ *and an equilibrium credit line is*

$$(m_1, m_2) = \left((\omega_H - \omega_M)\, c_1^{HL}, (\omega_H - \omega_M)\, c_2^{HL} \right).$$

This proposition states that the interbank market is used to fully coinsure against the regional liquidity shocks whenever such coinsurance is possible, that is, in all the states in which the two regions are not hit by the same shock. In these states the consumption levels c_t^{HL} and c_t^{LH} are equalized. In the remainder of the paper we will referred to their common value as c_t^M.

This proposition allows us to restate problem (2) in the following form:

$$\max_{x,y,\{c_t^S\}} \quad p\left[\omega_M u\left(c_1^M\right) + (1-\omega_M)\, u\left(c_2^M\right)\right] + \tag{3}$$

$$(1-p)\left\{\frac{1}{2}\left[\omega_H u\left(c_1^{HH}\right) + (1-\omega_H)\, u\left(c_2^{HH}\right)\right] + \frac{1}{2}\left[\omega_L u\left(c_1^{LL}\right) + (1-\omega_L)\, u\left(c_2^{LL}\right)\right]\right\}$$

[5] In other words, our credit lines are state-contingent securities that banks trade at date 0, with the following payoff matrix (for the bank in region A):

S	$t=1$	$t=2$
HH	0	0
HL	m_1	$-m_2$
LH	$-m_1$	m_2
LL	0	0

Proposition 2 shows that if all possible securities of this form are traded (i.e., all $(m_1, m_2) \in R_+^2$) then there is an equilibrium in which: (i) they all trade at price 0 at date 0, (ii) all banks in region A buy one unit of some security (m_1, m_2) and all banks in region B sell one unit of the same security.

In our simple symmetric environment these securities are sufficient to achieve a complete market allocation.

subject to

$$
\begin{aligned}
\omega_M c_1^M &\leq y, \qquad (1-\omega_M)\, c_2^M \leq R\,(1-y) + y - \omega_M c_1^M, \\
\omega_s c_1^{ss} &\leq y, \qquad (1-\omega_s)\, c_2^{ss} \leq R\,(1-y) + y - \omega_s c_1^{ss}, \qquad s = H, L.
\end{aligned}
$$

Notice that problem (3) coincides with the problem of a social planner who gives equal weights to consumers in all regions, proving that the combination of regional deposit contracts and cross-border contingent credit lines is sufficient to achieve a Pareto efficient allocation.

Program (3) can be used to obtain the following characterization of the equilibrium allocation.

Proposition 3 *The equilibrium allocation under financial integration satisfies:*

$$c_1^{HH} < c_1^M \leq c_1^{LL} \leq c_2^{LL} \leq c_2^M < c_2^{HH}.$$

Positive rollover can occur: (i) in states LL, HL and LH, in which case $c_1^M = c_2^M = c_1^{LL} = c_2^{LL}$; (ii) only in state LL, in which case $c_1^{LL} = c_2^{LL}$; or (iii) never.

As in the autarky case rollover never occurs in the less liquid state of the world (here state HH). However, rollover can occur in the state where both regions are hit by the low liquidity shock and also in the intermediate states where only one region is hit by the high shock.

6 Integration and illiquidity

In the rest of the paper we want to analyze the effects of financial integration by comparing the autarky case of Section 4 with the integrated economy of Section 5. First, we analyze how financial integration affects the banks' holdings of liquid assets. In the next section, we will analyze how it affects the severity of systemic crises.

In this section, we will use both analytical results and numerical examples to show that, under realistic parameter configurations, banks tend to hold less liquid assets under financial integration. The basic idea is the following: under financial integration there is more scope for coinsurance and banks are less concerned about holding a buffer of liquid resources, because they expect to be able to borrow from banks located in the other region in states of the world in which the regional shocks are uncorrelated. While this argument is intuitive, the result is, in fact, non-obvious because two forces are at work. On the one hand, integration means that banks can borrow on the interbank market when they are hit by a *high* (uncorrelated) liquidity shock. On the other hand, integration also means that banks can lend their excess liquidity on the interbank market when they are hit by a *low* (uncorrelated) liquidity shock. The first effect lowers the ex ante value of liquidity in period 1, reducing the banks' incentives to hold liquid reserves. But the second effect goes in the opposite direction. In the rest of this section, we derive conditions under which the first effect dominates.

To analyze the incentive to invest in liquid assets at date 0, it is useful to introduce the value function $V(y, \omega)$, which captures the optimal expected utility of consumers in period 1 when there are ω early consumers and y units of liquid asset available. Formally, define

$$V(y, \omega) \equiv \max_{c_1, c_2} \{\omega u(c_1) + (1-\omega) u(c_2) \text{ s.t. } \omega c_1 \leq y \text{ and } (1-\omega) c_2 \leq R(1-y) + y - \omega c_1\}. \tag{4}$$

The following lemma summarizes some useful properties of V.

Lemma 4 *The value function $V(y, \omega)$ is continuous, differentiable and strictly concave in y and $\partial V(y, \omega)/\partial y$ is non-decreasing in ω.*

Problem (1) can be restated compactly in terms of the value function V as the problem of maximizing $1/2V(y, \omega_H) + 1/2V(y, \omega_L)$. The optimal level of liquid investment y in autarky is then characterized by the first order condition[6]

$$\frac{1}{2}\frac{\partial V(y, \omega_H)}{\partial y} + \frac{1}{2}\frac{\partial V(y, \omega_L)}{\partial y} = 0. \tag{5}$$

Similarly, we can restate problem (3) in terms of the value function V and obtain the first order condition

$$p\frac{\partial V(y, \omega_M)}{\partial y} + (1-p)\left(\frac{1}{2}\frac{\partial V(y, \omega_H)}{\partial y} + \frac{1}{2}\frac{\partial V(y, \omega_L)}{\partial y}\right) = 0. \tag{6}$$

Comparing the expressions on the left-hand sides of (5) and (6) shows that the difference between the marginal value of liquidity under integration and in autarky is captured—after some rearranging—by the expression:

$$p\frac{1}{2}\left(\frac{\partial V(y, \omega_M)}{\partial y} - \frac{\partial V(y, \omega_H)}{\partial y}\right) + p\frac{1}{2}\left(\frac{\partial V(y, \omega_M)}{\partial y} - \frac{\partial V(y, \omega_L)}{\partial y}\right).$$

The two expressions in brackets are the formal counterpart of the two forces discussed above, one making liquidity less valuable under financial integration, the other making it more valuable. Take a given amount of liquidity y. For a region hit by the high liquidity

[6] The Inada condition for $u(.)$ ensures that we always have an interior optimum.

shock ω_H, financial integration leads to a reduction of the marginal value of liquidity, captured by the difference

$$\frac{\partial V(y,\omega_M)}{\partial y} - \frac{\partial V(y,\omega_H)}{\partial y} \leq 0. \tag{7}$$

This difference is non-positive by the last property in Lemma 4 and the fact that $\omega_M < \omega_H$. Intuitively, when a region hit by the shock ω_H is integrated in the world economy and the world-average liquidity shock is $\omega_M < \omega_H$, integration reduces the marginal value of a unit of liquidity. At the same time, for a region hit by the low liquidity shock ω_L, the marginal gain from being able to share its liquidity with a region hit by the high liquidity shock is captured by the difference

$$\frac{\partial V(y,\omega_M)}{\partial y} - \frac{\partial V(y,\omega_L)}{\partial y} \geq 0. \tag{8}$$

This difference is non-negative by the same reasoning made above. Therefore, a marginal unit of liquidity is less valuable under financial integration if the difference in (7) is larger (in absolute value) than the difference in (8). We will now provide conditions for this to be true.

The next proposition shows that a sufficient condition for investment in the liquid asset to be lower under financial integration is a sufficiently low value of the rate of return R.

Proposition 5 *All else equal, if R is smaller than some cutoff $\hat{R} > 1$ then the equilibrium investment in the short asset y is lower under financial integration than in autarky.*

To capture the intuition behind this proposition, notice that when R is low enough, the equilibrium under financial integration will feature positive rollover in all states except state HH. In other words, the aggregate liquidity constraint in period 1, $\omega c_1 \leq y$, will only be binding in that state. This happens because when R is close to 1, the cost of holding liquid resources is relatively low and so it is socially optimal to have excess liquidity in all states except one. However, in states with positive rollover it is possible to show that the marginal value of liquidity is equal to

$$\frac{\partial V(y,\omega)}{\partial y} = (1-R)\, u'(y + R(1-y)),$$

which is independent of ω. Once there is excess liquidity, the optimal thing is to equalize the consumption of early and late consumers, setting it equal to $y + R(1-y)$, irrespective of the fraction of early consumers. Therefore, if there is positive rollover when $\omega = \omega_M$ and $\omega = \omega_L$, it means that $\partial V(y,\omega_M)/\partial y$ and $\partial V(y,\omega_L)/\partial y$ are equal and so the expression

(8) is zero. At the same time, the expression in (7) is strictly negative, because the liquidity constraint is binding when $\omega = \omega_H$. Since the first effect is strictly negative and the second effect is zero, the first effect obviously dominates. This implies that the marginal value of liquidity is lower under integration, leading to the conclusion that investment in the liquid asset is lower under integration.

As the discussion above shows, the condition in Proposition 5 is a relatively stringent sufficient condition, since it essentially ensures that the second effect is zero. A weaker sufficient condition is given by the following proposition.

Proposition 6 *Let y^I denote the equilibrium investment in the short asset under financial integration. If the marginal value of liquidity $\partial V\left(y^I, \omega\right)/\partial y$ is convex in the liquidity shock ω on $[\omega_L, \omega_H]$ then the equilibrium investment in the short asset is lower under financial integration than in autarky.*

Unfortunately, it is not easy to derive general conditions on fundamentals which ensure the convexity of $\partial V\left(y^I, \omega\right)/\partial y$. Therefore, we now turn to numerical examples to show that our result holds for a realistic set of parameters.

Let us assume a CRRA utility function with relative risk aversion equal to γ. Let y^A and y^I denote the equilibrium investment in the short asset, respectively, in autarky and under integration. Figure 5 shows for which values of R and γ we obtain $y^I < y^A$. We fix the difference between the liquidity shocks to be $\omega_H - \omega_L = 0.2$ and we explore what happens for different values of the average liquidity shock ω_M. The area to the left of the lines labeled $\omega_M = 0.6, 0.7, 0.8$ is where $y^I < y^A$ holds in the respective cases.[7] The figure shows that if the coefficient of risk aversion is greater than 1 the condition is always satisfied for reasonably low values of the return of the long term asset—in particular for $R \leq 1.5$. For very high values of R—for $R > 1.5$—then the condition $y^I < y^A$ is satisfied only if agents are sufficiently risk averse.

Figure 6 is analogous to Figure 5, except that we use a higher variance for the liquidity shocks, setting $\omega_H - \omega_L = 0.4$. An increase in the variance of ω tends to enlarge the region where $y^I < y^A$ holds.

We conclude with comparative statics with respect to p. We use the notation $y^I(p)$ to denote the dependence of optimal investment under integration on the parameter p.

Proposition 7 *If $y^I(p_0) < y^A$ for some $p_0 \in (0, 1]$ then the equilibrium investment in the short asset under integration $y^I(p)$ is decreasing in p and is smaller than the autarky level y^A for all $p \in (0, 1]$.*

[7] Notice that when either $\gamma < 1$ or $\omega_M \leq 0.5$ the condition $y^I < y^A$ holds for all R.

As p increases, the probability of a correlated shock decreases and the scope for coinsurance increases. Therefore, the marginal value of liquidity falls and banks hold smaller liquidity buffers. Notice also that as $p \to 0$ the possibility of coinsurance disappears and the integrated economy converges to the autarky case. Combining propositions 5 and 7, shows that if R is below some cutoff $\hat{R}$ then investment in the liquid asset under integration is everywhere decreasing in p.

7 The depth of systemic crises

In this section we analyze the implications of financial integration for the depth of systemic crises. Here we simply identify a systemic crisis with a worldwide liquidity shock, that is with a realization of state HH. We first consider the effects of this shock on prices, looking at interest rates on deposit contracts and on the interbank market. Then we look at the effect of the shock on quantities, focusing on the response of period 1 consumption.

7.1 The price of liquidity

In the context of our model, we consider three different but related notions of the price of liquidity in period 1. First, we look at the terms of the deposit contracts. A deposit contract offers the option to withdraw c_1^S in period 1 or c_2^S in period 2. So the implicit gross interest rate on deposits is

$$r^S = \frac{c_2^S}{c_1^S}. \tag{9}$$

Second, we look at the terms of the interbank credit lines. Proposition 3 shows that in the integrated economy the rate of interest on these credit lines is equal to

$$\frac{m_2}{m_1} = \frac{c_2^M}{c_1^M}$$

in states HL and LH. This interest rate is the same as the interest rate on deposits. The only problem is that interbank credit lines are absent in autarky and even in the integrated economy they are only active when the regions are hit by uncorrelated shocks. However, it is easy to add some heterogeneity in the model, introducing different subregions within each region A and B, and assuming that the banks in each subregion are identical ex ante and are hit by asymmetric shocks ex post. In this extension, the aggregate behavior of the two regions is identical to the baseline model, but within-region interbank markets are always active and the interbank interest rate is always equal to the interest rate on deposit contracts. Therefore, in general, both deposit interest rates and interbank interest rates are equal to r, defined in (9). Notice that the fact that interest rates on deposits and interbank credit lines are equalized is a consequence of our assumption of fully state contingent deposit contracts. As argued in Section 3, this is not a realistic feature of the model, but it helps to analyze our mechanism in an environment with minimal distortions.

An alternative approach is to look at the shadow price of liquidity in the bank's problem. That is, the price that a bank would be willing to pay, in terms of period 2 consumption, for an extra unit of liquid resources in period 1. This shadow price is given by

$$\tilde{r}^S = \frac{u'\left(c_1^S\right)}{u'\left(c_2^S\right)}.$$

With CRRA utility this shadow price is a simple monotone transformation of the interest rate r on deposits and credit lines. In the special case of log utility, the two interest rates r and $\tilde{r}$ are identical.[8]

Notice that the equilibrium characterization in Proposition 1 (for the autarky case) and in Proposition 3 (for the case of integration) imply that both r and $\tilde{r}$ are always greater than or equal to 1. Furthermore, both are equal to 1 if there is positive rollover and are greater than 1 if the liquidity constraint $\omega c_1 \leq y$ is binding.

Notice also that in autarky the interest rates can be different in the two regions, so we now focus on the equilibrium behavior of the interest rate in region A. By symmetry, identical results hold for region B, inverting the role of states HL and LH.

The following proposition holds both for the interest rate r and for the shadow interest rate $\tilde{r}$.

Proposition 8 *All else equal, if R is below some cutoff $\hat{R}$ the equilibrium interest rate r ($\tilde{r}$) is higher under integration than in autarky in state HH, it is lower under integration in state HL, and is equal in states LH and LL.*

In state HL the banks in region A reap the benefits of integration as they are allowed to borrow at a lower interbank rate than in autarky. This is the stabilizing effect of financial integration. However, when the correlated shock HH hits, the price of liquidity increases more steeply than in autarky due to a worldwide shortage of liquidity. This shortage is simply a result of optimal ex ante investment decisions in the integrated financial system.

Let us analyze the effects of this liquidity shortages on the equilibrium distribution of interest rates. The comparative static result in Proposition 7 implies that for larger values

[8]Notice that here we are focusing on the ex post price of liquidity, i.e., the price at $t = 1$. Ex ante, i.e. at $t = 0$, the opportunity cost of investing in the short asset coincides with the missed return on the long asset and so is equal to R. Therefore, the ex ante price of liquidity is completely determined by the technology and is independent of the degree of financial integration.

of p the spike in the price of liquidity in state HH will be worse, as banks will hold less liquidity ex ante. At the same time, when $p \to 1$, the probability of a systemic event goes to zero, so the spike happens with smaller probability. The combination of these effects suggests that the volatility of the interest rate may increase or decrease with p, while the distribution will tend to be more positively skewed when p is larger. We now explore these effects formally.

First, let us look at a numerical example. The utility function is CRRA with relative risk aversion equal to $\gamma = 1$, the rate of return on the illiquid asset is $R = 1.15$ and the liquidity shocks are $\omega_L = 0.4$ and $\omega_H = 0.6$. The first panel of Figure 7 plots the interest rate in states HH, M and LL for different values of the parameter p. The second panel plots the standard deviation of r and the third panel plots the skewness of r, measured by its third standardized moment:[9]

$$sk\left(r\right) \equiv E\left[\left(r - E\left[r\right]\right)^3\right] / \left(Var\left[r\right]\right)^{3/2}.$$

For comparison, it is useful to recall that the case $p = 0$ coincides with the autarky case. The example shows that it is possible for financial integration to make interest rates both more volatile and more skewed. Clearly, if $p = 1$ interest rate volatility disappears with integration as banks can perfectly coinsure their local shocks. However, when there is a sufficient amount of residual aggregate uncertainty, i.e., when p is sufficiently smaller than 1, financial integration increases interest rate volatility. Moreover, for all levels of $p < 1$ financial integration makes the interest rate distribution more skewed.

We now provide some sufficient conditions for having an increase in volatility and in skewness under integration. First, we show that for low values of p the increase in banks' illiquidity identified in the previous section dominates the stabilizing effects of integration and volatility is higher under integration than in autarky.

Proposition 9 *Suppose rollover is optimal in all states except HH for all p in some interval $(0, p_0]$, then there is a $p < p_0$ such that financial integration increases the equilibrium volatility of the interest rate:*

$$Var(r^I) > Var(r^A).$$

When rollover is optimal in all states except HH the interest rate has a binary distribution taking the value $r^{HH}\left(p\right) > 1$ with probability $\left(1 - p\right)/2$ and the value 1 with

[9] Notice that alternative skewness indexes (e.g., Pearson's skewness coefficients) will have the same sign as $sk\left(r\right)$ (although, clearly, different magnitudes).

probability $1-(1-p)/2$. Therefore, the variance is

$$\begin{aligned} Var\left(r^I\right) &= \left(1-\frac{1-p}{2}\right)\frac{1-p}{2}\left(r^{HH}(p)-1\right)^2 = \\ &= \frac{1}{4}\left(1-p^2\right)\left(r^{HH}(p)-1\right)^2. \end{aligned} \tag{10}$$

Moreover, as $p \to 0$ the interest rate distribution converges to its autarky value

$$Var\left(r^A\right) = \frac{1}{4}\left(r^{HH}(0)-1\right)^2.$$

Therefore, to prove Proposition 9 it is enough to prove that the expression (10) is strictly increasing in p at $p=0$. Since the changes in $(1-p^2)$ are of second order at $p=0$, this is equivalent to proving that the crisis interest rate $r^{HH}(p)$ is strictly increasing in p. This can be proved by an argument similar to the one behind Proposition 7: as p increases banks' liquidity holdings are reduced and so the crisis interest rate is higher. The complete formal proof is in the appendix.

Notice that the hypothesis of the proposition—positive rollover in all states except HH—holds when R is sufficiently close to 1 (see the proof of Proposition 5 for a formal argument). The example in Figure 7 satisfies this hypothesis, and, indeed, displays increasing volatility for low levels of p. Notice also that the relation between p and interest rate volatility cannot be everywhere increasing, because the variance is positive as $p \to 0$ and goes to 0 as $p \to 1$. Therefore, under the assumptions of Proposition 9 there is a non-monotone relation between p and the variance of r: increasing for low values of p and eventually decreasing.

Let us now look at the model implication for skewness. In autarky, the interest rate follows a symmetric binary distribution, so in this case the skewness $sk\left(r^A\right)$ is zero. Under integration, the interest rate takes the three values r^{HH}, r^M and, r^{LL} with probabilities, respectively, $(1-p)/2$, p, and $(1-p)/2$. The following lemma allows us to characterize the sign of the skewness of this distribution.

Lemma 10 *The skewness of the interest rate distribution is positive if and only if*

$$r^{HH} - r^M > r^M - r^{LL}.$$

Notice that if roll over is optimal in states LL and M, then $r^M - r^{LL} = 0$ and $r^{HH} - r^M > 0$ so the lemma immediately implies that the distribution is positively skewed. However, the result is more general, as shown in the following proposition.

Proposition 11 *The interest rate distribution is positively skewed under financial integration.*

While skewness always increases with integration, the magnitude of the response clearly depends on the strength of the illiquidity effect studied in Section 6, which tends to magnify the spike in interest rates in a crisis.

7.2 Consumption and welfare

To assess the real consequences of integration it is useful to look at the effects of a systemic crisis on consumption in period 1. We will see that the implications on the real side are similar to those obtained in terms of prices: financial integration can make the distribution of consumption more volatile and more negatively skewed.

Let us begin with a numerical example. Figure 8 characterizes the distribution of consumption in period 1 in the same example used for Figure 8. The volatility of consumption is non-monotone in p and is higher than in autarky for intermediate values of p. The distribution of consumption is symmetric in autarky and negatively skewed in integration, with more negative skewness for larger values of p.

From an analytical point of view, it is easy to obtain the analog of Proposition 9 for consumption and show that when rollover is optimal in all states except HH consumption variance is always larger under integration if p is not too large. This helps us understand the increasing portion of the relation in panel (b) of Figure 8.

In terms of skewness, we know, exactly as for the interest rate, that consumption has a symmetric binary distribution in autarky. To show that the consumption distribution becomes negatively skewed under integration we need some restrictions on parameters, as shown in the following proposition.

Proposition 12 *Consumption is negatively skewed under integration iff*

$$y^I > \hat{y} = \frac{\omega_M \omega_H R}{\omega_M \omega_H R + 2\omega_H - \omega_M(1+\omega_H)}. \tag{11}$$

To illustrate numerically when condition (11) is satisfied we assume again a CRRA utility with relative risk aversion γ. In many examples, it turns out that increasing γ and holding fixed the other parameters the condition is satisfied. Table 2 shows the thresholds for γ in some numerical examples. It is evident that for $R \leq 3$ a value of $\gamma > 3$ is sufficient to obtain negative skewness.

It is important to notice that the increase in consumption volatility and skewness that may derive from financial integration are fully efficient in our model. Moreover, a higher level of p, by increasing the possibility for coinsurance is always beneficial in terms of ex ante welfare, as shown in the following proposition.

Proposition 13 *The ex ante expected utility of consumers is higher under integration for all $p \in (0, 1]$ and is strictly increasing in p.*

To have an intuition for this result, remember that in autarky consumers cannot be insured against the liquidity shock and, therefore, first period consumption is higher in state L than in state H, while second period consumption is higher in state H than in state L. Hence, it would be welfare improving to smooth both first and second period consumption levels across states. In an economy with no residual aggregate uncertainty, the interbank deposit market can provide consumers with full insurance. With some residual aggregate uncertainty, consumption smoothing (across states) is only available when the integrated regions are hit by asymmetric liquidity shocks, while symmetric shocks cannot be diversified away. As a consequence, conditional on being hit by asymmetric liquidity shocks, consumers welfare can be improved, but ex-ante this happens with probability p, so that the larger p the higher the consumers welfare.

Summing up the findings this section, we have shown that financial integration can increase both the volatility and skewness of consumption and, at the same time, increase welfare ex ante.

8 Conclusions

In this paper we focused on a simple frictionless model of inter-regional banking to explore a basic positive question: how does financial integration affect the optimal liquid holdings of banks ex ante? We show that financial integration, by reducing aggregate uncertainty, increases welfare but induces banks to reduce their liquidity holdings, increasing the severity of extreme events. That is, financial integration makes extreme events less likely but more extreme.

Introducing frictions in our environment would open the door to welfare improving regulation. In particular, we could get inefficient underprovision of liquidity as in Allen and Gale [5], Lorenzoni [19], Fahri, Golosov and Tsyvinski [12]. An open question for future research is how financial integration affects the need for regulation, and how this depends on the amount of aggregate uncertainty left in the system.

Appendix

Throughout the appendix, we will make use of the value function (4) defined in the text and we will use $C_1(y, \omega)$ and $C_2(y, \omega)$ to denote the associated optimal policies. The following lemma is an extended version of Lemma 4.

Lemma 14 *The value function $V(y,\omega)$ is strictly concave, continuous and differentiable in y, with*

$$\partial V(y,\omega)/\partial y = u'(C_1(y,\omega)) - Ru'(C_2(y,\omega)). \tag{12}$$

The policies $C_1(y,\omega)$ and $C_2(y,\omega)$ are given by

$$\begin{aligned} C_1(y,\omega) &= \min\left\{\frac{y}{\omega}, y + R(1-y)\right\}, \\ C_2(y,\omega) &= \max\left\{R\frac{1-y}{1-\omega}, y + R(1-y)\right\}. \end{aligned}$$

The partial derivative $\partial V(y,\omega)/\partial y$ is non-increasing in ω.

Proof. Continuity and weak concavity are easily established. Differentiability follows using concavity and a standard perturbation argument to find a differentiable function which bounds $V(y,\omega)$ from below. From the envelope theorem

$$\partial V(y,\omega)/\partial y = \lambda + (1-R)\mu,$$

where λ and μ are the Lagrange multipliers on the two constraints. The problem first order conditions are

$$\begin{aligned} u'(C_1(y,\omega)) &= \lambda + \mu, \\ u'(C_2(y,\omega)) &= \mu, \end{aligned}$$

which substituted in the previous expression give (12). Considering separately the cases $\lambda > 0$ (no rollover) and $\lambda = 0$ (rollover), it is then possible to derive the optimal policies. Strict concavity can be proven directly, substituting the expressions for $C_1(y,\omega)$ and $C_2(y,\omega)$ in (12) and showing that $\partial V/\partial y$ is strictly decreasing in y. This last step uses the strict concavity of $u(.)$ and $R > 1$. Substituting $C_1(y,\omega)$ and $C_2(y,\omega)$ in (12) also shows that $\partial V(y,\omega)/\partial y$ is non-decreasing in ω. ■

Lemma 15 *$C_1(y,\omega) \leq C_2(y,\omega)$ for all $y \geq 0$ and $\omega \in (0,1)$. In particular we distinguish two cases:*

(i) *If $y > \omega R/(1-\omega+\omega R)$ there is rollover and the following conditions hold*

$$\frac{y}{\omega} > C_1(y,\omega) = C_2(y,\omega) = y + R(1-y) > R\frac{1-y}{1-\omega},$$

(ii) *If $y \leq \omega R / (1-\omega+\omega R)$ there is no rollover and the following conditions hold*

$$C_1(y,\omega) = \frac{y}{\omega} \leq y + R(1-y) \leq R\frac{1-y}{1-\omega} = C_2(y,\omega),$$

where the inequalities are strict if $y < \omega R / (1-\omega+\omega R)$ and hold as equalities if $y = \omega R / (1-\omega+\omega R)$.

Proof. The proof follows from inspection of $C_1(y,\omega)$ *and* $C_2(y,\omega)$ in Lemma 14. ■

An immediate consequence of Lemma 15 is the following corollary.

Corollary 1 *If rollover is optimal in problem (4) for some pair (y,ω) then it is also optimal for any pair (y,ω') with $\omega' < \omega$.*

Proof of Proposition 1. Consider problem (1). Given the definition of the value function V in (4), it is easy to see that the liquidity level in autarky y^A solves

$$\max_y \frac{1}{2}V(y,\omega_L) + \frac{1}{2}V(y,\omega_H), \tag{13}$$

and optimal consumption in state s and time t is given by $C_t(y^A,\omega_s)$. The first order condition of this problem and Lemma 14 imply that y^A is characterized by

$$\frac{1}{2}\left(u'\left(C_1\left(y^A,\omega_L\right)\right) - Ru'\left(C_2\left(y^A,\omega_L\right)\right)\right) + \frac{1}{2}\left(u'\left(C_1\left(y^A,\omega_H\right)\right) - Ru'\left(C_2\left(y^A,\omega_H\right)\right)\right) = 0. \tag{14}$$

The Inada condition for $u(.)$ ensures that we have an interior solution $y^A \in (0,1)$. Note that if positive rollover is optimal in state H it is also optimal in state L by Corollary 1. But then Lemma 15 implies that the left-hand side of (14) is equal to

$$(1-R)\,u'\left(y^A + R\left(1-y^A\right)\right) < 0,$$

leading to a contradiction. Therefore no positive rollover occurs in state H. We can then distinguish two cases, which correspond to the two cases in Lemma 15.

(i) Solution with roll over in state L. In this case

$$c_1^L = c_2^L = y^A + R(1-y^A).$$

This condition together with (14) (and $R > 1$) implies

$$u'\left(c_1^H\right) - Ru'\left(c_2^H\right) > 0,$$

which in turns implies $c_1^H < c_2^H$ and, by Lemma 15, we have$c_1^H < y^A + R(1-y^A) < c_2^H$.

(ii) Solution without roll over in state L. Then $c_1^L = y^A/\omega_L > y^A/\omega_H = c_1^H$ and $c_2^H = R(1-y^A)/(1-\omega_H) > R(1-y^A)/(1-\omega_L) = c_2^L$. So we have

$$c_1^H < c_1^L \le c_2^L < c_2^H.$$

■

Proof of Proposition 2. Consider the program under integration (2). Let $\mu_t^{ss'}$ be the multiplier associated with the feasibility constraint at time t and state ss'. Among the first order conditions of the problem we have the following:

$$\begin{aligned}
c_1^{HL}: &\quad \tfrac{p}{2}u'(c_1^{HL}) = \mu_1^{HL} + \mu_2^{HL}, \\
c_1^{LH}: &\quad \tfrac{p}{2}u'(c_1^{LH}) = \mu_1^{LH} + \mu_2^{LH}, \\
c_2^{HL}: &\quad \tfrac{p}{2}u'(c_2^{HL}) = \mu_2^{HL}, \\
c_2^{LH}: &\quad \tfrac{p}{2}u'(c_2^{LH}) = \mu_2^{LH}, \\
m_1: &\quad \mu_1^{HL} = \mu_1^{LH}, \\
m_2: &\quad \mu_2^{HL} = \mu_2^{LH}.
\end{aligned}$$

An immediate implication is that $c_t^{HL} = c_t^{LH}$ for $t = 1, 2$. Summing period 1 constraints in HL and LH (using $c_1^{HL} = c_1^{LH}$) gives

$$(\omega_H + \omega_L)\, c_1^{HL} \le 2y. \tag{15}$$

If this condition holds as an equality at the solution, then also the individual constraints must hold as equalities. In this case, m_1 and m_2 are uniquely determined and

$$\omega_H c_1^{HL} = y + m_1 = (\omega_H + \omega_L)\, c_1^{HL}/2 + m_1,$$

which yields

$$m_1 = (\omega_H - \omega_M)\, c_1^{HL}.$$

The expression for m_2 is found analogously. If instead (15) holds as an inequality, the optimal values of m_1 and m_2 are not unique, but the values $(\omega_H - \omega_M)\, c_1^{HL}$ and $(\omega_H - \omega_M)\, c_2^{HL}$ are still one possible solution. ■

Proof of Proposition 3. Consider the reduced program under integration (3). Optimal liquidity under integration y^I solves

$$\max_y pV(y, \omega_M) + (1-p)\left[\frac{1}{2}V(y, \omega_H) + \frac{1}{2}V(y, \omega_L)\right]. \tag{16}$$

Using Lemma (14) the first order condition for this problem can be written as

$$\begin{aligned}
&p\left[u'(C_1(y, \omega_M)) - Ru'(C_2(y, \omega_M))\right] + \\
&\frac{1-p}{2}\left[u'(C_1(y, \omega_H)) - Ru'(C_2(y, \omega_H)) + u'(C_1(y, \omega_L)) - Ru'(C_2(y, \omega_L))\right] = 0
\end{aligned} \tag{17}$$

The Inada condition for $u(.)$ ensures that we have an interior solution $y^I \in (0,1)$. Optimal consumption in states HH and LL is given, respectively, by $C_t(y^I, \omega_H)$ and $C_t(y^I, \omega_L)$. Optimal consumption in states HL and LH is given by $C_t(y^I, \omega_M)$. Note that, by an argument similar to that used in Proposition 1, rollover cannot be optimal in state HH. Note also that, if rollover is optimal in state HL and LH then it is also optimal in LL. Hence, we distinguish three cases:

(i) Rollover in states LL, HL and LH. Note that, since

$$c_1^M = c_1^{LL} = c_2^{LL} = c_2^M = y^I + R\left(1 - y^I\right),$$

$c_1^{HH} = c_2^{HH}$ is incompatible with (17), so it must be $c_1^{HH} < c_2^{HH}$ and, given Lemma 15, we have

$$c_1^{HH} < c_1^M = c_1^{LL} = c_2^{LL} = c_2^M < c_2^{HH}.$$

(ii) Rollover only in state LL. From Lemma 15 we have

$$c_1^M = \frac{y^I}{\omega_M} \leq y^I + R\left(1 - y^I\right) \leq R\frac{1 - y^I}{1 - \omega_M} = c_2^M$$

and, since $\omega_H > \omega_M$, it immediately follows that

$$c_1^{HH} < c_1^M \leq c_1^{LL} = c_2^{LL} \leq c_2^M < c_2^{HH}.$$

(iii) No rollover. Applying Lemma 15 and $\omega_H > \omega_M > \omega_L$ yields

$$c_1^{HH} < c_1^M < c_1^{LL} \leq c_2^{LL} < c_2^M < c_2^{HH}.$$

■

Proof of Lemma 4. See Lemma 14 above. ■

Proof of Proposition 5. The proof is in two steps.

Step 1. First, we show that there is a cutoff $\hat{R} > 1$ such that if $R < \hat{R}$, then rollover occurs in all states except HH. Consider the optimality condition (17) and let y^I denote the optimal level of investment in the liquid asset. Lemma 15 gives us explicit expressions for the functions $C_t(y, \omega)$, for $t = 1, 2$ and $\omega \in \{\omega_H, \omega_L, \omega_M\}$. Substituting in (17), some algebra then shows that as $R \to 1$ (from above) we have $y^I \to \omega_H$ (from below) and $C_t\left(y^I, \omega\right) \to 1$ for all t and all ω. That is, as the return on the long asset approaches 1, the optimal solution is to hold the minimal amount of liquidity that ensures that all consumers consume 1 in all states of the world. Clearly, at the limit, the liquidity constraints $\omega c_1 < y$

are slack for $\omega \in \{\omega_L, \omega_M\}$ since $\omega_M \cdot 1 < \omega_H = y^I$ and $\omega_L \cdot 1 < \omega_H = y^I$. A continuity argument then shows that the same constraints are slack (and there is positive rollover) for $\omega \in \{\omega_L, \omega_M\}$, for all R below some cutoff $\hat{R}$.

Step 2. Consider an equilibrium under integration where rollover is positive in all states except HH. In this case the optimal liquidity y^I satisfies the first order condition

$$\frac{1-p}{2}\left(u'\left(C_1\left(y^I,\omega_H\right)\right)-Ru'\left(C_2\left(y^I,\omega_H\right)\right)\right)+\left(\frac{1}{2}+\frac{p}{2}\right)(1-R)\,u'\left(y^I+R\left(1-y^I\right)\right)=0.$$

Now $R>1$ implies $u'\left(C_1\left(y^I,\omega_H\right)\right) > Ru'\left(C_2\left(y^I,\omega_H\right)\right)$. Therefore, we obtain

$$\frac{1}{2}\left(u'\left(C_1\left(y^I,\omega_H\right)\right)-Ru'\left(C_2\left(y^I,\omega_H\right)\right)\right)+\frac{1}{2}(1-R)\,u'\left(y^I+R\left(1-y^I\right)\right)>0.$$

Since $C_1\left(y^I,\omega_L\right) = C_2\left(y^I,\omega_L\right) = y^I + R\left(1-y^I\right)$, this condition can be rewritten, using Lemma 14, as

$$\frac{1}{2}\frac{\partial V\left(y^I,\omega_H\right)}{\partial y}+\frac{1}{2}\frac{\partial V\left(y^I,\omega_L\right)}{\partial y}>0. \tag{18}$$

Now consider the autarky problem written in the form (13). Since the problem is concave condition (18) implies that the optimal solution y^A must be to the right of y^I. ■

Proof of Proposition 6. The result follows from a comparison of the first order conditions (5) and (6). Since y^A solves (5), the convexity assumption implies that

$$\frac{\partial V\left(y^A,\omega_M\right)}{\partial y}<\frac{1}{2}\frac{\partial V\left(y^A,\omega_H\right)}{\partial y}+\frac{1}{2}\frac{\partial V\left(y^A,\omega_L\right)}{\partial y}=0.$$

Substituting in the expression on the left-hand side of (6) shows that

$$p\frac{\partial V\left(y^A,\omega_M\right)}{\partial y}+(1-p)\left(\frac{1}{2}\frac{\partial V\left(y^A,\omega_H\right)}{\partial y}+\frac{1}{2}\frac{\partial V\left(y^A,\omega_L\right)}{\partial y}\right)<0.$$

Since the problem under integration is concave, this implies that the optimal solution y^I must be to the left of y^A, i.e., $y^A > y^I$. ■

Proof of Proposition 7. Suppose $y_0^I = y^I(p_0)$ is optimal under integration for $p = p_0$. If $y_0^I < y^A$ the optimality condition in autarky and the strict concavity of V imply that

$$\frac{1}{2}\frac{\partial V\left(y_0^I,\omega_H\right)}{\partial y}+\frac{1}{2}\frac{\partial V\left(y_0^I,\omega_L\right)}{\partial y}>0.$$

This inequality, combined with optimality under integration (6), implies that

$$\frac{\partial V\left(y_0^I,\omega_M\right)}{\partial y}<0.$$

Now consider an integrated economy with any $p > p_0$. The previous inequalities and optimality under integration (6) imply that

$$\begin{aligned}
p\frac{\partial V\left(y_0^I,\omega_M\right)}{\partial y}+(1-p)\left(\frac{1}{2}\frac{\partial V\left(y_0^I,\omega_H\right)}{\partial y}+\frac{1}{2}\frac{\partial V\left(y_0^I,\omega_L\right)}{\partial y}\right) &< \\
p_0\frac{\partial V\left(y_0^I,\omega_M\right)}{\partial y}+(1-p_0)\left(\frac{1}{2}\frac{\partial V\left(y_0^I,\omega_H\right)}{\partial y}+\frac{1}{2}\frac{\partial V\left(y_0^I,\omega_L\right)}{\partial y}\right) &= 0.
\end{aligned}$$

The concavity of V then implies that $y^I(p) < y^I(p_0)$ which immediately implies that $y^I(p) < y^A$. It remains to show that $y^I(p) < y^A$ and that $y^I(p)$ is decreasing in p for $p < p_0$. Now suppose, by contradiction that for some $0 < p' < p_0$ we have $y^I(p') \geq y^A$. Then an argument symmetric to the one above shows that $y^I(p)$ must be non-decreasing in p for all $p > p'$. This implies $y^I(p_0) \geq y^I(p') > y^A$, a contradiction. It follows that $y^I(p) < y^A$ for all $p \in (0,1]$. The argument made above starting at p_0 then applies to all points in $(0,1]$, so $y^I(p)$ is everywhere decreasing in p. ■

Proof of Proposition 8. As in the proof of Proposition 5, the condition $R < \hat{R}$ implies that, under integration, there is positive rollover in all states except HH. This implies that the interest rate is $r^{S,I} = 1$ in all states except HH. In autarky there is positive rollover in both states HL and HH, this immediately implies that $r^{HL,I} = 1 < r^{HL,A}$. To complete the proof it remains to show that $r^{HH,I} > r^{HH,A}$, but this follows immediately from the fact that $y^I < y^A$ (from Proposition 5) and that no rollover in state HH implies

$$r^{HH,I} = \frac{R\left(1-y^I\right)/\left(1-\omega_H\right)}{y^I/\omega_H} > \frac{R\left(1-y^A\right)/\left(1-\omega_H\right)}{y^A/\omega_H} = r^{HH,A}.$$

The argument for $\tilde{r}$ is analogous. ■

Proof of Proposition 9. To complete the argument in the text we need to prove that $(1-p^2)\left(r^{HH}(p)-1\right)^2$ is increasing in p at $p = 0$. Since

$$\frac{d}{dp}\left(1-p^2\right)\left(r^{HH}(p)-1\right)^2 = 2p\left(r^{HH}(p)-1\right)^2 + 2\left(1-p^2\right)\left(r^{HH}(p)-1\right)\frac{dr^{HH}(p)}{dp},$$

and the first term on the right-hand side goes to zero as $p \to 0$, we need to prove that

$$\lim_{p\downarrow 0} r^{HH}(p) > 1 \tag{19}$$

and

$$\lim_{p\downarrow 0} \frac{dr^{HH}(p)}{dp} > 0. \tag{20}$$

Denoting by $y^I(p)$ optimal investment under integration we have

$$r^{HH}(p) = \frac{C_2\left(y^I(p), \omega_H\right)}{C_1\left(y^I(p), \omega_H\right)} = \frac{R\left(1 - y^I(p)\right) / \left(1 - \omega_H\right)}{y^I(p)/\omega_H}. \tag{21}$$

To prove (19) notice that $\lim_{p \downarrow 0} y^I(p) = y^A$ and that optimality in autarky implies that the liquidity constraint is binding in state H. Given (21), to prove (20) is equivalent to prove that $\lim_{p \downarrow 0} dy^I(p)/dp < 0$. This is true if rollover is optimal in states LL, LH and HL. In this case, differentiating with respect to $p \in (0,1)$ the first order condition (17) yields

$$\frac{dy^I(p)}{dp} = \frac{u'\left(\frac{y^I(p)}{\omega_H}\right) - Ru'\left(R\frac{1-y^I(p)}{1-\omega_H}\right) - (1-R)\,u'\left(R(1-y^I(p)) + y^I(p)\right)}{\frac{1-p}{\omega_H}u''\left(\frac{y^I(p)}{\omega_H}\right) + \frac{1-p}{1-\omega_H}R^2u''\left(R\frac{1-y^I(p)}{1-\omega_H}\right) + (1-R)^2(1+p)u''\left(R(1-y^I(p)) + y^I(p)\right)}.$$

The denominator of the expression on the right-hand side is strictly negative for $p \downarrow 0$. Therefore, it remains to prove that the numerator is positive in the limit, that is,

$$u'\left(\frac{y^A}{\omega_H}\right) - Ru'\left(R\frac{1-y^A}{1-\omega_H}\right) - (1-R)u'\left(R(1-y^A) + y^A\right) > 0. \tag{22}$$

This follows from the autarky first order condition (14) which, with rollover in state L, is equal to

$$u'\left(\frac{y^A}{\omega_H}\right) - Ru'\left(\frac{1-y^A}{1-\omega_H}R\right) + (1-R)u'\left(R(1-y^A + y^A\right) = 0,$$

and implies that the right-hand side of (22) is equal to $2(R-1)u'\left(R(1-y^A + y^A\right) > 0$. ■

Proof of Lemma 10. The third moment around the mean is

$$\frac{p(1-p)(r^{LL} + r^{HH} - 2r^M)}{8}\left[(2p-1)(r^{LL} + r^{HH} - 2r^M)^2 + 3(r^{HH} - r^{LL})^2\right].$$

We want to show that this quantity has the same sign of $r^{LL} + r^{HH} - 2r^M$. To this end, we need to show that the following expression is positive

$$(2p-1)\left(r^{LL} + r^{HH} - 2r^M\right)^2 + 3\left(r^{HH} - r^{LL}\right)^2.$$

This can be proved observing that this expression is increasing in p and is positive at $p = 0$. To prove the last claim notice that $r^{LL} \leq r^M$ implies $r^{HH} - 2r^{LL} \geq r^{HH} - 2r^M$ which implies $r^{HH} - r^{LL} \geq r^{HH} + r^{LL} - 2r^M$ which, together with $r^{LL} < r^{HH}$, implies

$$3\left(r^{HH} - r^{LL}\right)^2 - \left(r^{LL} + r^{HH} - 2r^M\right)^2 > 0.$$

■

Proof of Proposition 11. From Proposition 3 three cases are possible. In the first case, rollover occurs in states LL, LH and HL and $r^{HH} + r^{LL} - 2r^{M} = r^{HH} - 1 > 0$. In the second case, rollover only occurs in state LL and we have

$$\begin{aligned} r^{HH} &= \frac{R\left(1-y^{I}\right)}{y^{I}} \frac{\omega_H}{1-\omega_H}, \\ r^{M} &= \frac{R\left(1-y^{I}\right)}{y^{I}} \frac{\omega_M}{1-\omega_M}, \\ r^{LL} &= 1 \geq \frac{R\left(1-y^{I}\right)}{y^{I}} \frac{\omega_L}{1-\omega_L}. \end{aligned}$$

Then

$$r^{HH} + r^{LL} - 2r^{M} \geq \frac{R\left(1-y^{I}\right)}{y^{I}} \left(\frac{\omega_H}{1-\omega_H} + \frac{\omega_L}{1-\omega_L} - 2\frac{\omega_M}{1-\omega_M} \right) > 0,$$

where the inequality follows from the convexity of the function $\omega/\left(1-\omega\right)$ and the fact that $\omega_M = (1/2)\,\omega_H + (1/2)\,\omega_L$. A similar argument applies in the third case, in which rollover never occurs. ■

Proof of Proposition 12. Notice first that Lemma 10 applies to any variable taking three values $x^L < x^M < x^M$ with probabilities $(1-p)/2, p, (1-p)/2$. So the distribution of consumption is negatively skewed iff

$$c^{HH} + c^{LL} - 2c^{M} < 0. \tag{23}$$

From Lemma 15, we have positive rollover in states LL, HL and LH if

$$y^{I} > \frac{\omega_M R}{1 + R(1-\omega_M)}.$$

Since in this case $c^{HH} < c^{LL} = c^{M}$ and condition (23) is immediately satisfied. If instead

$$\frac{\omega_L R}{1 + R(1-\omega_L)} < y^{I} \leq \frac{\omega_M R}{1 + R(1-\omega_M)}$$

positive rollover is optimal in state LL but not in states HL and LH and HH, so

$$c_1^{LL} = (1-y^{I})R + y^{I}, \quad c_1^{M} = y^{I}/\omega_M, \quad c_1^{HH} = y^{I}/\omega_H.$$

Substituting in (23) yields

$$(1-y^{I})R + y^{I} + y^{I}/\omega_H - 2y^{I}/\omega_M < 0,$$

and rearranging gives $y^{I} > \hat{y}$ (with $\hat{y}$ defined in the statement of the proposition). Finally, if

$$y^{I} < \frac{\omega_L R}{1 + R(1-\omega_L)}$$

then rollover is never optimal and we have

$$c_1^{LL} = y^I/\omega_L, \quad c_1^M = y^I/\omega_M, \quad c_1^{HH} = y^I/\omega_H.$$

In this case consumption is always positively skewed, because the convexity of the function $1/\omega$ implies

$$1/\omega_L + 1/\omega_H > 2/\omega_M.$$

Combining the three cases discussed above and noticing that

$$\hat{y} \in \left(\frac{\omega_L R}{1 + R(1-\omega_L)}, \frac{\omega_M R}{1 + R(1-\omega_M)} \right),$$

shows that $y^I > \hat{y}$ is a necessary and sufficient condition for negative skewness. ■

Proof of Proposition 13. Take any $y \in (0,1)$ and let $\left(c_1^H, c_2^H\right)$ and $\left(c_1^L, c_2^L\right)$ be optimal consumption allocations for the problem in (4) with, respectively, $\omega = \omega_H$ and $\omega = \omega_L$. Inspecting the constraints shows that the following are feasible consumption allocations for the same problem with $\omega = \omega_M$:

$$\begin{aligned}
\hat{c}_1 &= \frac{\omega_H}{\omega_H + \omega_L} c_1^H + \frac{\omega_L}{\omega_H + \omega_L} c_1^L, \\
\hat{c}_2 &= \frac{1-\omega_H}{2 - \omega_H - \omega_L} c_2^H + \frac{1-\omega_L}{2 - \omega_H - \omega_L} c_2^L.
\end{aligned}$$

The strict concavity of $u(.)$ implies that

$$\begin{aligned}
&\omega_M u(\hat{c}_1) + (1-\omega_M) u(\hat{c}_1) \geq \\
&\geq \omega_M \left[\frac{\omega_H}{\omega_H + \omega_L} u\left(c_1^H\right) + \frac{\omega_L}{\omega_H + \omega_L} u\left(c_1^L\right) \right] + (1-\omega_M) \left[\frac{1-\omega_H}{2-\omega_H-\omega_L} u\left(c_2^H\right) + \frac{1-\omega_L}{2-\omega_H-\omega_L} u\left(c_2^L\right) \right] \\
&= \frac{1}{2} V(y, \omega_H) + \frac{1}{2} V(y, \omega_L),
\end{aligned}$$

where the inequality holds strictly if $c_1^H \neq c_1^L$. Since $(\hat{c}_1, \hat{c}_2)$ is feasible, we further have

$$V(y, \omega_M) \geq \omega_M u(\hat{c}_1) + (1-\omega_M) u(\hat{c}_1).$$

We conclude that

$$V(y, \omega_M) \geq \frac{1}{2} V(y, \omega_H) + \frac{1}{2} V(y, \omega_L),$$

with strict inequality if the optimal consumption levels for the problems associated to $V(y, \omega_H)$ and $V(y, \omega_L)$ satisfy $c_1^H \neq c_1^L$.

For any $p \in [0,1]$ let $W(p)$ denote the expected utility of consumers:

$$W(p) = \max_y pV(y, \omega_M) + (1-p) \left(\frac{1}{2} V(y, \omega_H) + \frac{1}{2} V(y, \omega_L) \right).$$

The envelope theorem implies

$$W'(p) = V\left(y^I(p), \omega_M\right) - \frac{1}{2}V\left(y^I(p), \omega_H\right) - \frac{1}{2}V\left(y^I(p), \omega_L\right). \tag{24}$$

Moreover, Proposition 3 (and Proposition 1 for the case $p = 0$) show that the solutions to the problems associated to $V\left(y^I(p), \omega_H\right)$ and $V\left(y^I(p), \omega_L\right)$ yield $c_1^H < c_1^L$. We conclude that the expression on the right-hand side of (24) is strictly positive, so $W'(p) > 0$ for all $p \in [0, 1]$. Since $W(0)$ corresponds to the expected utility in autarky, this proves both statements in the proposition. ■

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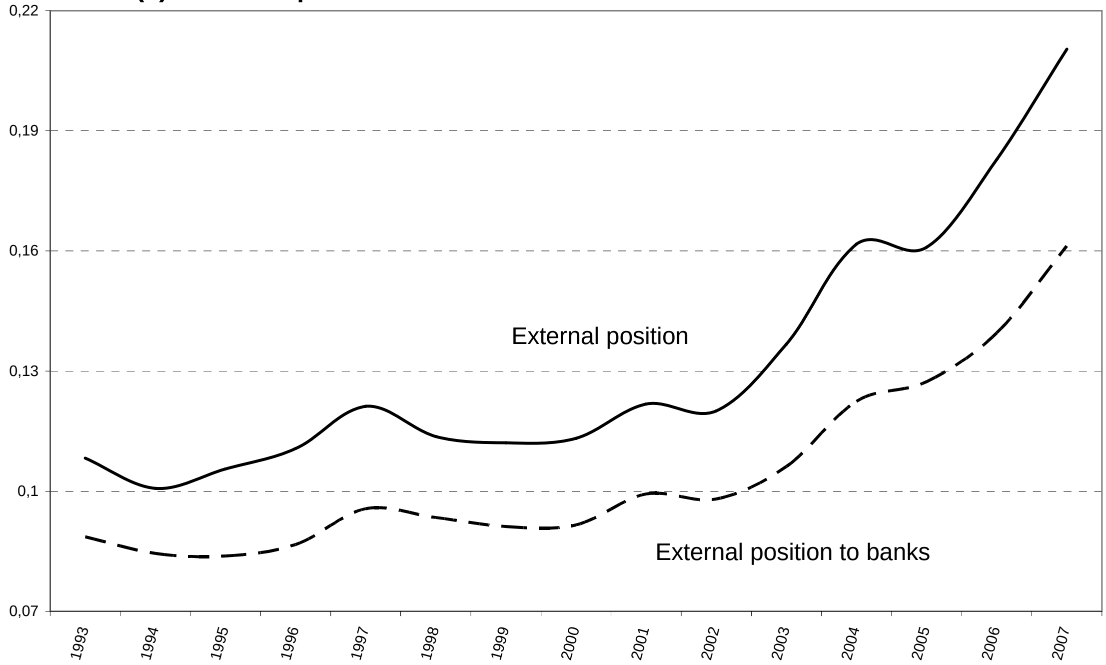
Figure 1

Panel (a): External position over domestic credit of US banks



Panel (b): Liquidity holdings over total deposits of US banks

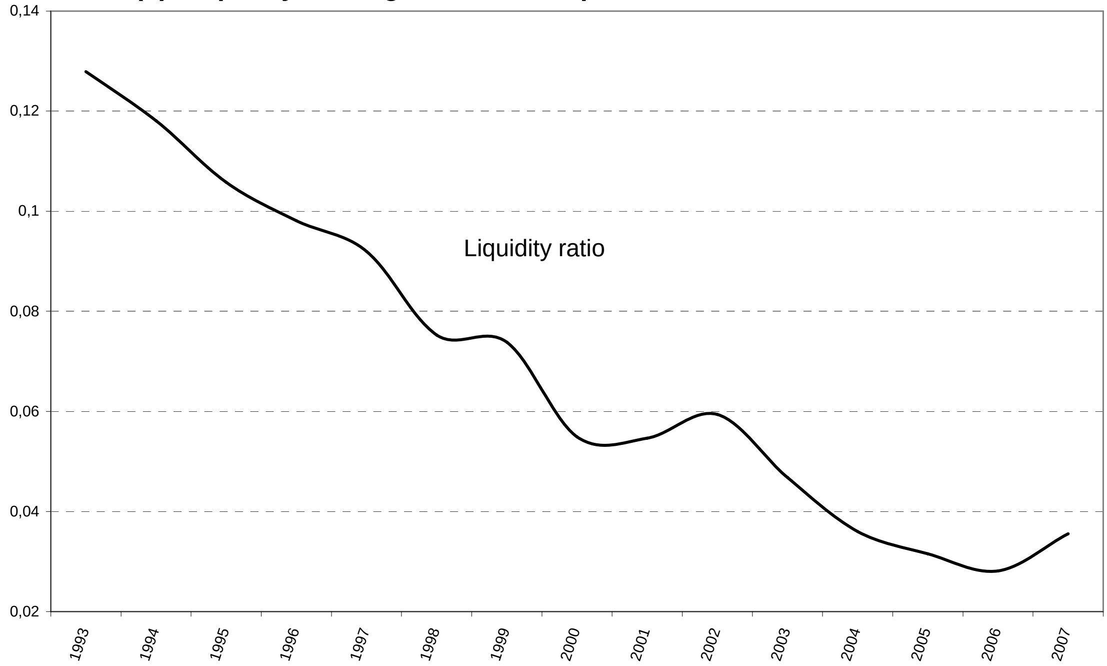
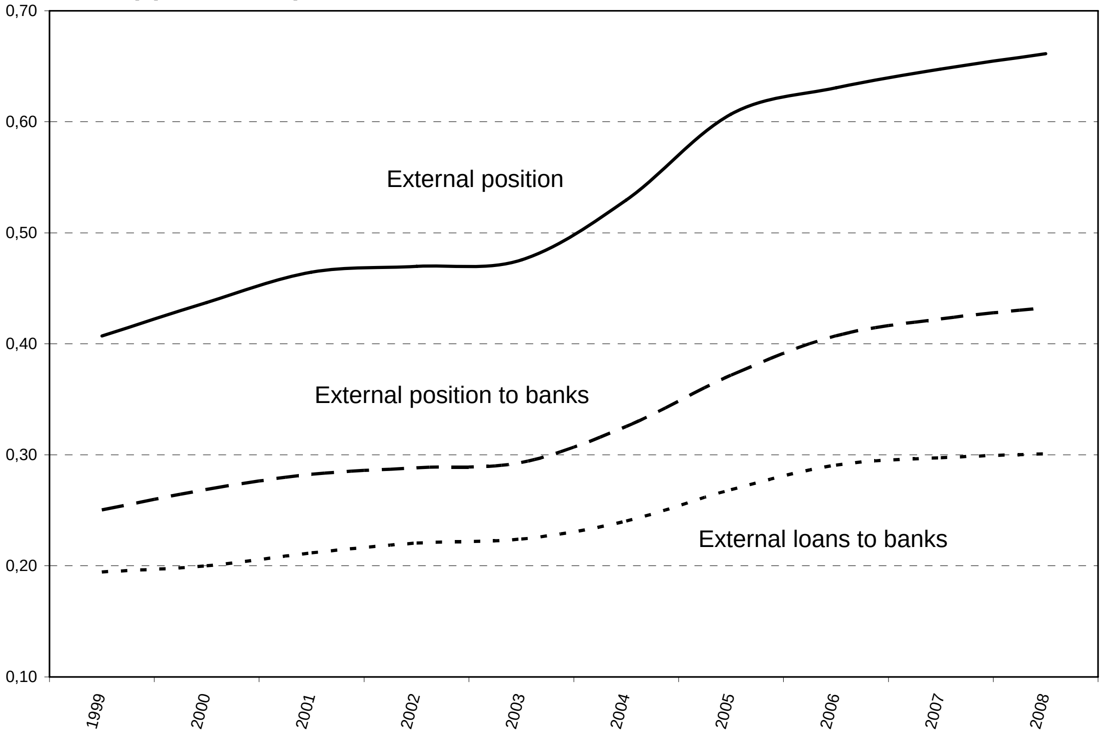


Figure 2

Panel (a): External position over domestic credit of Euro-area banks



Panel (b): Liquidity holdings over total deposits of Euro-area banks

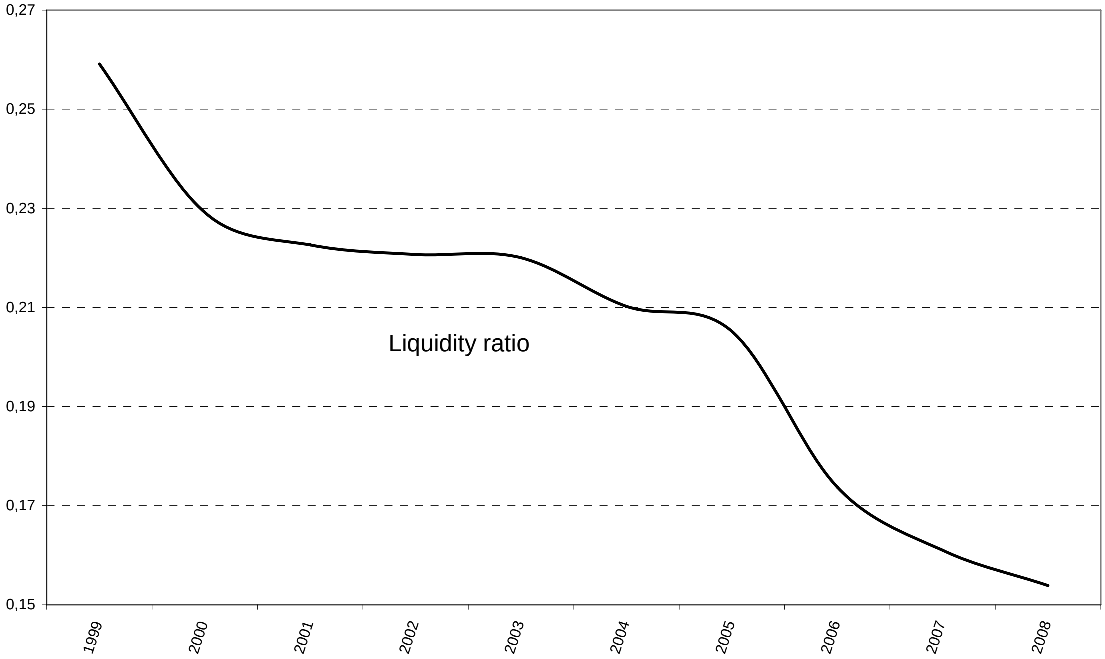
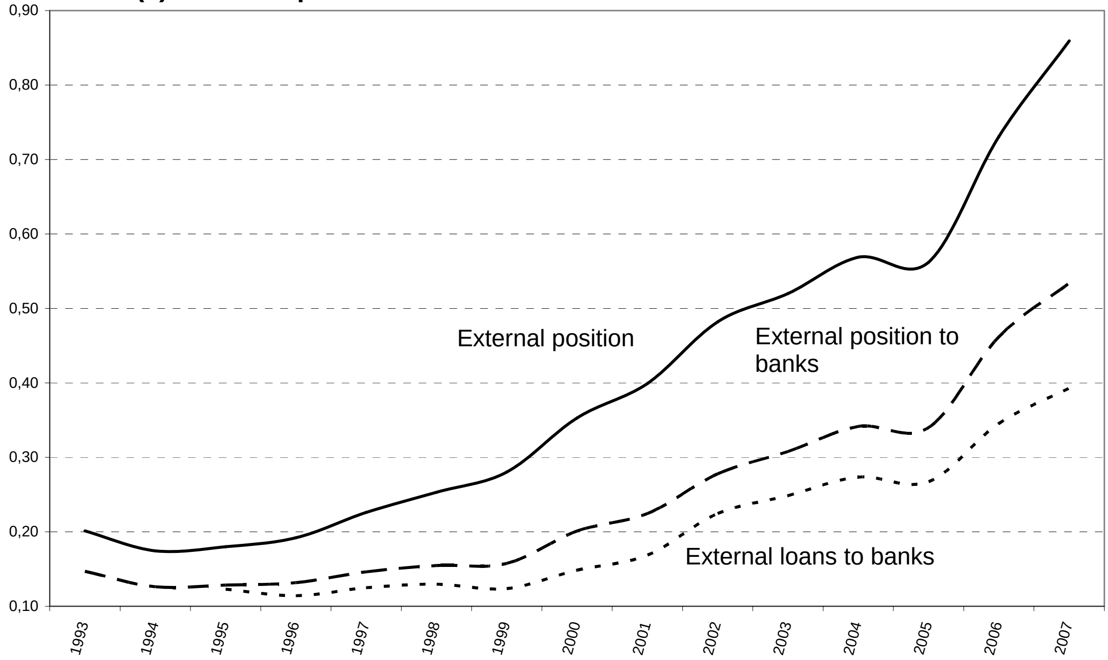


Figure 3

Panel (a): External position over domestic credit of German banks



Panel (b): Liquidity holdings over total deposits of German banks

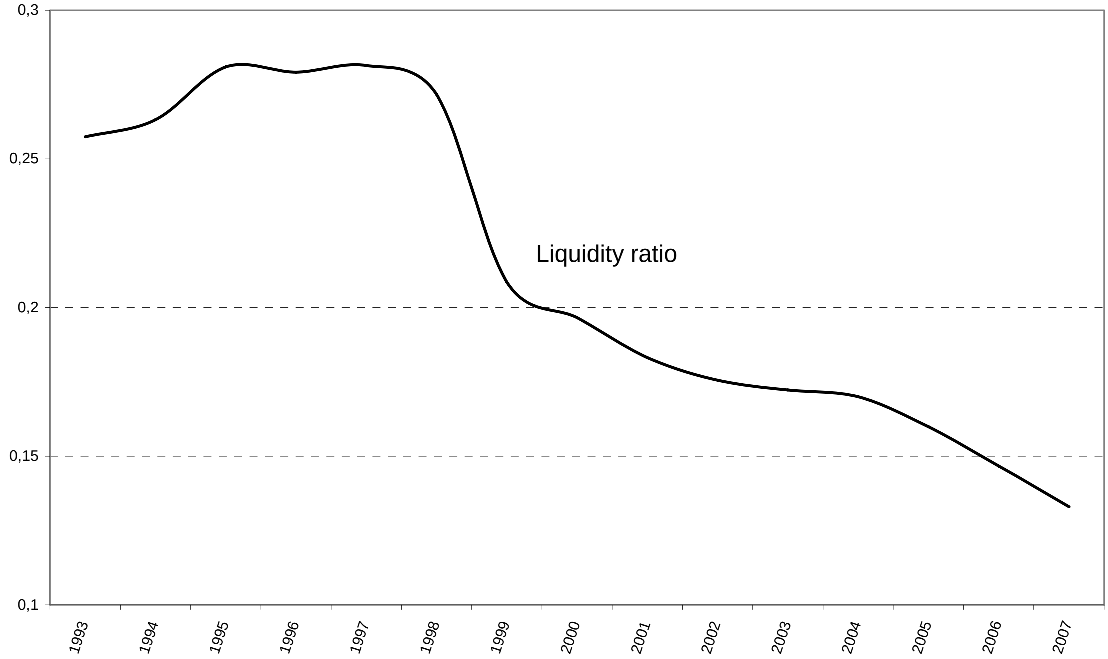
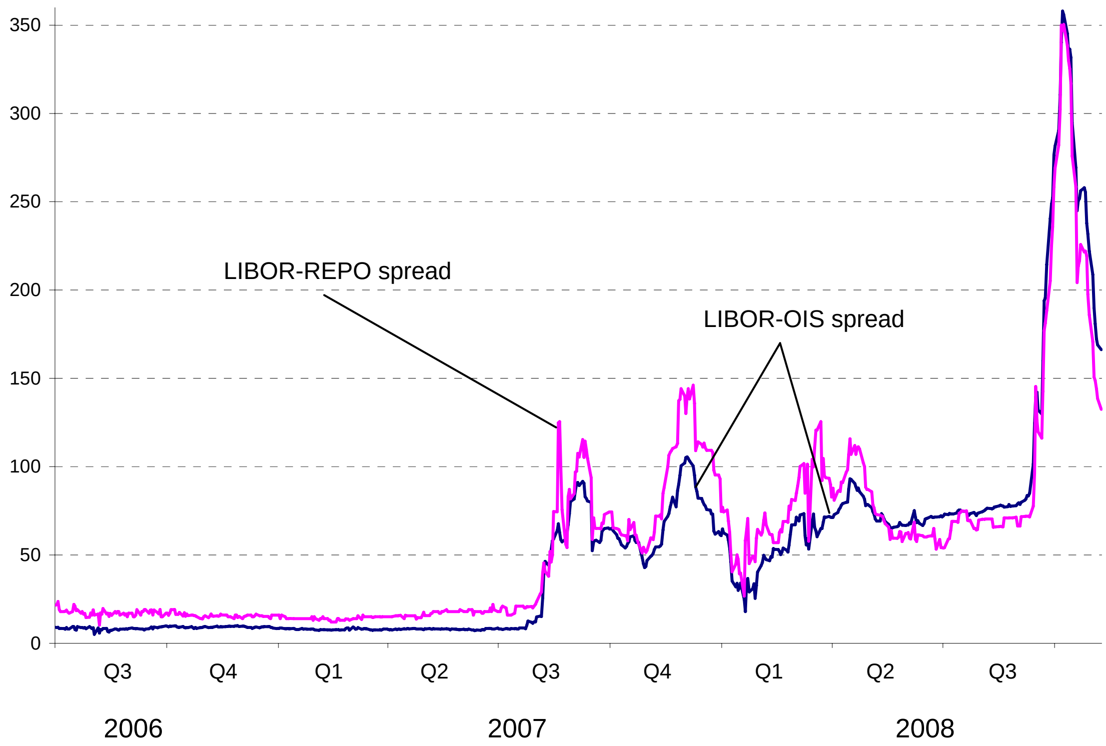
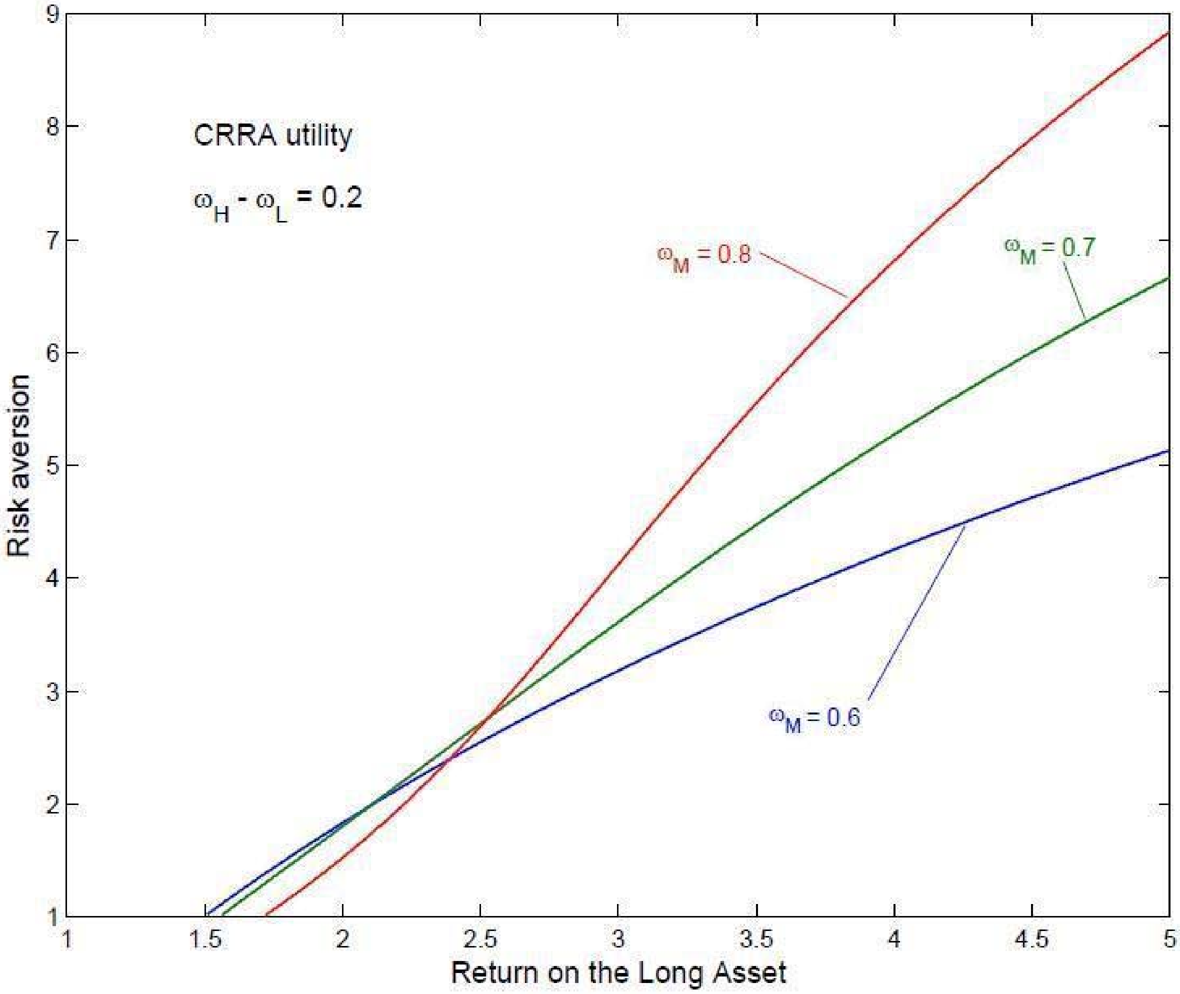


Figure 4



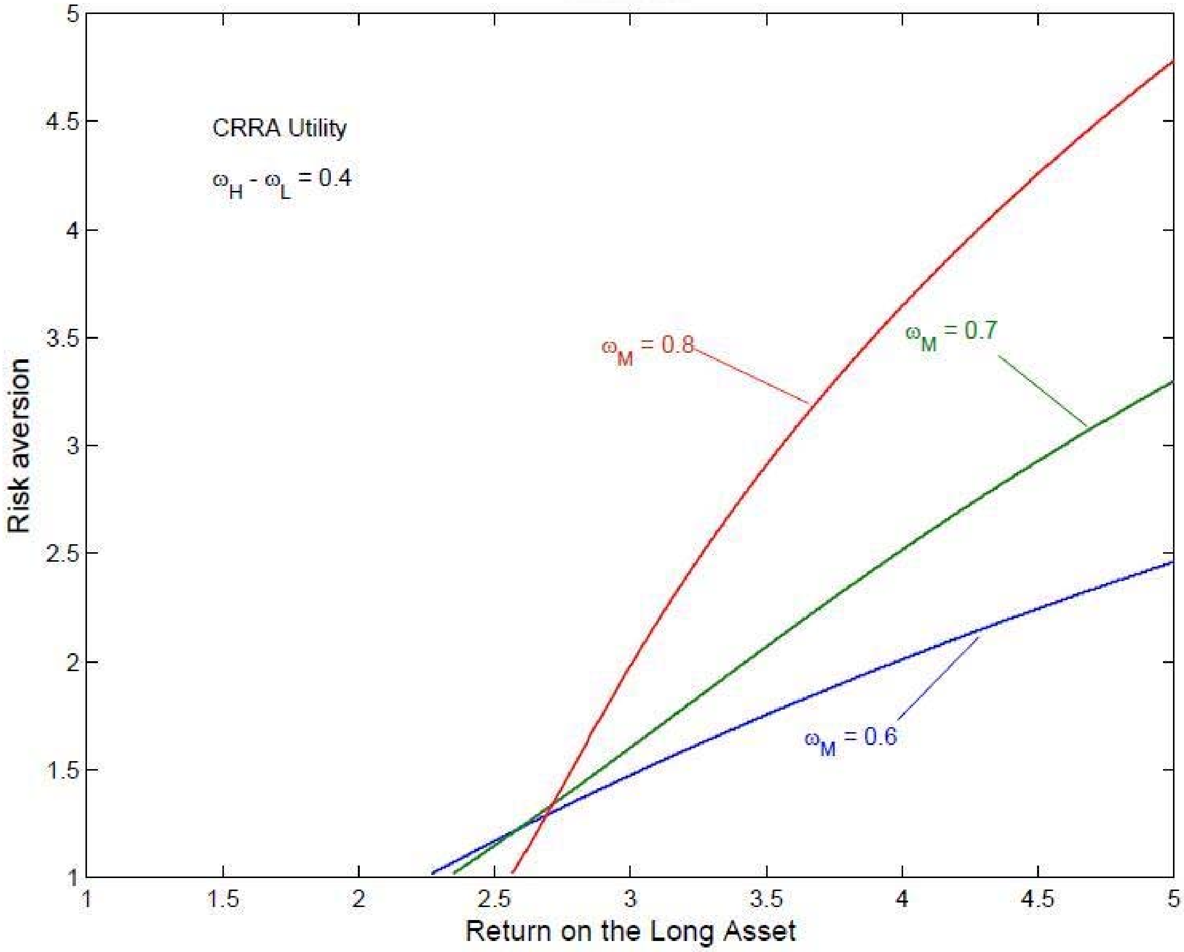
350
300
250
200
150
100
50
0
LIBOR-REPO spread
LIBOR-OIS spread
Q3
Q4
Q1
Q2
Q3
Q4
Q1
Q2
Q3
2006
2007
2008

Figure 5



CRRA utility
ωH - ωL = 0.2
ωM = 0.8
ωM = 0.7
ωM = 0.6
Risk aversion
Return on the Long Asset
9
8
7
6
5
4
3
2
1
1
1.5
2
2.5
3
3.5
4
4.5
5

Figure 6



CRRA Utility
$\omega_H - \omega_L = 0.4$
$\omega_M = 0.8$
$\omega_M = 0.7$
$\omega_M = 0.6$
Risk aversion
Return on the Long Asset
5
4.5
4
3.5
3
2.5
2
1.5
1
1
1.5
2
2.5
3
3.5
4
4.5
5

Figure 7

Interest rate distribution under financial integration for different values of *p*. Parameters: $\gamma = 1$, $R = 1.15$, $\omega_H = 0.6$, $\omega_L = 0.4$

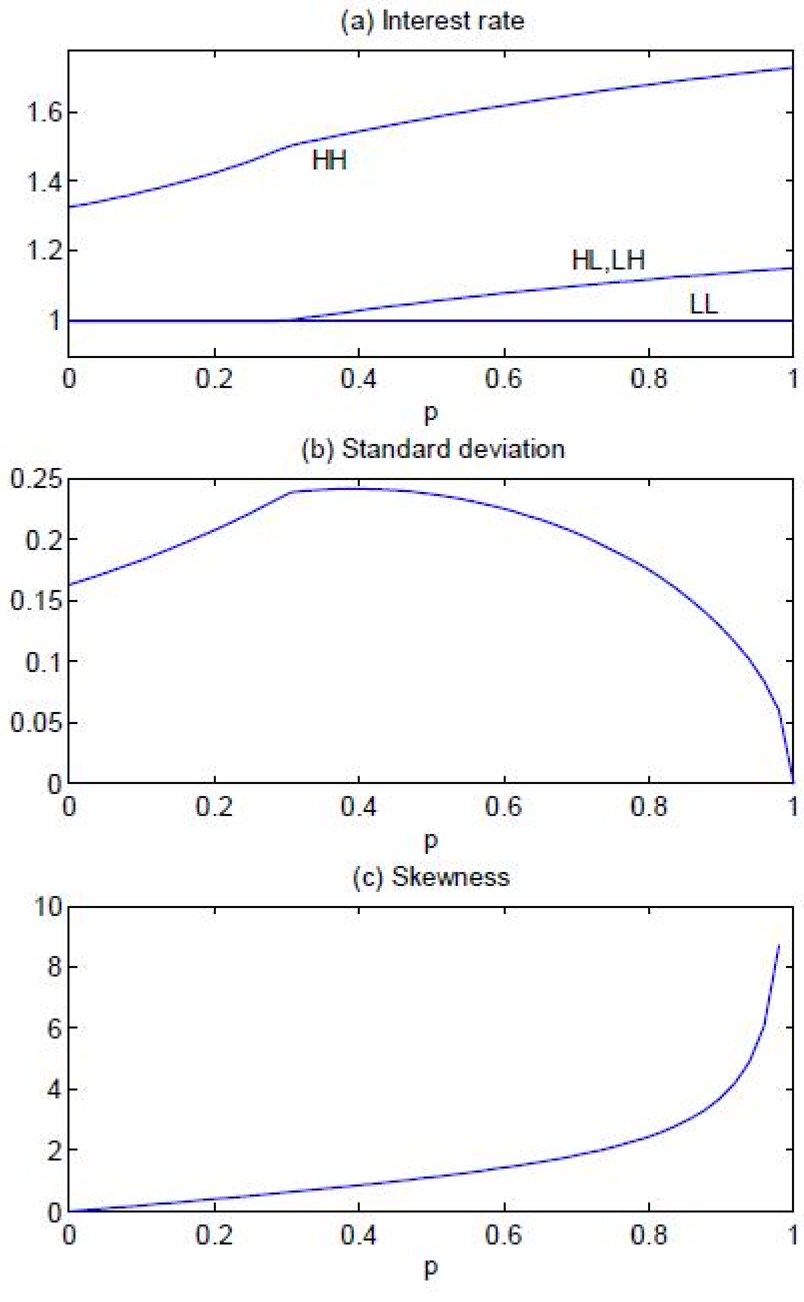


Figure 8

First period consumption distribution under financial integration for different values of *p*.
Parameters: $\gamma = 1$, $R = 1.15$, $\omega_H = 0.6$, $\omega_L = 0.4$

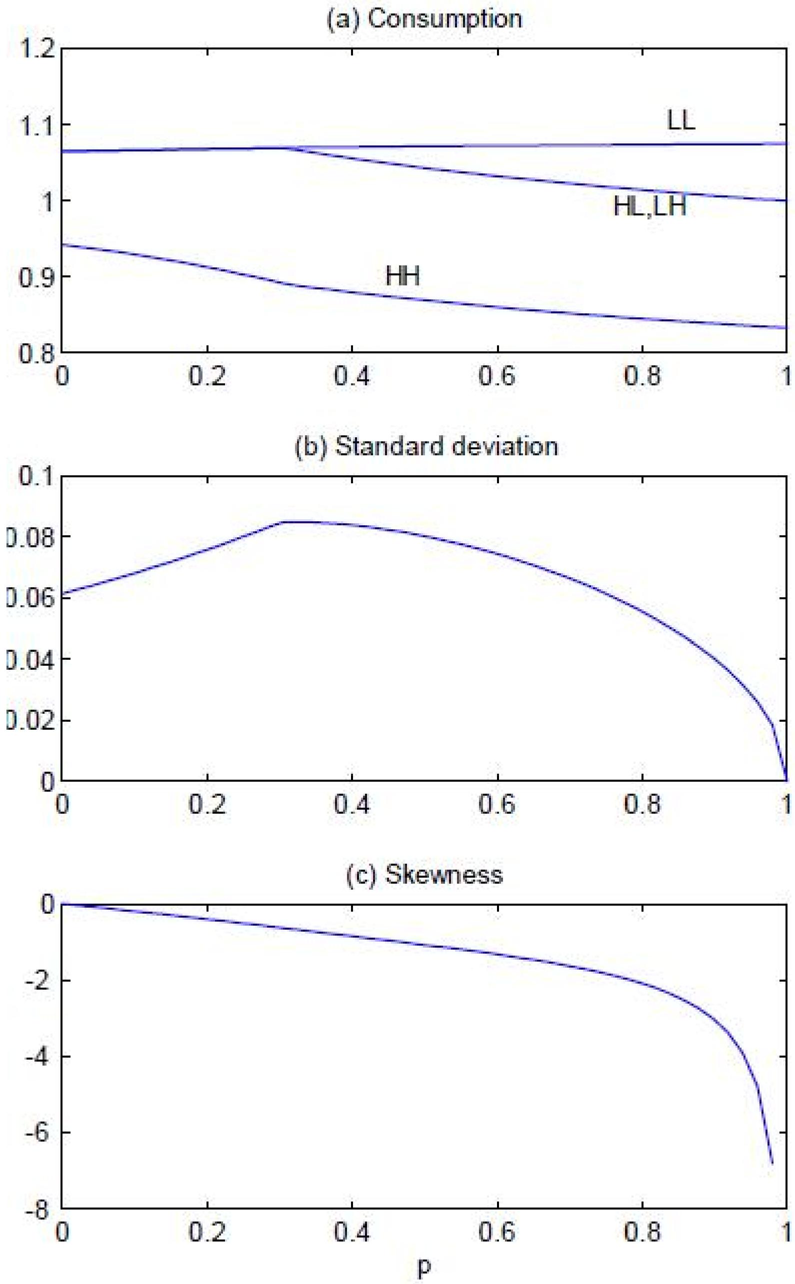


Table 2*

	$\omega_L = 0.35$; $\omega_H = 0.65$		$\omega_L = 0.1$; $\omega_H = 0.9$	
Long Return	Switching threshold from case 3 to case 2	Switching threshold from case 2 to case 1	Switching threshold from case 3 to case 2	Switching threshold from case 2 to case 1
1.1	0.19	0.25	0.07	0.15
1.2	0.36	0.48	0.13	0.28
1.4	0.68	0.88	0.25	0.52
1.8	1.19	1.56	0.46	0.92
2.2	1.61	2.07	0.64	1.24
2.6	1.97	2.52	0.81	1.50
3.0	2.28	2.89	0.95	1.72
4.0	2.89	3.65	1.27	2.18
5.0	3.36	4.24	1.52	2.53

Table 3

$R = 1.4$; $\omega_L = 0.35$; $\omega_H = 0.65$;

	Risk aversion = 4 (case 1)		Risk aversion = 0.75 (case 2)		Risk aversion = 0.5 (case 3)	
p	Std	Skew	Std	Skew	Std	Skew
0	0.036	0	0.204	0	0.275	0
0.1	0.040	-0.201	0.200	-0.032	0.262	0.091
0.2	0.045	-0.408	0.194	-0.040	0.248	0.192
0.3	0.050	-0.629	0.186	-0.028	0.233	0.306
0.4	0.058	-0.873	0.175	0.002	0.217	0.438
0.5	0.066	-1.155	0.162	0.053	0.199	0.596
0.6	0.077	-1.500	0.147	0.128	0.178	0.794
0.7	0.089	-1.960	0.129	0.237	0.155	1.062
0.8	0.080	-2.653	0.107	0.411	0.127	1.477
0.9	0.058	-4.023	0.076	0.763	0.090	2.334
1	0	0	0	0	0	0

* In tables 2,3 and 4, case 1 means that consumption skewness is negative for all *p*. Case 2 means that consumption skewness is negative for p below some threshold and positive above. Case 3 means that consumption skewness is positive for all *p*. Fixed *p*, as risk aversion increases we switch from case 3 to 2, and eventually to case 1. Table 2 reports some of such switching thresholds in several numerical examples.

Table 4

R = 2.6; ω_L = 0.1; ω_H = 0.9

	Risk aversion = 4 (case 1)		Risk aversion = 1.2 (case 2)		Risk aversion = 0.5 (case 3)	
p	Std	Skew	Std	Skew	Std	Skew
0	0.073	0	0.312	0	0.885	0
0.1	0.095	-0.201	0.354	-0.177	0.866	0.179
0.2	0.118	-0.408	0.368	-0.277	0.841	0.369
0.3	0.142	-0.629	0.368	-0.313	0.809	0.577
0.4	0.165	-0.873	0.359	-0.295	0.770	0.809
0.5	0.185	-1.155	0.343	-0.226	0.721	1.080
0.6	0.203	-1.500	0.319	-0.103	0.661	1.413
0.7	0.215	-1.960	0.286	0.089	0.587	1.860
0.8	0.191	-2.650	0.241	0.393	0.490	2.543
0.9	0.137	-3.941	0.176	0.988	0.354	3.956
1	0	0	0	0	0	0