Financial Stability in Open Economies*

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Abstract

This paper investigates the implications of an internationally integrated financial market and its intrinsic frictions for the monetary policy. When there is no other distortion than the financial market imperfections in the form of the international staggered loan contracts, the inward-looking financial stability, namely eliminating the inefficient fluctuations of the loan premiums, is the optimal monetary policy in open economies irrespective of the existence of the policy coordination. Yet, optimality of inward-looking, i.e., independent, monetary policy requires an additional condition to the previous studies on optimal monetary policy in open economies. For the coincidence of allocations between cooperative and noncooperative monetary policy, the exchange rate risk must be perfectly covered by the banks. Otherwise, each central bank has an additional incentive to stabilize nominal exchange rate only to the favor of firms in her country to reduce the exchange rate risk.

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1 Introduction

Financial globalization have been expanding quite rapidly. We can easily observe this trend from recent financial and economic developments. For example, many banks in the world now suffer from losses stemming from the US subprime loan crisis. Gadanecz (2004), McGuire and Tarashev (2006), and Lane and Milesi-Ferretti (2007, 2008) formally show that more funds from foreign countries are flowing into the domestic financial markets of many countries. Although we can find several studies investigating the implications of the goods market integration for monetary policy, summarized in Woodford (2007), very few studies have focused on monetary policy under global banking or internationally integrated financial markets.

Dose the international financial stability matter for the central banks? How does the international financial market developments alter the form of the optimal monetary policy? Should the central banks cooperatively conduct the monetary policy or not under the internationally integrated financial markets?

In order to answer these questions, we construct a new open economy macroeconomics (NOEM) model that incorporates the international loan contracts as in Fujiwara and Teranishi (2008). In our model, financial market are captured by the staggered loan contracts following Calvo (1983) - Yun (1996) framework. The stickiness in the loan rate contract is reported by many studies as Slovin and Sushka (1983) and Berger and Udell (1992) for US economy, Sorensen and Werner (2006) and Gambacorta (2008) for euro economy, and BOJ (2007) and BOJ (2008) for the Japanese economy.\textsuperscript{1} For the detailed modeling of the financial market, it is popular to incorporate the financial accelerator as in Bernanke, Gertler, and Gilchrist (1999) in the dynamic stochastic general equilibrium model, where

\textsuperscript{1}For the US, using micro level data, Slovin and Sushka (1983) and Berger and Udell (1992) show that it takes two or more quarters for the private banks to adjust the loan interest rates. For the Euro area, Sorensen and Werner (2006) estimate the incompleteness in the pass-through from the policy interest rate to the loan interest rates by the error correction model using macro data. They further show that the degree of the incomplete pass-through significantly differs among countries. Gambacorta (2008) conducted similar analysis for Germany and show the existence of the sticky adjustment in the loan interest rate. For Japan, according to BOJ (2007) and BOJ (2008), the major city banks need five quarters and the local banks need seven quarters to adjust the loan interest rates.
the net worth as the state variable causes the deviations of loan rates from the policy interest rate. The staggered loan contract model can be considered simplified or another type of the financial market friction. We aim to capture the dynamics of loan rates by the staggered loan contracts instead of the net worth dynamics. In our model, the wedge between the loan rate and the policy rate is due to the imperfect competition among banks following Sander and Kleimeier (2004), Gropp, Sorensen, and Lichtenberger (2006), van Leuvensteijn, Sorensen, Bikker, and van Rixtel (2008) and Gropp and Kashyap (2008), that point out the importance of bank competitions on the staggered loan rate setting. The end consequences are the same irrespective of these two ways of modeling. A shock related to the financial market imperfections eventually results in the increase in the costs of goods production. What is the most advantageous in our approach is that we can analyze the nature of optimal monetary policy analytically and therefore more intuitively.

Welfare analysis shows that the central banks should stabilize the international financial disturbance, implying the central bank should care about the international financial market heterogeneity between domestic and foreign countries. Most notably, when there is no other distortion than the staggered loan contracts as examined in this paper, the inward-looking financial stability, namely eliminating the inefficient fluctuation of the loan premiums stemming from the financial market imperfections, is the optimal monetary policy in open economies irrespective of the existence of the cooperation between central banks. There, each central bank should aim at stabilizing the loan premium to firms in its own country. Yet, optimality of inward-looking, i.e., independent, monetary policy requires an additional condition to the previous studies on optimal monetary policy in open economies. Otherwise, each central bank has an additional incentive to stabilize nominal exchange rate only to the favor of firms in her country to reduce the exchange rate risk.

2Yet, initial responses of loan rates to the monetary policy shock are quite different. In the financial accelerator model, the response becomes much larger than in our model. Where this difference is coming from is, however, not a trivial question. Morozumi (2008) shows that the financial accelerator mechanism itself does not amplify the responses but just makes them persistent to monetary policy shock. To investigate which is superior for empirical accounting is left for our future research.

3There exist models where financial market imperfections affect the aggregate TFP. See, for example, Chari, Kehoe, and McGrattan (2007).
These are quite new findings not considered in such previous studies that investigate the optimal monetary policy in open economies as Obstfeld and Rogoff (2002), Clarida, Galí, and Gertler (2002), Benigno and Benigno (2003), Devereux and Engel (2003), and Corsetti and Pesenti (2005).

The structure of the paper is as follows. Section 2 shows the model used for the analyses in this paper. Then, in Section 3, we derive the loss function that the central bank should minimize. Section 4 investigates the nature of the optimal monetary policy in internationally integrated financial markets. Section 5 gives a short discussion. Finally, Section 6 summarizes the findings of this paper.

## 2 Model

The model consists of two countries. There are four types of agent in each country—consumers, firms, private banks and the central bank—as depicted in Figure 1.
2.1 Consumer

A representative consumer plays four roles: (1) to consume differentiated goods determined through two-step cost minimization problems on both home- and foreign-produced consumer goods; (2) to choose the amount of aggregate consumption, bank deposits and investment in risky assets given a deposit interest rate set by the central bank; (3) with monopolistic power on labor supply, to provide differentiated labor services that belong to either the domestically financially supported (DFS) or the internationally financially supported (IFS) groups and to offer wages to those differentiated types of labor; and (4) to own banks and firms and to receive dividends in each period. Role (3) is crucial in staggered loan contracts. Thanks to this differentiated labor supply, the demand for loans is differentiated without assuming any restrictions on aggregate loans or loan interest rates.\(^4\)

2.1.1 Cost Minimization

The utility of the representative consumer in the home country \(H\) is increasing and concave in the aggregate consumption index \(C_t\).\(^5\) The consumption index that consists of bundles of differentiated goods produced by home and foreign firms is expressed as

\[
C_t = \frac{C_{H,t}^{\psi} C_{F,t}^{1-\psi}}{\psi^\psi (1-\psi)^{1-\psi}},
\]

where \(\psi (0 \leq \psi \leq 1)\) is a preference parameter that expresses the home bias, which is set to be 0.5 in this paper for no home bias.\(^6\) Here, \(C_{H,t}\) and \(C_{F,t}\) are consumption subindices of the continuum of differentiated goods produced by firms in the home country and the foreign country, respectively. They are defined as

\[
C_{H,t} = \left[ \int_0^1 c_t(f) \frac{\sigma-1}{\sigma} \, df \right]^{\frac{\sigma}{\sigma-1}},
\]

and

\[
C_{F,t} = \left[ \int_0^1 c_t(f^*) \frac{\sigma-1}{\sigma} \, df^* \right]^{\frac{\sigma}{\sigma-1}},
\]

\(^4\)For details, see Teranishi (2007).

\(^5\)The same optimal allocations are obtained even by assuming that each homogenous consumer provides differentiated labor supply to each firm.

\(^6\)Here, we follow Obstfeld and Rogoff (2000).
where \( c_t(f) \) is the demand for a good produced by firm \( f \) in the home country and \( c_t(f^*) \) is the demand for a good produced by a firm \( f^* \) in the foreign country, where the asterisk denotes foreign variables. Following the standard cost minimization problem on the aggregate consumption index of home and foreign goods as well as the consumption subindices of the continuum of differentiated goods, we can derive the consumption-based price indices:

\[
P_t \equiv P_{H,t}^{\frac{1}{2}} P_{F,t}^{\frac{1}{2}},
\]

with

\[
P_{H,t} \equiv \left[ \int_0^1 p_t(f)^{1-\sigma} \, df \right]^{\frac{1}{1-\sigma}},
\]

and

\[
P_{F,t} \equiv \left[ \int_0^1 p_t(f^*)^{1-\sigma} \, df^* \right]^{\frac{1}{1-\sigma}},
\]

where \( p_t(f) \) is the price on \( c_t(f) \), and \( p_t(f^*) \) is the price on \( c_t(f^*) \). Then, we can obtain the following Hicksian demand functions for each differentiated good given the aggregate consumption:

\[
c_t(f) = \frac{1}{2} \left[ \frac{p_t(f)^{1-\sigma}}{P_{H,t}} \right] C_t,
\]

and

\[
c_t(f^*) = \frac{1}{2} \left[ \frac{p_t(f^*)^{1-\sigma}}{P_{F,t}} \right] C_t.
\]

Here, as in other applications of the Dixit and Stiglitz (1977) aggregator, consumers’ allocations across differentiated goods at each time are optimal in terms of cost minimization.

We can derive similar optimality conditions for the foreign counterpart. For example, the demand functions for each differentiated good given the aggregate consumption are expressed as

\[
c_t^*(f) = \frac{1}{2} \left[ \frac{p_t^*(f)^{1-\sigma}}{P_{H,t}^*} \right] C_t^*,$

and

\[
c_t^*(f^*) = \frac{1}{2} \left[ \frac{p_t^*(f^*)^{1-\sigma}}{P_{F,t}^*} \right] C_t^*.
\]

6
2.1.2 Utility Maximization

A representative consumer in the home country maximizes the following utility function:

\[ U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ U(C_T) - \int_0^n V([l_T(h)]) dh - \int_1^n V([l_T(h)]) dh \right\}, \]

where \( E_t \) is the expectation operator conditional on the state of nature at date \( t \) and \( \beta \) is the subjective discount factor. The budget constraint of the consumer is given by

\[ P_t C_t + E_t [X_{t,t+1} B_{t+1}] + D_t \leq B_t + (1 + i_{t-1}) D_{t-1} + \int_0^n w_t(h) l_t(h) dh + \int_1^n w_t(h) l_t(h) d\bar{h} + \Pi_t^{PB} + \Pi_t^F - T_t, \]

(5)

where \( B_t \) is a set of risky asset, \( D_t \) is the deposit to private banks, \( i_t \) is the nominal deposit interest rate set by a central bank from \( t-1 \) to \( t \), \( w_t(h) \) is the nominal wage for labor supplied from the DFS \( l_t(h) \), \( \Pi_t^{PB} = \int_0^1 \Pi_t^{PB}(h) dh \) is the nominal dividend stemming from the ownership of both local and international banks in the home country, \( \Pi_t^F = \int_0^1 \Pi_t^F(h) df \) is the nominal dividend from the ownership of the firms in the domestic country, \( X_{t,t+1} \) is the stochastic discount factor and \( T_t \) is the lump sum tax.\(^7\) Here, because we assume a complete financial market between the two countries, the consumer in each country can internationally buy and sell the state contingent securities to insure against country-specific shocks. Consequently, there only exists a unique discount factor. The relationship between the deposit interest rate and the stochastic discount factor is now expressed as

\[ \frac{1}{1 + i_t} = E_t [X_{t,t+1}]. \]

(6)

Given the optimal allocation of differentiated consumption expenditures, the consumer now optimally chooses the total amount of consumption, risky assets and deposits in each period. Necessary and sufficient conditions, when the transversality condition is satisfied, for those optimizations are given by

\[ U_C(C_t) = \beta(1 + i_t) E_t \left[ U_C(C_{t+1}) \frac{P_t}{P_{t+1}} \right], \]

(7)

\(^7\)For simplicity, we do not explicitly include the amount of contingency claims under complete financial markets.
Together with equation (6), we see that the condition given by equation (7) defines the intertemporally optimal allocation on aggregate consumption. Then, the standard new Keynesian IS curve for the home country, by log-linearizing equation (7) around steady states, is obtained as follows:

$$
\bar{C}_t = E_t \bar{C}_{t+1} - \nu \left( \bar{\pi}_t - E_t \bar{\pi}_{t+1} \right),
$$

where aggregate inflation in the home country is $$\bar{\pi}_t \equiv \ln \frac{P_t}{P_{t-1}}$$ and $$\nu \equiv -\frac{\pi}{\bar{\pi}}$$. Each variable is defined as the log deviation from its steady-state value, where the log-linearized version of variable $$x_t$$ is expressed by $$\hat{x}_t = \ln \left( x_t / \bar{x} \right)$$, except for $$\pi_t$$, given that $$\bar{x}$$ is the steady-state value of $$x_t$$.

In this model, a representative consumer provides all types of differentiated labor to each firm and therefore maintains some monopoly power over the determination of his own wage, as in Erceg, Henderson, and Levin (2000). There are two types of labor group: the DFS and the IFS. The workers populated on $$[0, n)$$ belong to the DFS, and other labor populated on $$[n, 1]$$ belongs to the IFS.\(^8\) We assume that each firm hires all types of labor in the same proportion from the two groups. The consumer sets each wage $$w_t(h)$$ for any $$h$$ and $$w_t(\bar{h})$$ for any $$\bar{h}$$ to maximize its utility subject to the budget constraint given by equation (5) and the labor demand functions given by equations (24) and (25) in the next section. Here, although differentiated labor supply is assumed in this paper, consumers change wages in a flexible manner. Then we have the optimality conditions for labor supply as follows:

$$
\frac{w_t(h)}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{V_t[l_t(h)]}{U_C(C_t)},
$$

and

$$
\frac{w_t(\bar{h})}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{V_t[l_t(\bar{h})]}{U_C(C_t)},
$$

\(^8\)The difference between these two groups is characterized by somewhat wider properties of workers, like English speaking or Japanese speaking, though the differences between workers within each group are characterized by narrower properties of workers, like a person who has knowledge of accounting in banks or a person who has the skills to build automobiles in a plant.
where \( \varepsilon \) is the elasticity of substitution among differentiated labor. As written above, thanks to this heterogeneity in labor supply, we can model the differentiated demand for loans without assuming any restrictions on aggregate loans and loan interest rates. In this paper, consumers supply their labor only for firms, not for banks.

Similar to the above case with cost minimization, we can derive the optimality conditions for the foreign counterpart. For example, the standard new Keynesian IS curve for the foreign country is

\[
\hat{C}^*_t = E_t \hat{C}^*_{t+1} - \nu \left( \hat{i}_t - E_t \pi^*_t \right).
\]

(11)

### 2.1.3 Exchange Rate

Under complete financial markets with a symmetric initial state:

\[
U_C^*(C_t^*) = \frac{S_t P^*_t}{P_t} U_C(C_t),
\]

and

\[
Q_t = \frac{S_t P^*_t}{P_t},
\]

where \( S \) is the nominal exchange rate while \( Q \) is the real exchange rate. As we will see below, because of the symmetry in the home bias parameter and because no nominal rigidities are assumed in this paper:

\[
C_t^* = C_t \equiv C_t^W,
\]

(12)

where \( C_t^W \) is the world consumption and

\[
Q_t = 1.
\]

However, reflecting the nominal interest rate set by the central bank, nominal exchange rates can fluctuate according to the UIP condition as

\[
E_t \Delta S_{t+1} = \hat{i}_t - \hat{\pi}_t^*,
\]

(13)

which can also be expressed as

\[
\Delta \hat{S}_t = \pi_t - \pi_t^*.
\]

(14)
2.2 Firms

There exists a continuum of firms populated over unit mass \([0, 1]\) in each country. Each firm plays two roles. First, each firm decides the amount of differentiated labor to be employed from both the DFS and IFS groups, through the two-step cost minimization problem on the production cost. Part of the costs of labor must be financed by external loans from banks. For example, in country \(H\), to finance the costs of hiring workers from the DFS, the firm must borrow from local banks in the home country. However, to finance the costs of hiring workers from the IFS, the firm must borrow from international banks in the foreign country. The grounds for such heterogeneous sources of funds are as follows. First, Gadanecz (2004), McGuire and Tarashev (2006), Lane and Milesi-Ferretti (2007), and Lane and Milesi-Ferretti (2008) show that firms tend to borrow funds from both domestic and foreign banks; i.e., a bank lends funds to both domestic and foreign firms. Second, we also know the existence of the project finance, that firms borrow funds with many different loan interest rates at the same time depending on the nature of projects. In this paper, these project differences are stemming from the types of labor, which is immobile between the two countries. Since it is assumed that firms must use all types of labor, they borrow from both local and international banks.\(^9\)

The structure of the exchange rate risk sharing is as follows. Domestic firms borrows \(\xi \times 100\) percent of loans in their own currency from international banks in the foreign country. Thus, the exchange rate risk is shared by the firm in the home country and the international banks in the foreign country with \(\xi\) and \(1 - \xi\).

2.2.1 Cost Minimization

Firms in both the home and foreign countries optimally hire differentiated labor as price takers. This optimal labor allocation is also carried out through two-step cost minimization problems. Domestic firm \(f\) hires all types of labor from both the DFS and IFS groups. When hiring from the DFS group, \(\gamma\) portion of the labor cost associated with labor type \(h\) is financed by borrowing from the local bank \(h\). Then, the first-step cost minimization

\(^9\) The same structure is assumed for employment in Woodford (2003).
problem on the allocation of differentiated labor from the DFS is given by

$$\min_{l_t(h,f)} \int_0^h \left[ 1 + \gamma r_t(h) \right] w_t(h) l_t(h,f) dh,$$

subject to the subindex regarding labor from DFS to firm $f$:

$$L_t(f) = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\xi}} \int_0^h l_t(h,f) \frac{t+1}{t} dh \right]^{\frac{1}{1-\xi}}, \quad (15)$$

where $r_t(h)$ is the loan interest rate applied to employ a particular labor type $h$ applied to differentiated labor supply. There $l_t(h,f)$ denotes type of labor $h$ employed by firm $f$. The local bank $h$ has some monopoly power over setting loan interest rates. The relative demand on differentiated labor is given as follows:

$$l_t(h,f) = \frac{1}{n} L_t \left\{ \frac{[1 + \gamma r_t(h)] w_t(h)}{\Omega_t} \right\}^{-\xi}, \quad (16)$$

where

$$\Omega_t = \left\{ \frac{1}{n} \int_0^h \left[ [1 + \gamma r_t(h)] w_t(h) \right]^{1-\xi} dh \right\}^{\frac{1}{1-\xi}}. \quad (17)$$

Then, the first-step cost minimization problem on the allocation of differentiated labor from the IFS is given by

$$\min_{l_t(h,f)} \int_0^h \left[ S_t + \frac{S_t+1}{S_t} \xi + (1-\xi) \right] \left[ 1 + \gamma r_t(\bar{h}) \right] w_t(\bar{h}) l_t(\bar{h},f) d\bar{h},$$

subject to the subindex regarding labor from DFS to firm $f$:

$$L_t(f) = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\xi}} \int_n^1 l_t(\bar{h},f) \frac{t+1}{t} d\bar{h} \right]^{\frac{1}{1-\xi}},$$

where $\xi$ is a ratio of borrowing the loan by own currency. Through a similar cost minimization problem, we can derive the relative demand for each type of differentiated labor from the IFS as

$$l_t(\bar{h},f) = \frac{1}{1-n} L_t(f) \left\{ \frac{S_t+1}{S_t} \xi + (1-\xi) \right\} \left[ 1 + \gamma r_t(\bar{h}) \right] w_t(\bar{h}) l_t(\bar{h},f) \Omega_t^{-\xi}.$$

$$\Omega_t = \left\{ \frac{1}{1-n} \int_n^1 \left[ [1 + \gamma r_t(\bar{h})] w_t(\bar{h}) \right]^{1-\xi} d\bar{h} \right\}^{\frac{1}{1-\xi}}. \quad (18)$$
According to the above two optimality conditions, firms optimally choose the allocation of differentiated workers between these two groups. Because firms have some preference \( n \) to hire workers from the DFS and \((1 - n)\) to hire workers from the IFS, the second-step cost minimization problem describing the allocation of differentiated labor between these two groups is given by

\[
\min_{L_t, \tilde{L}_t} \Omega_t L_t (f) + \Omega_t \tilde{L}_t (f),
\]

subject to the aggregate labor index:

\[
\tilde{L}_t (f) = \frac{[L_t (f)]^n \{L_t (f)\}^{1-n}}{n^n (1-n)^{1-n}}.
\] (20)

Then, the relative demand functions for each differentiated type of labor are derived as follows:

\[
L_t (f) = n \tilde{L}_t (f) \left( \frac{\Omega_t}{\Omega_t} \right)^{-1},
\] (21)

\[
\tilde{L}_t (f) = (1 - n) \tilde{L}_t (f) \left( \frac{\Omega_t}{\Omega_t} \right)^{-1},
\] (22)

and

\[
\tilde{\Omega}_t \equiv \Omega_t \tilde{\Omega}_t^{1-n}.
\] (23)

Therefore, we can obtain the following equations:

\[
l_t (h, f) = \left\{ \frac{1 + \gamma r_t (h) w_t (h)}{\Omega_t} \right\}^{-\varepsilon} \left( \frac{\Omega_t}{\Omega_t} \right)^{-1} \tilde{L}_t (f),
\] (24)

and

\[
l_t (\tilde{h}, f) = \left\{ \frac{1 + \gamma r_t (\tilde{h}) w_t (\tilde{h})}{\tilde{\Omega}_t} \right\}^{-\varepsilon} \left( \frac{\Omega_t}{\tilde{\Omega}_t} \right)^{-1} \tilde{L}_t (f),
\] (25)

from equations (16), (18), (21), and (22). We can now clearly see that the demand for each differentiated worker depends on wages and loan interest rates, given the total demand for labor.

Finally, by the assumption that firms finance part of the labor costs by loans, we can derive

\[
q_t (h, f) = \gamma w_t (h) l_t (h, f) = \gamma w_t (h) \left\{ \frac{1 + \gamma r_t (h) w_t (h)}{\Omega_t} \right\}^{-\varepsilon} \left( \frac{\Omega_t}{\Omega_t} \right)^{-1} \tilde{L}_t (f),
\]
and
\[
q_t(h, f) = \gamma w_t(h) \left( \frac{S_t^{\xi} + (1 - \xi)}{S_t} \right) \left[ 1 + \gamma r_t^* (h) \right] w_t(h) \left( \frac{\Omega_t}{\Omega_t} \right)^{-1} \tilde{L}_t(f).
\]

$q_t(h, f)$ and $q_t(h, f)$ denote the amounts of loan borrowed by firm $f$ to the labor types $h$ and $\overline{h}$, respectively. These conditions demonstrate that the demands for each differentiated loan also depend on the wages and loan interest rates, given the total labor demand.

We can obtain similar conditions for the foreign country.

### 2.2.2 Price Setting (Profit Maximization)

In this paper, to understand the role of international staggered loan contracts, we do not assume any price rigidities. Therefore, each firm $f$ resets its price $p_t(f)$ and $p_t^*(f)$ to maximize the present profit, which is given by

\[
(1 + \tau) p_t(f) c_t(f) + (1 + \tau) S_t p_t^*(f) c_t^*(f) - \tilde{\Omega}_t \tilde{L}_t(f),
\]

where $\tau$ is the rate of subsidy, $S_t$ is the nominal exchange rate and is the sales subsidy to eliminate the monopolistic rents in the steady state.\(^{10}\) By substituting equations (3) and (4), we can obtain

\[
(1 + \tau) p_t(f) \frac{1}{2} \left[ \frac{p_t(f)}{P_{H,t}} \right]^{\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t + (1 + \tau) S_t p_t^*(f) \frac{1}{2} \left[ \frac{p_t^*(f)}{P_{H,t}} \right]^{\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t^* - \tilde{\Omega}_t \tilde{L}_t(f).
\]

The optimal price setting is given by

\[
(1 + \tau) \frac{\sigma - 1}{\sigma} p_t(f) = \tilde{\Omega}_t \frac{\partial \tilde{L}_t(f)}{\partial c_t(f)},
\]

where we use equation (3). By further substituting equations (9), (10), (17), (19) and (23), the above optimality condition can be now rewritten as

\[
1 = (1 + \tau) \frac{\sigma}{\sigma - 1} Z, \quad (26)
\]

\(^{10}\)As is standard with new Keynesian models, fiscal policy eliminates the steady-state markup from goods production.
\[ Z_t = \left\{ \frac{1}{n} \int_0^n \left\{ \frac{1 + \gamma t (h)}{\varepsilon - 1} \left( \frac{V_i [l_t (h)]}{P_t} \frac{\partial L_t (f)}{\partial c_t (f)} \right) \right\}^{1 - \varepsilon} \frac{1}{1 - n} \left\{ \frac{S_t + 1}{S_t} \cdot \xi (1 - \xi) \right\}^{1 - \varepsilon} \int_0^1 \left\{ \frac{1 + \gamma t (\hbar)}{\varepsilon - 1} \left( \frac{V_i [l_t (\hbar)]}{P_t} \frac{\partial L_t (f)}{\partial c_t (f)} \right) \right\}^{1 - \varepsilon} \mathrm{d} \hbar \right\}^{\frac{1}{1 - n}} \]

because without nominal rigidities,

\[ P_{H,t} = p_t (f). \]

By log-linearizing equation (26), we can derive

\[ \Theta_1 \hat{R}_{H, t} + \Theta_2 \hat{R}_{H, t} + \hat{m}_c + (1 - n) \xi \left( \hat{t}_t - \hat{t}_t^* \right) = 0, \quad (27) \]

where we use equation (13) and \( \Theta_1 \equiv n \frac{\gamma (1 + R_{SS})}{1 + \gamma R_{SS}} \) and \( \Theta_2 \equiv (1 - n) \frac{\gamma (1 + R_{SS})}{1 + \gamma R_{SS}} \) are positive parameters, and under symmetric equilibrium, the aggregate real marginal cost is given by

\[ \hat{m}_c \equiv \int_0^n \hat{m}_c (h) \mathrm{d} h + \int_n^1 \hat{m}_c (\hbar) \mathrm{d} \hbar, \quad (28) \]

where

\[ m_c (h) \equiv \frac{P_t \cdot V_i [l_t (h)]}{P_{H,t} \cdot U_C (C_t)} \frac{\partial L_t (f)}{\partial c_t (f)}, \quad (29) \]

and

\[ m_c (\hbar) \equiv \frac{P_t \cdot V_i [l_t (\hbar)]}{P_{H,t} \cdot U_C (C_t)} \frac{\partial L_t (f)}{\partial c_t (f)}. \quad (30) \]

We here also define the aggregate loan interest rate by local banks in the home country \( R_{H,t} \) and the aggregate loan interest rate by international banks in the home country \( R^*_{H,t} \) as

\[ R_{H,t} \equiv \frac{1}{n} \int_0^n r_t (h) \mathrm{d} h, \]

and

\[ R^*_{H,t} \equiv \frac{1}{1 - n} \int_n^1 r^* (\hbar) \mathrm{d} \hbar. \]

Similarly, regarding the optimal price setting of \( p_t^* (f) \), we can derive

\[ \hat{m}_c^* + \Theta_1^* \hat{R}_{F,t} + \Theta_2^* \hat{R}_{F,t} - (1 - n^*) \xi^* \left( \hat{t}_t - \hat{t}_t^* \right) = 0, \quad (31) \]

where we use equation (13) and \( \Theta_1^* \equiv \frac{n^* \gamma (1 + R_{SS})}{1 + \gamma R_{SS}} > 0 \) and \( \Theta_2^* \equiv (1 - n^*) \frac{\gamma (1 + R_{SS})}{1 + \gamma R_{SS}} \). \( R_{F,t} \) is the aggregate loan interest rate by international banks in the foreign country and \( R^*_{F,t} \) is the aggregate loan interest rate by local banks in the foreign country.
2.2.3 Marginal Cost

Here, we derive the equations for $\widetilde{mc}_t$ and $\widetilde{mc}_t^*$. By linearizing equation (20) under symmetric equilibrium, we can obtain

$$\widehat{L}_t \equiv n\widehat{L}_t + (1-n)\widehat{L}_t.$$  

Because the production function of each firm is assumed to be

$$y_t(f) = f\left(\widehat{L}_t(f)\right),$$  \hspace{2cm} (32)

where $f(\cdot)$ is an increasing and concave function. The aggregate domestic production function is now expressed as

$$Y_{H,t} = f\left(\widehat{L}_t\right).$$

By log-linearization, this can be transformed into

$$\widehat{Y}_{H,t} = \mu \left[n\widehat{L}_t(h) + (1-n)\widehat{L}_t(\overline{h})\right],$$  \hspace{2cm} (33)

where $\mu \equiv \frac{\overline{L}}{f}$. Now, by using equations (29) (30), and (33), equation (28) is transformed into

$$\widetilde{mc}_t = \left(\theta + \frac{\eta}{\mu}\right)\widehat{Y}_{H,t} + \frac{1}{\nu}\widehat{C}_t - \widehat{p}_{H,t},$$  \hspace{2cm} (34)

where

$$p_{H,t} = \frac{P_{H,t}}{P_t},$$

and $\eta \equiv \frac{V_d}{V}$ and $\theta \equiv \frac{-\frac{1}{\nu}Y_H}{K_V}$. Without nominal rigidities, because the country size is the same, the demand function is given by

$$Y_{H,t} = (1+\tau)p_{H,t}^{-1}c_t^{W},$$  \hspace{2cm} (35)

where we use equation (12). This can be linearly approximated as

$$\widehat{Y}_{H,t} = \widehat{C}_t^{W} - \widehat{p}_{H,t}.$$

At the same time, from equation (2),

$$\widehat{p}_{H,t} = -\frac{1}{2}\overline{T}oT_t,$$  \hspace{2cm} (36)
where we define

\[ T_{oT_t} = \frac{P_{F,t}}{P_{H,t}}. \]

Therefore, we can rewrite equation (34) as

\[
\tilde{m}_c_t = \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \tilde{C}_t^W + \left( \theta + \frac{\eta}{\mu} + 1 \right) \frac{1}{2} T_{oT_t} \\
= \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \tilde{Y}_{H,t} + \left( 1 - \frac{1}{v} \right) \frac{1}{2} T_{oT_t},
\]

(37)

where we use the relation of \( \tilde{C}_t^W = \tilde{Y}_{H,t} - \frac{1}{2} T_{oT_t} \).

Similarly, we can obtain the linearized equation for the foreign marginal cost as

\[
\tilde{m}_c^* = \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \tilde{C}_t^W - \left( \theta + \frac{\eta}{\mu} + 1 \right) \frac{1}{2} T_{oT_t} \\
= \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \tilde{Y}_{H,t} - \left( 1 - \frac{1}{v} \right) \frac{1}{2} T_{oT_t},
\]

(38)

where we use the relation of \( \tilde{C}_t^W = \tilde{Y}_{F,t} + \frac{1}{2} T_{oT_t} \), which can be derived under the specification of the Cobb-Douglas aggregator in equation (1).

### 2.3 Private Banks

There exists a continuum of private banks populated over \([0, 1]\). There are two types of banks in each country: local banks populated over \([0, n]\) and international banks populated over \([n, 1]\). Each private bank plays two roles: (1) to collect the deposits from consumers in its country, and (2) under the monopolistically competitive loan market, to set differentiated nominal loan interest rates according to their individual loan demand curves, given the amount of their deposits. We assume that each bank sets the differentiated nominal loan interest rate according to the types of labor force as examined in Teranishi (2007). Staggered loan contracts between firms and private banks produce a situation in which the private banks fix the loan interest rates for a certain period. A local bank lends only to firms when they hire labor from the DFS. However, an international bank only provides loans to firms when they hire labor from the IFS. The lending structure is shown in Table 1.

First, we describe the optimization problem of an international bank in the home country. In this case, the exchange rate risk is shared by the firm in the foreign country and
Table 1: Lending structure

<table>
<thead>
<tr>
<th></th>
<th>Local bank</th>
<th>International bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home country for f</td>
<td>to hire h</td>
<td>for f* to hire h*</td>
</tr>
<tr>
<td>Foreign country</td>
<td>for f* to hire h*</td>
<td>for f to hire h</td>
</tr>
</tbody>
</table>

the international banks in the domestic country with $1 - \xi^*$ and $\xi^*$. Each international bank can reset loan interest rates with probability $1 - \overline{\phi}^*$ following the Calvo (1983) – Yun (1996) framework. Under the segmented environment stemming from differences in labor supply, private banks can set different loan interest rates depending on the types of labor. As a consequence, the private bank holds some monopoly power over the loan interest rate to firms. Therefore, the international bank $h^*$ chooses the loan interest rate $r_t(h^*)$ to maximize the present discounted value of profit:

$$E_t \sum_{T=1}^{\infty} \left( \frac{\phi}{\phi} \right)^{T-t} X_{t,T,q,U} \left( h^*, f^* \right) \left\{ \left[ \frac{S_{T+1}}{S_T} (1 - \xi^*) + \xi^* \right] \left[ 1 + r_T(h^*) \right] - (1 + i_T) \right\}.$$  

The optimal loan condition is now given by

$$E_t \sum_{T=t}^{\infty} (\phi \beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T)}{U_C(C_t)} q_{t,T} \left( h^* \right) \left\{ \varepsilon \gamma S_T (1 + i_T) \right\} = 0.$$  

Because the international private banks that have the opportunity to reset their loan interest rates will set the same loan interest rate, the solution of $r_t(h^*)$ in equation (39) is expressed only with $\overline{\eta}_t$. In this case, we have the following evolution of the aggregate loan interest rate index by international banks in the home country:

$$1 + R_{F,t} = \overline{\phi}^* (1 + R_{F,t-1}) + \left( 1 - \overline{\phi}^* \right) (1 + \tau_t).$$  

By log-linearizing equations (39) and (40), we can determine the relationship between the

---

11 The staggered loan contracts in this paper work in the same way as the staggered wage contracts model in Erceg, Henderson, and Levin (2000), but the crucial difference between them is the existence of international linkages in the international staggered loan contracts model in this paper.
loan and deposit interest rate as follows:

\[
\hat{R}_{F,t} = \lambda_1 E_t \hat{R}_{F,t+1} + \lambda_2 \hat{R}_{F,t-1} + \lambda_3 \left[ \hat{\xi}_t - (1 - \xi^*) E_t \Delta S_{t+1} \right] + \bar{u}_t
\]

\[
= \lambda_1 E_t \hat{R}_{F,t+1} + \lambda_2 \hat{R}_{F,t-1} + \lambda_3 \left[ \xi^* \hat{\xi}_t + (1 - \xi^*) \hat{i}_t \right] + \bar{u}_t, \tag{41}
\]

where we use equation (13), \(\lambda_1 \equiv \frac{\phi \beta}{1 + \phi \beta} \), \(\lambda_2 \equiv \frac{\phi}{1 + \phi \beta} \), and \(\lambda_3 \equiv \frac{1 - \phi}{1 + \phi \beta} \frac{\epsilon}{\varepsilon - 1} \frac{(1 - \beta \phi)(1 + \beta \phi)(1 + \text{iss})}{1 + \beta \phi} \) are positive parameters, \(\bar{u}_t \) is the shock to this loan rate curve, and \(\text{iss} \) and \(\beta \) denote steady state nominal interest rates and loan rates respectively. This equation describes the foreign country’s loan interest rate (supply) curve by the international bank in the home country.\(^{12}\)

Similarly, from the optimization problem of a local bank \(h \) in the home country, we can obtain the relationship between loan and deposit interest rates as follows:

\[
\hat{R}_{H,t} = \lambda_1 E_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{\xi}_t + u_t, \tag{42}
\]

where \(\lambda_1 \equiv \frac{\phi \beta}{1 + \phi \beta} \), \(\lambda_2 \equiv \frac{\phi}{1 + \phi \beta} \) and \(\lambda_3 \equiv \frac{1 - \phi}{1 + \phi \beta} \frac{\epsilon}{\varepsilon - 1} \frac{(1 - \beta \phi)(1 + \beta \phi)(1 + \text{iss})}{1 + \beta \phi} \) are positive parameters, and \(u_t \) is the shock to this loan rate curve. This equation describes the home country’s loan interest rate (supply) curve by the local bank in the home country. Note that two loan interest rates, \(\hat{R}_{H,t} \) and \(\hat{R}_{F,t} \), are the same when \(\xi^* = 1 \), \(\lambda_1 = \lambda_1^* \), \(\lambda_2 = \lambda_2^* \), and \(\lambda_3 = \lambda_3^* \), and \(u_t = \bar{u}_t^* \). This is a case of the low of one price in the loan interest rates set by home private banks.

For international banks in the foreign country, we can derive the following loan interest rate curve:

\[
\hat{R}_{H,t} = \lambda_1 E_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \left[ (1 - \xi) \hat{\xi}_t + \hat{i}_t \right] + \bar{u}_t, \tag{43}
\]

where we use equation (13), \(\lambda_1 \equiv \frac{\phi \beta}{1 + \phi \beta} \), \(\lambda_2 \equiv \frac{\phi}{1 + \phi \beta} \), and \(\lambda_3 \equiv \frac{1 - \phi}{1 + \phi \beta} \frac{\epsilon}{\varepsilon - 1} \frac{(1 - \beta \phi)(1 + \beta \phi)(1 + \text{iss})}{1 + \beta \phi} \) are positive parameters, and \(\bar{u}_t \) is the shock to this loan rate curve. This equation describes the home country’s loan interest rate (supply) curve by the international bank in the foreign country. Similarly, for local banks in the foreign country, we can obtain

\[
\hat{R}_{F,t}^* = \lambda_1^* E_t \hat{R}_{F,t+1} + \lambda_2^* \hat{R}_{F,t-1} + \lambda_3^* \hat{i}_t^* + u_t^*, \tag{44}
\]

\(^{12}\)We assume that this shock is a purely nominal shock, which does not alter the allocations under the flexible price equilibrium.
where \( \lambda_1^* \equiv \frac{\phi^* \beta}{1 + (\phi^*)^2 \beta} \), \( \lambda_2^* \equiv \frac{\phi^*}{1 + (\phi^*)^2 \beta} \) and \( \lambda_3^* \equiv \frac{1 - \phi^*(1 + i_{SS})}{(1 + (\phi^*)^2 \beta)} \) are positive parameters, and \( u^*_t \) is the shock to this loan rate curve. This equation describes the foreign country’s loan interest rate (supply) curve by the local bank in the foreign country. It should be noted that the four types of private bank in both the home and foreign countries can have different probabilities for resetting their loan interest rates.

2.4 System of Equation

The linearized system of equations consists of eight equations: (27), (31), (37), (38), (41), (42), (43), (44), and two optimal monetary policies derived in the following sections for 10 endogenous variables: \( \hat{C}^W \), \( \hat{ToT} \), \( \hat{m}_c \), \( \hat{m}_c^* \), \( \hat{R}_H \), \( \hat{R}_H^* \), \( \hat{R}_F \), \( \hat{i} \) and \( \hat{i}^* \).\(^{13}\) Except for the two optimal monetary policies \( \hat{i} \) and \( \hat{i}^* \), the variables are summarized in Table 2. A very straightforward explanation is possible for this system. Equations (41) to (44) determine the cost of borrowing, and these combined define the marginal costs in equations (27) and (31). The aggregate consumption and the terms of trade are solely determined by these marginal costs as in equations (37) and (38).

<table>
<thead>
<tr>
<th>Table 2: System of Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (27): ( \Theta_1 \hat{R}<em>{H,t} + \Theta_2 \hat{R}</em>{H,t}^* + \hat{m}_c t + (1 - n) \xi (\hat{i}_t - \hat{i}_t^*) = 0 )</td>
</tr>
<tr>
<td>Eq. (31): ( \Theta_1 \hat{R}<em>{F,t} + \Theta_2 \hat{R}</em>{F,t}^* + \hat{m}_c t^* - (1 - n^<em>) \xi^</em> (\hat{i}_t - \hat{i}_t^*) = 0 )</td>
</tr>
<tr>
<td>Eq. (37): ( \hat{m}_c t = \left( \frac{1}{b} + \theta + \frac{\mu}{\mu} \right) \hat{C}^W_t + \frac{1}{2} \left( \theta + \frac{\mu}{\mu} + 1 \right) \hat{ToT}_t )</td>
</tr>
<tr>
<td>Eq. (38): ( \hat{m}_c^* t = \left( \frac{1}{b} + \theta + \frac{\mu}{\mu} \right) \hat{C}^W_t + \frac{1}{2} \left( \theta + \frac{\mu}{\mu} + 1 \right) \hat{ToT}_t )</td>
</tr>
<tr>
<td>Eq. (41): ( \hat{R}<em>{F,t} = \lambda_1^* E_t \hat{R}</em>{F,t+1} + \lambda_2^* \hat{R}_{F,t-1}^* + \lambda_3^* \left[ \xi^* \hat{i}_t + (1 - \xi^<em>) \hat{i}_t^</em> \right] + \pi^*_t )</td>
</tr>
<tr>
<td>Eq. (42): ( \hat{R}<em>{H,t} = \lambda_1^* E_t \hat{R}</em>{H,t+1} + \lambda_2^* \hat{R}_{H,t-1}^* + \lambda_3^* \hat{i}_t + u_t )</td>
</tr>
<tr>
<td>Eq. (43): ( \hat{R}<em>{H,t}^* = \lambda_1^* E_t \hat{R}</em>{H,t+1} + \lambda_2^* \hat{R}_{H,t-1}^* + \lambda_3^* \left[ (1 - \xi) \hat{i}_t + \xi^* \hat{i}_t^* \right] + \bar{u}_t )</td>
</tr>
<tr>
<td>Eq. (44): ( \hat{R}<em>{F,t} = \lambda_1^* E_t \hat{R}</em>{F,t+1} + \lambda_2^* \hat{R}_{F,t-1}^* + \lambda_3^* \hat{i}_t + u_t^* )</td>
</tr>
</tbody>
</table>

\(^{13}\)If we further add equations (8), (11), (12) and (14), we can derive the optimal responses in \( \pi, \pi^* \), and \( S \) as shown in figures below.
3 Welfare Analysis

3.1 Preference

We assume that \( U(\cdot), U^*(\cdot), V(\cdot) \) and \( V^*(\cdot) \) are isoelastic functions as
\[
U(X) = U^*(X) = \frac{X^{1-\frac{1}{\eta}}}{1-\zeta},
\]
and
\[
V(X) = V^*(X) = \frac{X^{1+\eta}}{1+\eta},
\]
where \( \nu \) is the intertemporal elasticity of substitution in consumption and \( \eta \) is the Frisch elasticity of labor supply.\(^{14}\) In the following analysis, we assume \( \nu = 1 \), namely the log utility, and the linear production function as \( Y_{H,t} = \bar{L}_t \) and \( Y_{F,t} = \bar{L}^*_t \). We choose this parametric assumption since we would like to solely focus on the implications of an internationally integrated financial market and its intrinsic frictions for the monetary policy. As already shown in Obstfeld and Rogoff (2002), Clarida, Galí, and Gertler (2002), Benigno and Benigno (2003), and Corsetti and Pesenti (2005), under the assumption of the log utility together with the Cobb-Douglas aggregator in equation (1), the optimal allocations under cooperative monetary policy coincides with those under noncooperative monetary policy when there is no international loan contracts. Furthermore, the inward-looking monetary policy that responds only to the domestic variable becomes optimal and there is no gain by targeting the exchange rate. The reason behind this optimality of independent and inward-looking monetary policy is as follows. There exists no direct effects of foreign activities on the domestic marginal cost since the terms of trade and risk sharing effects cancel. Mathematically, under the log utility function with \( \nu = 1 \), the terms of trade disappear in equations (37) and (38). As a result, each central bank has no incentive to manipulate the exchange rate, namely the terms of trade, so that it can shifts the burden of production to the foreign country. Hence, by having such parametric assumption as above, we can investigate whether the newly introduced international financial market imperfections has some new implications, which has not been studied, on the monetary policy cooperation and the exchange rate targeting.

\(^{14}\) \( \nu \equiv -\frac{U_{xu}}{U_{ccc}} \) and \( \eta \equiv \frac{U_{xT}}{U_T} \).
### 3.2 Noncooperative Allocation

We derive a second-order approximation to the welfare function for each country following Woodford (2003). To eliminate the linear term in the quadratic approximation in the noncooperative allocation stemming from the difference between consumption and output in open economies, we follow Clarida, Galí, and Gertler (2002), where output and the policy interest rate in the foreign country is assumed to given for the home central bank and fiscal authority sets the optimal subsidy in the noncooperative manner.\(^{15}\) Furthermore, as is standard in the New Keynesian models for the cost push shock, we assume that the shocks to the loan interest rates do not alter the output in the flexible price equilibrium. The details of the derivation is shown in the Appendix.

The consumer welfare in the home country is given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log (C_t) - \int_0^n l_t(h)^{1+\eta} \, dh - \int_n^1 l_t(\bar{h})^{1+\eta} \, d\bar{h} \right].
\]

Then, we have a second-order approximated loss function for the home country as follows:

\[
L_t \approx \lambda_{YH} \bar{Y}_{H,t}^2 + \lambda_H \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \lambda^*_H \left( \hat{R}_{H,t}^* - \hat{R}_{H,t-1}^* \right)^2 + \lambda_{HH} \left( \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \right)^2,
\]

where \(\lambda_{YH} \equiv \frac{\gamma + 1}{2}, \lambda_H \equiv n \left( \frac{\gamma (1+R_{SS})}{1+\gamma R_{SS}} \right)^2 \frac{\phi}{1+\eta (1-\phi)(1-\phi)},\)

\(\lambda^*_H \equiv (1-n) \left( \frac{\gamma (1+R_{SS})}{1+\gamma R_{SS}} \right)^2 \frac{\phi}{1+\eta (1-\phi)(1-\phi)},\) \(\text{and} \lambda_{HH} \equiv n (1-n) \frac{1}{2(1+\eta)}.\)

There are several intriguing points to be noted. First, the central bank has to stabilize the financial market as captured by the last three terms in equation (45). The central bank dislikes the dispersions in loan rates from both the home and the foreign banks, as the second and the third terms of loan rate fluctuations and the last term of the credit spread illustrate. Second, the existence of the policy interest rate in the last term implies that the central bank has an incentive to stabilize the nominal exchange rate. This is because if the

\(^{15}\)This problem does not occur under the strict parametric assumption as employed in Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2005) where analytical solution of optimal monetary policy is available. Another method to eliminate the linear term in the quadratic approximation is found in Benigno and Benigno (2003). We will show that under some special conditions, since the financial stability becomes independent optimal monetary policy, we can derive the optimal noncooperative monetary policy following Benigno and Benigno (2003).
firms are not free from the exchange rate risk, the central bank has an additional incentive to stabilize nominal exchange rate to reduce it to lower the marginal cost. Third, the heterogeneity in the financial market makes the monetary policy complicated. The central bank faces a trade-off in the international financial market with different speeds of loan rate adjustments. For example, when there is no asymmetry in loan rates which domestic firms face with respect to structural parameters and size of shocks, namely $\lambda_1 = \bar{\lambda}_1^*, \lambda_2 = \bar{\lambda}_2^*, \lambda_3 = \bar{\lambda}_3^*, n = 0.5$, and $u_t = \pi_t^*$, and domestic firms are free from the exchange rate risks, namely $\xi = 0$, the credit spread term is disappeared and the loss function in equation (45) is reduced to

$$L_t \simeq \lambda_{Y_H} \tilde{Y}_{H,t}^2 + \lambda \left( \tilde{R}_t - \tilde{R}_{t-1} \right)^2,$$

where $\tilde{R}_t = \tilde{R}_{H,t} = \tilde{R}_{H,t}^*$ and $\lambda = \lambda_H = \lambda_H^*$. Forth, the central bank in the home country needs to monitor the lending behavior of private banks in the foreign country. As the second term in equation (45) shows, as the speed of the adjustment in loan rates of the foreign private banks changes, the optimal path of the policy interest rates set by the central bank in the home country should also be altered. When there is not the international loan contracts, i.e., $n = 1$, the central bank does not take account of the loan rates set by the foreign private banks.

The optimal monetary policy in this situation aims at minimizing the home loss function subject to the equations (27), (42), and (43) as in the closed economy model. We will come back to this point in the following section.

Through a similar procedure, we can derive a second-order approximated loss function for the foreign country as follows:

$$L_t^* \simeq \lambda_{Y_F} \tilde{Y}_{F,t}^2 + \lambda_F \left( \tilde{R}_{F,t} - \tilde{R}_{F,t-1} \right)^2 + \lambda_{F_F} \left( \Theta_1^* \tilde{R}_{F,t} - \Theta_2^* \tilde{R}_{F,t-1} + \xi \tilde{\pi}_t^* \right)^2,$$

where $\lambda_{Y_F} \equiv \frac{n+1}{2}$, $\lambda_F \equiv n^* \left[ \frac{\gamma (1 + R_{SS})}{1 + \gamma R_{SS}} \right]^2 \frac{\epsilon}{1 + \gamma \xi \frac{\phi^*}{(1 - \phi^*) (1 - \phi^*)}}$, $\lambda_{F_F} \equiv (1 - n^*) \left[ \frac{\gamma (1 + R_{SS})}{1 + \gamma R_{SS}} \right]^2 \frac{\epsilon}{1 + \gamma \xi \frac{\phi^*}{(1 - \phi^*) (1 - \phi^*)}}$, and $\lambda_{F_F} \equiv n^*(1 - n^*) \frac{1}{2(1 + \eta)}$. The optimal monetary policy in the foreign country minimizes this foreign loss function subject to the equations (31), (41), and (44).
3.3 Cooperative Allocation

Similarly, we can derive the world loss function which the central banks in policy cooperation aim to minimize.\textsuperscript{16} In the case under noncooperative monetary policy, we follow Clarida, Galí, and Gertler (2002) and compute the optimal subsidy under cooperative fiscal policy. The derived loss function is given by

\begin{equation}
L_t^W = L_t + L_t^* = \lambda_{Y_H} \hat{Y}_{H,t}^2 + \lambda_{Y_F} \hat{Y}_{F,t}^2 + \lambda_H \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \lambda_H^* \left( \hat{R}_{H,t} - \hat{R}_{H,t-1}^* \right)^2 + \lambda_{HH} \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{i} - \hat{i}^* \right) \right]^2 + \lambda_F \left( \hat{R}_{F,t} - \hat{R}_{F,t-1} \right)^2 + \lambda_F^* \left( \hat{R}_{F,t} - \hat{R}_{F,t-1}^* \right)^2 + \lambda_{FF} \left[ \Theta_1 \hat{R}_{F,t} - \Theta_2 \hat{R}_{F,t}^* + \xi \left( \hat{i} - \hat{i}^* \right) \right]^2. 
\end{equation}

Note that nothing is given in this loss function under the cooperative monetary policy. Similarly to the case with noncooperative monetary policy, the central banks in cooperation aim at minimizing the world loss function subject to the equations (27), (42), (43), (31), (41), and (44).

3.4 Welfare Weight

Here, we show how the weights, namely $\lambda_H$ and $\lambda_{HH}$ as the ratio over $\lambda_{Y_H}$, in the social loss function given by equation (47) change as the parameters for financial openness $n$ and loan rate stickiness $\phi$ are altered. We aim at understanding whether the financial market integration under a heterogenous degree of financial market imperfections alters the nature of optimal monetary policies. We use the parameters in Table 3, most of which are from Woodford (2003).

Figure 2 shows the case with changing $n$. Here a larger $n$ means lower financial openness. Under symmetric assumptions except for the altered parameters between the two countries, $\lambda_{Y_H}$ does not move with changes in $n$ and $\phi$. $\lambda_H$, which measures the importance of the welfare loss stemming from the loan rate stickiness of the domestic (foreign) banks’ loan to domestic firms, naturally increases (decreases) as the financial dependency on the domestic (foreign) banks becomes larger (smaller). Although the loss from the relative marginal cost dispersion measured by $\lambda_{HH}$ is very small under the assumption of $\phi = \phi = \phi^* = \overline{\phi} = 0.5$,

\textsuperscript{16} For details, see Appendix.
Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>Dependence on external finance</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>7.66</td>
<td>Elasticity of substitution among differentiated labor</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>Elasticity of the desired real wage to the quantity of labor demanded</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0</td>
<td>Elasticity of marginal cost with respect to $y$ regarding production</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1</td>
<td>Elasticity of the output to additional labor input</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>7.66</td>
<td>Elasticity of substitution among differentiated goods</td>
</tr>
<tr>
<td>$\phi, \phi^<em>, \phi, \phi^</em>$</td>
<td>0.5</td>
<td>Calvo parameters for loan interest rates</td>
</tr>
<tr>
<td>$n, n^*$</td>
<td>0.5</td>
<td>Preference for DFS labor</td>
</tr>
</tbody>
</table>

the response for the changes in $n$ is non-monotonic. The term $\left( \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* \right)^2$ is the loan rate difference between borrowings from domestic and foreign banks. In extreme cases where $n = 1$ or $0$, there is no such dispersion. With $n$ being between $0$ and $1$, the relative marginal cost dispersion stemming from borrowing exists, and it becomes largest when $n = 0.5$. This distortion becomes relatively important when there exists less stickiness in the loan contracts as Figure 3 below shows.

Figure 3 illustrates the case when the loan stickiness in domestic banks’ lending is increased. Naturally, $\lambda_H$ becomes larger as the loan rate stickiness of the domestic banks is increased, because this makes the relative loan rate dispersion among domestic firms larger. An important implication of this exercise is that asymmetry in the loan rate stickiness between domestic and foreign banks alters the weights in the social loss functions and may have significant implications for the optimal conduct of monetary policy cooperation.

24
Figure 2: Loss Weights with Different $n$.

Figure 3: Loss Weights with Different $\varphi$. 

4 Financial Stability as Optimal Monetary Policy

We investigate the properties of the optimal monetary policy under internationally integrated financial markets. As equations (45), (46) and (47) show, the financial stability means to minimize the dispersions among different loan rates. By minimizing the dispersions in loan rates, the central bank tries to reduce the markup fluctuations so that the disutility from labor of consumers is also lowered. Thus, as a general principle, in the absence of the distortions other than the staggered loan contracts, we have

**Proposition 1** Irrespective of the existence of the cooperation, central banks aim at achieving the financial stability.

Yet, whether the financial stability is always the sole target by the central bank depends on the assumption about model structures, namely parameters and shocks. Another interesting question is whether we can obtain the standard results in NOEM literatures as the optimality of the inward-looking and independent monetary policy with the flexible exchange rate.

Before showing other propositions, for convenience, we rewrite the optimality conditions in Table 2 by lag (L) and forward (F) operators and substituting them into loss functions. Then, equations (45) and (46) collapse to

\[
L_t = \lambda_{YH} \left\{ \frac{(1 + \eta)^{-1} \Theta_1 (1 - \lambda_1 F - \lambda_2 L) \lambda_3 \left( \hat{i}_t + u_t \right)}{1 - \lambda_1 F - \lambda_2 L} \right\}^2 + \left(1 - n \right) \xi \left( \hat{i}_t - \hat{i}_t^* \right) + \left(1 + n \right) \xi \left( \hat{i}_t^* + \hat{\pi}_t^* \right) + (1 - n) \xi \left( \hat{i}_t^* - \hat{\pi}_t^* \right)
\]

\[
+ \lambda_H \left\{ \frac{(1 - L) \lambda_3 \left( \hat{i}_t + u_t \right)}{1 - \lambda_1 F - \lambda_2 L} \right\}^2 + \lambda_H^* \left\{ \frac{(1 - L) \lambda_3 \left( 1 - \xi \right) \hat{i}_t + \xi \hat{i}_t^* + \hat{\pi}_t}{1 - \lambda_1 F - \lambda_2 L} \right\}^2
\]

\[
+ \lambda_{HH} \left\{ \Theta_1 \frac{\lambda_3 \left( \hat{i}_t + u_t \right)}{1 - \lambda_1 F - \lambda_2 L} - \Theta_2 \frac{\lambda_3 \left( 1 - \xi \right) \hat{i}_t + \xi \hat{i}_t^* + \hat{\pi}_t}{1 - \lambda_1 F - \lambda_2 L} + \xi \left( \hat{i} - \hat{i}_t^* \right) \right\}^2
\]
where \( \hat{i}_t \) is considered to be given, and

\[
L_t^* = \lambda_{Y_F} \left\{ \frac{(1 + \eta)^{-1} \Theta_1^* (1 - \lambda_1^* F - \lambda_2^* L) \lambda_3^* \left( \hat{i}_t + u^*_t \right)}{1 - \lambda_1^* F - \lambda_2^* L} \right\}^2 \\
+ \lambda_F \left\{ \frac{(1 - L) \lambda_3^* \left( \hat{i}_t + u^*_t \right)}{1 - \lambda_1^* F - \lambda_2^* L} \right\}^2 \\
+ \lambda_{FF} \left\{ \Theta_1^* \frac{\lambda_3^* \left( \hat{i}_t + u^*_t \right)}{1 - \lambda_1^* F - \lambda_2^* L} - \Theta_2^* \frac{\lambda_3^* \left( \xi^* \hat{\pi}_t + (1 - \xi^*) \hat{i}_t + \pi^*_t \right)}{1 - \lambda_1^* F - \lambda_2^* L} + \xi^* \left( \hat{i} - \hat{i}^* \right) \right\}^2,
\]

where \( \hat{i}_t \) is considered to be given. Furthermore, naturally,

\[
L_t^W = L_t + L_t^*.
\]

where no endogenous variables are considered to be given. By this transformation, we can analyze the nature of optimal monetary policy in internationally integrated financial market more intuitively.

**Proposition 2** Even under the internationally integrated financial markets, where banks lends both domestic and foreign countries, there is no gain from cooperation among central banks if the exchange rate risks are completely covered by banks, i.e., \( \xi = \xi^* = 0 \).

When \( \xi = \xi^* = 0 \), the international banks take all risks stemming from exchange rate fluctuations. As a result, equation (48) does not contain the foreign policy interest rate and *vice versa*. Then, each central bank does not have any incentive to manipulate nominal exchange rates so that firms in her country does not suffer from exchange rate risks. Therefore, in this situation, as long as we assume the log utility, the linear production function and the Cobb-Douglas aggregator as in Obstfeld and Rogoff (2002), and Corsetti and Pesenti (2005), the existence of financial market imperfections does not alter the optimality of independent monetary policy. It is worth mentioning the reason why the domestic central bank does not need any assistance from the foreign central bank, even though some portion of lending are from the foreign banks whose cost is the policy rate in the foreign country. This is because of the UIP condition. For the foreign international
bank lends to the home firms, the cost including the all risks in exchange rate fluctuations becomes the domestic policy interest rate, as equations (42) and (43) illustrate. Even under complicated financing as we can see as of now, as long as the exchange rate risks are completely covered by the lending banks and the UIP conditions hold, the domestic central bank can completely control the loan rates from the foreign international banks. Thus, we can also have

**Proposition 3** Optimal monetary policy is inward-looking, if the exchange rate risks are completely covered by banks, i.e., $\xi = \xi^* = 0$. Each central bank aims at stabilizing loan rates applied to firms in her country by manipulating the policy interest rate.

Consequently, as long as $\xi = \xi^* = 0$, we can derive the standard theoretical prescriptions on optimal monetary policy in open economies as independent policy with flexible exchange rates.

Another intriguing point is whether the complete stabilization of loan interest rates is possible. In other words, can monetary policy achieve zero social loss? Equations (48) and (49) clarify this point. By setting the policy interest rates as

$$i_t = -u_t = -\bar{u}_t,$$

and

$$i_t^* = -u_t^* = -\bar{u}_t^*,$$

the social losses in both countries become zero. Expected changes in exchange rate moves in accordance with above two monetary policy following the UIP condition in equation (13). This, however, does not cause any welfare deterioration since any movements in nominal exchange rates do not have any impact on the marginal costs in both countries. Then, we can have

**Proposition 4** When the exchange rate risks are completely covered by banks, i.e., $\xi = \xi^* = 0$, and the economic structures (parameters) are the same between two countries, complete stabilization becomes possible regardless of noncooperative or cooperative if firms in one country face the same size of loan rate shocks, namely $u_t = \bar{u}_t$ or $u_t^* = \bar{u}_t^*$.  

28
In this case, since attaining the complete financial stability is optimal and possible, we can also derive the optimal noncooperative policy following Benigno and Benigno (2003).

When $0 < \xi, \xi^* \leq 1$, the international banks and the firms share the risks from the exchange rate fluctuations. Interestingly, although the setting of this paper is the same as the previous studies for the optimality of independent and inward-looking monetary policy, there exist gains from cooperation. As shown in equations (48) and (49), both contains the uncontrollable policy interest rate set by the central bank in the opposite country. Since the monetary cooperation can internalize all the policy interest rates, it can attain higher social welfare in both countries than two independent monetary policy. Thus, the below is very much a new feature in the literatures on the optimal monetary policy in open economies.

**Proposition 5** When the risks from the exchange rate fluctuations are shared between the international banks and the firms, i.e., $0 < \xi, \xi^* \leq 1$, there exist gains from cooperation.

When $0 < \xi, \xi^* \leq 1$, firms suffer from the future exchange rate fluctuations and therefore the marginal cost becomes higher than when they are free from any exchange rate risks. In order to lower the marginal cost to increase the social welfare, the central bank without cooperation faces the trade-off between stabilizing the financial market imperfections and nominal exchange rates. This mechanism is similar to the case for the fixed exchange rate under the local currency pricing as analyzed in Devereux and Engel (2003) and Corsetti and Pesenti (2005). Under the local currency pricing, since exporting firms face the exchange rate risks, they set higher markups than under the producer currency pricing. Although firms end up with higher markups due to the exchange rate fluctuations, the exchange rate risk affects the marginal cost through the demand channel in our paper instead of the supply channel under the local currency pricing. As a result, we also have

**Proposition 6** When the risks from the exchange rate fluctuations are shared between the international banks and the firms, i.e., $0 < \xi, \xi^* \leq 1$, there exist gains from joint nominal exchange rate management.
5 Discussion

Many studies and data show that firms and governments borrow money in foreign currency, i.e., foreign currency denominated debt. For instance, Claessens, Klingebiel, and Schmukler (2003) show that some parts of government debt are issued by foreign currency. They report that this tendency is stronger in emerging countries, like Argentina, Mexico, and Brazil, than in developed countries, like United State, Japan, and Italy. Jeanne (2002) reports that the firms borrow large part of debt in foreign currency in emerging countries. The ratio of foreign currency borrowing to total debt is around 60% in Argentina, 40% in Mexico, and 20% in Brazil. Rosenberg and Tirpak (2008) show that the new member states of the euro largely rely on the foreign currency borrowing. Surprisingly, the ratio of foreign currency debt to GDP is 70% in Latvia and Estonia and 30% even in Hungary and Bulgaria, for example.

Along with our conclusion, these empirical facts imply that the cooperative monetary policy can improve the world welfare to the financial market disturbances. This is not surprising implication since each central bank can not escape from the monetary policy of other countries when the exchange rate risks are shared by countries and the productive activities depend on the monetary policy of foreign countries.

6 Conclusion

In this paper we have built up the NOEM model with the international financial frictions and have analyzed the optimal monetary policy. We show that the central banks should stabilize the international financial disturbances under the financially open economy. There the heterogeneity in the international financial markets makes the monetary policy very complicated. For the equivalence between cooperative and noncooperative allocations, the international exchange risk sharing is an additional key condition. Only when one country takes all risks on the exchange rate, two allocations are coincidence.

Our future research agenda is first to incorporate sticky prices as in Clarida, Galí, and Gertler (2002) and Benigno and Benigno (2003) so that we can quantitatively investigate a policy trade-off between the goods and financial markets. In particular, we would like
to obtain robust policy prescriptions, as in the form of the Ramsey optimal policy, in an economy under global banking in a more realistic situation.
References


Appendix

A Derivation of the Loss Function

In this section, we derive a second-order approximation to the welfare function following Woodford (2003).

A.1 Noncooperative case

The consumer welfare in the home country is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t) - \int_0^n V[l_t(h)] \, dh - \int_n^1 V[l_t(\bar{h})] \, d\bar{h} \right\}. \tag{50}$$

The first term of equation (50) can be approximated up to the second order as

$$U(C_t) = \bar{C}U_c \left[ \hat{C}_t + \frac{1}{2} (1 - v^{-1}) \hat{C}_t^2 \right] + \text{t.i.p.} + \text{Order}\left( \| \xi \|^3 \right), \tag{51}$$

where Order(\| \xi \|^3) expresses higher-order terms than the second-order approximation.

The second and third terms of equation (50) are also approximated as

$$\frac{1}{n} \int_0^n V[l_t(h)] \, h = \bar{L}V_l \left[ \hat{L}_t + \frac{1}{2} (1 + \eta) \hat{L}_t^2 + \frac{1}{2} \left( \eta + \frac{1}{\varepsilon} \right) \text{var}_h \hat{L}_t(h) \right] \tag{52},$$

and

$$\frac{1}{1 - n} \int_n^1 V[l_t(\bar{h})] \, d\bar{h} = \bar{L}V_l \left[ \hat{L}_t + \frac{1}{2} (1 + \eta) \hat{L}_t^2 + \frac{1}{2} \left( \eta + \frac{1}{\varepsilon} \right) \text{var}_h \hat{L}_t(\bar{h}) \right] \tag{53},$$

Here, we use the labor aggregator as in equation (15) in the second-order approximation such as

$$\hat{L}_t = \mathbb{E}_h \hat{L}_t(h) + \frac{1}{2} \frac{\varepsilon - \frac{1}{\varepsilon}}{\varepsilon} \text{var}_h \hat{L}_t(h) + \text{Order}\left( \| \xi \|^3 \right).$$

This combined with equations (52) and (53) yields

$$\int_0^n V[l_t(h)] \, dh + \int_n^1 V[l_t(\bar{h})] \, d\bar{h} \tag{54}$$

$$= \bar{L}V_l \left[ \hat{L}_t + \frac{1+n}{2} \hat{L}_t^2 + n \left( 1 - n \right) \frac{1+n}{2} \left( \hat{L}_t - \hat{L}_t \right)^2 \right]$$

$$+ \frac{\varepsilon}{2} \left( \eta + \frac{1}{\varepsilon} \right) \text{var}_h \hat{L}_t(h) + \frac{1-n}{2} \left( \eta + \frac{1}{\varepsilon} \right) \text{var}_h \hat{L}_t(\bar{h})$$

$$+ \text{t.i.p.} + \text{Order}\left( \| \xi \|^3 \right).$$
where we use the approximation for equation (20). From equation (32), the condition that the demand of labor is equal to the supply of labor is given by

\[ \tilde{L}_t = \int_0^1 \tilde{L}_t(f) \, df = \int_0^1 f^{-1} [y_t(f)] \, df, \]

whose second-order approximation becomes

\[ \hat{L}_t = \frac{1}{\mu} \left( \hat{Y}_{H,t} - a_t \right) + \frac{1}{2} \left( 1 + \theta - \frac{1}{\mu} \right) \frac{1}{\mu} \left( \hat{Y}_{H,t} - a_t \right)^2 + \text{Order} (\| \xi \|^3). \]

By substituting this, we can now transform equation (54) into

\[ \int_0^n V [l_t(h)] \, dh + \int_n^1 V [l_t(h)] \, d\tilde{h} \]

\[ = \overline{C} U_c \left[ \hat{Y}_{H,t} + \frac{1}{2} \left( 1 + \theta + \frac{n}{\mu} \right) \hat{Y}_{H,t} \right.
\[ + \frac{n}{2} \mu \left( \eta + \frac{1}{2} \right) \text{var}_h \ln l_t(h) + \frac{n}{2} \mu \left( \eta + \frac{1}{2} \right) \text{var}_{\tilde{h}} \ln l_t(\tilde{h})
\[ + n(1-n) \frac{1}{(1+n)} \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{i} - \hat{i}^* \right) \right]^2 \]
\[ + \text{t.i.p.} + \text{Order} (\| \xi \|^3), \]

where we use the following:

\[ \hat{L}_t - \hat{L}_t = \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{i} - \hat{i}^* \right) \right] - \eta \left( \hat{L}_t - \hat{L}_t \right), \]

which are derived using equations (9), (10), (16), (17), (18), (19), (21), and (22). Furthermore, following Clarida, Galí, and Gertler (2002), we replace \( \frac{1}{\mu} L_i V_i \) by \( \frac{1}{2} \overline{C} U_c \) thanks to the social planner’s optimization problem.\(^{17} \)

Then we can combine equation (51) and equation (55) as

\[ U_t = \overline{C} U_c \left[ \frac{1}{2} (1 - v^{-1}) \hat{C}_t^2 + \hat{C}_t \right.
\[ - \hat{Y}_{H,t} - \frac{1}{2} \left( 1 + \theta + \frac{n}{\mu} \right) \hat{Y}_{H,t} + \left( \theta + \frac{n}{\mu} \right) q_t \hat{Y}_{H,t}
\[ - \frac{n}{2} \eta \text{var}_h \ln l_t(h) - \frac{n}{2} \eta \text{var}_{\tilde{h}} \ln l_t(\tilde{h})
\[ - n(1-n) \frac{1}{(1+n)} \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{i} - \hat{i}^* \right) \right]^2 \]
\[ + \text{t.i.p.} + \text{Order} (\| \xi \|^3), \]

where \( \eta_t \equiv \mu \left( \eta + e^{-1} \right) \). To transform \( \text{var}_h \hat{i}_t(h) \) and \( \text{var}_{\tilde{h}} \hat{i} \) further, we use the optimal conditions of labor supply and demand functions given by equations (9), (10), (24) and

\(^{17}\text{The social planner optimizes the following problem:}
\]

\[ \max_{C,L} U(C) - V(L) \text{ s.t. } C = \hat{L}^{\delta} \left( Y_F \right)^{\gamma}, \text{where } Y_F \text{ is exogeneously given.} \]

37
(25). Approximation of these equations leads to

$$\text{var}_h \ln l_t(h) = \Xi \text{var}_h \ln [1 + r_t(h)] + \text{Order}(\| \xi \|^3),$$

and

$$\text{var}_h \ln l_t(\overline{h}) = \Xi^* \text{var}_\overline{h} \ln [1 + r_t(\overline{h})] + \text{Order}(\| \xi \|^3),$$

where $\Xi \equiv \Theta^2 \frac{\epsilon^2}{(1+\eta \epsilon)^2}$ and $\Xi^* \equiv (\Theta^*)^2 \frac{\epsilon^2}{(1+\eta \epsilon)^2}$. Then, equation (56) is further transformed into

$$U_t = \frac{1}{2} \overline{C} U_c \begin{bmatrix} -\frac{1}{2} (1 - v^{-1}) \tilde{C}_t^2 - \tilde{C}_t \\ + \tilde{Y}_{H,t} + \frac{1}{2} \left( 1 + \theta + \frac{2}{n} \right) \tilde{Y}_{H,t}^2 \\ + n (1 - n) \left( \frac{1}{2} \epsilon (1 + \eta \epsilon) \right) \left[ \Theta_1 \tilde{R}_{H,t} - \Theta_2 \tilde{R}_{H,t}^* - \xi \left( \varepsilon_1 - \varepsilon_2 * \right) \right]^2 \\
+ n \eta_r \text{var}_h \ln [1 + r_t(h)] + (1 - n) \eta_r^* \text{var}_\overline{h} \ln [1 + r_t(\overline{h})] \\ + \text{t.i.p.} + \text{Order}(\| \xi \|^3) \end{bmatrix},$$

(57)

where $\eta_r \equiv \Xi \eta_l = \mu \Theta^2 \frac{\epsilon}{1 + \eta \epsilon}$ and $\eta_r^* \equiv \Xi^* \eta_l = \mu \Theta^2 \frac{\epsilon}{1 + \eta \epsilon}$. Here, we also assume that the central bank aims at stabilizing the deviations from the nonstochastic efficient steady state. The remaining important part is to transform $\text{var}_h \ln [1 + r_t(h)]$ in equation (57). Following Woodford (2003), we define $\tilde{R}_{H,t}$ and $\Delta^R_t$ as

$$\tilde{R}_{H,t} \equiv \text{E}_h \ln [1 + r_t(h)],$$

and

$$\Delta^R_t \equiv \text{var}_h \ln [1 + r_t(h)].$$

Then, we can derive

$$\Delta^R_t = \phi \Delta^R_{t-1} + \frac{\phi}{1 - \phi} \left( \tilde{R}_{H,t} - \tilde{R}_{H,t-1} \right)^2.$$  

(58)

Furthermore, the following is also derived from the log-linear approximation:

$$\tilde{R}_{H,t} = \ln (1 + R_{H,t}) + \text{Order}(\| \xi \|^3),$$

(59)

where we make use of the definition of the aggregate loan rates:

$$1 + R_{H,t} \equiv \int_0^1 q_t(h) [1 + r_t(h)] dh.$$
Then, from equations (58) and (59) we obtain
\[
\Delta_t^R = \phi \Delta_{t-1}^R + \frac{\phi}{1-\phi} \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2,
\]
(60)
where
\[
\hat{R}_{H,t} = \ln \frac{1+R_{H,t}}{1+R_H}.
\]
The forward iteration of equation (60) leads to
\[
\Delta_t^R = \phi^{t-1} \Delta_{t-1}^R + \sum_{s=0}^{t} \phi^{t-s} \left( \frac{\phi}{1-\phi} \left( \hat{R}_{H,s} - \hat{R}_{H,s-1} \right)^2 \right).
\]
Then, we have
\[
\sum_{t=0}^{\infty} \beta^t \Delta_t^R = \frac{\phi}{(1-\phi)(1-\phi\beta)} \sum_{t=0}^{\infty} \beta^t \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \text{t.i.p.} + \text{Order}(\| \xi \|^3).
\]
(61)

Then, we have
\[
U_t = -\Lambda \left[ \lambda_{Y_H} \hat{Y}_{H,t}^2 + \lambda_H \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \lambda_H^* \left( \hat{R}_{H,t}^* - \hat{R}_{H,t-1}^* \right)^2 + \lambda_{HH} \left( \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \hat{\xi} \hat{t} \right)^2 \right] + \text{t.i.p.} + \text{Order}(\| \xi \|^3),
\]
where we assume parameters in Table 3 and the given foreign output and policy rate. We have similar procedures to derive the loss function of the foreign country under noncooperative allocation.

A.2 Cooperative case

In this case, the following conditions are newly necessary.

(1) the output in abroad is not given for two countries,

(2) following Clarida, Galí, and Gertler (2002), we replace \( \frac{1}{\mu} \) by \( \overline{C}U_c \) thanks to the social planner’s optimization problem.\(^{18}\)

Then, the world loss function \( L_t^W = L_t + L_t^* \) is given by
\[
L_t^W = L_t + L_t^* = \lambda_{Y_H} \hat{Y}_{H,t}^2 + \lambda_H \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \lambda_H^* \left( \hat{R}_{H,t}^* - \hat{R}_{H,t-1}^* \right)^2 + \lambda_{HH} \left( \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \hat{\xi} \hat{t} \right)^2 + \lambda_{FF} \left( \Theta_1 \hat{R}_{F,t} - \Theta_2 \hat{R}_{F,t}^* + \hat{\xi} \hat{t} \right)^2.
\]

\(^{18}\)The social planner optimizes the following problem:
\[
\max_{C,\hat{X},\hat{L},\hat{M}} \frac{1}{2} \left( U(C) - V(\hat{L}) \right) + \frac{1}{2} \left( U(C) - V(\hat{L}) \right) \text{ s.t. } C = \frac{\hat{L}^2}{\hat{M}} \left( \hat{L} \right)^{\frac{1}{2}}.
\]