BUSINESS CYCLE IMPLICATIONS
OF INTERNAL CONSUMPTION HABIT
FOR NEW KEYNESIAN MODELS

Takashi Kano†
Faculty of Economics
Graduate School of Economics
The University of Tokyo
7-3-1 Hongo
Tokyo, JAPAN
email: tkano@e.u-tokyo.ac.jp

James M. Nason
Research Department
Federal Reserve Bank of Atlanta
1000 Peachtree Street, NE
Atlanta, GA
U.S.A. 30309–4470
email: jim.nason@atl.frb.org

Current Draft: December 1, 2008

Abstract

The inability of a wide array of dynamic stochastic general equilibrium models to produce dynamics that resemble actual aggregate fluctuations has lead to the use of consumption habit in preferences. This paper compares the fit of new Keynesian dynamic stochastic general equilibrium (NKDSGE) models with and without internal consumption habit to output and consumption growth spectral densities. We conduct model evaluation using Bayesian Monte Carlo methods. Simulation experiments show that the match to the moments is improved by including internal consumption habit in NKDSGE models. Nonetheless, the fit of habit NKDSGE models is vulnerable to the source of the nominal rigidity, often improves on moments limited to the business frequencies, and to relying exclusively on moments connected to transitory monetary policy shocks.

Key Words: Consumption Habit; New Keynesian; Bayesian Calibration; Monte Carlo Simulation.

JEL Classification Number: E10, E20, E32.

†We thank Jeannine Bailliu, Hafedh Bouakez, Toni Braun, Youngsung Chang, Mick Devereux, Bill Dupor, John Geweke, Tasos Karantounias, Thomas Lubik, Christian Matthes, Ricardo Nunes, B. Ravikumar, Pedro Silos, Larry Schembri, Tan Wang, Tao Zha, and seminar and session participants at the Bank of Japan, Bank of Canada, 2004 Far Eastern Meetings of the Econometric Society, 10th International Conference on Computing in Economics and Finance (July 2004), 2007 Summer Meetings of the Econometric Society, HEC Montréal, Hitotsubashi University, University of Tokyo, Federal Reserve Bank of Atlanta’s macro lunch workshop, Yonsei University, and Workshop on Methods and Applications for DSGE Models at the Federal Reserve Bank of Cleveland (October 2008) for useful and helpful discussions and suggestions. A previous version of this paper circulated under the title, “Business Cycle Implications of Internal Habit Formation”. The views in this paper represent those of the authors and are not those of either the Federal Reserve Bank of Atlanta or the Federal Reserve System.
1. INTRODUCTION

It is a ‘folk-theorem’ of macroeconomics that, “All models are false.” A sufficiently rich collection of stylized facts will cause dynamic stochastic general equilibrium (DSGE) models to be rejected by the data. One response is to find the most powerful moments to evaluate a DSGE model, which follows methods begun by Hansen (1982). Another approach is to focus on sample moments most relevant for students of the business cycle.

This paper takes the latter tack to assess the business cycle implications of internal consumption habit in new Keynesian (NK) DSGE models. Consumption habits have become a key component of business cycle propagation and monetary transmission mechanisms. A leading example is Christiano, Eichenbaum, and Evans (2005). They observe that internal consumption habit and sticky wages are needed to obtain hump-shaped responses to a monetary policy shock in NK DSGE models. Sticky prices are only a minor component of the mechanism that propagates a monetary policy shock in the view of Christiano, Eichenbaum, and Evans (CEE).

The role of consumption habit in macro models is still debated despite this success. Otrok, Ravikumar, and Whiteman (2002) show that habit solves risk-free rate and equity premium puzzles habit by creating a dislike for high-frequency consumption movements by the

\[ \text{Ryder and Heal (1973) is the first growth model with consumption habit. Nason (1988), Sundaresan (1989),} \]
\[ \text{and Constantinides (1990) are early examples that exploit consumption habit to solve risk-free rate and equity} \]
\[ \text{premium puzzles. More recently, Jermann (1998) studies asset pricing and macro predictions of a real business} \]
\[ \text{cycle (RBC) model with consumption habit, while Boldrin, Christiano, and Fisher (2001) find that consumption} \]
\[ \text{habit resolve several macro and asset pricing puzzles in a two-sector RBC model. Schmitt-Grohé and Uribe (2007) provide an excellent survey of consumption habit in finance and macro; also see Nason (1997). However, Pollak} \]
\[ \text{(1976) shows that consumption habit can impose unusual restrictions on utility functions in the long-run. Rozen} \]
\[ \text{(2008) addresses such issues with an axiomatic treatment of habit formation. Chetty and Szeidl (2005) and Ravn,} \]
\[ \text{Schmitt-Grohé, and Uribe (2006) aim to deepen the micro foundations of consumption habit.} \]

This paper is inspired by Lettau and Uhlig to compare habit and non-habit NKDSGE models. We evaluate NKDSGE model fit with moments motivated by business cycle theory and the permanent income hypothesis (PIH). Galí (1991) notes that actual data is at odds with the PIH prediction that the entire consumption growth spectral density (SD) is flat. Cogley and Nason (1995b) observe that it is problematic for DSGE models to match the U.S. output growth SD because it peaks between seven and two years per cycle. They and Nason and Cogley (1994) find that DSGE models cannot replicate the response of output to permanent and transitory shocks generated by structural vector moving averages (SVMAs).

An innovation of this paper is to judge NKDSGE model fit with output and consumption growth SDs. These moments are also used to depict propagation and monetary transmission in NKDSGE models. We borrow NKDSGE models from CEE. Besides a transitory monetary policy shock, a random walk total factor productivity (TFP) shock is added to drive NKDSGE models. This impulse structure restricts output and consumption to react only to permanent real shocks in the long-run, which is the long-run monetary neutrality (LRMN) proposition. LRMN just-identifies SVMAs that break output and consumption growth into univariate SMAs tied to permanent TFP or transitory monetary policy shocks. The SMAs are employed to parameterize permanent and transitory output and consumption growth SDs.

We adapt Bayesian calibration methods developed by DeJong, Ingram, and Whiteman (1996) and Geweke (2007) to assess NKDSGE model fit to SDs. This approach uses moments
computed from atheoretic econometric models to link DSGE models to actual data. Geweke calls this the minimal econometric approach to DSGE model evaluation because it relies neither on likelihood-based tools nor arbitrarily decides which sample moments are relevant to gauge DSGE model fit. The minimal econometric approach evaluates DSGE model fit with theoretical distributions of population moments and empirical distributions of sample moments.

The SVMAs are the econometric models that tie NKDSGE models to sample permanent and transitory output and consumption growth SDs. Actual data, a SVMA, its priors, and a Markov chain Monte Carlo (MCMC) simulator yield a posterior from which empirical distributions of sample SDs are obtained. Theoretical distributions of population SDs are obtained from a SVMA estimated on synthetic data generated by simulations of a NKDSGE model that draw priors on its parameters at each replication. NKDSGE model fit is judged on the overlap of theoretical and empirical distributions of Kolmogorov-Smirnov and Cramer-von Mises goodness of fit statistics constructed from theoretical and empirical SD distributions. This collapses multidimensional SDs into scalar goodness of fit statistics, which is an advantage of our approach to NKDSGE model evaluation. We also compare mean empirical and theoretical SDs to study propagation and monetary transmission in habit and non-habit NKDSGE models.

This paper reports evidence that favors retaining internal consumption habit in NKDSGE models. The evidence affirms CEE because the fit of habit NKDSGE models dominate non-habit versions. We also find that stripping NKDSGE models of sticky prices, but not sticky wages, better replicates SDs identified by transitory monetary policy shocks. This further supports CEE. Nonetheless, sticky wage only NKDSGE models cannot match permanent output and consumption growth SDs. These moments are matched by NKDSGE models in which the only nominal rigidity is sticky prices, but the improved fit only occurs at the business cycle frequencies.
The rest of the paper is constructed as follows. Section 2 discusses internal consumption habit and NKDSGE models. The Bayesian approach to DSGE model evaluation is reviewed in section 3. Results appear in section 4. Section 5 concludes.

2. Internal Consumption Habit New Keynesian DSGE Models

This sections reviews internal consumption habit, studies its propagation mechanism, and outlines the NKDSGE model. Our choice of NKDSGE model is motivated by CEE who, among others, argue that internal consumption habit is key to propagation and monetary transmission in this class of DSGE model.

2.1 Internal consumption habit

DSGE models include a typical household that garners utility conditional on internal consumption habit. This paper adopts internal consumption habit because, among others, CEE have found that it helps NKDSGE models match actual data. Internal habits operate on lagged household consumption, unlike ‘external habit’ or the ‘catching-up-with-the-Joneses’ specification of Abel (1990) which assume lags of aggregate consumption appear in utility. Under internal consumption habit, household preferences are intertemporally separable and assume separability of (net) consumption flow, labor disutility and real balances

\[
U(c_t, c_{t-1}, n_t, H_t/P_t) = \ln[c_t - hc_{t-1}] - \frac{n_t^{1+\gamma}}{1+\gamma} + \ln\left[\frac{H_t}{P_t}\right], \quad 0 < \gamma,
\]

where \(c_t, n_t, H_t, P_t,\) and \(H_t/P_t,\) are household consumption, labor supply, the household’s stock of cash at the end of date \(t - 1,\) the aggregate price level, and real balances, respectively.

We also maintain that \(h \in (0, 1)\) and \(0 < c_t - hc_{t-1}, \forall t.\) Since internal habit ties current consumption to past consumption, households treat near dated consumption as complements in period utility function in studies that seek to improve the fit of DSGE models to the data. However, Dunn and Singleton (1986), Eichenbaum...
household consumption choice to its past consumption, the marginal utility of consumption is forward-looking, 
\[ \lambda_t = \frac{1}{c_t - h c_{t-1}} - E_t \left( \frac{\beta h}{c_{t+1} - h c_t} \right), \]
where \( \beta \in (0, 1) \) is the household discount factor and \( E_t \{ \cdot \} \) is the mathematical expectation operator given date \( t \) information.

2.2 The internal consumption habit propagation mechanism

Forward-looking marginal utility suggests internal habit alters the relationship between consumption and interest rates. We study this relationship with a log linear approximation of the Euler equation
\[ \lambda_t = P_t E_t \left\{ \lambda_{t+1} R_{t+1} / \pi_{t+1} \right\}, \]
where \( R_t \) is the nominal rate and \( \pi_{t+1} (= P_{t+1} / P_t) \) is date \( t + 1 \) inflation. The log linear approximation gives a second order stochastic difference equation for demeaned (but not detrended) consumption growth, \( \Delta \tilde{c}_t \), whose solution is

\[ \Delta \tilde{c}_t = \varphi_1 \Delta \tilde{c}_{t-1} + \frac{\Psi}{\varphi_2} \sum_{j=0}^{\infty} \varphi_2^{-j} E_t \tilde{q}_{t+j}, \tag{2} \]

where the stable and unstable roots are \( \varphi_1 = h \alpha^* - 1 \) and \( \varphi_2 = \alpha^* (\beta h)^{-1} \), \( \alpha^* \) is the steady state growth rate of the economy, the demeaned real rate is \( \tilde{q}_t = \tilde{R}_t - \frac{\pi^*}{1 + \pi^*} \tilde{\pi}_t \), \( \pi^* \) is mean inflation, and \( \Psi \) is a constant that is nonlinear in model parameters.\(^3\)

The consumption growth generating equation \( (2) \) sets a quasi-difference of consumption growth, \( \Delta c_t - \varphi_1 \Delta c_{t-1} \), to the ‘permanent income’ of the expected discounted stream of future real rates. Since \( \Delta c_t - \varphi_1 \Delta c_{t-1} \) is smoothed by this present discounted value, it places restrictions on the path of \( \Delta c_t \). Contrast this with the non-habit model, \( h = 0 \), in which a linear approximation of the Euler equation sets \( E_t \Delta \tilde{c}_{t+1} = \tilde{q}_t \). In the non-habit model, the dynamic properties of the real rate are inherited by expected consumption growth.

\(^3\)We focus on consumption growth because the next section builds NKDSGE models with unit root TFP shocks.
The internal consumption habit propagation mechanism is discussed by CEE. They point out that in their NKDSGE model, in which \( h \) is estimated to be about 0.65, this mechanism produces a hump-shaped consumption response to a shock that lowers the real interest rate. The reason is that period utility function (1) implies that as \( h \) rises from zero toward one the household considers dates \( t \) and \( t + j \) consumption to switch from being complements to becoming substitutes. This suggests that internal consumption habit acts as a propagation mechanism when \( h \) is closer to one.

The consumption growth generating equation (2) is used to quantify the internal consumption habit propagation mechanism. We quantify this mechanism with impulse response functions (IRFs) generated by equation (2) given a one percent real rate shock. The calibration sets \( h = [0.15 \ 0.35 \ 0.50 \ 0.65 \ 0.85] \) and \( \tilde{q}_t \) to a first order autoregression, AR(1).

Figure 1 reports consumption growth IRFs. At impact, \( \tilde{q}_t \) drives consumption growth higher, but its response falls from about one percent \( (h = 0.15) \) to 0.11 percent \( (h = 0.85) \). The consumption growth increase shrinks at impact as \( h \) moves toward one because the household substitutes future consumption for current consumption. This substitutability shows up next in figure 1 in higher more humped shaped IRF peaks that are shifted further to the right and decay more slowly in response to \( \tilde{q}_t \) as \( h \) moves toward one. Thus, the internal consumption habit propagation mechanism relies on \( h \geq 0.5 \) to produce a humped shaped IRF with a peak at or beyond two quarters. This paper asks whether NKDSGE models need internal consumption habit for the propagation of TFP and transmission of monetary policy shocks.

---

4The real federal funds rate defines \( \tilde{q}_t \) with the quarterly nominal federal funds rate implicit GDP deflator inflation. The SIC selects the AR(1) over any lag length up to ten on a 1954Q1-2002Q4 sample. The estimated AR1 coefficient is 0.87. The rest of the calibration is \( \beta = 0.993 \) and \( \alpha^* = \exp(0.004) \), which are degenerate priors for these parameters and are described in section 3.1.
2.3 A new Keynesian DSGE model

We adapt the NKDSGE model of CEE. The model contains (a) internal consumption habit, (b) capital adjustment costs, (c) variable capital utilization, (d) fully indexed Calvo-staggered price setting by monopolistic final goods firms, and (e) fully indexed Calvo-staggered wage setting by monopolistic households with heterogenous labor supply.

Households reside on the unit circle with addresses \( l \in [0, 1] \). The budget constraint of household \( l \) is

\[
\frac{H_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} + c_t + x_t + a(u_t)k_t + \tau_t = r_t u_t k_t + \frac{W_t(\ell)}{P_t} n_t(\ell) + \frac{H_t}{P_t} + R_t \frac{B_t}{P_t} + D_t, \tag{3}
\]

where \( B_{t+1} \) is the stock of government bonds the household carries from date \( t \) into date \( t + 1 \), \( x_t \) is investment, \( k_t \) is household capital at the end of date \( t \), \( \tau_t \) is a lump sum government transfer, \( r_t \) is the real rental rate of \( k_t \), \( W_t(\ell) \) is the nominal wage paid to household \( l \), \( R_t \) is the nominal return on \( B_t \), \( D_t \) is dividends received from firms, \( u_t \in (0, 1) \) is the capital utilization rate, and \( a(u_t) \) is its cost function. At the steady state, \( u^* = 1 \), \( a(1) = 0 \) and \( \frac{a''(1)}{a'(1)} = 0.01 \) as in CEE. Note that \( u_t \) forces household \( l \) to forgo \( a(\cdot) \) units of consumption per unit of capital.

We place the CCE adjustment costs specification into the law of motion of household capital

\[
k_{t+1} = (1 - \delta)k_t + \left[ 1 - S \left( \frac{1}{\alpha} \frac{x_t}{x_{t-1}} \right) \right] x_t, \quad \delta \in (0, 1), \quad 0 < \alpha, \tag{4}
\]

where \( \delta \) is the capital depreciation rate and \( \alpha \) is deterministic TFP growth. The cost function \( S(\cdot) \) is strictly convex, where \( S(1) = S'(1) = 0 \) and \( S''(1) = \varpi > 0 \). In this case, the steady state is independent of the adjustment cost function \( S(\cdot) \).

Given \( k_0, B_0, \) and \( c_{-1}, \) the expected discounted lifetime utility function of household \( l \)

\[
E_t \left\{ \sum_{i=0}^{\infty} \beta^i U \left( c_{t+i}, c_{t+i-1}, n_{t+i}(\ell), \frac{H_t}{P_t} \right) \right\}, \tag{5}
\]
is maximized by choosing $c_t, k_{t+1}, H_{t+1}, B_{t+1}$, and $W_t(\ell)$ subject to period utility \([1]\), budget constraint \([3]\), the law of motion of capital \([4]\), and downward sloping labor demand.

Monopolistically competitive firms produce the final goods that households consume. The consumption aggregator is $c_t = \left[ \int_0^1 y_{D,t}(j)^{(\xi - 1)/\xi} \, dj \right]^{\xi/(\xi - 1)}$, where $y_{D,t}(j)$ is household final good demand for a firm with address $j$ on the unit interval. Final good firm $j$ maximizes its profits by setting its price $P_t(j)$, subject to $y_{D,t}(j) = \left[ P_t/P_t(j) \right]^\xi Y_{D,t}$, where $\xi$ is the price elasticity, $Y_{D,t}$ is aggregate demand, the price index is a $P_t = \left[ \int_0^1 P_t(j)^{1-\xi} \right]^{1/(1-\xi)}$.

The $j$th final good firm mixes capital, $K_t(j)$, rented and labor, $N_t(j)$, hired from households (net of fixed cost $N_0$) with labor-augmenting TFP, $A_t$, in the constant returns to scale technology, $[u_t K_t(j)]^\psi \left( (N_t(j) - N_0) A_t \right)^{1-\psi}$, $\psi \in (0, 1)$, to create output, $\gamma_t(j)$. TFP is a random walk with drift, $A_t = A_{t-1} \exp \{ \alpha + \varepsilon_t \}$, with $\varepsilon_t$ its Gaussian innovation, $\varepsilon_t \sim \mathcal{N}(0, \sigma^2_\varepsilon)$.

Calvo-staggered price setting restricts a firm to update to optimal price $P_{c,t}$ at probability $1 - \mu_P$. Or with probability $\mu_P$, firms are stuck with date $t-1$ prices scaled by inflation of the same date, $\pi_{t-1}$. This gives the price aggregator $P_t = \left[ (1 - \mu_P) P_{c,t}^{1-\xi} + \mu_P (\pi_{t-1} P_t - 1)^{1-\xi} \right]^{1/(1-\xi)}$.

Under full price indexation, Calvo-pricing yields the optimal forward-looking price

$$P_{c,t}/P_{t-1} = \left( \frac{\xi}{\xi - 1} \right)^{\xi/(\xi - 1)} \frac{\mathbb{E}_t \sum_{i=0}^\infty (\beta \mu_P)^i \lambda_{t+i} \phi_{t+i} Y_{D,t+i} \pi_{t+i}^{\xi}}{\mathbb{E}_t \sum_{i=0}^\infty (\beta \mu_P)^i \lambda_{t+i} Y_{D,t+i} \pi_{t+i}^{\xi-1}}$$

of a firm able to update its price.

Households offer differentiated labor services to firms in a monopsonistic market in which a Calvo staggered nominal wage mechanism. We assume the labor supply aggregator $N_t(j) = \left[ \int_0^1 n_t(\ell)^{(\theta - 1)/\theta} \, d\ell \right]^{\theta/(\theta - 1)}$, where $\theta$ is the wage elasticity. Labor market monopsony force firms to face downward sloping labor demand schedules for differentiated labor services, $n_t(\ell) = \left[ W_t/W_t(\ell) \right]^\theta N_t(j)$, where the nominal wage index is $W_t = \left[ \int_0^1 W_t(\ell)^{1-\theta} \, d\ell \right]^{1/(1-\theta)}$. The
nominal wage aggregator is $W_t = \left[(1 - \mu_W)W_{c,t}^{1-\theta} + \mu_W(\alpha^*\pi_{t-1}W_{t-1})^{1-\theta}\right]^{1/(1-\theta)}$, which has households updating their desired nominal wage $W_{c,t}$ at probability $1 - \mu_W$. With probability $\mu_W$, households receive the date $t - 1$ nominal wage indexed by steady state TFP growth, $\alpha^* = \exp(\alpha)$, and lagged inflation. In this case, the optimal nominal wage condition is

$$\left[\frac{W_{c,t}}{P_t}\right]^{1+\theta/y} = \left(\frac{\theta}{\theta - 1}\right)\frac{E_t}{\theta} \sum_{i=0}^{\infty} \left[\beta \mu_W \alpha^{*(1-\theta)}\right]^{i} \left[\frac{W_{t+i}}{P_{t+i}}\right]^{\theta} N_{t+i}^{-1} N_{t+i},$$

because households solve a fully indexed Calvo-pricing problem.

We close the NKDSGE model with one of two monetary policy rules. CEE identify monetary policy with a money growth process that is a structural infinite order moving average, SMA($\infty$). As CEE note, the SMA($\infty$) is equivalent to the AR(1) money growth supply rule

$$\ln M_{t+1} - \ln M_t = m_{t+1} = (1 - \rho_m)m^* + \rho_m m_t + \mu_t, \quad \left|\rho_m\right| < 1, \quad \mu_t \sim \mathcal{N}\left(0, \sigma^2_{\mu}\right),$$

where $m^*$ is mean money growth and the money growth innovation is $\mu_t$. NKDSGE-AR defines models with the money growth rule (8). Monetary policy is described with the Taylor rule

$$(1 - \rho_R)R_t = (1 - \rho_R)\left(R^* + a_\pi E_t \pi_{t+1} + a_\tilde{Y}_t \tilde{Y}_t\right) + \nu_t, \quad \left|\rho_R\right| < 1, \quad \nu_t \sim \mathcal{N}\left(0, \sigma^2_{\nu}\right),$$

in NKDSGE-TR models, where $R^* = \pi^*/\beta$ and $\pi^* = \exp(m^* - \alpha)$. Under the interest rate rule (9), the monetary authority obeys the ‘Taylor’ principle, $1 < a_\pi$, and sets $a_\tilde{Y} \in (0, 1)$. This assumes the monetary authority computes private sector inflationary expectations, $E_t \pi_{t+1}$, and mean-zero transitory output, $\tilde{Y}_t$, without inducing measurement errors.

The government finances $B_t$, interest on $B_t$, and a lump-sum transfer $\tau_t$ with new bond issuance $B_{t+1} - B_t$, lump-sum taxes $\tau_t$, and money creation, $M_{t+1} - M_t$. Under either monetary policy rule, the government budget constraint is $P_t \tau_t = [M_{t+1} - M_t] + [B_{t+1} - (1 + R_t)B_t]$. 

9
Government debt is in zero net supply, $B_{t+1} = 0$ and the nominal lump-sum transfer equals the monetary transfer, $P_t \tau_t = M_{t+1} - M_t$, along the equilibrium path at all dates $t$.

Equilibrium requires goods, labor, and money markets clear in the decentralized economy. This occurs when $K_t = k_t$ given $0 < r_t$, $N_t = n_t$ given $0 < W_t$, $M_t = H_t$, and also requires $P_t$, and $R_t$ are strictly positive and finite. This leads to the aggregate resource constraint, $Y_t = C_t + I_t + a(u_t)K_t$, where aggregate consumption $C_t = c_t$ and aggregate investment $I_t = x_t$. A rational expectations equilibrium equates, on average, firm and household subjective forecasts of $r_t$ and $A_t$ to the objective outcomes generated by the decentralized economy. We add to this list $\mu_t$ and $R_t$, $v_t$, $P_t$, or $W_t$ under the money growth rule (8), the interest rate rule (9), a flexible price regime, or a competitive labor market, respectively.

3. **Bayesian Monte Carlo Strategy**

This section describes our Bayesian Monte Carlo approach to NKDSGE model fit. We adapt methods DeJong, Ingram, and Whiteman (1996) and Geweke (2007) develop to compare DSGE models to actual data. DeJong, Ingram, and Whiteman (DIW) and Geweke eschew standard calibration and likelihood-based DSGE model evaluation. Instead, they see a DSGE model as lacking implications for moments of actual data without an econometric model that link the two. We connect population output and consumption growth SDs of NKDSGE models to sample SDs with SVMAs that are just-identified with a LRMN restriction. The SVMAs produce theoretical and empirical distributions of population and sample SDs using Bayesian simulation methods. NKDSGE model fit is judged by the overlap of theoretical and empirical SD distributions.

3.1 **Solution methods and Bayesian calibration of the DSGE models**

It takes several steps to solve and simulate NKDSGE models. Since the models have permanent shocks, optimality and equilibrium conditions are stochastically detrended before
log-linearizing around the deterministic steady state. We engage algorithms of Sims (2002) to solve for linear approximate equilibrium decision rules of the NKDSGE models. These decision rules yield synthetic samples by feeding in TFP and monetary policy shocks, given initial conditions and draws from priors on NKDSGE model parameters.

Priors embed our uncertainty about NKDSGE model parameters, which endows the population SD with theoretical distributions. Table 1 lists these priors. For example, table 1 reports an uninformative prior of $h$ is drawn from an uniform distribution with end points 0.05 and 0.95. The uninformative prior reflects an objective attitude toward any $h \in (0, 1)$. Non-habit NKDSGE models are defined by the degenerate prior $h = 0$.

Priors are also taken from earlier DSGE model studies. We place degenerate priors on $[\beta \gamma \delta \alpha \psi]' = [0.9930 1.3088 0.0200 0.0040 0.3500]'$ that are consistent with the Cogley and Nason (1995b) calibration. Uncertainty about $[\beta \gamma \delta \alpha \psi]'$ is captured by 95 percent coverage intervals, which include values in Nason and Cogley (1994), Hall (1996), and Chang, Gomes, and Shorfheide (2002). We set the prior of the investment cost of adjustment parameter $\omega$ to estimates reported by Bouakez, Cardia, and Ruge–Murcia (2005). An uninformative prior is imposed on the standard deviation of TFP shock innovations, $\sigma_\epsilon$. The RBC literature suggests that any $\sigma_\epsilon \in [0.0070, 0.0140]$ is equally fair, which motivates our choice of this prior.

There are four sticky price and wage parameters to calibrate. Sticky price and wage parameter prior means are $[\xi \mu_P \theta \mu_w]' = [8.0 0.55 15.0 0.7]'$. The mean of $\xi$ implies a steady state price markup, $\xi/(\xi - 1)$, of 14 percent with a 95 percent coverage interval that runs from 11 to 19 percent. This coverage interval blankets estimates found in Basu and Fernald (1997) and CEE. More uncertainty surrounds the priors of $\mu_P$, $\theta$, and $\mu_w$. For example, Sbordone (2002),

---

\(^5\)The appendix contains details of the solution methods.
Nason and Slotsve (2004), Lindé (2005), and CEE suggest a 95 percent coverage interval for \( \mu_p \) of \([0.45, 0.65]\). Likewise, a 95 percent coverage interval of \([0.04, 0.25]\) suggests substantial uncertainty around the seven percent prior mean household wage markup, \( \theta/(\theta - 1) \). However, the degenerate mean of \( \mu_w \) and its 95 percent coverage interval reveals stickier nominal wages than prices, as found for example by CEE, but with the same degree of uncertainty.

The money growth rule \((8)\) is calibrated to estimates from a 1954Q1–2002Q4 sample of M1. The estimates are degenerate priors for \( \begin{bmatrix} m^* & \rho_m & \sigma_\mu \end{bmatrix}' = \begin{bmatrix} 0.015 & 0.627 & 0.006 \end{bmatrix}' \). Precision of these estimates yield narrow 95 percent coverage intervals. For \( \rho_m \), the lower end of its interval is near 0.5. CEE note that \( \rho_m \approx 0.5 \) implies the money growth rule \((8)\) mimics their identified monetary policy shock process.

The calibration of the interest rate rule \((9)\) obeys the Taylor principle and \( a_y \in (0, 1) \). The degenerate prior of \( a_\pi \) is 1.80. We assign a small role to movements in transitory output, \( \tilde{Y} \), with a prior mean of 0.05 for \( a_y \). The 95 percent coverage intervals of \( a_\pi \) and \( a_y \) rely on estimates that Smets and Wouters (2007) report. The interest rate rule \((9)\) is also calibrated to smooth \( R_t \) given a prior mean of 0.65 and a 95 percent coverage interval of \([0.55, 0.74] \). Ireland (2001) is the source of the prior mean of the standard deviation of the monetary policy shock, \( \sigma_\nu = 0.0051 \), and its 95 percent coverage interval, \([0.0031, 0.0072] \). We assume all shock innovations are uncorrelated at all leads and lags (i.e., \( \mathbb{E}\{\varepsilon_{t+i}, u_{t+q}\} = 0 \), for all \( i, q \)).

### 3.2 Output and consumption moments

We evaluate NKDSGE model fit with output and consumption growth SDs. The SDs are calculated from just-identified SVMAs. The SVMAs are identified with an LRMN restriction that is embedded in the NKDSGE model of section 2. In this model, LRMN ties the TFP innovation \( \varepsilon_t \) to the permanent shock. The transitory shock is identified with the money growth innovation
μt or Taylor rule innovation \( u_t \). We recover the SVMAs from unrestricted VARs with the Blanchard and Quah (1989) decomposition. The VARs are estimated for \( [\Delta \ln Y_t \ \Delta \ln P_t] \)' and \( [\Delta \ln C_t \ \Delta \ln P_t] \)' using 1954Q1–2002Q4 and synthetic samples.

We employ just-identified SVMAs to compute permanent and transitory output and consumption growth SDs. This mapping relies on Blanchard and Quah (BQ) decomposition assumptions to map from just-identified SVMAs to permanent and transitory output and consumption SDs. Since the BQ decomposition assumes shock innovations are uncorrelated at all leads and lags, just-identified SVMAs can be broken into univariate output and consumption growth SMAs. The SMAs parameterize permanent and transitory output and consumption growth SDs, which extends ideas found in Akaike (1969) and Parzen (1974).

The SDs are estimated as if the univariate SMAs are estimated directly. Consider the \( [\Delta \ln Y_t \ \Delta \ln P_t] \)' information set where the interest rate rule \( \text{9} \) is the source of the transitory monetary policy innovation \( u_t \). In this case, the just-identified SVMA is

\[
\begin{bmatrix}
\Delta \ln Y_t \\
\Delta \ln P_t
\end{bmatrix} = \sum_{h=0}^{\infty} B_h \begin{bmatrix}
\xi_{t-h} \\
u_{t-h}
\end{bmatrix}, \quad \text{where} \quad B_h = \begin{bmatrix}
B_{\Delta Y,\xi,h} & B_{\Delta Y,\nu,h} \\
B_{N,\xi,h} & B_{N,\nu,h}
\end{bmatrix}.
\]

Given the BQ decomposition assumptions, we decompose the SVMA \( \text{10} \) into univariate infinite order SMAs of output growth, \( B_{\Delta Y,\xi}(L)\xi_t \) and \( B_{\Delta Y,\nu}(L)\nu_t \). The former (latter) SMA is the IRF of output growth with respect to the permanent TFP \( \xi_t \) (transitory Taylor rule monetary policy shock \( \nu_t \)). The SVMA \( \text{10} \) is also a Wold representation of \( [\Delta \ln Y_t \ \Delta \ln P_t] \)' whose spectrum (at

6The appendix shows that applying the LRMN restrictions to the SVMAs recover the NKDSGE model shocks which satisfies the ABCs and Ds of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007).

7VAR lag length is chosen using likelihood ratio statistics and the actual data testing down from a maximum of ten lags. These tests settle on a lag length of five.

8LRMN restricts output to be independent of the Taylor rule shock, \( v_t \), at the infinite horizon, \( B_{Y,\nu}(1) = 0 \).
frequency $\omega$ is $S_{\Delta Y \Delta P}(\omega) = (2\pi)^{-1} \Gamma_{\Delta Y \Delta P} \exp(-i\omega)$, where $\Gamma_{\Delta Y \Delta P}(\ell) = \sum_{h=0}^{\infty} b_h b'_{h-\ell}$.

Expanding the convolution $\Gamma_{\Delta Y \Delta P}(\ell)$ at horizon $h$ gives

$$\mathbb{B}_h b'_{h-\ell} = \begin{bmatrix} b_{\Delta Y,\epsilon,h} b_{\Delta Y,\epsilon,h-\ell} + b_{\Delta Y,\epsilon,v} b_{\Delta Y,v,h-\ell} + b_{\Delta Y,\epsilon,\ell} b_{\Delta Y,\epsilon,h} & b_{\Delta Y,\epsilon,\ell} b_{\Delta Y,v,h} + b_{\Delta Y,\epsilon,\ell} b_{\Delta Y,\epsilon,h} \\ b_{\Delta P,\epsilon,h} b_{\Delta Y,\epsilon,h-\ell} + b_{\Delta P,\epsilon,v} b_{\Delta Y,v,h-\ell} + b_{\Delta P,\epsilon,\ell} b_{\Delta Y,\epsilon,h} & b_{\Delta P,\epsilon,\ell} b_{\Delta Y,v,h} + b_{\Delta P,\epsilon,\ell} b_{\Delta Y,\epsilon,h} \end{bmatrix},$$

whose off-diagonal elements imply output growth and employment cross-covariances and, therefore, co- and quad-spectra, while the upper left diagonal elements contain output growth autocovariances $b_{\Delta Y,\epsilon,h} b_{\Delta Y,\epsilon,h-\ell}$ and $b_{\Delta Y,v,h} b_{\Delta Y,v,h-\ell}$. These autocovariances suggest treating the univariate output growth SMAs $b_{\Delta Y,\epsilon}(L) \epsilon_t$ and $b_{\Delta Y,\epsilon}(L) \nu_t$ as estimated objects whose innovations are the permanent TFP shock $\epsilon_t$ and transitory Taylor rule shock $\nu_t$. We employ these SMAs to parameterize permanent and transitory output growth SDs. Given the BQ decomposition assumption $\sigma^2_{\epsilon} = 1$, this gives us the output growth SD at frequency $\omega$

$$S_{\Delta Y,\epsilon}(\omega) = \frac{1}{2\pi} \left| b_{\Delta Y,\epsilon,0} + b_{\Delta Y,\epsilon,1} e^{-i\omega} + b_{\Delta Y,\epsilon,2} e^{-i2\omega} + \ldots + b_{\Delta Y,\epsilon,j} e^{-ij\omega} + \ldots \right|^2, \ t = \epsilon, \nu.$$

Before computing $S_{\Delta Y,\epsilon}(\omega)$, we truncate its polynomial at $j = 40$, a ten year horizon.

### 3.3 Bayesian simulation methods

We use MCMC software created by Geweke (1999) and McCausland (2004) to generate posteriors of SVMAs given priors and a 1954Q1–2002Q4 sample ($T = 196$) of U.S. output, consumption, and price growth. The posteriors are the source of $J = 5000$ replications of SVMA parameters used to compute empirical, $T$, distributions of the sample moments.

The SVMAs are also needed to construct theoretical, $T$, distributions of population moments. The $T$ moment distributions rely on $J$ synthetic samples of length $M \times T$ ($M = 5$) that are simulated from a linearized NKDSGE model conditional on priors placed on its parameters.

---

9The software is found at [http://www2.cirano.qc.ac.ca/~bacc](http://www2.cirano.qc.ac.ca/~bacc), while the appendix describes the data.
On each artificial sample of length $M \times T$, SVMAs are estimated and $T$ moments are computed. NKDSGE models are judged on the overlap of distributions of $T$ and $E$ moments.

3.4 Measures of fit

Our metric for judging the fit of a NKDSGE model begins with Cogley and Nason (1995a). They measure the fit of DSGE models to samples moments using Kolmogorov-Smirnov ($KS$) and Cramer-von Mises ($CVM$) goodness of fit statistics. We employ the same statistics, but in the context of Bayesian calibration experiments. The $KS$ and $CVM$ statistics are centered on the sample output (or consumption) growth SD, $\hat{I}_T(\omega)$, which is constructed from SVMAs estimated on the actual data. At frequency $\omega$, the $j$th draw from the ensemble of $E$ SDs of output growth (or consumption growth) is $I_{E,T,j}(\omega)$. The associated $T$ distribution is $I_{T,T,j}(\omega)$. Define the ratio $R_{D,T,j}(\omega) = \hat{I}_T(\omega) / I_{D,T,j}(\omega)$ at replication $j$, as well as its partial sum $V_{D,T,j}(2\pi q/T) = 2\pi \sum_{\ell=1}^{q} R_{D,T,j}(2\pi \ell/T)$, where $D = E, T$. The partial sum serves to construct $B_{T,D,j}(\kappa) = 0.5\sqrt{2T} \left[ V_{T,D,j}(\kappa \pi) - \kappa V_{T,D,j}(\pi) \right] / \pi$, $\kappa \in [0, 1]$. If the ‘partial’ differences $B_{T,D,j}(\cdot)$, $j = 1, \ldots, J$, are small, the sample and $D$ spectra are close. Vectors of ‘partial’ differences $\left\{ B_{T,D,j}(\cdot) \right\}_{j=1}^{J}$ are collected to form $KS_{D,j} = \max \left| B_{T,D,j}(\kappa) \right|$ and $CVM_{D,j} = \int_{0}^{1} B_{T,D,j}(\kappa)^2 d\kappa$.

Although $KS$ and $CVM$ statistics measure the distance between sample and $E$ or $T$ spectra, we employ distributions of $E$ or $T$ $KS$ and $CVM$ statistics to gauge the fit of the NKDSGE models. NKDSGE model fit is judged on the overlap of $E$ and $T$ distributions of $KS$ and $CVM$ statistics. Substantial overlap of these distributions indicate a good fit of a NKDSGE model.

DIW propose the confidence interval criterion (CIC) to quantify the intersection of $E$ and $T$ distributions. The CIC measures the fraction of a $T$ distribution that occupies an interval defined by lower and upper quantiles of the relevant $E$ distribution, conditional on a $1 - p$.

\footnote{Since $V_T(\omega)$ is the sum of the ratio $R_T(\omega)$, a linear filter applied to the actual and synthetic data has no effect on $B_{T,D,j}(\kappa)$. Hence, linear filtering has no impact on $KS$ and $CVM$ statistics and NKDSGE model evaluation.}
percent confidence level. We set \( p = 0.05 \). If a habit NKDSGE model yields a \( CIC > 0.3 \) (as DIW suggest), say, for the transitory output growth SD and the non-habit model’s \( CIC \leq 0.3 \), the data view habit as a more plausible for this moment. We also compute densities of \( T \) and \( T\ KS \) and \( CVM \) statistic distributions to examine visually the fit of NKDSGE models.

We calculate SDs on the entire spectrum and only on business cycle horizons from eight to two years per cycle. By restricting attention to business cycle fluctuations, we build on an approach to model evaluation suggested by Diebold, Ohanian, and Berkowitz (1998). Their insight is that concentrating on business cycle frequencies can matter for DSGE model evaluation when model misspecification (i.e., ‘all models are false’) corrupt short- and long-run output and consumption growth movements. We address such problems by ignoring low and high frequency output and consumption growth fluctuations when we report on the fit of NKDSGE models.

4. Habit and Non-Habit NKDSGE Model Evaluation

This section presents evidence about habit and non-habit NKDSGE model fit to \( E \) permanent and transitory output and consumption growth SDs. Mean \( E \) SDs appear in figure 2. We report \( CIC \) statistics in table 2. Figures 3–8 give visual evidence about NKDSGE model fit.

4.1 Business cycle moments: Output and consumption growth spectral densities

Figure 2 contains mean permanent and transitory \( E \) output and consumption growth SDs. The top (bottom) panel of figure 2 contains mean \( E \) permanent (transitory) output and consumption growth SDs. Mean \( E \) output and consumption growth SDs appear as solid (blue) lines, but the latter SDs also have plots with ‘\( \times \)’ symbols.

The SDs decompose variation in output and consumption growth frequency by frequency

\[ 1^{11} \text{DIW set the } CIC \text{ of } Q \text{ to } \frac{1}{1 - p} \int_a^b T(Q_j) dQ_j, \text{ given a } 1 - p \text{ percent confidence level, where } a (b) \text{ is the lower } 0.5p (\text{upper } 1 - 0.5p) \text{ quantile. The } CIC \text{ is normalized by } 1 - p \text{ to equal } \int_a^b T(Q_j) dQ_j. \]
in response to permanent and transitory shocks. The former shock yields mean permanent $T$ output and consumption growth SDs that display greatest power at frequency zero as shown in the top panel of figure 2. However, the consumption growth SD exhibits only about a third of the amplitude (i.e., volatility) that is seen in output growth at the long run. The permanent shock also produces lesser peaks around four years per cycle in output and consumption growth SDs that suggest economically important fluctuations at business cycle frequencies.

The bottom panel of figure 2 shows that mean transitory $T$ output and consumption growth SDs peak in the business cycle frequencies. The transitory shock generates greatest power in output growth at just under four years per cycle. The peak is closer to six years per cycle for the mean transitory $T$ consumption growth SD. At these peaks, output growth is nearly four times more volatile than consumption growth.

We view the mean permanent and transitory $T$ output and consumption growth SDs as a challenge to NKDSGE models. For example, mean $T$ consumption growth SDs appear to vary enough at growth and business cycle frequencies to reject the PIH. Thus, NKDSGE models must violate the PIH in the long and medium run. Output growth SDs confront NKDSGE models with fluctuations at the lowest and business cycle frequencies that suggest the need for economically meaningful propagation and monetary transmission mechanisms to match these moments.

4.2 Habit and non-habit NKDSGE model fit: Evaluation by CIC

Table 2 reports $CIC$s that evaluate NKDSGE model fit. The top panel has $CIC$s of habit and non-habit sticky price and wage (baseline), sticky price only (SPrice), and sticky wage only (SWage) NKDSGE-AR models (the money growth rule (8) defines monetary policy).[^12] The lower

[^12]: The SWage NKDSGE model requires the degenerate prior $\mu_p = 0$ with fixed markup $\phi = (\xi - 1)/\xi$. When the nominal wage is flexible, households set their optimal wage period by period in SPrice NKDSGE models. The markup in the labor market is fixed at $(\theta - 1)/\theta$, which equals $n^{-1/y}$, given $\mu_w = 0.$
panel includes $CIC$s of NKDSGE-TR models (the Taylor rule (9) replaces the money growth rule). Columns headed $\infty : 0$ and $8 : 2$ contain $CIC$s that measure the overlap of $E$ and $T$ $KS$ statistic distributions based on the entire spectrum and eight to two years per cycle, respectively.

The $CIC$s show that habit NKDSGE models yield greater overlap of $E$ and $T$ $KS$ statistic distributions for output and consumption growth SDs. Habit NKDSGE models yield $CIC$s of at least 0.3 in 24 of 48 simulation experiments, while non-habit NKDSGE models are responsible for only 12 such $CIC$s. When habit and non-habit NKDSGEs models generate $CIC$s $\geq$ 0.3 of $KS$ statistic distributions for the same SD, habit model $CIC$s are larger in ten of 12 cases. We view these results as evidence that internal consumption habit improves NKDSGE model fit.$^{13}$

Habit NKDSGE models often offer a better fit to $E$ transitory output and consumption growth SDs than to those identified by the permanent TFP shock. The best fit to transitory $E$ SDs is obtained by baseline habit NKDSGE-TR and SWage habit NKDSGE-TR models. These models are responsible for four $CIC$s $\geq$ 0.35 that measure the intersection of $E$ and $T$ $KS$ statistic distributions of transitory SDs integrated over the entire spectrum or constrained to eight and two years per cycle. The remaining habit NKDSGE models produce $CIC$s $\geq$ 0.3 in at most three of the four relevant cases.

A striking feature of table 2 is that the fit of the baseline habit NKDSGE-TR model dominates that of the baseline habit NKDSGE-AR model. Although the simulation experiments show that baseline habit NKDSGE models possess economically important monetary transmission mechanisms under either the money growth rule (8) or the Taylor rule (9), the latter monetary rule moves the baseline habit NKDSGE model closer to $E$ transitory SDs. The baseline habit NKDSGE models lack a mechanism that propagate TFP innovations into output and consump-

$^{13}$The $CIC$s are nearly unchanged when the uniform prior on $h$ is replaced with a prior drawn from a beta distribution with mean, standard deviation, and 95 percent coverage interval of 0.65, 0.15, and [0.38, 0.88].
tion growth fluctuations that match those observed in actual data.

Table 2 also provides information about the impact of sticky prices on NKDSGE model fit. Only SPrice habit NKDSGE models yield $CIC_s > 0.3$ for KS statistic distributions of the permanent output and consumption growth SDs. However the KS statistics must be limited to eight to two years per cycle for the SPrice habit NKDSGE models to generate $CIC_s$ of this size. Thus there is evidence that internal consumption habit and fully indexed Calvo staggered pricing combine to propagate TFP shocks into economically meaningful output and consumption growth fluctuations, but only at the business cycle frequencies.

4.3 Baseline habit NKDSGE model fit: The role of monetary policy rules

This section studies propagation and monetary transmission mechanisms of baseline habit and non-habit NKDSGE models. Besides internal consumption habit, these NKDSGE models differ by the money growth rule (8) or the Taylor rule (9) that defines monetary policy.

We plot mean $E$ and $T$ SDs and KS statistic densities of the baseline NKDSGE models in figures 3 and 4. Mean $E$ and $T$ habit and non-habit output and consumption growth SDs appear in the first column of figures 3 and 4. The second (third) column contains densities of KS statistics that are constructed over the entire spectrum (limited to eight to two years per cycle). The KS statistic densities also appear with the relevant $CIC_s$. From top to bottom, the rows of figures 3 and 4 present results for permanent output, transitory output, permanent consumption, and transitory consumption growth SDs. We denote mean $E$ SDs and KS statistic densities with (blue) solid lines, mean $T$ non-habit SDs and KS statistic densities with (green) dashed lines, and mean $T$ habit SDs and KS statistic densities with (red) dot-dash lines in figures 3 and 4. The four remaining figures employ the same layout.

Baseline habit and non-habit NKDSGE models fail to replicate permanent $E$ output and
consumption growth SDs fluctuations. One reason is that mean permanent $T$ habit and non-habit SDs peak between four and two years per cycle as shown in the odd numbered windows of the first column of figures 3 and 4. This creates large gaps between mean permanent $T$ and $T$ output and consumption growth SDs. The gaps are especially wide at $T$ SD peaks, which signals a poor fit to this moment by baseline habit and non-habit NKDSGE models. The poor fit is reflected in $T$ KS statistic densities that are flat or far to the right of associated $E$ densities as seen in the second and third columns of the odd numbered rows of figures 3 and 4. Note that internal consumption habit combined with either of the monetary policy rules is unable to improve the fit of baseline NKDSGE models to permanent $E$ SDs.

The choice of monetary policy rule matters for baseline NKDSGE model fit to the transitory $E$ SDs. The even numbered rows of figures 3 and 4 show that baseline habit NKDSGE-AR and non-habit NKDSGE-TR models yield a good match to the transitory $T$ output growth SD, but not to the transitory $T$ consumption growth SD. Only the combination of internal consumption habit and the Taylor rule $[9]$ is able to replicate transitory $T$ output and consumption growth SDs. For example, the baseline habit NKDSGE-TR model produces $T$ KS statistic densities of the transitory SDs that exhibit substantial overlap with $E$ KS statistic densities in the second and third columns of the even numbered rows of figure 4.

The Taylor rule $[9]$ improves the fit of the baseline habit NKDSGE model to $E$ transitory output and consumption growth SDs. Relative to the baseline habit NKDSGE-AR model, the baseline habit NKDSGE-TR model responds to a monetary policy shock by dampening output and consumption growth fluctuations between eight and two years a cycle. Thus, the baseline habit NKDSGE-TR model yields transitory $T$ SDs whose volatility is muted by a factor of four or more compared to those of the baseline habit NKDSGE-AR model. The even numbered windows
of the first column figure 4 show that within the business cycle frequencies this moves mean
transitory $T$ SDs of the baseline habit NKDSGE-TR model closer to mean transitory $E$ SDs.

In summary, internal consumption habit and the Taylor rule (9) contribute to a better
match between NKDSGE models and $E$ moment distributions. Arguments in Poole (1970) sug-
gest that the Taylor rule (9) can damp output and consumption growth fluctuations in baseline
NKDSGE models when real side shocks are more volatile than are monetary shocks. Internal
consumption habit also generate mean transitory $E$ SDs with less volatility, especially in the
business cycle and higher frequencies, in the baseline habit NKDSGE models. These results
are reminiscent of Otrok, Ravikumar, and Whiteman (2002) who find that consumption habit
creates a distaste by households for high frequency fluctuations.

4.4 Propagation and transmission in NKDSGE models: Habit and nominal rigidities

This section studies the role played by internal consumption habit and nominal rigidities
in the propagation and transmission of TFP and monetary policy shocks. Erceg, Henderson,
and Levin (2000) recognize that sticky prices and sticky wages matter for monetary policy
evaluation. However, this relies on sticky prices and wages being economically important for
the propagation of TFP shocks and transmission of monetary shocks to the real economy.

Table 2 shows that stripping out sticky nominal wages ($\mu_W = 0$) or sticky prices ($\mu_P = 0$)
have disparate effects on the NKDSGE models. Retaining sticky prices as the only nominal
rigidity leads the habit SPrice NKDSGE models to match better to $E$ permanent output and
consumption growth SDs than the baseline or SWage NKDSGE models. However, it is only on
the business cycle frequencies that the SPrice habit NKDSGE models are able to replicate the
$E$ permanent SDs. The SWage NKDSGE models have difficulties matching these moments, but
are more successful at duplicating $E$ transitory output and consumption growth SDs. These
moments are best fit by the SWage habit NKDSGE-TR model.

Figures 5–8 give visual evidence about the fit of the SPrice and SWage NKDSGE models. We present mean $\mathcal{E}$ and $\mathcal{T}$ permanent and transitory SDs and KS statistic densities for the SPrice NKDSGE-AR and -TR models in figures 5 and 6, respectively. The same plots appear in figures 7 and 8 for the SWage NKDSGE-AR and -TR models.

The first column of figure 5 contain mean $\mathcal{T}$ permanent and transitory output and consumption growth SDs that describe theoretical propagation and monetary transmission mechanisms of the habit and non-habit SPrice NKDSGE-AR models. These models produce mean $\mathcal{T}$ permanent SDs that rise from the low frequencies to a single peak around two years per cycle before a slow loss of power into the highest frequencies as the odd numbered rows of the first column of figure 5 show. Although the baseline and SPrice NKDSGE-AR models yield similar shaped mean $\mathcal{T}$ permanent output and consumption growth SDs, without sticky nominal wages NKDSGE-AR models yield mean $\mathcal{T}$ SDs whose peaks occur at higher frequencies and exhibit less amplitude at those peaks.

The SPrice habit NKDSGE-AR model has a theoretical propagation mechanism which better replicates the $\mathcal{E}$ permanent output and consumption growth SDs. Evidence of the better match to these moments is the overlap of the $\mathcal{E}$ and habit $\mathcal{T}$ KS statistic densities that appear in the first and third rows of the third column of figure 5. However, this improved fit is limited to the business cycle frequencies because the first and third rows of the middle column of figure 5 plot $\mathcal{T}$ KS statistic densities computed on the entire spectrum that are flat and far to the right of the associated $\mathcal{E}$ KS statistic densities.

The habit and non-habit SPrice NKDSGE-AR models are responsible for mean $\mathcal{T}$ transitory output and consumption growth SDs that differ from those of the baseline NKDSGE-AR
models. The latter NKDSGE models are responsible for mean $\mathcal{T}$ transitory SDs that peak at lower frequencies, between four and three years per cycle, and then fall steadily into the high frequencies as shown in the even numbered rows of the first column of figure 3. The even numbered rows of the first column of figure 5 reveal that the SPrice NKDSGE-AR models generate mean $\mathcal{T}$ transitory SDs with peaks around two years per cycle. Subsequent to this peak, these SDs plateau rather than fall at the higher frequencies. The SPrice NKDSGE-AR models also produce mean $\mathcal{T}$ transitory SDs with less amplitude in the business cycle and higher frequencies. Thus, stripping out nominal wage stickiness creates a monetary transmission in the SPrice NKDSGE-AR models that yields less volatility in transitory output and consumption growth fluctuations, especially in the business cycle frequencies.

Removing nominal wage stickiness conveys a theoretical propagation and monetary transmission mechanisms that provides about the same match quality to the transitory $\mathcal{E}$ SDs as found for the baseline NKDSGE-AR models. For these moments, the fit of the habit and non-habit SPrice NKDSGE-AR models continues to be best at the business cycle frequencies. The caliber of this match is readily apparent in the even numbered windows of the third column of figure 5. These windows include plots of $\mathcal{E}$ and $\mathcal{T}$ KS statistic densities limited to eight to two years per cycle that exhibit substantial overlap. The fit of the SPrice NKDSGE-AR models falters when asked to replicate the transitory $\mathcal{E}$ SDs on the entire spectrum as shown in the middle column of figure 5. In either case, internal consumption habit does little to push the fit of the SPrice NKDSGE-AR model closer to these SDs compared to the baseline NKDSGE-AR models.

Figure 6 shows that, when limited to the business cycle frequencies, the habit SPrice NKDSGE-TR model better replicates $\mathcal{T}$ permanent output and consumption growth SDs compared to the non-habit SPrice NKDSGE-TR model. According to the first and third windows of
the first column of figure 6, the habit SPrice NKDSGE-TR model produces mean $T$ permanent SDs that are closer to the associated $E$ SDs, especially between four and two years per cycle. These mean $T$ SDs rely on distributions that map into $KS$ statistic densities with significant overlap with $E$ $KS$ statistic densities at the business cycle frequencies as shown in the odd numbered rows of the third columns of figure 4. The same $E$ $KS$ statistic densities are not connected with $T$ $KS$ statistic densities tied to the non-habit SPrice NKDSGE-TR model.

The absence of sticky nominal wages results in a propagation mechanism for the SPrice habit NKDSGE-TR model that flattens output and consumption growth fluctuations frequency by frequency. The SPrice habit NKDSGE-TR model is responsible for mean $T$ permanent output and consumption growth SDs that are closer to $E$ permanent output and consumption growth SDs within the business cycle frequencies. This is the source of the superior fit of the habit SPrice NKDSGE-TR model compared to the baseline habit NKDSGE-TR model. Thus, the habit NKDSGE models create more plausible propagation mechanisms when the only nominal rigidity is sticky prices because TFP shocks generate less volatile permanent output and consumption growth fluctuations within the business cycle frequencies.

The even numbered rows of figure 6 provide more evidence that stripping out sticky nominal wages has little effect on the fit of NKDSGE models. The SPrice NKDSGE-TR models have monetary transmission mechanisms that reduce volatility frequency by frequency and smooth out humps observed in mean $T$ transitory output and consumption growth SDs compared to those of baseline NKDSGE-TR models. However, the second and third columns of the even numbered rows of figure 6 present $T$ $KS$ statistic densities that indicate baseline and SPrice NKDSGE-TR models yield about the same fit to $E$ transitory output and consumption growth SDs. The former models match better to $E$ transitory output growth SDs, while the
SPrice NKDSGE-TR models show more ability to replicate $T$ transitory consumption growth SDs. These results indicate that NKDSGE-TR models which lack sticky nominal wages have no more credibility with the data at transmitting Taylor rule shocks into output and consumption growth fluctuations than do the baseline NKDSGE-TR models.

Next, we study the implications of nominal sticky wages for permanent and transitory output and consumption growth fluctuations. Figures 7 and 8 report results for the SWage NKDSGE-AR and NKDSGE-TR models. The evidence is that these models have problems matching $T$ permanent output and consumption growth SDs, but that the SWage habit NKDSGE-TR model produces the best match to the $T$ transitory SDs.

The SWage habit NKDSGE models yields a poor match to $T$ permanent SDs. Figures 7 and 8 reveal, in the first and third windows of their first column, that these SDs are only near those produced by SWage habit NKDSGE models at the higher frequencies. The $T$ habit permanent output and consumption growth SDs have peaks in the business cycle frequencies not observed in the $T$ permanent SDs. Without sticky prices, NKDSGE models produce too much volatility in the business cycle frequencies in response to permanent TFP shocks. The distance between $T$ and $T$ permanent SDs is reflected in the lack of overlap of the $T$ and $T$ KS statistic densities in the second and third columns of the odd number rows of figures 7 and 8.

The even number rows of figure 7 and 8 testify to the good fit of SWage habit NKDSGE models to $T$ transitory output and consumption growth SDs. These models produce mean $T$ transitory SDs with maximum power at business cycle frequencies consistent with that found for $T$ transitory growth SDs as seen in the second and fourth windows of the first column of figures 7 and 8. The resulting $T$ and $T$ KS statistic densities display substantial overlap over the entire spectrum or when limited to eight to two years per cycle. However, only the SWage
habit NKDSGE-TR model matches the $T$ transitory SDs on the entire spectrum and when limited to just the business cycle frequencies.

The SWage habit NKDSGE models produce monetary transmission mechanisms that are economically meaningful. Whether it is the innovation $\mu_t$ to the money growth supply rule (8) or the Taylor rule (9) innovation $\nu_t$, these monetary shocks are transmitted by SWage habit NKDSGE models into $T$ transitory output and consumption growth SDs with peaks in the business cycle frequencies. The key though is that without sticky prices these peaks lack the volatility often produced by other NKDSGE models, especially the non-habit versions.

5. Conclusion

This paper studies the business cycle implications of internal consumption habit in new Keynesian dynamic stochastic general equilibrium (NKDSGE) models. The NKDSGE models include Calvo staggered price and nominal wage mechanisms, along with internal consumption habit and other real rigidities. We confront these NKDSGE models with output and consumption growth spectra identified by permanent productivity and transitory monetary policy shocks.

The fit of habit and non-habit NKDSGE models is explored using Bayesian calibration and simulation methods. The simulations show that internal consumption habit has subtle effects on NKDSGE model fit to permanent and transitory output and consumption growth spectra. We obtain a poor match for NKDSGE models to spectra identified by the permanent technology shock with one exception. The fit to this moment improves if the only nominal rigidity is Calvo staggered prices and NKDSGE model evaluation is judged only on business cycle frequencies. Habit NKDSGE models are more successful at replicating spectra identified by a monetary policy shock. The match is better for habit NKDSGE models that describe monetary policy with a Taylor rule rather than a money growth rule. We also obtain evidence that internal
consumption habit together with sticky wages push the NKDSGE model closer to transitory output and consumption growth spectra. Thus, we find support for Christiano, Eichenbaum, and Evans (2005) who argue that this reduced form nominal rigidity moves NKDSGE models closer to the data.

Our results raise at least two questions for business cycle research. First, DSGE models take internal consumption habit literally as a part of household preferences instead of treating it as a reduced-form for producing real frictions that improve model fit. For example, work by Chetty and Sziedl (2005) and Ravn, Schmitt-Grohè, and Uribe (2006) aim to deepen micro foundation for consumption habit while Rozen (2008) develops axioms for it. We believe that incorporating these ideas into DSGE models is important for future business cycle research.

We also report evidence that reveal vulnerabilities in NKDSGE model fit. It seems puzzling that NKDSGE models match better to transitory output and consumption growth spectra than to permanent spectra. Similar issues of NKDSGE model fit are raised by Dupor, Han, and Tsai (2007) and Del Negro and Schorfheide (2008). Dupor, Han, and Tsai obtain limited information estimates of NKDSGE models that suggest the moments used for identification affects inference about the role nominal rigidities play in propagation and monetary transmission. In contrast, Del Negro and Schorfheide argue that Bayesian likelihood methods and the aggregate data cannot distinguish between competing nominal rigidities in NKDSGE models. We think future research should study identification schemes and estimators to understand better which real and nominal rigidities matter most for propagation and monetary transmission.

References


Dupor, B., Han, J., Tsai, Y.C., 2007. What do technology shocks tell us about the new Keynesian paradigm? Manuscript, Department of Economics, Ohio State University, Columbus, OH.


### Table 1: Bayesian Calibration of NKDSGE Models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>95 Percent Cover Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>Internal Consumption Habit</td>
<td>Uniform</td>
<td>—</td>
<td>[0.0500, 0.9500]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>H’hold Subjective Discount</td>
<td>Beta</td>
<td>0.9930</td>
<td>[0.9886, 0.9964]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Labor Supply Elasticity</td>
<td>Normal</td>
<td>1.3088</td>
<td>[0.7831, 1.8345]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>Beta</td>
<td>0.0200</td>
<td>[0.0122, 0.0297]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Deterministic Growth Rate</td>
<td>Normal</td>
<td>0.0040</td>
<td>[0.0015, 0.0065]</td>
</tr>
<tr>
<td>$\varpi$</td>
<td>Capital Adjustment Costs</td>
<td>Normal</td>
<td>4.7710</td>
<td>[3.0834, 6.4586]</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Capital’s Share of Output</td>
<td>Beta</td>
<td>0.3500</td>
<td>[0.2554, 0.4509]</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>TFP Growth Shock Std.</td>
<td>Uniform</td>
<td>—</td>
<td>[0.0070, 0.0140]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Final Good Dmd Elasticity</td>
<td>Normal</td>
<td>8.0000</td>
<td>[6.1907, 9.8093]</td>
</tr>
<tr>
<td>$\mu_P$</td>
<td>No Price Change Probability</td>
<td>Beta</td>
<td>0.5500</td>
<td>[0.4513, 0.6468]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Labor Demand Elasticity</td>
<td>Normal</td>
<td>15.0000</td>
<td>[8.9633, 21.0367]</td>
</tr>
<tr>
<td>$\mu_W$</td>
<td>No Wage Change Probability</td>
<td>Beta</td>
<td>0.7000</td>
<td>[0.5978, 0.7931]</td>
</tr>
<tr>
<td>$m^*$</td>
<td>$\Delta \ln M$ Mean</td>
<td>Normal</td>
<td>0.0152</td>
<td>[0.0142, 0.0162]</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>$\Delta \ln M$ AR1 Coef.</td>
<td>Beta</td>
<td>0.6278</td>
<td>[0.5355, 0.7162]</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>$\Delta \ln M$ Shock Std.</td>
<td>Normal</td>
<td>0.0064</td>
<td>[0.0048, 0.0080]</td>
</tr>
<tr>
<td>$a_{\pi}$</td>
<td>Taylor Rule $E_t \pi_{t+1}$ Coef.</td>
<td>Normal</td>
<td>1.8000</td>
<td>[1.4710, 2.1290]</td>
</tr>
<tr>
<td>$a_{\gamma}$</td>
<td>Taylor Rule $\hat{Y}_t$ Coef.</td>
<td>Normal</td>
<td>0.1000</td>
<td>[0.0524, 0.1476]</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Taylor Rule AR1 Coef.</td>
<td>Beta</td>
<td>0.6490</td>
<td>[0.5512, 0.7417]</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Taylor Rule Shock Std.</td>
<td>Normal</td>
<td>0.0051</td>
<td>[0.0031, 0.0072]</td>
</tr>
</tbody>
</table>

The calibration relies on existing DSGE model literature; see the text for details. For a non-informative prior, the right most column contains the lower and upper end points of the uniform distribution. When the prior is based on the beta distribution, its two parameters are $a = \Gamma_{i,n} \left( 1 - \Gamma_{i,n} \right) \Gamma_{i,n} / STD(\Gamma_{i,n})^2 - 1$ and $b = a (1 - \Gamma_{i,n}) / \Gamma_{i,n}$, where $\Gamma_{i,n}$ is the degenerate prior of the $i$th element of the parameter vector of model $n = 1, \ldots, 4$, and its standard deviation is $STD(\Gamma_{i,n})$. 

31
### Table 2: CICs of Kolmogorov-Smirnov Statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Delta Y ) w/r/t Trend Sh’k</th>
<th>( \Delta Y ) w/r/t Transitory Sh’k</th>
<th>( \Delta C ) w/r/t Trend Sh’k</th>
<th>( \Delta C ) w/r/t Transitory Sh’k</th>
</tr>
</thead>
<tbody>
<tr>
<td>NKDSGE-AR</td>
<td>( \infty : 0 ) 8 : 2</td>
<td>( \infty : 0 ) 8 : 2</td>
<td>( \infty : 0 ) 8 : 2</td>
<td>( \infty : 0 ) 8 : 2</td>
</tr>
<tr>
<td>Baseline</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Habit</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>Habit</td>
<td>0.00</td>
<td>0.07</td>
<td>0.39</td>
<td>0.46</td>
</tr>
<tr>
<td>SPrice</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Habit</td>
<td>0.01</td>
<td>0.54</td>
<td>0.00</td>
<td>0.86</td>
</tr>
<tr>
<td>Habit</td>
<td>0.12</td>
<td>0.75</td>
<td>0.50</td>
<td>0.51</td>
</tr>
<tr>
<td>SWage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Habit</td>
<td>0.02</td>
<td>0.16</td>
<td>0.00</td>
<td>0.74</td>
</tr>
<tr>
<td>Habit</td>
<td>0.02</td>
<td>0.21</td>
<td>0.59</td>
<td>0.77</td>
</tr>
<tr>
<td>NKDSGE-TR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Habit</td>
<td>0.00</td>
<td>0.00</td>
<td>0.46</td>
<td>0.67</td>
</tr>
<tr>
<td>Habit</td>
<td>0.00</td>
<td>0.05</td>
<td>0.69</td>
<td>0.55</td>
</tr>
<tr>
<td>SPrice</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Habit</td>
<td>0.00</td>
<td>0.78</td>
<td>0.40</td>
<td>0.29</td>
</tr>
<tr>
<td>Habit</td>
<td>0.23</td>
<td>0.95</td>
<td>0.48</td>
<td>0.14</td>
</tr>
<tr>
<td>SWage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Habit</td>
<td>0.00</td>
<td>0.15</td>
<td>0.34</td>
<td>0.58</td>
</tr>
<tr>
<td>Habit</td>
<td>0.03</td>
<td>0.30</td>
<td>0.65</td>
<td>0.41</td>
</tr>
</tbody>
</table>

NKDSGE-AR and NKDSGE-TR denote the NKDSGE model with the AR(1) money supply rule \( (8) \) and the Taylor rule \( (9) \), respectively. Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The column heading \( \infty : 0 \) (8 : 2) indicates that CICs measure the intersection of distributions of KS statistics computed over the entire spectrum (from eight to two years per cycle).
Figure 1: \( \Delta C \) Response to Real Interest Rate Shock
FIGURE 2: MEAN STRUCTURAL $\mathcal{E}$ SPECTRA OF $\Delta Y$ AND $\Delta C$
Figure 3: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Model with AR(1) Money Growth Rule

Mean Spectral Densities

KS Densities: Entire Spectrum

KS Densities: Business Cycle Frequencies (8 to 2 years per cycle)
Figure 4: Mean Structural $F$ and $T$ SDs and KS Densities for Baseline NKDSGE Model with Taylor Rule

Mean Spectral Densities

Δ$Y$ w/r/t Permanent Shock

KS Densities: Entire Spectrum

Non-Habit: CIC = 0.00
Habit: CIC = 0.00

KS Densities: Business Cycle Frequencies (8 to 2 years per cycle)

Non-Habit: CIC = 0.00
Habit: CIC = 0.05

Δ$Y$ w/r/t Transitory Shock

KS Densities: Entire Spectrum

Non-Habit: CIC = 0.46
Habit: CIC = 0.69

Δ$C$ w/r/t Permanent Shock

KS Densities: Entire Spectrum

Non-Habit: CIC = 0.00
Habit: CIC = 0.05

Δ$C$ w/r/t Transitory Shock

KS Densities: Entire Spectrum

Non-Habit: CIC = 0.01
Habit: CIC = 0.33

Empirical Non-Habit Habit

0 2 4 6 8
0 0.48 0.96 1.44

Years per cycle
Figure 5: Mean Structural $\mathcal{E}$ and $\mathcal{I}$ SDs and KS Densities for NKDSGE Model with AR(1) Money Growth Rule and Only Sticky Prices

Mean Spectral Densities

KS Densities: Entire Spectrum

Non-Habit: CIC = 0.01
Habit: CIC = 0.12

KS Densities: Business Cycle Frequencies (8 to 2 years per cycle)

Non-Habit: CIC = 0.54
Habit: CIC = 0.75

KS Densities: Business Cycle Frequencies

Non-Habit: CIC = 0.86
Habit: CIC = 0.51

KS Densities: Business Cycle Frequencies

Non-Habit: CIC = 0.32
Habit: CIC = 0.68

Empirical
Non-Habit
Habit
**Figure 6: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Model with Taylor Rule and only Sticky Prices**

Mean Spectral Densities

Δ$Y$ w/r/t Permanent Shock

- **Empirical**: Blue
- **Non-Habit**: Green
- **Habit**: Red

KS Densities: Entire Spectrum

- Non-Habit: CIC = 0.00
- Habit: CIC = 0.23

KS Densities: Business Cycle Frequencies (8 to 2 years per cycle)

- Non-Habit: CIC = 0.78
- Habit: CIC = 0.95

Δ$Y$ w/r/t Transitory Shock

- Non-Habit: CIC = 0.40
- Habit: CIC = 0.48

Δ$C$ w/r/t Permanent Shock

- Non-Habit: CIC = 0.00
- Habit: CIC = 0.16

Δ$C$ w/r/t Transitory Shock

- Non-Habit: CIC = 0.06
- Habit: CIC = 0.48

Empirical

Non-Habit
Habit
Figure 7: Mean Structural $E$ and $T$ SDs and KS Densities for NKDSGE Model with AR(1) Money Growth Rule and Only Sticky Wages
Figure 8: Mean Structural $\mathcal{E}$ and $T$ SDs and KS Densities for NKDSGE Model with Taylor Rule and only Sticky Wages

- Mean Spectral Densities
- KS Densities: Entire Spectrum
- KS Densities: Business Cycle Frequencies (8 to 2 years per cycle)

- $\Delta Y$ w.r.t. Permanent Shock
- $\Delta Y$ w.r.t. Transitory Shock
- $\Delta C$ w.r.t. Permanent Shock
- $\Delta C$ w.r.t. Transitory Shock

Empirical
Non-Habit
Habit

Non-Habit: CIC = 0.00
Habit: CIC = 0.03

Non-Habit: CIC = 0.00
Habit: CIC = 0.08

Non-Habit: CIC = 0.00
Habit: CIC = 0.22

Non-Habit: CIC = 0.15
Habit: CIC = 0.30

Non-Habit: CIC = 0.34
Habit: CIC = 0.65

Non-Habit: CIC = 0.34
Habit: CIC = 0.65

Non-Habit: CIC = 0.57
Habit: CIC = 0.41

Non-Habit: CIC = 0.72
Habit: CIC = 0.80

Non-Habit: CIC = 0.72
Habit: CIC = 0.80